Nuclear Gravitational Form Factor

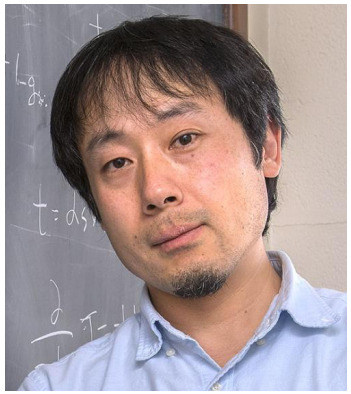
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Nuclear Gravitational Form Factor

- **Y. Hatta (BNL), T. Oishi (Ibaraki Kosen), M. Oka**
  "Mass radius and D-term of atomic nuclei in relativistic mean field theory", Phys. Rev. C114, 024343 (2026), arXiv:2605.12023



Yoshitaka Hatta



Tomohiro Oishi

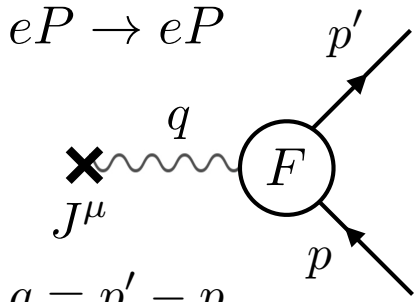
– Mechanical (energy-momentum) structure of nuclei studied by relativistic mean field theory (Walecka model) with Dirac nucleons + (S+V) meson mean field

EM Form Factor

- **EM form factors of proton (P)**

$$\langle p' | J^\mu(0) | p \rangle = \bar{u}(p') \left[\gamma^\mu F_1(t) + \frac{i\sigma^{\mu\nu} q_\nu}{2M} F_2(t) \right] u(p)$$

EM current: source of EM fields



$q = p' - p$

$t = q^2$

- **Electric form factor ⇔ FT of charge density (at Breit frame)**

$$G_E(t) = F_1(t) + \frac{t}{4M^2} F_2(t)$$

$$= \int \rho_{\rm ch}(\boldsymbol{r}) e^{i\boldsymbol{q}\cdot\boldsymbol{r}}\, d\boldsymbol{r} = Q - \frac{1}{6}\boldsymbol{q}^2 \int r^2 \rho_{\rm ch}(\boldsymbol{r})\, d\boldsymbol{r} + \ldots$$

total charge *mean square radius*

- **Magnetic form factor ⇔ magnetic moment**

$$G_M(t) = F_1(t) + F_2(t) \qquad G_M(0) = 2.79$$

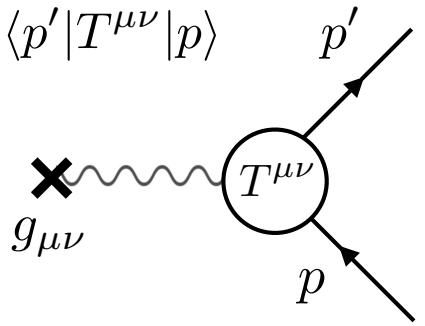
- **EM form factors represent charge and current distributions.**

Gravitational Form Factor (GFF)

- **Source of Gravitation: Energy-momentum tensor (EMT)**

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

- **$T^{\mu\nu}$ ($\mu \leftrightarrow \nu$) symmetric tensor (BR improved)**



energy density *momentum density*

$$\begin{pmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{pmatrix}$$

trace-part: pressure
traceless-part: shear

stress tensor (momentum flux)

- **Gravitational Form Factors of proton:**

$$\langle p'|T^{\mu\nu}|p\rangle = \bar{u}(p')\left[A(t)\gamma^{(\mu}P^{\nu)} + B(t)\frac{P^{(\mu}i\sigma^{\nu)\rho}q_\rho}{2M} + D(t)\frac{q^\mu q^\nu - g^{\mu\nu}q^2}{2M}\right]u(p)$$

Form factors: $A(t), B(t), D(t)$ $\quad P = \frac{p+p'}{2}, q = p' - p,\ t = q^2$

Gravitational Form Factor (GFF)

- **Many studies for the proton (+other hadrons)**

  ***Review*: M. V. Polyakov, P. Schweitzer, *Forces inside hadrons: pressure, surface tension, mechanical radius and all that*, Int. J. Mod. Phys. A 33 (2018) 1830025, arXiv:1805.06596.**

- **Model calculations (examples)**

  M. J. Neubelt, A. Sampino, J. Hudson, K.Tezgin, P. Schweitzer, *Energy momentum tensor and the D term in the bag model*, PR D101 (2020) 034013.

  M. Fujita, Y. Hatta, S. Sugimoto, T. Ueda, *Nucleon D-term in holographic quantum chromodynamics*, PTEP 2022 (2022) 093B06.

- **Other studies:**

  – Individual contributions of quarks and gluons, scale invariant and non-inv. terms, *etc.*

  – Relations to Generalized Parton Distributions (GPD)

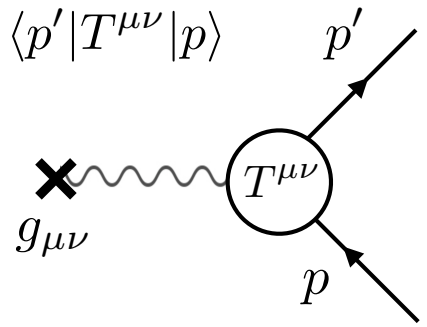
Gravitational Form Factor (GFF)

