Flavor hierarchies and the Strong CP

Based on: JHEP 04 (2025) 170

New perspectives on flavor and symmetries in particle physics

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Outline

- Flavour Problem
- Radiative mass generation mechanism
- RMGM and the Strong CP
- Summary

The Flavor problem

Recent reviews: Feruglio (2015), Altmannshofer et al.(2025)

- Large hierarchies in fermion mass spectrum

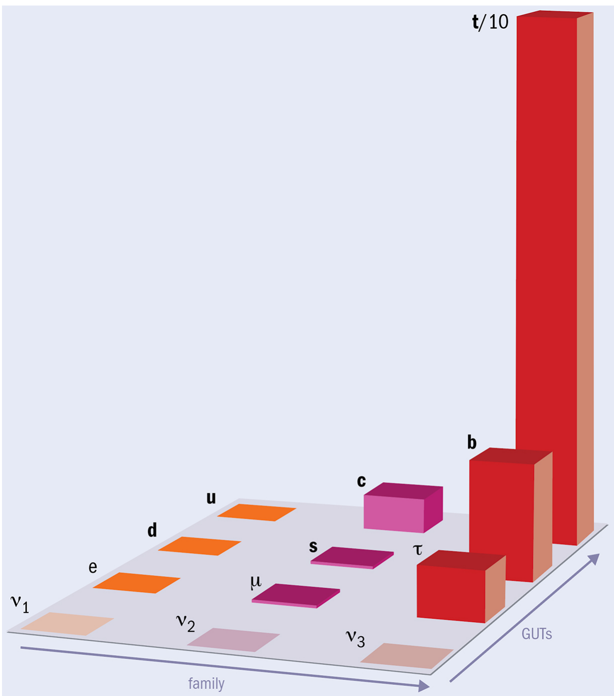


Image credit: CERN

In the SM, the masses varies $10^{-6} \sim 1$ in EW scale

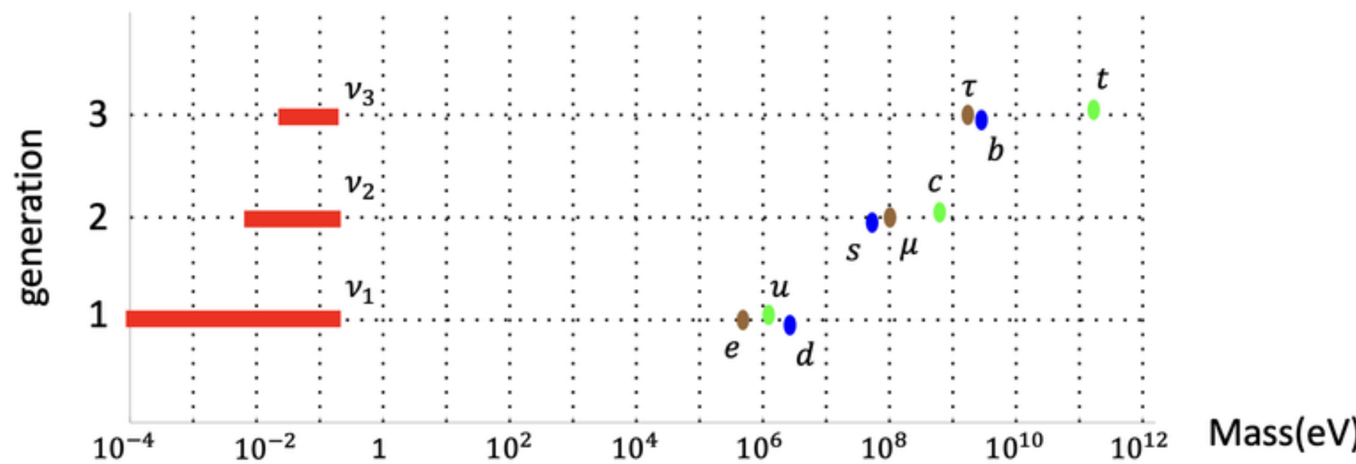


Image credit: Feruglio (2015)

