Probing Nucleon Resonance Structure with Transition GPDs:

Exclusive $N\pi\gamma$ electroproduction, competing mechanisms, and final-state interactions

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The Roper Resonance

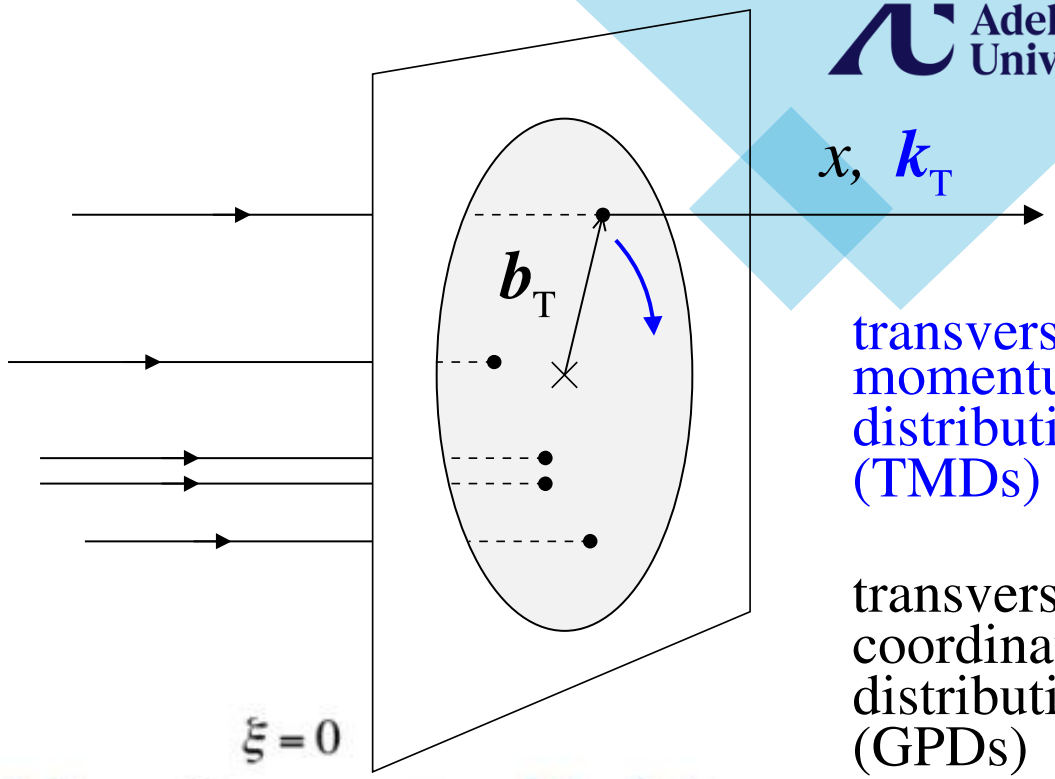
- Nucleon Resonance of mass $M \approx 1440$ and width $\Gamma \approx 300\ \mathrm{MeV}/c^2$
- Total Spin-Parity $J = {}^1/_2{}^+$
- Isospin $I = {}^1/_2$

Can be decomposed as

$$P_{11}(1440)\rangle = \alpha|uud\rangle + \beta|N\pi\rangle + \cdots$$

radially-excited three-quark state
$n = 2$ (radial quantum no.)

molecule-like bound state of a nucleon and a pion



transverse momentum distributions (TMDs)

transverse coordinate distributions (GPDs)

$W(x,b_T,k_T)$
Wigner Distributions

$\int d^2b_T$ $\int d^2k_T$

$f(x,k_T)$ $f(x,b_T)$

transverse momentum distributions (TMDs)
semi–inclusive processes

impact parameter distributions

Fourier trf.
$b_T \leftrightarrow \Delta$

$H(x,0,t)$
$t=-\Delta^2$

$\xi=0$

$H(x,\xi,t)$

generalized parton distributions (GPDs)
exclusive processes

$\int d^2k_T$ $\int d^2b_T$

$f(x)$
parton densities
inclusive and semi–inclusive processes

$\int dx$

$F_1(t)$
form factors
elastic scattering

$\int dx x^{n-1}$

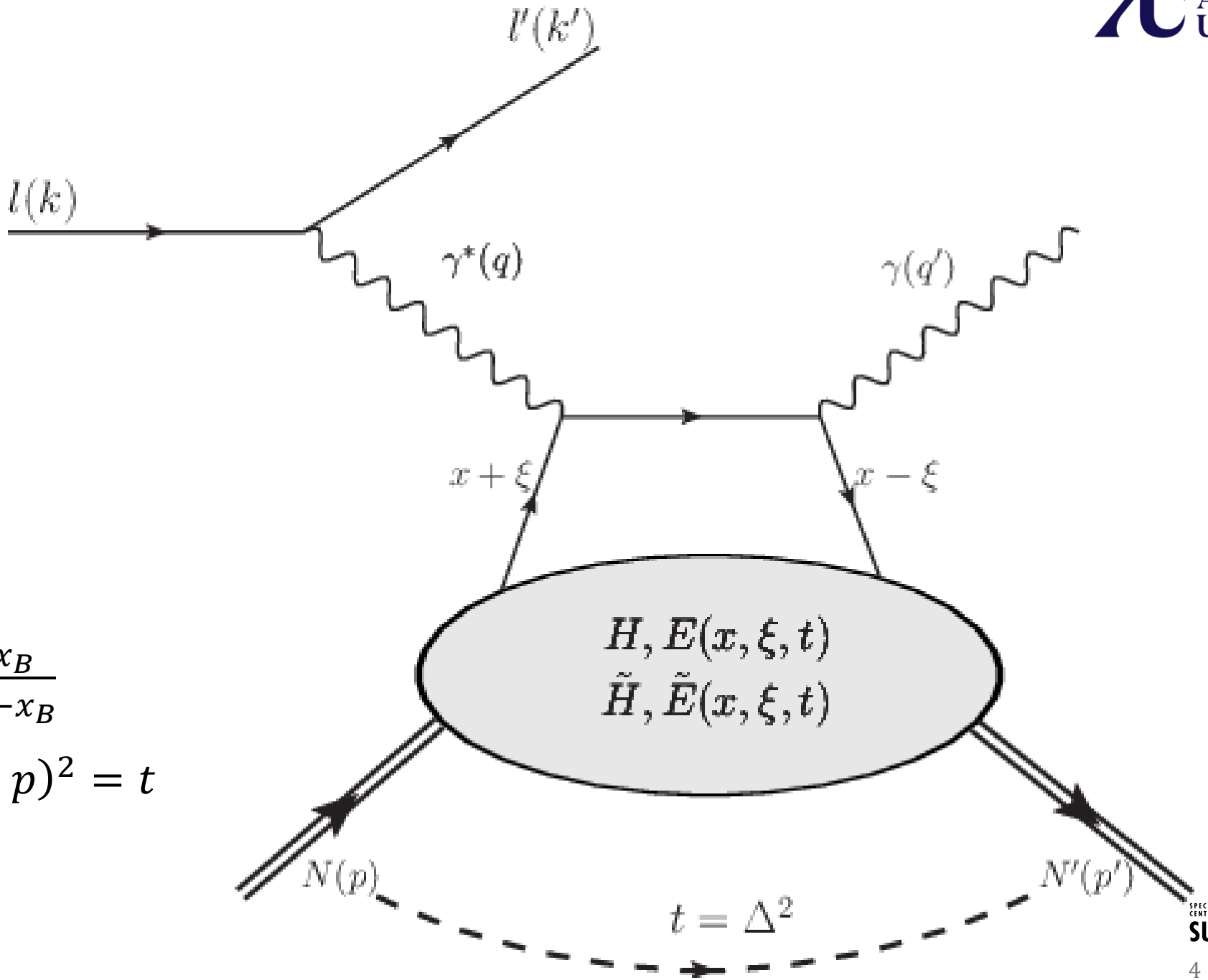
$A_{n,0}(t)+4\xi^2 A_{n,2}(t)+....$
generalized form factors
lattice calculations

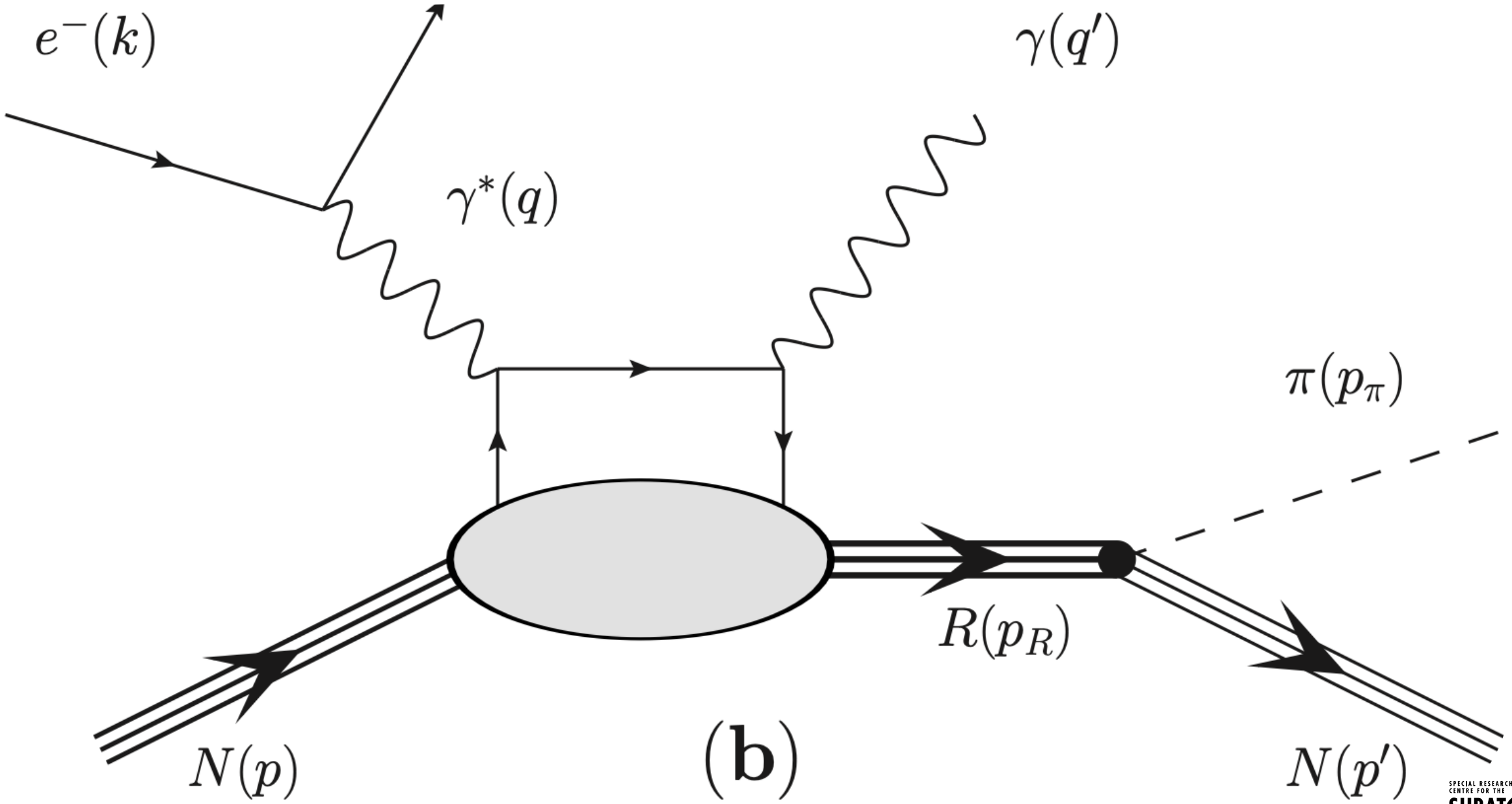
DVCS

$$\ell N \to \ell' N' \gamma$$

skewness: $\xi = \frac{x_i - x_f}{2} \to \frac{x_B}{2 - x_B}$

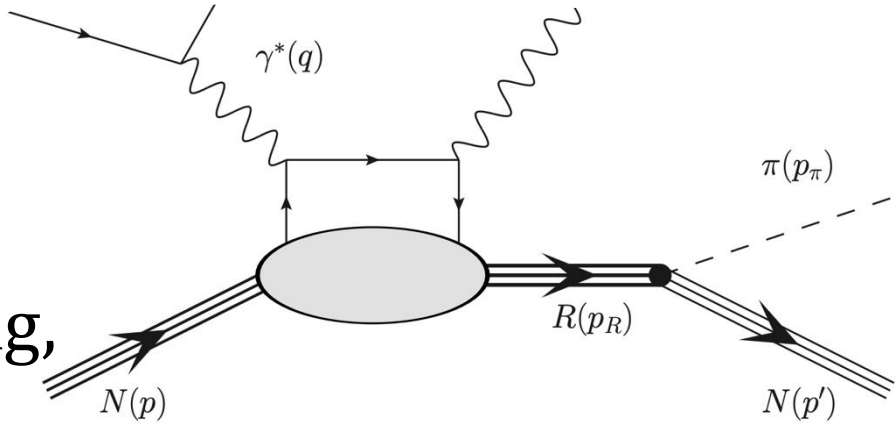
forwardness: $\Delta^2 = (p' - p)^2 = t$




$e^-(k)$
$\gamma(q')$
$\gamma^*(q)$
$\pi(p_\pi)$
$R(p_R)$
$N(p)$
(b)
$N(p')$

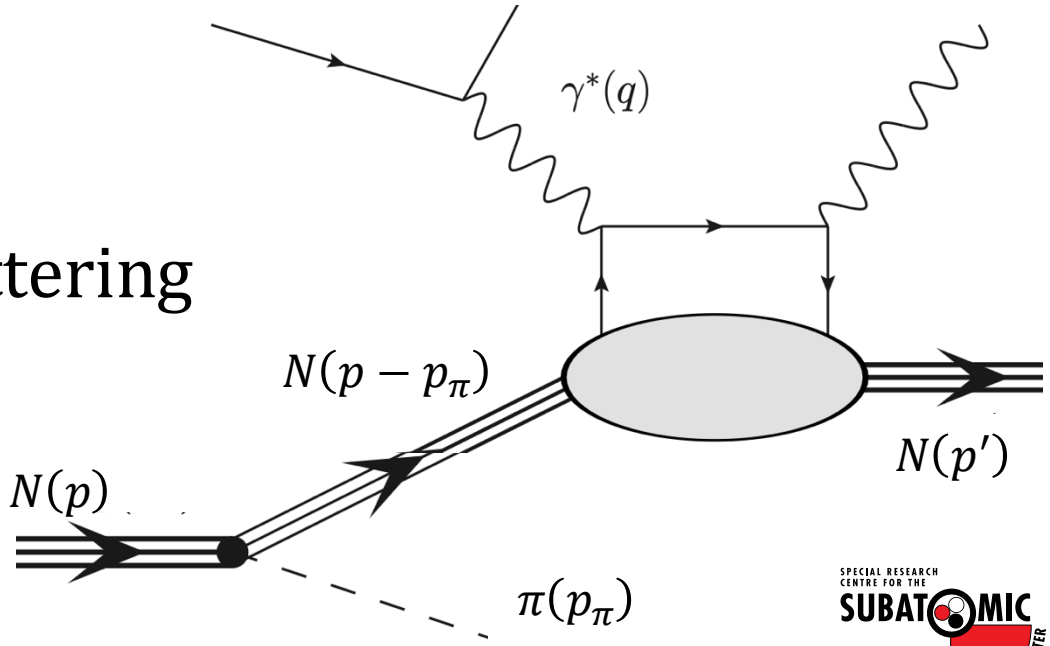
final state $N + \pi$ can be achieved by

