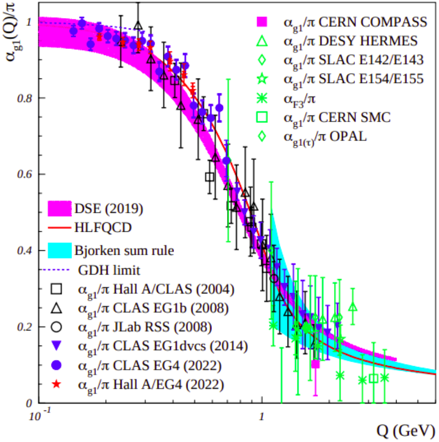
The low-energy Compton amplitude from lattice Feynman-Hellmann

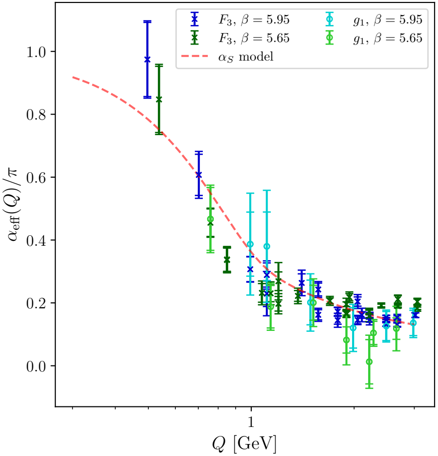
Alec Hannaford Gunn

For the CSSM/QCDSF/UKQCD collaboration

23rd September, 2026



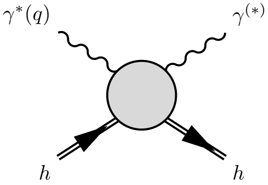
Experimental results for the g_1 effective strong coupling. Source: Deur et al., arXiv:2303.00723.



Our lattice results for the g_1 and F_3 effective strong couplings.

1. Introduction to the Compton amplitude and the lattice Feynman-Hellmann approach
2. Feynman-Hellmann calculations of the F_3 and g_1 structure functions
3. Outline of low-Q^2 contaminations
4. New Results — Preliminary!

The Compton amplitude



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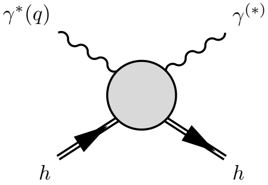

High-energy ($Q^2 = -q^2 \gtrsim 2$ GeV^2)

- Scattering off quarks/gluons
- Parton distributions (PDFs, GPDs): properties of quarks/gluons in a hadron

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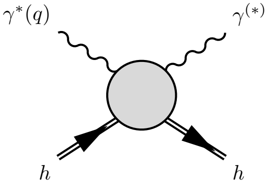

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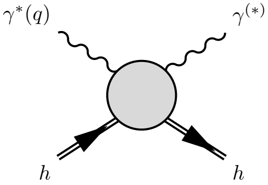

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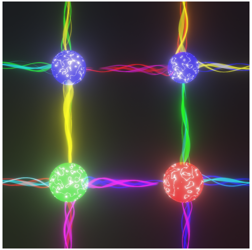
Intermediate energy ($Q^2 \approx 0.1$–2 GeV^2)

- Sensitive to whole resonance structure of QCD $\rightarrow$ harder to model
- Transition energy region: asymptotic freedom and confinement
- subtraction function, effective strong coupling

Lattice QCD and spectroscopy

- QCD: almost all ME non-pert (part)
- Lattice QCD: the only (1) systematically improveable and (2) first principles



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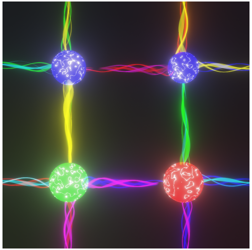

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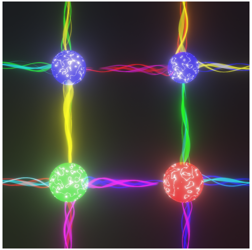

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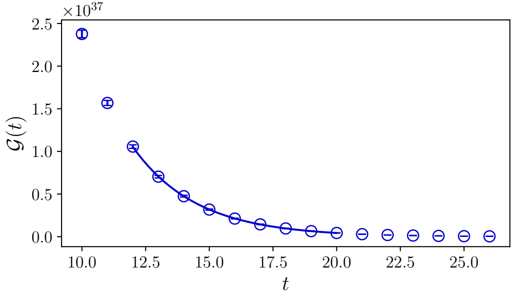

n-point function

$$\langle \underbrace{\chi(t)}_{\text{annihilation}} \underbrace{\mathcal{O}_1(t_1)\ldots\mathcal{O}_{n-2}(t_{n-2})}_{n-2 \text{ interactions}} \underbrace{\bar{\chi}(t_0)}_{\text{creation}} \rangle$$

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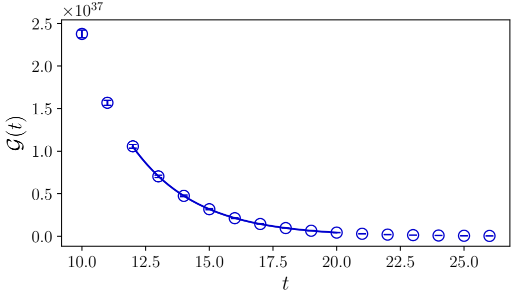

Nucleon two-point function

$$\mathcal{G}(t) = \langle \chi(t)\bar{\chi}(0)\rangle = A_N e^{-E_N t} + A_{N^*} e^{-E_{N^*} t} + \ldots$$