- The incalculable parameters

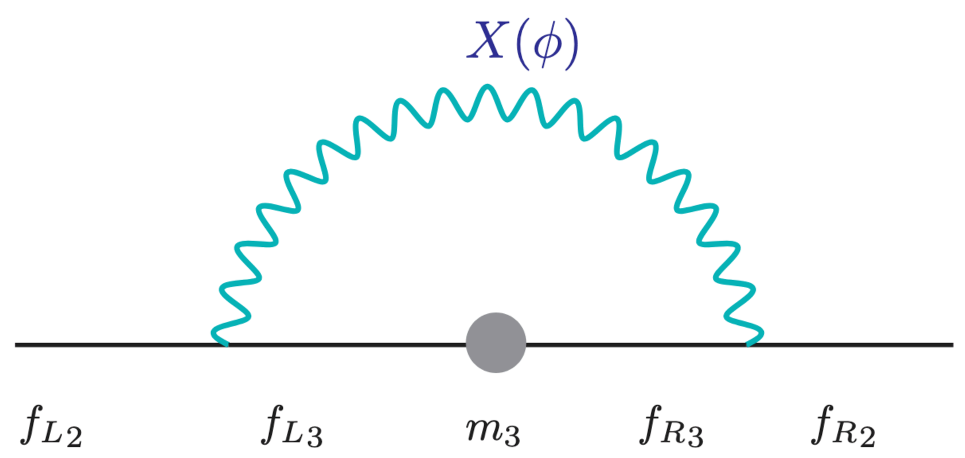
$$-\mathscr{L}_y = Y_i^u \bar{Q}_{Li}\tilde{H}u_{Ri} + Y_{ij}^d \bar{Q}_{Li}Hd_{Rj} + Y_i^e \bar{L}_{Li}He_{Ri} + h.c$$

  - 19 unknown real parameters in the unbroken phase

    In 2506.06423, some of the Yukawa are constrained through inequalities
  - In the limit $Y = 0$, the action has enhanced global symmetry $[U(3)]^5$: masses are Technical natural parameters
  - Approximate flavor symmetries implies, existence of UV physics

Possible Explanation

- The well known theories of flavor: FN mechanism, Extra-dimensions, Clockwork mechanism etc, explains the flavor hierarchies, but not the computability
- The radiative mass generation mechanism: Mass generation through quantum corrections can explain both of these issues



- At the tree-level only third generation fermions are massive, fermion self-energy corrections induces mass for light fermions
- Loop suppression $\frac{1}{16\pi^2}$ explains the intergenerational hierarchy; Masses becomes partially computable parameters

Weinberg (1972), Georgi et al. (1973), Mohapatra (1974), Zee & Barr (1978).. Balakrishna & Mohapatra (1988),....

- Flavor violating couplings with BSM particles X or ϕ are necessary

Weinberg (2020), Jana et al. (2022,2024), Mohanta & Patel (2022,2023,2024,2025), Bonila et al.(2023)...

Radiative mechanism

Rank 1 : SM fermions + VLF

$$\mathscr{L}_m \supset \mu_{Li}\,\bar{f}_{Li}F_R + \mu_{Ri}\,\bar{F}_L f_{Ri} + m_F\,\bar{F}_L F_R + h.c$$

