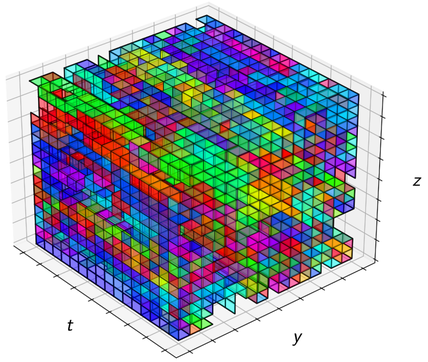
Topological Data Analysis of Centre Vortices in Lattice QCD

Thakur **Giri**raj Hiranandani, Jackson A. Mickley, Riordan de Vries, Ryan Bignell, Derek B. Leinweber, Vanessa Robins, and Chris Allton



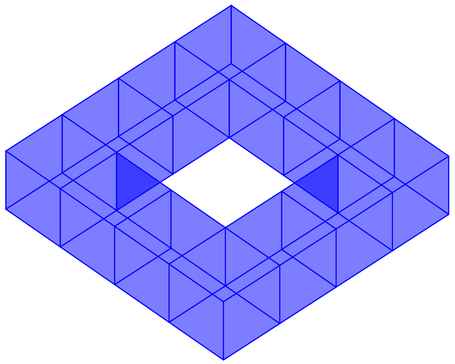
University of Queensland, Brisbane

Adelaide University, Adelaide

Swansea University, Swansea,

Trinity College, Dublin

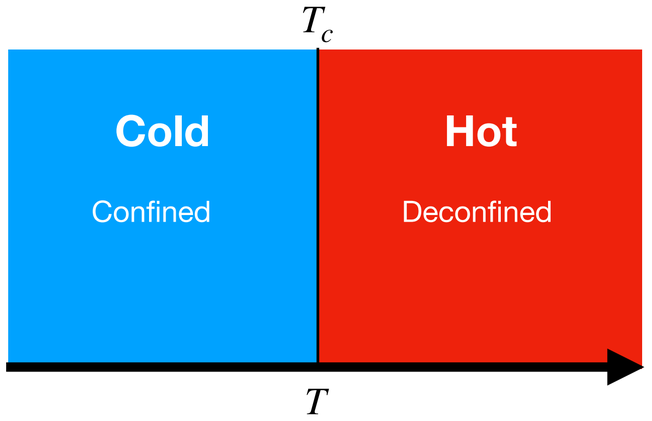
Australian National University, Canberra



Previously, on QCD

- The problem of confinement is one of the long-standing questions in QCD
- In the quenched theory, confinement is lost when chiral symmetry is restored
- Conventional wisdom in the dynamical theory indicates that the theory is deconfined at the chiral restoration temperature T_c

Quenched theory



Aarts et al., EPJ A 60, 59 (2024).

Rothkopf, Phys. Rep. 858 (2020).

Previously, on QCD

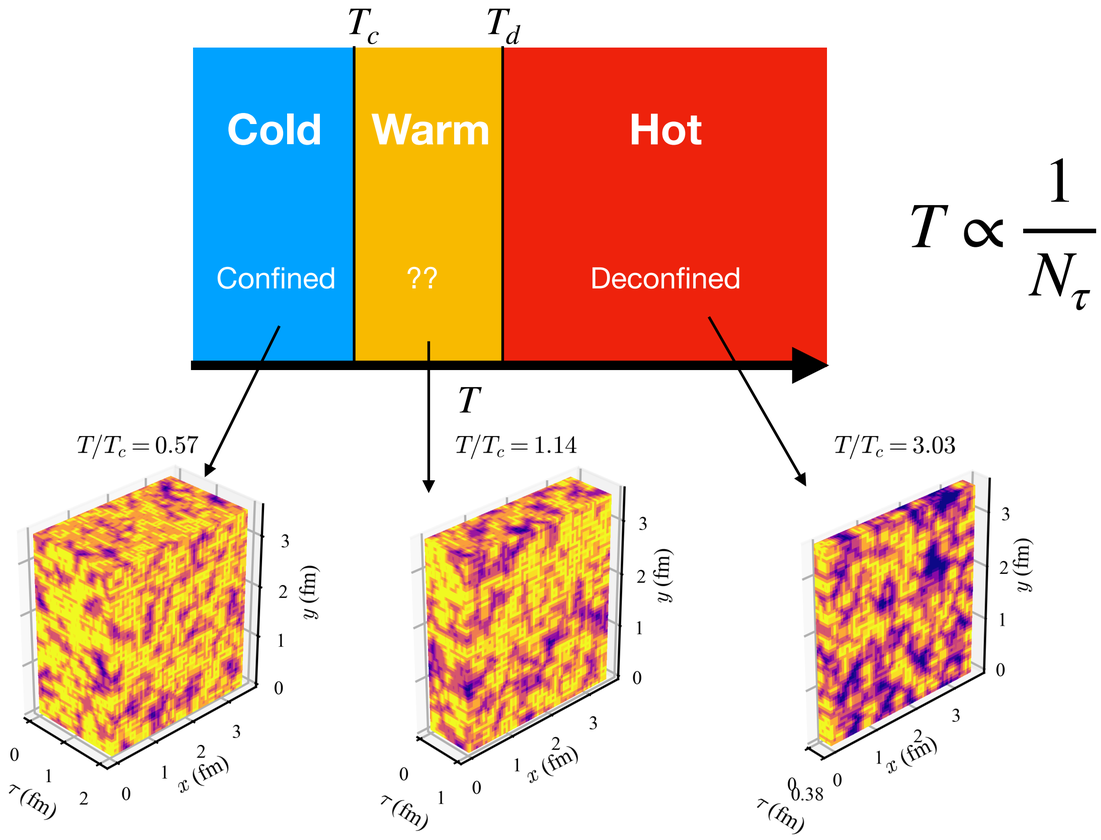
- In the dynamical theory, a nonzero string tension persists past T_c
- It was found by Mickley et al. deconfinement happens later at T_d

$$T_c = 167(3)\ \text{MeV},$$
$$T_d = 321(6)\ \text{MeV}$$

Mickley et al. (2025), *Phys. Rev. D* **111**, 034508.

