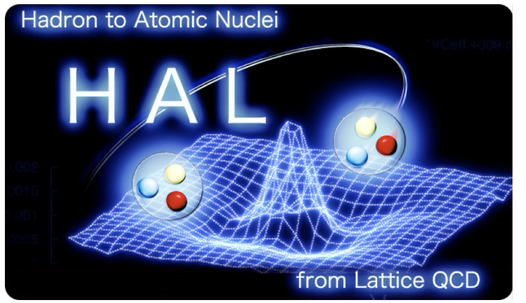
Lattice QCD studies on the $\Lambda(1405)$ baryon in the flavor SU(3) limit

Kotaro Murakami
Institute of Science Tokyo/RIKEN iTHEMS

Institute of SCIENCE TOKYO

iTHEMS
RIKEN Center for Interdisciplinary Theoretical and Mathematical Sciences



based on

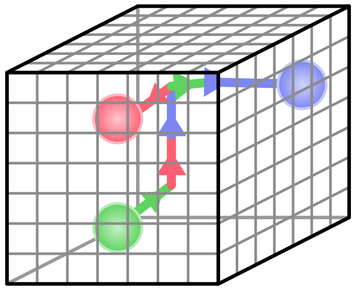
- KM, S. Aoki, PoS LATTICE2024, 101 (2025)
- KM, S. Aoki, in preparation

Fundamental Problems in Subatomic Physics 2026, Cairns, Australia, Sep. 23–27, 2026

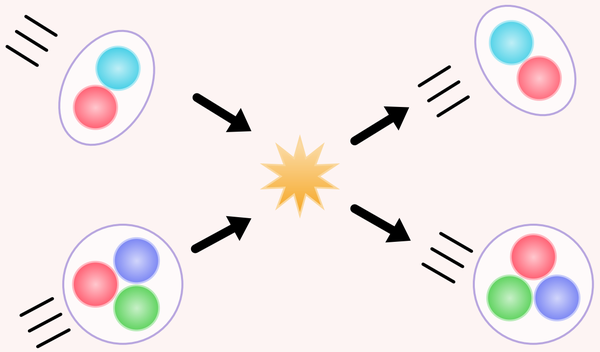
Hadron interactions from lattice QCD

- our goal: understand **exotic hadrons/nuclei/matters** via hadron interactions using lattice QCD

Lattice QCD



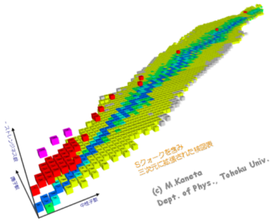
Hadron interactions (scatterings)



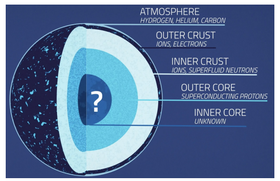
Excited hadrons
Exotic hadrons



(Exotic) nuclei



Nuclear matter
Neutron stars



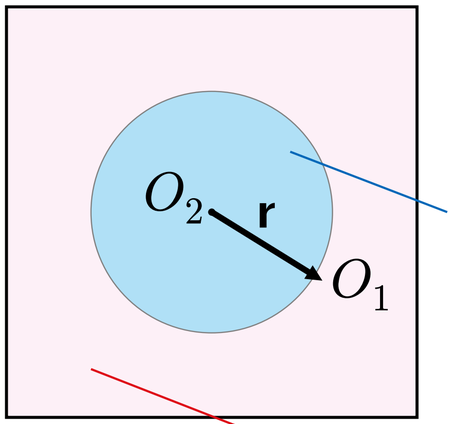
Hadron scatterings in lattice QCD

- key quantity: Equal-time Nambu-Bethe-Salpeter (NBS) wave function

$$\Psi^W(\mathbf{r}) = \langle 0|\, \hat{O}_1(\mathbf{r})\hat{O}_2(\mathbf{0})\, |1,2;W\rangle \qquad (W = \sqrt{k^2+m_1^2}+\sqrt{k^2+m_2^2})$$

$$\xrightarrow[r\to\infty]{} A\,\frac{\sin\left(kr - \frac{l}{2}\pi + \delta^l(k)\right)}{kr}\, e^{i\delta^l(k)}\, Y_{l,m}(\Omega)$$

[Lin et al., NPB (2001)]



not asymptotic
$(\frac{\nabla^2}{2\mu} + \frac{k^2}{2\mu})\Psi^W(\mathbf{r}) \neq 0$

asymptotic: $(\frac{\nabla^2}{2\mu} + \frac{k^2}{2\mu})\Psi^W(\mathbf{r}) = 0$

- methods based on NBS wave function:

Finite-volume method [Lüscher, NPB (1991)]

finite-volume energy $\longrightarrow$ phase shift

quantization condition

HAL QCD method [Ishii, Aoki, Hatsuda, PRL (2007)]

NBS wave function $\longrightarrow$ phase shift

interaction potentials

$$\int d^3r'\, \boxed{U(\mathbf{r},\mathbf{r}')}\, \Psi^W(\mathbf{r}') = \left(\frac{\nabla^2}{2\mu} + \frac{k^2}{2\mu}\right) \Psi^W(\mathbf{r})$$

Time-dependent HAL QCD method

[Ishii et al., PLB (2012)]

- ratio of 4pt (3pt) and 2pt functions (R-correlator):

$$R(\mathbf{r},t) = \frac{\langle O_1(\mathbf{r},t) O_2(\mathbf{0},t) \bar{J}(0)\rangle}{\langle O_1(t)\bar{O}_1(0)\rangle \langle O_2(t)\bar{O}_2(0)\rangle} \approx \sum_n C_{\bar{J},n}\, \Psi^{W_n}(\mathbf{r})\, e^{-(W_n - m_1 - m_2)t}$$

NBS wave function

- time-dependent equation (at large t) (μ: reduced mass)

$$\int d^3r'\, U(\mathbf{r},\mathbf{r}')\, R(\mathbf{r}',t) \simeq \Big(\frac{\nabla^2}{2\mu} \underbrace{- \frac{\partial}{\partial t} + \frac{1}{8\mu}\frac{\partial^2}{\partial t^2}}_{\left(=\frac{k^2}{2\mu}\right)}\Big) R(\mathbf{r},t)$$

$$U(\mathbf{r},\mathbf{r}') \approx V(\mathbf{r})\,\delta^{(3)}(\mathbf{r}-\mathbf{r}')$$

