A Study of Electric Form Factor of Proton in a Parity Doublet Model based on the Two-Parameter Formula

Masayasu Harada (Nagoya University)

@ Workshop on Fundamental Problems in Subatomic Physics

(September 24, 2026)

Based on

- M. Harada, Journal of Subatomic Particles and Cosmology 6, 100512 (2026).
- M. Harada and M. Rho, Phys. Rev. D 83, 114040 (2011)

See also

- Y. Motohiro, Y.Kim, M.Harada, Phys. Rev. C 92, 025201 (2015); Erratum: Phys. Rev. C 95, 059903 (2017).
- T. Minamikawa, T. Kojo and M. Harada, Phys. Rev. C 103, 045205 (2021).
- Y.-K. Kong, T. Minamikawa and M. Harada, Phys. Rev. C 108, 055206 (2023).
- B. Gao, W.L.Yuan, M. Harada and Y.L. Ma, Phys. Rev. C 110, 045802 (2024).

1. Introduction

Origin of Mass ?

1. Introduction

of Hadrons

Origin of Mass ?

1. Introduction

of Hadrons

Origin of Mass ?

of Us

1. Introduction

of Hadrons

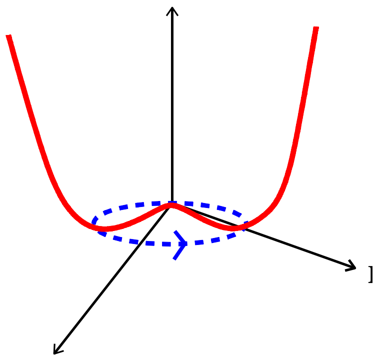
Origin of Mass ?

of Us

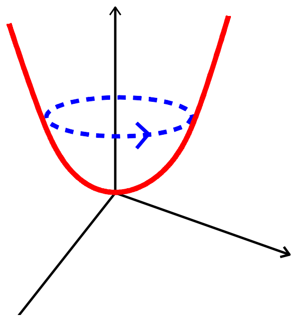
II

One of the Interesting problems of QCD

Spontaneous chiral symmetry breaking



chiral symmetry broken phase at vacuum

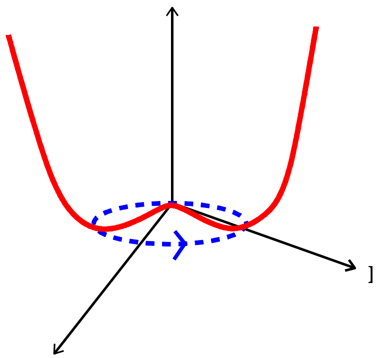


chiral symmetric phase at high T and/or density

$\langle \bar{q}q \rangle \neq 0$ (chiral condensate)

$\langle \bar{q}q \rangle = 0$

Spontaneous chiral symmetry breaking



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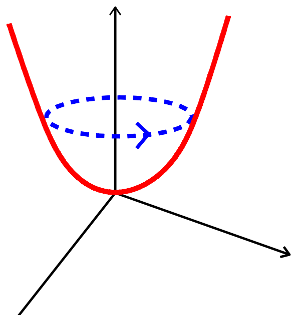

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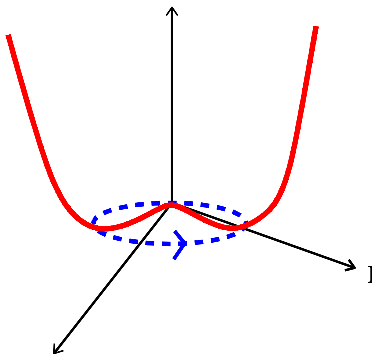
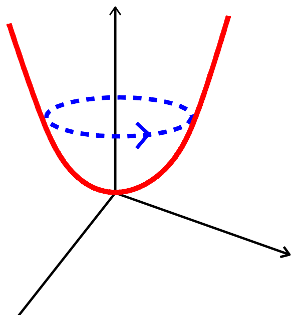
- The spontaneous chiral symmetry breaking is expected to generate a part of hadron masses.
- It causes mass difference between chiral partners.

