Exploring non-perturbative fermion-photon interactions through the Schwinger-Dyson approach.

C. J. Carrington A. Kizilersu A. W. Thomas

Centre for the Subatomic Structure of Matter
Adelaide University

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Connection to hadronic physics

Addressing the issues of

- Confinement
- Dynamical chiral symmetry breaking (DχSB)
- Hadron spectroscopy

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Perturbative

- $\alpha \ll 1$
- QCD: high Q^2 (UV), quarks and gluons *asymptotically* free
- QED: low Q^2 (IR)

Non-perturbative

- $\alpha \geq 1$
- QCD: low Q^2 (IR), confinement and chiral symmetry breaking
- QED: high Q^2 (UV)

Current methods

Some non-perturbative methods include

- Lattice (discretised)

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- most continuum approaches start from quark-gluon correlation functions

$\rightarrow$ Functional methods can access regions presently inaccessible to lattice.

Example: DχSB

In vacuum, the fermion propagator has a general structure

$$
\begin{aligned}
S_{\mathrm{F}}(p) &= \frac{1}{\not{p}A(p^2) - B(p^2)\mathbb{1}} \\
&= \frac{F(p^2)}{\not{p} - M(p^2)\mathbb{1}},
\end{aligned}
$$

Example: DχSB

In vacuum, the fermion propagator has a general structure

$$S_{\mathrm{F}}(p) = \frac{1}{\not{p}A(p^2) - B(p^2)\mathbb{1}} = \frac{F(p^2)}{\not{p} - M(p^2)\mathbb{1}},$$

and $M(p^2) \neq 0$ when $m_0 \to 0$ is a direct signature for DχSB.

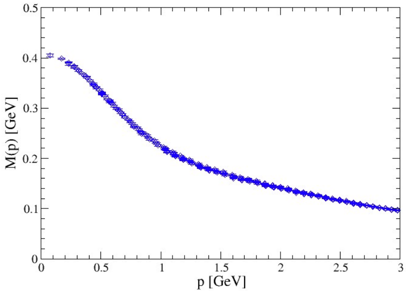

Figure: Quark mass function with p

Unpublished, courtesy of A. Kizilersu and J. Skullerud from their paper in preparation.

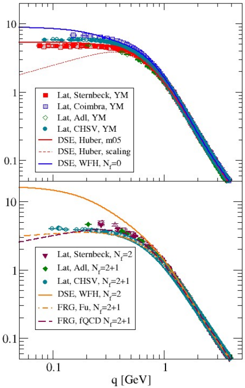

Figure: Gluon propagator [1-9].

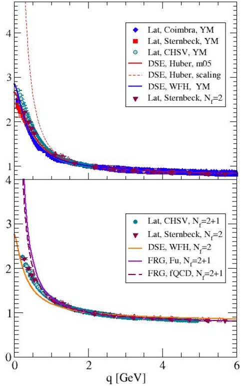

Figure: Ghost propagator

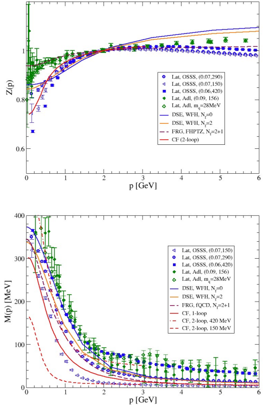

Figure: Quark propagator [10-13].

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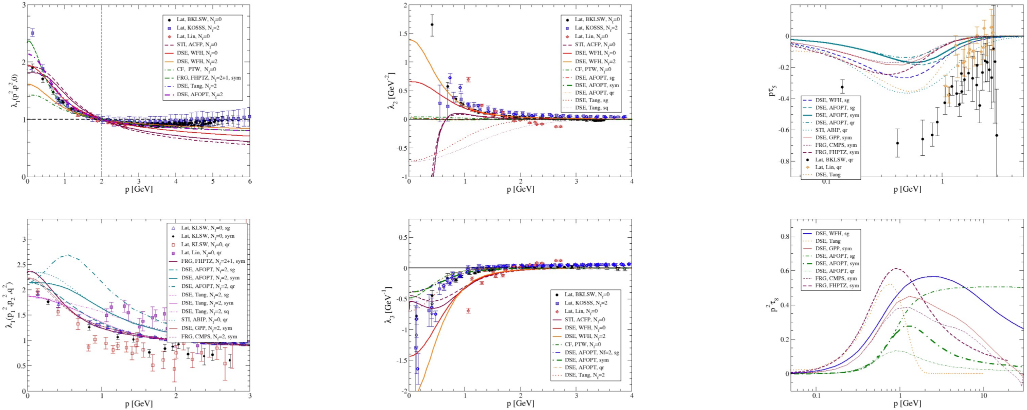