- $N(p) \rightarrow N(p_I) + \pi(k_\pi)$ *before* the lepton scattering,

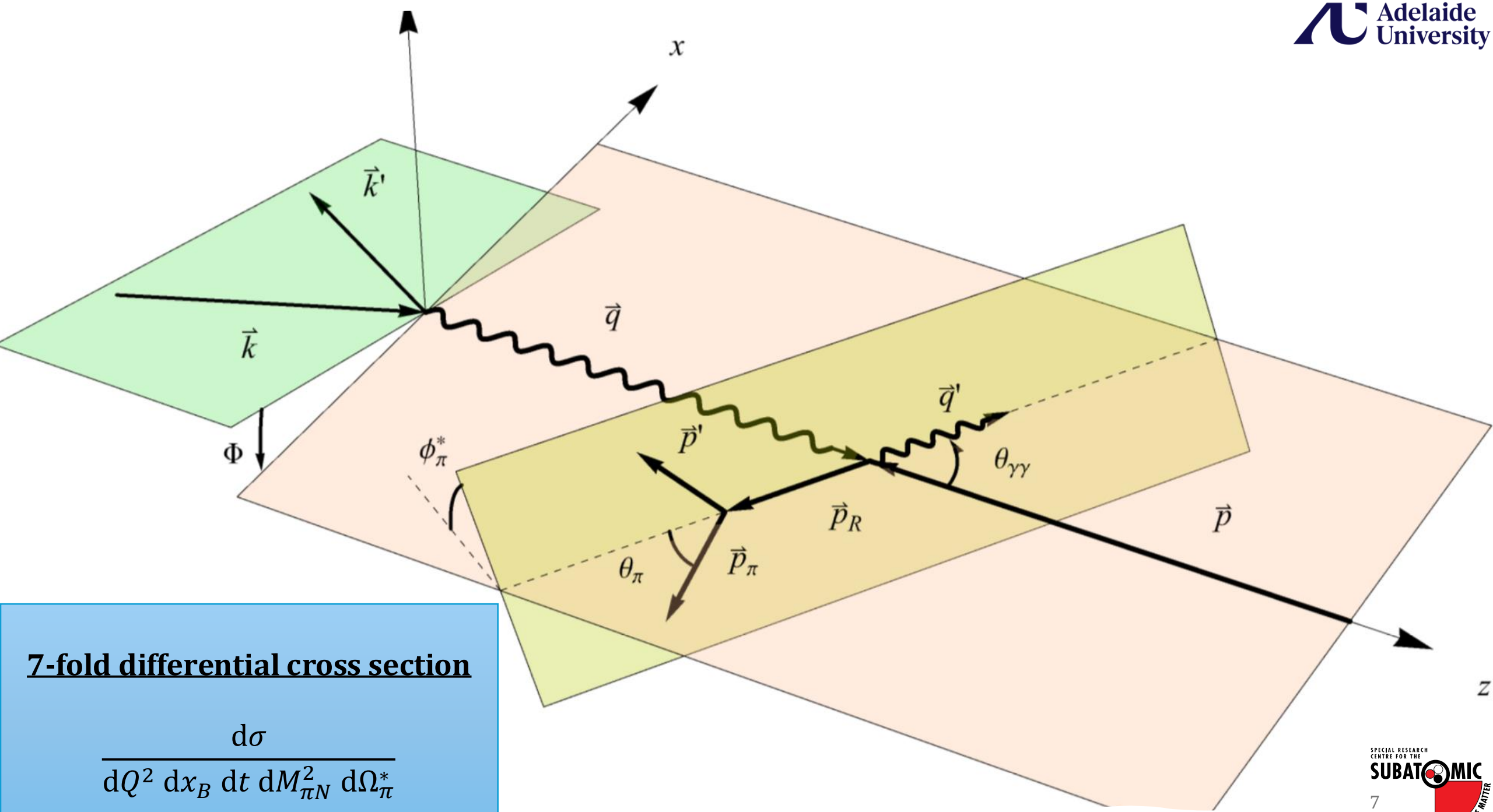


OR

- $R(p_R) \rightarrow N(p') + \pi(k_\pi)$ *after* the lepton scattering



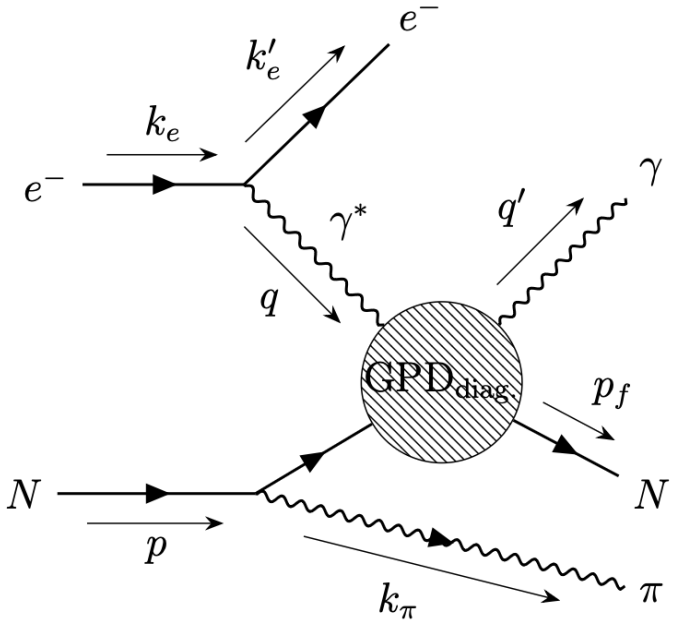
where $p' + p_\pi = p_R$



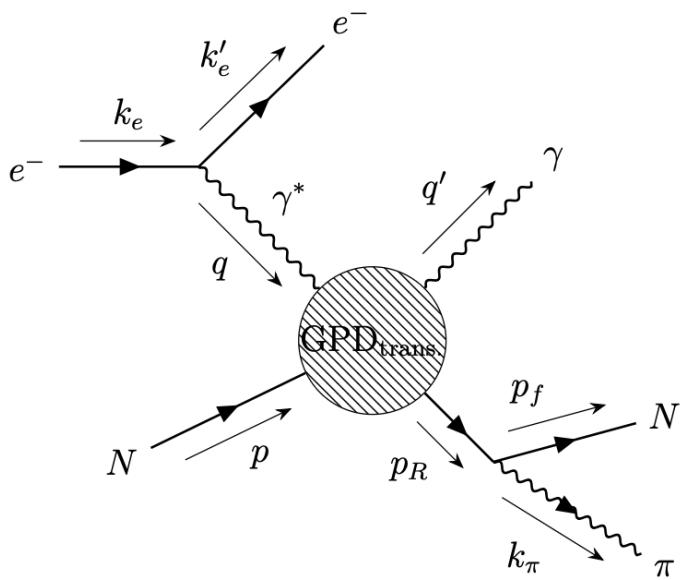
7-fold differential cross section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\ \mathrm{d}x_B\ \mathrm{d}t\ \mathrm{d}M_{\pi N}^2\ \mathrm{d}\Omega_\pi^*}$$

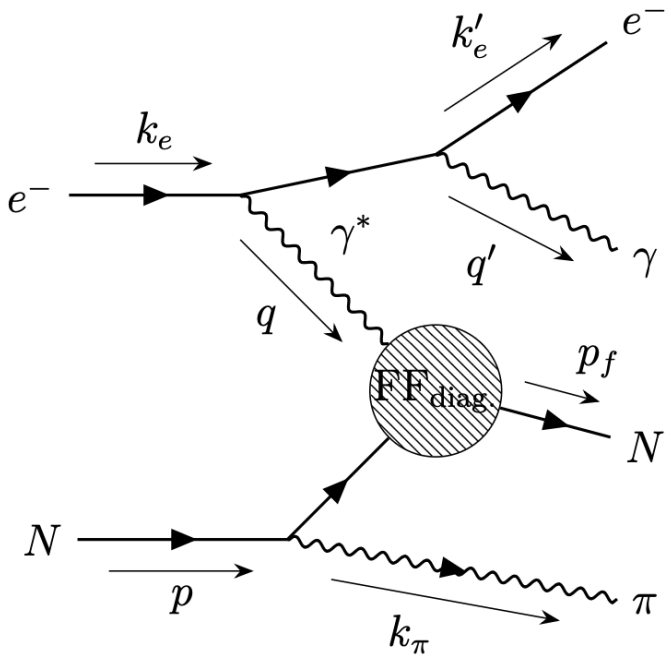
Must also include Bethe-Heitler versions of both topologies



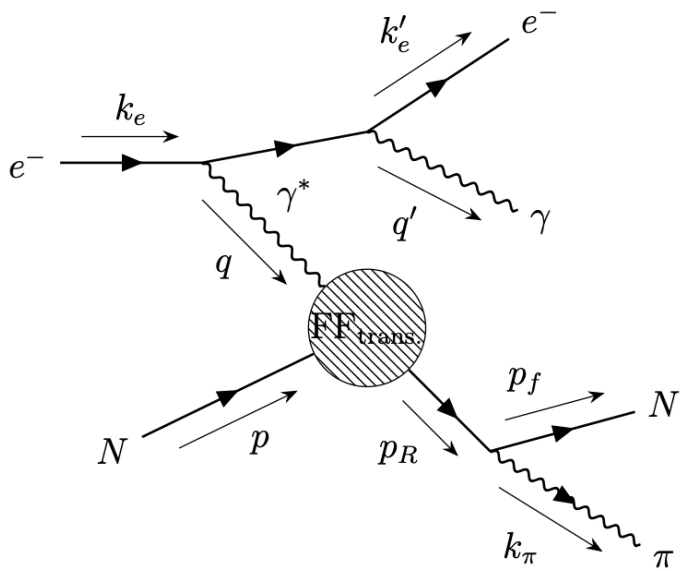
(a) Diagonal DVCS



(b) Transition DVCS



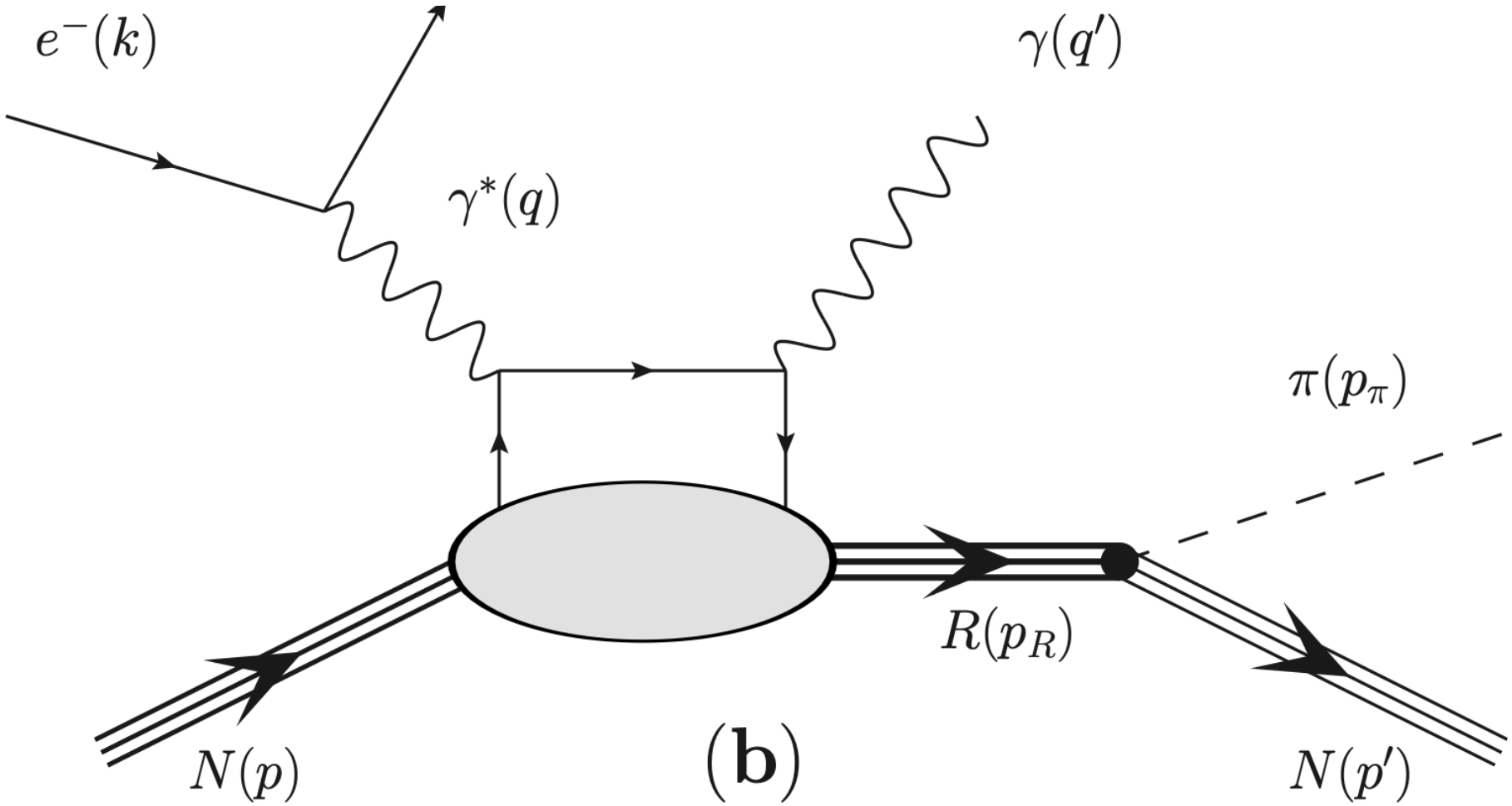
(c) Diagonal BH



(d) Transition BH

Transition DVCS Amplitude

$$J^{\mu}_{\text{trans.}} \sim \bar{u}(p_f)\,\Gamma_{\pi N N^*}(p_R, k_\pi, p_f)\,S_R(p_R)\,\varepsilon^*_\nu(q')\,T^{\mu\nu}_{N\to N^*}(p, p_R, \Delta)\,u(p),$$



Transition DVCS Tensor $H_{NR}^{\mu\nu}$

$$H_{NR}^{\mu\nu} = \frac{1}{2}(-g_\perp^{\mu\nu})\int_{-1}^{1} dx \left[\frac{1}{x-\xi+i\epsilon}+\frac{1}{x+\xi-i\epsilon}\right] \int \frac{d\lambda}{2\pi} e^{i\lambda x} \sum_q e_q^2 \langle R(p_R,s_R)|\bar{q}\left(-\frac{\lambda n}{2}\right)\gamma\cdot n\, q\left(\frac{\lambda n}{2}\right)|N(p,s_N)\rangle$$

$$+ \frac{i}{2}(\varepsilon_\perp^{\mu\nu})\int_{-1}^{1} dx \left[\frac{1}{x-\xi+i\epsilon}-\frac{1}{x+\xi-i\epsilon}\right] \int \frac{d\lambda}{2\pi} e^{i\lambda x} \sum_q e_q^2 \langle R(p_R,s_R)|\bar{q}\left(-\frac{\lambda n}{2}\right)\gamma\cdot n\gamma_5\, q\left(\frac{\lambda n}{2}\right)|N(p,s_N)\rangle,$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \sum_q e_q^2 \langle R(p_R,s_R)|\bar{q}\left(-\frac{\lambda n}{2}\right)\gamma\cdot n\, q\left(\frac{\lambda n}{2}\right)|N(p,s_N)\rangle$$

$$= H_1^{pP_{11}}(x,\xi,\Delta^2)\,\bar{R}(p_R,s_R)\left(n^\nu - \frac{n\cdot\Delta}{\Delta^2}\Delta^\nu\right)\gamma_\nu N(p,s_N) + H_2^{pP_{11}}(x,\xi,\Delta^2)\,\bar{R}(p_R,s_R)\frac{i\sigma_{\nu\kappa}n^\nu\Delta^\kappa}{M_R+M_N}N(p,s_N).$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \sum_q e_q^2 \langle R(p_R,s_R)|\bar{q}\left(-\frac{\lambda n}{2}\right)\gamma\cdot n\gamma_5\, q\left(\frac{\lambda n}{2}\right)|N(p,s_N)\rangle$$

$$= \tilde{H}_1^{pP_{11}}(x,\xi,\Delta^2)\,\bar{R}(p_R,s_R)\gamma\cdot n\gamma_5 N(p,s_N) + \tilde{H}_2^{pP_{11}}(x,\xi,\Delta^2)\,\bar{R}(p_R,s_R)\frac{\Delta\cdot n}{M_R+M_N}\gamma_5 N(p,s_N).$$

Need to model these GPDs: $H_1(x,\xi,\Delta^2)$, $H_2(x,\xi,\Delta^2)$, $\widetilde{H_1}(x,\xi,\Delta^2)$, $\widetilde{H_2}(x,\xi,\Delta^2)$.

