河南師範大學
HENAN NORMAL UNIVERSITY

The Minimal Supersymmetric Standard Model with Non-Invertible Selection Rules

Yoshihiro Shigekami

(Henan Normal University)

with

Yuichiro Nakai, Zhihao Zhang (TDLI, SJTU)

Hajime Otsuka (Kyushu University)

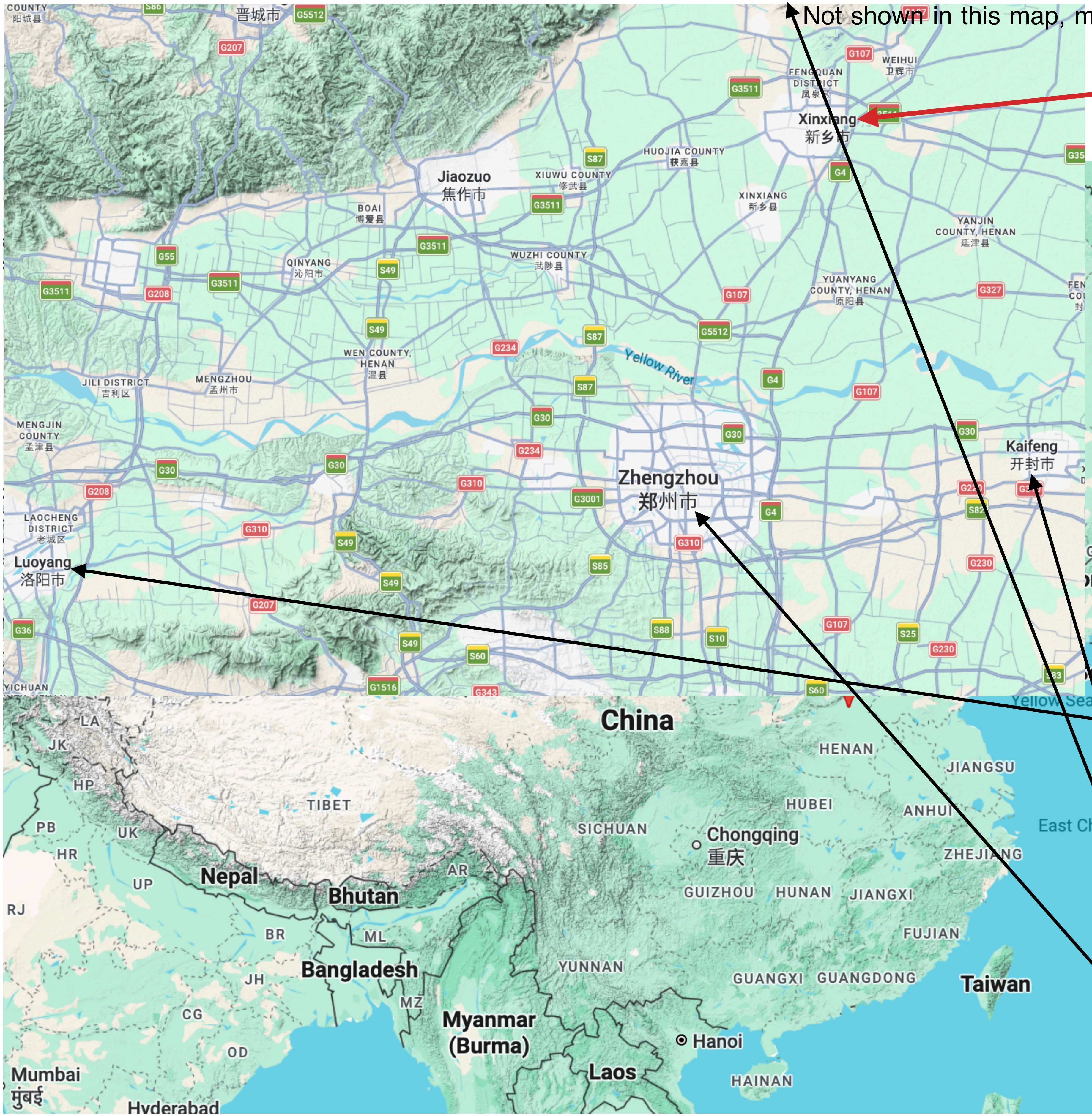
Based on 2512.21509

New perspectives on flavor and symmetries in particle physics@APCTP, Pohang, June 4, 2026

河南師範大學
HENAN NORMAL UNIVERSITY

Not shown in this map, more north part

河南師範大學
HENAN NORMAL UNIVERSITY



中国十大古都

1、西安(陕西省):十三朝古都
(西周、秦、西汉、新朝、东汉、西晋、前赵、前秦、后秦、西魏、北周、隋、唐)

2、洛阳(河南省):十三朝古都
(夏、商、西周、东周、东汉、曹魏、西晋、北魏、隋、唐、后梁、后唐、后晋)

3、南京(江苏省):十朝古都
(东吴、东晋、南朝宋、南朝齐、南朝梁、南朝陈、南唐、明朝、太平天国、民国)

4、开封(河南省):八朝古都
(夏、战国时期的魏、五代时期的后梁、后晋、后汉、后周、北宋、金)

5、安阳(河南省):七朝古都
(商朝、曹魏、后赵、冉魏、前燕、东魏、北齐)

6、北京(北京市):六朝古都
(燕国、辽国、金国、元朝、明朝、清朝)

7、郑州(河南省):五朝古都
(夏朝、商朝、西周管国、春秋郑国、战国韩国)

8、成都(四川省):五朝古都
(古蜀、蜀汉、成泛、前蜀、后蜀)

9、大同(山西省):三朝古都(北魏、辽国、金国)

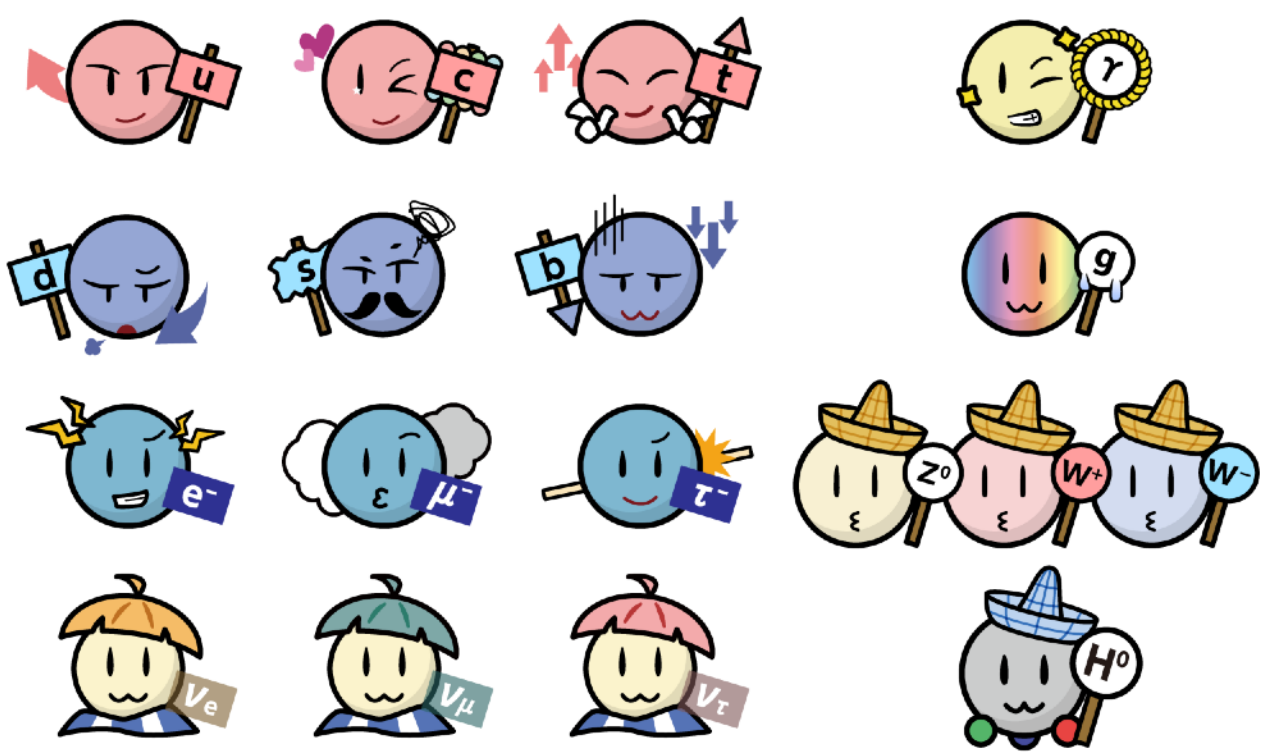
10、杭州(浙江省):两朝古都(吴越、南宋)

Introduction

- Supersymmetry (SUSY) is one of the promising extensions of the Standard Model (SM)