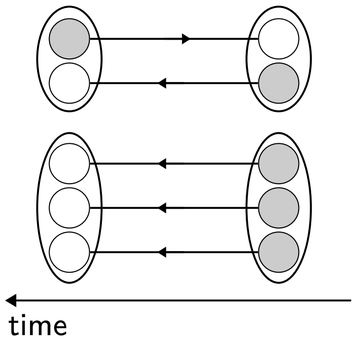
local (leading-order) approximation

$$\Rightarrow \boxed{V(\mathbf{r}) \simeq \frac{1}{R(\mathbf{r},t)} \left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t} + \frac{1}{8\mu}\frac{\partial^2}{\partial t^2}\right) R(\mathbf{r},t)}$$

HAL QCD studies without quark-pair annihilations

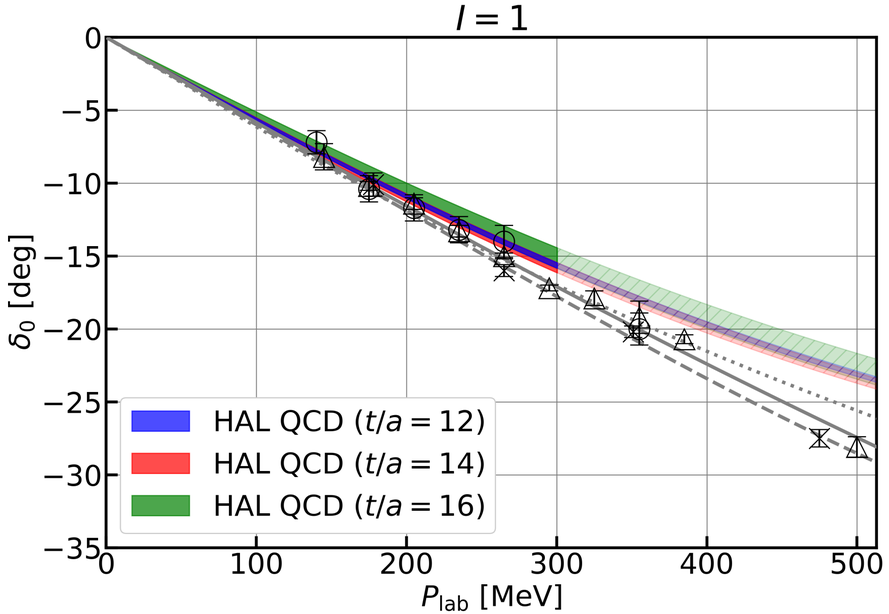
- studies **without quark-pair annihilations** ongoing

  at the **physical point** ($m_\pi \approx 137$ MeV) [Aoyama et al. (HAL QCD), PRD (2024)]



- baryon-baryon systems
  - $N\Omega_{ccc}$ $(C=+3)$ [Zhang et al., PLB (2025)]
  - ΛN, ΣN $(S=-1)$ [Murase et al., in preparation]
  - $\Lambda\Lambda$, ΞN $(S=-2)$ [Sasaki et al., in preparation]
  - $N\Omega$ $(S=-3)$ [Zhang et al., in preparation]
- meson-baryon systems
  - KN $(S=+1)$ [KM et al., PRD (2026); Erratum: PRD (2026)]
  - $\bar{D}N$ $(C=-1)$ [Yamada et al., arXiv:2603.12251]

KN phase shift $(I=1)$ at the physical point



[KM et al., PRD (2026); Erratum: PRD (2026)]

HAL QCD studies with quark pair annihilations

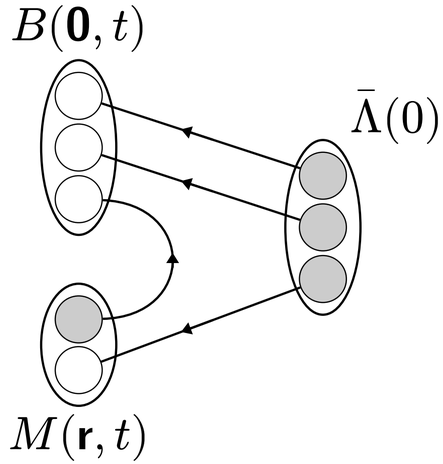
- excited hadrons possess **quark-pair annihilation diagrams**

  $\Rightarrow$ very high computational cost $\longleftarrow \times \mathcal{O}(L^4)$ larger

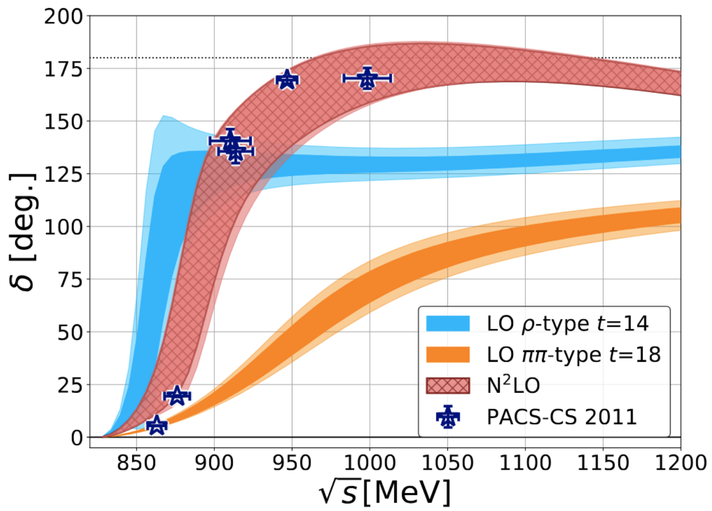
- **efficient calc. technique** in HAL QCD [Akahoshi, Aoki, Doi, PRD (2021)]

  noise vectors (one-end trick) + all-mode averaging (AMA)