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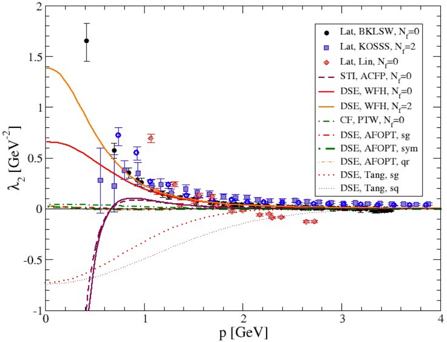

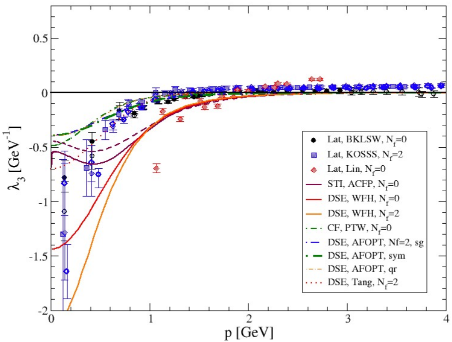

$$\Gamma^\mu = \sum_{i=1}^{4} \lambda_i L_i^\mu + \sum_{i=1}^{8} \tau_i T_i^\mu$$

Why QED?

The QCD vertex actually contains *abelian* substructure, which can be described by QED.

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- Exploring the QED vertex provides a useful, simpler approach for constraining the quark-gluon vertex.
- Even in the massless limit, by constraining the relevant terms we subsequently constrain *other* terms proportional to mass.

The general procedure

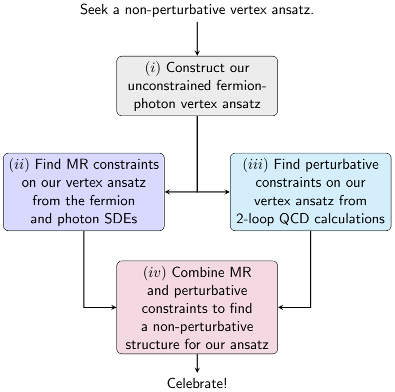

Figure: The general procedure in constructing the vertex ansatz.

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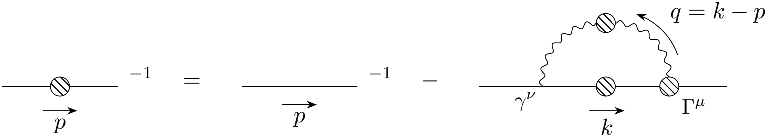
- They are a non-perturbative *continuum* approach.
- They relate n-point to $(n+1)$-point Green's functions.
- They encode the *quantum* into our Green's functions.

The 2-point SDEs



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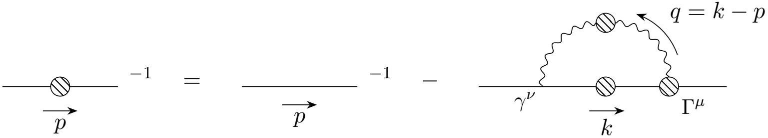