Spontaneous chiral symmetry breaking



chiral symmetry broken phase at vacuum



chiral symmetric phase at high T and/or density

$\langle \bar{q}q \rangle \neq 0$ (chiral condensate)

$\langle \bar{q}q \rangle = 0$

- The spontaneous chiral symmetry breaking is expected to generate a part of hadron masses.
- It causes mass difference between chiral partners.

- How much mass of nucleon is from the spontaneous chiral symmetry breaking ?
- What is the chiral partner of the nucleon ?

Parity Doublet models for nucleons

- **How much mass of nucleon is from the spontaneous chiral symmetry breaking ?**

- A Parity doublet model for light baryons
  - In [C.DeTar, T.Kunihiro, PRD39, 2805 (1989)], N(1535) is regarded as the chiral partner to the N(939) having the chiral invariant mass.

$$\boldsymbol{m_N = m_0 + m_{\langle \bar{q}q \rangle}}$$

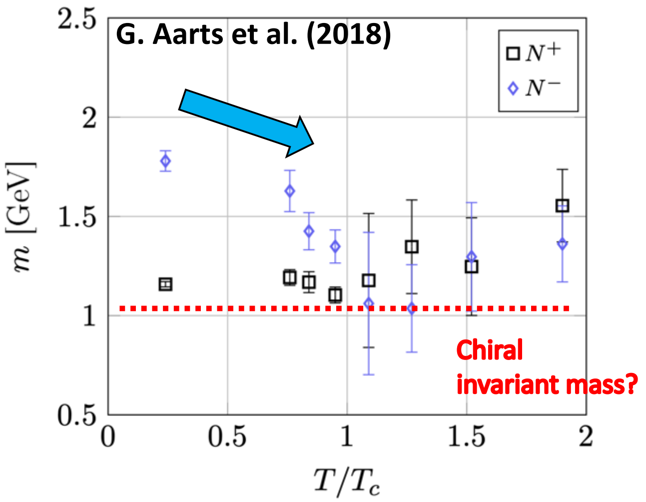
spontaneous chiral symmetry breaking

chiral invariant mass

- A Lattice QCD analysis at non-zero T supports parity doublet structure.

⇨ Constraint to m_0 from phenomenological analysis ?

e.g. Finite-T lattice calculation



Constraint to m_0 from Dense Nuclear Matter & Neutron Star Observation

- PDM with σ^6 term

  Y.Motohiro, Y.Kim, M.Harada, Phys. Rev. C 92, 025201 (2015)

  - $K_0 \approx 240$ MeV for $500 \lesssim m_0 \lesssim 900$ MeV

- PDM (low density) - NJL (high density) crossover model

  T. Minamikawa, T. Kojo and M.H., Phys. Rev. C 103, 045205 (2021).

  - use $M_{max} \gtrsim 2M_\odot$ & GW170817 & NICER
  - obtain $600 \lesssim m_0 \lesssim 900$ MeV

- PDM - NJL crossover model with iso-triplet a_0 meson

  Y.-K. Kong, T. Minamikawa and M. Harada, Phys. Rev. C 108, 055206 (2023).

  - use $M_{max} \gtrsim 2M_\odot$ & GW170817 & NICER
  - obtain $560 \lesssim m_0 \lesssim 840$ MeV

- PDM - NJL 1st-order model

  B. Gao, W. L. Yuan, M.H. and Y.L. Ma, Phys. Rev. C 110, 045802 (2024).

  - Constraint to m_0 is not affected very much.

Constraint to m_0 from Electric form factor of proton?