- $E_N < E_{N^*} < \ldots$

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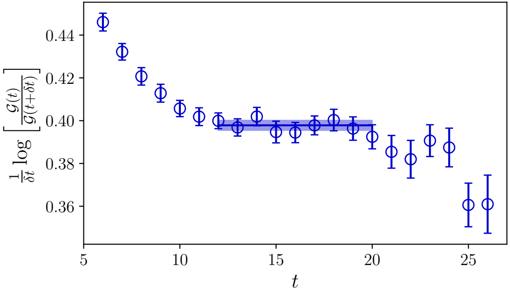

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- t too large $\to$ signal decay; t too small $\to$ excited state contamination

The Compton amplitude from lattice QCD

Motivation for lattice Compton amplitude:

- Many physically important kinematic limits are experimentally inaccessible
- Lattice results used to guide or compare to experimental determinations

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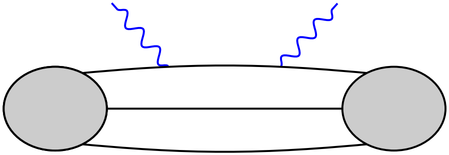

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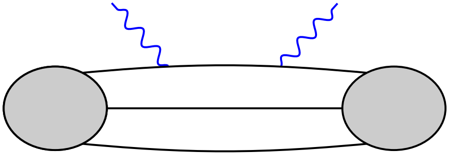

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Direct calculation problems:

- Recall: for n-point functions, larger n $\Rightarrow$ excited state control harder
- Large time-extent needed: $t_{\text{snk}} \gg t_1$, $t_{\text{src}} \ll t_2$ — expensive!
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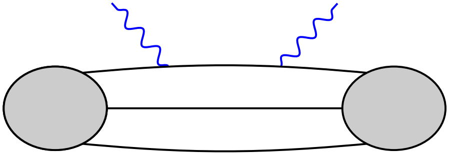

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Most other methods (e.g. quasi/pseudo-distributions) aim to determine light-cone parton distribution — not the physical amplitude — hence only relevant at high-energies.

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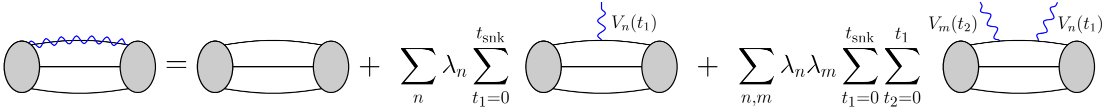

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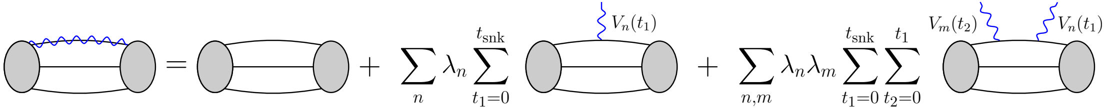

More exactly, we can show

$$\mathcal{G}_\lambda(t) = A_\lambda e^{-E_\lambda t} = A_\lambda \exp\left\{ - \left(E_N + \lambda E^{(1)} + \lambda^2 E^{(2)} + \mathcal{O}(\lambda^3) \right) t \right\}$$

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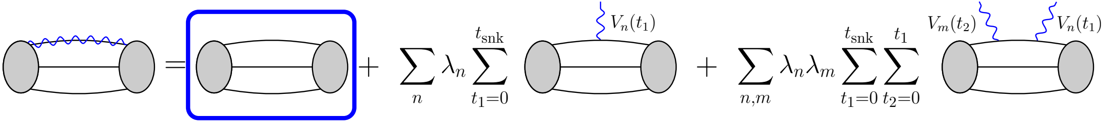

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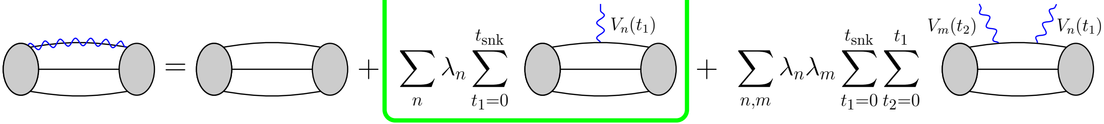

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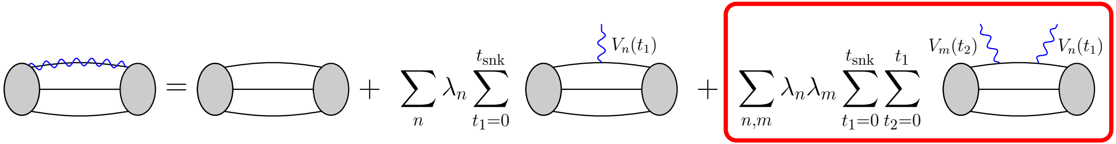

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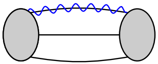
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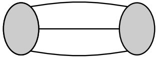
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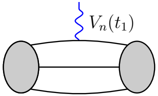

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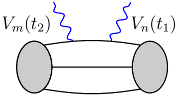

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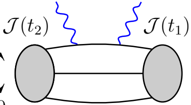

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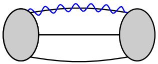
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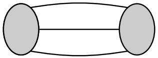
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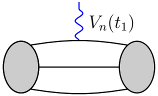

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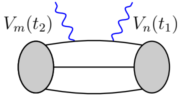

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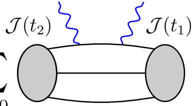


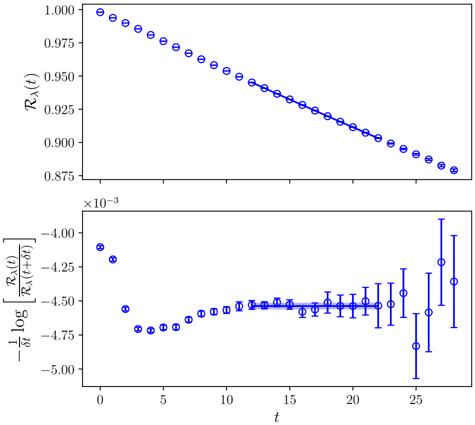
Compton amplitude (four-point), from a two-point function $\rightarrow$ simple excited state control