- Percolation in the centre vortex sheets are only lost at T_d

Dyanamical theory



$$T \propto \frac{1}{N_\tau}$$

Centre vortices in lattice QCD

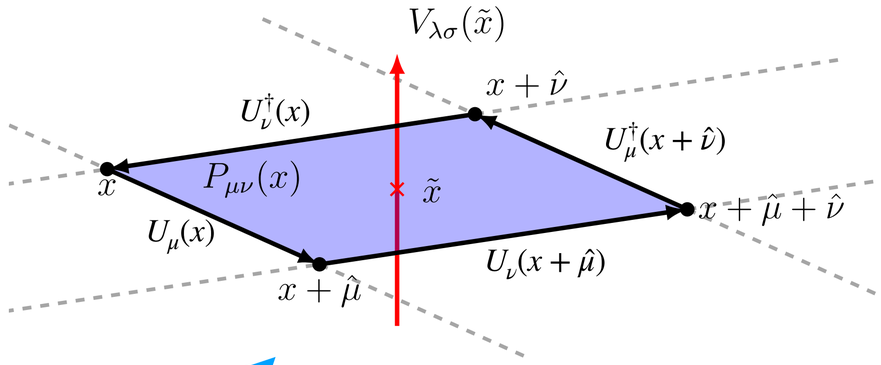
- Topological defects of the QCD vacuum
- Centre vortices play an important role in nonperturbative effects, including confinement
- Vortices are dual to plaquettes, and hence orthogonal to them

$U_\mu(x) \in \mathrm{SU}(3)$

Gauge Fixing

Maximal Centre Gauge + Centre Projection

$U_\mu \in \mathrm{Z}(3) = \{I, e^{\frac{2\pi i}{3}} I, e^{-\frac{2\pi i}{3}} I\}$



$$V_{\mu\nu}(\tilde{x}) = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta} P_{\alpha\beta}(x)$$

Center vortex structure

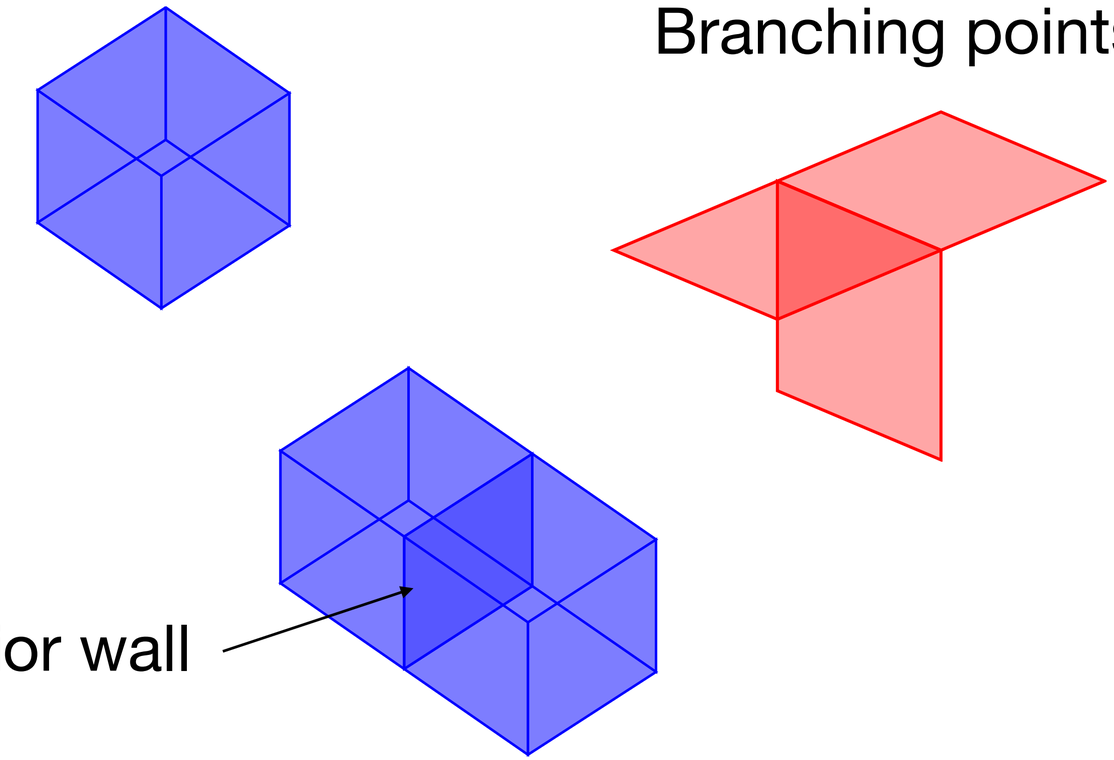
- By construction these vortices have a conservation law

$$\partial_\mu V_{\mu\nu}(x) = 3n, \quad n \in \mathbb{Z}$$

- The vortices $V_{\mu\nu}$ form closed two dimensional sheets

- Due to $Z(3)$, branching points can occur

- This makes the vortex sheets harder to study

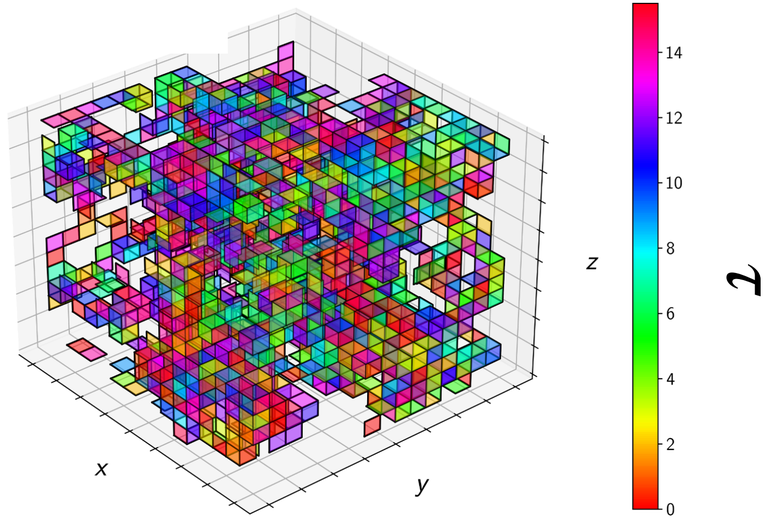


Centre vortices in four dimensions

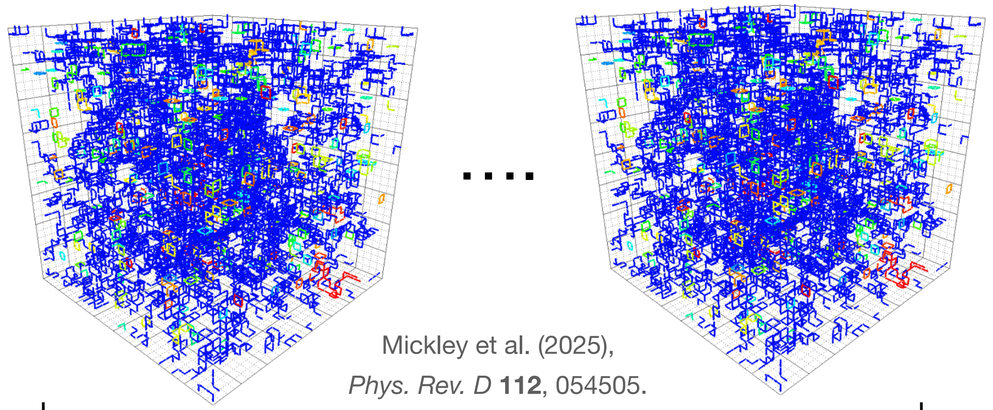
- In practice, these vortex sheets are much more complicated and dense
- The full four dimensional structure cannot be visualised (yet)
- Three dimensional visualisations are possible with projection or “slicing” or holding coordinate axis fixed
- Emergence of a giant “primary cluster” making up ~90% of the vortex sheet

