

Flavor Physics and CP from Non-Invertible Selection Rules

Hajime Otsuka (Kyushu University)



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Outline :

1. Coupling selection rules (review)
2. Yukawa textures
3. CP
4. Strong CP
5. String compactifications
6. Conclusions

In a 4D theory with group-like symmetries,
a field corresponds to (a rep. of) a group element

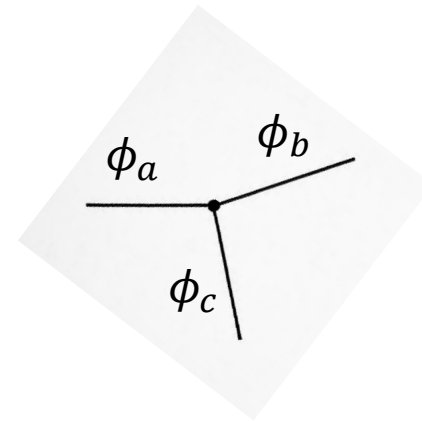
- Transformations of fields:

$$\phi \xrightarrow{g} \rho(g)\phi \quad g \in G$$

- Suppose that fields $\{\phi_a, \phi_b, \phi_c\}$ correspond to a rep. of G

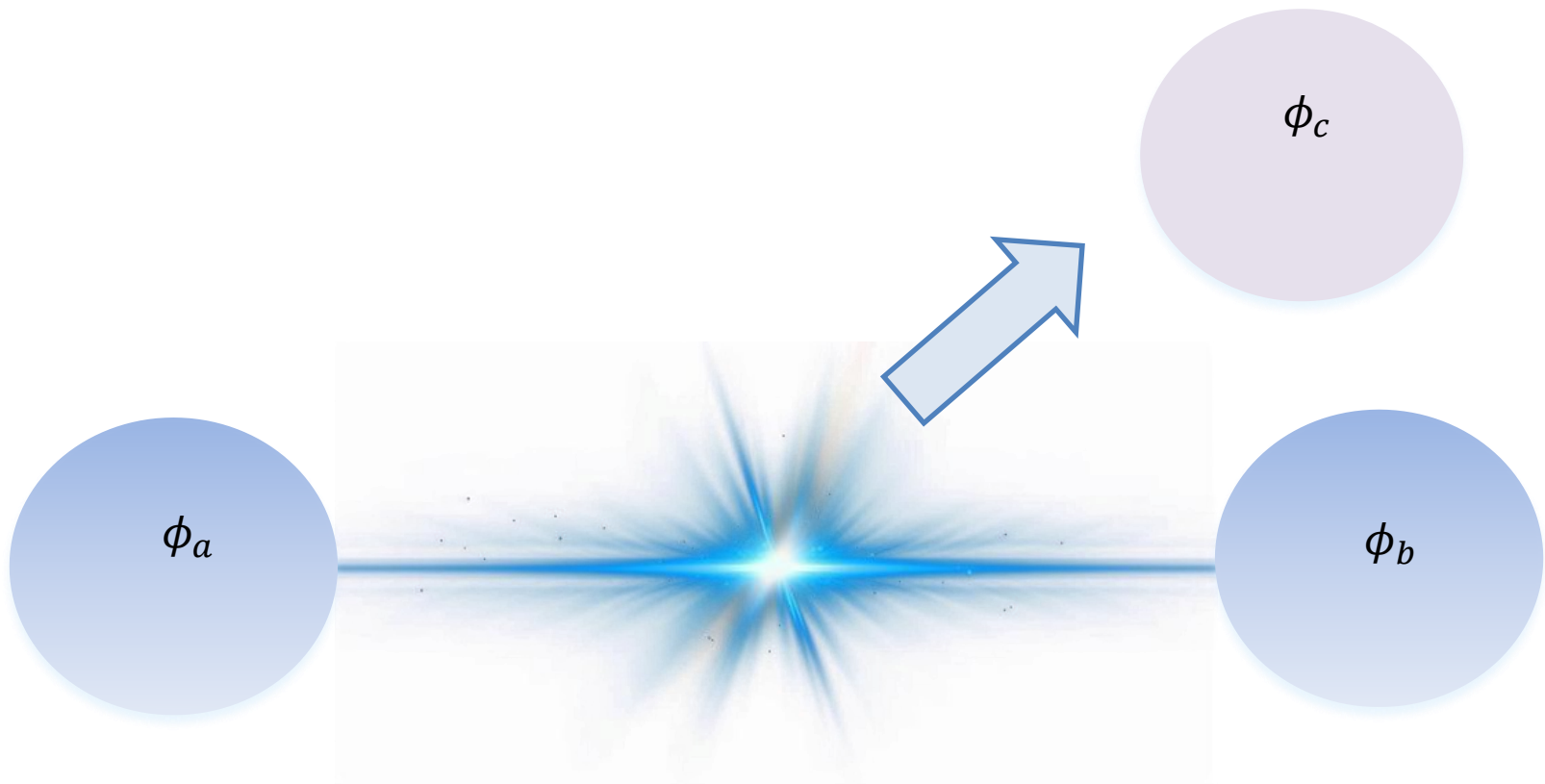
if $\rho_a \otimes \rho_b \otimes \rho_c \supset \mathbb{I} \rightarrow \phi_a \phi_b \phi_c$ is allowed

if $\rho_a \otimes \rho_b \otimes \rho_c \not\supset \mathbb{I} \rightarrow \phi_a \phi_b \phi_c$ is forbidden

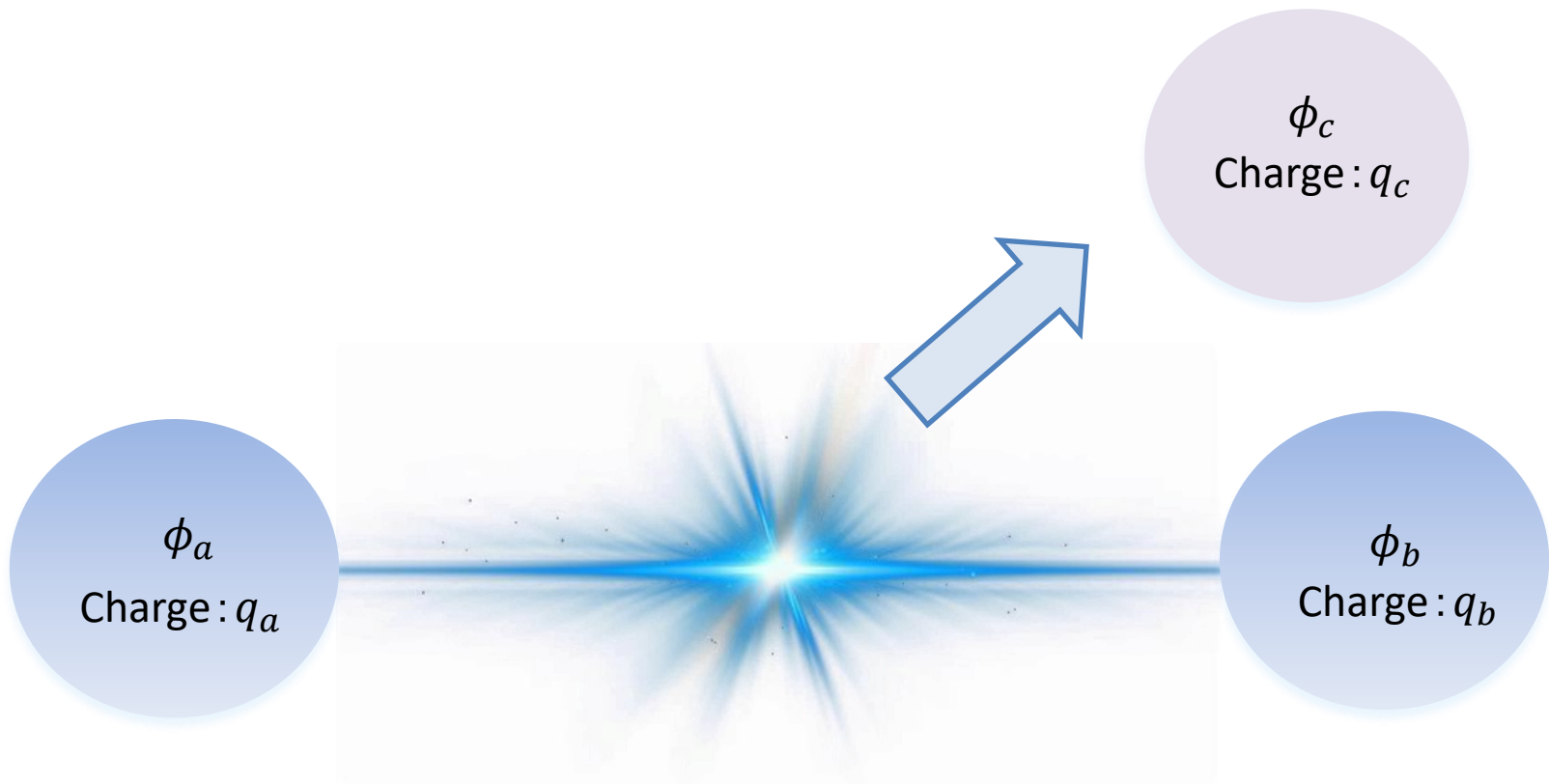


“Coupling selection rules”

(When G is Abelian, it is the charge conservation law)