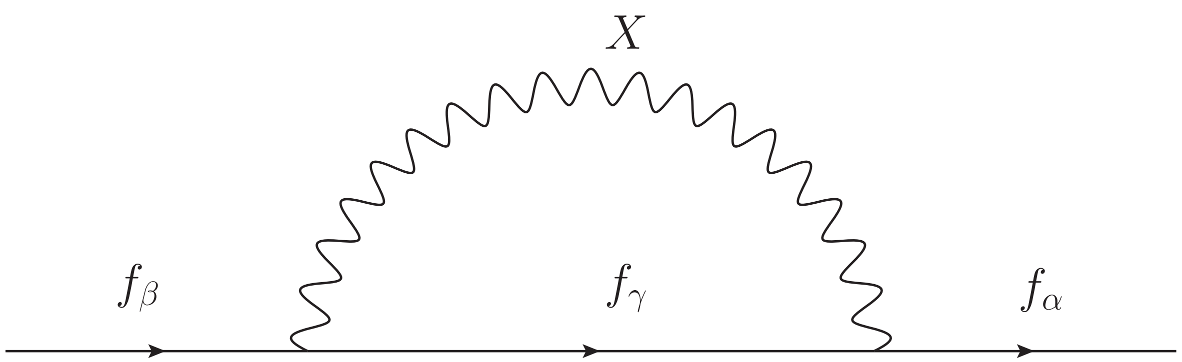
$$\mathscr{M} = \begin{pmatrix} 0_{3\times 3} & \mu_L \\ \mu_R & M_F \end{pmatrix}$$

$$\Longrightarrow \quad M^{(0)}_{ij} = -\frac{\mu_{Li}\mu_{Rj}}{M_F};$$

New FC interactions

$$\mathscr{L}_X \supset g_X X_\mu \left(q_{L\alpha}\,\bar{f}_{L\alpha}\gamma^\mu f_{L\alpha} + q_{R\alpha}\,\bar{f}_{R\alpha}\gamma^\mu f_{R\alpha}\right),$$

with flavor non-universal $q_{L,R}$



$$(\delta M)_{ij} = \frac{{g_X}^2}{4\pi^2}\, q_{Li}\, M^{(0)}_{ij} q_{Rj} \left(B_0[M_X, m_3] - B_0[M_X, m_F]\right)$$

$$B_0[M,m] = \Delta_\epsilon + 1 - \frac{M^2 \ln\frac{M^2}{\mu^2} - m^2 \ln\frac{m^2}{\mu^2}}{M^2 - m^2}$$

$$M^{(1)}_{ij} = M^{(0)}_{ij}\left(1 + C\, q_{Li}q_{Rj} + C_1\, q_{Li}q_{R4} + C_2\, q_{L4}q_{Rj} + C\,C_1C_2\, q_{Li}q_{Rj}\, q_{L4}q_{R4}\right)$$

- For Abelian gauge extensions, det. $M^{(1)}$= 0. One of the state remains massless; Higher order corrections may induce it's mass.
- The loop-induced corrections are sensitive only to the relative hierarchy among the new-physics scales, rather than to their absolute values.

The Strong CP puzzle

- CP violation in Strong sector $\Delta\mathscr{L}_{\text{CPV}} = \frac{\theta_{QCD}\, g_s^{\,2}}{32\pi^2}\,\epsilon^{\mu\nu\rho\sigma}\,G^a_{\mu\nu}G^a_{\rho\sigma} + m_q e^{i\theta_q\gamma_5}\,\bar{q}_L q_R$

- The physical strong CP parameter is $\bar{\theta} = \theta_{QCD} + Arg(Det(\mathscr{M}_u \mathscr{M}_d))$

From Neutron EDM: $\bar{\theta} \leq 10^{-10}$ **"Strong CP problem"**

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Popular solutions:

- Massless quark
- Pecci-Quinn Axion
- Symmetry based solutions
  - CP symmetry — Nelson-Barr mechanism (1984)
  - P symmetry — Babu, Mohapatra: PRL(1989)

- In Kaplan, Melia and Rajendran (2025) it is shown that only dynamical solutions solve the strong CP problem.

- Joshua et al. (2025) showed gauging P/CP solves the puzzle.

Radiative mechanism and Strong CP

- In radiative mass mechanism in an Abelian extension of the SM
  - $M^{(0)}$ and $M^{(1)}$ has vanishing determinant. Massless quark solution holds.
  - $\det M^{(2)}$ is not real, and induces large $\bar{\theta}$.