- based on the "two-parameter formula" which I proposed in 2011

PHYSICAL REVIEW D **83,** 114040 (2011)

Integrating holographic vector dominance to hidden local symmetry for the nucleon form factor

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(Received 17 March 2011; published 22 June 2011)

We derive a two-parameter formula for the electromagnetic form factors of the nucleon described as an instanton by "integrating out" all Kaluza-Klein modes other than the lowest mesons from the infinite tower of vector mesons in holographic QCD while preserving hidden local symmetry for the resultant vector fields. With only two parameters, the proton Sachs form factors can be fit surprisingly well to the available experimental data for momentum transfers $Q^2 \lesssim 0.5\ \mathrm{GeV}^2$ with $\chi^2/\mathrm{dof} \lesssim 2$. We interpret this agreement as indicating the importance of an infinite tower in the soliton structure of the nucleon. The

Outline

1. Introduction
2. Two-Parameter Formula
3. Parity Doublet Model
4. Constraint to m_0 from NS observations
5. Electric Form Factor of Proton
6. Summary

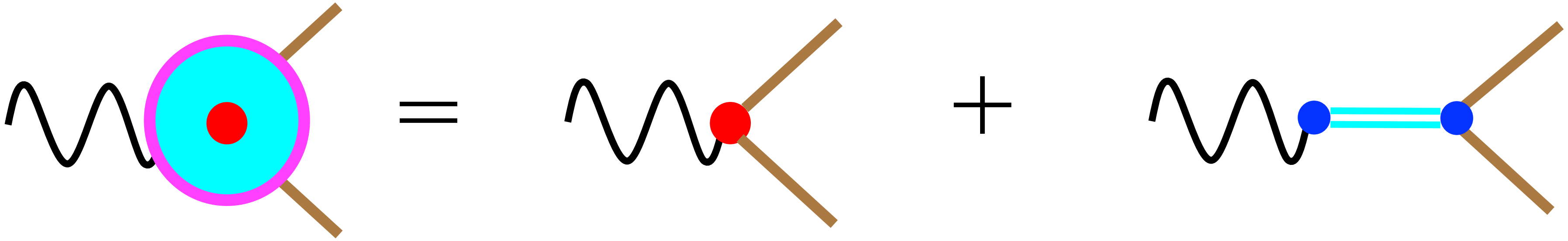
2. Two-Parameter Formula

M. Harada and M. Rho, Phys. Rev. D 83, 114040 (2011)

Two-Parameter Formula for Nucleon Form Factor

M. Harada and M. Rho, Phys. Rev. D 83, 114040 (2011)

$$G_E^p = \underbrace{1 - \frac{\tilde{a}}{2} + \tilde{z}\frac{Q^2}{m_V^2}}_{\text{Core}} + \underbrace{\frac{\tilde{a}}{2}\frac{m_V^2}{m_V^2 + Q^2}}_{\text{meson-cloud}}$$