- Its minimal extension is known as MSSM

  Same set of copy (but different spin by ±1/2) is introduced



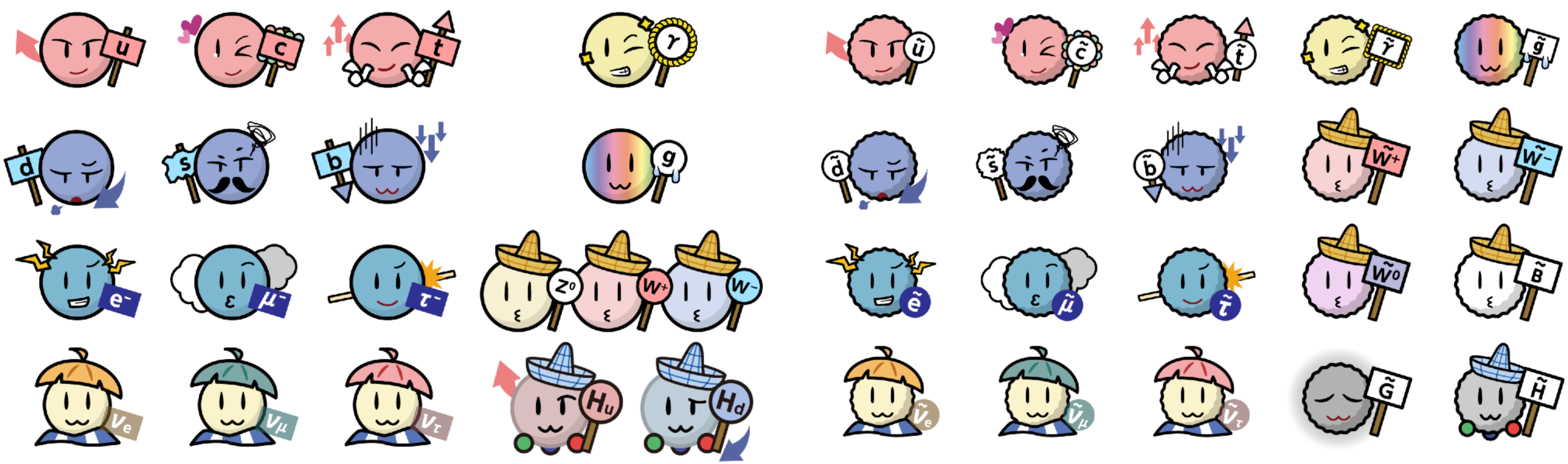
https://higgstan.com/

Introduction

- Supersymmetry (SUSY) is one of the promising extensions of the Standard Model (SM)

- Its minimal extension is known as MSSM

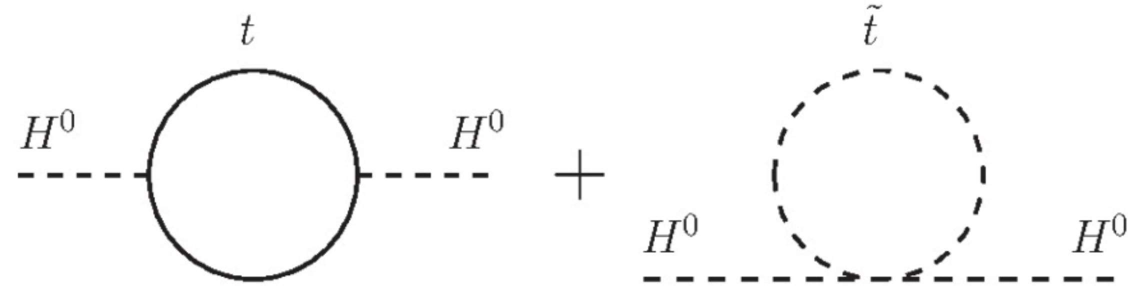
  Same set of copy (but different spin by ±1/2) is introduced

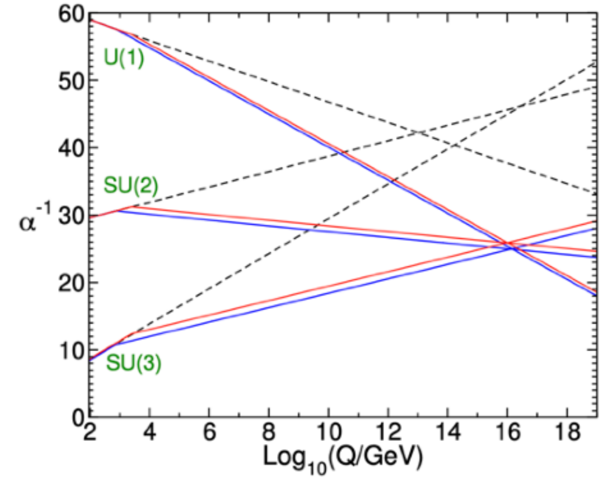


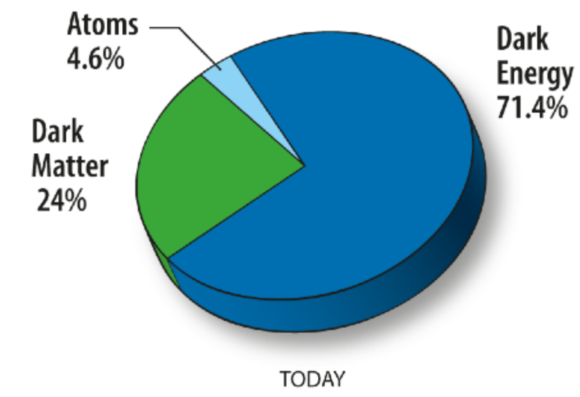
https://higgstan.com/

Introduction

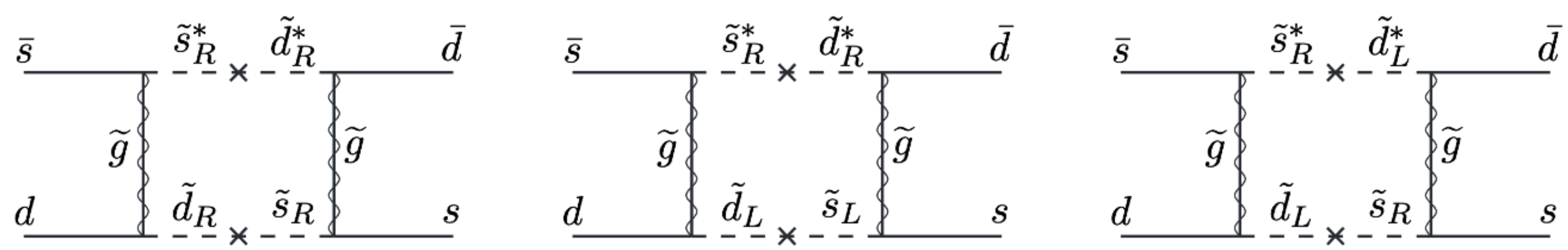
- SUSY is attractive because of
  - ✓ Solving EW naturalness problem
  - ✓ Realizing gauge coupling unification
  - ✓ Existing dark matter candidates
  - ✓ …







- But in SUSY models, we should solve
  - ▸ SUSY flavor problem
  - ▸ SUSY CP problem
  - ▸ μ problem
  - ▸ …



$$W \supset \mu H_u H_d \;\Rightarrow\; \mu \sim M_{\mathrm{Pl,GUT}} \overset{?}{\longleftrightarrow} \mu \sim 0.1\text{-}1\mathrm{TeV} \text{ for EWSB}$$

- SUSY is interesting, but still challenging

Some figures are borrowed from S.P. Martin's SUSY primer

Introduction

- SUSY FCNC … originates from soft-SUSY breaking para.

$$\mathcal{L}_{\rm soft} \supset -\sum_f (M_f^2)_{ij}\tilde{f}_i^\dagger \tilde{f}_j - \left[(A_u)_{ij}\tilde{Q}_i H_u \tilde{u}_j + (A_d)_{ij}\tilde{Q}_i H_d \tilde{d}_j + (A_e)_{ij}\tilde{L}_i H_d \tilde{e}_j + \text{c.c.}\right]$$

Generally have off-diag. elements

Not diagonalize simultaneously with Yukawas

- Flavor-changing parameters are severely constrained when $m_{\rm soft}$ ~ TeV scale

In particular for flavor-changing related with first and second gens.

D^0–$\overline{D^0}$, K^0–$\overline{K^0}$, $\mu \to e\gamma, \cdots$

- Easy solution: taking higher SUSY scale

Not interesting both for theoretical and experimental reasons

Th) Loss of the motivation:
Arise little hierarchy problem

Ex) Loss of the testability:
Higher energy and/or luminosity

Introduction

- SUSY FCNC … originates from soft-SUSY breaking para.

$$\mathcal{L}_{\rm soft} \supset -\sum_f (M_f^2)_{ij}\tilde{f}_i^\dagger \tilde{f}_j - \Big[(A_u)_{ij}\tilde{Q}_i H_u \tilde{u}_j + (A_d)_{ij}\tilde{Q}_i H_d \tilde{d}_j + (A_e)_{ij}\tilde{L}_i H_d \tilde{e}_j + \text{c.c.}\Big]$$

Generally have off-diag. elements

Not diagonalize simultaneously with Yukawas

- Other solution: alignment mechanism

NPB398(1993)319, PLB309(1993)337, NPB420(1994)468, PLB332(1994)100, NPB448(1995)30, PLB382(1996)363

Suppose we have U(1) horizontal symmetries

☆ Proper charge assignments lead to: $\begin{cases}\text{correct mass hierarchies}\\ \text{correct mixing angles}\end{cases}$ for SM fermions

via Froggatt-Nielsen mechanism together with holomorphic nature of W

- Off-diagonal elements of soft-SUSY breaking para. are small even in the mass basis of SM fermions!

✓ We can safely set $M_{\rm SUSY} \sim O(\text{TeV})$ in the sense of SUSY FCNC bounds

Introduction

- Is the horizontal symmetry only possibility to do that?

⚠ No!

- Another way to realize alignment mechanism:

Non-Invertible Symmetry

✓ Specific "selection rules"
✓ Non-trivial textures can be obtained

Yukawa textures are consistent with exp.:
JHEP11(2024)120, JHEP12(2024)117,
JHEP05(2025)177, JHEP08(2025)189

⇨ Totally different from ordinary group-based sym.