  [Foster, Michael, PRD (1999)] [Bali, Collins, Schäfer, CPC (2010)]
  [Blum, Izubuchi, Shintani, PRD (2013)]

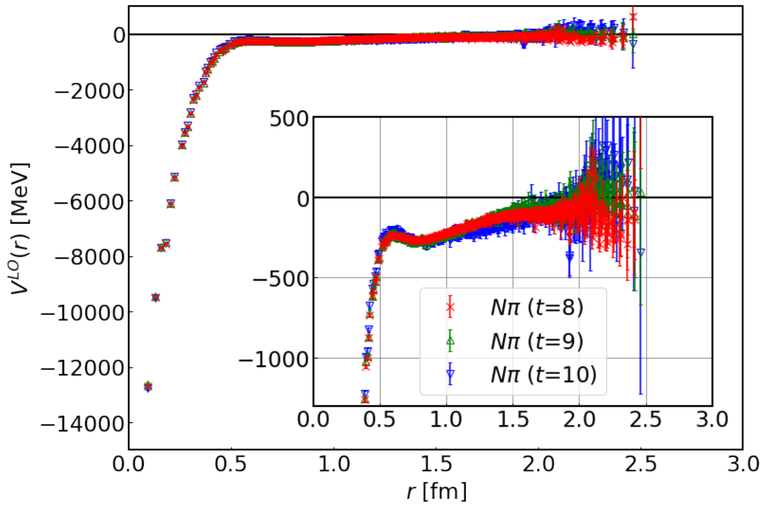


- $\pi\pi \to \rho$ (resonance)
  $(m_\pi \approx 410$ MeV$)$



[Akahoshi, Aoki, Doi, PRD (2021)]

- $N\pi \to \Delta$ (stable)
  $(m_\pi \approx 410$ MeV$)$



[KM et al., PTEP (2023)]

- next step:
  excited hadrons
  $(\Lambda(1405)$ etc.$)$

$\Lambda(1405)$

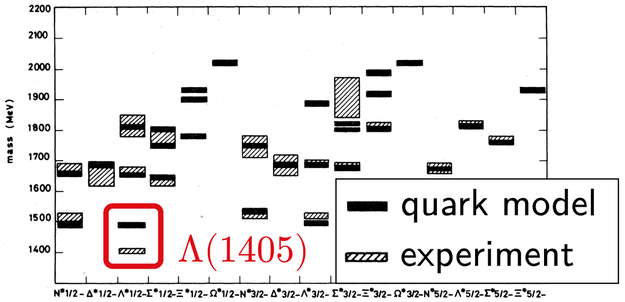
- $I = 0$, $J^P = 1/2^-$ baryon known from 1950s

  [Dalitz, Tuan, PRL (1959)]

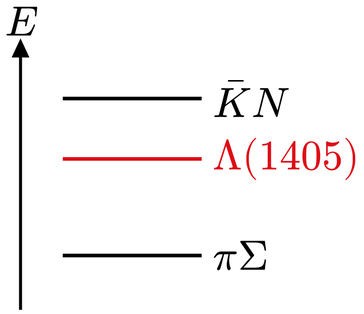
- not a simple uds baryon
- (mainly) couple to $\bar{K}N$ and $\pi\Sigma$
- **two pole structure?**
  - $\Lambda(1405)$ and $\Lambda(1380)$?

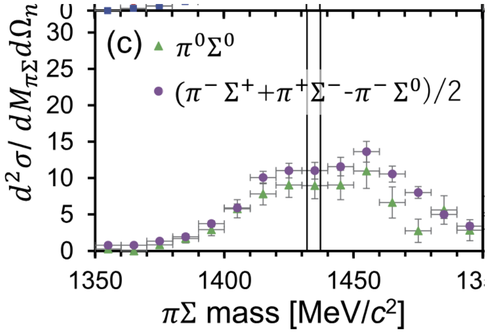
    [Oller, Meißner, PLB (2001); Jido et al., NPA (2003)]

- understand the nature of $\Lambda(1405)$ from the **first-principle calculation (lattice QCD)**

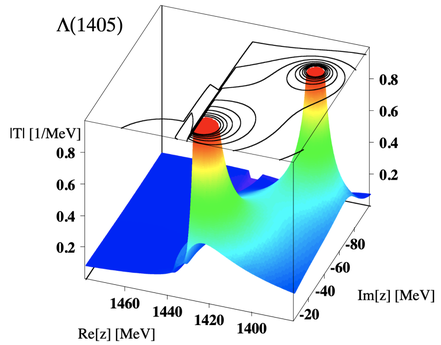


[Isgur, Karl, PRD (1978)]





[Aikawa et al. (J-PARC E31), PLB (2023)]



[Hyodo, Jido, PPNP (2012)]

$\Lambda(1405)$ from lattice QCD

- early studies of $\Lambda(1405)$ spectrum from two-point functions

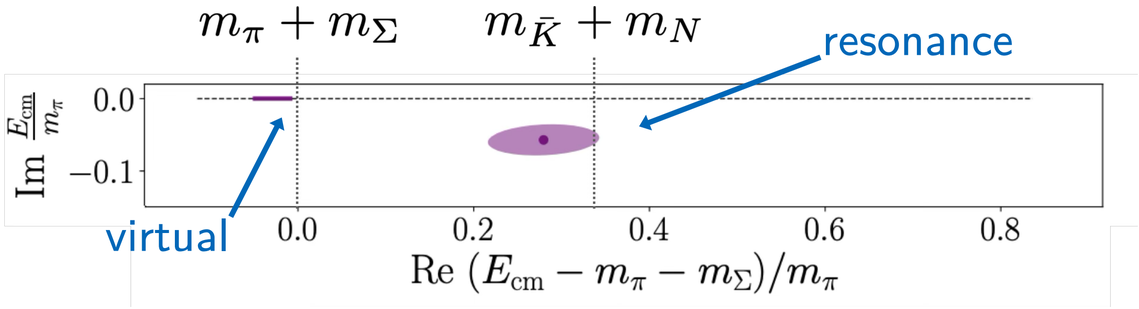
  [Melnitchouk et al. (2003); Nemoto et al. (2003); Burch et al. (BGR) (2006); Ishii et al. (2007);

  Takahashi, Oka (2010); Menadue et al. (2012); Engel, Lang, Schäfer (BGR) (2013); Gubler, Takahashi, Oka (2016)]

- EM form factor [Hall et al., PRL (2015)]

- study from $\bar{K}N$–$\pi\Sigma$ scatterings in finite-volume method at $m_\pi \approx 200$ MeV

  [Bulava et al. (BaSc), PRL (2024)]



- our goal: **study in the HAL QCD approach**
  - how does the interaction behave to generate $\Lambda(1405)$?