$4D \rightarrow 3D$

Projection



“bubbly, foam-like structure”

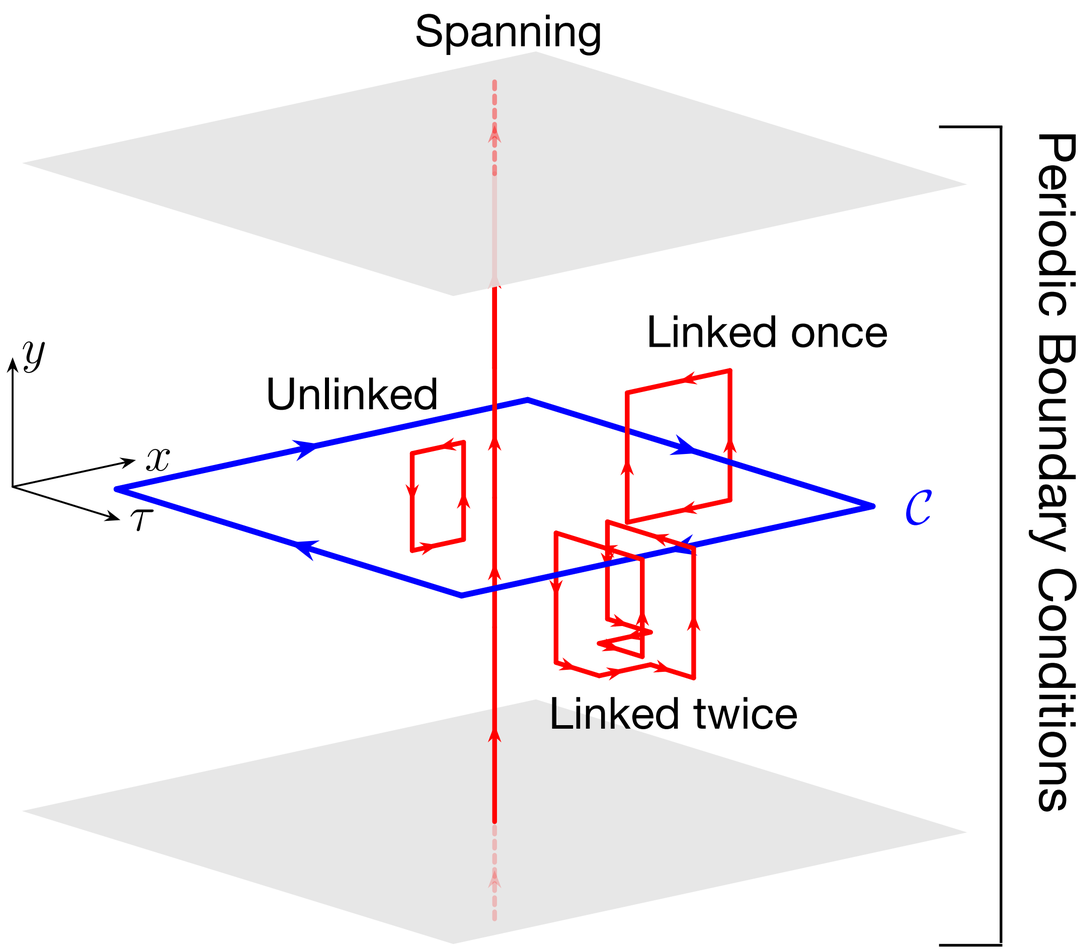


Mickley et al. (2025),
Phys. Rev. D **112**, 054505.

N_z Slices

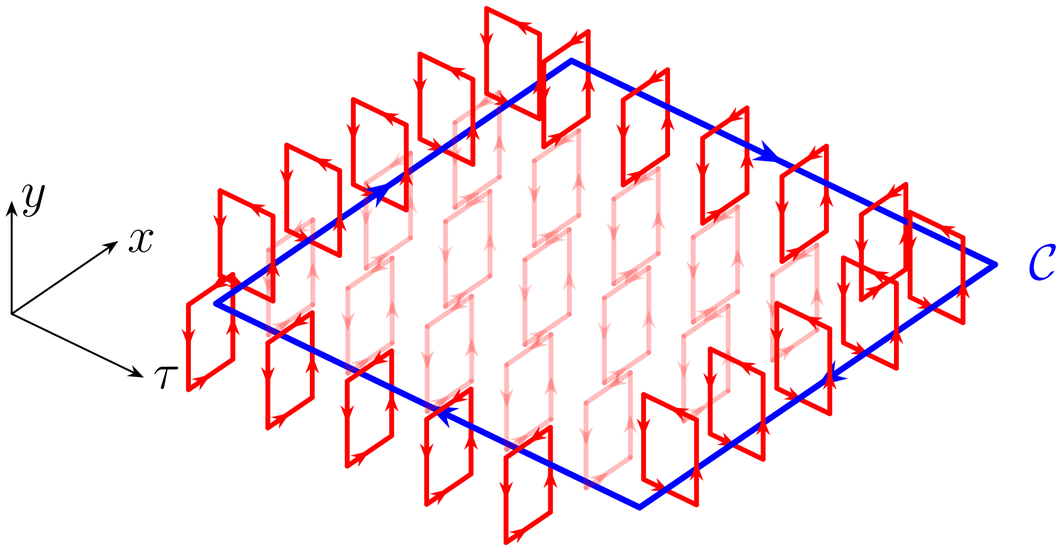
Centre Vortex Confinement

- The static quark potential is related to the expectation of the Wilson loop $\langle W[C] \rangle$
- Only vortex loops **linked** to the Wilson loop contribute
- Small vortices only link along the perimeter and generate a **perimeter law**
- Spanning vortices link through the area and generate an **area law**