- Lots of applications to phenomenological models

PRL129(2022)161601, PRX13(2023)011034, PRX14(2024)031033, PRX15(2025)031011, 2503.19964, PLB868(2025)139706, JHEP12(2025)111, 2506.10241, 2506.16706, PRD113(2026)056028, 2507.10299, NPB1025(2026)117391, 2507.16198, PRD113(2026)055016, 2508.14970, 2508.16174, 2510.01680, PRD112(2025)115029, 2510.17156, 2510.17292, 2512.16376, 2512.20891, 2512.21509, 2601.15749, 2602.24214, 2604.04423

→ Most of them focus on non-SUSY model

- It may give hints for the Yukawa hierarchies and textures

Introduction

- We apply this NIS to MSSM

  Correct mass hierarchies and mixing angles for SM (charged) fermions

  Alignment mechanism works when SUSY breaking sector respects NIS

- NIS should be applied at high scale (M_{Pl}, M_{GUT})

  Taking account RGE effects on MSSM parameters

- Some questions are

  ? Textures are stable against RGEs or not

  ? SUSY FCNCs are enough suppressed or not

  Crucial points of this work

- If yes, NIS offers a robust and attractive framework!

  Both for addressing origin of flavor structures and ensuring flavor safety

Non-Invertible Symmetry

More details: e.g., Kobayashi, Otsuka JHEP11(2024)120
Dong et al. PRD113(2026)056028

- We focus on Z_2 gauging of Z_N

  Generators of Z_N, g, obey $g^{k_1} g^{k_2} = g^{k_1+k_2}$ for $k_{1,2} = 0, 1, \cdots, N-1$

  It allows n-point interaction only when $k_1 + \cdots + k_n = 0 \mod N$

- Considering Z_2 (generator r) outer automorphism

$$r^2 = e \text{ (identity)}, \quad r g^k r^{-1} = g^{-k}$$

- We can define “classes” of Z_2 gauging of Z_N

$$[g^k] = \{h g^k h^{-1} | h = e, r\} = \{g^k, g^{-k}\} \qquad k = 0, 1, \cdots, \begin{cases} \frac{N-1}{2} & \text{for odd } N \\ \frac{N}{2} & \text{for even } N \end{cases}$$

  E.g.
  - Z_2 gauging of Z_3 has two classes, $[g^0]$, $[g^1]$
  - Z_2 gauging of Z_4 has three classes, $[g^0]$, $[g^1]$, $[g^2]$
  - Z_2 gauging of Z_5 has three classes, $[g^0]$, $[g^1]$, $[g^2]$

- This is related to high energy theory (such as string theory)

Theoretically grounded path

Non-Invertible Symmetry

- The "classes" of Z_2 gauging of Z_N

$$[g^k] = \{hg^k h^{-1} | h = e, r\} = \{g^k, g^{-k}\}$$

$$k = 0, 1, \cdots, \begin{cases} \frac{N-1}{2} & \text{for odd } N \\ \frac{N}{2} & \text{for even } N \end{cases}$$

- Fusion rule: $[g^{k_1}] \otimes [g^{k_2}] =$???

Non-Invertible Symmetry

- The "classes" of Z_2 gauging of Z_N

$$[g^k] = \{hg^kh^{-1} | h = e, r\} = \{g^k, g^{-k}\}$$

$$k = 0, 1, \cdots, \begin{cases} \frac{N-1}{2} & \text{for odd } N \\ \frac{N}{2} & \text{for even } N \end{cases}$$

- Fusion rule: $[g^{k_1}] \otimes [g^{k_2}] = [g^{k_1+k_2}] \oplus [g^{k_1-k_2}]$

Z_N rule:

	g^{k_1}	g^{-k_1}
g^{k_2}	$g^{k_1+k_2}$	$g^{-k_1+k_2}$
g^{-k_2}	$g^{k_1-k_2}$	$g^{-k_1-k_2}$

One "class" has some set of elements

- This leads a crucial consequence

We can construct models with non-ordinary procedure!

Non-trivial textures can be obtained!

Non-Invertible Symmetry

- What's the essence?

Fusion rule: $[g^{k_1}] \otimes [g^{k_2}] = [g^{k_1+k_2}] \oplus [g^{k_1-k_2}]$

In general, allow when $\sum_i \pm k_i = 0 \mod N$

Term is allowed when one of these is identity class (= $[g^0]$)

- Example: 2x2 submatrix of Yukawa … Try to get $\begin{pmatrix} \text{forbid} & \text{allow} \\ \text{allow} & \text{allow} \end{pmatrix}$

Group-based case

each field has charge *q* …

$$\begin{pmatrix} q_1^Q + q_1^D + q^{H_d} & q_1^Q + q_2^D + q^{H_d} \\ q_2^Q + q_1^D + q^{H_d} & q_2^Q + q_2^D + q^{H_d} \end{pmatrix}$$

$$\begin{cases} q_1^Q + q_2^D + q^{H_d} = 0 \\ q_2^Q + q_1^D + q^{H_d} = 0 \\ q_2^Q + q_2^D + q^{H_d} = 0 \end{cases}$$ is required, but this means

$$q_1^Q + q_1^D + q^{H_d} = 0$$

If three elements are allowed, remaining one is also allowed!

$$\begin{pmatrix} \text{allow} & \text{allow} \\ \text{allow} & \text{allow} \end{pmatrix}$$

Non-invertible symmetry case (N=5)

each field has class $[g^k]$ …

2+2+1=5

$$\begin{pmatrix} [g^{k_{Q_1}}] \otimes [g^{k_{D_1}}] \otimes [g^{k_{H_d}}] & [g^{k_{Q_1}}] \otimes [g^{k_{D_2}}] \otimes [g^{k_{H_d}}] \\ [g^{k_{Q_2}}] \otimes [g^{k_{D_1}}] \otimes [g^{k_{H_d}}] & [g^{k_{Q_2}}] \otimes [g^{k_{D_2}}] \otimes [g^{k_{H_d}}] \end{pmatrix}$$

±2+0±1=±3
±2+0∓1=±1

1+0-1=0

1-2+1=0

e.g. $\{k_{Q_1}, k_{Q_2}, k_{D_1}, k_{D_2}, k_{H_d}\} = \{2, 1, 0, 2, 1\}$

✓

$$\begin{pmatrix} \text{forbid} & \text{allow} \\ \text{allow} & \text{allow} \end{pmatrix}$$

Yukawa textures thanks to NISR

Nakai, Otsuka, YS, Zhang
2512.21509

- Charge assignments for each (super-)field:

NIS	$Q_{1,2,3}$	$U_{1,2,3}$	$D_{1,2,3}$	$L_{1,2,3}$	$E_{1,2,3}$	H_u	H_d
$\tilde{\mathbb{Z}}_5^{(1)}$	$[g^2],[g^1],[g^2]$	$[g^1],[g^0],[g^1]$	$[g^0],[g^2],[g^2]$	$[g^0],[g^1],[g^2]$	$[g^1],[g^0],[g^1]$	$[g^1]$	$[g^1]$
$\tilde{\mathbb{Z}}_5^{(2)}$	$[g^2],[g^1],[g^1]$	$[g^1],[g^2],[g^0]$	$[g^2],[g^2],[g^0]$	$[g^2],[g^1],[g^1]$	$[g^1],[g^2],[g^0]$	$[g^1]$	$[g^1]$

- Check each term:

$$(Y_u)_{11}\cdots\tilde{\mathbb{Z}}_5^{(1)}:[g^2]\otimes[g^1]\otimes[g^1]\supset[g^0]\,;\ \tilde{\mathbb{Z}}_5^{(2)}:[g^2]\otimes[g^1]\otimes[g^1]\supset[g^0]\ \to\ \text{allowed}$$

$$(Y_u)_{12}\cdots\tilde{\mathbb{Z}}_5^{(1)}:[g^2]\otimes[g^0]\otimes[g^1]\not\supset[g^0]\,;\ \tilde{\mathbb{Z}}_5^{(2)}:[g^2]\otimes[g^2]\otimes[g^1]\supset[g^0]\ \to\ \text{forbidden}$$

$$(Y_u)_{13}\cdots\tilde{\mathbb{Z}}_5^{(1)}:[g^2]\otimes[g^1]\otimes[g^1]\supset[g^0]\,;\ \tilde{\mathbb{Z}}_5^{(2)}:[g^2]\otimes[g^0]\otimes[g^1]\not\supset[g^0]\ \to\ \text{forbidden}$$

- Finally, we obtain diagonal $Y_{u,e}$ while non-trivial Y_d at $\mu = M_U$

$$Y_u^{\mathrm{MSSM}}(M_{\mathrm{U}})\simeq\begin{pmatrix}2.89\times10^{-6}&0&0\\0&1.46\times10^{-3}&0\\0&0&0.527\end{pmatrix},\ Y_d^{\mathrm{MSSM}}(M_{\mathrm{U}})\simeq\begin{pmatrix}0&0.0256&0\\0.0109&0.118e^{1.09i}&0.0199\\0&2.53&1.27\end{pmatrix}\times10^{-2}$$

$$Y_e^{\mathrm{MSSM}}(M_{\mathrm{U}})\simeq\mathrm{diag}\left(1.02\times10^{-5},2.15\times10^{-3},3.66\times10^{-2}\right)$$

Note: Y_d texture cannot be obtained in ordinary symmetry models

RGE effect on the Yukawa textures

- Yukawas at SUSY scale (3 TeV, for tanβ = 5)

$$|Y_u(M_{\rm SUSY})| \simeq \begin{pmatrix} 5.78\times10^{-6} & 4.55\times10^{-10} & 2.59\times10^{-6} \\ 9.01\times10^{-13} & 2.92\times10^{-3} & 1.25\times10^{-5} \\ 1.46\times10^{-11} & 3.55\times10^{-8} & 0.816 \end{pmatrix}$$

$$|Y_d(M_{\rm SUSY})| \simeq \begin{pmatrix} 2.58\times10^{-11} & 1.29\times10^{-4} & 5.75\times10^{-8} \\ 5.51\times10^{-5} & 5.94\times10^{-4} & 1.00\times10^{-4} \\ 2.40\times10^{-9} & 1.17\times10^{-2} & 5.88\times10^{-3} \end{pmatrix}$$