GPD - GFF correspondence

- **GPD (quarks) from DVCS**

  $H_q(x,\xi,t)$ **and** $E_q(x,\xi,t)$ **(unpolarized)**

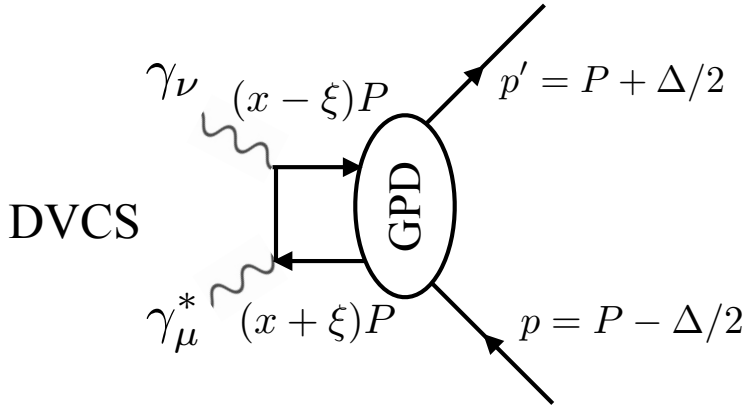
  (helicity-non-flip) (flip)



- **GPDs are related to the EM form factor and GFF (quark part) as**

$$\sum_q e_q \int_{-1}^{1} dx H_q(x,\xi,t) = F_1(t)$$

$$\int_{-1}^{1} dx\, x H_q(x,\xi,t) = A_q(t) + \xi^2 D_q(t)$$



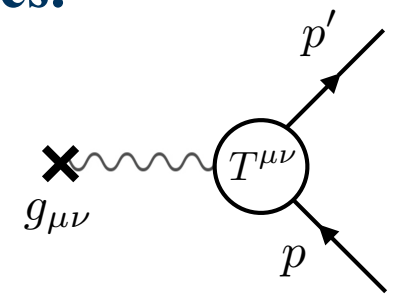
$$\Delta = p' - p$$

$$t = \Delta^2$$

Why? Nuclear GFF

- **Nucleus is an isolated self-bound system by the strong interaction. Its mechanical structure is interesting because the nucleus shows various quantum phenomena, such as deformation, effects of non-central spin-orbit and tensor forces, paring correlations, clustering, and so on.**
- **Nuclear GFFs represent distributions of mechanical forces and structures and bring new aspects in nuclear physics.**
- **For $J = 0$ nuclei**

$$\langle p'|T^{\mu\nu}|p\rangle = 2A(t)P^\mu P^\nu + \frac{1}{2}D(t)(q^\mu q^\nu - g^{\mu\nu}q^2)$$



$$P = \frac{p+p'}{2}, q = p' - p,\ t = q^2$$

$A(t)$ and $D(t)$ are GFFs of nuclei

- **In the forward scattering limit:**

  momentum conservation $A(t=0) = 1$

  new constant (D-term) $D(t=0) = D$

Nuclear GFF

- **Relativistic mean field (RMF) formalism of nuclear structure**

  Serot-Walecka (1979) (Walecka model)

  nucleons as Dirac particles

  scalar mean field (attraction) + vector mean field (repulsion)

  ⇒ nucleon effective mass, relativistic spin-orbit force

- **Local Lagrangian form gives covariant energy-momentum tensor and its expectation values as functions of spatial coordinates.**

- **Realistic model**

  Niksic, Vretenar, Ring, *PPNP 66, 519 (2011)*

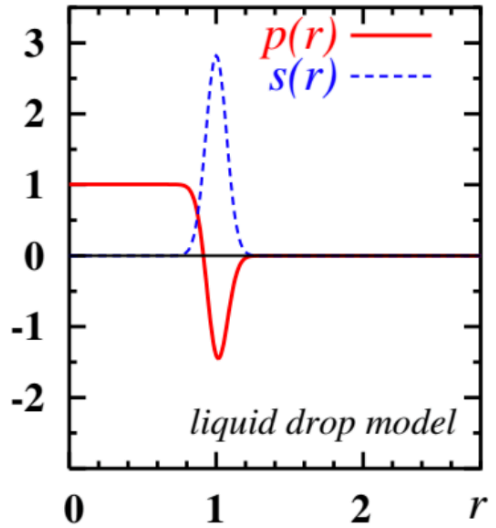
  DIRHB program code with a parameter set DD-ME2

  by *Niksic et al., Comp. Phys. Comm. 185, 1808 (2014)*

Nuclear GFF

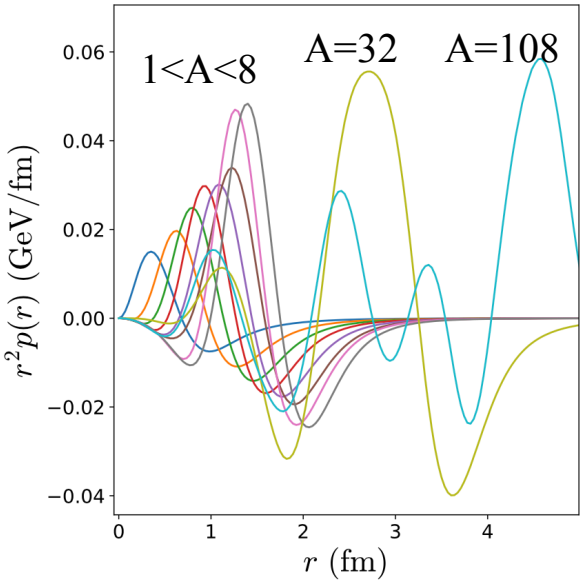
- **Liquid drop model**
  M. V. Polyakov, *Phys. Lett. B 555, 57 (2003)*

$$D = -\frac{16\pi}{15} M\gamma R^4 \propto A^{7/3}$$



- **Skyrme model**
  Garcia Martín-Caro, Huidobro, Hatta
  PR D 110 (2024) 034002

- **Deuteron**
  Freese, Cosyn, *PR D 106 (2022) 114013*
  He, Zahed, *PR C 109 (2024) 045209*