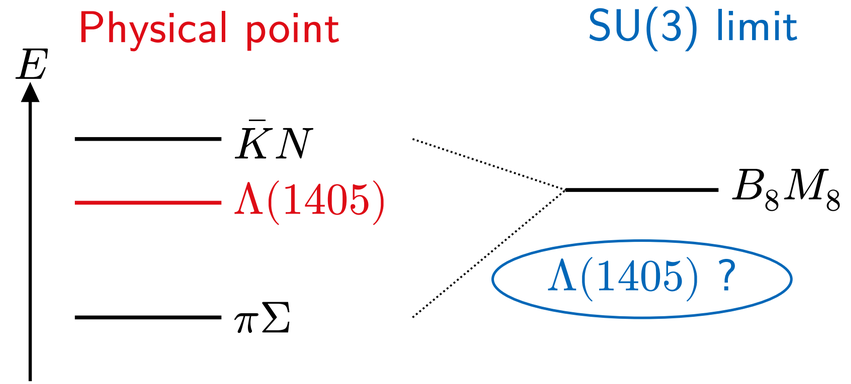
$\Lambda(1405)$ in flavor SU(3) limit

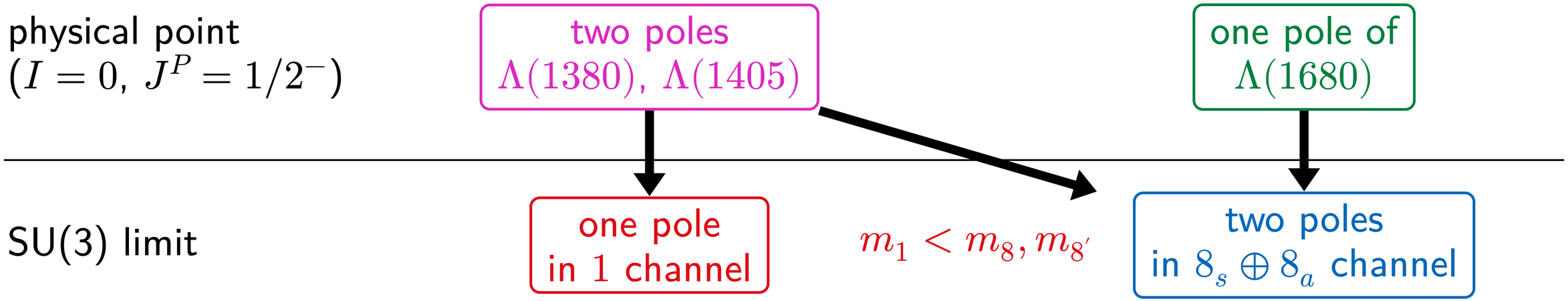
- start with **flavor SU(3) limit** $m_u = m_d = m_s$
- channels:

$$\underset{\text{meson}}{8} \otimes \underset{\text{baryon}}{8} = 27 \oplus 10 \oplus 10^* \oplus \underline{8_s \oplus 8_a \oplus 1}$$

- predictions by chiral unitary model

  [Jido et al., NPA (2003); Bruns, Cieplý, NPA (2022); Guo et al., PLB (2023)]

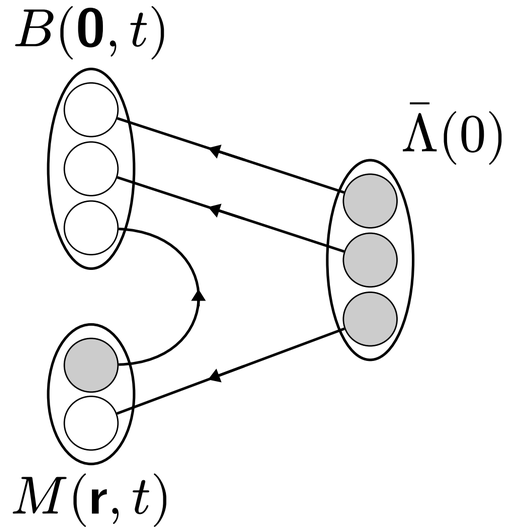




Setups

- SU(3) conf. w/ $a \approx 0.12$ fm, 32^4 lattices, $m_M \approx 670$ MeV, $m_B \approx 1489$ MeV
  [Inoue (HAL QCD), PoS CD15, 020 (2016)]
- ratio of correlators (rep $= 1, 8_s, 8_a$)

$$R^{(\text{rep})}(\mathbf{r}, t) = \frac{\langle (M(\mathbf{r}, t)B(\mathbf{0}, t))_{(\text{rep})}\, \bar{\Lambda}(0)\rangle}{\langle M(t)\bar{M}(0)\rangle\langle B(t)\bar{B}(0)\rangle}, \qquad \bar{\Lambda}(0) \sim \sum_{\mathbf{z}} \bar{u}(\mathbf{z})\bar{d}(\mathbf{z})\bar{s}(\mathbf{z})$$



- smearing quark operator at sink
  to smooth the correlators [KM et al., PTEP (2023)]
- **bound state in each channel from $\langle \Lambda^{(8)}(t)\bar{\Lambda}^{(8)}(0)\rangle$ and $\langle \Lambda^{(1)}(t)\bar{\Lambda}^{(1)}(0)\rangle$**
- **neglect coupling between 8_s and 8_a in this work**

$$\begin{pmatrix} V_{8_s8_s}(r) & V_{8_s8_a}(r) \\ V_{8_a8_s}(r) & V_{8_a8_a}(r) \end{pmatrix} \approx \begin{pmatrix} V_{8_s8_s}(r) & 0 \\ 0 & V_{8_a8_a}(r) \end{pmatrix}$$