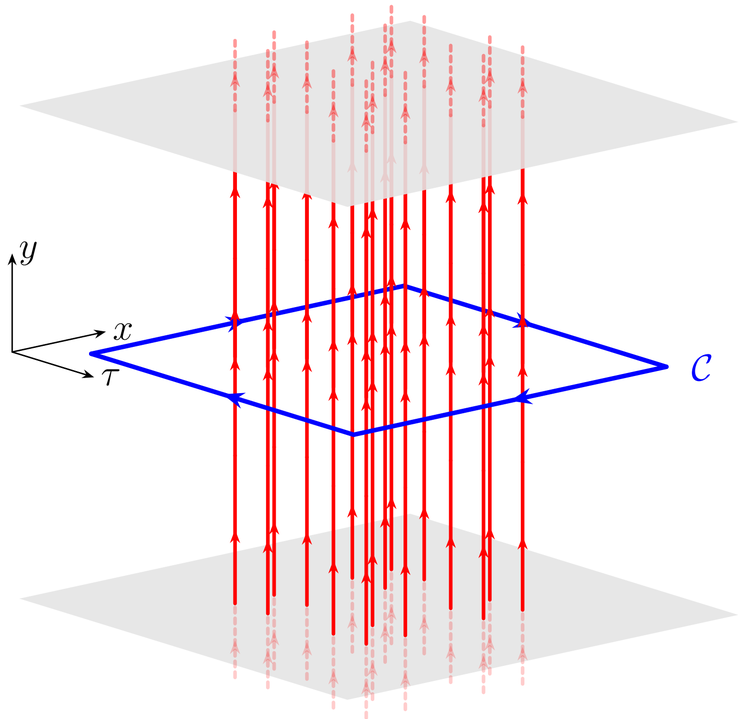
Perimeter vs Area law scaling

Small vortex loops



Number of vortex piercings proportional to **perimeter**

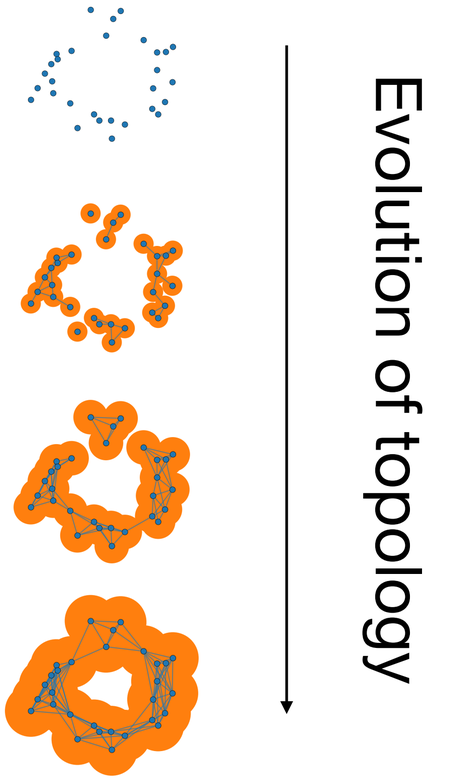
Spanning vortex loops



Number of vortex piercings proportional to **area**

What is topological data analysis (TDA)?

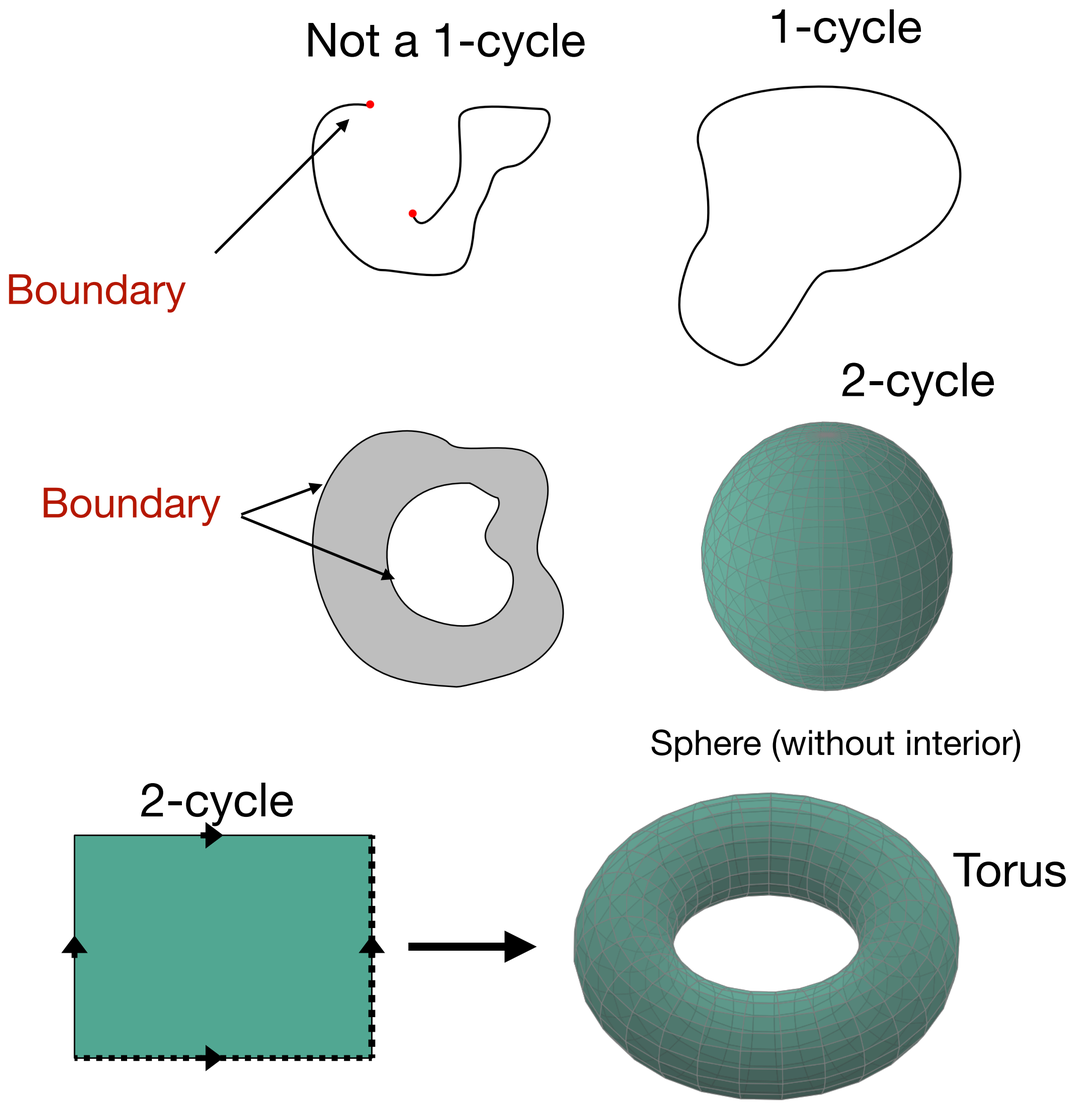
- **Topological data analysis (TDA)** studies the shape of data: how points connect, cluster, and form loops or voids across different scales
- A relatively young field, with roots in the early 2000s (~20 years), built largely on persistent homology
- Applied across many fields, including biology, materials science, neuroscience, and physics
- Especially useful for complex and high-dimensional data
- Well suited to lattice data and centre vortex data



Database of Original & Non-Theoretical Uses of Topology (DONUT)

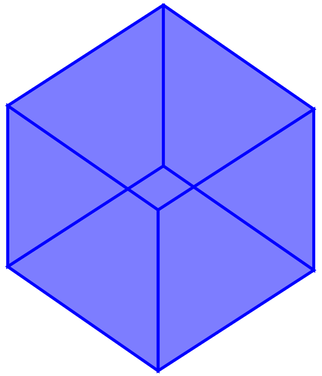
A brief introduction to topological

- Topology is the study of the connectedness space
- We study the homology → the number of **holes** in the space
- A k**-cycle** is k-dimensional object without a boundary
- k-cycles enclosed $(k+1)$ - dimensional region
- What is a k-hole? A k-hole is the number of k -cycles
- Betti Number β_k = number of k-holes