Feynman-Hellmann: excited state control

In Feynman-Hellmann, we look at the ratio

$$R_\lambda = \frac{\mathcal{G}_{(\lambda,\lambda)}\mathcal{G}_{(-\lambda,-\lambda)}}{\mathcal{G}_{(\lambda,-\lambda)}\mathcal{G}_{(-\lambda,\lambda)}} \simeq A_\lambda \exp\left\{-\frac{\partial^2 E_\lambda}{\partial\lambda_1\partial\lambda_2}t\right\}$$

- $\therefore$ can fit as a simple decaying exponential — very well-studied in lattice QCD
- no need for very large time extent
- simple control of excited states



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Feynman-Hellmann Compton amplitude so far...

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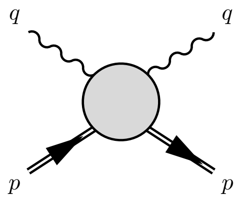

Scaling behaviour for moments of DIS structure functions, F_1 and F_2

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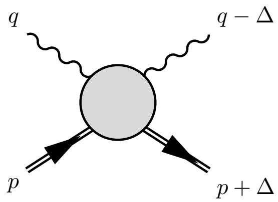

Off-forward Compton amplitude $\rightarrow$ GPD moments

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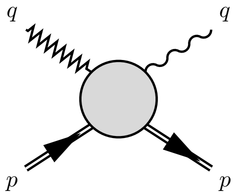
- **Parity-odd structure function:**
  - Can et al. (2025) PRD 111, 114505
  - Jordan McKee (talk later today)



Parity-odd structure function, F_3

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  - Batelaan et al. (2023) PRD 107
- **Off-forward Compton amplitude:**
  - Hannaford-Gunn et al. (2021) PRD 105
  - Hannaford-Gunn et al. (2024) PRD 110
- **Parity-odd structure function:**
  - Can et al. (2025) PRD 111, 114505
  - Jordan McKee (talk later today)
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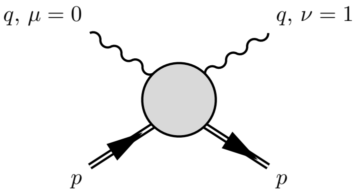

Spin-dependent structure function, g_1

Feynman-Hellmann Compton amplitude so far...

- **Forward Compton amplitude:**
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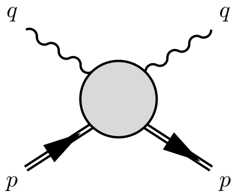
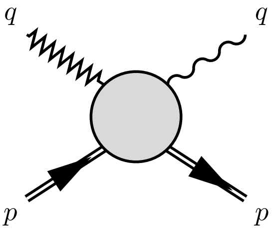


Identified and corrected major lattice artefact $\rightarrow$ allowed S_1 calculation

The parity-odd structure function: F_3



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Motivation:

- Ideal to determine (effective) strong coupling
- Radiative corr. precision physics
- Lack of low-energy expt. measurements and no lattice determinations

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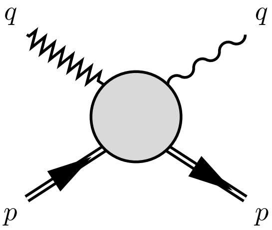

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Feynman-Hellmann details

- Background fields:

$$2\lambda_1 \cos(\vec{q}\cdot\vec{x}) j_1^{\mathcal{V}}(x) + 2\lambda_2 \sin(\vec{q}\cdot\vec{x}) j_3^{\mathcal{A}}(x),$$

where $Q^2 = |\vec{q}|^2$ is our hard scale.

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$$\left.\frac{\partial^2 E_\lambda}{\partial\lambda_1\partial\lambda_2}\right|_{\lambda_1=\lambda_2=0} = \frac{q_2}{Q^2} M_1^{F_3}(Q^2),$$

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Parity-odd structure function [Can et al. (2025) PRD 111]

Calculation details

- Spacings: $a = 0.068$ vs 0.052[fm]
- Sites: $N_L^3 \times N_T = 48^3 \times 96$
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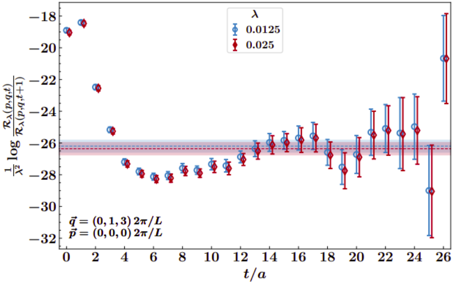

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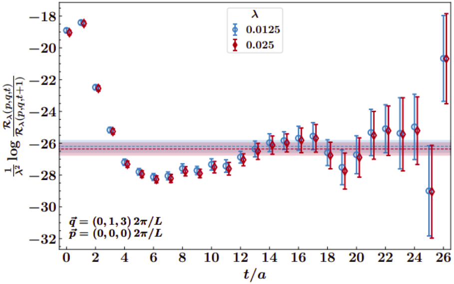
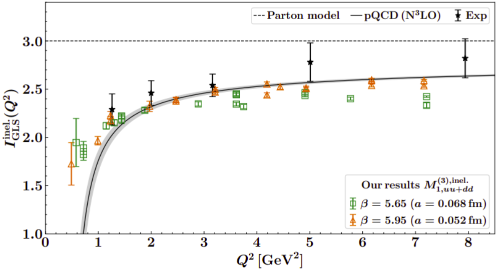
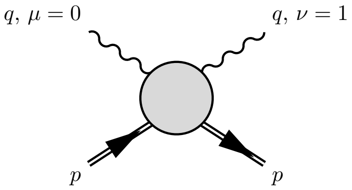