Doesn't solve the strong CP problem of the SM

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Doesn't solve the strong CP problem of the SM

- **Parity Solutions/ Babu-Mohapatra solutions:** LRSM with parity invariance

$$\bar{f}_L \, \mathscr{M} \, f_R = \begin{pmatrix} \bar{u}_L & \bar{T}_L \end{pmatrix} \begin{pmatrix} 0_{3\times 3} & y v_L \\ y^\dagger v_R & M_T \end{pmatrix} \begin{pmatrix} u_R \\ T_R \end{pmatrix}$$

Parity invariance

- Parity sets $\bar{\theta}_{QCD} = 0$
- VEVs are real.
- Determinant is real. "Hermitian Type" matrix

$$\bar{\theta}_{\text{tree}} = Arg\,.\,\text{Det}\,.\,(\mathscr{M}) = 0$$

- $\bar{\theta}$ appears at two loop level and $\bar{\theta} < 10^{-10}$
- Fermion masses are generated through universal seesaw mechanism. Yukawa coupling hierarchy reduces to 10^{-3} to 1.

Babu, Mohapatra P.R.L(1989)

Parity invariant LRSM with Radiative masses

$U(1)_{2-3}$ Model

Particles	$\mathcal{G}_{LRSM}$	G_F
$Q_{Li} = \begin{pmatrix} u \\ d \end{pmatrix}_{Li}$	$(3,2,1,\frac{1}{3})$	$\{0,1,-1\}$
$Q_{Ri} = \begin{pmatrix} u \\ d \end{pmatrix}_{Ri}$	$(3,1,2,\frac{1}{3})$	$\{0,1,-1\}$
$L_{Li} = \begin{pmatrix} \nu \\ e \end{pmatrix}_{Li}$	$(1,2,1,-1)$	$\{0,1,-1\}$
$L_{Ri} = \begin{pmatrix} \nu \\ e \end{pmatrix}_{Ri}$	$(1,1,2,-1)$	$\{0,1,-1\}$
H_{Li}	$(1,2,1,1)$	$\{0,1,-1\}$
H_{Ri}	$(1,1,2,1)$	$\{0,1,-1\}$
$U_{L,R}$	$(3,1,1,\frac{4}{3})$	0
$D_{L,R}$	$(3,1,1,-\frac{2}{3})$	0
$E_{L,R}$	$(1,1,1,-2)$	0

GM: JHEP (2025)

- Parity symmetry

$$\Psi_{Li} \leftrightarrow \Psi_{Ri}$$

$$H_{Li} \leftrightarrow H_{Ri}$$

- The scalar potential

$$V = \mu^2_{Li} H^\dagger_{Li} H_{Li} + \mu^2_{Ri} H^\dagger_{Ri} H_{Ri} + (\lambda)_{ij} \left[(H^\dagger_{Li} H_{Li})(H^\dagger_{Lj} H_{Lj}) + (H^\dagger_{Ri} H_{Ri})(H^\dagger_{Rj} H_{Rj}) \right]$$
$$+ (\tilde{\lambda})_{ij} \left[(H^\dagger_{Li} H_{Lj})(H^\dagger_{Lj} H_{Li}) + (H^\dagger_{Ri} H_{Rj})(H^\dagger_{Rj} H_{Ri}) \right] + (\lambda^4)_{ij} (H^\dagger_{Li} H_{Li})(H^\dagger_{Rj} H_{Rj})$$
$$+ (\tilde{\lambda}^4)_{ij} (H^\dagger_{Li} H_{Lj})(H^\dagger_{Rj} H_{Ri})$$

  - Potential has all real parameters; Implies real vevs for the scalars

- The Yukawa Lagrangian

$$\mathscr{L}_q = y_{di} \left(\bar{Q}_{Li} H_{Li} D_R + \bar{Q}_{Ri} H_{Ri} D_L \right) + y_{ei} \left(\bar{Q}_{Li} H_{Li} E_R + \bar{Q}_{Ri} H_{Ri} E_L \right)$$
$$+ y_{u1} \left(\bar{Q}_{L1} \tilde{H}_{L1} U_R + \bar{Q}_{R1} \tilde{H}_{R1} U_L \right) + y_{u2} \left(\bar{Q}_{L3} \tilde{H}_{L2} U_R + \bar{Q}_{R3} \tilde{H}_{R2} U_L \right)$$
$$+ y_{u3} \left(\bar{Q}_{L2} \tilde{H}_{L3} U_R + \bar{Q}_{R2} \tilde{H}_{R3} U_L \right) + M_U \bar{U}_L U_R + M_D \bar{D}_L D_R$$
$$+ M_E \bar{E}_L E_R + H.c.$$