- **Walecka model**
  Guzey, Siddikov, *J. Phys. G32, 251 (2006)*

Relativistic Mean Field

- **Nuclear Lagrangian**

  for (point Dirac) nucleons, and mean-fields σ, ω, and ρ.

$$\begin{aligned}\mathcal{L} &= \bar{\psi}(i\partial\!\!\!/ - m)\psi - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{m_\omega^2}{2}\omega_\mu\omega^\mu - g_\omega\omega_\mu\bar{\psi}\gamma^\mu\psi \\ &-\frac{1}{4}\vec{R}_{\mu\nu}\cdot\vec{R}^{\mu\nu} + \frac{m_\rho^2}{2}\vec{\rho}_\mu\cdot\vec{\rho}^\mu - g_\rho\vec{\rho}_\mu\cdot\bar{\psi}\gamma^\mu\vec{\tau}\psi \\ &+\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{m_\sigma^2}{2}\sigma^2 - g_\sigma\sigma\bar{\psi}\psi\end{aligned}$$

- **Eq. motion for static mean fields of mesons**

$$i\gamma^\mu\partial_\mu\psi - (m + g_\sigma\sigma)\psi - (g_\omega\omega_\mu + g_\rho\vec{\rho}_\mu\cdot\vec{\tau} + \Sigma_\mu^R)\gamma^\mu\psi = 0$$

$$(\boldsymbol{\nabla}^2 - m_\sigma^2)\sigma = g_s\sum\bar{\psi}\psi \qquad (\boldsymbol{\nabla}^2 - m_V^2)V_\mu = -g_V\sum\bar{\psi}\gamma_\mu\psi$$

rearrangement term Σ_μ^R generated by density dependent coupling constants according to DIRHB (DD-ME2).

Relativistic Mean Field

- **DD-ME2: Density dependent couplings to fit observed nuclear binding energies and radii.**

$$g_\sigma = g_\sigma(\rho_B) = g_\sigma(\rho_{B,sat}) a_\sigma \frac{1 + b_\sigma (x + d_\sigma)^2}{1 + c_\sigma (x + d_\sigma)^2}$$

$$g_\omega = g_\omega(\rho_B) = g_\omega(\rho_{B,sat}) a_\omega \frac{1 + b_\omega (x + d_\omega)^2}{1 + c_\omega (x + d_\omega)^2},$$

$$g_\rho = g_\rho(\rho_B) = g_\rho(\rho_{B,sat}) \exp\left[-a_\rho (x - 1)\right],$$

$$x = \frac{\rho_B(r)}{\rho_{B,\mathrm{sat}}}$$

m [MeV]	m_σ [MeV]	m_ω [MeV]	m_ρ [MeV]	$g_{\sigma,sat}$	$g_{\omega,sat}$	$g_{\rho,sat}$
939	550.1238	783	763	10.5396	13.0189	3.6836

a_σ	b_σ	c_σ	d_σ	a_ω	b_ω	c_ω	d_ω	a_ρ
1.3881	1.0943	1.7057	0.4421	1.3892	0.9240	1.4620	0.4775	0.5647

Relativistic Mean Field

- **Solutions for**

  Even-even nuclei ($J = 0$) with *spherical* mean fields,

  $$\sigma(r),\ \omega(r) = \langle \omega^0 \rangle,\ \rho(r) = \langle \rho_3^0 \rangle$$

- **Partial wave solutions**

  $$\psi_a = \begin{pmatrix} f_a(r)\Phi^{\pm}_{jm} \\ i g_a(r)\Phi^{\mp}_{jm} \end{pmatrix} \qquad j = \ell \pm \frac{1}{2} \qquad \kappa = \mp\left(j + \frac{1}{2}\right)$$

  spinor spherical harmonics

  $$\Phi^{\pm}_{jm}(\theta,\varphi) = \sum_{m_s,m_l} \langle \frac{1}{2}, m_s; l = j \mp \frac{1}{2}, m_l \,|\, jm \rangle \, \chi_{m_s} Y_{l m_l}(\theta,\varphi)$$

- **Slater determinant wave function for *A*=*Z*+*N* nucleons**

  (No deformation, no paring, no SR correlation)

Energy-Momentum tensor

- **Energy momentum tensor for the RMFT**

$$T_{\mu\nu} = \bar{\psi} i\gamma_{(\mu} \overleftrightarrow{\partial}_{\nu)} \psi - g_{\mu\nu}\bar{\psi}\Sigma^R_\alpha\gamma^\alpha\psi \quad + \partial_\mu\sigma\partial_\nu\sigma - g_{\mu\nu}\left(\frac{1}{2}\partial_\alpha\sigma\partial^\alpha\sigma - \frac{m_\sigma^2}{2}\sigma^2\right)$$
$$- \Omega_{\mu\rho}\Omega_\nu{}^\rho + m_\omega^2\omega_\mu\omega_\nu - g_\omega\omega_{(\mu}\bar{\psi}\gamma_{\nu)}\psi - g_{\mu\nu}\left(-\frac{1}{4}\Omega_{\alpha\beta}\Omega^{\alpha\beta} + \frac{m_\omega^2}{2}\omega_\alpha\omega^\alpha\right)$$
$$- \vec{R}_{\mu\rho}\cdot\vec{R}_\nu{}^\rho + m_\rho^2\vec{\rho}_\mu\cdot\vec{\rho}_\nu - g_\rho\vec{\rho}_{(\mu}\cdot\bar{\psi}\gamma_{\nu)}\vec{\tau}\psi - g_{\mu\nu}\left(-\frac{1}{4}\vec{R}_{\alpha\beta}\cdot\vec{R}^{\alpha\beta} + \frac{m_\rho^2}{2}\vec{\rho}_\alpha\cdot\vec{\rho}^\alpha\right)$$