- Captures Q^2 behaviour, especially finer spacing
- Statistical errors much smaller than expt.
- Limited low-Q^2results, with large error bars

The spin-dependent structure function: g_1



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Motivation:

- Like F_3, can be used to determine (effective) strong coupling
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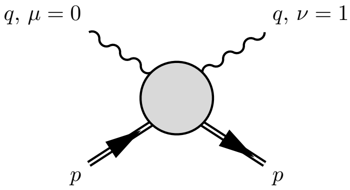

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$$M_1^{g_1}(Q^2) = \int_0^1 dx g_1(x, Q^2).$$

Notice: almost identical to F_3 relations.

Spin-dependent structure function [Crawford et al. in prep]

Calculation details

- Only one set of gauge configs
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- Sites: $N_L^3 \times N_T = 48^3 \times 96$
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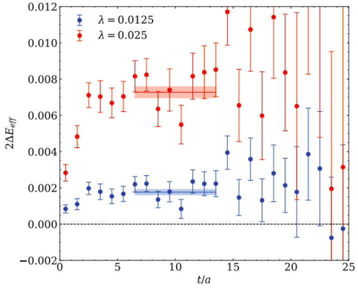

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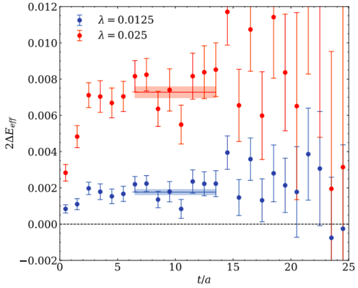
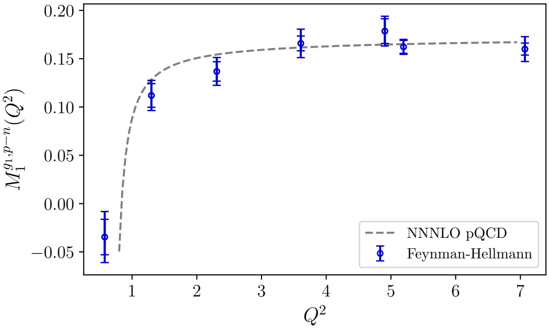
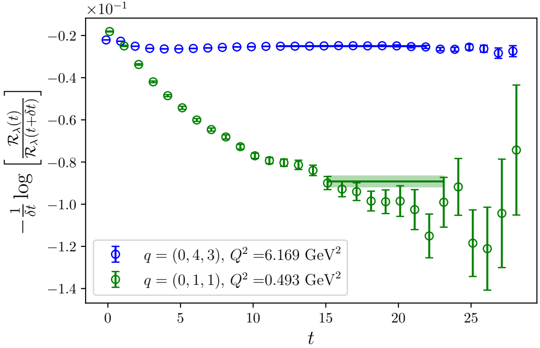




- Again, successful describing Q^2 shape
- Small statistical errors
- Only one low-Q^2 value, with larger error bars

Systematics at low-energy

Problem:



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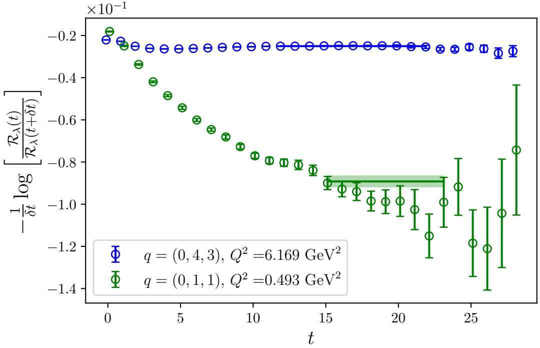

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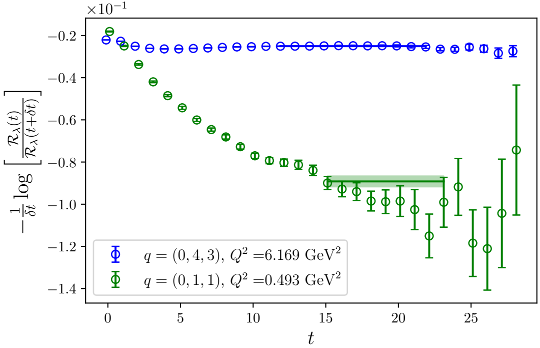

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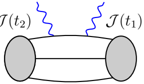
- Large error bars
- Fails for smaller Q^2

Need to understand what's going on

1. Introduction to the Compton amplitude and the lattice Feynman-Hellmann approach
2. Feynman-Hellmann calculations of the F_3 and g_1 structure functions
3. Outline of low-Q^2 contaminations
4. New Results — Preliminary!