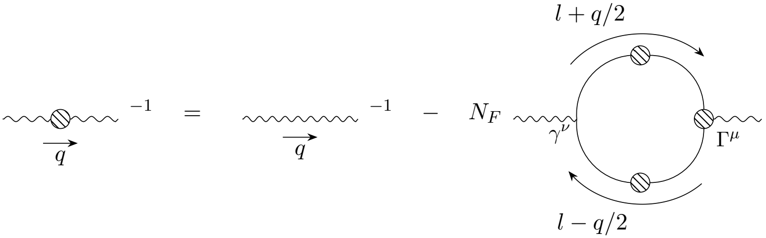

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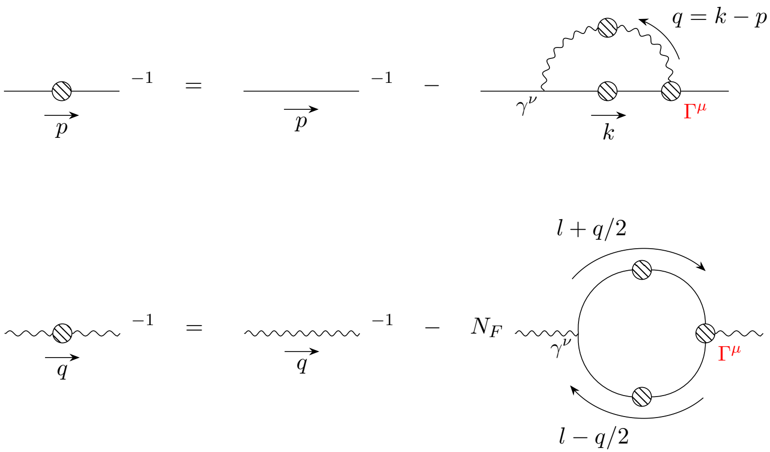

The 2-point SDEs

Fermion equation:

$$-iS_{\mathrm{F}}^{-1}(p) = -i{S_{\mathrm{F}}}_0^{-1}(p) - (-ie)^2 \int_{\mathrm{M}} \frac{d^4k}{(2\pi)^4} \underline{\Gamma^\mu(p,k;q)} \big(iS_{\mathrm{F}}(k)\big)\gamma^\nu \big(i\Delta_{\mu\nu}(q)\big),$$

Photon equation:

$$-i\Delta_{\mu\nu}^{-1}(q) = -i{\Delta_{\mu\nu}^{-1}}_0(q) + (-ie)^2 N_F \int_M \frac{d^4l}{(2\pi)^4} \mathrm{Tr}\left\{ \underline{\Gamma^\mu(l_-,l_+;q)} iS_{\mathrm{F}}(l_+)\gamma^\nu iS_{\mathrm{F}}(l_-)\right\}.$$

Convention: propagators and the vertex

We may write our dressed propagators as

$$iS_{\mathrm{F}}(p) = i\frac{F(p^2)}{\not p - M(p^2)\mathbb{1}}, \qquad i\Delta_{\mu\nu}(q) = -\frac{i}{q^2}\left[G(q^2)\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) + \xi\frac{q_\mu q_\nu}{q^2}\right]$$

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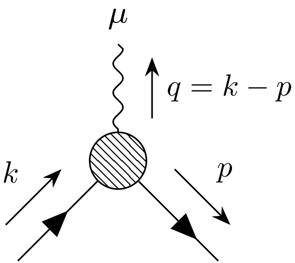
and the vertex may be decomposed as

$$\Gamma^\mu(p,k;q) = \sum_{i=1}^{4} \lambda_i^{\mathrm{M}}(p^2,k^2,q^2)L_i^\mu(p,k;q) \leftarrow \text{Ball-Chiu}$$
$$+ \sum_{i=1}^{8} \tau_i^{\mathrm{M}}(p^2,k^2,q^2)T_i^\mu(p,k;q).$$

Convention: vertex momenta

We define our vertex with the configuration

$$\Gamma^\mu(p,k;q) \quad \longleftrightarrow$$



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Cut-off dependence

In solving the SDEs, we encounter infinite integrals which must be *regulated*. By implementing a momentum *cut-off* Λ^2, we transform our momentum integrals as

$$\int_0^\infty dk^2 \to \int_0^{\Lambda^2} dk^2,$$

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Certainly, the *renormalised* forms of these functions ought to be independent of Λ^2, such that we may recover $\Lambda^2 \to \infty$.