  - Redefinition of fields leaves only 2 complex Yukawa couplings

1-Loop

- Fermions mass matrix after SSB

$$\mathscr{M} = \begin{pmatrix} 0_{3\times 3} & \mu \\ \mu' & M_F \end{pmatrix},\qquad \mu_i = y_i v_{Li}, \quad \mu'_i = y_i^\dagger v_{Ri}$$

$$\Rightarrow \quad M_{ij}^{(0)} = -\frac{v_{Li} v_{Ri}}{M_F}\, y_i\, y_j^*$$

This implies $\bar{\theta}_{tree}$ is unphysical

- 1-loop corrected fermion mass matrix has the form

$$M_{ij}^{(1)} = M_{ij}^{(0)} + \delta M_{ij}^{(0)} = M_{ij}^{(0)}(1 + C\, q_{Li} q_{Rj})$$

with $$(\delta M)_{ij} = \frac{{g_X}^2}{4\pi^2}\, q_{Li}\, M_{ij}^{(0)} q_{Rj} \left(B_0[M_X, m_3] - B_0[M_X, m_F]\right)$$

$$\det . M^{(1)} = 0.$$

$\bar{\theta}_{1-loop}$ is unphysical.

2-Loop

The 2-loop corrected mass matrix

$$\left(M_f^{(2)}\right)_{ij} = \left(M_f^{(0)}\right)_{ij}\left(1+\frac{g_X^2}{16\pi^2}q_{Li}\,q_{Rj}\,(b_0[M_X,m_{f3}^{(1)}]-b_0[M_X,m_F])\right)$$
$$+\left(\delta M_f^{(0)}\right)_{ij}\left(1+\frac{g_X^2}{16\pi^2}q_{Li}\,q_{Rj}\,b_0[M_X,m_{f3}^{(1)}]\right)$$
$$+\frac{g_X^2}{16\pi^2}q_{Li}\,q_{Rj}\left(U_{fL}^{(1)}\right)_{i2}\left(U_{fR}^{(1)}\right)^*_{j2}m_{f2}^{(1)}(b_0[M_X,m_{f2}^{(1)}]-b_0[M_X,m_{f3}^{(1)}])$$

- We choose phase of y_i as θ_i, Then phase of y_i^* : $-\theta_i$. Then μ_i : θ_i, μ'_i : $-\theta_i$

$$M_{ij}^{(0)} \sim e^{i\theta_{ij}}$$
$$\delta M_{ij}^{(0)} \sim e^{i\theta_{ij}}$$

"Hermitian Type Matrix"

→

$$M_{ij}^{(1)} \sim e^{i\theta_{ij}}$$

"Hermitian Type Matrix"

where $\theta_{ij} = \theta_i - \theta_j$

- The sum of two hermitian type matrices X and Y is also a hermitian type matrix only when the phase factor of the elements X_{ij} and Y_{ij} are the same.

Diagonalising matrices

U_L and U_R are obtained from biunitary diagonalisation eqns.

$$U_L^\dagger M M^\dagger U_L = D^2 ,$$
$$U_R^\dagger M^\dagger M U_R = D^2 ,$$

$$D = \text{Diag}\,.(m_k)$$

$$\left(MM^\dagger\right)_{ij} = c_{ij}\, e^{i\theta_{ij}} \qquad \left(M^\dagger M\right)_{ij} = d_{ij}\, e^{i\theta_{ij}}$$

Now inverting the diagonalisation eqn.