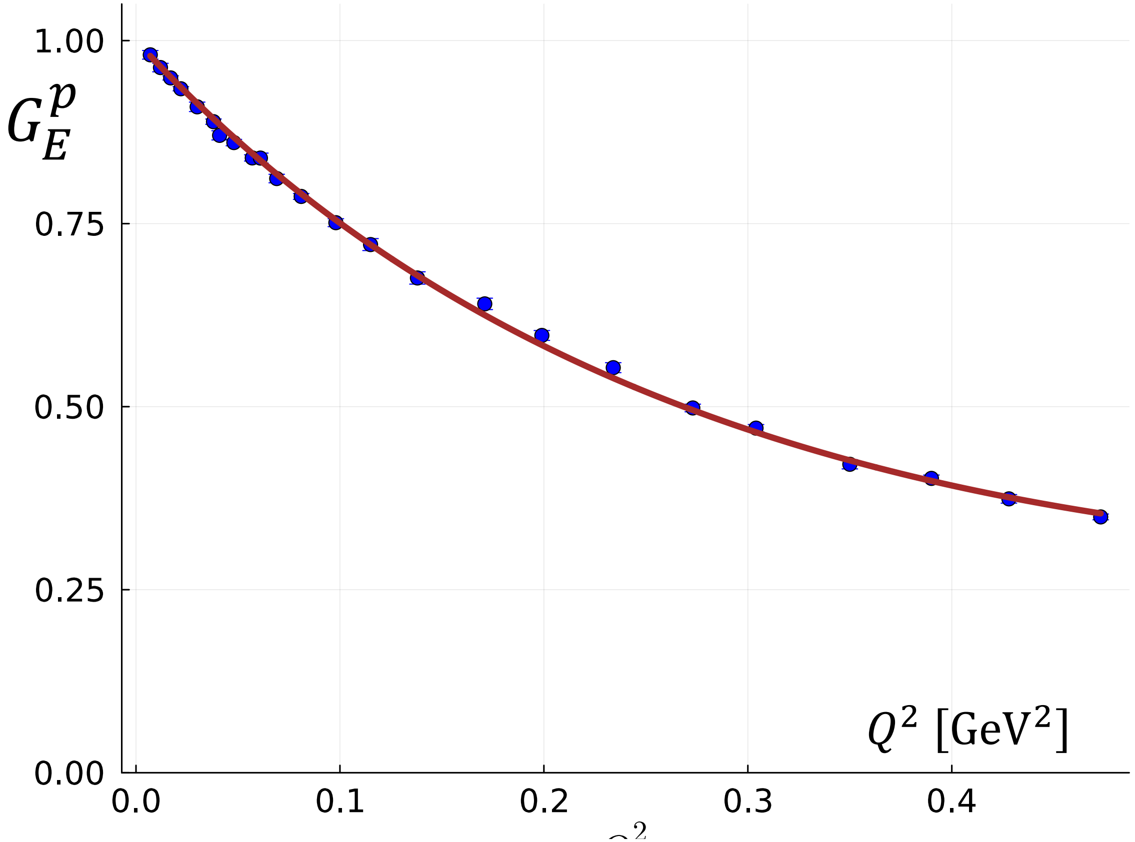
Comparison of G_E^p with EXP

M. Harada and M. Rho, Phys. Rev. D 83, 114040 (2011)

$$G_E^p = 1 - \frac{\tilde{a}}{2} + \tilde{z}\frac{Q^2}{m_V^2} + \frac{\tilde{a}}{2}\frac{m_\rho^2}{m_V^2 + Q^2}$$

best fit: $\chi^2/\mathrm{dof} = 32.3/22 = 1.47$

$\tilde{a} = 4.45\ ;\ \ \tilde{z} = 0.43$

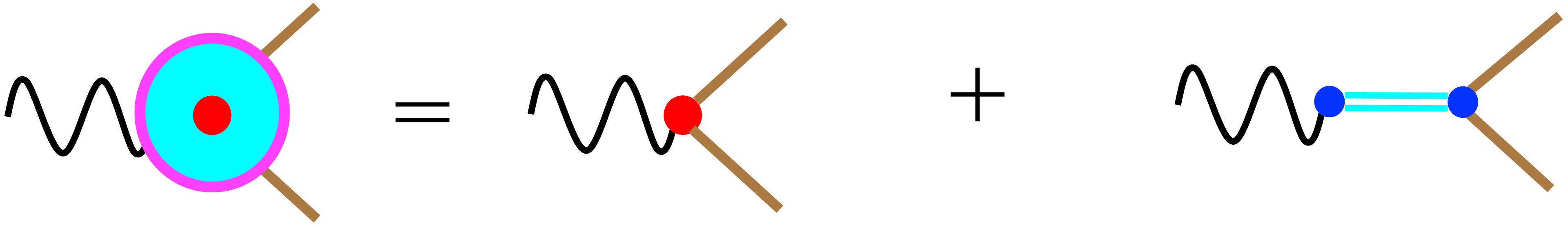


The two-parameter formula reproduces experimental data very well.

Structure of Core & meson-cloud

$$G_E^p = 1 - \frac{\tilde{a}}{2} + \tilde{z}\frac{Q^2}{m_V^2} + \frac{\tilde{a}}{2}\frac{m_V^2}{m_V^2 + Q^2}$$