Return to Feynman-Hellmann theory

$$\sum_{t_1=0}^{t_{\rm snk}} \sum_{t_2=0}^{t_1}$$



$$\sim \underbrace{\sum_X \frac{\langle N(p)|\mathcal{J}|X(p+q)\rangle\langle X(p+q)|\mathcal{J}|N(p)\rangle}{E_X(p+q)(E_X(p+q)-E(p))}}_{\text{Compton amplitude}} \times \underbrace{\left[t_{\rm snk} + \frac{e^{-(E_X(p+q)-E_N(p))t_{\rm snk}}}{E_X(p+q)-E_N(p)}\right]}_{\text{time dep.}}$$

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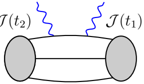

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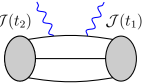

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Explains why high-energy effective mass looked fine, but low-energy doesn't plateau

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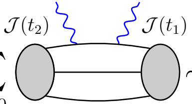

$$\sim \mathcal{A}^{\text{full}} t_{\text{snk}} + \mathcal{A}^{\text{elastic}} \frac{e^{-(E_N(p+q)-E_N(p))t_{\text{snk}}}}{E_N(p+q) - E_N(p)}$$

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- Throw away excited state terms to rewrite. Then

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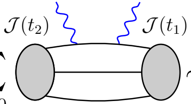

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Therefore results will $\rightarrow 0$ instead of to the elastic curve

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If our contaminations are proportional to $\mathcal{A}^{\text{elastic}}$, then we can construct:

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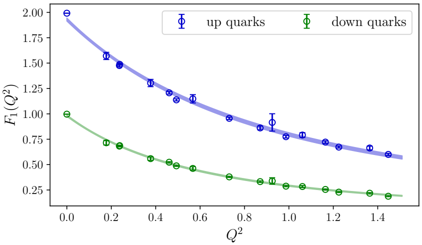

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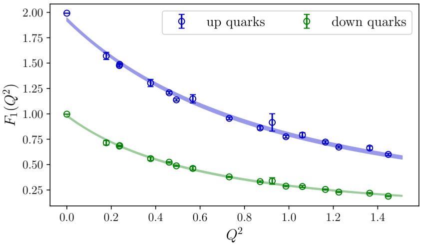

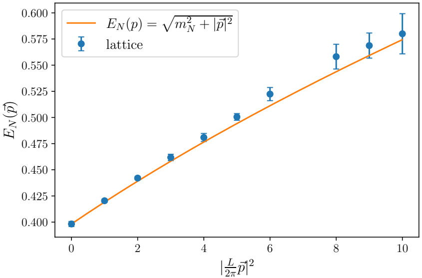

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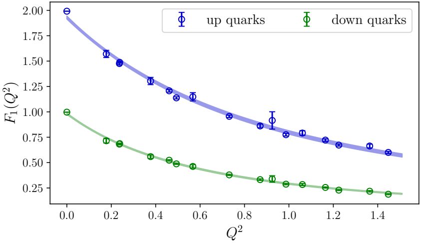
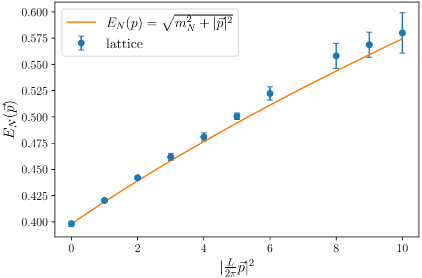
Construct contaminations $\rightarrow$ subtract from correlator





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Free Wilson fermion propagator $S(p)$ has analytic solution $\Rightarrow$ ideal testing ground:

$$\mathcal{R}(t) = \sum_{t_1,t_2} S(\vec{p}, t-t_1)\Gamma_1 S(\vec{p}+\vec{q}, t_1-t_2)\Gamma_2 S(\vec{p}, t_2)$$

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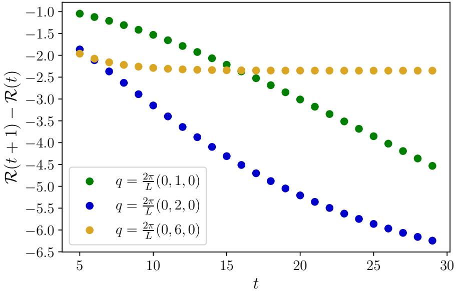

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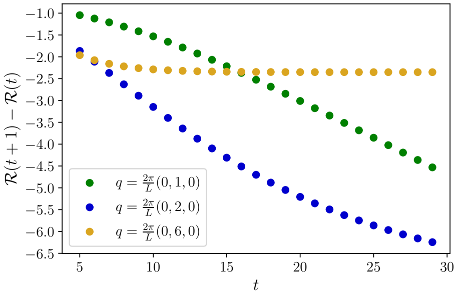

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Contamination ($F_2 = 0$ and $F_1^{\text{bare}} = 1/Z_V$):

$$\text{Contam} = -\frac{e^{-(E(p+q)-E(p))t} - 1}{Z_V^2 E(p+q)}$$

Improved signal: $\mathcal{R} - \text{Contam}$

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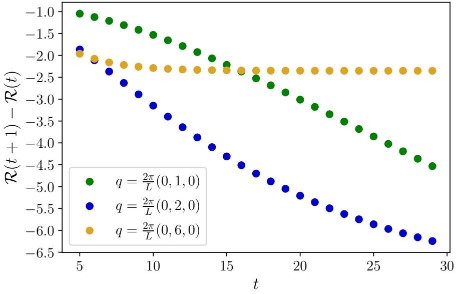
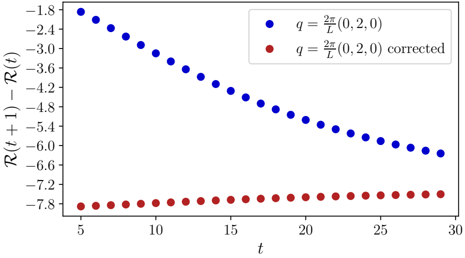


Slope comparable to effective mass — should be flat!