- **Mean fields for "spherical" $N \neq Z$ nuclei,**

$$\mathcal{H} = T_{00} = \sum_{a=1}^{A}\epsilon_a\psi_a^\dagger(r)\psi_a(r) + \left[\frac{1}{2}\left(\frac{d\omega}{dr}\right)^2 + \frac{m_\omega^2}{2}\omega^2 - g_\omega\omega\bar{\psi}\gamma^0\psi\right] + \left[\frac{1}{2}\left(\frac{d\sigma}{dr}\right)^2 + \frac{m_\sigma^2}{2}\sigma^2\right]$$
$$+ \left[\frac{1}{2}\left(\frac{d\rho}{dr}\right)^2 + \frac{m_\rho^2}{2}\rho^2 - g_\rho\rho\bar{\psi}\gamma^0\tau^3\psi\right]$$

$$M(\text{nucleus}) = \int T^{00}d^3\boldsymbol{r} = Am + (-B)$$

$$T_\mu^\mu = m\bar{\psi}\psi - m_\omega^2\omega^2 + \left(\frac{d\sigma}{dr}\right)^2 + 2m_\sigma^2\sigma^2 + g_\sigma\sigma\bar{\psi}\psi - m_\rho^2\rho^2$$

Energy-Momentum tensor

- **Stress tensor decomposed into traceless, $s(r)$, and trace part, $p(r)$.**

$$T^{ij} = \left(\frac{r^i r^j}{r^2} - \frac{\delta^{ij}}{3} \right) s(r) + \delta^{ij} p(r)$$

- **Pressure and shear force distributions**

$$T_{ii} = 3p(r) = \frac{1}{4\pi} \sum_{a=1}^{A} \left[f_a(r) \frac{dg_a(r)}{dr} - \frac{df_a(r)}{dr} g_a(r) - 2\kappa_a \frac{f_a(r) g_a(r)}{r} \right]$$
$$+ \frac{1}{2} \left(\frac{d\omega}{dr} \right)^2 + \frac{3m_\omega^2}{2} \omega^2 - \frac{1}{2} \left(\frac{d\sigma}{dr} \right)^2 - \frac{3m_\sigma^2}{2} \sigma^2 + \frac{1}{2} \left(\frac{d\rho}{dr} \right)^2 + \frac{3m_\rho^2}{2} \rho^2$$

$$s(r) \equiv \frac{3}{2} \left(\frac{r^i r^j}{r^2} - \frac{\delta^{ij}}{3} \right) T_{ij} = \frac{1}{4\pi} \sum_{a=1}^{A} \left[f_a(r) \frac{dg_a(r)}{dr} - \frac{df_a(r)}{dr} g_a(r) + \kappa_a \frac{f_a(r) g_a(r)}{r} \right]$$
$$+ \left(\frac{d\sigma}{dr} \right)^2 - \left(\frac{d\omega}{dr} \right)^2 - \left(\frac{d\rho}{dr} \right)^2$$

$$\psi_a = \begin{pmatrix} f_a(r) \Phi_{jm}^{\pm} \\ i g_a(r) \Phi_{jm}^{\mp} \end{pmatrix} \qquad j = \ell \pm \frac{1}{2} \qquad \kappa_a = \mp \left(j + \frac{1}{2} \right)$$

Energy-Momentum tensor

- **Stability condition (von Laue condition)**

$$\int d^3r\, p(r) = \frac{1}{3}\int d^3r\, T_{ii}(r) = 0$$

- **Scale invariance**

  For a scale invariant theory, dilatation current $D^\mu = x_\nu T^{\mu\nu}$ **is conserved and the trace of EMT is zero.** $\partial_\mu D^\mu = T^\mu_\mu = 0$

- **The total energy (mass) is given by**

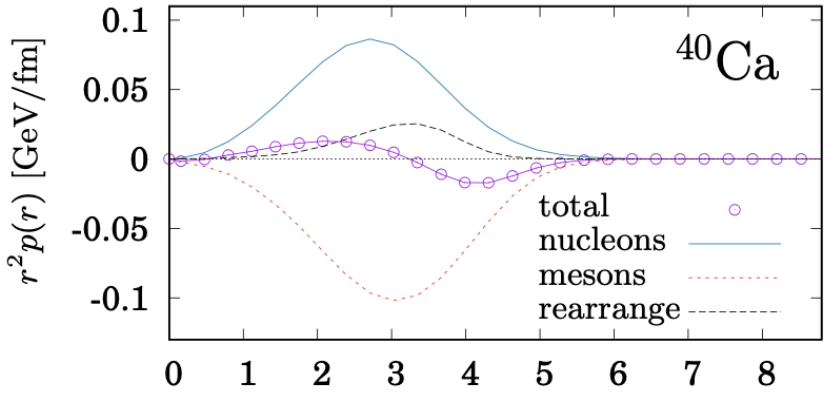
$$M = \int d^3r\, T^{00} = \int d^3r\, T^\mu_\mu$$

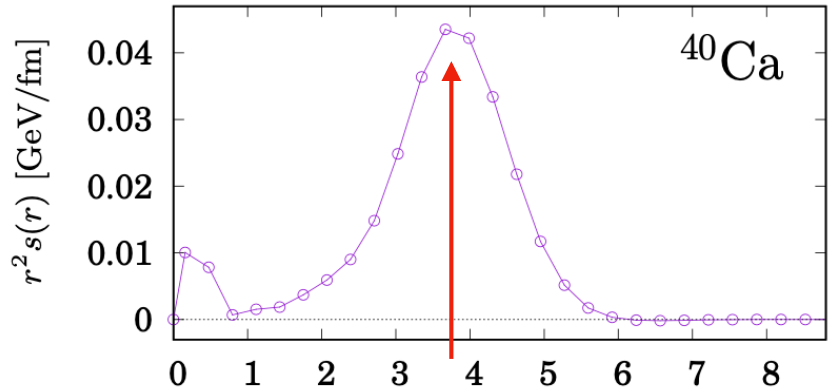
ex $$T^\mu_\mu(\mathrm{QCD}) = \sum_q m_q \bar{q}q + \frac{\beta(g)}{2g} G^{a\mu\nu} G^a_{\mu\nu}$$

