Ω-baryon excited-state contamination in lattice calculations using Heavy Baryon χPT

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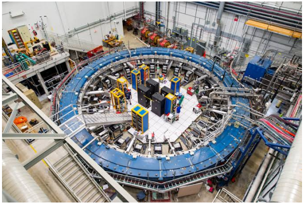
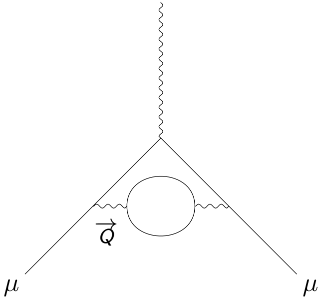
Motivation - Muon g-2

- Hadronic vacuum polarisation (HVP) dominates the uncertainty of g-2
- Total error $\approx 0.5\%$ of value
- Lattice spacing error $\approx 0.3\%$
- **Lattice spacing is 40% of total variance!** [Nature volume 653, pages 373–377 (2026)]

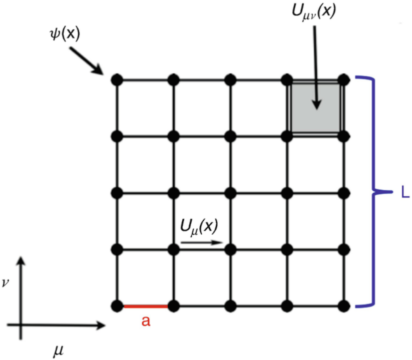


[Fermi National Accelerator Laboratory 2017]



Motivation - Muon g-2

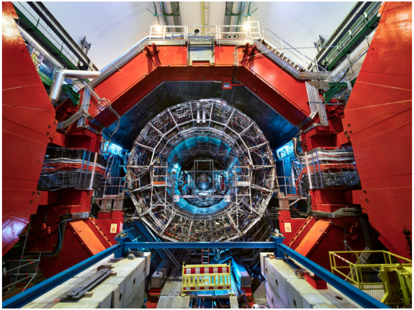
- Lattice spacing a is distance between two lattice points
- Choose Omega mass to set scale
- $$a = \frac{[am_\Omega]_{lattice}}{[m_\Omega]_{physical}}.$$
- Want to know a very precisely



[Ratti, C., Bellwied, R. (2021)]

Why the Omega baryon

- The Ω baryon is composed of three strange quarks: $\Omega = sss$.
- Ω is significantly heavier than the nucleon — useful for reducing relative errors in lattice calculations.
- Ω is stable under strong interactions — no decay to lighter baryons.



[ALICE Detector]

- Experimental value:

$$m_\Omega = 1\,672.511 \pm 0.033_{\mathrm{stat.}} \pm 0.102_{\mathrm{syst.}}\ \mathrm{MeV}/c^2$$

Correlation Functions and Excited States

Two point Function:

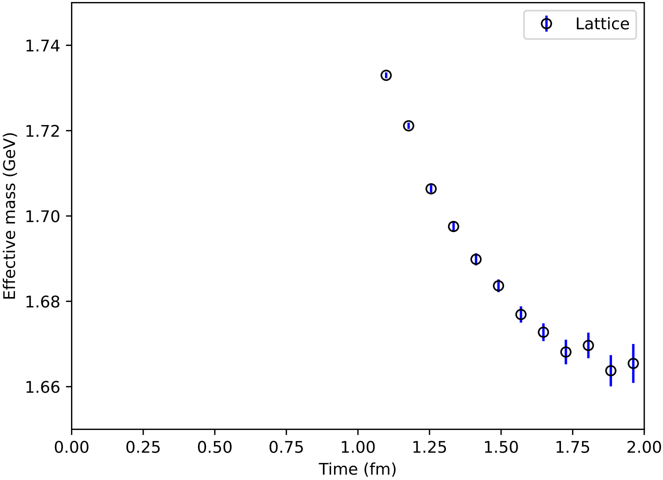
$$G(t) = \sum_{\vec{x}} < 0|\Omega(t)\Omega^{\dagger}(0)|0 > \tag{1}$$

$$G(t) = \underbrace{A_0 e^{-m_\Omega t}}_{\text{Ground state}} + \underbrace{\int dE\, \rho(E)\, e^{-Et}}_{\text{Excited states}} \tag{2}$$

- Two types of excited states
  - Single particle excitations of the Omega
  - Multi-particle states
- $\rho(\vec{q})$: spectral density describing the coupling to each excited state.
- At large Euclidean times, lower-energy contributions dominate.
- Goal: isolate the ground-state Ω and suppress excited-state contamination

Lattice calculation of the Omega baryon

- Effective mass
- Want a plateau at ground state mass
- Large excited states shown by decaying exponential
- We want to remove these



Generalized Eigenvalue Problem (GEVP)

Smearing:

- Point Operators:
  - Three quarks at the same point
- Smeared operators:
  - Three quarks spread out from each other
  - Looks more like a physical Ω
- Different smearing cause different couplings to different states

Time Shifts:

- Correlator looks approximately like

$$G(t) \approx \sum_i A_i^2 \; e^{-E_i t}$$

- Shift time t to time $t + 2\tau$

$$G(t{+}2\tau) \approx \sum_i A_i^2 \; e^{-E_i(t+2\tau)}$$

$$G(t + 2\tau) \approx \sum_i D_i^2 \; e^{-E_i t}$$

- Shift with different τ to have different Euclidean time behaviour

Generalized Eigenvalue Problem (GEVP)

- GEVP equation:

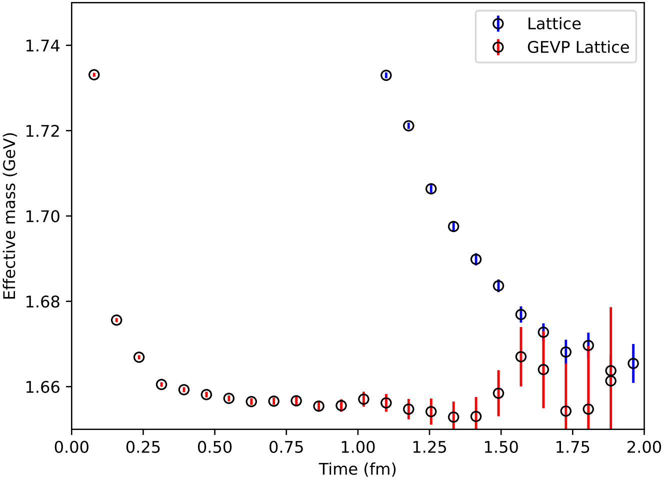
$$\boldsymbol{G}(t_a)v(t_a, t_b) = \lambda(t_a, t_b)\boldsymbol{G}(t_b)v(t_a, t_b), \tag{3}$$

- Construct a matrix of correlators using multiple operators
- Find optimal linear combinations that maximise overlap onto individual states such as ground state

- Reduce contamination from excited states

Lattice calculation of the Omega baryon - with the GEVP

- Reaches a plateau
- Large Euclidean times, there is large amounts of noise
- Small Euclidean times, there are still some excited states



Finite Volume Correlation functions

$$G(t) = e^{-m_\Omega t} + \underbrace{\int d^3q\, \rho(\vec{q}) e^{-(E_K + E_\Xi)t}}_{K\Xi \text{ scattering states}} \tag{4}$$

In Finite Volume:

$$G(t) = e^{-m_\Omega t} + \underbrace{\frac{1}{L^3} \sum_{\vec{q}} \rho(\vec{q}) e^{-(E_K + E_\Xi)t}}_{K\Xi \text{ scattering states}} \tag{5}$$

where,

$$\vec{q} = \frac{2\pi}{L} \vec{n}, \quad \vec{n} \in \mathbb{Z}^3.$$

Heavy Baryon Chiral Perturbation Theory

Goal: Model the $K\Xi$ scattering-state contamination in the Ω correlator

- Effective field theory of QCD in low energies
- Describes interactions between baryons and pseudoscalar mesons
- Leading-order (LO) calculation performed in the heavy-baryon limit

$$m_B \to \infty$$

- Calculate the contribution of intermediate $K\Xi$ states
- Use lattice masses for the Ω and K
- Apply the same GEVP projection used in the lattice calculation.

Provides a prediction for the excited-state contamination observed in lattice QCD.

Heavy baryon chiral perturbation theory Feynman diagrams

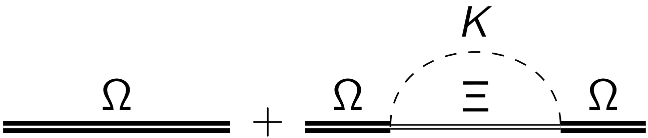


Figure: Feynman diagram showing the process $\Omega \to \Xi K \to \Omega$.

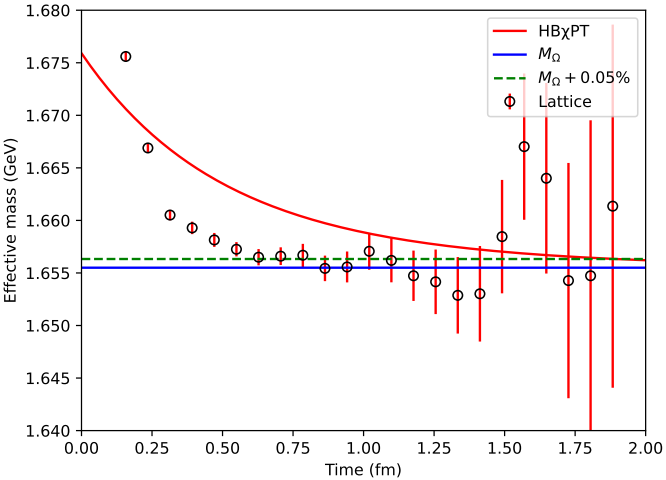
Contributions

- Tree-level Ω propagation
- One-loop $K\Xi$ contribution
- Intermediate $K\Xi$ states generate excited-state contamination

$$\mathcal{L}_{\mathrm{int}} = \frac{C}{f}\bar{\Omega}^{\mu}\,\partial_{\mu}K\,\Xi$$

Comparison of Lattice GEVP Effective Mass and HBχPT GEVP Effective Mass

- Reaches the 0.05% at $\approx$ 2fm.
- Poor agreement with lattice data.
- Over-estimates excited states.