Constructing GPDs

- Diagonal $N \to N$ GPDs are relatively well constrained.
- Transition $N \to N^*$ GPDs are more difficult.
  - Resonance electroproduction typically constrains FFs or Helicity Amplitudes rather than quark-level distributions.
  - Provides information on moments or projections of the transition dynamics, not the direct x-, ξ-, and t-dependence.

Constructing a GPD (PART 1)

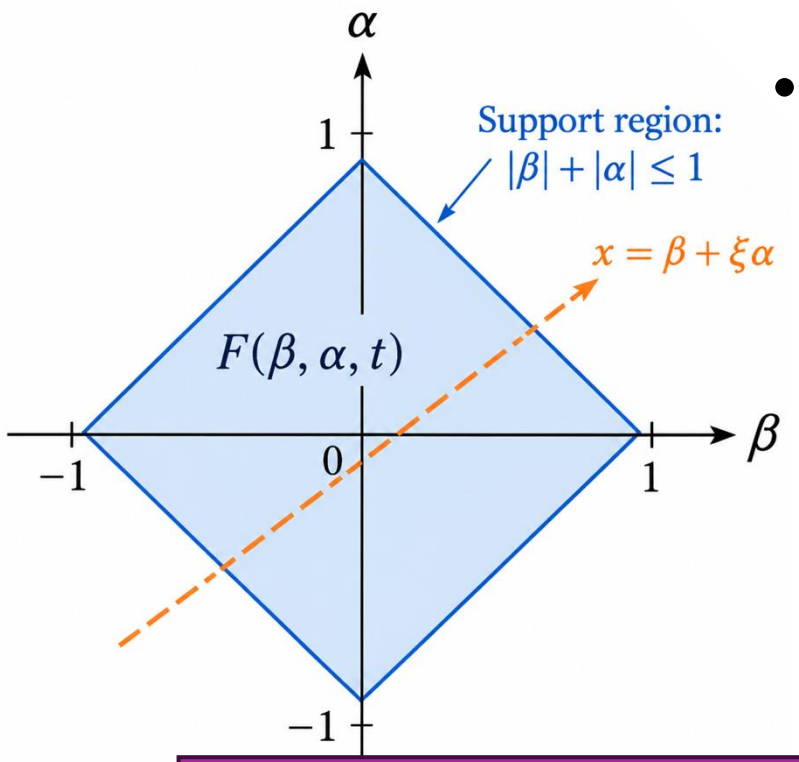
POLYNOMIALITY

- Mellin Moments in x must be expressible in terms of local twist-2 operator matrix elements between the corresponding states.
  - Implies that the n-th Mellin moment of a GPD must be polynomial in ξ with coefficients given by generalised form factors.
  - This forces a non-arbitrary interdependence between x and ξ or risk violating Lorentz symmetry at the local operator moment level.

use a Double Distribution

Double Distributions

β	"PDF-like" Parton Momentum Fraction
α	Sharing of Longitudinal Momentum Transfer
t	Invariant Momentum Transfer



- Introduce an auxiliary two-variable distribution, $F(\beta, \alpha, t)$.
- Integrate along the line $x = \beta + \xi\alpha$ to get the GPD.

$$H_i^q(x,\xi,t) = \int_0^1 \mathrm{d}\beta \int_{-(1-\beta)}^{1-\beta} \mathrm{d}\alpha\, \delta(x-\beta-\xi\alpha)\, F_i^q(\beta,\alpha,t)\,,$$

$$\int_{-1}^{1} \mathrm{d}x\, x^n H_i^q(x,\xi,t) = \int \mathrm{d}\beta\, \mathrm{d}\alpha\, (\beta+\xi\alpha)^n\, F_i^q(\beta,\alpha,t)$$
$$= \sum_{k=0}^{n} \binom{n}{k} \xi^k \int \mathrm{d}\beta\, \mathrm{d}\alpha\, \beta^{n-k}\alpha^k F_i^q(\beta,\alpha,t)\,.$$

the DD builds the required x-ξ correlation to ensure polynomiality

Double Distributions

β	"PDF-like" Parton Momentum Fraction
α	Sharing of Longitudinal Momentum Transfer
t	Invariant Momentum Transfer

$$F_i^q(\beta, \alpha, t) = h(\beta, \alpha)\ f_i^q(\beta, t)$$

profile function

forward-like input

Profile Function

use the Radyushkin profile function with $b_{\mathrm{DD}} = 1$ and normalised to 1

$$h(\beta, \alpha) = \frac{\Gamma(2b_{\mathrm{DD}} + 2)}{2^{2b_{\mathrm{DD}}+1}\,\Gamma^2(b_{\mathrm{DD}} + 1)} \frac{\left[(1 - |\beta|)^2 - \alpha^2\right]^{b_{\mathrm{DD}}}}{(1 - |\beta|)^{2b_{\mathrm{DD}}+1}}$$

$$\to \frac{3}{4} \frac{(1 - \beta)^2 - \alpha^2}{(1 - \beta)^3}.$$

Constructing a GPD (PART 2)

FORWARD LIMITS

Set $\xi, t = 0$ and recover

$$H^q(x,0,0) = q(x), \qquad \widetilde{H}^q(x,0,0) = \Delta q(x)$$

SUM RULES

$$\int_{-1}^{1} \mathrm{d}x\, H_1(x,\xi,t) = F_1(t), \qquad \int_{-1}^{1} \mathrm{d}x\, \widetilde{H}_1(x,\xi,t) = G_A(t),$$

$$\int_{-1}^{1} \mathrm{d}x\, H_2(x,\xi,t) = F_2(t), \qquad \int_{-1}^{1} \mathrm{d}x\, \widetilde{H}_2(x,\xi,t) = G_P(t),$$

many distributions satisfy these conditions

Forward Limits

- Diagonal GPDs have PDFs as forward limits:

$$H^q(x,0,0) = q(x), \qquad \widetilde{H}^q(x,0,0) = \Delta q(x)$$

- Must model a phenomenological forward limit for Transition GPDs as no known constraint exists

$\therefore$ use $s_q(\beta) \propto \beta^{-\alpha_0}(1-\beta)^{n_q}$

normalised to $\int_0^1 \mathrm{d}\beta\, s_q(\beta) = 1$

β	"PDF-like" Parton Momentum Fraction
$\alpha_0 = 0.5$	gives Regge-inspired low-β behaviour
$n_u = 3$	valence endpoint behaviour / high-β suppression
$n_d = 4$	

- The β-t dependence is introduced as

$$T_i^q(\beta,t) = e^{b_i^q(1-\beta)t}$$

Forward Limits

- Our forward-like input is then

$$f_i^q(\beta, t) = C_i^q(t)\, s_q(\beta)\, T_i^q(\beta, t)$$

with the normalisation function $C_i^q(t) = \dfrac{\mathcal{F}_i^q(t)}{\displaystyle\int_0^1 \mathrm{d}\beta\, s_q(\beta)\, T_i^q(\beta, t)}$

- Since the GPD is already normalised in α, this means we obey the sum rule by construction

$$\int_{-1}^{1} \mathrm{d}x\, H_i^q(x, \xi, t) = \mathcal{F}_i^q(t)$$

Constraining Transition FFs: MAID

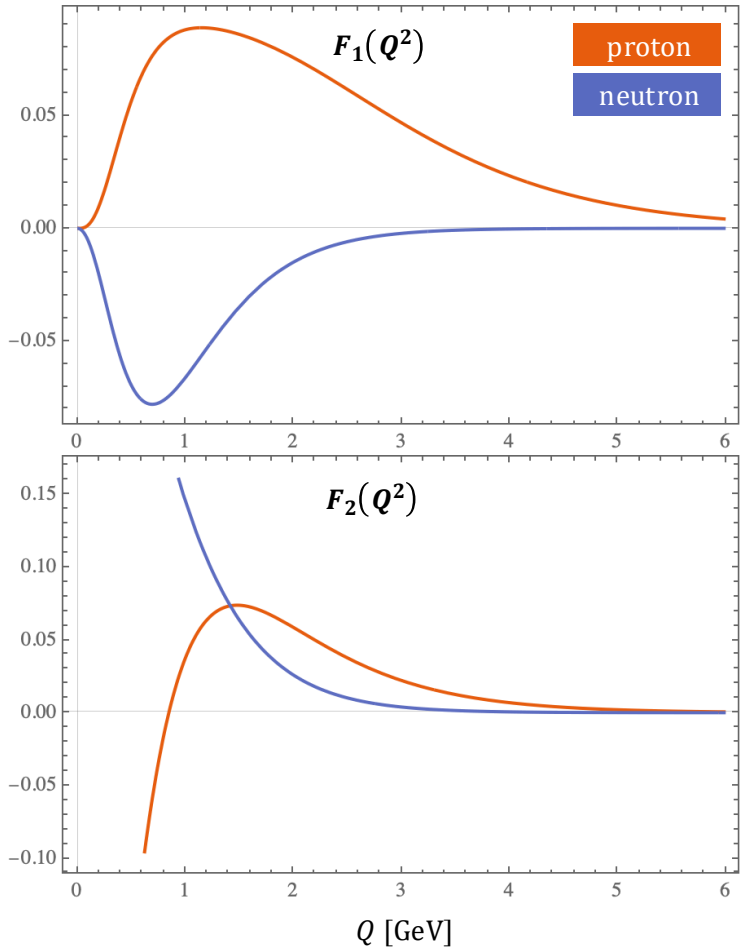
$$A_{1/2} = \sqrt{\frac{2\pi\alpha}{k_R}} \frac{1}{(M_R + M_N)} \left(F_1(Q^2) + \frac{(M_R - M_N)}{2M_N} F_2(Q^2) \right)$$

$$S_{1/2} = \sqrt{\frac{2\pi\alpha}{k_R}} \frac{\sqrt{Q^2}}{(M_R + M_N)} \left(F_1(Q^2) - \frac{(M_R + M_N)}{2M_N} F_2(Q^2) \right)$$

$$\bar{\mathcal{A}}_\alpha(Q^2) = \bar{\mathcal{A}}_\alpha(0)(1 + a_1 Q^2 + a_2 Q^4 + a_3 Q^6 + a_4 Q^8)\, e^{-b_1 Q^2}.$$

$N^*,\ \Delta^*$		$\bar{\mathcal{A}}_\alpha(0)$	a_1	a_2	a_4	b_1
$P_{11}(1440)p$	$A_{1/2}$	-61.4	0.871	-3.516	-0.158	1.36
	$S_{1/2}$	4.2	40.	0	1.50	1.75

N^*		$\bar{\mathcal{A}}_\alpha(0)$	a_1	b_1
$P_{11}(1440)n$	$A_{1/2}$	54.1	0.95	1.77
	$S_{1/2}$	-41.5	2.98	1.55



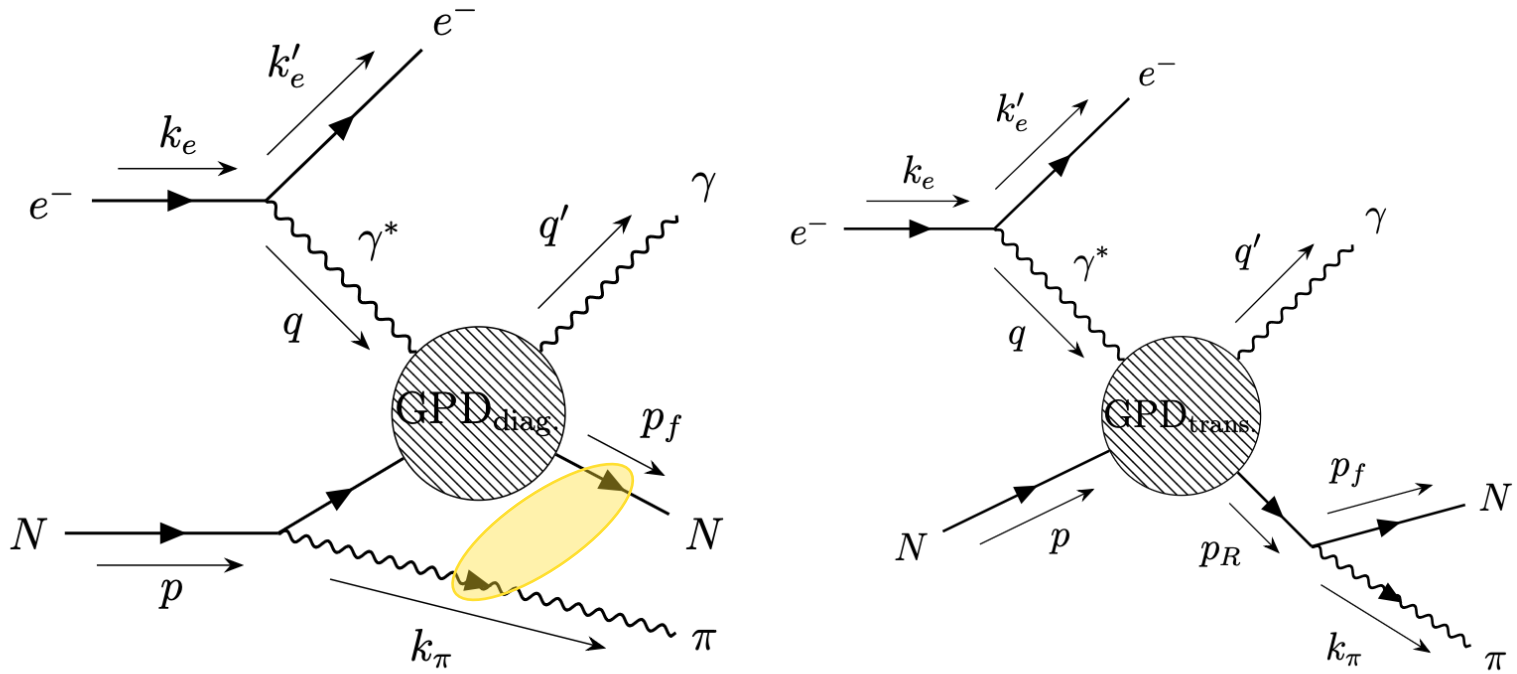
(Dreschel, D., Kamalov, S. S., Tiator L. 2007 Eur.Phys.J.A. 34 69-97)

Constructing a GPD (PART 3)

	Diagonal GPD	Transition GPD
Polynomiality	satisfied by Double Distribution form	
Forward Limit	Quark PDFs $s_q(\beta) = q(\beta)$	Functional Form $s_q(\beta) \propto \beta^{-\alpha_0}(1-\beta)^{n_q}$
Sum Rules	Form Factors $\mathcal{F}(t)$	Transition Form Factors $\mathcal{F}(t)$

Final State Interactions

- Transition mechanism contains the resonance propagator $S_F(p_R)$
  - already includes $N\pi$ rescattering
  - requires a width $\Gamma(s)$ (preferably energy-dependent)
- Diagonal mechanism requires re-scattering to be added manually.



(a) Diagonal DVCS **(b)** Transition DVCS

Final State Interaction Options

- Phase only: multiply by $e^{i\delta_s}$.
  - Works okay for P_{11} but less so for S_{11}

$\therefore$ use an effective coupled-channel K-Matrix treatment instead.

This will also require a running width.

The Running Width $\Gamma(M_{\pi N})$

Blatt-Weisskopf Barrier Factor

- General Partial Width for $R \to ab$ decay:

$$\Gamma_{ab}(M) = \Gamma_R \mathcal{B}_{ab} \left(\frac{q_{ab}(M)}{q_{ab}(M_R)} \right)^{2L+1} \frac{M_R}{M} \left(\frac{X_R^2 + q_{ab}^2(M_R)}{X_R^2 + q_{ab}^2(M)} \right)^{L}$$

  - M is the total mass $M > m_a + m_b$. Width set to 0 below threshold.
  - $\mathcal{B}_{ab}$ is the branching fraction at the pole.
  - $X_R = 0.5$ GeV is the centrifugal-barrier scale.