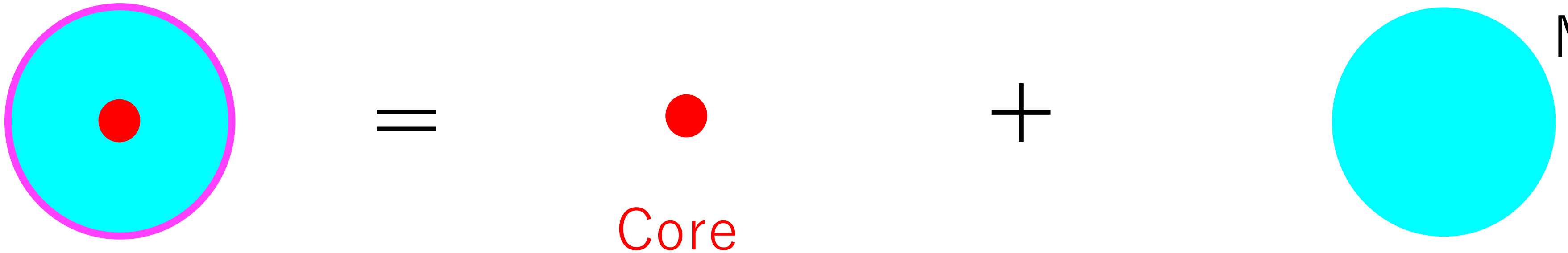
$\tilde{a} = 4.45\,;\ \ \tilde{z} = 0.43$



Near $Q^2 \approx 0$

-1

+2

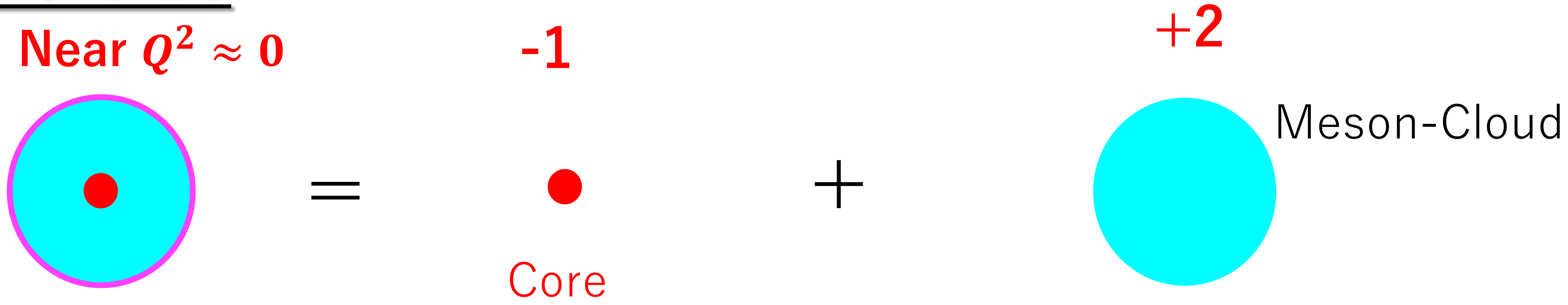


Meson-Cloud

Core

Core carries charge of -1 and meson-cloud carries +2

Core size



Fitted energy region $\lesssim m_\rho \approx 770\mathrm{MeV}$
→ Core size ~ $1/m_\rho \sim 0.3\ \mathrm{fm}$

Structure for size $\lesssim 0.3\ \mathrm{fm}$
→ Need to study higher energy region, $Q \gtrsim m_\rho$
→ Need to add heavier vector mesons such as ρ'
This is beyond the scope of two-parameter formula.

3. Parity Doublet Model

Parity Doublet Model (PDM)

C.DeTar, T.Kunihiro, PRD39, 2805 (1989)
D.Jido, M.Oka, A.Hosaka, PTP106, 873 (2001)

- N(1535) = chiral partner to N(939)
- N(939) Normal assignment $\psi_1 = \psi_{1l} + \psi_{1r}$
  - The transformation property under the chiral group is assigned with the chirality.

$$\gamma_5 \psi_{1l} = -\psi_{1l}; \quad \psi_{1l} \to g_L \psi_{1l}; \quad g_L \in SU(2)_L$$

$$\gamma_5 \psi_{1r} = +\psi_{1r}; \quad \psi_{1r} \to g_R \psi_{1r}; \quad g_R \in SU(2)_R$$