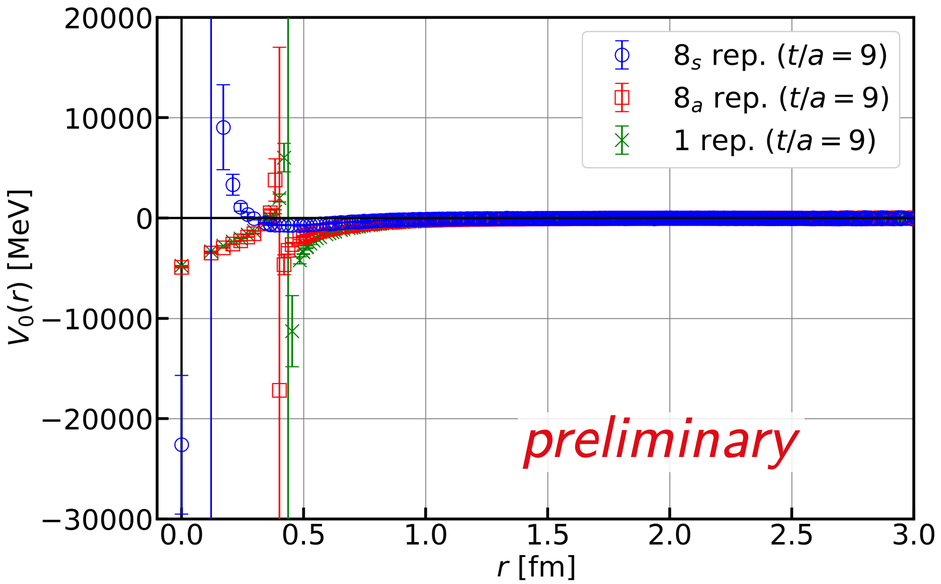
cf.) chiral perturbation theory with Weinberg–Tomozawa term:

- **no coupling between 8_s and 8_a**
- same interactions for 8_s and 8_a

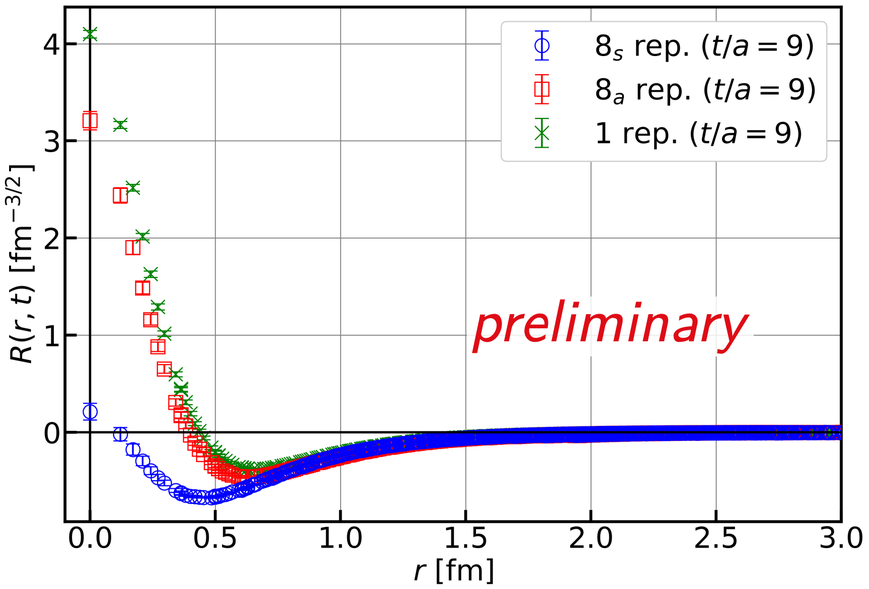
Local potentials

$$V(\mathbf{r}) = \frac{\mathcal{D}R(\mathbf{r},t)}{R(\mathbf{r},t)}, \qquad \mathcal{D}R(\mathbf{r},t) = \left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t} + \frac{1}{8\mu}\frac{\partial^2}{\partial t^2}\right) R(\mathbf{r},t)$$

local potentials $V(\mathbf{r})$



$R(\mathbf{r},t)$ (NBS w.f.)



- singular behavior in all channels because of $R(\mathbf{r},t)$ crossing zero
  - (probably) not problematic in principle, [Aoki, Yazaki, PTEP (2022)]
    but difficult to obtain reliable results
- take alternative approach: **separable potential**

Separable potentials in the HAL QCD method

- local approximation

$$\left(R(\mathbf{r},t) = \frac{\langle O_1(\mathbf{r},t) O_2(\mathbf{0},t) \bar{J}(0) \rangle}{\langle O_1(t) \bar{O}_1(0) \rangle \langle O_2(t) \bar{O}_2(0) \rangle} \qquad \mathcal{D}R(\mathbf{r},t) = \left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t} + \frac{1}{8\mu} \frac{\partial^2}{\partial t^2} \right) R(\mathbf{r},t) \right)$$

$$U(\mathbf{r},\mathbf{r}') \approx V(\mathbf{r})\, \delta^{(3)}(\mathbf{r}-\mathbf{r}') \quad \Rightarrow \quad V(\mathbf{r}) = \frac{\mathcal{D}R(\mathbf{r},t)}{R(\mathbf{r},t)}$$