RGE-induced entries

Enough small!

$$|Y_e(M_{\rm SUSY})| \simeq \mathrm{diag}\left(2.840\times10^{-6}, 5.982\times10^{-4}, 1.016\times10^{-2}\right)$$

- Out textures are essentially stable against RGE

This results in …

Small FCNC in up-type sector, safe for any FCNC processes

Enough large FCNC in down-type sector, leading to testable predictions

No charged lepton flavor violating (cLFV) processes

We mainly focus on quark FCNCs

Soft-SUSY breaking parameters

- We have following soft-SUSY breaking terms:

$$\mathcal{L}_{\rm soft} \supset -\sum_f M_f^2 \tilde{f}^\dagger \tilde{f} - \left(A_u \tilde{Q} H_u \tilde{u} + A_d \tilde{Q} H_d \tilde{d} + A_e \tilde{L} H_d \tilde{e} + {\rm c.c.} \right)$$

- Scalar trilinear couplings (A-terms) have same textures

$$A_u = \begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix}, \quad A_d = \begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}, \quad A_e = \begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix}$$

But, each entry doesn't need to be same as one corresponding in Y_f

- Soft mass-squared matrices are diagonal forms!

$$M_f^2 = {\rm diag}(m_{f1}^2, m_{f2}^2, m_{f3}^2)$$

But, in general, not proportional to identity matrix

- In the analysis, we just assume O(1) coefficient as like

$$(A_f(M_{\rm U}))_{ij} = \sigma_A M_{\rm SUSY} \mathcal{O}(1) (Y_f(M_{\rm U}))_{ij} \quad \text{and} \quad M_f^2(M_{\rm U}) = M_{\rm SUSY}^2 {\rm diag}\left(\mathcal{O}(1), \mathcal{O}(1), \mathcal{O}(1)\right)$$

+ or -

Note: other parameters … $M_{1,2,3}(M_U) = M_{\rm SUSY}\,, \quad m_{H_u,H_d}^2(M_U) = M_{\rm SUSY}^2$

Mass Insertion parameters

- 6x6 sfermion mass matrices: (similar for selectron sector)

$$\mathcal{M}_u^2 = \begin{pmatrix} \hat{M}_Q^2 + v_u^2 \hat{Y}_u \hat{Y}_u + \mathcal{D}_{\tilde{u}_L} & v_u \hat{A}_u^* - \mu v_d \hat{Y}_u \\ v_u \hat{A}_u^T - \mu^* v_d \hat{Y}_u & \hat{M}_u^2 + v_u^2 \hat{Y}_u \hat{Y}_u + \mathcal{D}_{\tilde{u}_R} \end{pmatrix} \equiv \begin{pmatrix} \Delta_{LL}^u & \Delta_{LR}^u \\ (\Delta_{LR}^u)^\dagger & \Delta_{RR}^u \end{pmatrix}$$

$$\mathcal{M}_d^2 = \begin{pmatrix} V_{\rm CKM}^\dagger \hat{M}_Q^2 V_{\rm CKM} + v_d^2 \hat{Y}_d \hat{Y}_d + \mathcal{D}_{\tilde{d}_L} & -v_d \hat{A}_d^* + \mu v_u \hat{Y}_d \\ -v_d \hat{A}_d^T + \mu^* v_u \hat{Y}_d & \hat{M}_d^2 + v_d^2 \hat{Y}_d \hat{Y}_d + \mathcal{D}_{\tilde{d}_R} \end{pmatrix} \equiv \begin{pmatrix} \Delta_{LL}^d & \Delta_{LR}^d \\ (\Delta_{LR}^d)^\dagger & \Delta_{RR}^d \end{pmatrix}$$

We take “super-CKM basis” $\hat{Y}_u \equiv L_u^\dagger Y_u R_u\,,\ \hat{M}_Q^2 \equiv L_u^\dagger M_Q^2 L_u\,,\ \hat{M}_u^2 \equiv R_u^\dagger M_u^2 R_u\,,\ \hat{A}_u \equiv L_u^\dagger A_u R_u$

D-term contribution: $\mathcal{D}_\phi = \left[\frac{g_2^2 + (3/5) g_1^2}{2} \left(T_3(\phi) - Q(\phi) \sin^2\theta_W \right) \cos(2\beta) v_H^2 \right] \times \mathbf{1}_{3\times 3}$

- MI parameters: $(\delta_{XY}^f)_{ij} \equiv \frac{(\Delta_{XY}^f)_{ij}}{m_{\tilde{f}}^2}$ for f = u, d, e and X, Y = L or R

- Non-zero off-diagonal elements lead to flavor changing processes

For example:

$(\delta_{XY}^u)_{12,21} \to D \text{ meson}\,,\ (\delta_{XY}^d)_{12,21} \to K \text{ meson}\,,$

$(\delta_{XY}^d)_{13,31} \to B_d \text{ meson}\,,\ (\delta_{XY}^d)_{23,32} \to B_s \text{ meson}$

Mass Insertion parameters

- Numerical results for MI parameters

✓ Note: no cLFV since no LFV sources

- Up-type sector

$$|\delta^u_{LL}| = \begin{pmatrix} 1.2(1.0,1.3) & 7.1(5.5,9.5)\times 10^{-7} & 1.3(1.0,1.8)\times 10^{-5} \\ 7.1(5.5,9.5)\times 10^{-7} & 1.2(1.0,1.3) & 6.0(4.6,8.2)\times 10^{-5} \\ 1.3(1.0,1.8)\times 10^{-5} & 6.0(4.6,8.2)\times 10^{-5} & 0.94(0.82,1.07) \end{pmatrix}$$

$$|\delta^u_{RR}| = \begin{pmatrix} 1.1(0.9,1.2) & 5.9(0.2,14.8)\times 10^{-11} & 6.0(0.2,16.2)\times 10^{-12} \\ 5.9(0.2,14.8)\times 10^{-11} & 1.1(0.9,1.2) & 8.1(5.9,10.5)\times 10^{-8} \\ 6.0(0.2,16.2)\times 10^{-12} & 8.1(5.9,10.5)\times 10^{-8} & 0.63(0.50,0.76) \end{pmatrix}$$

Tiny off-diag. elements
… due to diagonal forms

$$|\delta^u_{LR}| = \begin{pmatrix} 2.1(1.2,3.1)\times 10^{-7} & 4.8(2.7,7.0)\times 10^{-11} & 1.1(0.3,1.9)\times 10^{-7} \\ 9.7(5.5,13.7)\times 10^{-14} & 1.1(0.6,1.6)\times 10^{-4} & 1.1(0.7,1.5)\times 10^{-6} \\ 7.8(0.5,16.9)\times 10^{-13} & 3.0(1.9,4.2)\times 10^{-9} & 2.4(1.9,3.0)\times 10^{-2} \end{pmatrix}$$

- Down-type sector

$$|\delta^d_{LL}| = \begin{pmatrix} 1.1(1.0,1.2) & 1.97(0.08,4.94)\times 10^{-2} & 1.2(0.3,2.2)\times 10^{-3} \\ 1.97(0.08,4.94)\times 10^{-2} & 1.1(1.0,1.2) & 8.7(0.7,17.0)\times 10^{-3} \\ 1.2(0.3,2.2)\times 10^{-3} & 8.7(0.7,17.0)\times 10^{-3} & 0.89(0.77,1.01) \end{pmatrix}$$

$$|\delta^d_{RR}| = \begin{pmatrix} 0.98(0.86,1.10) & 1.71(0.07,4.25)\times 10^{-2} & 7.5(0.3,19.2)\times 10^{-3} \\ 1.71(0.07,4.25)\times 10^{-2} & 0.98(0.89,1.08) & 3.4(0.1,8.8)\times 10^{-2} \\ 7.5(0.3,19.2)\times 10^{-3} & 3.4(0.1,8.8)\times 10^{-2} & 0.98(0.88,1.08) \end{pmatrix}$$

Sizable off-diag. elements
… interesting predictions!

$$|\delta^d_{LR}| = \begin{pmatrix} 1.2(0.8,1.6)\times 10^{-6} & 6.8(5.1,8.4)\times 10^{-6} & 8.4(5.7,11.0)\times 10^{-6} \\ 6.5(4.2,8.9)\times 10^{-6} & 3.5(2.4,4.7)\times 10^{-5} & 5.0(3.9,6.2)\times 10^{-5} \\ 1.8(1.2,2.3)\times 10^{-4} & 8.2(5.7,10.7)\times 10^{-4} & 9.0(7.0,11.0)\times 10^{-4} \end{pmatrix}$$