Contamination ($F_2 = 0$ and $F_1^{\text{bare}} = 1/Z_V$):

$$\text{Contam} = -\frac{e^{-(E(p+q)-E(p))t} - 1}{Z_V^2 E(p+q)}$$

Improved signal: $\mathcal{R} - \text{Contam}$



Improvement + asymptoting same direction

Nucleon calculation set-up

Improved ratio

$$\hat{\mathcal{G}}_{(\lambda_1,\lambda_2)}(t) = \mathcal{G}_{(\lambda_1,\lambda_2)}(t) - \lambda_1\lambda_2\mathcal{C}(t)$$

$$\mathcal{R}(t) = \frac{\hat{\mathcal{G}}_{(\lambda,\lambda)}(t)\hat{\mathcal{G}}_{(-\lambda,-\lambda)}(t)}{\hat{\mathcal{G}}_{(\lambda,-\lambda)}(t)\hat{\mathcal{G}}_{(-\lambda,\lambda)}(t)},$$

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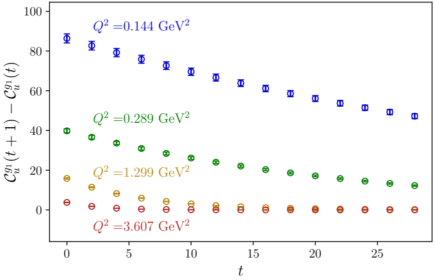
Use EFFs to reconstruct $\mathcal{C}(t)$



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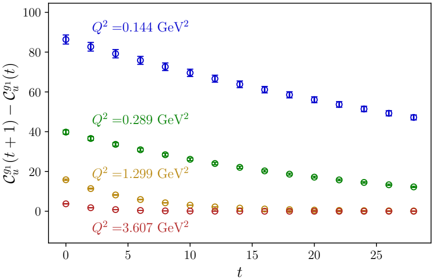


Parity-odd (F_3)

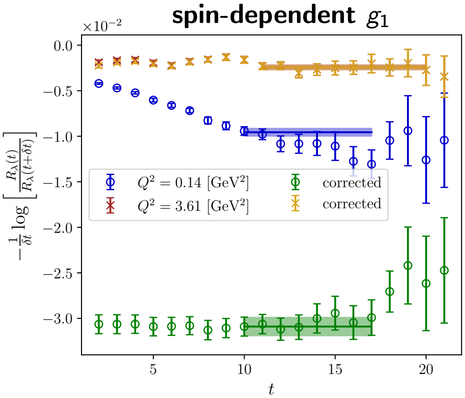
- Two configs: $a = 0.068$ vs 0.052 [fm]
- Sites: $N_L^3 \times N_T = 48^3 \times 96$
- Pion masses: $m_\pi \approx 3m_\pi^{\text{phys}}$
- Three additional low-Q^2 results

Spin-dependent (g_1)

- Two configs: $a = 0.068$ vs 0.052 [fm]
- Sites: $N_L^3 \times N_T = 48^3 \times 96$
- Pion masses: $m_\pi \approx 3m_\pi^{\text{phys}}$
- Additional low-Q^2 results

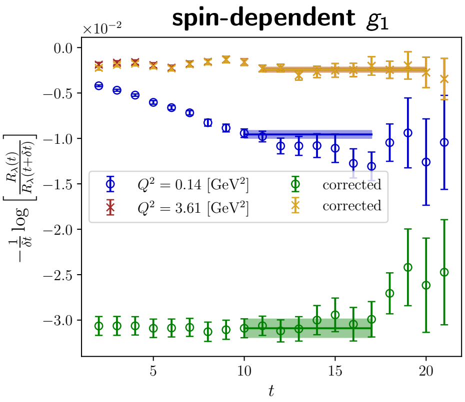
Use G_A and $F_{1,2}$ EFFs from three-pt, same configs

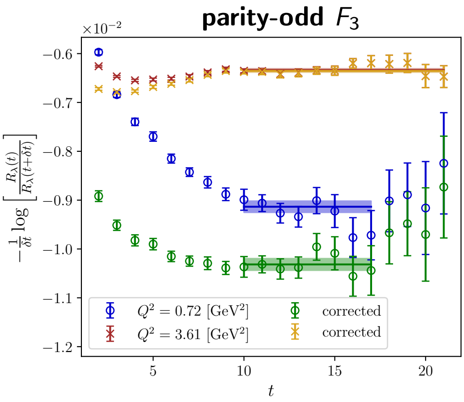
Results: effective masses



- High-Q^2: negligible change
- Low-Q^2: big improvement; uncorrected asymptoting before signal lost

Results: effective masses

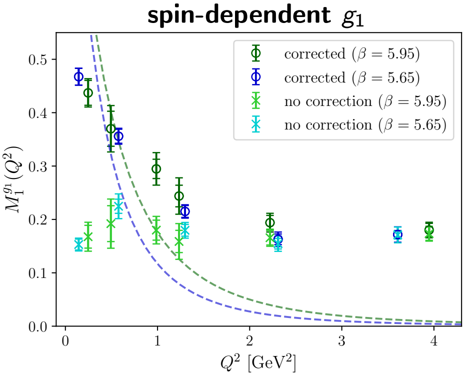




- High-Q^2: negligible change
- Low-Q^2: big improvement; uncorrected asymptoting before signal lost

- Both for coarser lattice spacing: $a = 0.068$ [fm]
- g_1 low-Q^2 significantly smaller

Results: structure function moments



- Elastic contribution: $\sim$ EFF^2
- Two points below elastic (both $0.1 - 0.2$ GeV^2)
- Expt. results lie above elastic

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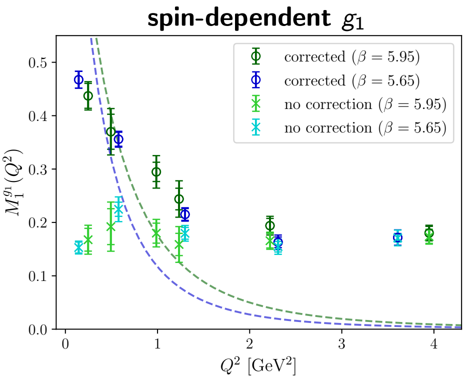

