Pole-Expansion of Two-Particle Imaginary-Time Correlation Function in the Infinite Volume and Its Finite-Volume Correction

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Abstract

In a series of papers [1–7] we have proposed and developed the idea of the pole expansion as a model-independent method to analyze the results of experiments and/or lattice QCD simulations, in particular for unstable states, that are more general than resonances. In the talk we report on the pole expansion of the two-hadron imaginary-time correlation function in the infinite volume and its finite-volume correction [6, 7].

First, in the infinite volume, by the Mittag-Leffler theorem, we express the imaginary-time correlation function as a sum of pole terms in terms of the uniformization variable, which makes the correlation function single-valued.

Secondly, in a finite volume, identifying unstable states as poles on the newly defined unphysical complex-energy sheet, we demonstrate that the finite-volume correction to the complex energy of unstable states (and the pole expansion of the correlation function) decreases exponentially as the volume size increases.

Also, we find that (imaginary time)/(volume size)^2 $\to$ 0 is the infinite-volume limit while (imaginary time)/(volume size)^2 $\to \infty$ is the lowest-mode-dominant limit. Therefore, in the former region we can obtain the pole positions and residues of unstable states by fitting the pole expansion of the imaginary-time correlation function to the results of lattice QCD simulations.

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