Experimental status for FCNC

- Relevant bounds on FCNC processes

Values from PDG2025

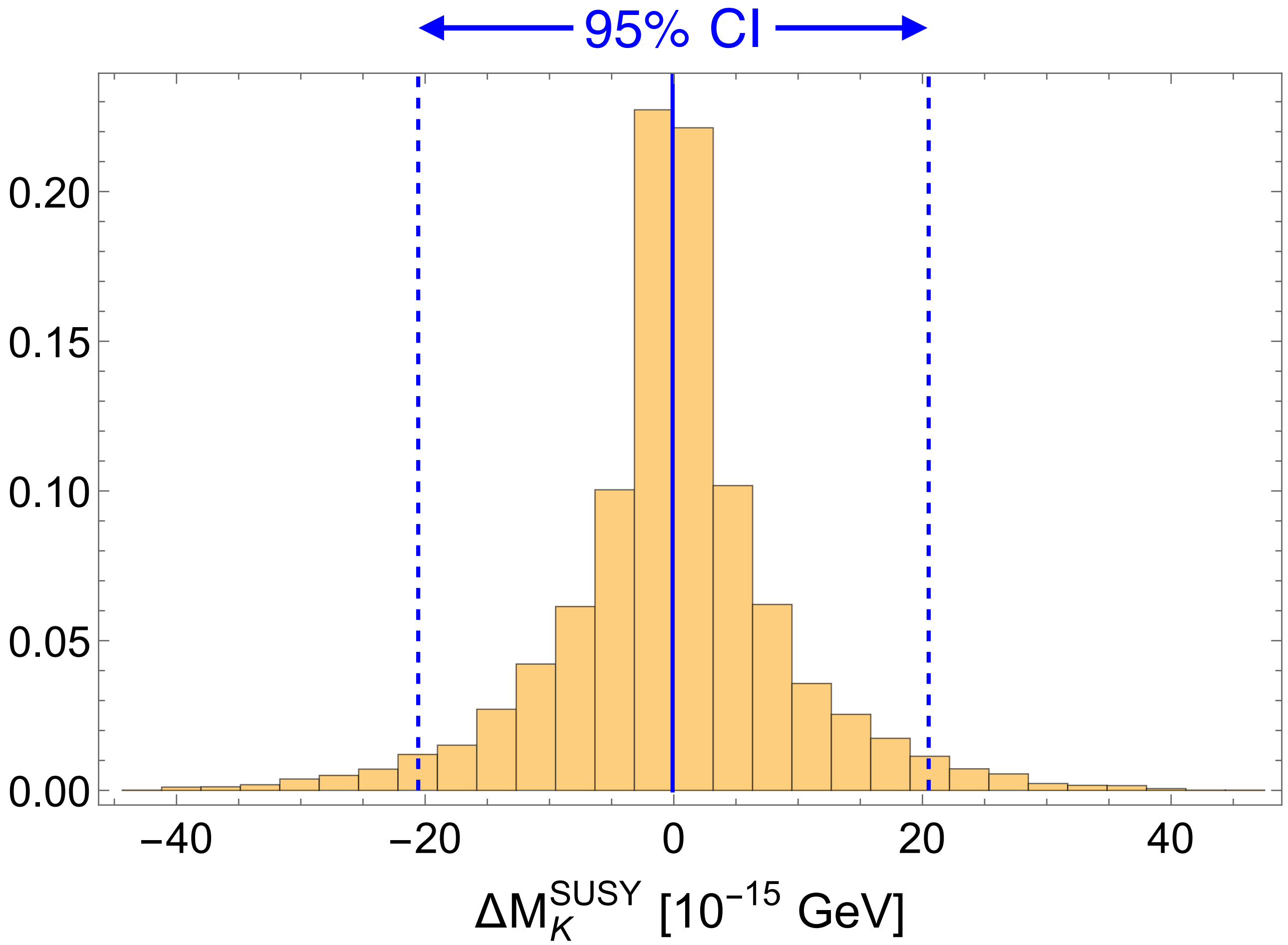
Observable	Exp. results/bounds
ΔM_K	$= 3.483(6) \times 10^{-15}\,\mathrm{GeV}$
ΔM_{B_d}	$= 3.33(1) \times 10^{-13}\,\mathrm{GeV}$
ΔM_{B_s}	$= 1.1700(4) \times 10^{-11}\,\mathrm{GeV}$
$\lvert\varepsilon_K\rvert$	$= 2.228(11) \times 10^{-3}$
$\mathrm{BR}(K^+ \to \pi^+ \nu\bar{\nu})$	$= (1.14^{+0.40}_{-0.33}) \times 10^{-10}$
$\mathrm{BR}(K_L \to \pi^0 \nu\bar{\nu})$	$< 3.0 \times 10^{-9}$
$\mathrm{BR}(B \to X_s \gamma)$	$= (3.49 \pm 0.19) \times 10^{-4}$
$\mathrm{BR}(B_d \to \ell\bar{\ell})\ (\ell = \{e, \mu, \tau\})$	$\{< 2.5 \times 10^{-9}, < 1.5 \times 10^{-10}, < 2.1 \times 10^{-3}\}$
$\mathrm{BR}(B_s \to \ell\bar{\ell})\ (\ell = \{e, \mu, \tau\})$	$\{< 9.4 \times 10^{-9}, = (3.34 \pm 0.27) \times 10^{-9}, < 6.8 \times 10^{-3}\}$
$\mathrm{BR}(B^+ \to \tau^+ \nu)$	$= (1.09 \pm 0.24) \times 10^{-4}$
$R(D) = \frac{\mathrm{BR}(B \to D\tau^- \bar{\nu}_\tau)}{\mathrm{BR}(B \to D\ell^- \bar{\nu}_\ell)}$	$= 0.342 \pm 0.026$
$R(D^*) = \frac{\mathrm{BR}(B \to D^*\tau^- \bar{\nu}_\tau)}{\mathrm{BR}(B \to D^*\ell^- \bar{\nu}_\ell)}$	$= 0.287 \pm 0.012$

⭐ SUSY FCNC predictions are calculated by susy_flavor_v2.54

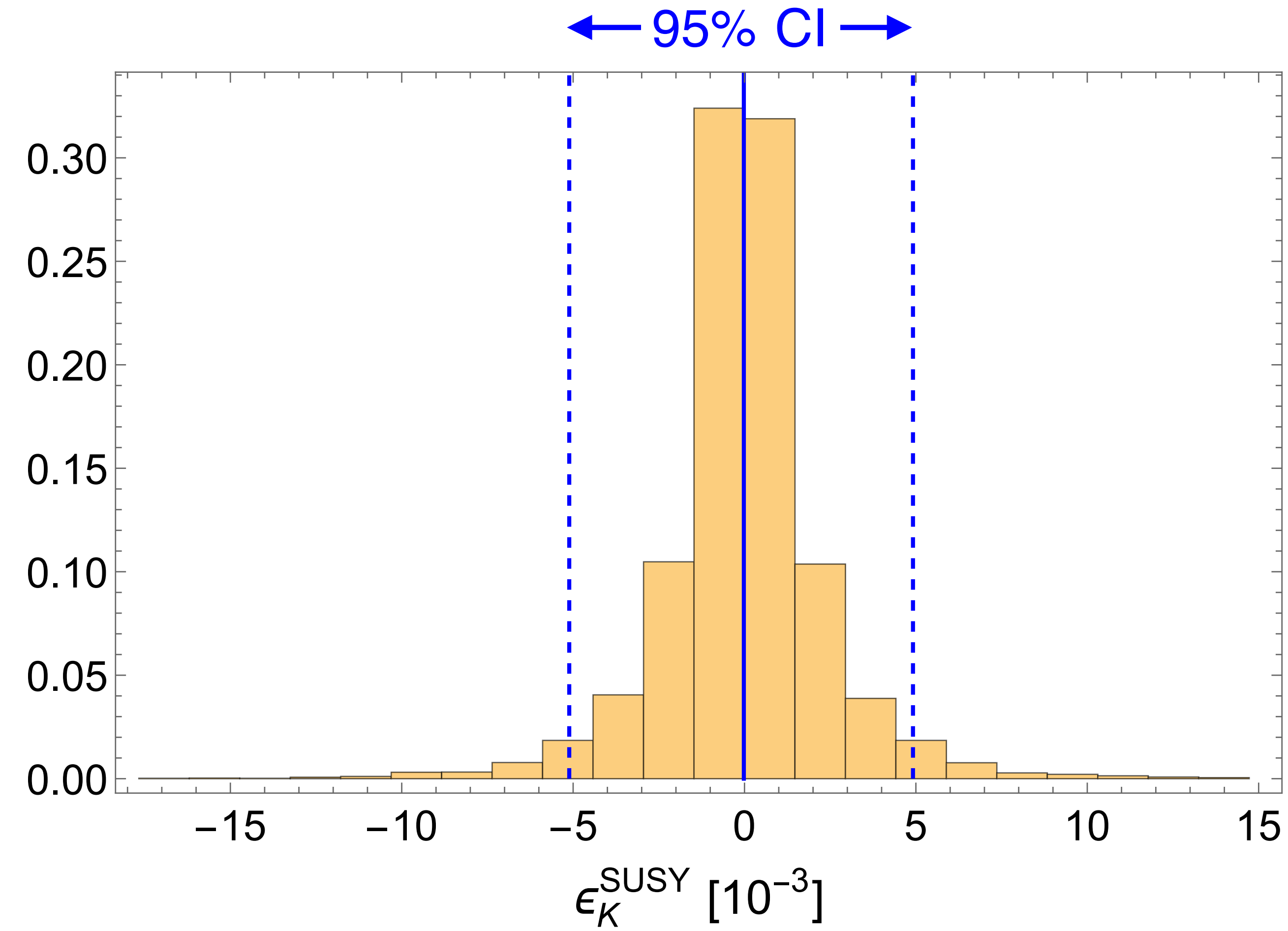
Result: Kaon mixing parameters

- Enough large to be tested

$$\Delta M_K^{\rm exp} = 3.483(6) \times 10^{-15}\,{\rm GeV}$$
$$|\varepsilon_K^{\rm exp}| = 2.228(11) \times 10^{-3}$$