$$\left(MM^\dagger\right)_{ij} = \left(U_L\, D^2\, U_L^\dagger\right)_{ij}$$
$$c_{ij} = \left(e^{-i\theta_{ik}}\,(U_L)_{ik}\right)\, m_k^2\, \left(e^{i\theta_{jk}}\,(U_L)^*_{jk}\right)$$
$$= \left(S_L\right)_{ik}\, m_k^2\, \left(S_L\right)^*_{ik}$$
$$\left(S_L\right)_{ik} = e^{-i\theta_{ik}}\,(U_L)_{ik}$$

$$\left(M^\dagger M\right)_{ij} = \left(U_R\, D^2\, U_R^\dagger\right)_{ij} ,$$
$$d_{ij} = \left(e^{-i\theta_{ik}}\,(U_R)_{ik}\right)\, m_k^2\, \left(e^{i\theta_{jk}}\,(U_R)^*_{jk}\right)$$
$$= \left(S_R\right)_{ik}\, m_k^2\, \left(S_R\right)^*_{ik}$$
$$\left(S_R\right)_{ik} = e^{-i\theta_{ik}}\,(U_R)_{ik}$$

Phase factor of U_L and U_R

$$(U_L)_{ik} \sim e^{i\theta_{ik}} , \qquad (U_R)_{ik} \sim e^{i\theta_{ik}} .$$

$$M_{ij}^{(2)} = r_{ij}\, e^{i\theta_{ij}} \quad \Rightarrow \quad \bar{\theta}_{2-loop} = Arg\,.\,\text{Det}\,.\,(M^{(2)}) = 0$$

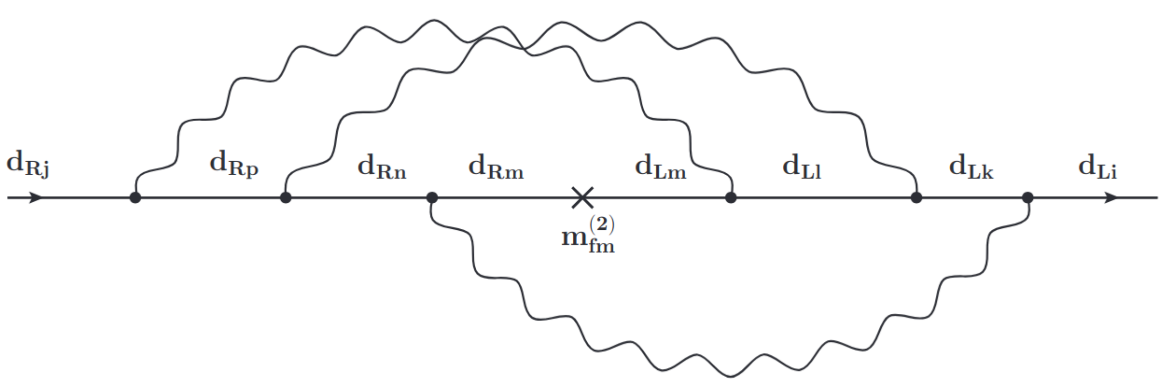
Qualitative Analysis

- In the mass basis of fermions

$$-\mathscr{L}_{\text{gauge}} = \frac{g_X}{2} X_\mu \left[(Q_f^{(2)}{}_L)_{ij}\, \bar{f}'_{Li}\gamma^\mu f'_{Lj} + (Q_f^{(2)}{}_R)_{ij}\, \bar{f}'_{Ri}\gamma^\mu f'_{Rj} \right]$$

$$Q_f^{(2)}{}_{L,R} = U_f^{(2)\dagger}{}_{L,R}\, q\, U_f^{(2)}{}_{L,R}\, ;$$

$$\left(Q_f^{(2)}{}_{L,R}\right)_{ij} \sim e^{i\theta_{ij}} \qquad \text{where} \quad \theta_{ij} = \theta_i - \theta_j$$