	$P_{11}(1440)$	$S_{11}(1535)$
πN	$L = 1, \quad \mathcal{B}_{\pi N}^{P_{11}} = 0.65$	$L = 0, \quad \mathcal{B}_{\pi N}^{S_{11}} = 0.42$
ηN	**N/A**	$L = 0, \quad \mathcal{B}_{\eta N}^{S_{11}} = 0.43$
$(\pi\pi)N$	stores remaining inelastic strength	

$$\Gamma_{\pi\pi N}(M) = \Gamma_R \mathcal{B}_{\pi\pi N} \left(\frac{q_{2\pi}(M)}{q_{2\pi}(M_R)} \right)^{2L+4} \left(\frac{X_R^2 + q_{2\pi}^2(M_R)}{X_R^2 + q_{2\pi}^2(M)} \right)^{L+2}$$

Final State Interactions by Partial Wave

Projecting the diagonal mechanism into partial waves is now useful in applying the correct K-Matrix to each partial wave.

For each partial wave R, the elastic amplitude is

$$T^R_{\pi N,\pi N}(M) = -\frac{M_R\,\Gamma^R_{\pi N}(M)}{M^2 - M_R^2 + iM_R\Gamma^R_{\rm tot}(M)},$$

$$S^R_{\pi N,\pi N}(M) = 1 + 2i\,T^R_{\pi N,\pi N}(M).$$

$$a^{\rm diag}_{R,M} \to F_R(M)\,a^{\rm diag}_{R,M}, \qquad F_R(M) = 1 + iT^R_{\pi N,\pi N}(M) = \frac{1 + S^R_{\pi N,\pi N}(M)}{2}$$

Kinematics for Evaluating the Cross Section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}x_B\,\mathrm{d}t\,\mathrm{d}\Phi\,\mathrm{d}M_{\pi N}^2\,\mathrm{d}\Omega_\pi^*}$$

integrated over

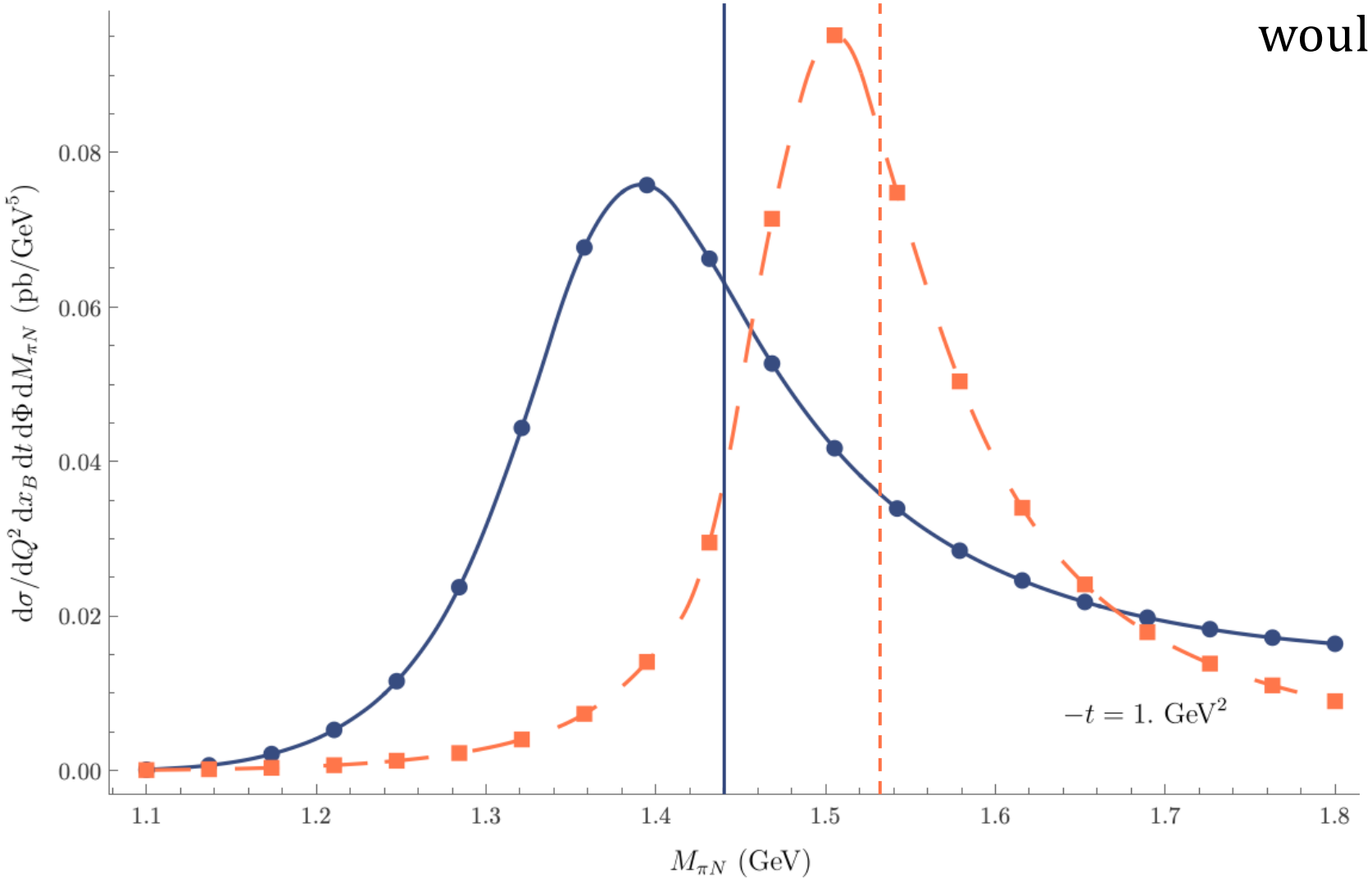
Beam Energy	$E_e = 10.6$ GeV
Virtuality	$Q^2 = 2.3$ GeV^2
Björken-x	$x_B = 0.25$
Momentum Transfer	scanned over
Hadronic Mass	threshold $\leq M_{\pi N} \leq 2.0$ GeV
Azimuth (Trento Angle)	90°

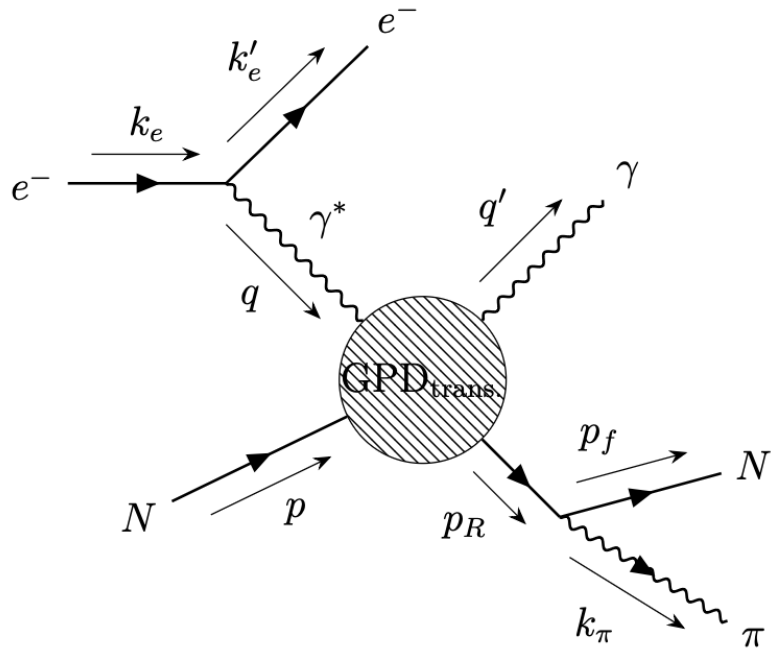
BH dominated by sin Φ term

choice of kinematics in CLAS12 ranges for experimental availability

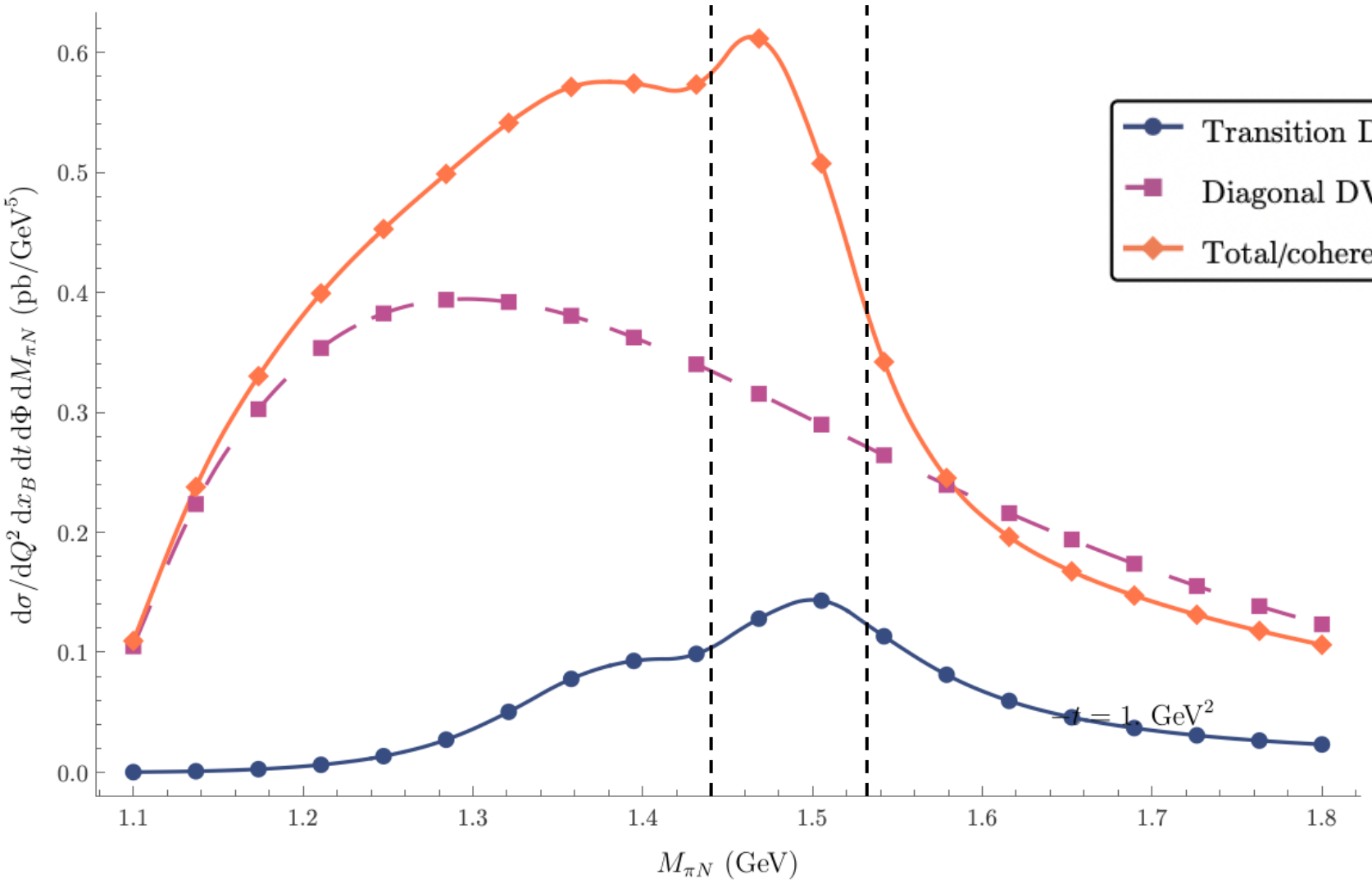
Transition DVCS Signal: $e^- p^+ \to e^- \gamma n^0 \pi^+$

would like to extract this signal





Add the Diagonal Signal: $e^- p^+ \to e^- \gamma n^0 \pi^+$



would like to extract competing background measured

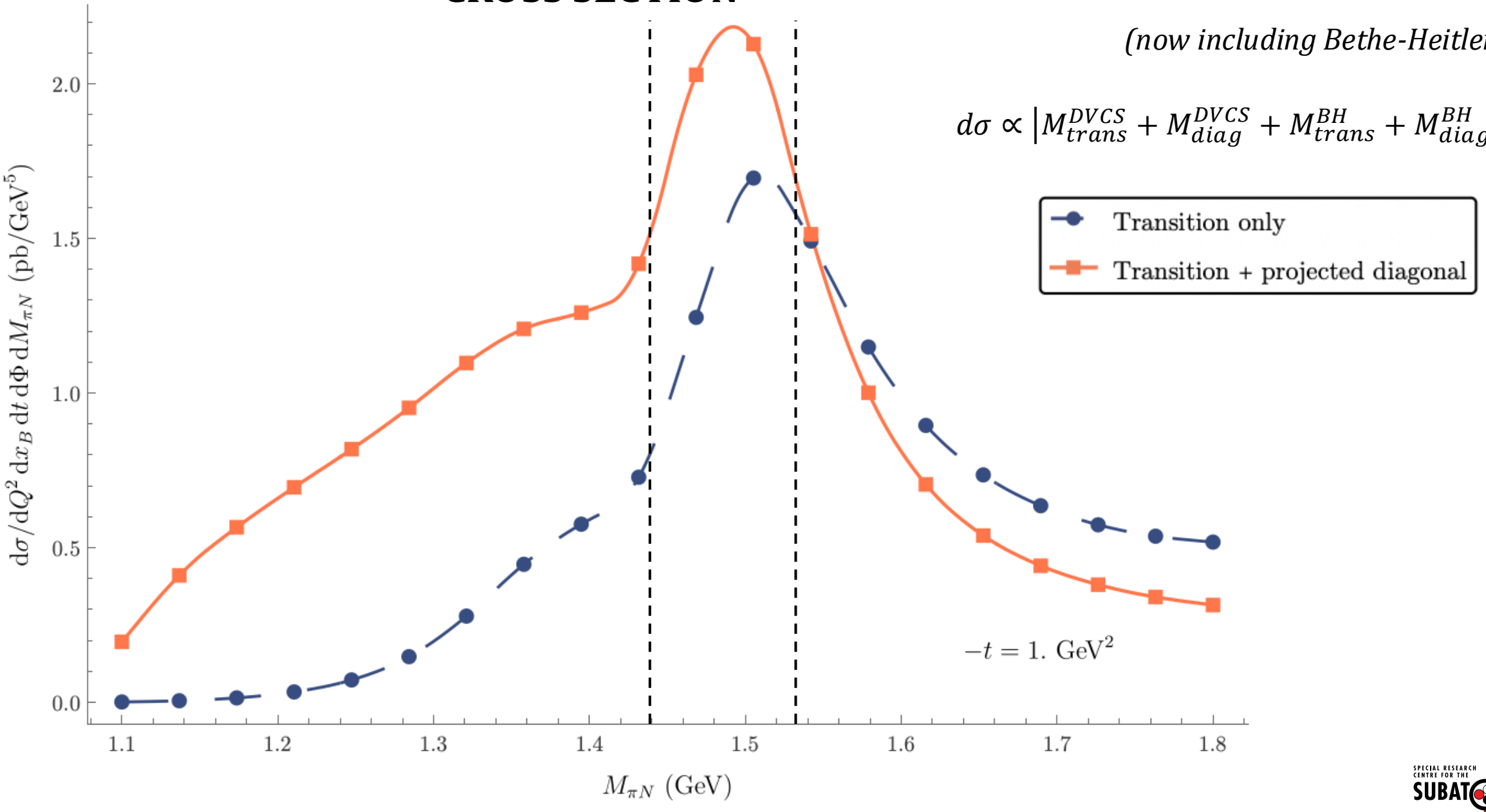
still not including the BH background

(Rumley, M., Thomas, A. W. 2026 Phys. Rev. D 113 054007)