$$\Delta M_K^{\rm SUSY} = -0.1\,(-20.6, 20.5) \times 10^{-15}\,{\rm GeV}$$



$$\varepsilon_K^{\rm SUSY} = -0.02\,(-5.11, 4.92) \times 10^{-3}$$

✓ Some samples can explain the discrepancy btw exp. and SM

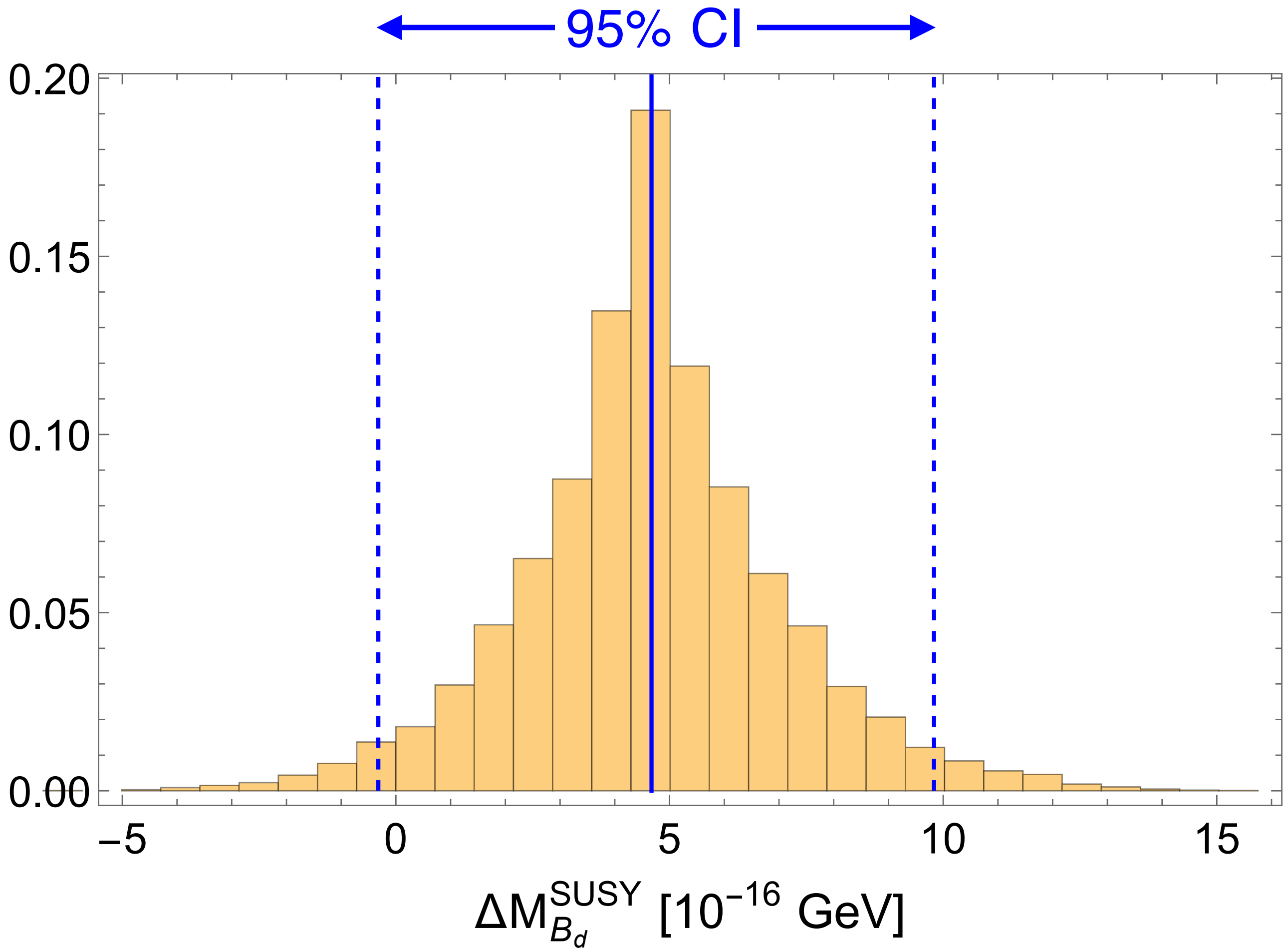
SM predictions calculated by susy_flavor_v2.54:

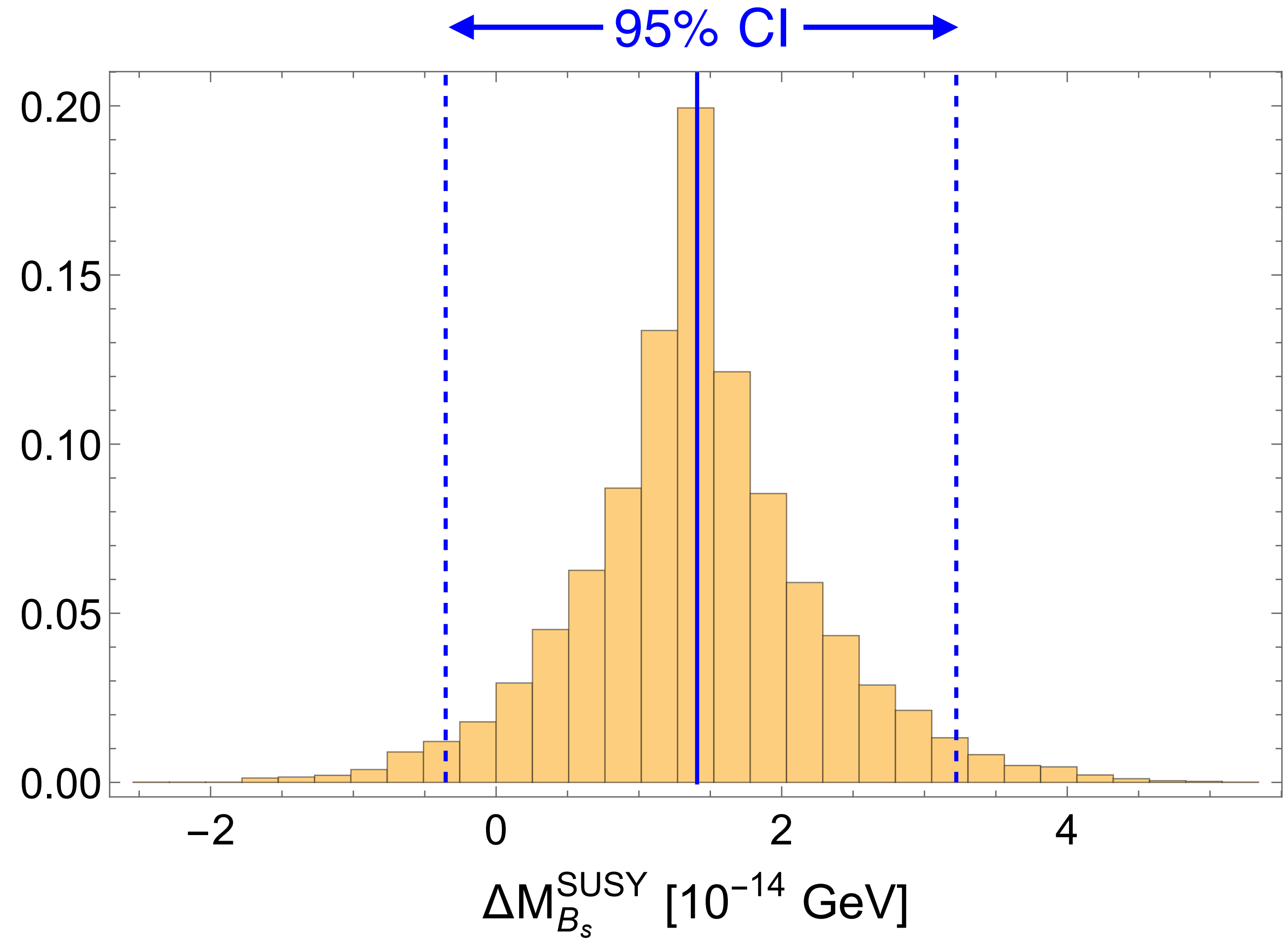
$$\Delta M_K^{\rm SM} = 2.73 \times 10^{-15}\,{\rm GeV}$$
$$\varepsilon_K^{\rm SM} = 2.34 \times 10^{-3}$$

Result: B meson mixing parameters

- Safely small

$$\Delta M_{B_d}^{\rm exp} = 3.33(1) \times 10^{-13}\,{\rm GeV}$$
$$\Delta M_{B_s}^{\rm exp} = 1.1700(4) \times 10^{-11}\,{\rm GeV}$$





$$\Delta M_{B_d}^{\rm SUSY} = 4.7\,(-0.3, 9.8) \times 10^{-16}\,{\rm GeV}$$
$$\Delta M_{B_s}^{\rm SUSY} = 1.4\,(-0.4, 3.2) \times 10^{-14}\,{\rm GeV}$$

✓ Below experimental constraints, non-zero mean values

SM predictions calculated by susy_flavor_v2.54:

$$\Delta M_{B_d}^{\rm SM} = 3.51 \times 10^{-13}\,{\rm GeV}$$
$$\Delta M_{B_s}^{\rm SM} = 1.20 \times 10^{-11}\,{\rm GeV}$$