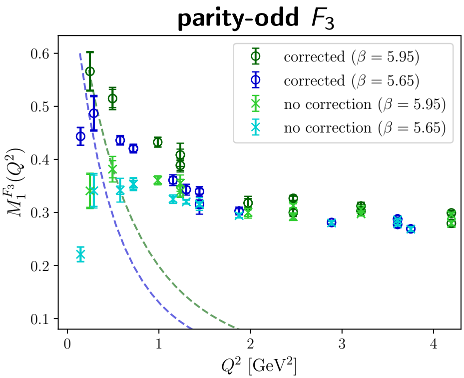

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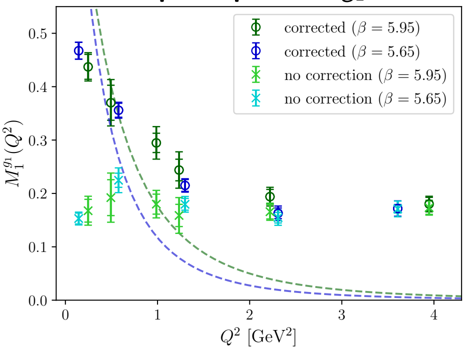

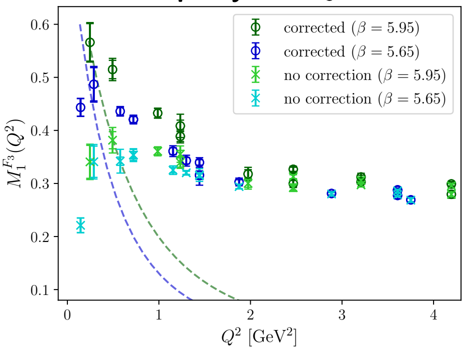

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- Two points below elastic (both $0.1 - 0.2$ GeV^2)
- Expt. results lie above elastic

Summary

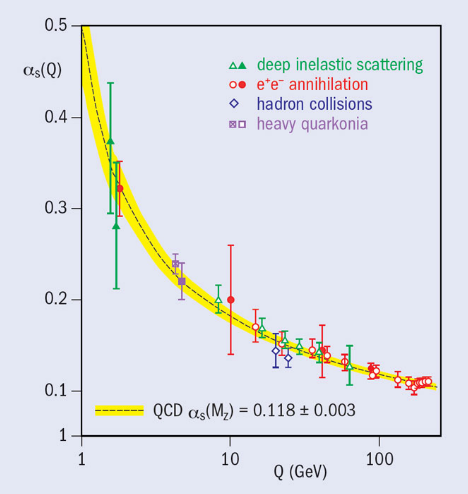
- Effective masses much flatter
- Q^2-space results meet elastic results
- Appears to be working well!

Effective charges: introduction

- To calculate the strong coupling α_s, need high-energy $\rightarrow$ perturbative domain
- First moment of F_3 and g_1 both clean probes of α_s:

$$M_1^{g_1(F_3)}(Q^2) = \left[1 - \sum_n C_n \left(\frac{\alpha_s(Q)}{\pi}\right)^n\right] g_A(N_q)$$

C_n calculated up to $n = 4$



Bethke & Zerwas, Physik Journal (2004).

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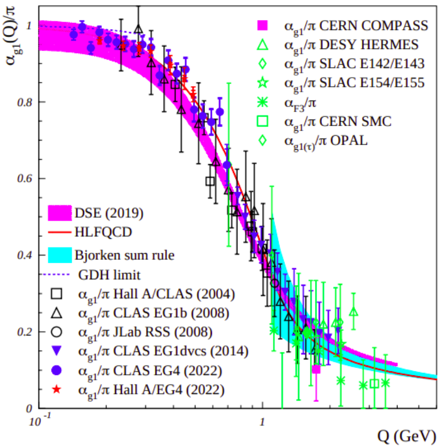
Effective strong charge/coupling truncates at $n = 1$:

$$M_1^{g_1(F_3)}(Q^2) = \left[1 - \frac{\alpha_{g_1(F_3)}(Q)}{\pi}\right] g_A(N_q)$$

Phenom. measure of QCD strength below pert. thresh. $\rightarrow$ no guarantee of coinciding



Deur et al., arXiv:2303.00723.

Effective charges: results

From the moments, we reconstruct α_{eff}:

$$\frac{\alpha_{g_1(F_3)}(Q)}{\pi} = 1 - \frac{M_1^{g_1(F_3)}(Q^2)}{g_A(N_q)}$$

Note: elastic subtracted: $M_1(Q^2) - M_1^{\text{el}}(Q^2)$

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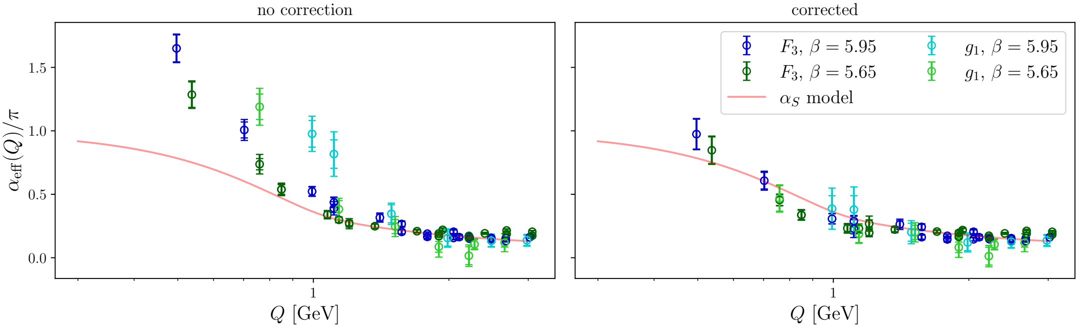
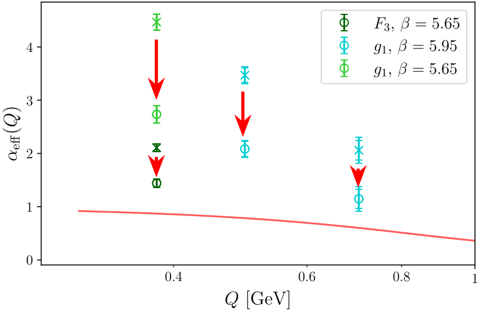
- Model good description of expt.
- Corrected results self-consistent and consistent with model
- Lattice and model discrepancy $Q \sim 0.7 - 1.2$ GeV ($m_\pi \approx 3m_\pi^{\text{phys}}$)