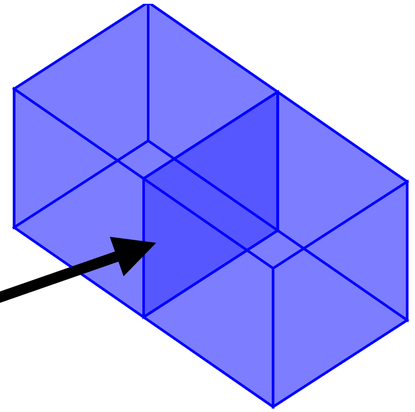
Examples of topologies on a cubical lattice

1)



$\beta_0 = 1, \quad \beta_1 = 0$
$\beta_2 = 1, \quad \beta_3 = 0$

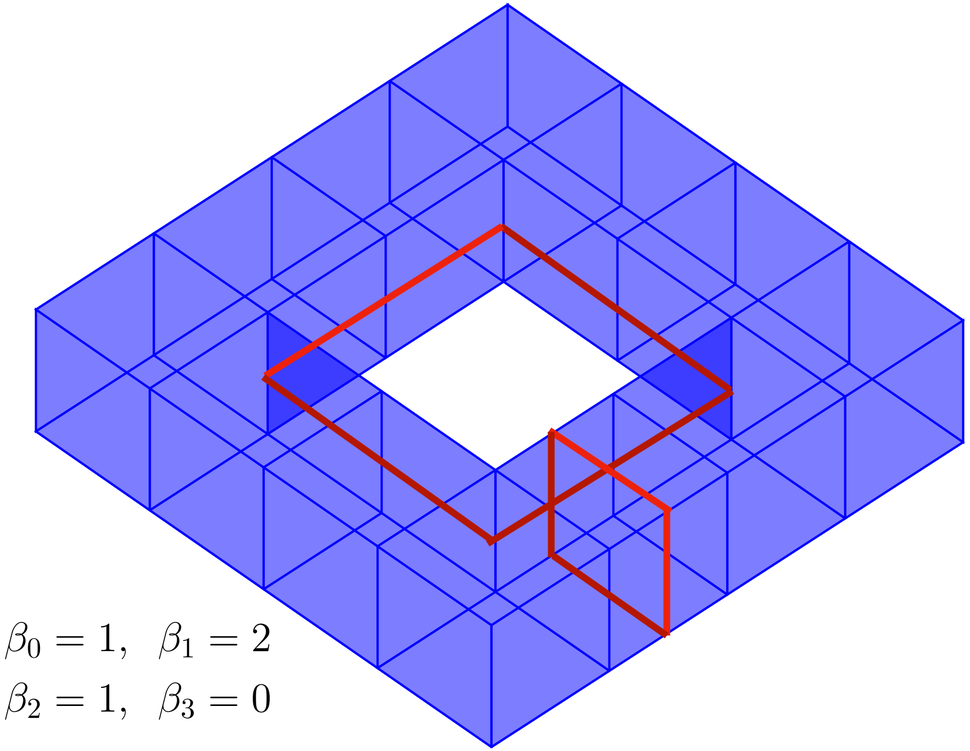
2)



$\beta_0 = 1, \quad \beta_1 = 0$
$\beta_2 = 2, \quad \beta_3 = 0$

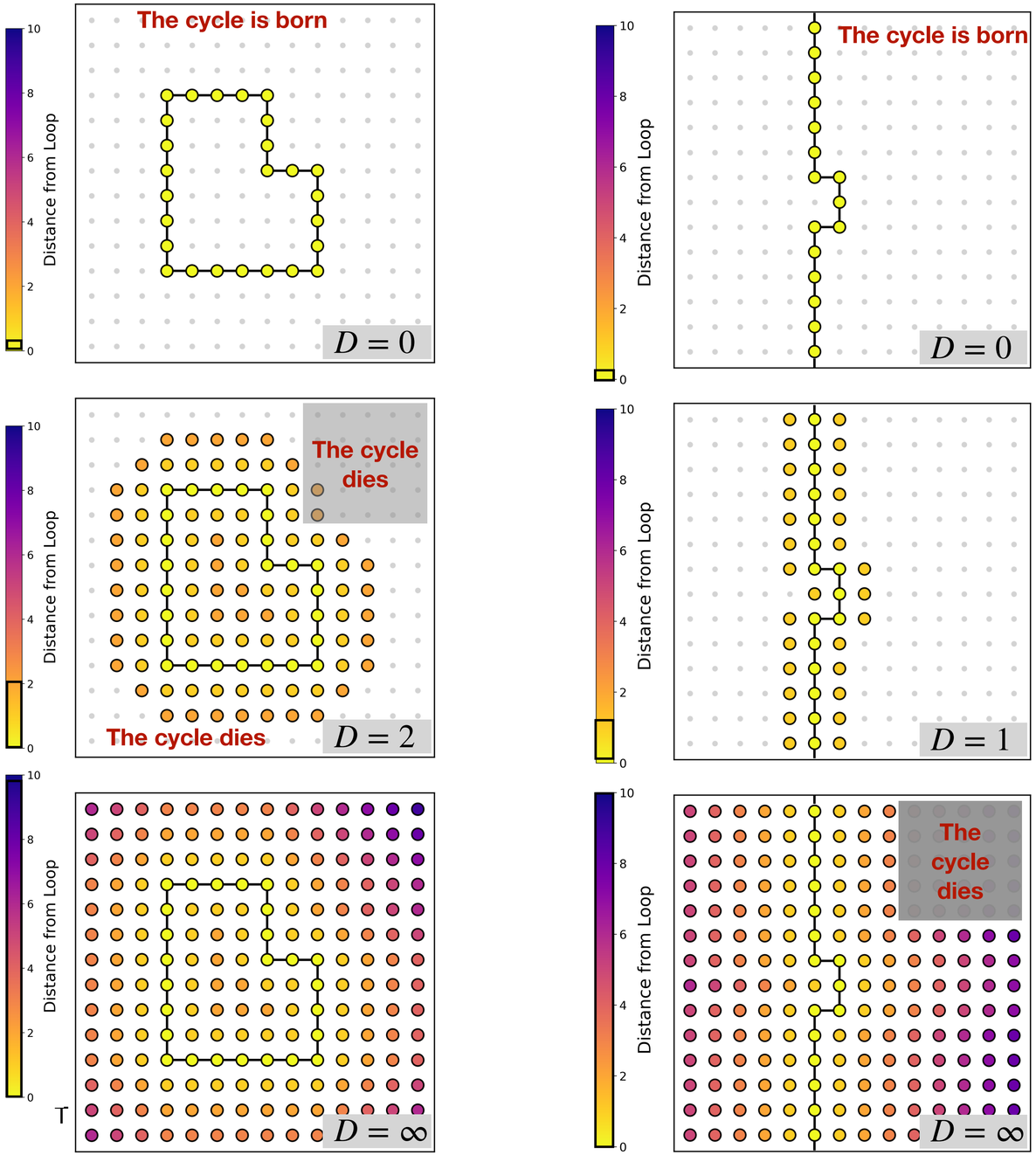
Interior wall

3)