Renormalisation

In multiplicative renormalisation (MR), a *non-perturbative* renormalisation scheme, our functions F and G should be renormalised according to

$$F_{\mathrm{R}}(p^2,\mu^2) = Z_2^{-1}(\mu^2,\Lambda^2)\ F(p^2,\Lambda^2) \qquad F_{\mathrm{R}}(\mu^2,\mu^2) = 1$$
$$G_{\mathrm{R}}(q^2,\mu^2) = Z_3^{-1}(\mu^2,\Lambda^2)\ G(q^2,\Lambda^2) \qquad G_{\mathrm{R}}(\mu^2,\mu^2) = 1,$$

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with

$$\alpha_{\mathrm{R}} = Z_3(\mu^2,\Lambda^2)\ \alpha_0$$
$$\xi_{\mathrm{R}} = Z_3^{-1}(\mu^2,\Lambda^2)\ \xi_0.$$

Finding the general MR forms

Let us allow all of our functions to have perturbative expansions (up to next-to-leading order) in terms of generic coefficients. For example, to leading order,

$$F(p^2, \Lambda^2) = 1 + \sum_{n=1}^{\infty} \alpha_0^n \left(A_{n,n} \ln^n \frac{p^2}{\Lambda^2} \right),$$

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where $A_{n,n} = a_{n,n}\xi^n + b_{n,n}\xi^{n-1} + \ldots$, and $\mathcal{A}_{n,n} = \mathfrak{a}_{n,n}\xi_{\mathrm{R}}^n + \mathfrak{b}_{n,n}\xi_{\mathrm{R}}^{n-1} + \ldots$.

Finding the general MR forms (continued)

We also define an expansion for Z_2 and Z_3, which we may write

$$Z_2^{-1}(\mu^2, \Lambda^2) = 1 + \sum_{n=1}^{\infty} \alpha_{\mathrm{R}}^n \left(y_{n,n} \ln^n \frac{\mu^2}{\Lambda^2} \right) ,$$

and

$$Z_3^{-1}(\mu^2, \Lambda^2) = 1 + \sum_{n=1}^{\infty} \alpha_0^n \left(z_{n,n} \ln^n \frac{\mu^2}{\Lambda^2} \right) ,$$

Finding the general MR forms (continued)

By inserting our expansions into the renormalisation equations, we can compare coefficients order-by-order, log-to-log, and solve. For example, at $\mathcal{O}\left(\alpha_R, \alpha_0\right)$, we see

$$1 + \alpha_{\mathrm{R}} \mathcal{A}_{1,1} \ln \frac{p^2}{\mu^2} = \left(1 + \alpha_{\mathrm{R}} y_{1,1} \ln \frac{\mu^2}{\Lambda^2}\right) \left(1 + \alpha_0 A_{1,1} \ln \frac{p^2}{\Lambda^2}\right),$$

hence

$$\alpha_{\mathrm{R}} \mathcal{A}_{1,1} \ln \frac{p^2}{\mu^2} = \alpha_{\mathrm{R}} y_{1,1} \ln \frac{\mu^2}{\Lambda^2} + \alpha_0 A_{1,1} \ln \frac{p^2}{\Lambda^2} + \mathcal{O}\left(\alpha^2\right).$$

Finding the general MR forms (continued)

But our coupling and gauge parameter are both renormalised by Z_3, hence we expand out

$$\alpha_{\mathrm{R}} \left(\mathfrak{a}_{1,1}\xi_{\mathrm{R}} + \mathfrak{b}_{1,1}\right) \ln \frac{p^2}{\mu^2} = \alpha_{\mathrm{R}} y_{1,1} \ln \frac{\mu^2}{\Lambda^2} + \alpha_{\mathrm{R}} \left(a_{1,1}\xi_{\mathrm{R}} + b_{1,1} Z_3^{-1}(\mu^2, \Lambda^2)\right) \ln \frac{p^2}{\Lambda^2},$$

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$$\alpha_{\mathrm{R}}\left(\mathfrak{a}_{1,1}\xi_{\mathrm{R}}+\mathfrak{b}_{1,1}\right)\ln\frac{p^2}{\mu^2}=\alpha_{\mathrm{R}}y_{1,1}\ln\frac{\mu^2}{\Lambda^2}+\alpha_{\mathrm{R}}\left(a_{1,1}\xi_{\mathrm{R}}+b_{1,1}Z_3^{-1}(\mu^2,\Lambda^2)\right)\ln\frac{p^2}{\Lambda^2},$$

hence

$$\begin{aligned}\alpha_{\mathrm{R}}\left(\mathfrak{a}_{1,1}\xi_{\mathrm{R}}+\mathfrak{b}_{1,1}\right)\ln\frac{p^2}{\mu^2}&=\alpha_{\mathrm{R}}\left(a_{1,1}\xi_{\mathrm{R}}+b_{1,1}\right)\ln\frac{p^2}{\mu^2}\\&+\alpha_{\mathrm{R}}\left(a_{1,1}\xi_{\mathrm{R}}+b_{1,1}+y_{1,1}\right)\ln\frac{\mu^2}{\Lambda^2}+\mathcal{O}\left(\alpha_R^2\right)\end{aligned}$$