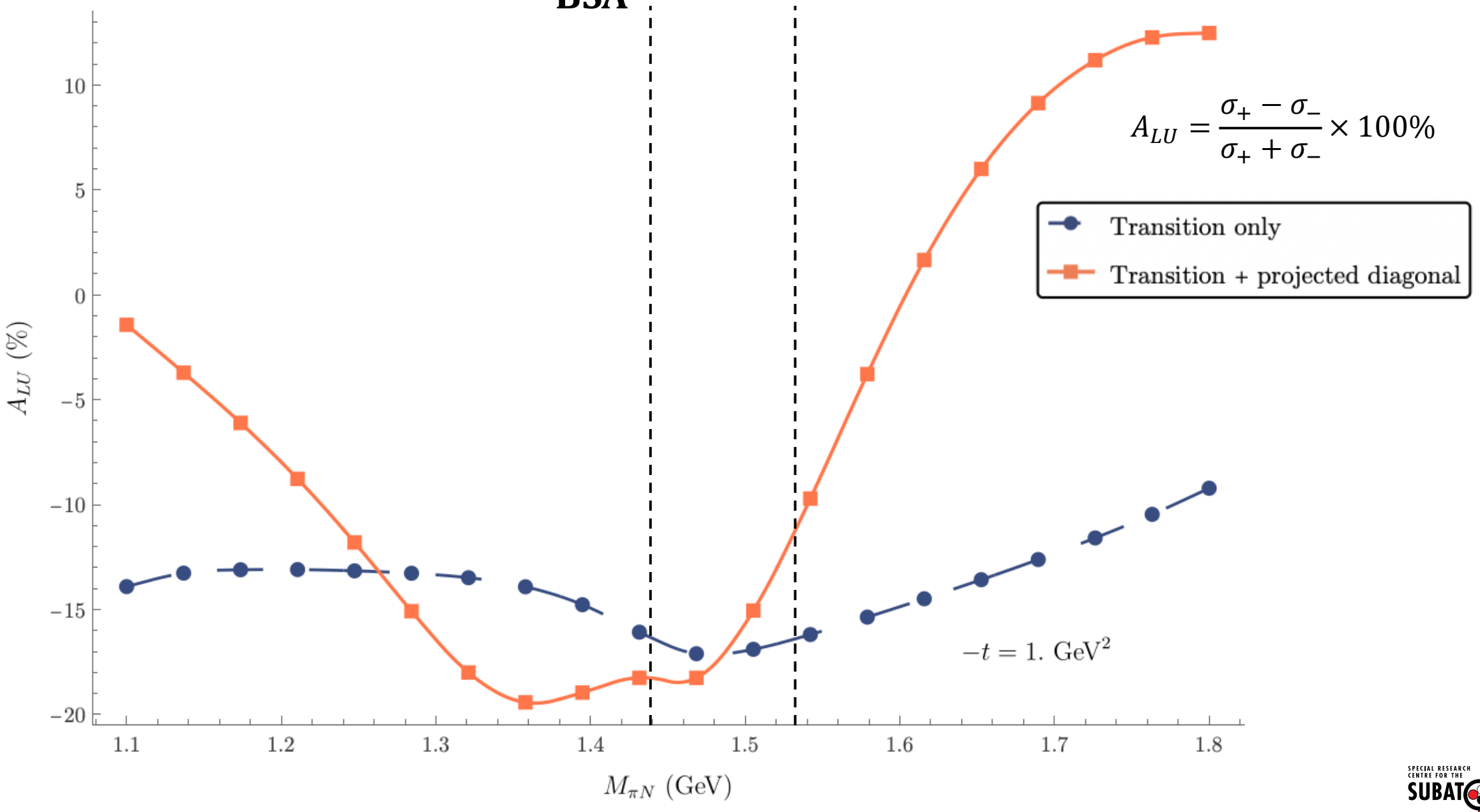
CROSS SECTION

(now including Bethe-Heitler)

$$d\sigma \propto \left| M^{DVCS}_{trans} + M^{DVCS}_{diag} + M^{BH}_{trans} + M^{BH}_{diag} \right|^2$$



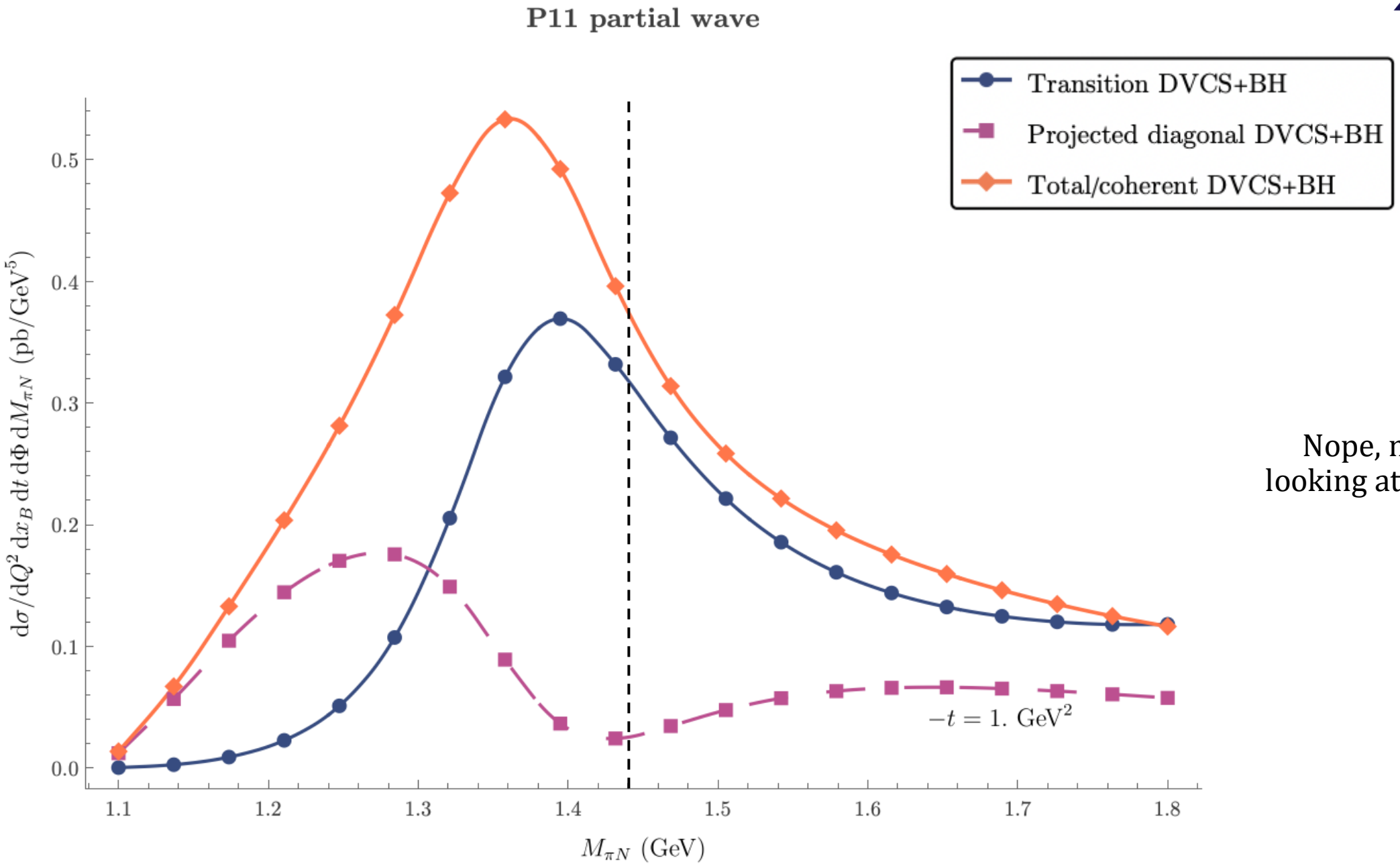
BSA


$A_{LU} = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-} \times 100\%$
Transition only
Transition + projected diagonal
$-t = 1.\ \mathrm{GeV}^2$
A_{LU} (%)
$M_{\pi N}$ (GeV)
10
5
0
−5
−10
−15
−20
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8

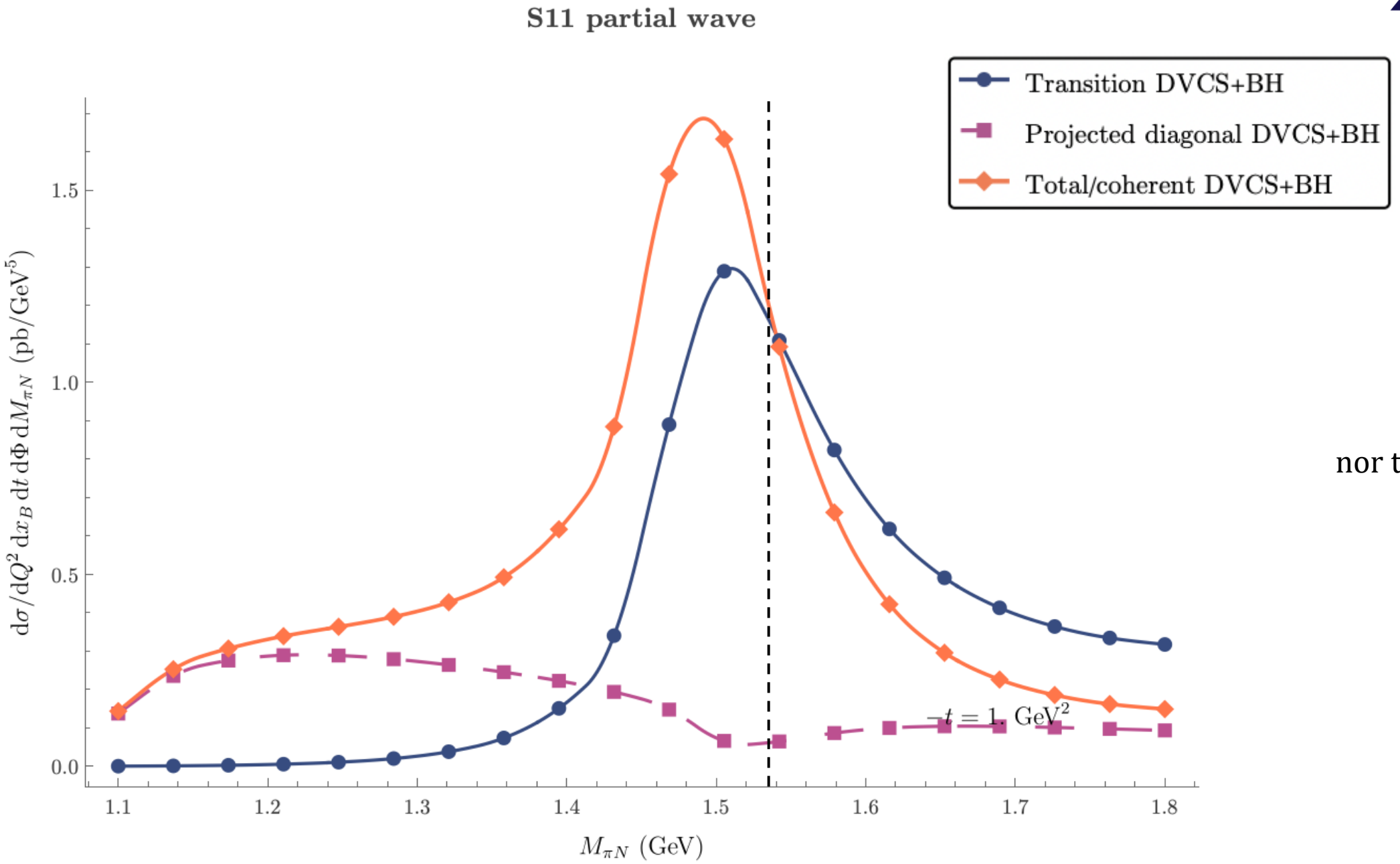
Extracting Transition GPDs

Clearly, the diagonal process interferes with our ability to extract the Transition GPDs significantly.

Try a partial wave analysis.



Nope, not better looking at just the P_{11}


S11 partial wave
Transition DVCS+BH
Projected diagonal DVCS+BH
Total/coherent DVCS+BH
$-t = 1.\ \mathrm{GeV}^2$
$\mathrm{d}\sigma/\mathrm{d}Q^2\,\mathrm{d}x_B\,\mathrm{d}t\,\mathrm{d}\Phi\,\mathrm{d}M_{\pi N}$ (pb/GeV^5)
$M_{\pi N}$ (GeV)
0.0
0.5
1.0
1.5
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8