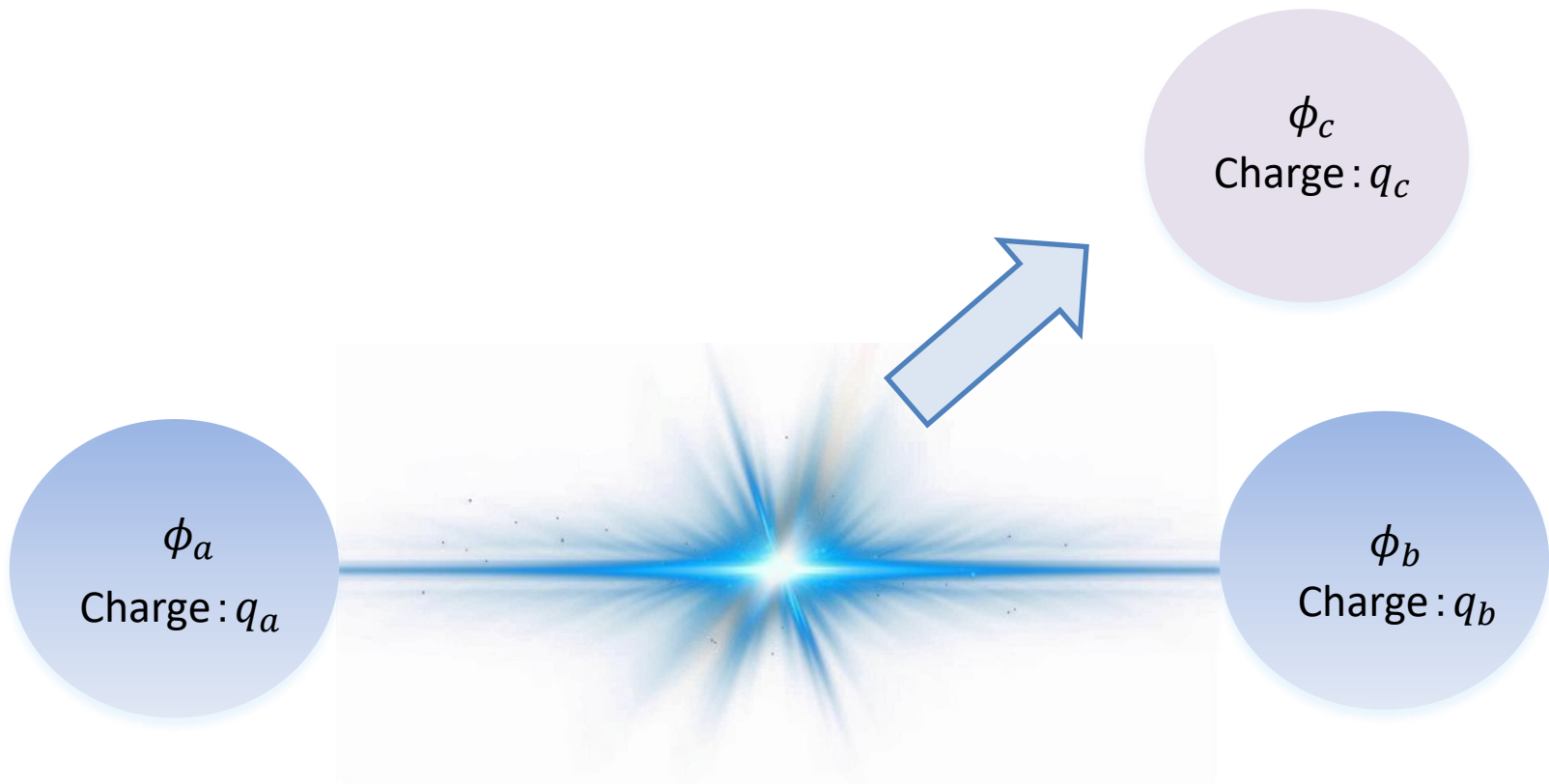


“**Selection rules**” based on a symmetry determine the presence or absence of interactions



Charge conservation law: $q_a + q_b = q_c$

“**Selection rules**” based on a symmetry determine the presence or absence of interactions

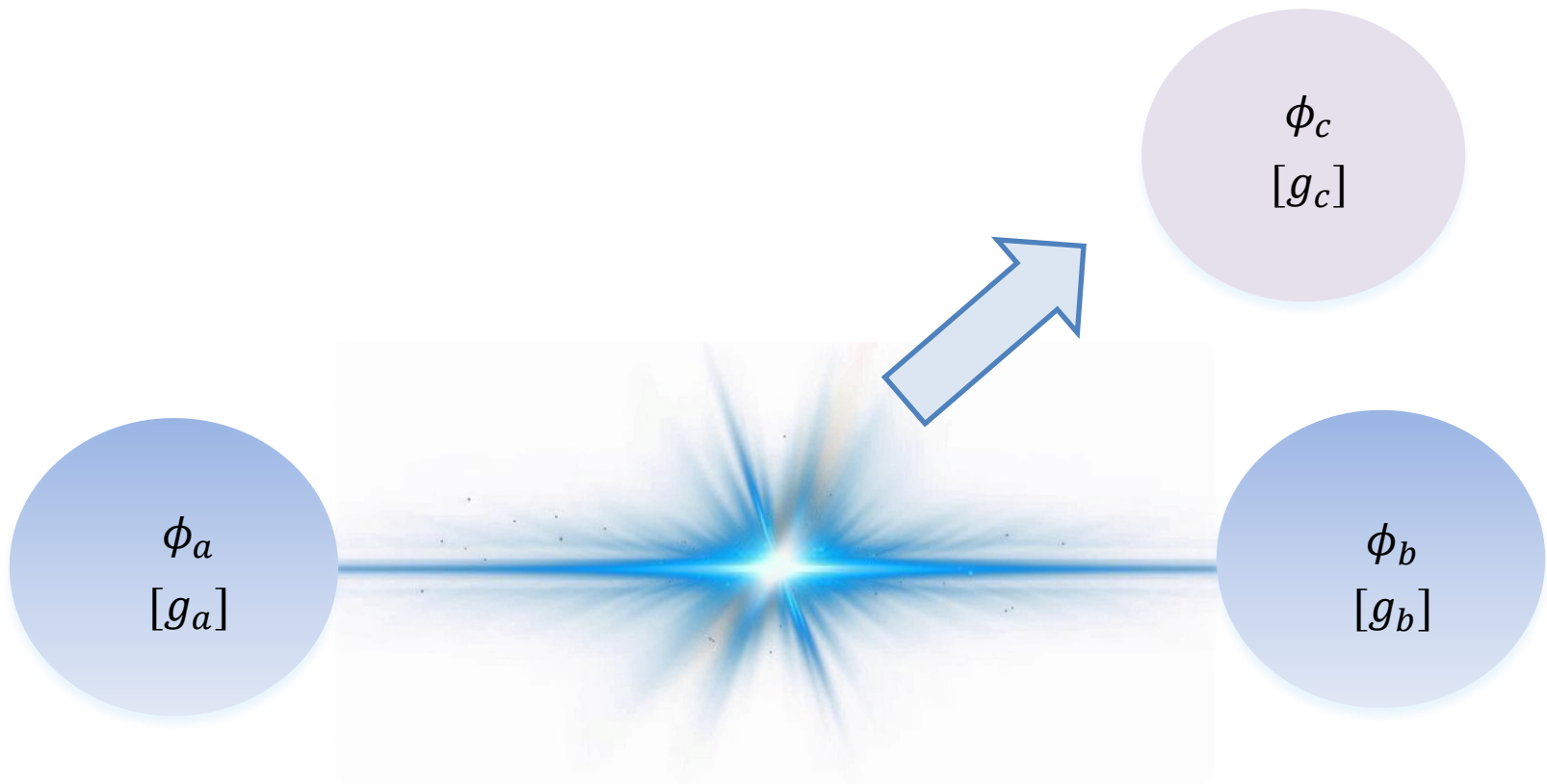


Charge conservation law: $q_a + q_b = q_c$

“**Selection rules**” based on a symmetry determine the presence or absence of interactions

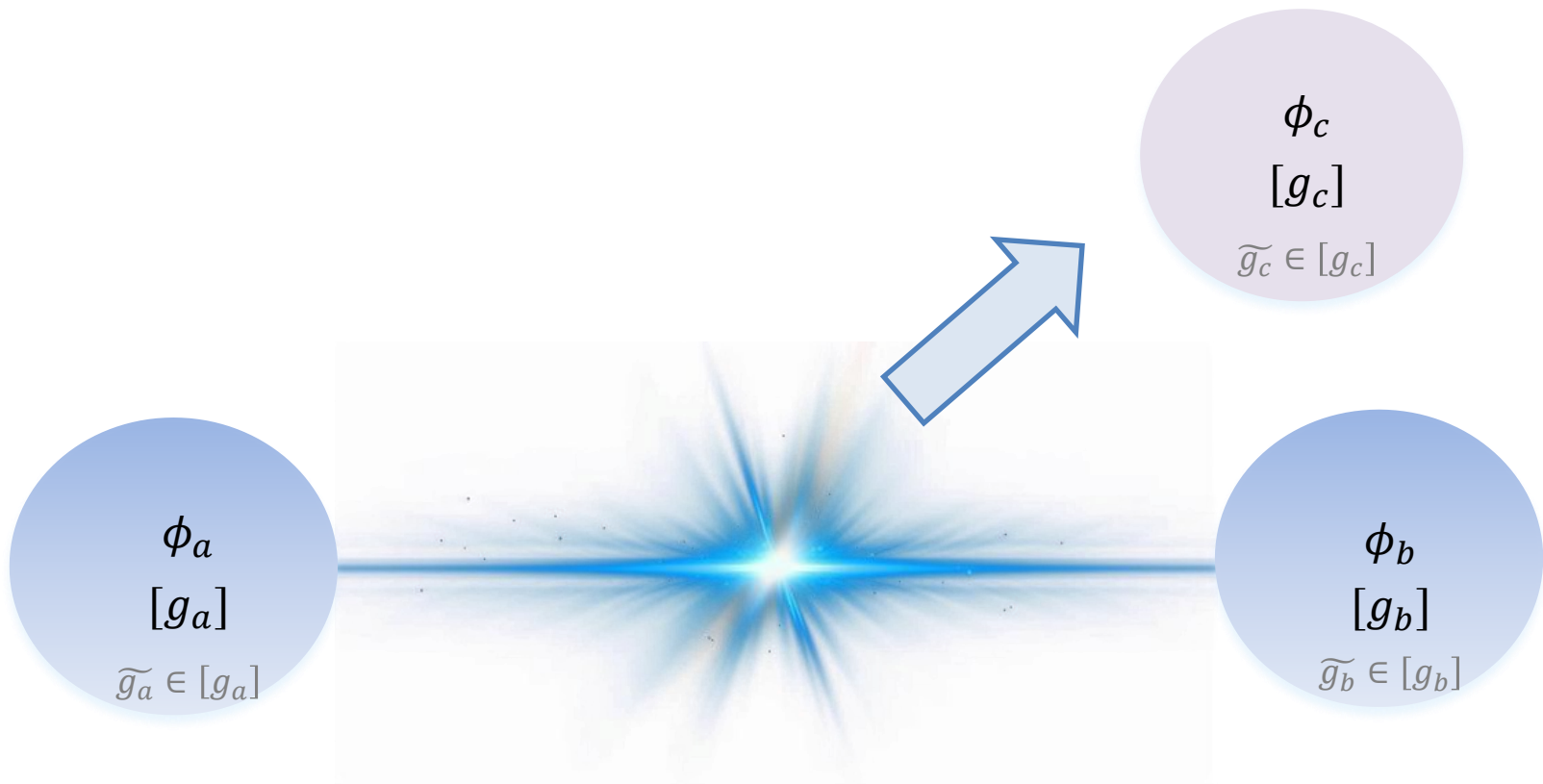
It was associated with conservation laws derived from group-theoretical symmetry