The general MR consistent forms

We extend this exact same procedure to next-to-leading order, as well as for the renormalisation of G. As it turns out, they follow the sum for (when $A_{1,0} = 0$ and $A_{1,1} = \xi/4\pi$),

$$F_{\mathrm{R}}(p^2,\mu^2) = \left(\frac{p^2}{\mu^2}\right)^{\frac{\alpha_{\mathrm{R}}\xi_{\mathrm{R}}}{4\pi}} \left[1 + \alpha_{\mathrm{R}}^2\xi_{\mathrm{R}}^2 a_{2,1}\ln\frac{p^2}{\mu^2} - \alpha_{\mathrm{R}}\xi_{\mathrm{R}}\left(\frac{b_{2,1}}{B_{1,1}}\right)\ln\left(1 - \alpha_{\mathrm{R}}B_{1,1}\ln\frac{p^2}{\mu^2}\right)\right.$$
$$\left. + \alpha_{\mathrm{R}}^2 c_{2,1}\frac{\ln\left(p^2/\mu^2\right)}{1 - \alpha_{\mathrm{R}}B_{1,1}\ln\left(p^2/\mu^2\right)}\right] + \mathrm{n^2lo},$$

Solving for MR consistent coefficients (continued)

and

$$G_{\mathrm{R}}^{-1}\left(q^2,\mu^2\right) = 1 - \alpha_{\mathrm{R}} B_{1,1} \ln \frac{q^2}{\mu^2} + \alpha_{\mathrm{R}} \left(\frac{B_{2,1} - 2B_{1,0}B_{1,1}}{B_{1,1}}\right) \ln\left(1 - \alpha_{\mathrm{R}} B_{1,1} \ln \frac{q^2}{\mu^2}\right).$$

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The unconstrained ansatz

By dimensional analysis, charge conjugation symmetry, and perturbation theory, we may write a general ansatz with

$$\tau_2^{\mathrm{M}} = \frac{2}{k^4 - p^4}\left[\beta_2 + \gamma_2 \frac{2k\cdot p}{k^2+p^2}\right]\tau_2^{\text{anti}} + \frac{2}{(k^2+p^2)^2}\left[\delta_2 + \epsilon_2 \frac{2k\cdot p}{k^2+p^2}\right]\tau_2^{\text{symm}}$$

$$\tau_3^{\mathrm{M}} = \frac{1}{k^2 - p^2}\left[\beta_3 + \gamma_3 \frac{2k\cdot p}{k^2+p^2}\right]\tau_3^{\text{anti}} + \frac{1}{k^2+p^2}\left[\delta_3 + \epsilon_3 \frac{2k\cdot p}{k^2+p^2}\right]\tau_3^{\text{symm}}$$

$$\tau_6^{\mathrm{M}} = \frac{1}{k^2 + p^2}\left[\beta_6 + \gamma_6 \frac{2k\cdot p}{k^2+p^2}\right]\tau_6^{\text{anti}} + \frac{k^2-p^2}{(k^2+p^2)^2}\left[\delta_6 + \epsilon_6 \frac{2k\cdot p}{k^2+p^2}\right]\tau_6^{\text{symm}}$$

$$\tau_8^{\mathrm{M}} = \frac{1}{k^2 - p^2}\left[\beta_8 + \gamma_8 \frac{2k\cdot p}{k^2+p^2}\right]\tau_8^{\text{anti}} + \frac{1}{k^2+p^2}\left[\delta_8 + \epsilon_8 \frac{2k\cdot p}{k^2+p^2}\right]\tau_8^{\text{symm}}.$$

The generic ansatz (continued)