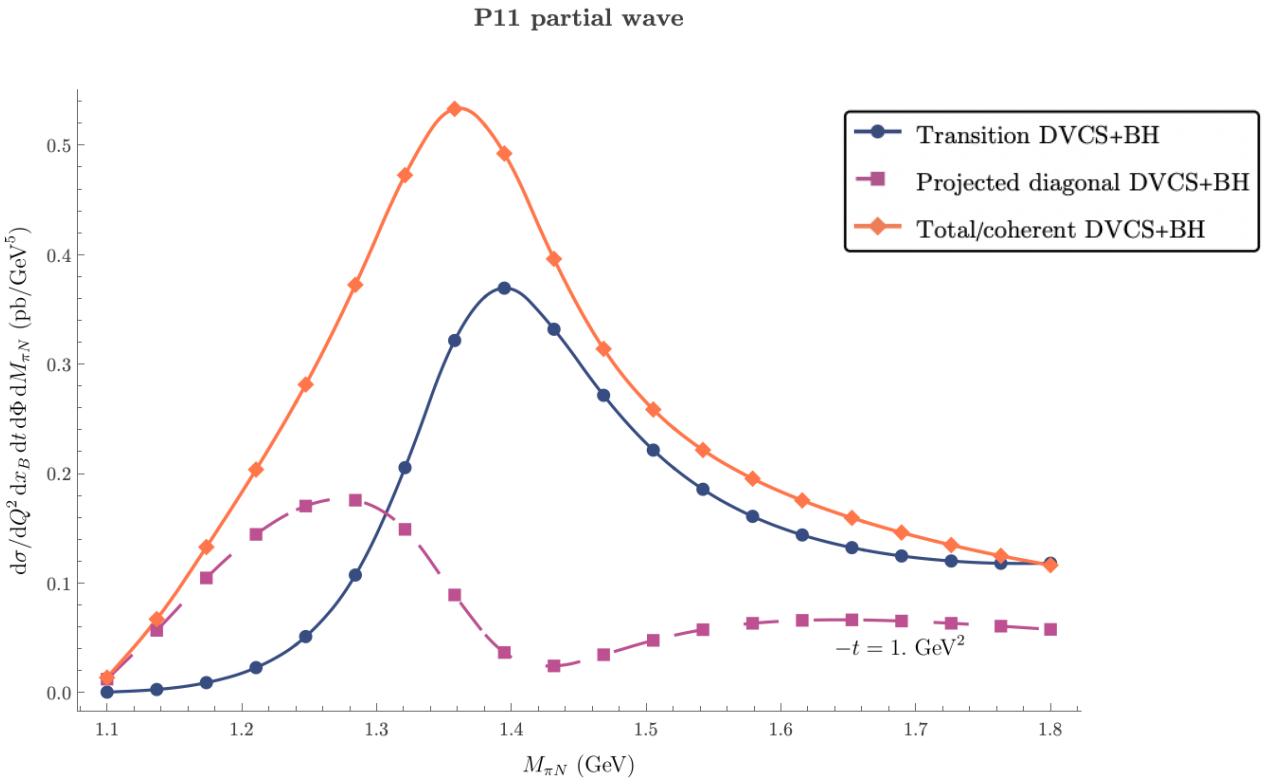
nor the S_{11}

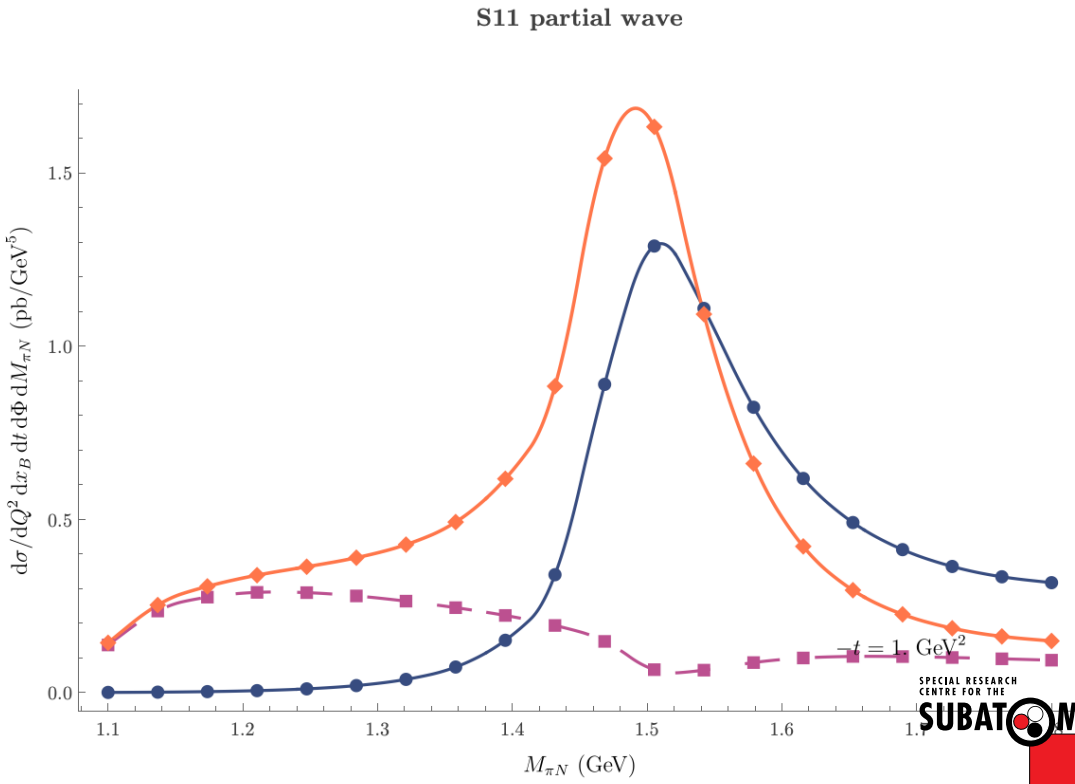
Isolating Partial Waves

Partial-wave selection isolates the relevant quantum numbers

Does not isolate the production mechanism

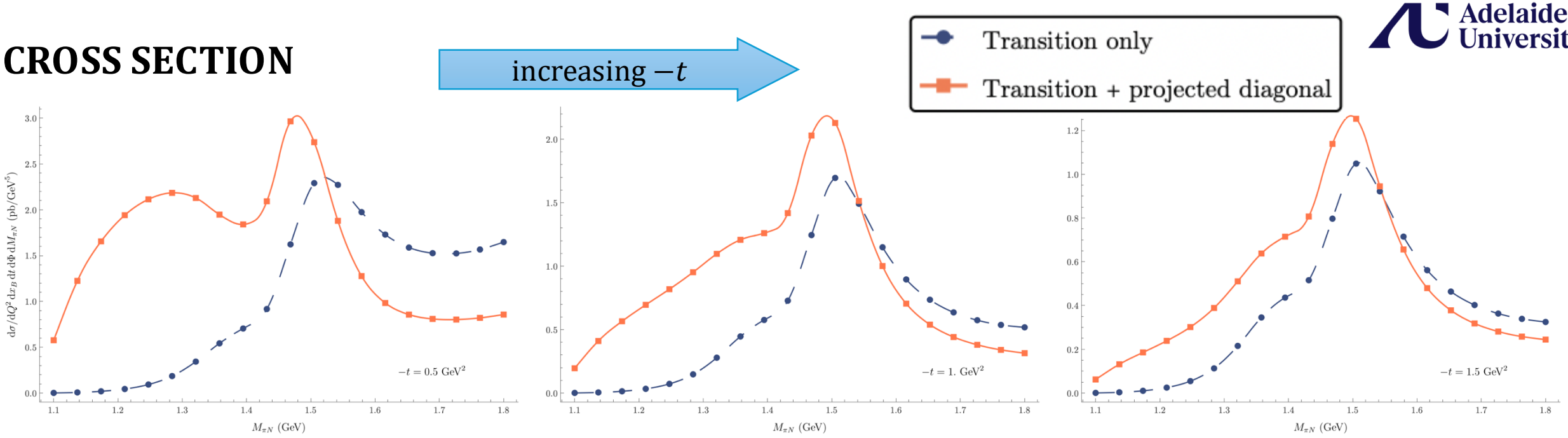
$\Longrightarrow$ bad for extracting GPDs