$$(Q_d^{(2)}{}_L)_{ik}(Q_d^{(2)}{}_L)_{kl}(Q_d^{(2)}{}_L)_{lm}(Q_d^{(2)}{}_R)_{mn}(Q_d^{(2)}{}_R)_{np}(Q_d^{(2)}{}_R)_{pj}$$

$$\propto \text{Constant} \times e^{i\theta_{ik}}\, e^{i\theta_{kl}}\, e^{i\theta_{lm}}\, e^{i\theta_{mn}}\, e^{i\theta_{np}}\, e^{i\theta_{pj}}$$

$$= \text{Constant} \times e^{i\theta_{ij}}$$

$\bar{\theta}$ doesn't get induced in all order of perturbation through gauge corrections .

Non-vanishing $\bar{\theta}$

$$\mathscr{L} \supset \mathscr{L}_y + \mathscr{L}_X + \mathscr{L}_{scalar}$$

Universal seesaw of Babu-Mohapatra model

Don't violate parity

Scalar induced corrections can break parity

$$\bar{\theta} \sim \left(\frac{1}{16\pi^2}\right)^2 \left(\frac{v_L}{v_R}\right)^2 \phi^2 .$$

Babu, Mohapatra PRL(1989)

- Here $v_R \propto M_X$. M_X is constrained from FV analysis. For $U(1)_{2-3}$ case

$$M_X \gtrsim 10^8 \ \text{GeV}$$

GM & KMP:JHEP(2024)

This gives

$$\bar{\theta} \sim 10^{-14}$$

Summary

- Radiative mass mechanism can accommodate the parity solutions to strong CP problem when implemented in a parity invariant LR model.
- This model has 6 less VL fermions, but 4 more scalars compared to Babu-Mohapatra model.
- In minimal model, $U(1)_F$ and RH sector breaking scale are at same level. The former constraints the latter.
- The model predicts very small strong CP phase $\bar{\theta} \sim 10^{-14}$. Nearly 4 order smaller compared to the present experimental limit.

Thank you

Backup slides

Gauge sector

Gauge Bosons: $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times U(1)_{2-3}$

Charged gauge bosons: $M^2_{W^\pm_L} = \frac{1}{4} g^2 \sum_i v^2_{Li}$, and $M^2_{W^\pm_R} = \frac{1}{4} g^2 \sum_i v^2_{Ri}$.

Neutral gauge bosons: $(W^3_{L\mu}, W^3_{R\mu}, B_\mu, X_\mu)$ mixing happens

$$M^2 = \frac{1}{4}\begin{pmatrix} g^2 \sum_i v^2_{Li} & 0 & -g\,g_1 \sum_i v^2_{Li} & g\,g_X(v^2_{L3} - v^2_{L2}) \\ 0 & g^2 \sum_i v^2_{Ri} & -g\,g_1 \sum_i v^2_{Ri} & g\,g_X(v^2_{R3} - v^2_{R2}) \\ -g\,g_1 \sum_i v^2_{Li} & -g\,g_1 \sum_i v^2_{Ri} & g_1^2 \sum_i (v^2_{Li} + v^2_{Ri}) & -g_1\,g_X(v^2_{L,R3} - v^2_{L,R2}) \\ g\,g_X(v^2_{L3} - v^2_{L2}) & g\,g_X(v^2_{R3} - v^2_{R2}) & -g_1\,g_X(v^2_{L,R3} - v^2_{L,R2}) & g_X^2(v^2_{R2} + v^2_{R3}) \end{pmatrix} \rightarrow \mathscr{D}^2 = \begin{pmatrix} 0 & & & \\ & m_Z^2 & & \\ & & m^2_{Z_R} & \\ & & & m_X^2 \end{pmatrix}$$

Has vanishing det.

Photon A_μ

Det. $\propto v^2_{Li}$.