We define τ_i^{anti} and τ_i^{symm} to have all possible antisymmetric or symmetric combinations of logs, for example

$$\tau_i^{\text{symm}} = \alpha_0 \left[S^i_{1000} \left(\ln^1 \frac{k^2}{\Lambda^2} + \ln^1 \frac{p^2}{\Lambda^2} \right) + S^i_{0100} \ln \frac{q^2}{\Lambda^2} + S_N{}^i_{0000} \right] + \mathcal{O}\left(\alpha_0^2\right)$$

$$\tau_i^{\text{anti}} = \alpha_0 A^i_{1000} \left(\ln^1 \frac{k^2}{\Lambda^2} - \ln^1 \frac{p^2}{\Lambda^2} \right) + \mathcal{O}\left(\alpha_0^2\right)$$

Solving the fermion equation

We may insert our generic vertex into the fermion SDE with the goal of finding its corresponding solution for F. We convert the Minkowski integral by:

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Solving the fermion equation (continued)

We subsequently obtain an integral equation of the form

$$\frac{1}{F(p^2,\Lambda^2)} = 1 + \frac{\alpha_0\xi_0}{2\pi^2}\int_0^{\Lambda^2} dk^2 \int_0^{\pi} d\Psi \ \sin^2\Psi \ \frac{F(k^2)}{F(p^2)}\frac{\left(k^2 - k\cdot p\right)}{q^4}$$
$$- \frac{\alpha_0}{2\pi^2 p^2}\int_0^{\Lambda^2} dk^2 \int_0^{\pi} d\Psi \ \sin^2\Psi \ F(k^2)G(q^2)\mathcal{F}'(p,k;q),$$

where

$$\mathcal{F}'(p,k;q) = \frac{1}{4}\mathrm{Tr}\left\{\not{p}\Gamma^{\mu}_{M=0}(p,k;q)\not{k}\gamma^{\nu}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\right\}$$

Solving the photon equation

To solve the photon SDE, we first project out the transverse component by acting across with

$$P_{\mu\nu} = \frac{1}{3q^4}\left(4q_\mu q_\nu - q^2 g_{\mu\nu}\right) ,$$

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$$\frac{1}{G(q^2,\Lambda^2)} = 1 - \frac{\alpha_0 N_F}{2\pi^2}\int_0^{\Lambda^2} dl^2 \int_0^{\pi} d\Psi \frac{\sin^2\Psi}{l_+^2 l_-^2} F(l_+^2)F(l_-^2)\mathcal{G}'(l_-,l_+;q),$$

where

$$\mathcal{G}'(l_-,l_+;q) = P_{\mu\nu}\,\mathrm{Tr}\left\{\Gamma^{\mu}_{M=0}(l_-,l_+;q)\not{l}_+\gamma^{\nu}\not{l}_-\right\}$$

Fermion SDE constraints

By inserting generic expansions for F and G, we may perform these integrals, *including* the relevant angular dependence to NLO, and thus compare with our MR forms to find constraints order by order. For example, at second order we have

$$\boldsymbol{\alpha_0^2 \ln^2 :} \quad \frac{1}{2}A_{1,1} = \overline{A}^f_{1000} - \overline{S}^f_{1000},$$

and

$$\boldsymbol{\alpha_0^2 \ln :} \quad A_{2,1} = \frac{3}{4\pi}\tilde{S}^f_{1000} - \frac{3}{8\pi}\left(B_{1,1} + \overline{S_N}^f_{1000}\right).$$

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2-loop QCD calculations

We make use of 2-loop perturbative QCD calculations of the quark-gluon vertex, from J. A. Gracey, "Off-shell two-loop QCD vertices", Physical Review D **90**, (2014) 025014, and *project* out the abelian structure by taking

$$C_A = 0, \qquad C_F = 1, \qquad T_F = 1, \qquad \text{and} \qquad g^2 \to \frac{\alpha_0}{4\pi}.$$

2-loop QCD calculations

We make use of 2-loop perturbative QCD calculations of the quark-gluon vertex, from J. A. Gracey, “Off-shell two-loop QCD vertices”, Physical Review D **90**, (2014) 025014, and *project* out the abelian structure by taking

$$C_A = 0, \qquad C_F = 1, \qquad T_F = 1, \qquad \text{and} \qquad g^2 \to \frac{\alpha_0}{4\pi}.$$

We then take the *fermion* and *photon* limits of both the 2-loop τ_i and the generic ansatz τ_i and compare.