→ Selection rules based on **non-invertible symmetries** have been developing



Ex., when fields are labeled by a conjugacy class of a group G , i.e., $[g_a] = \{hg_a h^{-1} | h \in G\}$

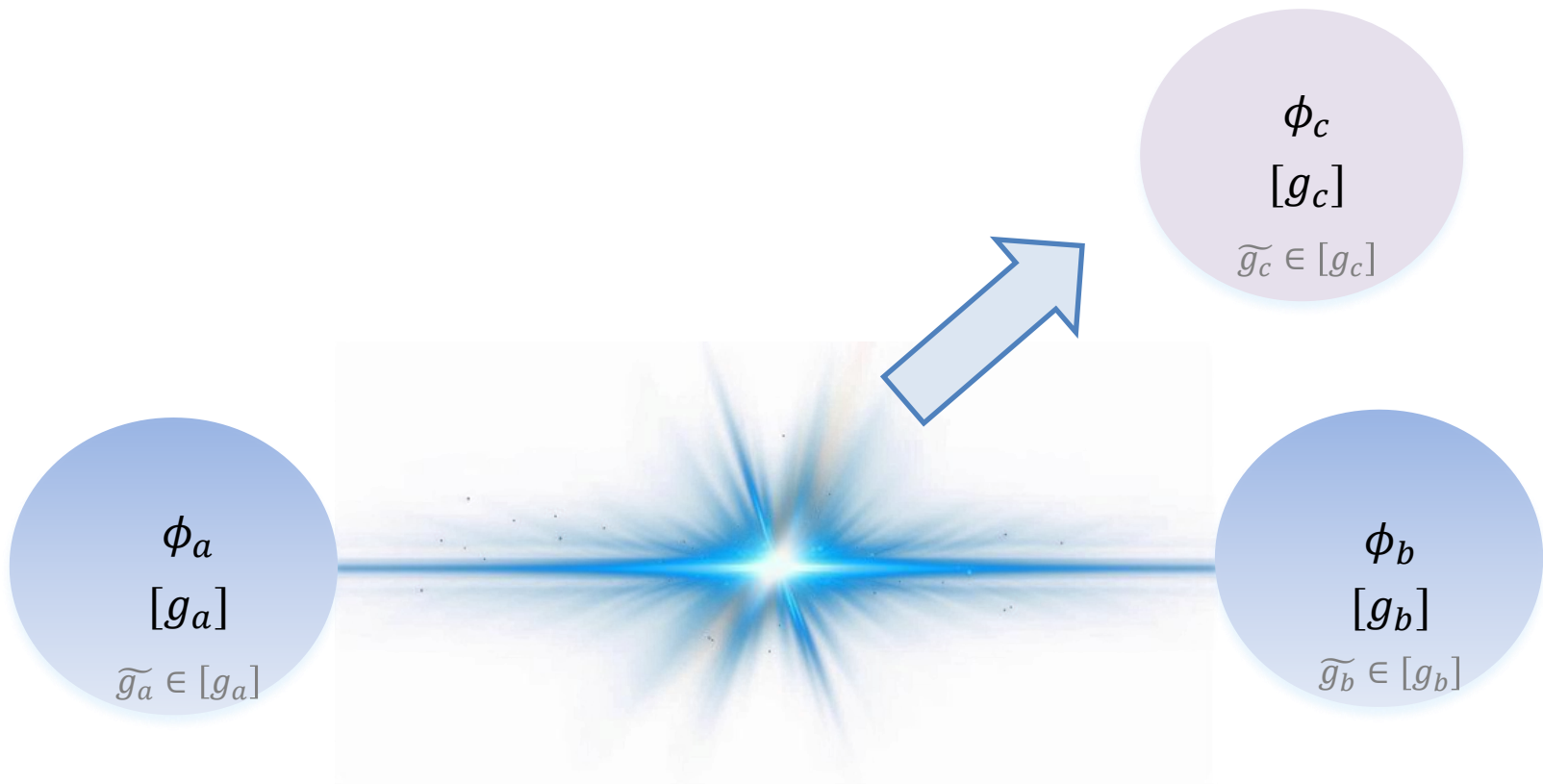
The tensor product of conjugacy classes are non-invertible, ex., $[g_a] \cdot [g_b] = \sum_c N_{ab}^c [g_c]$



Interactions $\phi_a\phi_b\phi_c$ are allowed when $\tilde{g}_a\tilde{g}_b\tilde{g}_c = e$ (identity)

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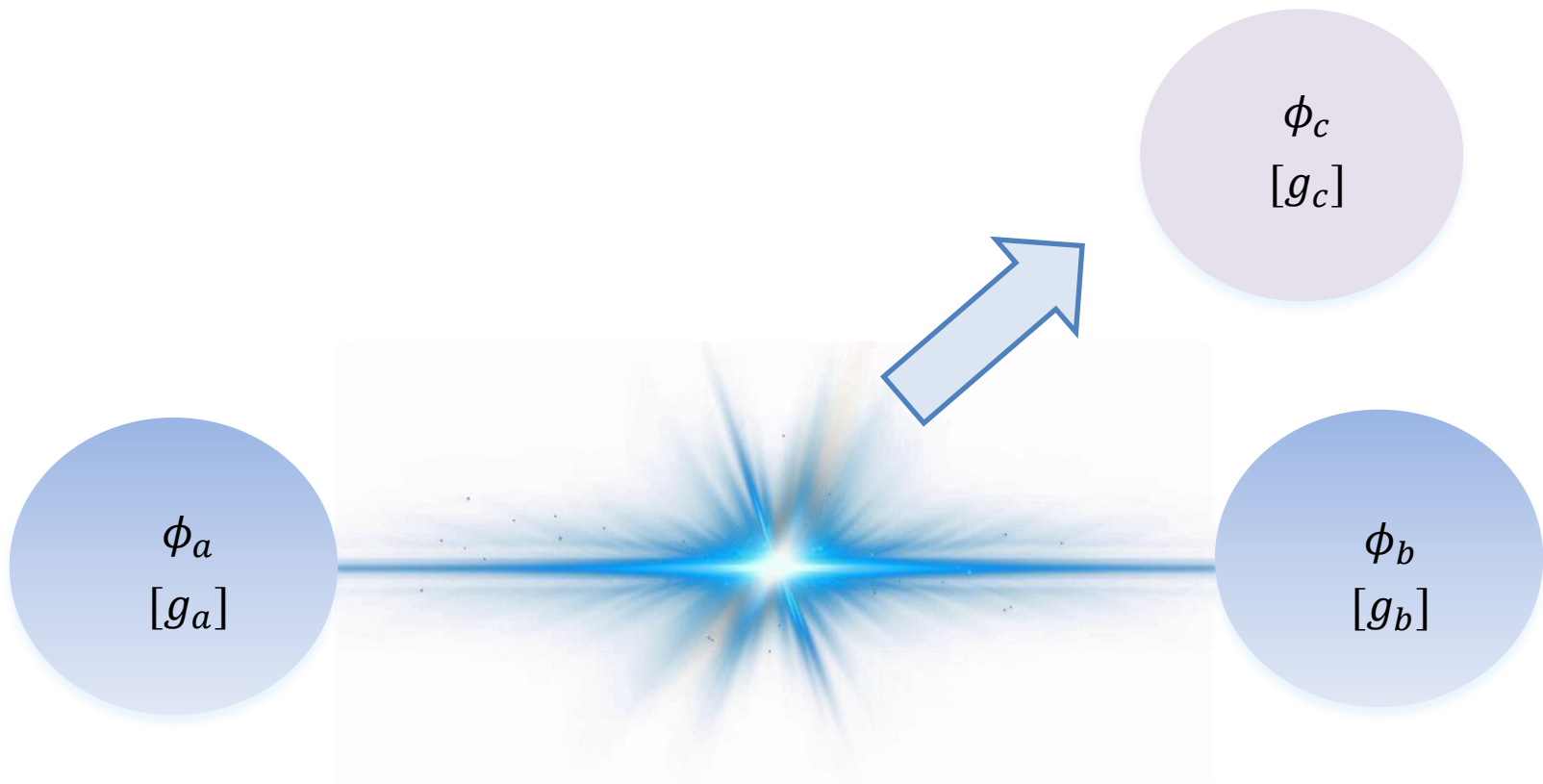


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Selection rules would be different from group-like ones since a field is labeled by a class



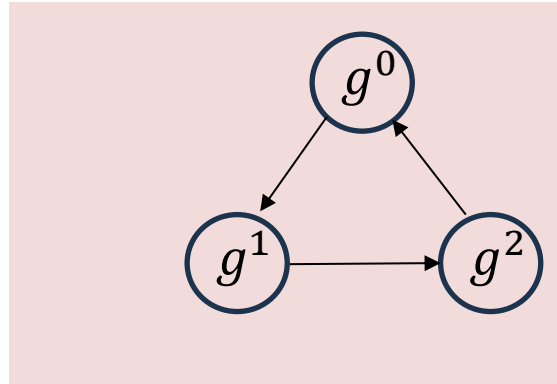
Main message :

1. Application of fusion rules ($[g_a] \cdot [g_b] = \sum_c N_{ab}^c [g_c]$)
→ Novel Yukawa textures and CP
2. String compactifications → non-invertible selection rules