- **separable approximation**

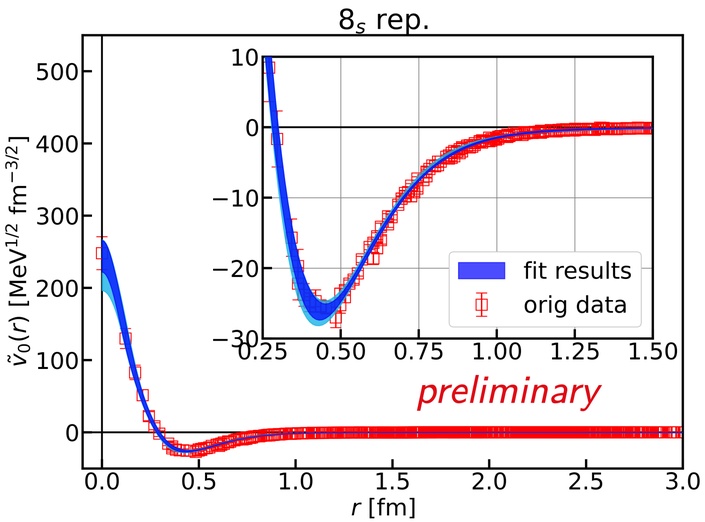
$$U(\mathbf{r},\mathbf{r}') \approx \eta\, v(\mathbf{r}) v(\mathbf{r}') \quad \Rightarrow \quad \begin{cases} v(\mathbf{r}) = \dfrac{\int d^3r\; R(\mathbf{r},t)\mathcal{D}R(\mathbf{r},t)}{\sqrt{\left| \int d^3r\; R(\mathbf{r},t)\mathcal{D}R(\mathbf{r},t) \right|^3}} \mathcal{D}R(\mathbf{r},t) \\ \eta = \dfrac{\int d^3r\; R(\mathbf{r},t)\mathcal{D}R(\mathbf{r},t)}{\left| \int d^3r\; R(\mathbf{r},t)\mathcal{D}R(\mathbf{r},t) \right|} (= \pm 1) \end{cases}$$

$\rightarrow$ **no singular points** in $v(\mathbf{r})$

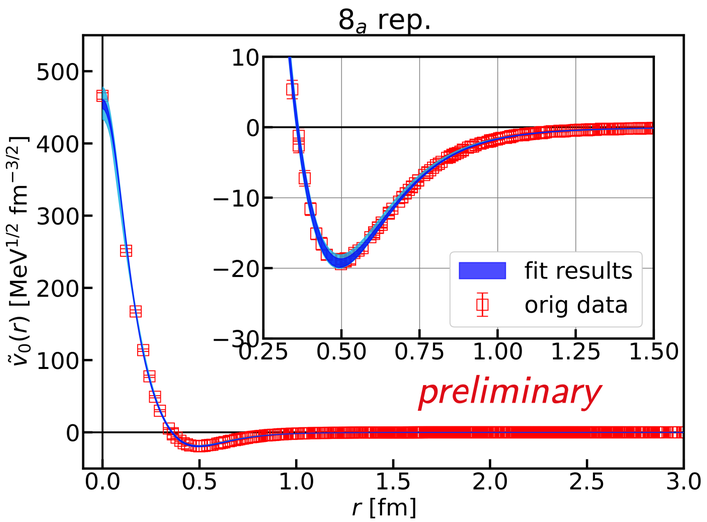
Results of separable potential

- $v(\mathbf{r})$ in $U(\mathbf{r},\mathbf{r}^{'}) \approx \eta\, v(\mathbf{r})v(\mathbf{r}^{'})$

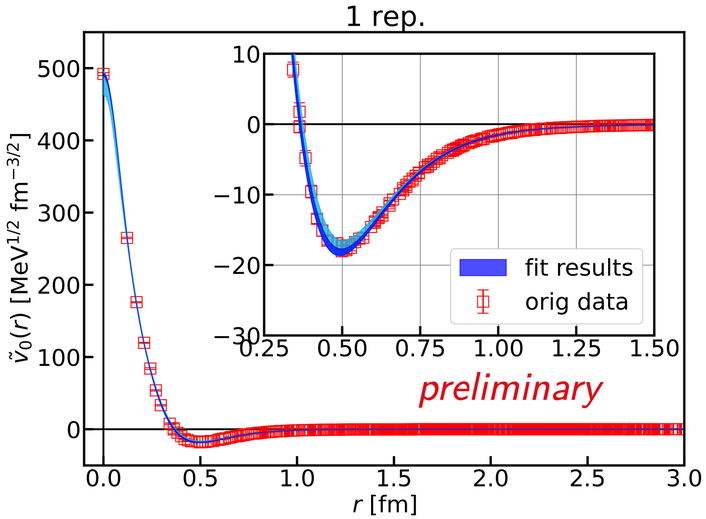
8_s channel



8_a channel



1 channel



red: discrete potential (central value) dark blue: fit results (central value) light blue: systematic uncertainties

- no singular points
- obtained $\eta = -1$ for all channels $\rightarrow$ **attractive**

Binding energies

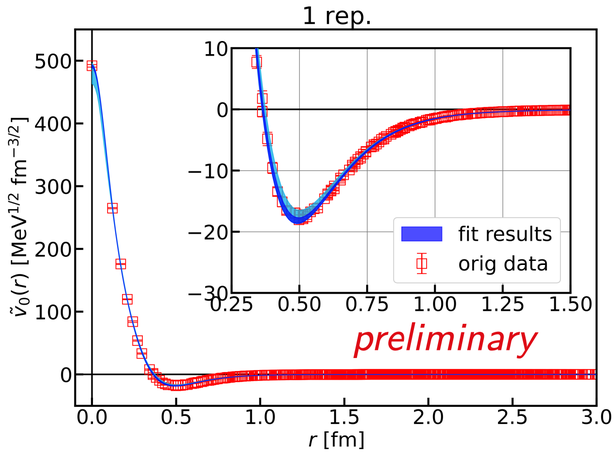
channel	E_{bind} [MeV]	from 2pt [MeV]
8_s	$157.3(16.3)(^{+20.7}_{-43.3})$	$154.7(7.8)$
8_a	$202.8(6.6)(^{+19.4}_{-33.8})$	
1	$249.8(9.0)(^{+23.7}_{-29.3})$	$223.8(6.1)$