Fermion limit $k^2 \approx q^2 \gg p^2$

Photon limit $k^2 \approx p^2 \gg q^2$

Example: τ_2 2-loop photon limits (to $\mathcal{O}(\alpha_0)$)

Comparing τ_2^E photon limit

$$\tau_2^E\Big|^{\gamma}_{\text{2-loop}} = -\frac{\alpha_0}{2p^4}\frac{\xi-1}{12\pi}\ln\left(\frac{q^4}{k^2p^2}\right) + \frac{\alpha_0}{2p^4}\frac{\xi+2}{36\pi}$$

$$\tau_2^E\Big|^{\gamma}_{\text{gen.}} = -\frac{\alpha_0}{2p^4}(\delta_2+\epsilon_2)S^2_{1000}\ln\left(\frac{q^4}{k^2p^2}\right) + \frac{\alpha_0}{2p^4}\left[2(\beta_2+\gamma_2)A^2_{1000} + (\delta_2+\epsilon_2){S_N}^2_{0000}\right]$$

hence

$$(\delta_2+\epsilon_2)S^2_{1000} = \frac{\xi-1}{12\pi}, \qquad 2(\beta_2+\gamma_2)A^2_{1000} + (\delta_2+\epsilon_2){S_N}^2_{0000} = \frac{\xi+2}{36\pi}$$

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Leading order

The leading order constraints may be *fully* satisfied by just F. We find that

$$\tau^{\text{anti}} = \left(\frac{1}{F(k^2)} - \frac{1}{F(p^2)}\right),$$

and, for example,

$$\tau^{\text{symm}} = \frac{1}{2}\left(\frac{1}{F(k^2)} + \frac{1}{F(p^2)}\right) \ln\left[\frac{F(q^2)}{2}\left(\frac{1}{F(k^2)} + \frac{1}{F(p^2)}\right)\right],$$

give fully consistent solutions to leading order.

Leading order (continued)

The $\beta_i, \gamma_i, \delta_i$, and ϵ_i coefficients are also partially constrained, allowing us to write, for example,

$$\tau_2^{\mathrm{M}} = \frac{2}{k^4 - p^4}\left[\left(\frac{1}{2} - \beta_3 - \beta_6 + \beta_8 + \gamma_3 - \gamma_6\right) + \gamma_2 \frac{2k\cdot p}{k^2+p^2}\right]\tau_2^{\text{anti}}$$
$$+ \frac{2}{(k^2+p^2)^2}\left[\left(-\frac{2}{3} - \delta_6 + \delta_8 - \frac{\epsilon_2}{2} + \epsilon_3 - \epsilon_6 + \frac{\epsilon_8}{2}\right) + \epsilon_2 \frac{2k\cdot p}{k^2+p^2}\right]\tau_2^{\text{symm}}$$
$$\tau_3^{\mathrm{M}} = \frac{1}{k^2 - p^2}\left[\beta_3 + \gamma_3 \frac{2k\cdot p}{k^2+p^2}\right]\tau_3^{\text{anti}}$$
$$+ \frac{1}{k^2+p^2}\left[\left(\frac{2}{3} + \frac{\epsilon_2}{2} - \frac{\epsilon_8}{2}\right) + \epsilon_3 \frac{2k\cdot p}{k^2+p^2}\right]\tau_3^{\text{symm}}$$

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Our new next-to-leading order constraints tell us that

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For example,

$$\tau^{\text{anti}} = \frac{1}{F(k^2)} - \frac{1}{F(p^2)} \to \left(\frac{1}{F(k^2)} - \frac{1}{F(p^2)}\right) \times f(F, G),$$

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with f potentially having intriguing candidates, such as

$$f(F,G) = F\left[\exp\left\{g\left(G\right)\right\}\right],$$

for some function g.

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What we did

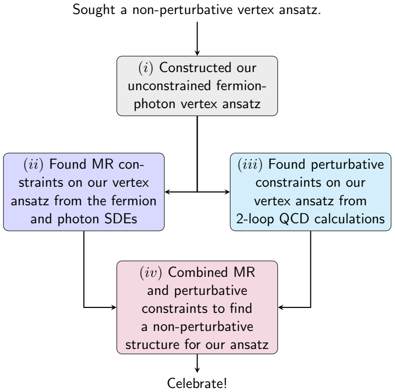

Figure: The general procedure in constructing the vertex ansatz.

Key findings

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Thank you!

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