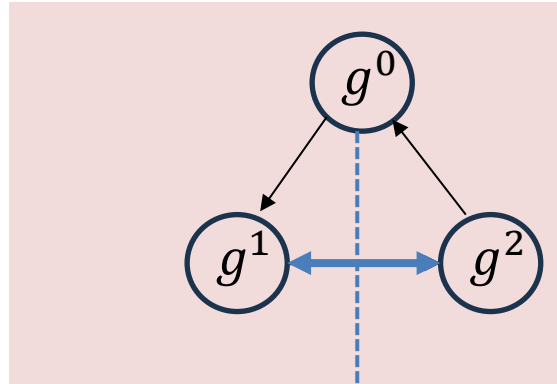
Non-invertible selection rules

arising from gauging outer automorphisms of a group

- Let us start with \mathbb{Z}_3 symmetry (generator : $g = e^{2\pi i/3}$)

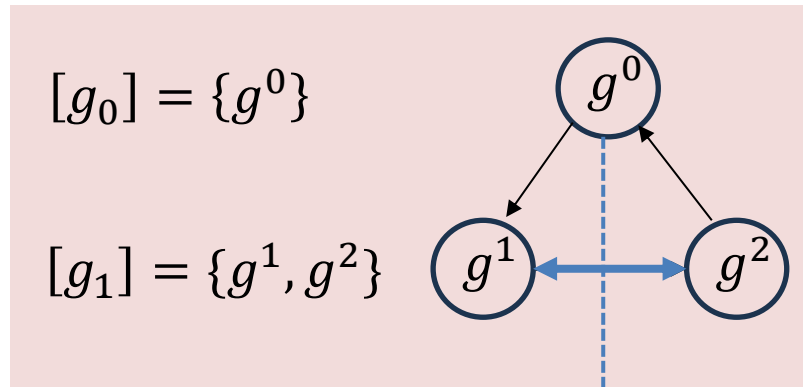


- Let us start with \mathbb{Z}_3 symmetry (generator : $g = e^{2\pi i/3}$)



\mathbb{Z}_2 gauging (automorphism of \mathbb{Z}_3)
(corresponding to the orbifold \mathbb{Z}_2 twist)

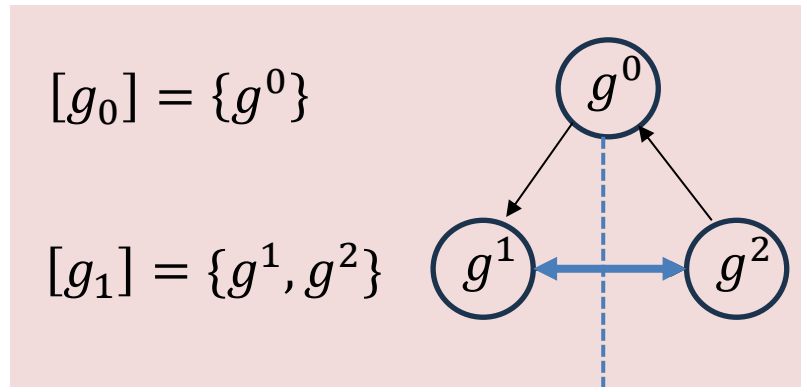
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\mathbb{Z}_2 gauging (automorphism of \mathbb{Z}_3)

$[g_k]$: \mathbb{Z}_2 invariant conjugacy class of $D_3 \cong \mathbb{Z}_3 \rtimes \mathbb{Z}_2$

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$[g_k]$: \mathbb{Z}_2 invariant conjugacy class of $D_3 \cong \mathbb{Z}_3 \rtimes \mathbb{Z}_2$

Fibonacci fusion rule :

$$[g_0] \cdot [g_0] = [g_0]$$

$$[g_0] \cdot [g_1] = [g_1]$$

$$[g_1] \cdot [g_1] = [g_0] \oplus [g_1]$$

$$\mathbb{I} \otimes \mathbb{I} = \mathbb{I}$$

$$\mathbb{I} \otimes \tau = \tau \otimes \mathbb{I} = \tau$$

$$\tau \otimes \tau = \mathbb{I} + \tau$$

- $[g_1]$ does not have the inverse

Fields Φ_i in 4D QFT :

Φ_i : labeled by the class $[g_i]$

$$\tilde{g}^i \in [g_i]$$

Selection rules :

An interaction in the classical Lagrangian

$$\mathcal{L} \supset \Phi_1 \cdots \Phi_n$$

is allowed when

$$\tilde{g}^1 \cdots \tilde{g}^n = \text{identity}$$

In standard group-theoretic frameworks, every field possesses a unique charge
A coupling is allowed iff the net charge vanishes

4D QFT with the Fibonacci fusion rule

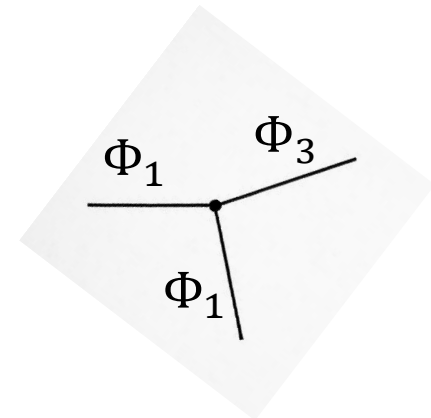
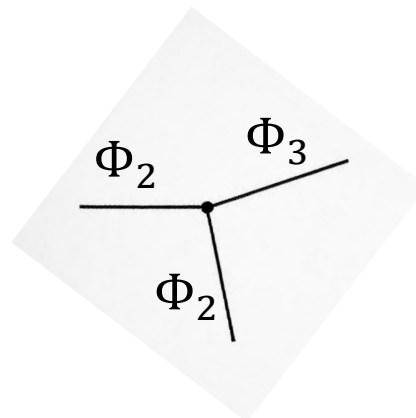
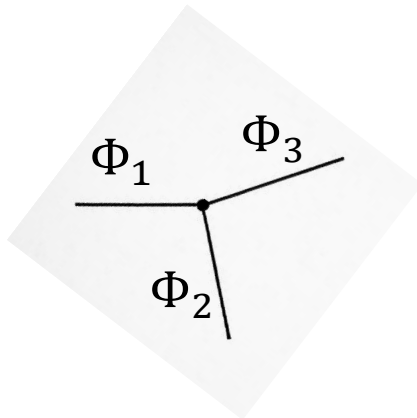
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- Suppose that fields are labeled by $\{\Phi_1, \Phi_2, \Phi_3\} = \{[g_0], [g_1], [g_1]\}$



4D QFT with the Fibonacci fusion rule

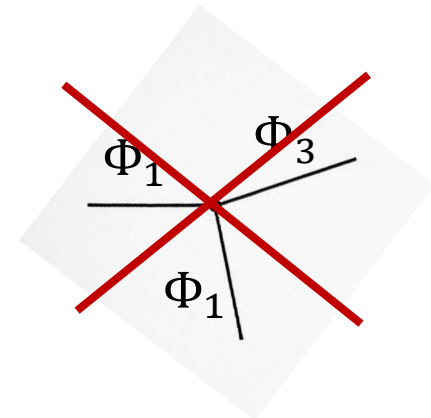
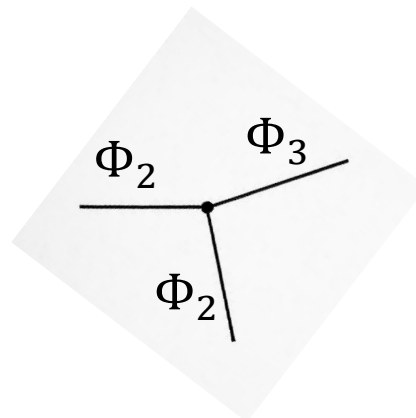
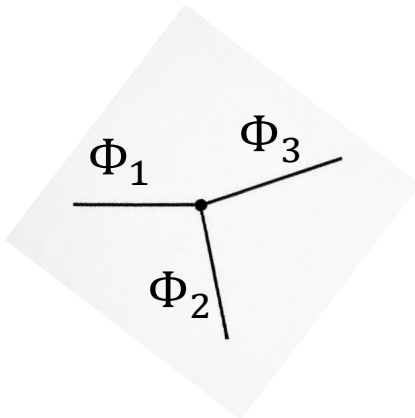
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- Suppose that fields are labeled by $\{\Phi_1, \Phi_2, \Phi_3\} = \{[g_0], [g_1], [g_1]\}$



2. Yukawa textures with two families



$$m_{\text{down}} = \begin{pmatrix} 0 & * \\ * & * \end{pmatrix}$$

S. Weinberg (79)

Texture zeros approach

Two generations of quarks

- In the basis in which the up-type quark mass matrix is diagonal,



$$m_{\text{down}} = \begin{pmatrix} 0 & m \\ m & M \end{pmatrix}$$

$$\xrightarrow{M \gg m} \begin{pmatrix} m^2/M & 0 \\ 0 & M \end{pmatrix} = \begin{pmatrix} m_d & 0 \\ 0 & m_s \end{pmatrix}$$

The Cabibbo angle is successfully predicted to be

$$\sin\theta_c \sim \tan\theta_c = m/M = \sqrt{m_d/m_s}$$

- Two generations of fermions $Y_{ij}\psi_i^L\psi_j^R H$ ($i,j=1,2$):

$Left : ([g_0], [g_1])$ $Right : ([g_0], [g_1])$ $Higgs : [g_1]$	$Y_{ij} = \begin{pmatrix} 0 & * \\ * & * \end{pmatrix}$	$* : \text{non-zero entries}$
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Fibonacci fusion rule

$$[g_0] \cdot [g_0] = [g_0]$$

$$[g_0] \cdot [g_1] = [g_1]$$

$$[g_1] \cdot [g_1] = [g_0] \oplus [g_1]$$

One cannot derive Weinberg texture from group-based symmetries, ex., $U(1)$ or \mathbb{Z}_M for any M

Yukawa textures with three families



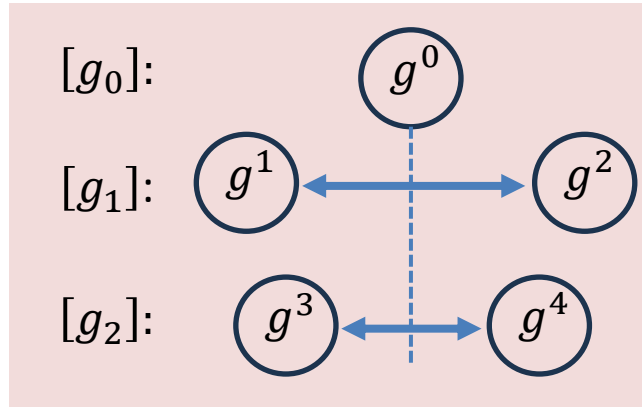
$$m^{(\text{Fritzsch,NNI})} = \begin{pmatrix} 0 & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix}$$