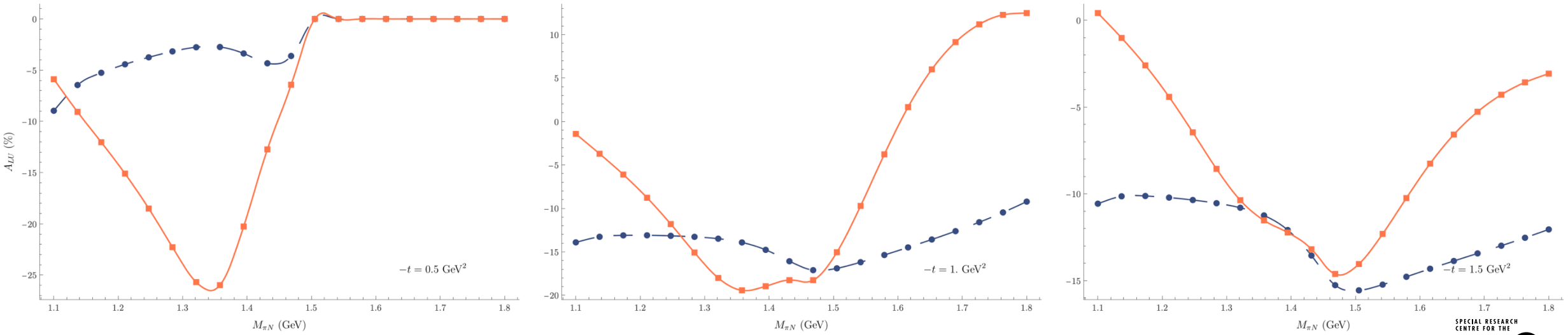


CROSS SECTION

increasing $-t$



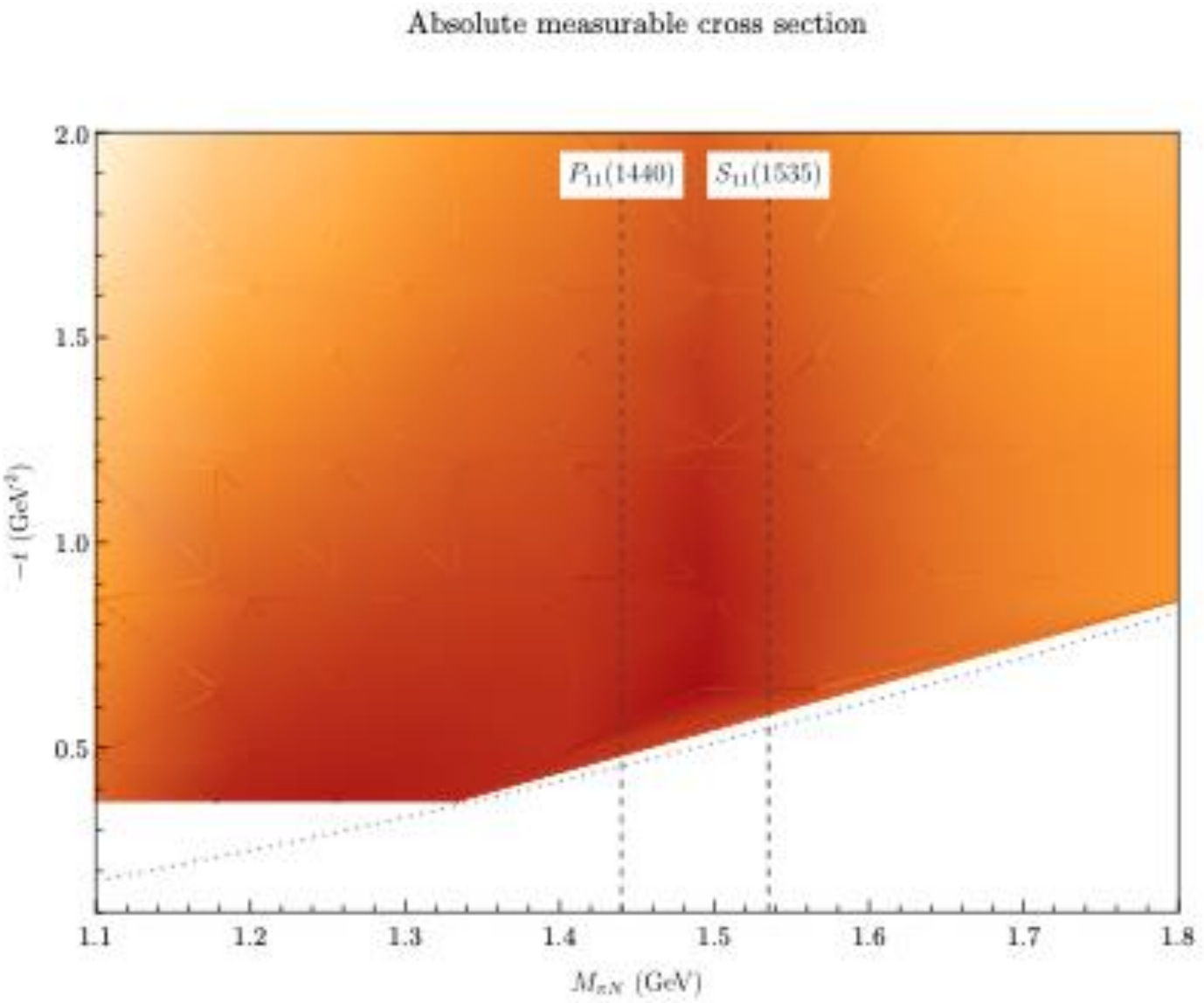
BSA



transition mechanism dominates at larger $-t$

BUT the event rate also decreases

ABSOLUTE CROSS SECTION

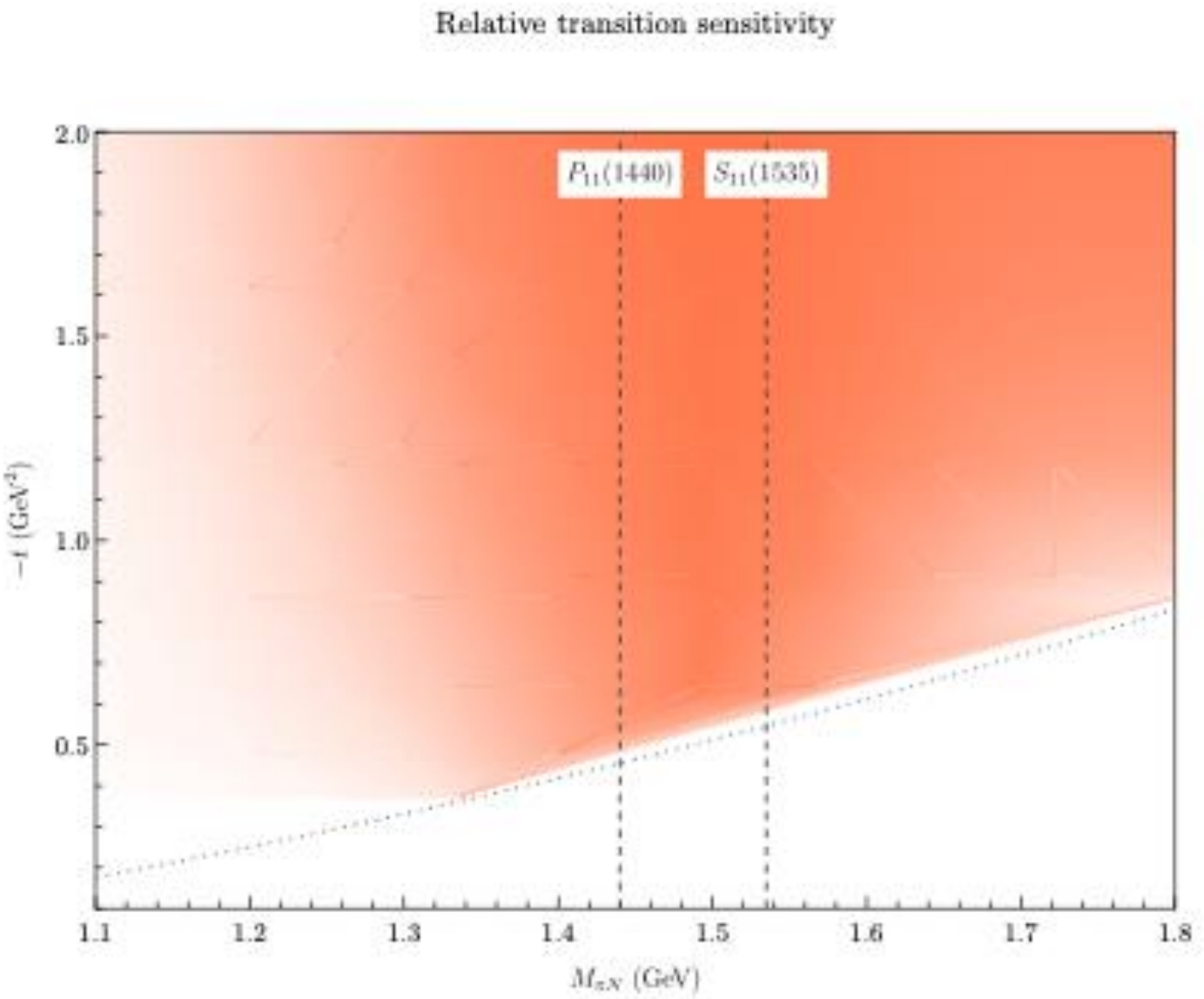


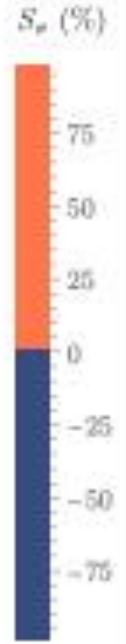


$\sigma_{\rm tot}(M_{\pi N}, t)$

unphysical regions are unmarked

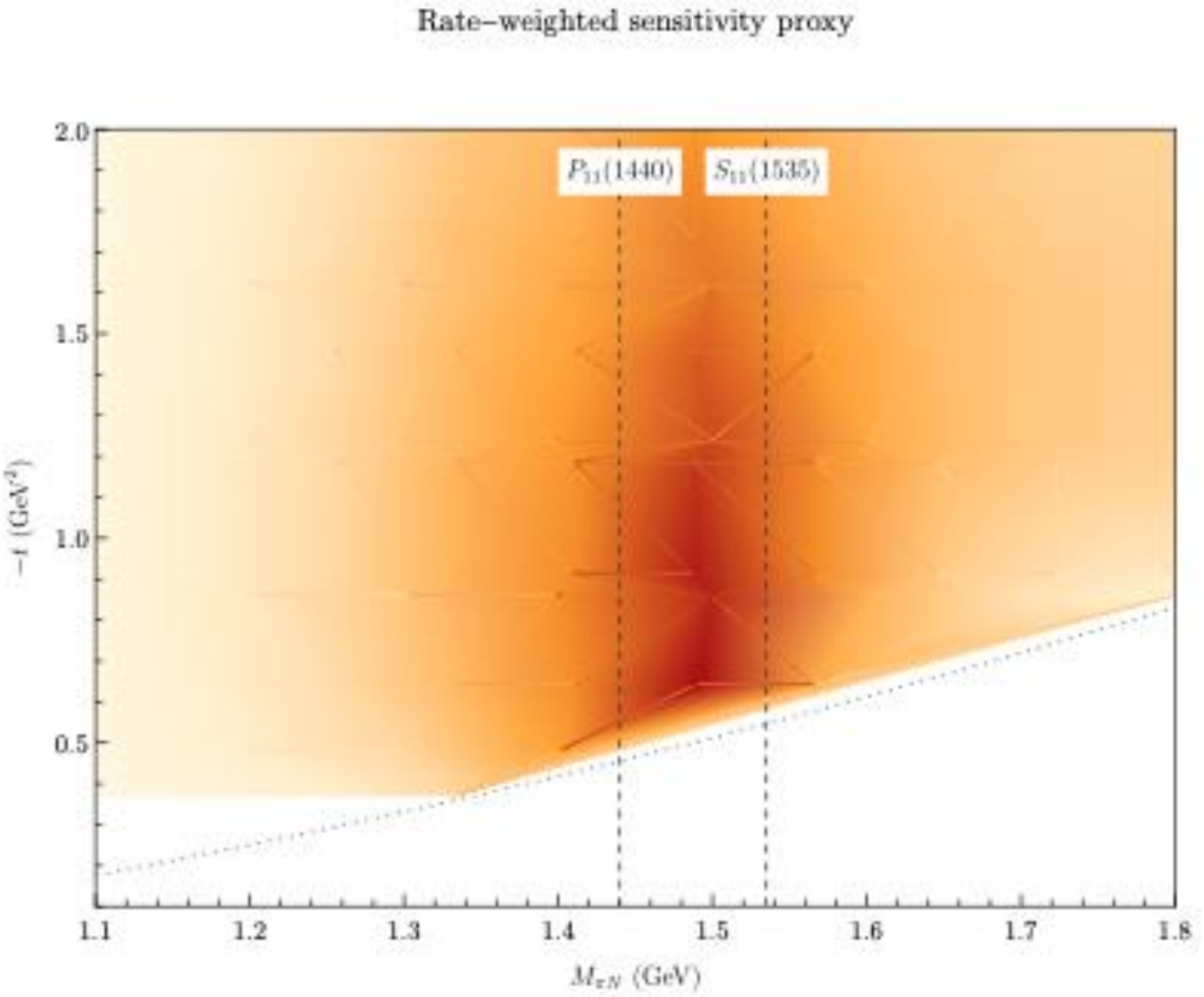
RELATIVE TRANSITION SENSITIVITY



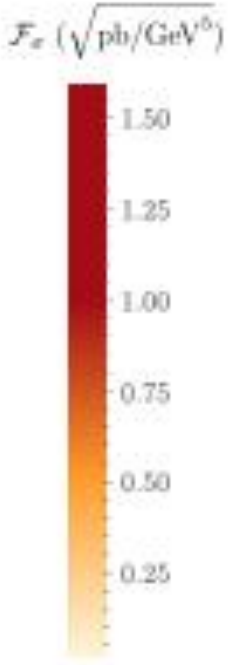


$$S_\sigma = \frac{\sigma_{\text{tot}} - \sigma_{\text{diag}}}{\sigma_{\text{tot}}} \times 100\%$$

measures the sensitivity to the transition mechanism



RATE-WEIGHTED TRANSITION SENSITIVITY



$$\mathcal{F}_\sigma = \frac{\left|\sigma_{\text{tot}} - \sigma_{\text{diag}}\right|}{\sqrt{\sigma_{\text{tot}}}}$$

indicates regions of high sensitivity while penalising regions with low event rates

NOT a direct observable

Future Work

- Vary GPD model parameters throughout the accepted range and explore how the sensitivity to the transition signal changes.
- Include nearby low-lying spin-$\frac{3}{2}$ resonances $\Delta(1232)$ and $D_{13}(1520)$.
- Since ηN couples strongly to the $S_{11}(1535)$ (with $\mathcal{B}_{\eta N}^{S_{11}} = 0.43$) but not to N, a similar reaction $e^- N \to e^- \gamma N \eta$ may provide stronger sensitivity to the $N \to S_{11}$ transition and motivate improvements to the transition GPD model for other low-lying resonances.

Summary

- Transition GPDs provide a new probe of nucleon resonance structure by extending the information available from transition form factors to partonic x, ξ, t dependence
- We cannot uniquely measure the transition mechanism and thus extract the transition GPD
  - Diagonal DVCS and BH processes both interfere
  - Partial-wave decomposition does not solve the extraction problem
- Sensitivity can be enhanced through kinematic selection
  - Increasing $-t$ improves relative transition sensitivity, at the cost of event rate
  - Cross section and BSA provide complementary sensitivity

The challenge is not simply to observe the resonance — it is to identify where the measured observables retain sensitivity to its transition GPD.

Probing Nucleon Resonance Structure with Transition GPDs:

Exclusive $N\pi\gamma$ electroproduction, competing mechanisms, and final-state interactions

Matthew Rumley

Adelaide University

BACKUP SLIDES

Further Reading

Inspired by:

- Semenov-Tian-Shansky, K.M., Vanderhaeghen, M. *Deeply virtual Compton process* $e^- N \rightarrow e^- \gamma \pi N$ *to study nucleon to resonance transitions*. Phys. Rev. D 108, 034021. https://doi.org/10.1103/PhysRevD.108.034021

Initial Results presented in:

- Rumley, M, Thomas, A. W. *Extracting nucleon resonance transition GPDs from* $e^- N \rightarrow e^- \gamma \pi N$ *deeply virtual Compton scattering*. Phys. Rev. D 113, 054007. https://doi.org/10.1103/j8cn-6m31

Transition Data from:

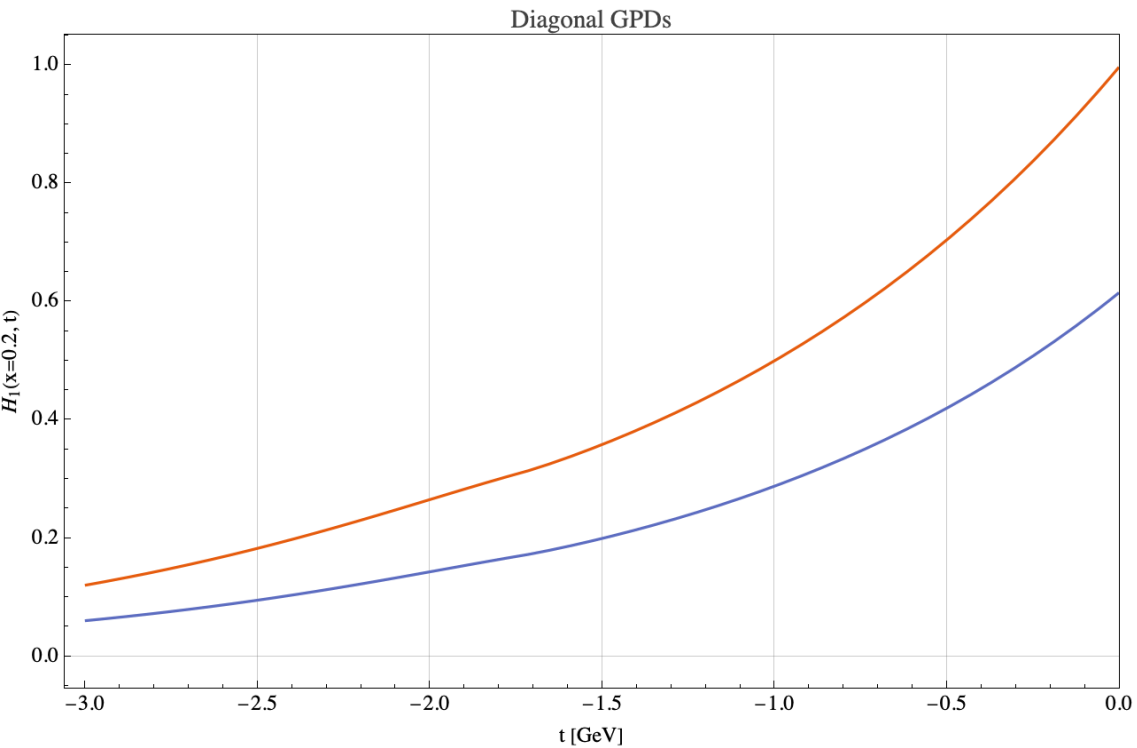
- Drechsel, D., Kamalov, S.S., Tiator, L. 2007 *Unitary Isobar Model - MAID2007*. Eur.Phys.J.A, 34 69-97 (2007). https://doi.org/10.1140/epja/i2007-10490-6
- Tiator, L., Drechsel, D., Kamalov, S.S. et al. *Electromagnetic excitation of nucleon resonances.* Eur. Phys. J. Spec. Top. 198, 141–170 (2011). https://doi.org/10.1140/epjst/e2011-01488-9

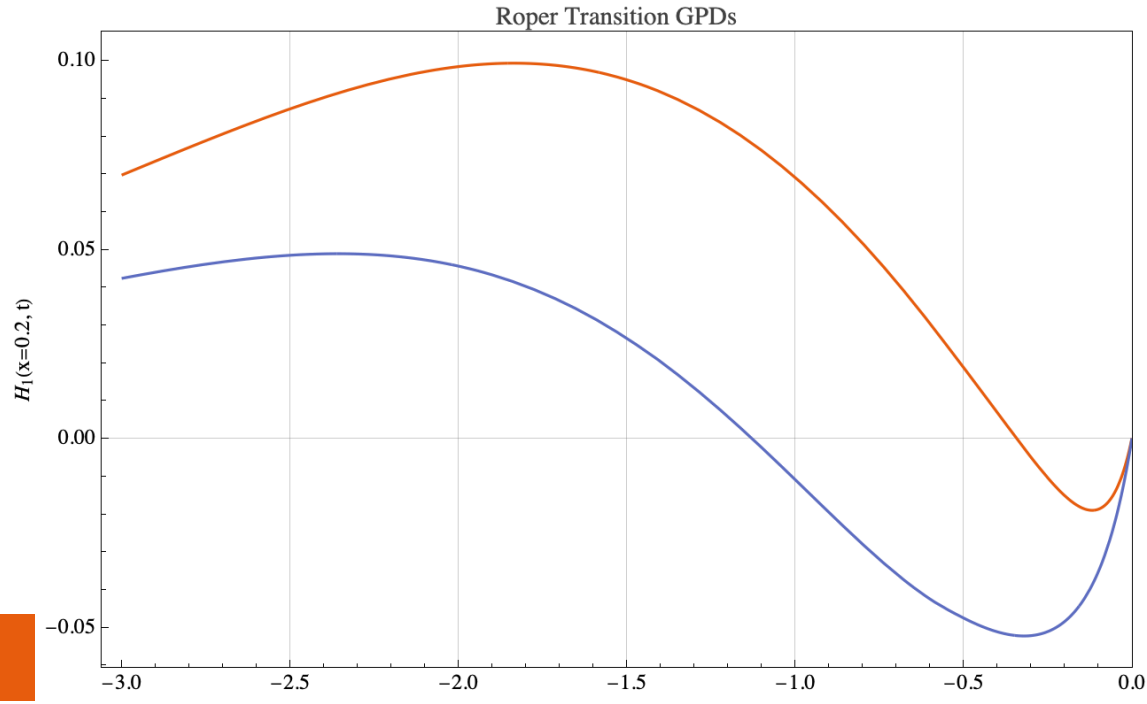
$$H_1(0.2, \xi', t)$$

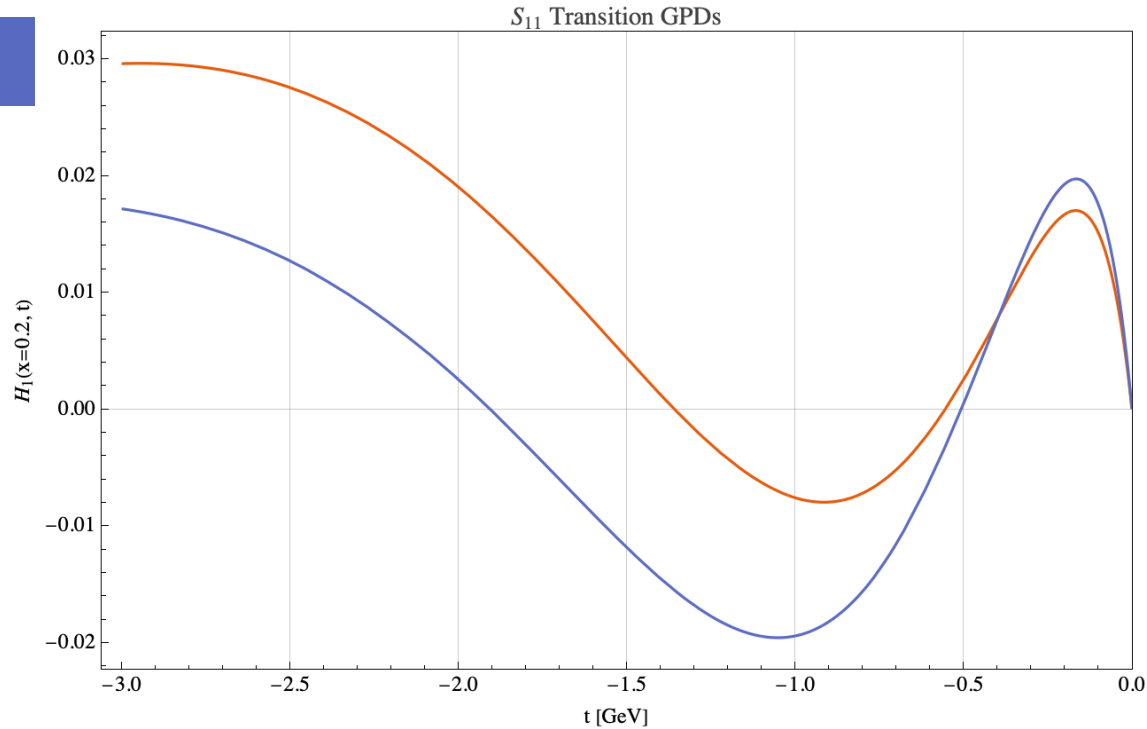
where $\xi \to \xi' = \frac{x}{2-x}\left(1 + \frac{M_2^2 - M_1^2 - t}{Q^2}\right)$ is the Björken limit.