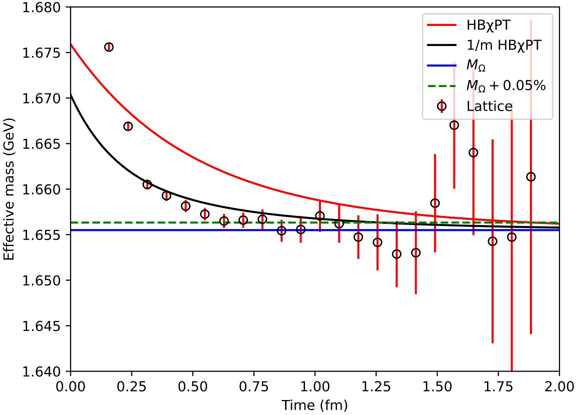


Comparison of Lattice GEVP Effective Mass and 1/m HBχPT GEVP Effective Mass

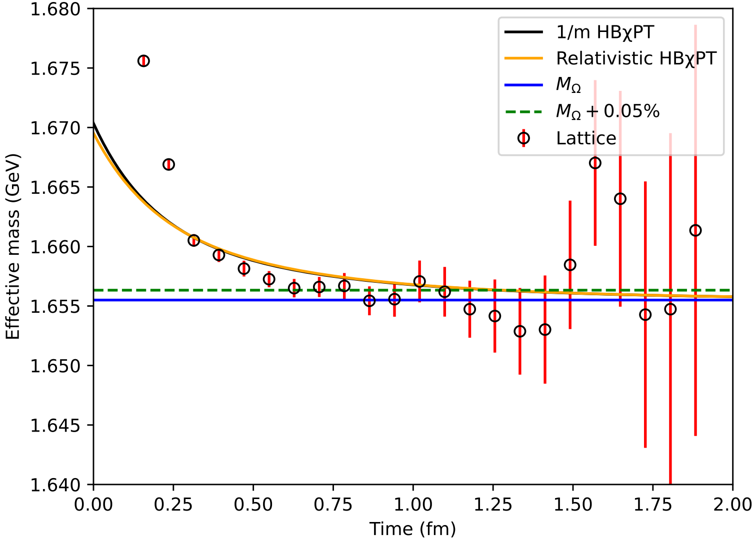
- Include baryon recoil:

$$E \approx m_{\Xi} \;\Rightarrow\; m_{\Xi} + \frac{q^2}{2m_{\Xi}}$$

- Recoil suppresses excited-state contamination through $e^{-q^2/(2m_{\Xi})t}$.
- Additional suppression from $\rho(E)$.
- Improves agreement with lattice data.



Comparison of 1/m HBχPT GEVP Effective Mass and relativistic HBχPT GEVP Effective Mass



Hamiltonian Effective Field Theory (HEFT)

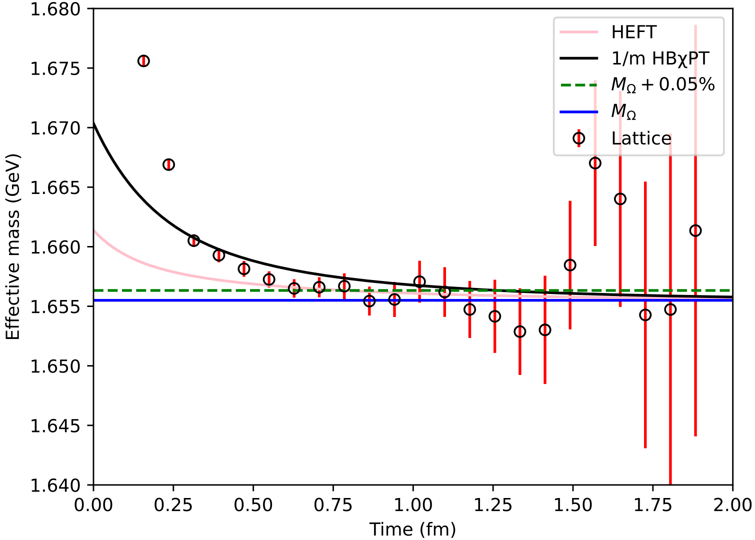
Work in collaboration with Lauzhau University: Zhan-Wei Liu and Fang-Chao Han

- HEFT is a relativistic calculation with all 1 loop reducible diagrams
- HEFT describes the coupling of a bare baryon state to meson-baryon channels through

$$H = H_0 + H_{\mathrm{int}}.$$

- Uses the same vertex term from HBχPT and coupling
- Provides a complementary description of scattering states

Comparison of 1/m HBχPT GEVP Effective Mass and HEFT



Conclusion

Key takeaways:

- Quantify $K\,\Xi$ scattering states contamination.
- Compare HBχPT with HEFT.
- Provides a framework to correct lattice data.
- Estimate the uncertainty.