H. Fritzsch (78), G.C.Branco, L.Lavoura, F.Mota (89),...

$M = 5$ (\mathbb{Z}_2 gauging of \mathbb{Z}_5)

T. Kobayashi, [H.O.](#), M. Tanimoto, 2409.05270 [hep-ph]

- Three different classes :

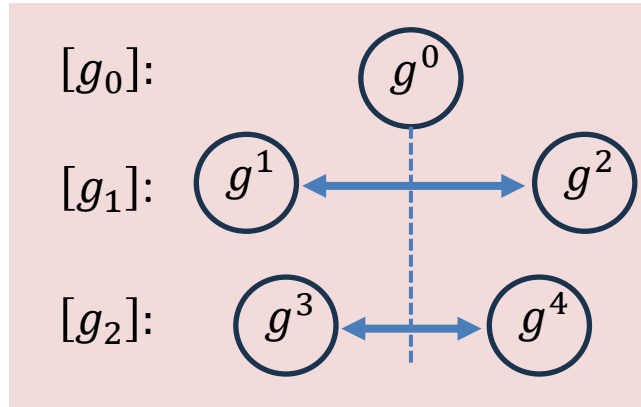


\mathbb{Z}_2 gauging (automorphism of \mathbb{Z}_5)

$M = 5$ (\mathbb{Z}_2 gauging of \mathbb{Z}_5)

T. Kobayashi, [H.O.](#), M. Tanimoto, 2409.05270 [hep-ph]

- Three different classes :



- Yukawa couplings (3 generations) :

Left : $([g_0], [g_1], [g_2])$
 Right : $([g_0], [g_1], [g_2])$
 Higgs : $[g_1]$

$$Y = \begin{pmatrix} 0 & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix}$$

* : non-zero entries

Fusion rules :

$$\begin{aligned} [g_0][g_0] &= [g_0] \\ [g_1][g_1] &= [g_0] + [g_2] \\ [g_2][g_2] &= [g_0] + [g_1] \\ [g_1][g_2] &= [g_1] + [g_2] \end{aligned}$$

“Nearest Neighbor Interaction”

Yukawa Textures for Leptons

- We classify the Yukawa textures of leptons with Weinberg op.:

$$Y_{ij}\bar{L}_i H e_j + \frac{C_{ij}}{\Lambda} L_i H L_j H$$

Ex.,

$$Y = \begin{pmatrix} 0 & 0 & * \\ 0 & * & * \\ * & * & * \end{pmatrix}, \begin{pmatrix} 0 & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix} \quad C = \begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix}$$

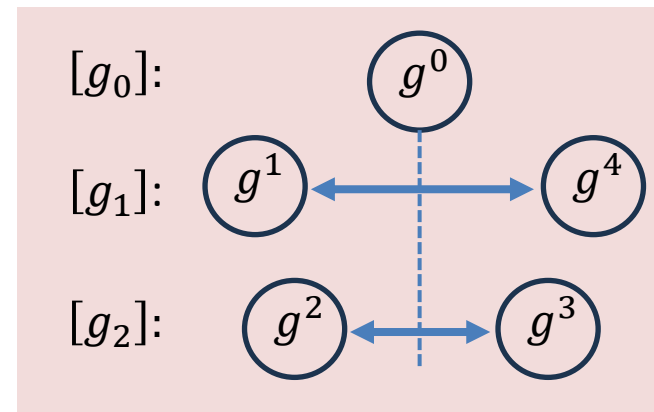
* : non-zero entries

- *Consistent with the charged lepton masses and PMNS matrix*
- *Other textures are discussed in*

3. CP from non-invertible selection rules

Group-like symmetries in non-invertible selection rules

Ex. \mathbb{Z}_2 gauging of \mathbb{Z}_5



Fusion rules :

$$[g_0][g_0] = [g_0]$$

$$[g_1][g_1] = [g_0] + [g_2]$$

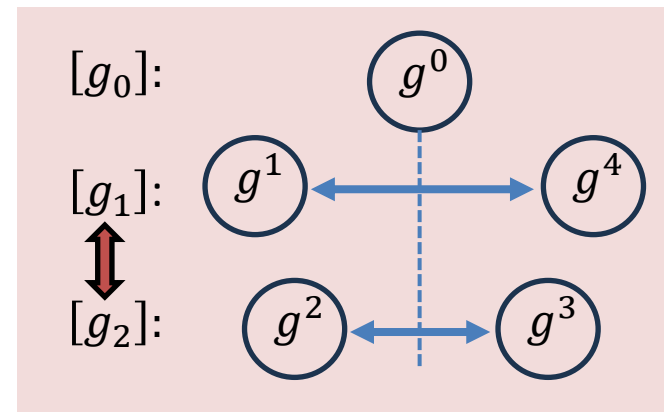
$$[g_2][g_2] = [g_0] + [g_1]$$

$$[g_1][g_2] = [g_1] + [g_2]$$

Group-like symmetries in non-invertible selection rules

Ex. \mathbb{Z}_2 gauging of \mathbb{Z}_5

\mathbb{Z}_2 symmetry : $[g_1] \leftrightarrow [g_2]$



Fusion rules :

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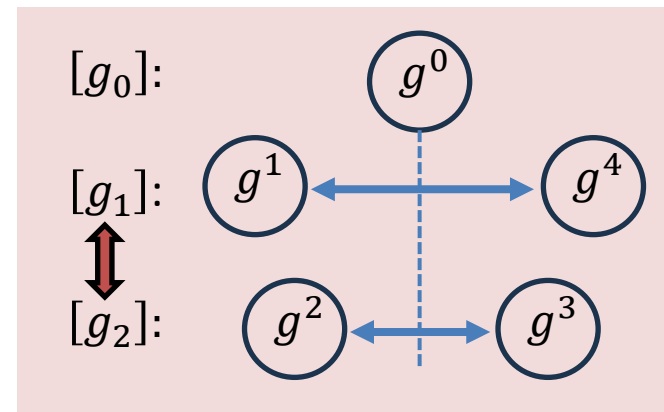
$$[g_1][g_2] = [g_1] + [g_2]$$

Group-like symmetries in non-invertible selection rules

Ex. \mathbb{Z}_2 gauging of \mathbb{Z}_5

\mathbb{Z}_2 symmetry : $[g_1] \leftrightarrow [g_2]$

\mathbb{Z}_2 flavor symmetry : $\Phi_1 \leftrightarrow \Phi_2$



Fusion rules :

$$[g_0][g_0] = [g_0]$$

$$[g_1][g_1] = [g_0] + [g_2]$$

$$[g_2][g_2] = [g_0] + [g_1]$$

$$[g_1][g_2] = [g_1] + [g_2]$$

Q : What is a class of Φ^* (the conjugate of Φ) ?

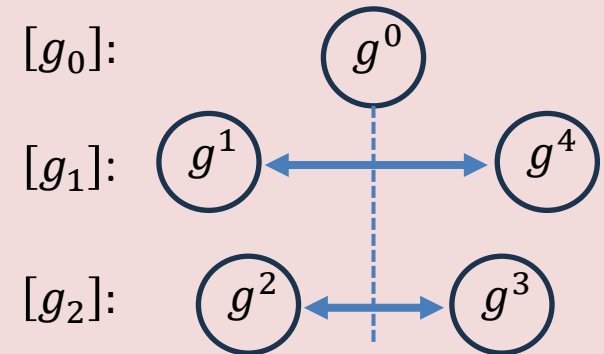
A : Φ^* is labeled by the inverse class $[g_i^{-1}]$

Ex. \mathbb{Z}_2 gauging of \mathbb{Z}_5

$$[g_1] = \{g^1, g^4\}$$

$$\begin{array}{l} \Downarrow \\ (g^1)^{-1} = g^4 \\ (g^4)^{-1} = g^1 \end{array}$$

$$[g_1^{-1}] = \{g^4, g^1\} = [g_1]$$

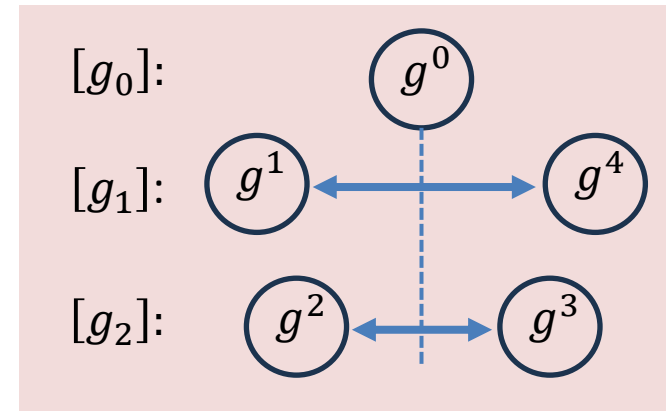


Ex. \mathbb{Z}_2 gauging of \mathbb{Z}_5

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$$\begin{array}{l} \downarrow \\ (g^1)^{-1} = g^4 \\ (g^4)^{-1} = g^1 \end{array}$$

$$[g_1^{-1}] = \{g^4, g^1\} = [g_1]$$



$$[g_i^{-1}] = [g_i] \text{ for } \mathbb{Z}_2 \text{ gauging of } \mathbb{Z}_N$$

- Φ and Φ^* are labeled by the same class

Q : Can we consider the different class between Φ and Φ^* ?