RMF $$T^\mu_\mu = m\bar{\psi}\psi - m_\omega^2\omega^2 + \left(\frac{d\sigma}{dr}\right)^2 + 2m_\sigma^2\sigma^2 + g_\sigma\sigma\bar{\psi}\psi - m_\rho^2\rho^2$$

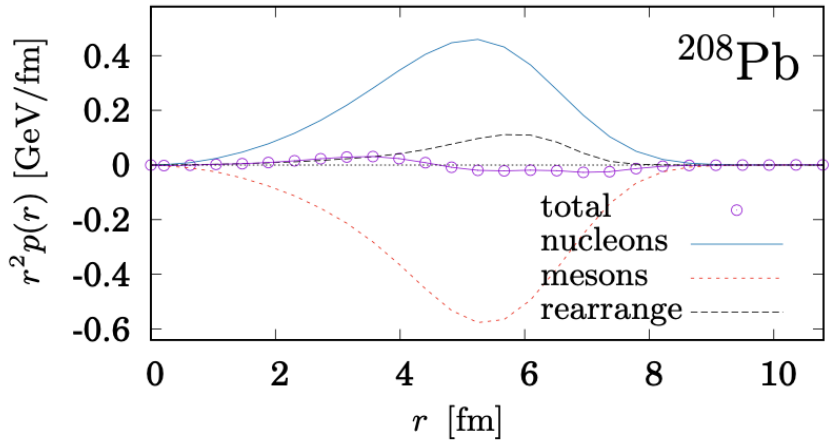
Energy-Momentum tensor

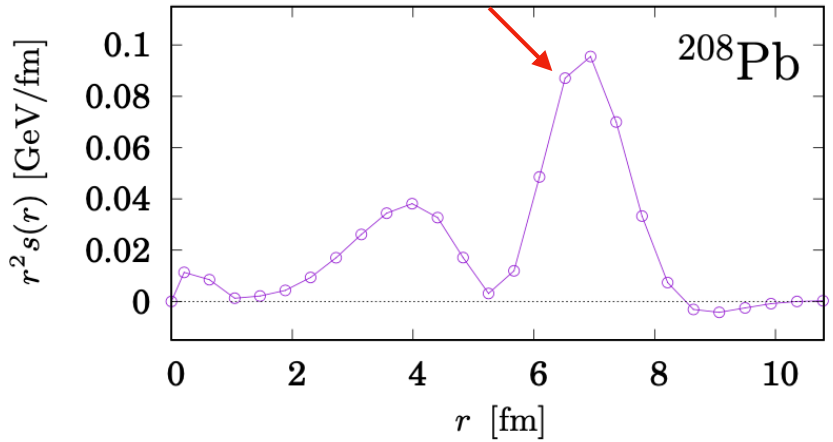
- **pressure and shear force distributions**





Contribution of the orbits with large *l*





prop to the matter density

large peak at the surface

Y. Hatta, T. Oishi, M. Oka, Phys. Rev. C 114, 024343 (2026)

Radius Zoo

Charge radius (proton)

$$\langle r^2 \rangle_c = \frac{\int d\boldsymbol{x} x^2 \rho_c(\boldsymbol{x})}{\int d\boldsymbol{x} \rho_c(\boldsymbol{x})} = \frac{6}{G_E(0)} \frac{dG_E(t)}{dt}\bigg|_{t=0}$$

Magnetic radius

$$\langle r^2 \rangle_M = \frac{6}{G_M(0)} \frac{dG_M(t)}{dt}\bigg|_{t=0}$$

Baryon number radius (proton+neutron)

$$\langle r^2 \rangle_B = \frac{\int d\boldsymbol{x} x^2 \rho_B(\boldsymbol{x})}{\int d\boldsymbol{x} \rho_B(\boldsymbol{x})}$$

Mass radius

$$\langle r^2 \rangle_m = \frac{\int d\boldsymbol{x}\, x^2 T^{00}(\boldsymbol{x})}{\int d\boldsymbol{x} T^{00}(\boldsymbol{x})} = 6\frac{dA(t)}{dt}\bigg|_{t=0} - \frac{3D(0)}{2M^2}$$

Scalar radius

$$\langle r^2 \rangle_s = \frac{\int d\boldsymbol{x}\, x^2 T^{\mu}_{\mu}(\boldsymbol{x})}{\int d\boldsymbol{x} T^{\mu}_{\mu}(\boldsymbol{x})} = 6\frac{dA(t)}{dt}\bigg|_{t=0} - \frac{9D(0)}{2M^2}$$

Tensor radius

$$\langle r^2 \rangle_t \equiv \frac{\int d\boldsymbol{x}\, x^2 \left(T^{00}(\boldsymbol{x}) + \frac{1}{2}T_{ii}(\boldsymbol{x})\right)}{\int d\boldsymbol{x} \left(T^{00} + \frac{1}{2}T_{ii}\right)} = 6\frac{dA(t)}{dt}\bigg|_{t=0}$$