$\beta_0 = 1, \quad \beta_1 = 2$
$\beta_2 = 1, \quad \beta_3 = 0$

$\beta_0 \rightarrow$ # Connected components
$\beta_1 \rightarrow$ # Enclosed Areas
$\beta_2 \rightarrow$ # Enclosed Volume
$\vdots$



Detecting Spanning Cycles using TDA

A cycle that is born at $D_b = 0$ and survives until $D_d = \infty$ is spanning

1. Take the your data; and calculate the topological invariants

2. Then fatten your dataset and recalculate the topological invariants

3. Bonus: By tracking when are born or die we can study the size of topological features

$|D_b - D_d| \approx$ scale of feature

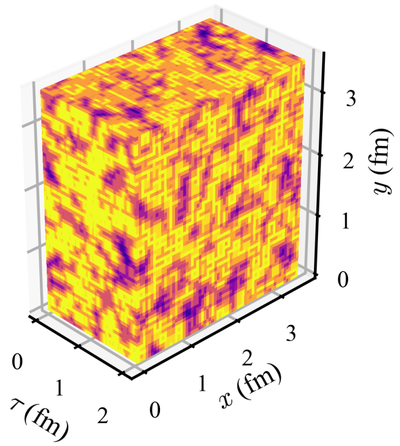
Studying the topology of centre vortices

FASTSUM, Aarts et al
Phys.Rev.D 105 (2022) 3, 034504

Dynamical fermion QCD configuration on an anisotropic lattice ($a_s/a_\tau = 3.45$)
$32^3 \times N_\tau, m_\pi = 239(1)$ MeV

Extract Centre Vortices

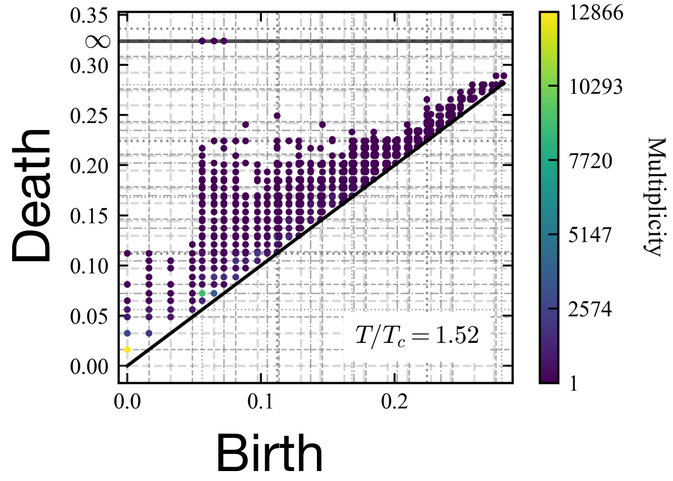
Mickley et al. (2025), *Phys. Rev. D* **111**, 034508.



TDA

GUDHI: Geometry Understanding in Higher Dimensions
(Maria et al., 2014).

Interpretation + Analysis



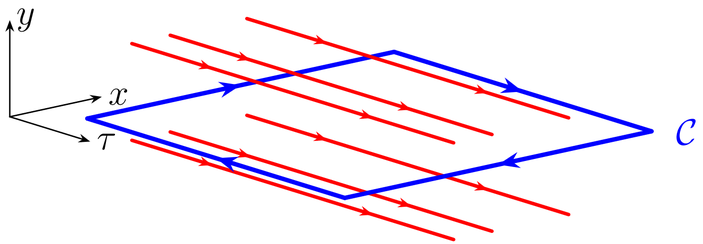
How does the centre vortex foam change with temperature?

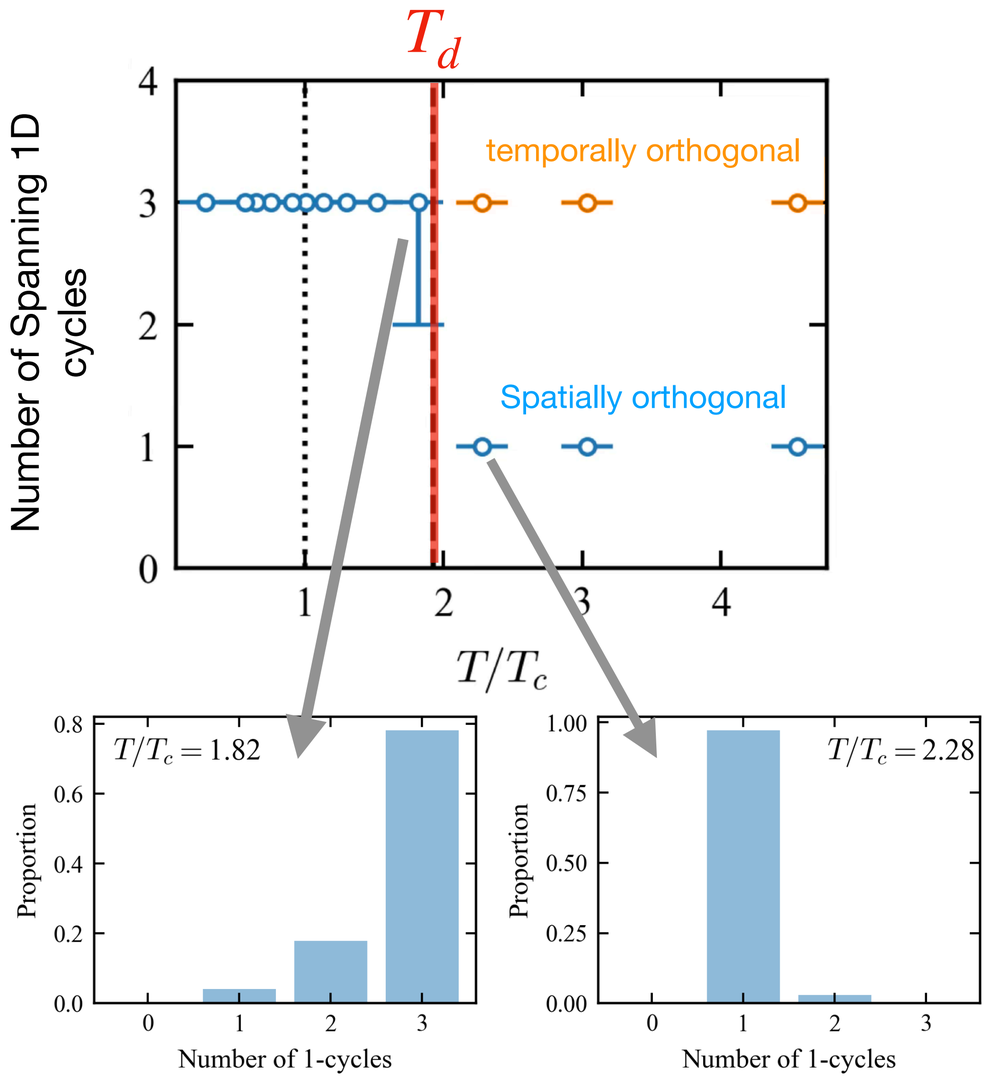
- Are they spanning?
- How many vortex sheets are there?
- How complicated are they?
- How big are the sheets?