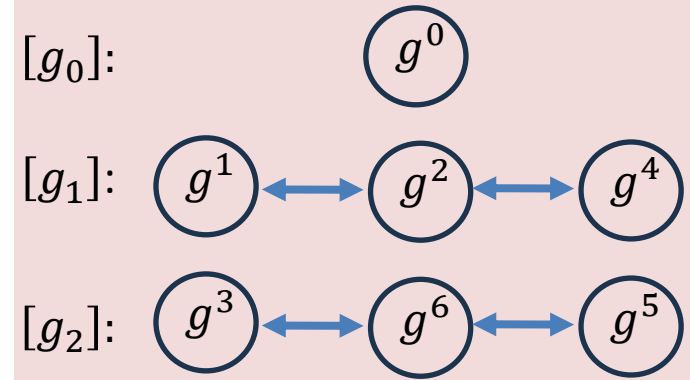
A : Different gauging of \mathbb{Z}_N

CP in non-invertible selection rules

T. Kobayashi, H.O., 2512.16376 [hep-ph]

Ex. \mathbb{Z}_3 gauging of \mathbb{Z}_7

$$g \equiv e^{2\pi i/7}$$



Fusion rules :

$$[g_0][g_0] = [g_0]$$

$$[g_1][g_1] = [g_1] + 2[g_2]$$

$$[g_2][g_2] = 2[g_1] + [g_2]$$

$$[g_1][g_2] = 3[g_0] + [g_1] + [g_2]$$

CP in non-invertible selection rules

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Ex. \mathbb{Z}_3 gauging of \mathbb{Z}_7

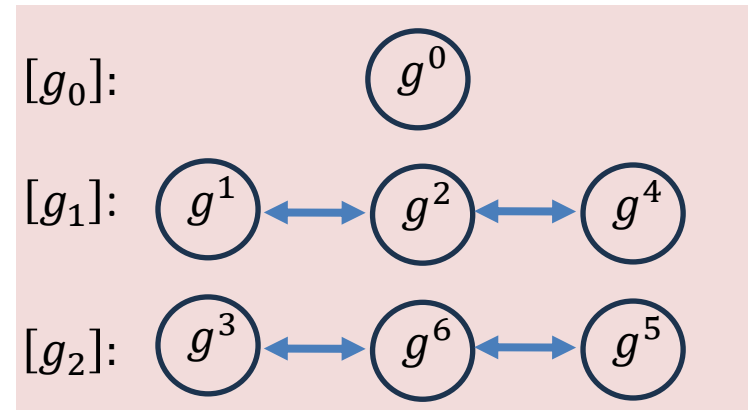
$$[g_1] = \{g^1, g^2, g^4\}$$



$$\begin{aligned} (g^1)^{-1} &= g^6 \\ (g^2)^{-1} &= g^5 \\ (g^4)^{-1} &= g^3 \end{aligned}$$

$$[g_1^{-1}] = \{g^6, g^5, g^3\} = [g_2]$$

$$g \equiv e^{2\pi i/7}$$



Fusion rules :

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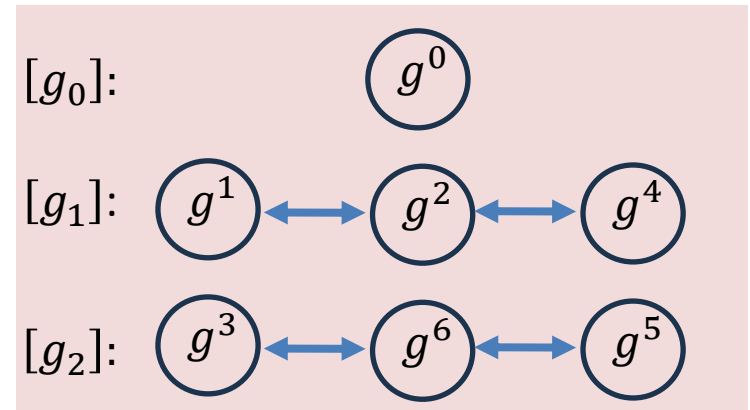
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- Φ and Φ^* are labeled by a different class, ex.,

$$\Phi : [g_1]$$

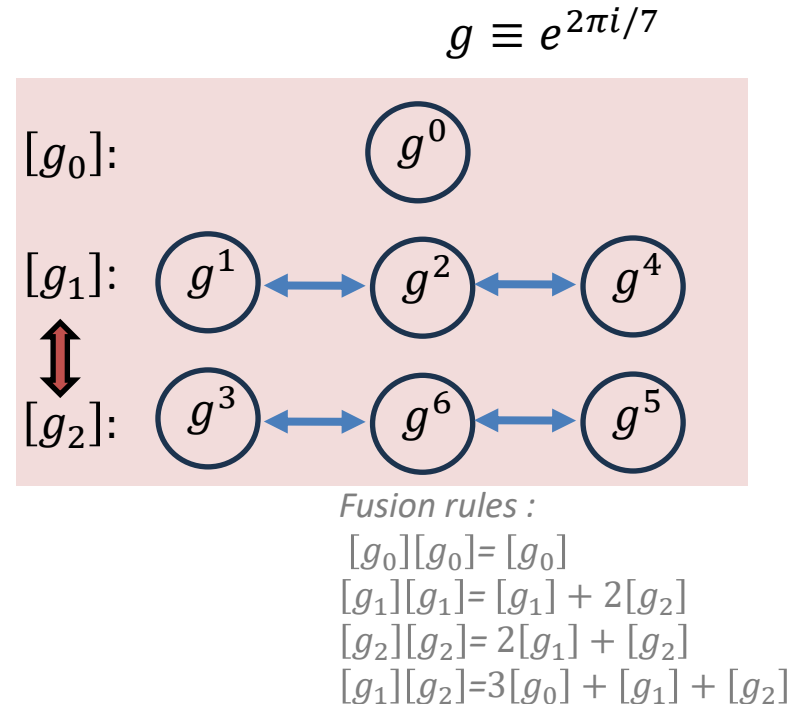
$$\Phi^* : [g_2]$$

CP in non-invertible selection rules

T. Kobayashi, H.O., 2512.16376 [hep-ph]

Ex. \mathbb{Z}_3 gauging of \mathbb{Z}_7

\mathbb{Z}_2 symmetry : $[g_1] \leftrightarrow [g_2]$



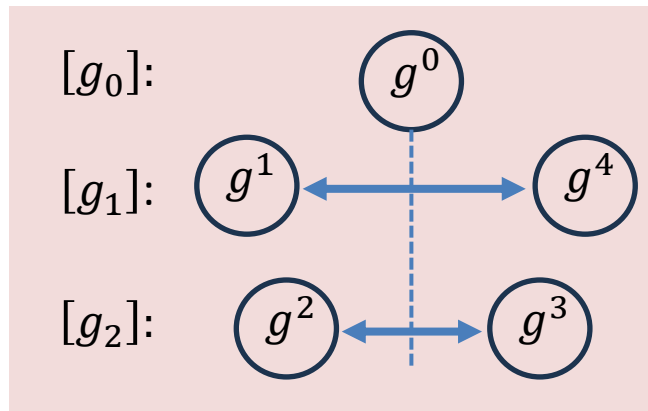
- Φ and Φ^* are labeled by a different class, ex.,
 $\Phi : [g_1]$
 $\Phi^* : [g_2]$
- CP transformation : $\Phi \leftrightarrow \Phi^*$

Flavor and CP in non-invertible selection rules

T. Kobayashi, H.O., 2512.16376 [hep-ph]

Combining two selection rule :

\mathbb{Z}_2 gauging of \mathbb{Z}_5



Yukawa Textures

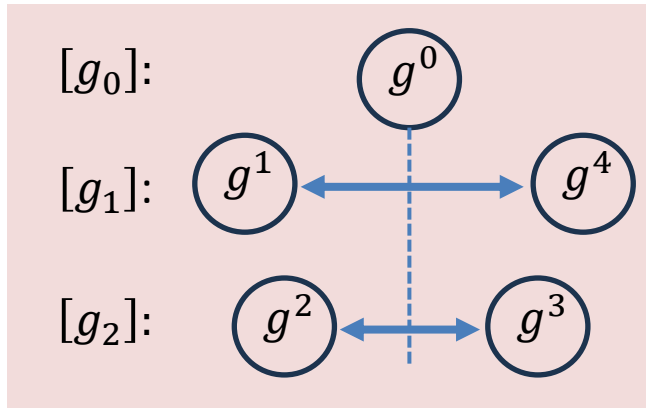
$$Y = \begin{pmatrix} 0 & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix}$$

Flavor and CP in non-invertible selection rules

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Combining two selection rule :

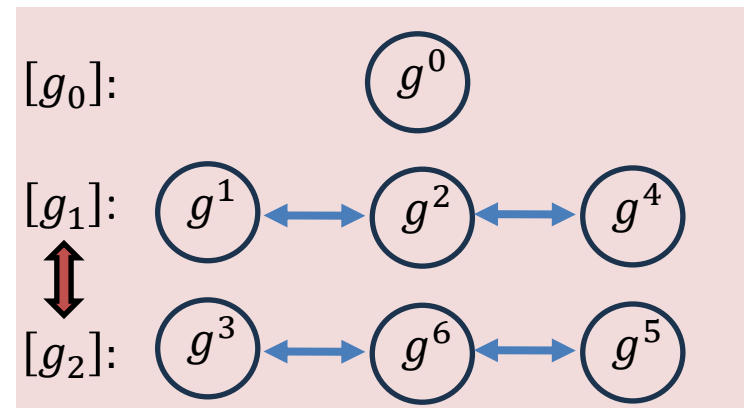
\mathbb{Z}_2 gauging of \mathbb{Z}_5



Yukawa Textures

$$Y = \begin{pmatrix} 0 & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix}$$

\mathbb{Z}_3 gauging of \mathbb{Z}_7



CP (real couplings)

$$Y = Y^*$$

Spontaneous CP violation

T. Kobayashi, H.O., 2512.16376 [hep-ph]

Ex. \mathbb{Z}_3 gauging of \mathbb{Z}_7

Φ : SM singlet field labeled by $[g_1]$

Fusion rules :

$$[g_0][g_0] = [g_0]$$

$$[g_1][g_1] = [g_1] + 2[g_2]$$

$$[g_2][g_2] = 2[g_1] + [g_2]$$

$$[g_1][g_2] = 3[g_0] + [g_1] + [g_2]$$

$$V = m^2 \Phi \Phi^* + \xi_1 \Phi \Phi^* (\Phi + \Phi^*) + \xi_2 (\Phi^3 + \Phi^{*3}) + \lambda_1 (\Phi \Phi^*)^2 + \lambda_2 \Phi \Phi^* (\Phi^2 + \Phi^{*2}) + \lambda_3 (\Phi^4 + \Phi^{*4}).$$