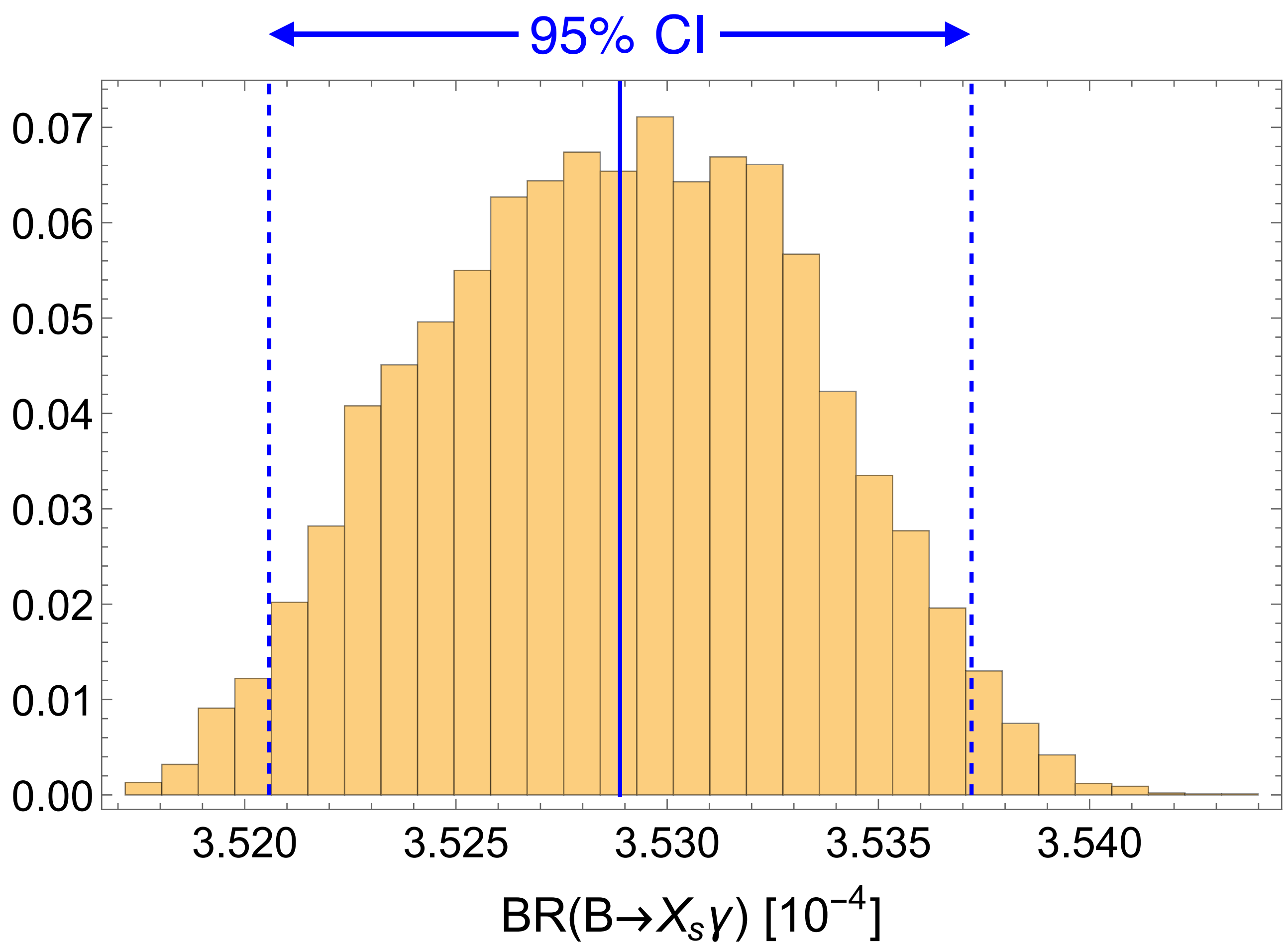
Result: b → sγ

$\mathrm{BR}(B \to X_s\gamma)_{\rm exp} = (3.49 \pm 0.19) \times 10^{-4}$

- Small SUSY contributions are expected, because …

$$\mathrm{BR}(b \to s\gamma)_{\rm SUSY} \simeq 6.25 \times 10^{-3} \times \left(\frac{3\,\mathrm{TeV}}{M_{\rm SUSY}}\right)^2 \left[|(\delta^d_{LR})_{23}|^2 + |(\delta^d_{LR})_{32}|^2\right] + \mathcal{O}\left(\frac{m_b^2}{M_{\rm SUSY}^2}\right)$$

$(\delta^d_{LR})_{23} = 5.0\,(3.9, 6.2) \times 10^{-5}, \quad (\delta^d_{LR})_{32} = 8.2\,(5.7, 10.7) \times 10^{-4}$ ⇨ $\mathrm{BR}(b \to s\gamma)_{\rm SUSY} \lesssim 10^{-8}$



SUSY contributions are small!

SM predictions calculated by

susy_flavor_v2.54: $\mathrm{BR}(b \to s\gamma)_{\rm SM} \sim 3.5 \times 10^{-4}$

$$|\mathcal{M}_{B\to X_s\gamma}|^2 \simeq |\mathcal{M}^{\rm SM}_{B\to X_s\gamma}|^2 + 2\mathrm{Re}\left[\mathcal{M}^{\rm SM}_{B\to X_s\gamma}\mathcal{M}^{\rm SUSY\,*}_{B\to X_s\gamma}\right]$$

$$\frac{2\mathrm{Re}\left[\mathcal{M}^{\rm SM}_{B\to X_s\gamma}\mathcal{M}^{\rm SUSY\,*}_{B\to X_s\gamma}\right]}{|\mathcal{M}^{\rm SM}_{B\to X_s\gamma}|^2} \sim 10^{-2}$$

$$\mathrm{BR}(B \to X_s\gamma) \approx \mathrm{BR}(b \to s\gamma) = 3.529\,(3.521, 3.537) \times 10^{-4}$$

Consistent with rough MI estimation!

Result: other processes

- ΔF = 1 in Kaon system

$$\mathrm{BR}(K^+ \to \pi^+ \nu\bar{\nu}) = 9.18\,(9.16, 9.21) \times 10^{-11}$$

$$\mathrm{BR}(K_L \to \pi^0 \nu\bar{\nu}) = 3.2435\,(3.2434, 3.2436) \times 10^{-11}$$

consistent with exp.

- Leptonic B meson decays

ℓ	e	μ	τ	95% CI
$\mathrm{BR}(B_d \to \ell^+\ell^-)$	2.91×10^{-15}	1.24×10^{-10}	2.60×10^{-8}	$-0.077\% \sim 0.080\%$
$\mathrm{BR}(B_s \to \ell^+\ell^-)$	9.84×10^{-14}	4.20×10^{-9}	8.92×10^{-7}	$-0.084\% \sim 0.086\%$

These may be tested! $\mathrm{BR}(B_d \to \mu^+\mu^-) < 1.5 \times 10^{-10}$ (90% CL)

$\mathrm{BR}(B_s \to \mu^+\mu^-) = (3.34 \pm 0.27) \times 10^{-9}$

JHEP04(2019)098, JHEP04(2020)188, PRL128(2022)041801, PLB842(2023)137955

- Other B physics … small due to same reason in b → sγ

$$\mathrm{BR}(B^+ \to \tau^+\nu) \simeq 8.8 \times 10^{-5}$$ with ±10^{-3}%

$$R_{D^{(*)}} \equiv \frac{\mathrm{BR}(B \to D^{(*)}\tau\nu)}{\mathrm{BR}(B \to D^{(*)}\ell\nu)}$$ are almost fixed by SM predictions

Note: charged Higgs mass is $m_{H^\pm} = 4.6\,(4.2, 5.1)\,\mathrm{TeV}$

Summary

- We proposed new method to suppress SUSY FCNC

Non-Invertible Symmetries …
✦ Not ordinary group-based symmetry
✦ Resultant selection rules control terms

- Not group-based selection rule

$$[g^{k_1}] \otimes [g^{k_2}] = [g^{k_1+k_2}] \oplus [g^{k_1-k_2}]$$

Due to fact that “class” has some elements

- We apply this to MSSM and check the FCNC predictions

All terms in superpotential and soft-breaking terms are controlled by NIS

- SUSY FCNC suppression is realized

M_{SUSY} = 3 TeV w/ tanβ = 5 is still consistent with experimental bounds!

NIS application to BSM is fascinating!

Thank you!

Back up

Calculation method

- (1) starting from gauge and Yukawa couplings at some scale … summarized in, e.g. Antusch, Hinze, Saad PRD113(2026)095011
- (2) running up gauge and Yukawa couplings at high scale (we set $M_U = 2\times10^{16}$ GeV)
- (3) take proper unitary rotations in order to get textures
- (4) set each O(1) coefficient for soft-SUSY breaking parameters
- (5) running down to SUSY scale
- Repeat (4), (5) with varying all O(1) coefficients

Note: for analysis, we generate 10,000 samples

Result: lepton sector

- In the current setup, no cLFV due to no sources

  Our predictions of cLFV = cLFV in the SM

- Still have predictions of g-2

  PRD53(1996)6565, PRD64(2001)035003

  Rough estimation: $$\frac{a_\ell^{\mathrm{SUSY}}}{m_\ell^2} \approx \frac{6.5 \times 10^{-10}}{\mathrm{GeV}^2} \times \left(\frac{\tan\beta}{5}\right)\left(\frac{3\,\mathrm{TeV}}{M_{\mathrm{SUSY}}}\right)^2$$

  Our predictions: $a_e^{\mathrm{SUSY}} \simeq 2.1 \times 10^{-16}\,,\ a_\mu^{\mathrm{SUSY}} \simeq 8.8 \times 10^{-12}\,,\ a_\tau^{\mathrm{SUSY}} \simeq 2.5 \times 10^{-9}$

  ⇨ Smaller than each experimental result

  PRL130(2023)071801, PRL135(2025)101802, PRL131(2023)151802, Rept.Prog.Phys.87(2024)107801

- Above results will be changed when we introduce Y_ν, A_ν

$$Y_\nu = \begin{pmatrix} 0 & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix}\,,\ A_\nu = \begin{pmatrix} 0 & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix}$$