Can we predict the regime (Cold, Warm or Hot) just based on the topological features?

Spanning vortex cycles

- Sliced configurations show 1D spanning
- There is a loss of spanning cycles across T_d
- Zero variance, except around T_d
- Spans only in the temporal direction

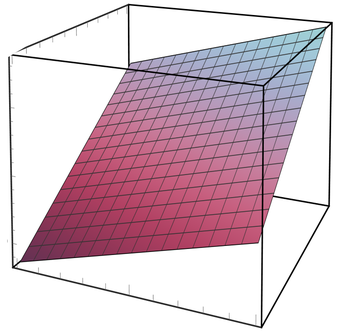


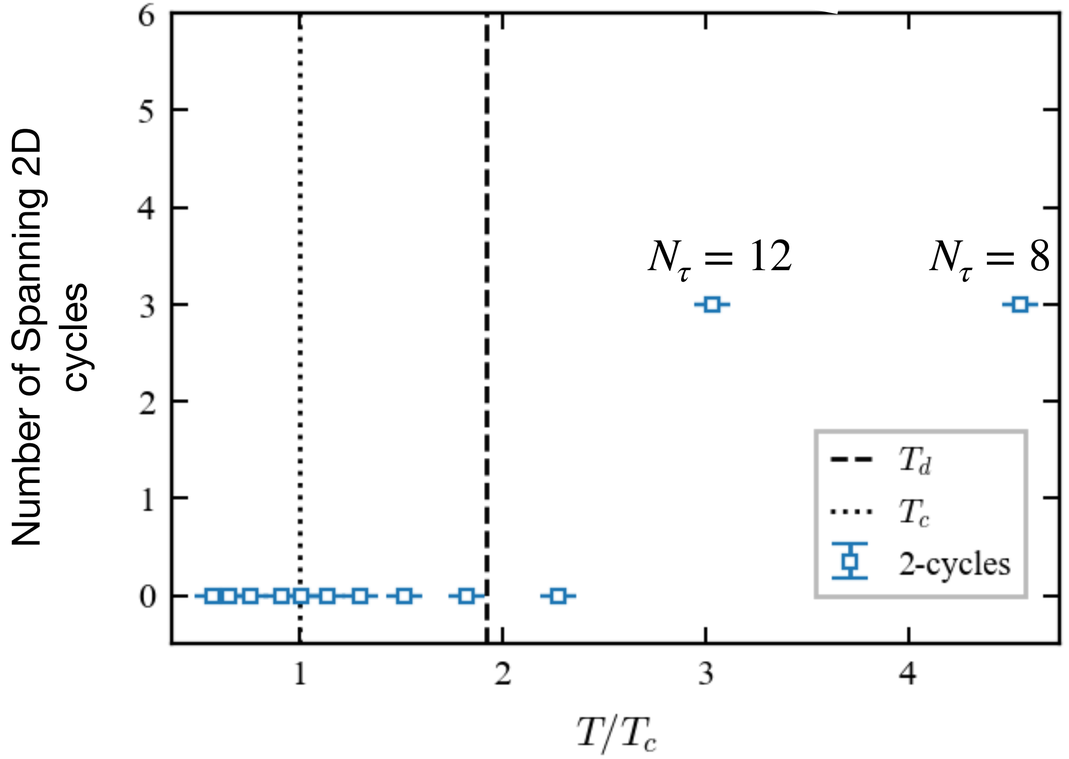


Spanning vortex surfaces

- No 2D spanning in the confined phase
- Only spanning at high temperatures in 3/6 of the planes
- Not a property of the deconfined phase. Finite lattice effect?
- **Conclusion**: 2D spanning is not a requirement for confinement