- $E_{\text{bind}}^{(8_s)} \lesssim E_{\text{bind}}^{(8_a)} \lesssim E_{\text{bind}}^{(1)}$? $\leftarrow$ consistent with UChPT ($m^{(8)}, m^{(8')} > m^{(1)}$) [Jido et al., 2003] etc.
- E_{bind} for 8_s agrees with that from 2pt functions w/ 3-quark operator
  i. 3-quark operator couples to the bound state in 8_s channel
  ii. E_{bind} for 8_a appears to be larger due to missing 8_s-8_a mixing effect
- large systematic uncertainty

Systematic uncertainties

sources		**estimated by**
(1) lattice discretization at short distances	$\rightarrow$	5pt/9pt Laplacian
(2) beyond the separable form	$\rightarrow$	$t/a = 8$–10, $\xi_s/a = 1.0, 1.3$ (sink smearing)
(3) inelastic contamination	$\rightarrow$	$t/a = 8$–10
(4) fit ansatz of $v(\mathbf{r})$	$\rightarrow$	$N/(N{+}1)$-range Gaussians
(5) finite-volume effect	$\rightarrow$	without/with mirror images

- main contributions to the syst. error of E_{bind}
  - 8_s channel: (2), (3)
  - 8_a, 1 channels: (1), (2), (3), (4)
- E_{bind} sensitive to the uncertainty of $v(\mathbf{r})$ at short distances



Summary

- we study $\Lambda(1405)$ in flavor SU(3) limit from the **meson-baryon scatterings in lattice QCD** using the HAL QCD method
- R-correlator has a zero point, which leads to singular behavior in the local potential
- we employ a **separable potential** and the results show **attractive interactions**
- the obtained binding energies indicate $E_{\text{bind}}^{(8_s)} \lesssim E_{\text{bind}}^{(8_a)} \lesssim E_{\text{bind}}^{(1)}$
- the binding energies have large systematic uncertainties, mostly coming from the fluctuation in short distances

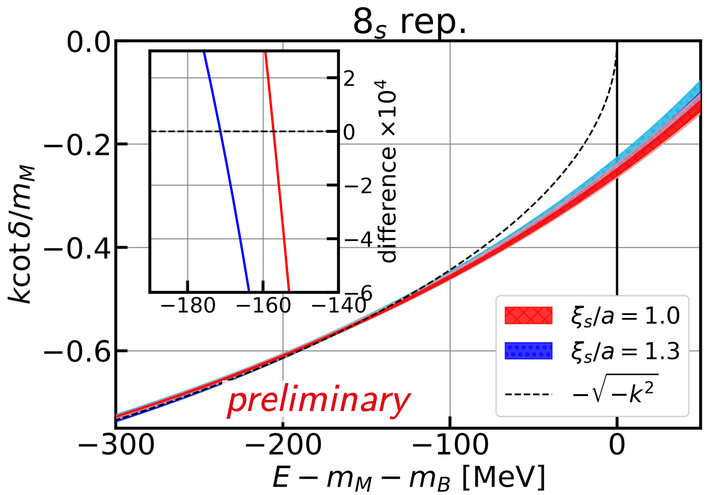
Future prospects

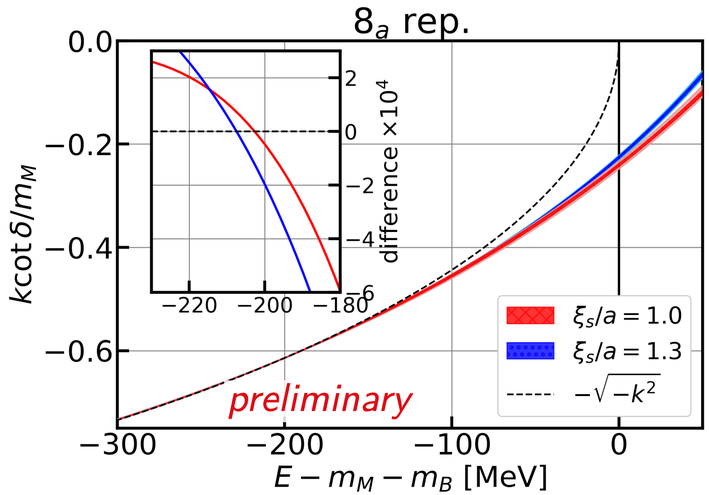
- to obtain precise signal of $\Lambda(1405)$, we need to control the fluctuations in short distances by employing···
  - finer lattice spacing, large temporal volume to suppress timeslice dependence, large statistics
  - potential form beyond the single separable, or other non-local potential forms (application of [Ernst, Shakin, Thaler, PRC (1973)])
- beyond SU(3) limit, simulation w/ lighter pion mass
  - application of separable potentials to coupled-channel case

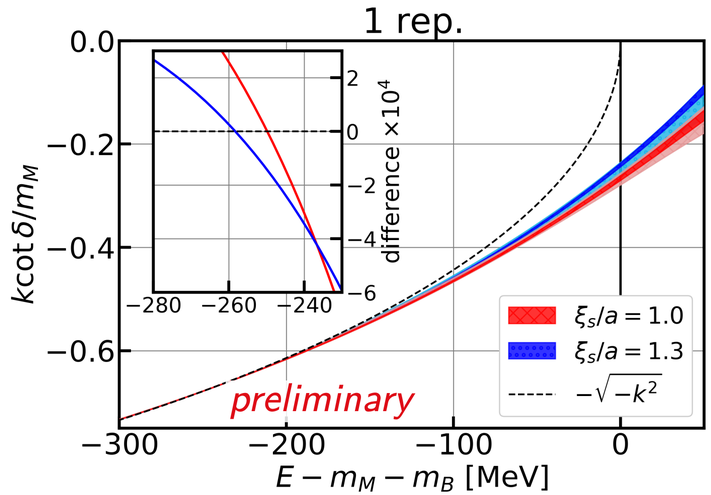
Backups

Phase shifts in each channel and each ξ_s

- $k \cot \delta$ (dashed line: bound-state condition $k \cot \delta = -\sqrt{-k^2}$)