proton

neutron





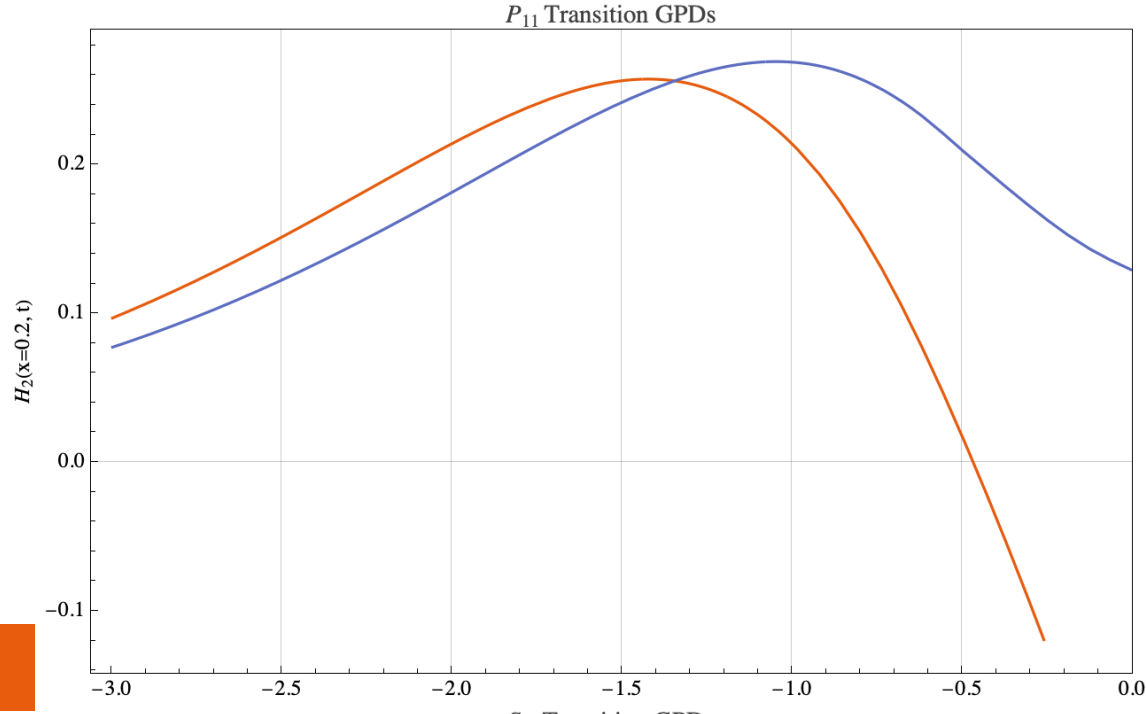


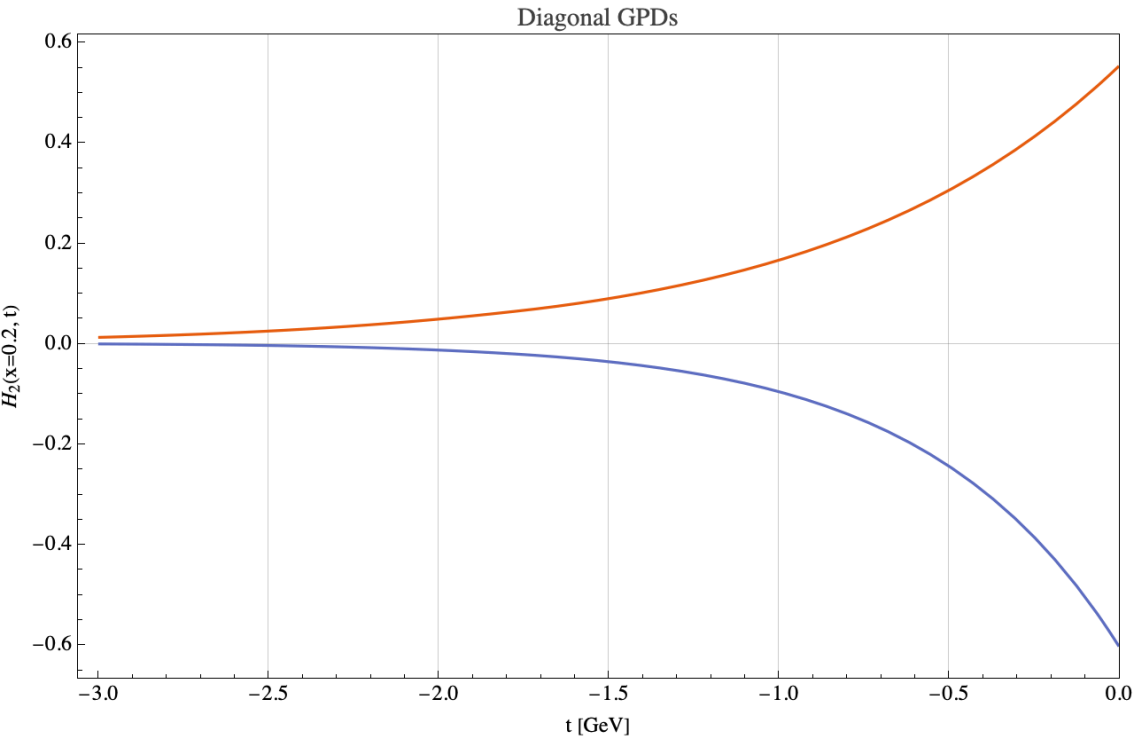
$H_2(0.2, \xi', t)$

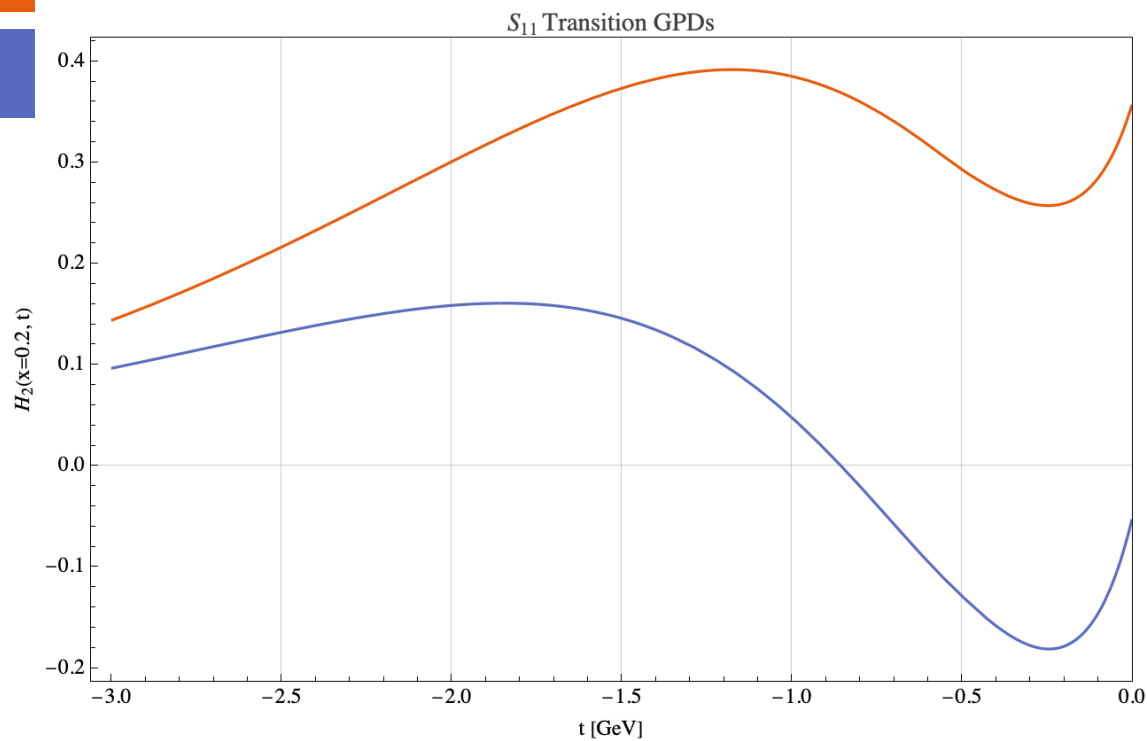
where $\xi \to \xi' = \frac{x}{2-x}\left(1 + \frac{M_R^2 - M_N^2 - t}{Q^2}\right)$
is the Björken limit.

proton

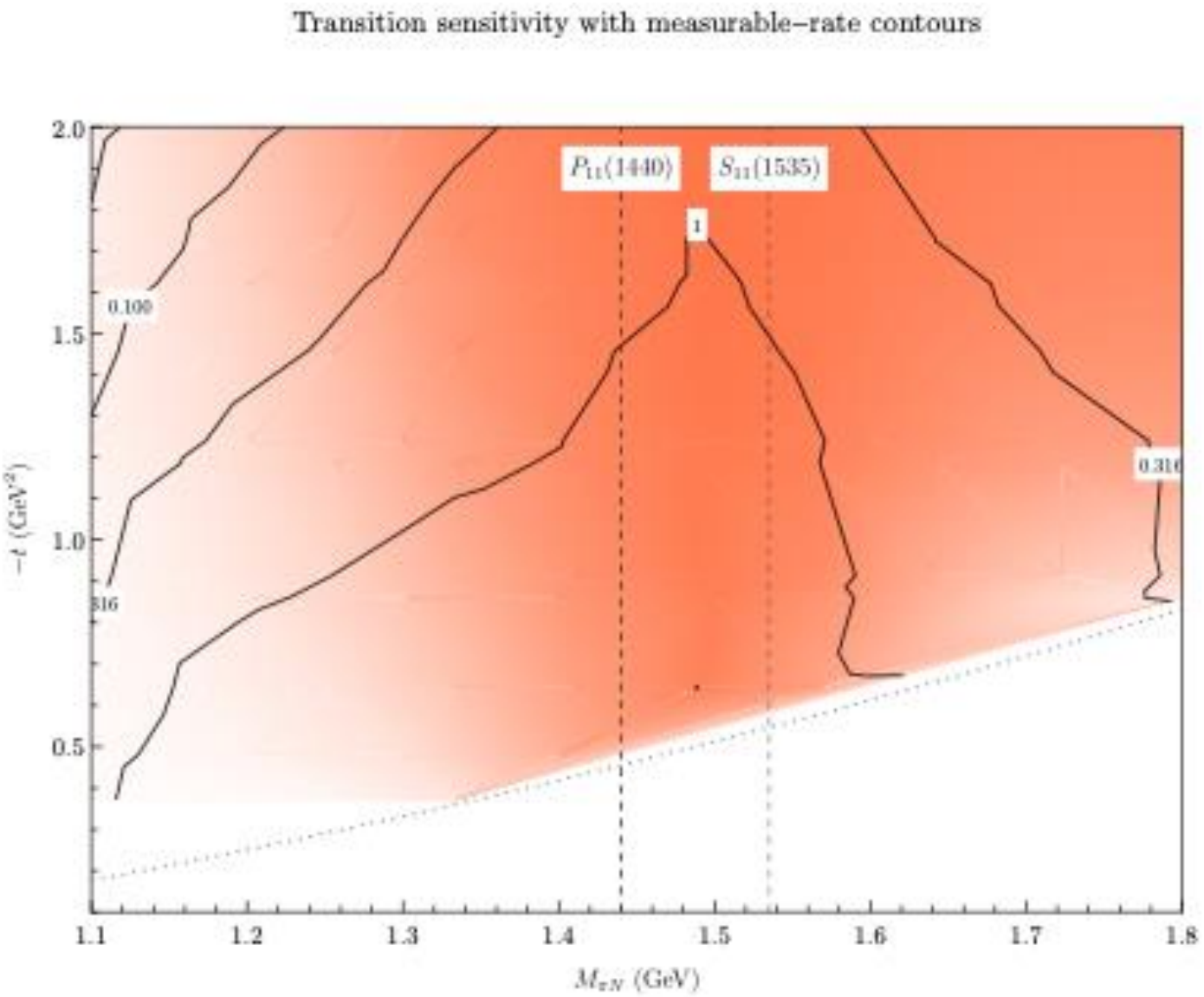
neutron

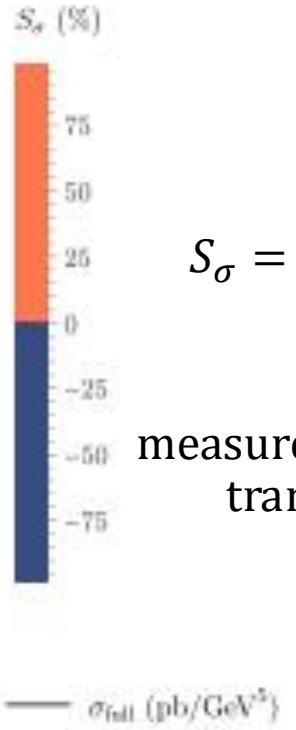






RELATIVE TRANSITION SENSITIVITY

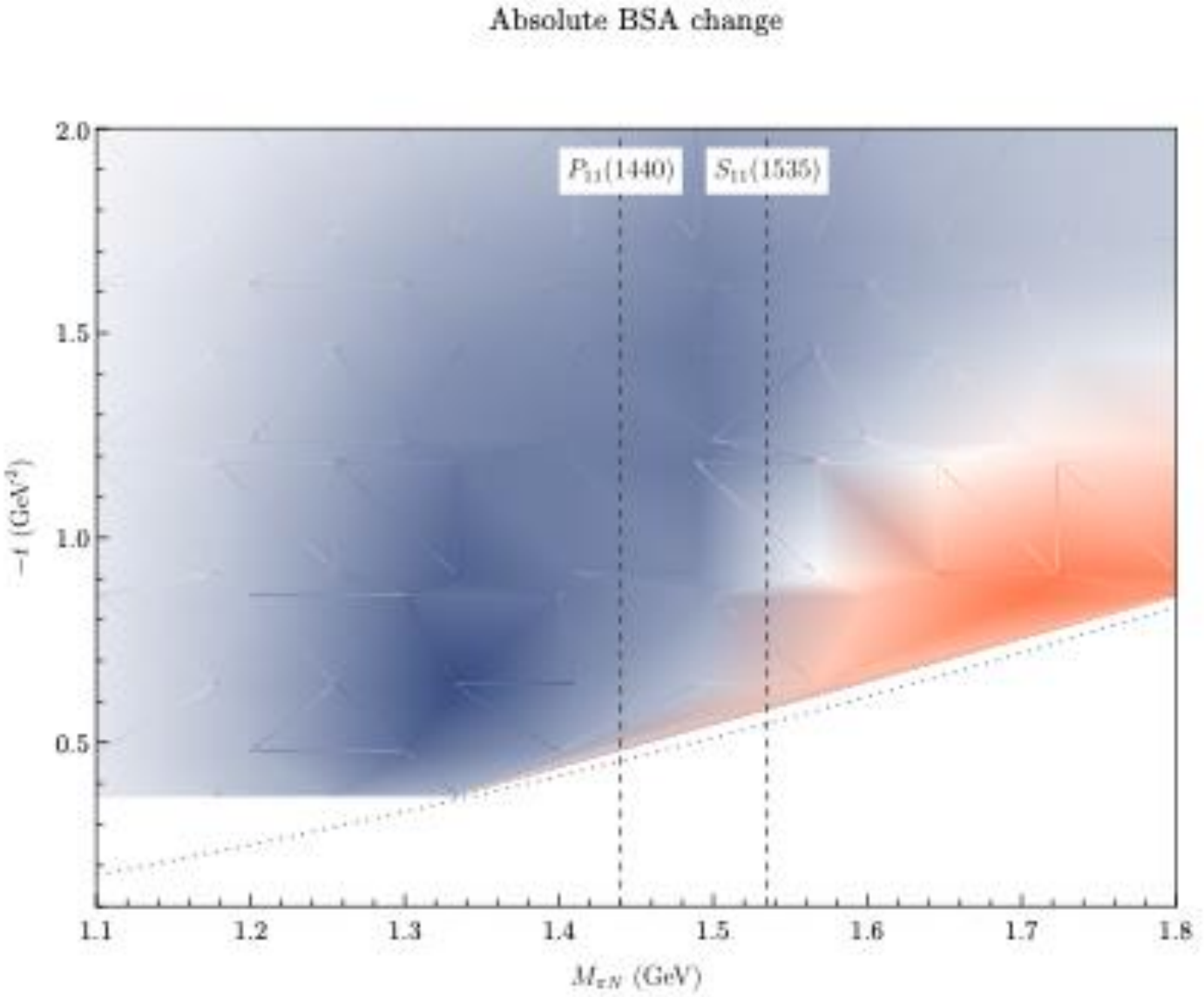




$$S_\sigma = \frac{\sigma_{\text{tot}} - \sigma_{\text{diag}}}{\sigma_{\text{tot}}} \times 100\%$$

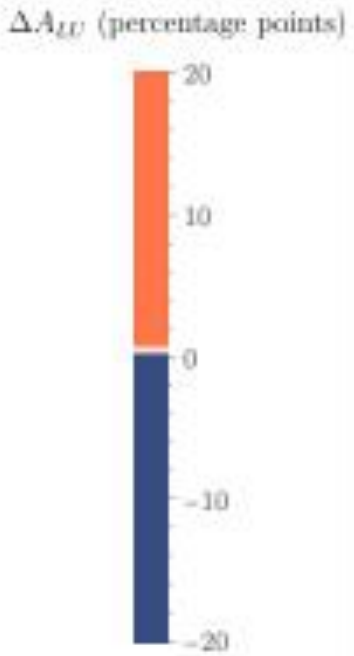
measures the sensitivity to the transition mechanism

now includes contours of constant number of events



BSA SENSITIVITY

$$\Delta A_{LU} = \left(A_{LU}^{\text{tot}} - A_{LU}^{\text{diag}} \right) \times 100\%$$



Axial Sector Inputs

Dipole form

$$G_A^{NR}(t) = \frac{G_A^{NR}(0)}{\left(1 - \frac{t}{M_{A,R}^2}\right)^2}$$

Transition	$M_{A,R}$ [GeV]	Axial isoscalar ratio
$N \to P_{11}$	1.00	3/5
$N \to S_{11}$	1.00	3/5

Pseudoscalar Transition from PCAC and π-pole dominance

$$G_P^{NP_{11}}(t) \simeq \frac{(M_{P_{11}} + M_N)^2}{m_\pi^2 - t} G_A^{NP_{11}}(t),$$

$$G_P^{NS_{11}}(t) \simeq \frac{M_{S_{11}}^2 - M_N^2}{m_\pi^2 - t} G_A^{NS_{11}}(t).$$