Shear radius

$$\langle r^2 \rangle_{shear} = \frac{\int d^3r\, r^2 s(r)}{\int d^3r\, s(r)} = -\frac{15}{4M}\frac{D}{\int d^3r\, s(r)}$$

by Y. Hatta

Nuclear Radii and D term

- **Results for double-magic (closed shell) nuclei**
  RMS radii

charge baryon tensor mass scalar shear

	r_{em}	r_B	r_t	r_m	r_s	r_{shear}	D	$\mathcal{E}/A$
$^{16}_{8}$O	2.685	2.563	2.569	2.574	2.583	2.886	-90.723	-8.418
$^{40}_{20}$Ca	3.387	3.291	3.295	3.303	3.318	3.766	-930.453	-10.408
$^{68}_{28}$Ni	3.800	3.862	3.870	3.874	3.881	4.323	-1903.734	-10.665
$^{90}_{40}$Zr	4.153	4.158	4.166	4.170	4.178	4.586	-4123.965	-11.524
$^{132}_{50}$Sn	4.591	4.728	4.737	4.739	4.744	4.973	-4192.501	-11.091
$^{208}_{82}$Pb	5.323	5.448	5.454	5.459	5.468	5.926	-25374.209	-11.912

- **Order of the various radii**

$$r_B < r_t < r_m < r_s \ll r_{\text{shear}}$$

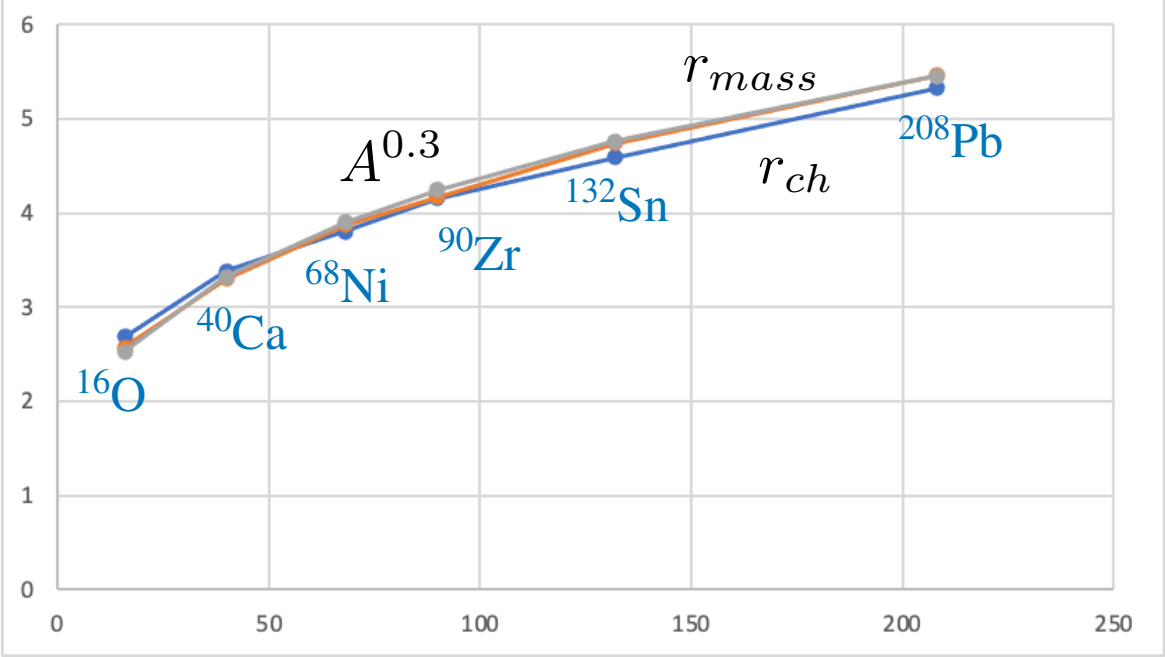
Shear distribution has large contribution from nuclear surface.

Y. Hatta, T. Oishi, M. Oka, Phys. Rev. C 114, 024343 (2026)

Charge/Mass Radii

- **Charge (proton) and Mass radii for doubly closed shell nuclei**

  They increase $\sim A^{0.3}$ for the mass number A.