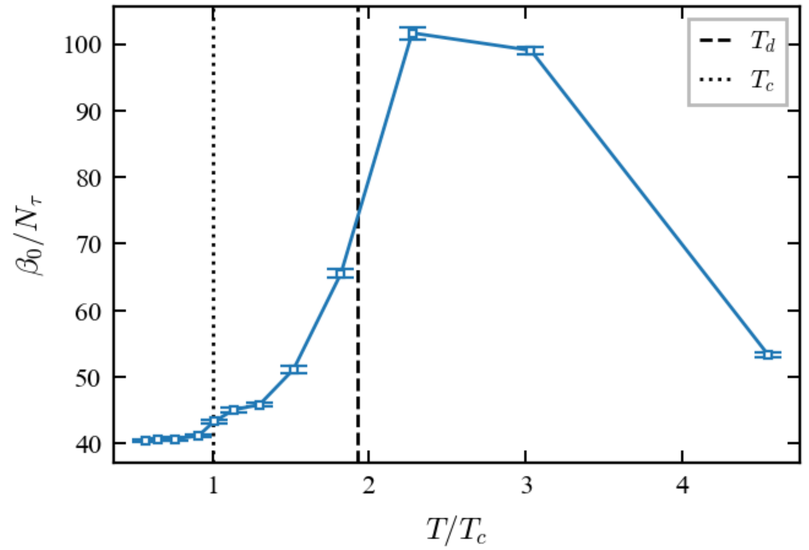
2D Spanning

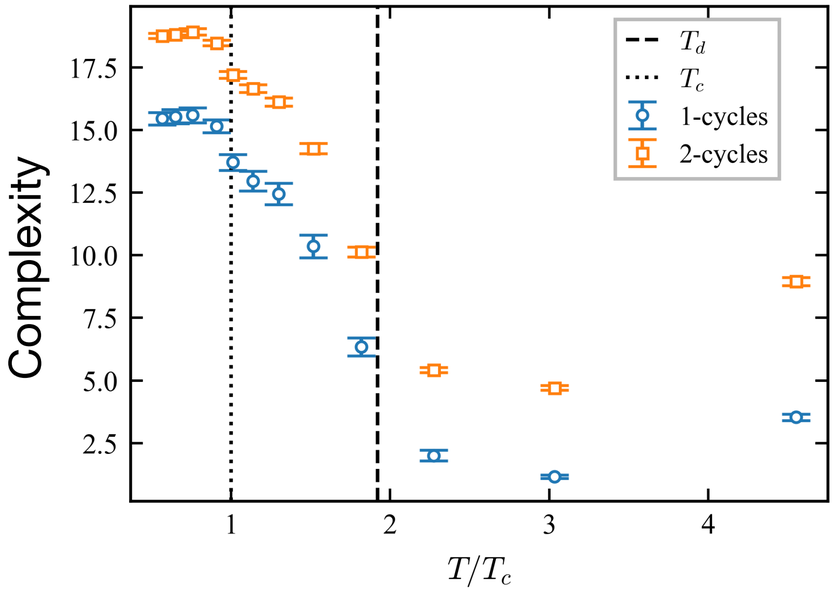




Complexity of the Vortex Sheet

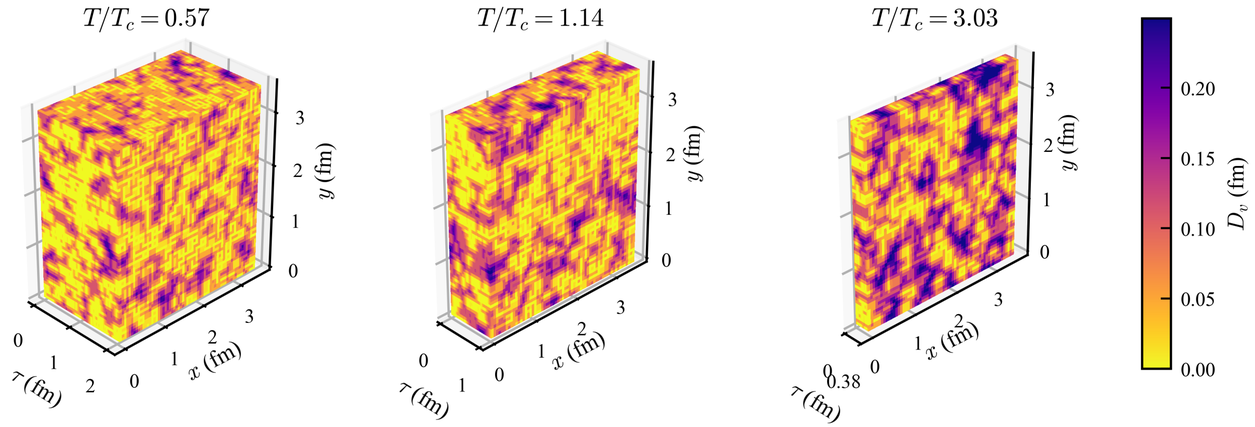
- Number of clusters increase sharply at T_c and then through T_d
- **Complexity** is the average number of cycles per connected component (β_k/β_0)
- Complexity is highest at low temperatures and minimal at high temperatures

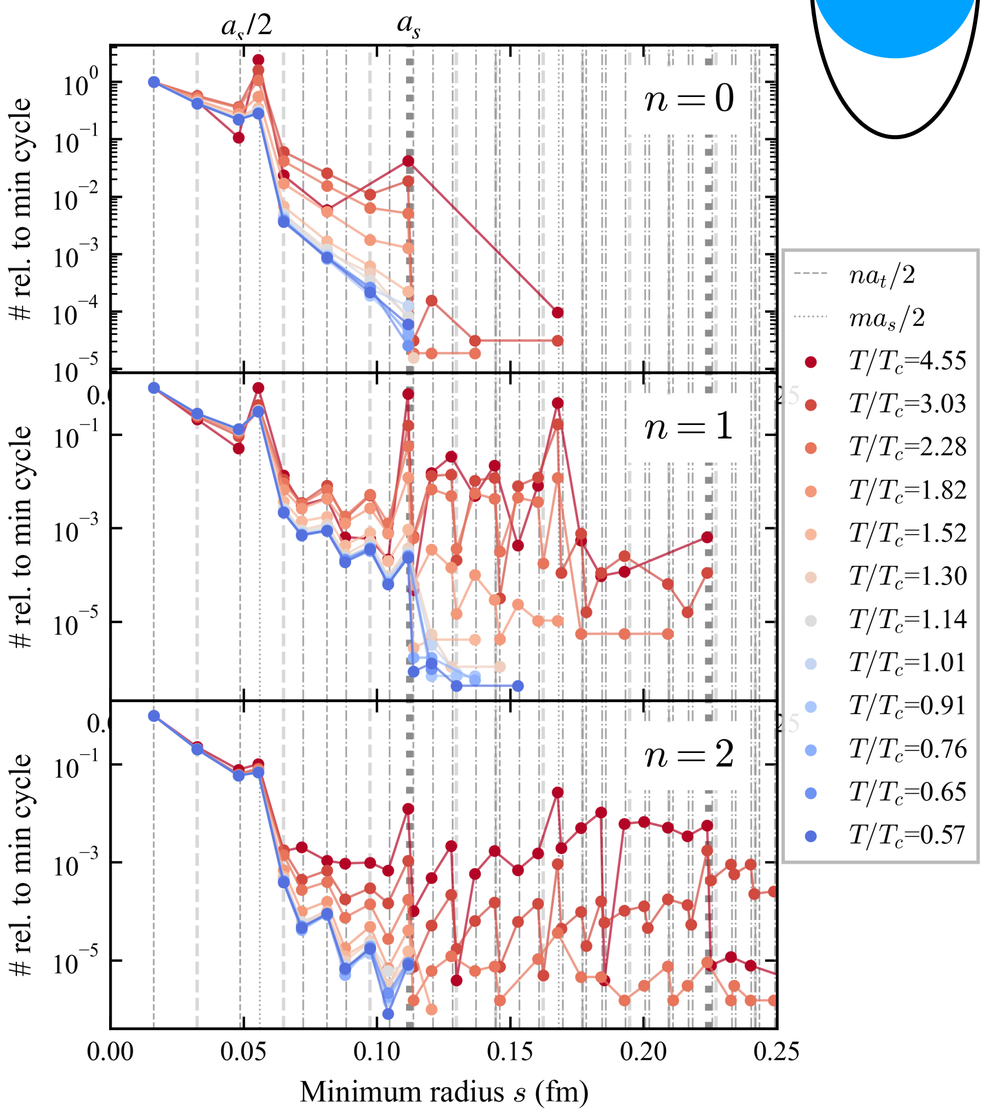


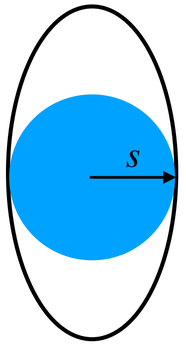


How big are the cycles?

- At all temperatures the vortex sheets are densely packed
- As temperature is increased the distance between vortex sheets increases
- However, the size of the topological features increase as well



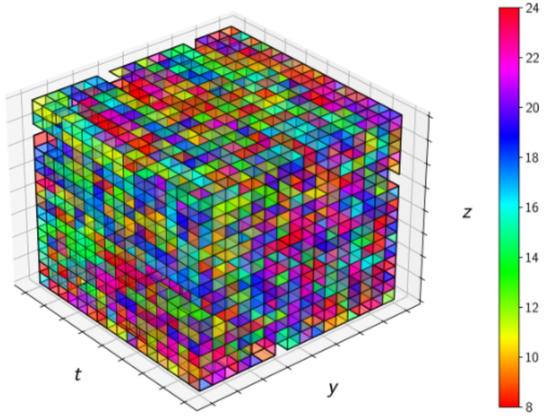




Summary & Conclusion

Do they span?

Centre vortices span the temporal direction at all temperatures, but only span in the spatial directions at low temperatures



How many vortex sheets are there?

The number of vortex sheets per N_τ increases with temperature

How complicated are they?

The vortex sheets become simpler, with fewer loops, with an increase in temperature

How big are the cycles?

As temperature increases so do the sizes of the cycles

Can we predict the regime (Cold, Warm or Hot) just based on the topological features?

Probably, but with different topological observables. Work in progress.