Spontaneous CP violation

T. Kobayashi, H.O., 2512.16376 [hep-ph]

Ex. \mathbb{Z}_3 gauging of \mathbb{Z}_7

Fusion rules :

$$[g_0][g_0] = [g_0]$$

$$[g_1][g_1] = [g_1] + 2[g_2]$$

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$$[g_1][g_2] = 3[g_0] + [g_1] + [g_2]$$

Φ : SM singlet field labeled by $[g_1]$

$$V = m^2 \Phi \Phi^* + \xi_1 \Phi \Phi^* (\Phi + \Phi^*) + \xi_2 (\Phi^3 + \Phi^{*3}) + \lambda_1 (\Phi \Phi^*)^2 + \lambda_2 \Phi \Phi^* (\Phi^2 + \Phi^{*2}) + \lambda_3 (\Phi^4 + \Phi^{*4}).$$

Spontaneous CP violation by Φ with $m^2, \xi_1 < 0, \xi_2, \lambda_i > 0$

$$\begin{aligned} \mathcal{L}_{\text{quark}} = & -(Y_u)_{ij} \bar{Q}_{L,i} H u_{R,j} - (Y_d)_{ij} \bar{Q}_{L,i} H^c d_{R,j} \\ & -(Y'_u)_{ij} \frac{\Phi}{\Lambda} \bar{Q}_{L,i} H u_{R,j} - (Y'_d)_{ij} \frac{\Phi}{\Lambda} \bar{Q}_{L,i} H^c d_{R,j} \end{aligned}$$

For some models, one can realize the same texture for $Y_{u,d}$ and $Y'_{u,d}$

4. Application to strong CP

Axion-less solution to the strong CP problem

- Physical strong CP phase :


$$\bar{\theta} = \theta + \text{argdet}[M_u M_d]$$

- To avoid the strong bound on the neutron EDM $|\bar{\theta}| < 10^{-11}$, we focus on *spontaneous CP violation*, providing an axionless solution to the strong CP problem (as in Nelson–Barr mechanism)

- i) CP is an exact symmetry at UV, i.e, $\theta = 0$ and all real couplings
- ii) Nonvanishing CKM phase with $\text{argdet}[M_u M_d] = 0$?

Corrections to $\bar{\theta} = 0$

T. Kobayashi, H.O., M. Tanimoto, T.T. Yanagida, 2510.01680[hep-ph]

- 
- GUT-inspired SM with two Higgs doublets
(controlled by non-invertible selection rules and CP)
 - Λ_{CP} Spontaneous CP breaking
 - m_{H_2} Heavy Higgs mass scale
 - EW SM corrections are negligible, i.e., $\bar{\theta} \sim 10^{-18}$ at four loops

Ellis, Gaillard, NPB **150** (1979) 141

Khriplovich, PLB **173** (1986) 193

Axion-less solution to the strong CP problem

T. Kobayashi, H.O., M. Tanimoto, T.T. Yanagida, 2510.01680[hep-ph]

- Consider a GUT–inspired SM with non–invertible selection rules
(all fermions belong to GUT rep.)

	Fermions			Scalars	
	T	\bar{F}	H_1	H_2	η
$SU(5) \times U(1)_{B-L}$	$(\mathbf{10}, -1)$	$(\bar{\mathbf{5}}, 3)$	$(\bar{\mathbf{5}}, -2)$	$(\bar{\mathbf{5}}, -2) + (\mathbf{45}, -2)$	$(\mathbf{1}, 0)$
$\tilde{Z}_5^{(1)}$	$([g^0], [g^1], [g^2])$	$([g^0], [g^1], [g^2])$	$[g^1]$	$[g^2]$	$[g^2]$
$\tilde{Z}_5^{(2)}$	$([g^0], [g^1], [g^2])$	$([g^0], [g^1], [g^2])$	$[g^1]$	$[g^0]$	$[g^1]$

- Yukawa interactoins ($\langle H_1 \rangle$:real, $\langle H_2 \rangle$:complex):

$$f_u T T H_1^\dagger + f_d T \bar{F} H_1 \quad f'_u T T H_2^\dagger + f'_d T \bar{F} H_2$$

$$f_{u,d} = \begin{pmatrix} 0 & \checkmark & 0 \\ \checkmark & 0 & \checkmark \\ 0 & \checkmark & \checkmark \end{pmatrix} \quad f'_{u,d} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \checkmark & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \Rightarrow \quad M'_{u,d} = \begin{pmatrix} 0 & a_{u,d} & 0 \\ a'_{u,d} & b_{u,d} e^{i\phi_{u,d}} & c_{u,d} \\ 0 & c'_{u,d} & d_{u,d} \end{pmatrix}$$

$\rightarrow \text{argdet}[M_u M_d] = 0$ and Realistic CKM phase

Axion-less solution to the strong CP problem

T. Kobayashi, H.O., M. Tanimoto, T.T. Yanagida, 2510.01680[hep-ph]

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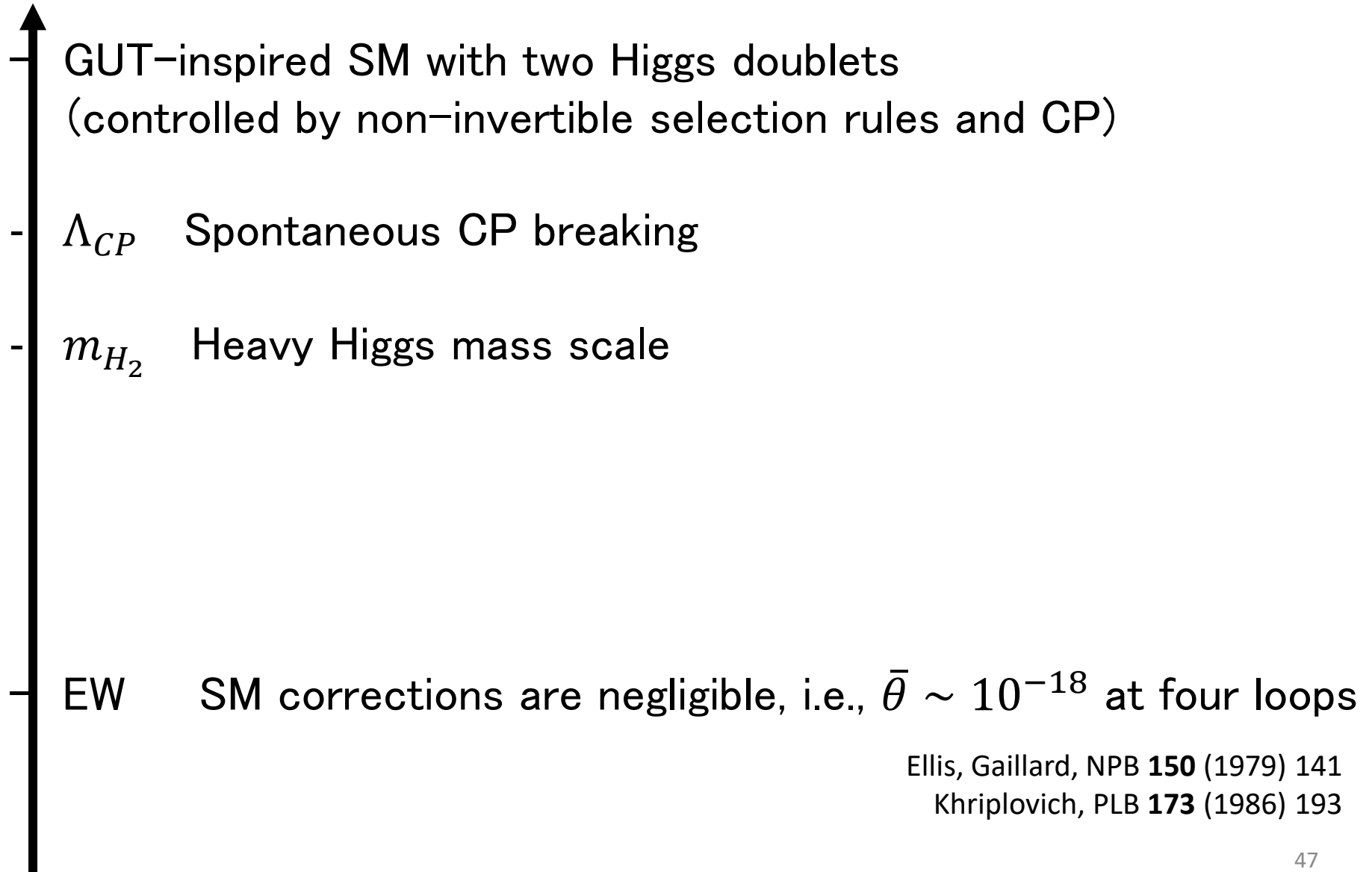
- CP phase is induced from CP violating sector (η):

$$V_\eta = -\mu\eta\eta^\dagger - \mu'(\eta^2 + \eta^{\dagger 2}) + \xi(\eta\eta^\dagger)^2 + \xi'(\eta^4 + \eta^{\dagger 4}) + \xi'_{33}(\eta^2 + \eta^{\dagger 2})\eta\eta^\dagger$$

$$V_{H-\eta} = \mu_{12}(H_1^\dagger H_2 \langle \eta \rangle + \text{h.c.}) + \mu_{21}(H_1^\dagger H_2 \langle \eta^\dagger \rangle + \text{h.c.})$$

Corrections to $\bar{\theta} = 0$

T. Kobayashi, H.O., M. Tanimoto, T.T. Yanagida, 2510.01680[hep-ph]

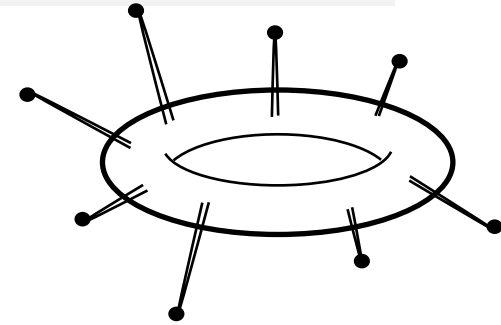


5. String compactifications → Non-invertible selection rules

- i) Heterotic string on toroidal orbifolds (2 slide)
- ii) Type IIB magnetized D-brane models on toroidal orbifolds

Heterotic string on toroidal orbifolds (1/2)