Z boson

$$m_X \sim M_{W_R} \sim m_{Z_R} \sim v_R$$

QFV

$$-\mathscr{L}_{Z_{1,2}} = \sum_{k=1,2} g_k \left(\left(X^{(k)}_{f_L}\right)_{ij} \overline{f_{Li}} \gamma^\mu f_{Lj} + \left(X^{(k)}_{f_R}\right)_{ij} \overline{f_{Ri}} \gamma^\mu f_{Rj} \right) Z_{k\mu} ,$$

- Quark flavour violations: Meson-antimeson oscillations

$$\mathscr{H}^{\rm eff}_M = \sum_{i=1}^{5} C_i^M Q_i^M + \sum_{i=1}^{3} \tilde{C}_i^M \tilde{Q}_i^M$$

$$\begin{aligned}
Q_1^{q_i q_j} &= \bar{q}^\alpha_{jL} \gamma_\mu q^\alpha_{iL} \bar{q}^\beta_{jL} \gamma^\mu q^\beta_{iL} , \\
Q_2^{q_i q_j} &= \bar{q}^\alpha_{jR} q^\alpha_{iL} \bar{q}^\beta_{jR} q^\beta_{iL} , \\
Q_3^{q_i q_j} &= \bar{q}^\alpha_{jR} q^\beta_{iL} \bar{q}^\beta_{jR} q^\alpha_{iL} , \\
Q_4^{q_i q_j} &= \bar{q}^\alpha_{jR} q^\alpha_{iL} \bar{q}^\beta_{jL} q^\beta_{iR} , \\
Q_5^{q_i q_j} &= \bar{q}^\alpha_{jR} q^\beta_{iL} \bar{q}^\beta_{jL} q^\alpha_{iR} .
\end{aligned}$$

JHEP 03 (2008) 049: UTfit

At TeV scale

$K^0 - \bar{K}^0$ mixing:

$$C_K^1 = \frac{g'^2}{M^2_{Z'}} \left[\left(X^{(1)}_{dL}\right)_{12} \right]^2 , \quad \tilde{C}_K^1 = \frac{g'^2}{M^2_{Z'}} \left[\left(X^{(1)}_{dR}\right)_{12} \right]^2 , \quad C_K^5 = -4 \frac{g'^2}{M^2_{Z'}} \left(X^{(1)}_{dL}\right)_{12} \left(X^{(1)}_{dR}\right)_{12}$$

$B^0_d - \bar{B}^0_d$ mixing:

$$C_{B_d}^1 = \frac{g_1^2}{M^2_{Z_1}} \left[\left(X^{(1)}_{d_L}\right)_{13} \right]^2 , \quad \tilde{C}_{B_d}^1 = \frac{g_1^2}{M^2_{Z_1}} \left[\left(X^{(1)}_{d_R}\right)_{13} \right]^2 , \quad C_{B_d}^5 = -4 \frac{g_1^2}{M^2_{Z_1}} \left(X^{(1)}_{d_L}\right)_{13} \left(X^{(1)}_{d_R}\right)_{13} ,$$

$B^0_s - \bar{B}^0_s$ mixing:

$$C_{B_s}^1 = \frac{g_1^2}{M^2_{Z_1}} \left[\left(X^{(1)}_{d_L}\right)_{23} \right]^2 , \quad \tilde{C}_{B_s}^1 = \frac{g_1^2}{M^2_{Z_1}} \left[\left(X^{(1)}_{d_R}\right)_{23} \right]^2 , \quad C_{B_s}^5 = -4 \frac{g_1^2}{M^2_{Z_1}} \left(X^{(1)}_{d_L}\right)_{23} \left(X^{(1)}_{d_R}\right)_{23} .$$

$D^0 - \bar{D}^0$ mixing:

$$C_D^1 = \frac{g_1^2}{M^2_{Z_1}} \left[\left(X^{(1)}_{u_L}\right)_{12} \right]^2 , \quad \tilde{C}_D^1 = \frac{g_1^2}{M^2_{Z_1}} \left[\left(X^{(1)}_{u_R}\right)_{12} \right]^2 , \quad C_D^5 = -4 \frac{g_1^2}{M^2_{Z_1}} \left(X^{(1)}_{u_L}\right)_{12} \left(X^{(1)}_{u_R}\right)_{12} .$$