NIS	$L_{1,2,3}$	$N_{1,2,3}$	H_u
$\tilde{\mathbb{Z}}_5^{(1)}$	$[g^0], [g^1], [g^2]$	$[g^0], [g^1], [g^2]$	$[g^1]$
$\tilde{\mathbb{Z}}_5^{(2)}$	$[g^2], [g^1], [g^1]$	$[g^0], [g^2], [g^2]$	$[g^1]$

✓ Should have off-diag. elements for neutrino mixing

Result: FCNC in up-type sector

- Top quark FCNC: extremely insensitive

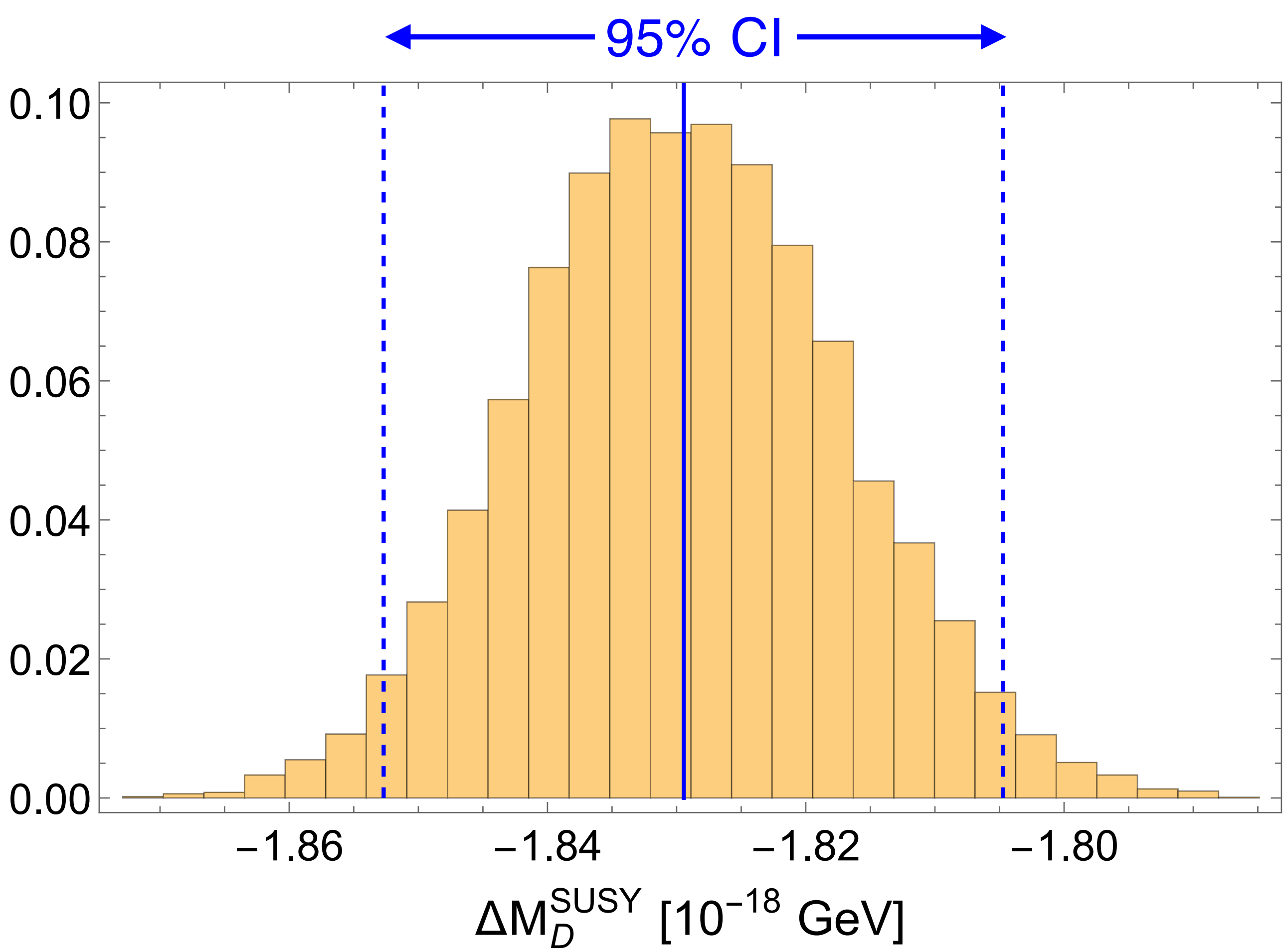
our predictions…

$$\mathrm{BR}(t \to ch) = 1.44\,(1.35, 1.53) \times 10^{-13}$$
$$\mathrm{BR}(t \to uh) = 1.15\,(1.08, 1.22) \times 10^{-15}$$

exp. bounds…

$$\mathrm{BR}(t \to ch) < 3.4 \times 10^{-4}$$
$$\mathrm{BR}(t \to uh) < 1.9 \times 10^{-4}$$
@ 95% CL

- D meson mixing: also small prediction



✓ Below experimental constraints and smaller than SM prediction

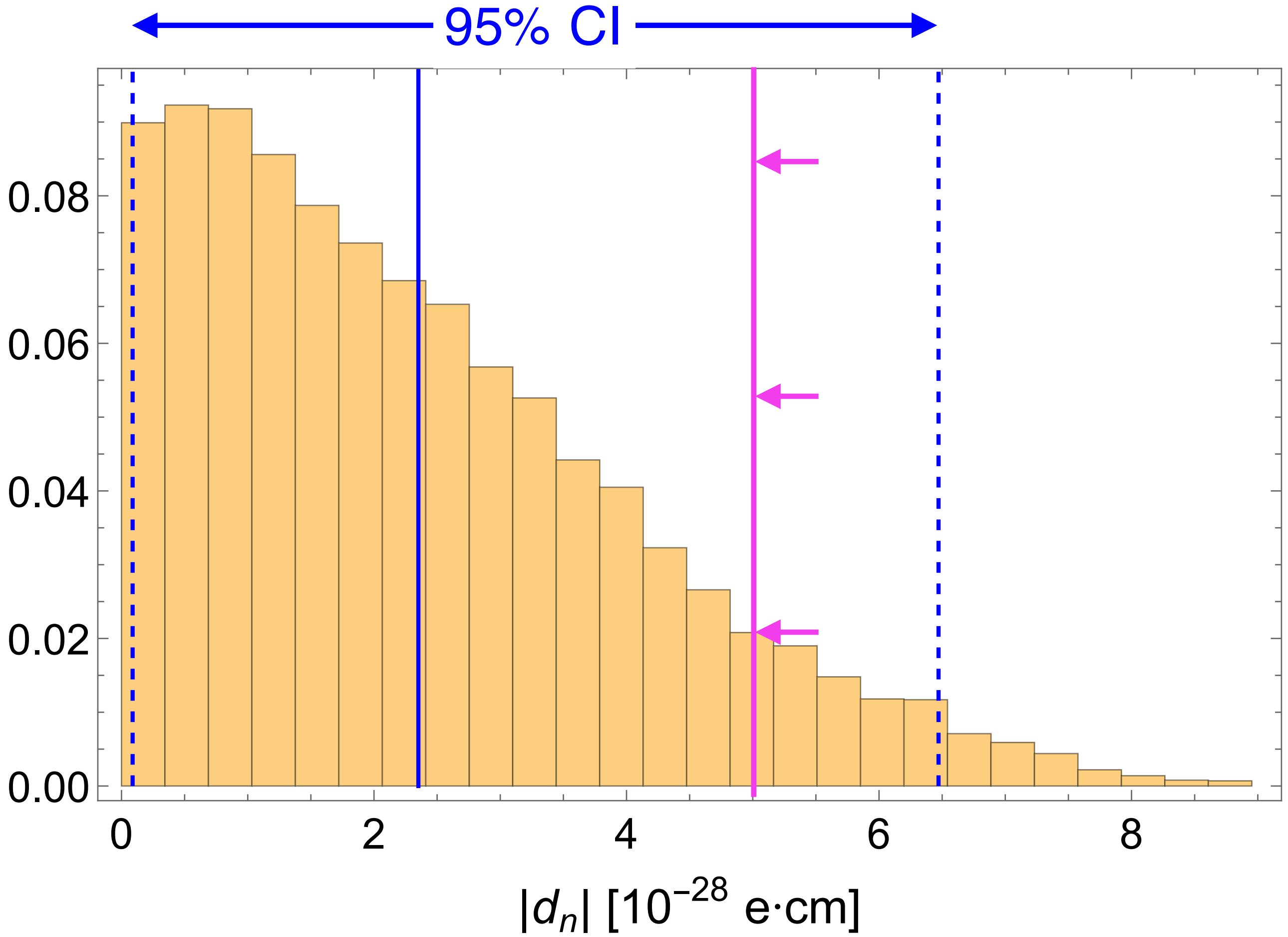
$$\Delta M_D^{\mathrm{SUSY}} = -1.83\,(-1.85, -1.80) \times 10^{-18}\,\mathrm{GeV}$$

$$\Delta M_D^{\mathrm{exp}} = 6.56(76) \times 10^{-15}\,\mathrm{GeV}$$

Result: neutron EDM

- Small, due to only source of CPV is the phase in CKM

This is just an assumption



$$|d_n| = 2.35\,(0.09, 6.47) \times 10^{-28}\,e\,\mathrm{cm}$$

Note: future n2EDM experiment sensitivity … ~ 5×10^{-28} e cm

The n2EDM experiment
EPJ Web Cong. 219 (2019) 02002

- Adding new CP phases in soft parameters enhances $|d_n|$
  → relates to other obs., such as the strong CP, charged lepton EDMs, …

Result: other process distributions