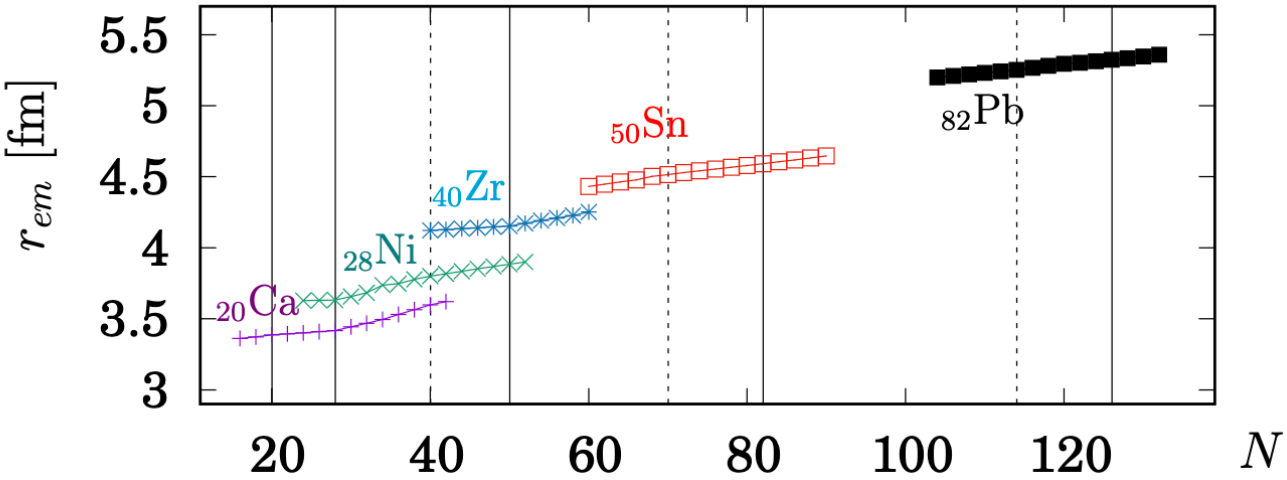
- **Nuclear saturation:**

  Volume $\propto$ Mass number, A, i.e., Radius, $R \propto A^{1/3}$

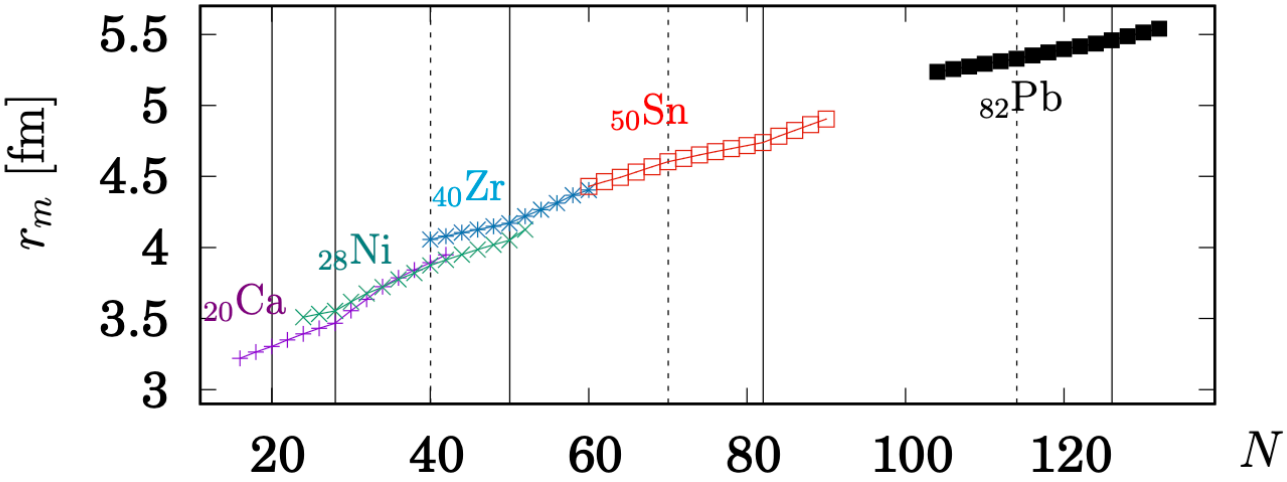
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Charge/Mass Radii

- Isotope dependencies of proton (charge) distribution



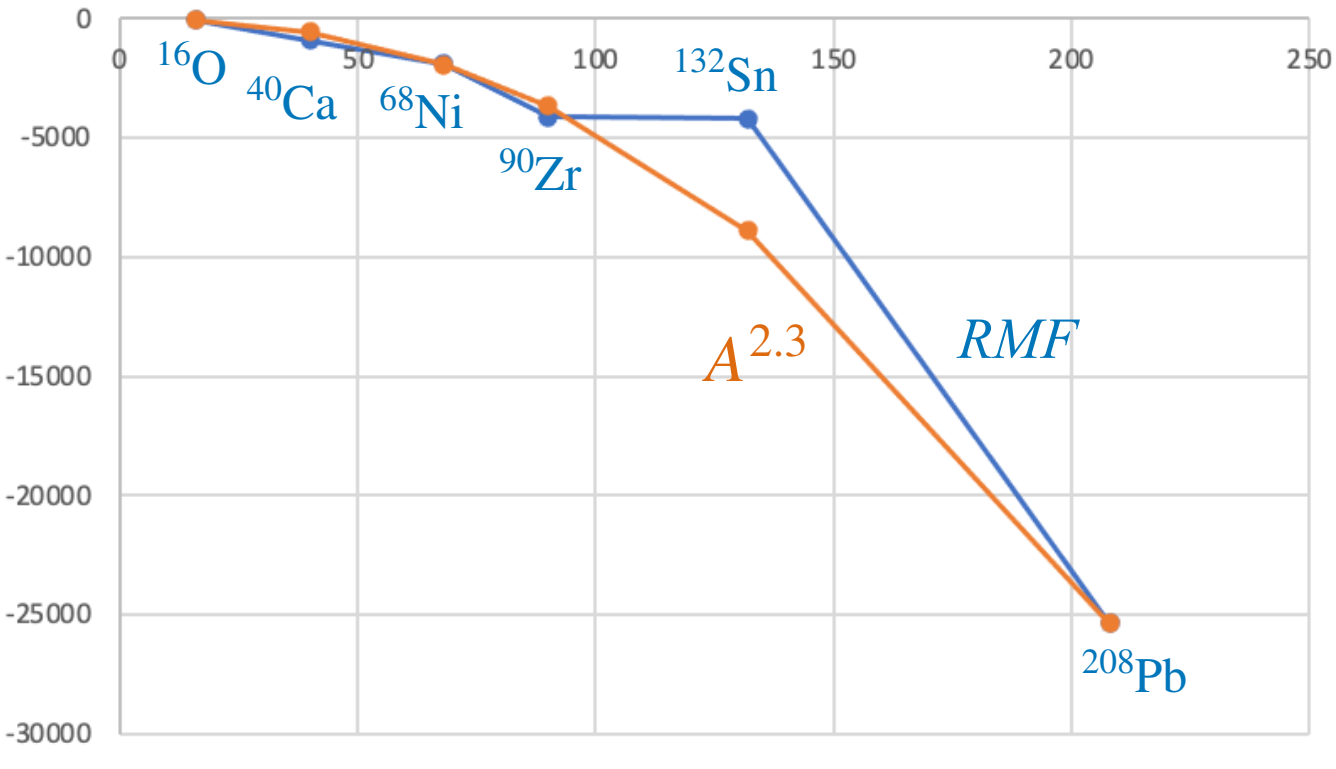
- Mass distribution



Y. Hatta, T. Oishi, M. Oka, Phys. Rev. C 114, 024343 (2026)

Nuclear D term

- **Double magic (closed shell) nuclei: Our fit** $\propto A^{\beta} \sim A^{2.3}$