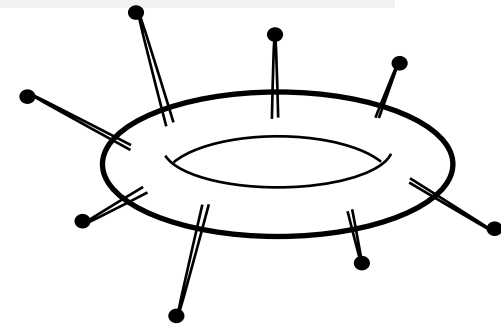
- Orbifold geometry requires for a string to be closed up to space group action $h = (\theta, v) \in S$



θ : Point group action
 v : Lattice translation

Heterotic string on toroidal orbifolds (1/2)

- Orbifold geometry requires for a string to be closed up to space group action $h = (\theta, v) \in S$
- Consider a complexified worldsheet boson Z

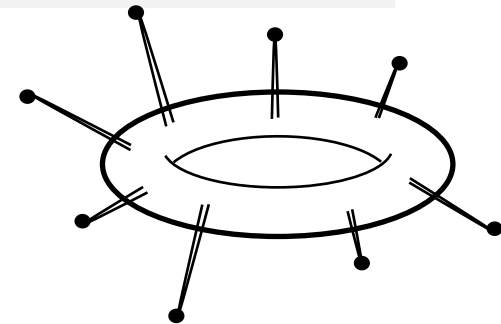


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$$Z(\sigma + \pi) = hZ(\sigma) = \theta Z + v$$

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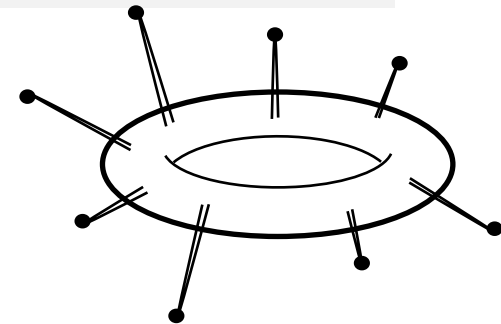
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- Consider another string gZ satisfying the same B.C. ($g \in S$)

$$gZ(\sigma + \pi) = hgZ(\sigma)$$

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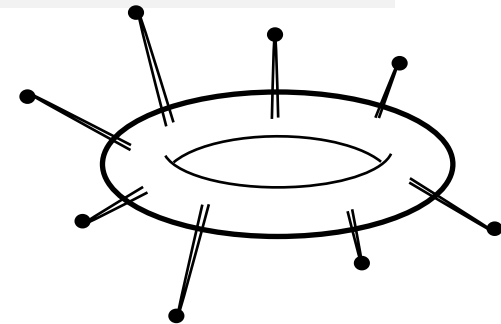
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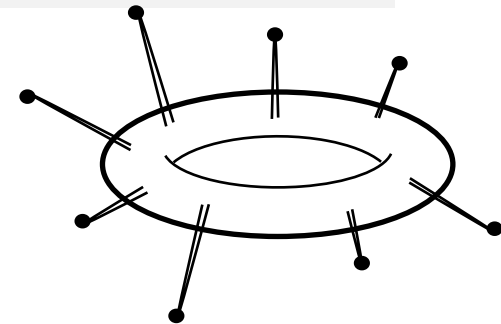
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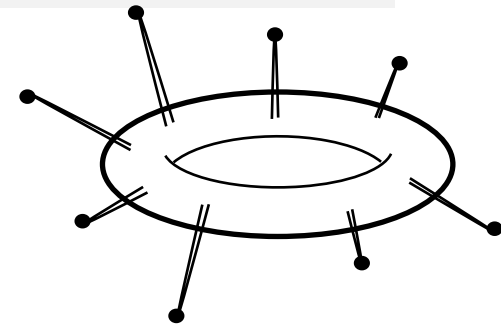
$$gZ(\sigma + \pi) = hgZ(\sigma)$$

$$\rightarrow Z(\sigma + \pi) = (g^{-1}hg)Z(\sigma)$$

- The Hilbert space is specified by

$$\mathcal{H}_{[h]} = \{Z \mid Z(\sigma + \pi) = hZ(\sigma), h \sim g^{-1}hg, \quad g, h \in S\}$$

Heterotic string on toroidal orbifolds (2/2)



- Twisted string is labeled by conjugacy group of the space group S :

$$\mathcal{H}_{[h]} = \{Z \mid Z(\sigma + \pi) = hZ(\sigma), h \sim g^{-1}hg, \quad g, h \in S\}$$

- Selection rules of twisted string :

$$\text{Fusion : } [g_i] \cdot [g_j] = \sum_k d_{ij}^k [g_k]$$

→ Non-invertible selection rules on (non) Abelian orbifolds

(\mathbb{Z}_N, S_3, T_7 orbifolds by T. Kobayashi, R. Nishida, H.O., 2509.10019[hep-th])

5. String compactifications → Non-invertible selection rules

- i) Heterotic string on toroidal orbifolds (2 slide)
- ii) Type IIB magnetized D-brane models on toroidal orbifolds (1 slide)

Overview

T. Kobayashi, [H.O.](#), 2408.13984 [hep-th]

(i) \mathbb{Z}_N symmetry :

$$\phi^j \rightarrow g^{kj} \phi^j$$

(ii) Gauging \mathbb{Z}_2 (the automorphism of \mathbb{Z}_N),
i.e., $D_N \cong \mathbb{Z}_N \rtimes \mathbb{Z}_2$

$$[g_k] = \{h g^k h^{-1} \mid h = e, r\}$$

$[g_k]$ includes g^k and $g^{-k} = g^{M-k}$

(iii) \mathbb{Z}_2 invariant modes labeled by $[g^k]$

$$\Phi_j = \phi_j + \phi_{M-j}$$

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T. Kobayashi, [H.O.](#), 2408.13984 [hep-th]

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Symmetry of chiral zero modes
on higher-dimensional Super Yang-
Mills on T^2 with magnetic fluxes
(EFT of type IIB Magnetized D-brane models)

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T. Kobayashi, [H.O.](#), 2408.13984 [hep-th]

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Non-invertible symmetries

→ Flavor symmetries of chiral zero modes (quarks/leptons)

Symmetry of chiral zero modes
on higher-dimensional Super Yang-
Mills on T^2 with magnetic fluxes
(EFT of type IIB Magnetized D-brane models)

T^2/\mathbb{Z}_2 orbifold with gauged \mathbb{Z}_2
(leave all operators invariant
under \mathbb{Z}_2)

Fusion rules of momentum ops.:

$$\widehat{U}_i^{(\lambda)} \widehat{U}_i^{(\lambda')} = \widehat{U}_i^{(\lambda+\lambda')} + \widehat{U}_i^{(\lambda-\lambda')}$$

$$i = 1, 2$$

\mathbb{Z}_2 invariant modes on T^2/\mathbb{Z}_2

Conclusions

Main message : Fusion rules : $[g_a] \cdot [g_b] = \sum_c N_{ab}^c [g_c]$
→ Novel Yukawa textures and CP

- leading to the non-trivial structure of mass matrices including



$$m_{\text{down}} = \begin{pmatrix} 0 & * \\ * & * \end{pmatrix}$$

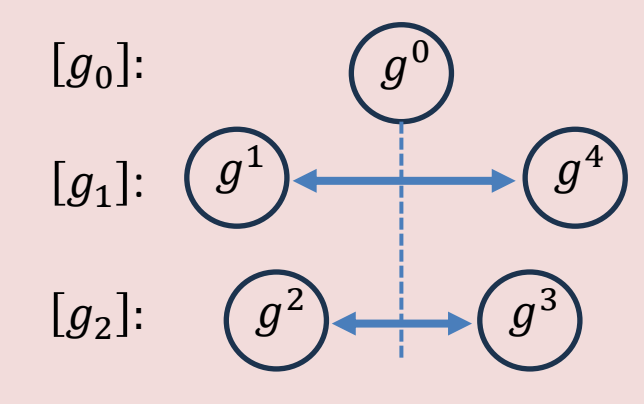


$$m^{(\text{Fritzsch, NNI})} = \begin{pmatrix} 0 & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix}$$

Conclusions

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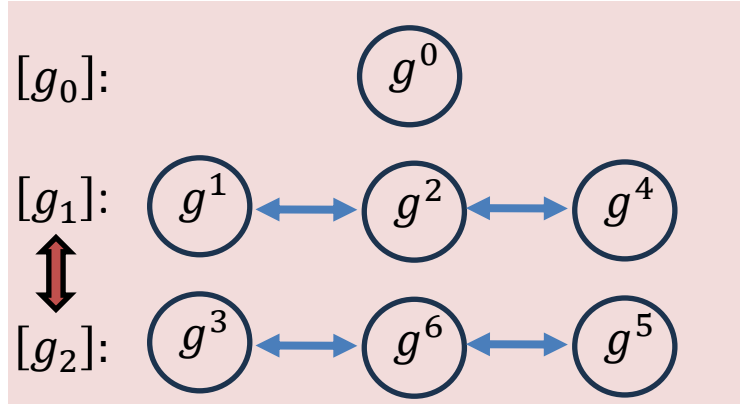
\mathbb{Z}_2 gauging of \mathbb{Z}_5



Yukawa Textures

$$Y = \begin{pmatrix} 0 & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix}$$

\mathbb{Z}_3 gauging of \mathbb{Z}_7



CP (real couplings)

$$Y = Y^*$$

1. Origin of non-invertible selection rules

- Heterotic string on toroidal orbifolds and Calabi-Yau threefolds
(Twisted states are labeled by conjugacy classes)

T. Kobayashi, [H.O.](#), R. Nishida, 2504.09773, 2509.10019

- Type IIB magnetized D-brane models on toroidal orbifolds

T. Kobayashi, [H.O.](#), 2408.13984 [hep-th]

2. Other non-invertible selection rules ?

- Selection rules of conjugacy classes are summarized in

J. Dong, T. Jeric, T. Kobayashi, R. Nishida, and [H.O.](#), 2507.02375 [hep-th]

3. Other phenomenological applications ?

- SUSY flavor
- Radiative seesaw models
- GUT-like models

Next Talk by Y. Shigekami

T. Kobayashi, [H.O.](#), T. Nomura, H. Okada, O. Popov, Y. Shigekami,.. ('25)

Q. Liang, T. T. Yanagida, T. Kobayashi, H.O., ...

⁶²
Thank you!