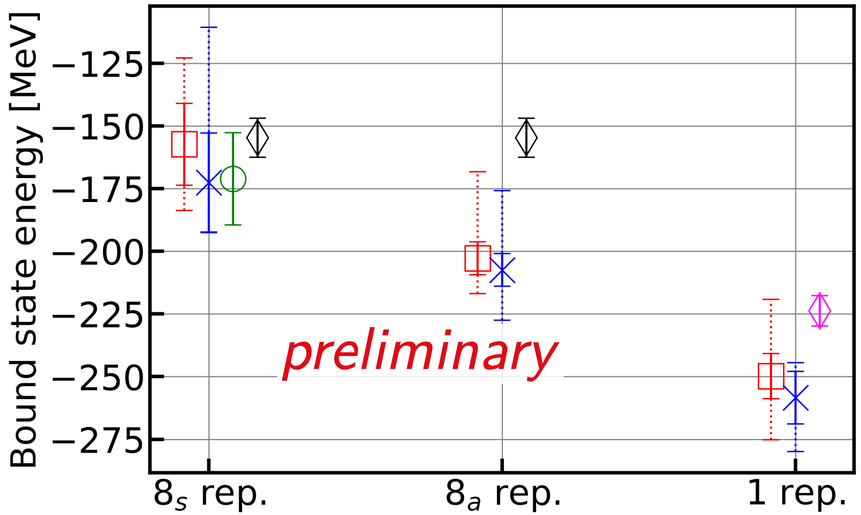


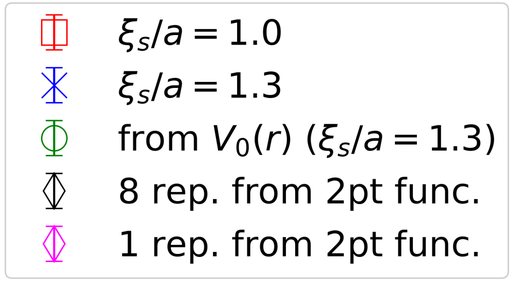


- ξ_s: range of smearing at sink (results with $\xi_s/a = 1.0$ and 1.3 agree with each other)
- attractive behavior, exhibiting **a bound state**

Binding energies in each channel and each ξ_s

- bound state energies





- ξ_s: range of smearing at sink
- solid (dotted) error bars: statistical (statistical + systematic) errors
- diamonds: estimates from 2pt functions

- deviation between 8_s and 8_a come from the effect of 8_s-8_a mixing?
- sensitive to short-distance behavior of $v(\mathbf{r})$ ($r \lesssim 0.5$ fm)

  (as in the case of a strong attractive local potential)

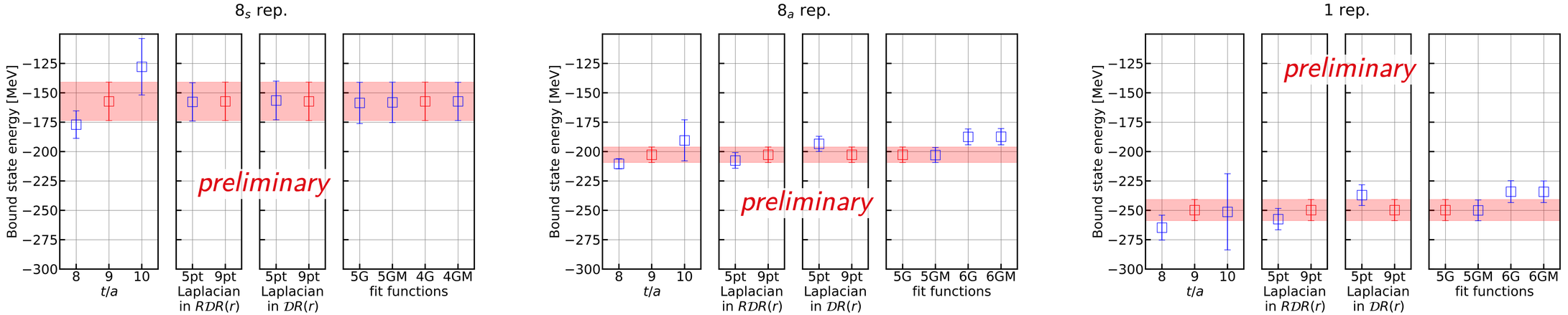
Breakdown of systematic uncertainties

sources		**estimated by** (underlined: central value)
lattice discretization at short distances	$\rightarrow$	5pt/<u>9pt</u> Laplacian (in $\mathcal{D}R$ and $\int d^3r\, R\mathcal{D}R$)
beyond the single-separable form	$\rightarrow$	$t/a = 8, \underline{9}, 10,\ \ \xi_s/a = \underline{1.0}, 1.3$ (sink smearing)
inelastic contamination	$\rightarrow$	$t/a = 8, \underline{9}, 10$
fit ansatz of $v(\mathbf{r})$	$\rightarrow$	$\underline{N}/(N{+}1)$-range Gaussians
finite-volume effect	$\rightarrow$	<u>without</u>/with mirror images

syst. error = spread over 96 combinations of the above

Breakdown of systematic uncertainties (cont.)

- main contributions to the syst. error of E_{bind} (syst. 10–30% vs stat. 3–10%)
  - 8_s: beyond the single-separable form, inelastic contamination (t-dependence alone: 13–21%)
  - 8_a, 1: lattice discretization at short distances, fit ansatz, beyond the single-separable form, inelastic contamination (3–8% from each setup, 10–17% combined)
  - finite-volume effect: negligible, sink smearing (ξ_s): 15, 6, 10 MeV for 8_s, 8_a, 1



- uncertainty of the potential at short distances contributes

Separable-potential approx. as an expansion

- What expansion is the separable-potential approx. based on?
  - one answer: sum of the separable potentials $U(\mathbf{r}, \mathbf{r}') = \sum_{i,j=0}^{\infty} \Lambda_{ij}\, v_i(\mathbf{r}) v_j^*(\mathbf{r}')$
  - systematic way to determine $v_i(\mathbf{r})$ and Λ_{ij} from a given set of wave functions

[Ernst, Shakin, Thaler, PRC (1973)]

  - application to HAL QCD has already been discussed,
    in which the leading-order analysis is equivalent to our current analysis

[slide by Lu Meng in Baryon2025]