Remaining problem (lowest Q^2)

In previous slide we dropped four Q values:



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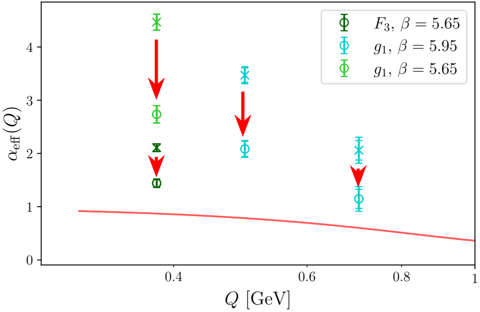

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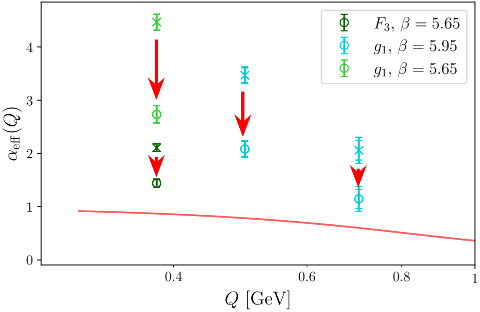
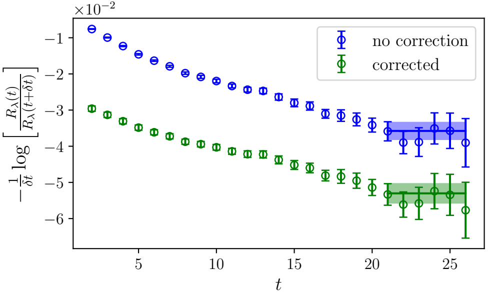


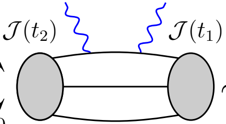


- Correction moves results closer to phenomen. expectation
- Three g_1 points affected, one F_3. All lowest Q^2

- For F_3 lowest Q^2, plateau or just flatter?
- Correction doesn't seem to flatten much

Subdominant contaminations

$$\sum_{t_1=0}^{t_{\rm snk}} \sum_{t_2=0}^{t_1}$$



$$\sim \mathcal{A}^{\rm full} t_{\rm snk} + \mathcal{A}^{\rm elastic} \frac{e^{-(E_N(p+q)-E_N(p))t_{\rm snk}}}{E_N(p+q)-E_N(p)}$$

$$+\mathcal{A}^{\text{non-fwd}} \left[\frac{e^{-(E_N(p+q)-E_N(p))t_{\rm snk}}}{E_N(p+q)-E_N(p)} - \frac{e^{-(E_X(p+2q)-E_X(p))t_{\rm snk}}}{E_X(p+2q)-E_N(p)} \right]$$

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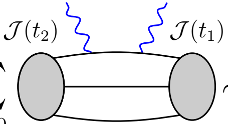

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- $\mathcal{A}^{\mathrm{elastic}}$ are the contaminations we have been modelling

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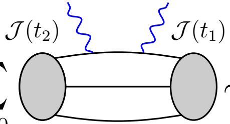

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- Also these "subdominant" contaminations $\mathcal{A}^{\text{non-fwd}}$ $\to$ result of source with higher momentum AND/OR excited state

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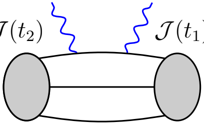

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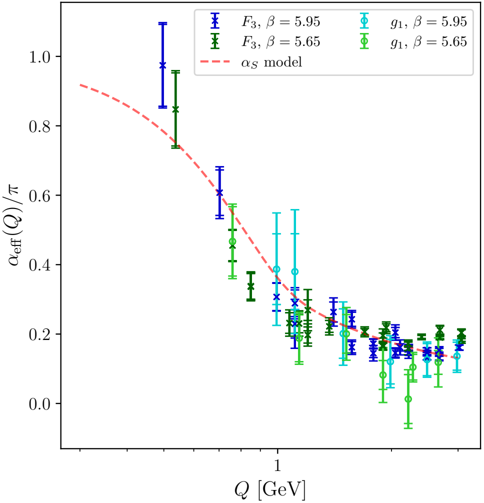
- $\mathcal{A}^{\text{elastic}}$ are the contaminations we have been modelling
- Also these "subdominant" contaminations $\mathcal{A}^{\text{non-fwd}}$ $\rightarrow$ result of source with higher momentum AND/OR excited state
- Expect to be relevant only at lowest momentum due to $\langle X(p+2q)|\bar{\chi}|\Omega\rangle$ suppression
- Hard to reconstruct $\rightarrow$ possibly Bayesian fit à la Bouchard et al. 2017 PRD 96

Conclusion

- Feynman-Hellmann Compton amplitude approach versatile and unique
- But low-energy contaminations are present
- We showed (1) where these come from and (2) how to fix them
- Allows calculation of low-Q^2 structure function moments $\rightarrow$ effective strong coupling
- At lowest Q^2 subdominant contaminations remain



Thank you for listening—questions?