$\beta = 7/3 = 2.33$ **(liquid drop, Polyakov)** $D = -\frac{16\pi}{15} M\gamma R^4 \propto A^{7/3}$

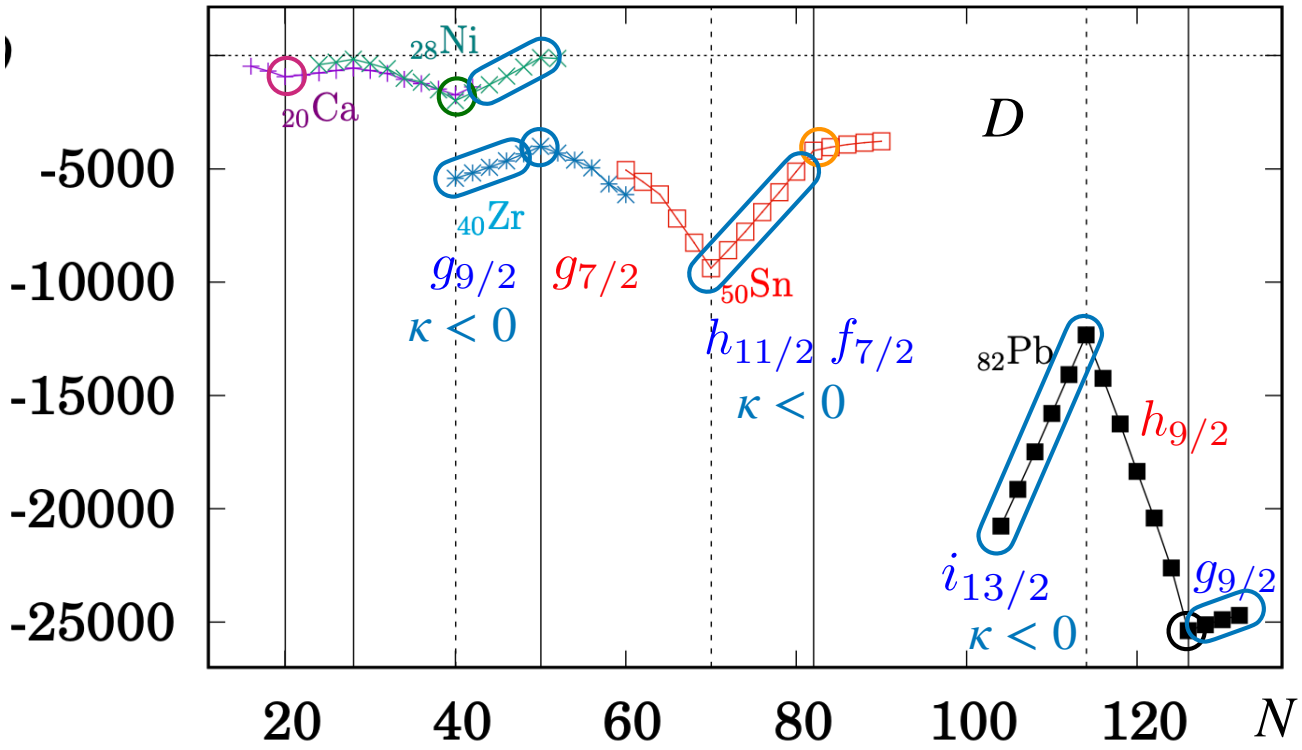
$\beta = 1.9$ **(Skyrme model, Martín-Caro, Huidobro, Hatta)**

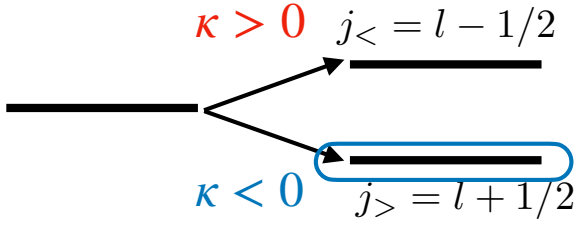
$\beta = 2.26$ **(Walecka model, Guzey, Siddikov)**

Y. Hatta, T. Oishi, M. Oka, Phys. Rev. C 114, 024343 (2026)

Nuclear D term

- Isotope and shell effects for proton-magic isotopes





$$\kappa_a = \mp\left(j+\frac{1}{2}\right) \qquad j = \ell \pm \frac{1}{2}$$

- D terms do not increase monotonically. ⇒ "*LS* splitting" (shell effect)

$$s(r) = \frac{1}{4\pi}\sum_{a=1}^{A}\left[f_a(r)\frac{dg_a(r)}{dr} - \frac{df_a(r)}{dr}g_a(r) + \boxed{\kappa_a\frac{f_a(r)g_a(r)}{r}}\right] + \left(\frac{d\sigma}{dr}\right)^2 - \left(\frac{d\omega}{dr}\right)^2 - \left(\frac{d\rho}{dr}\right)^2$$

Y. Hatta, T. Oishi, M. Oka, Phys. Rev. C 114, 024343 (2026)

Summary and future prospects

- **Gravitational form factors carry new information of mechanical structures of hadrons and nuclei.**
- **D-term is a new (coupling) constant, representing the shear-force structure of nuclei.**
- **Relativistic mean field (RMF) theory is applied to calculate the nuclear gravitational form factor.**
- **The D-terms of the closed shell nuclei show a strong mass-number dependence, $A^{7/3}$, while the mass and charge radii increase roughly as $A^{1/3}$.**
- **The isotope dependencies of the D-terms are not monotonic, depending on κ of the occupying shell, showing strong shell effects due to the *LS* coupling.**

Summary and future prospects

- **Future prospects**
  - **We so far have "neglected" various nuclear physics effects.**

    deformation → change shell structures

    pairing correlation (quasi-particle)

    Coulomb interaction
  - **Convolution of the NGFF with the nucleon/meson GFF**

    "EMC" effects on GFF !