- N(1535) Mirror assignment $\psi_2 = \psi_{2l} + \psi_{2r}$
  - The chiral transformation is assigned oppositely to the chirality

$$\gamma_5 \psi_{2l} = -\psi_{2l}; \quad \psi_{2l} \to g_R \psi_{2l}; \quad g_R \in SU(2)_R$$

$$\gamma_5 \psi_{2r} = +\psi_{2r}; \quad \psi_{2r} \to g_L \psi_{2r}; \quad g_L \in SU(2)_L$$

Linear sigma model (2 flavor)

- A scalar-pseudoscalar field : $M = \sigma + i\vec{\pi}\cdot\vec{\tau}$
  - Transforms as $M \to g_L M g_R^\dagger$; VEV $\langle M \rangle = \sigma_0 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

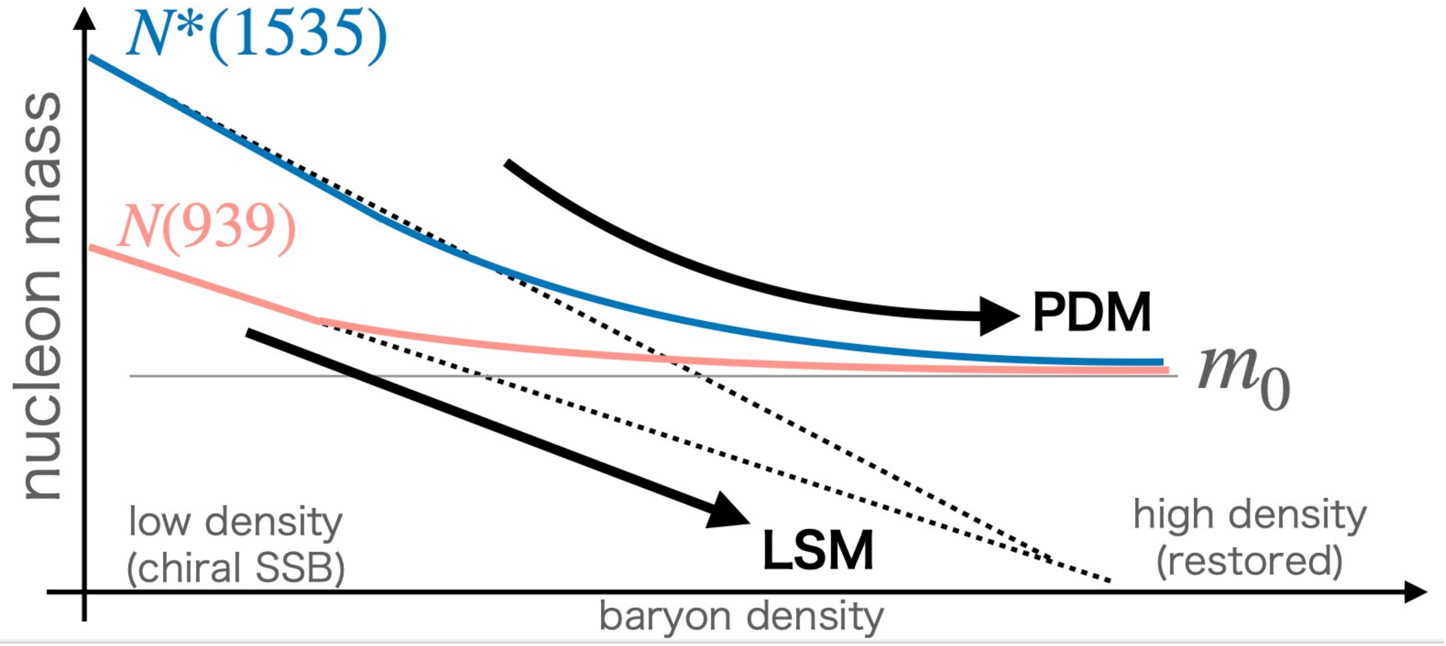
- A linear sigma model with 2 baryons

$$
\begin{aligned}
\mathcal{L}_N =& \bar{\psi}_{1r} i\gamma^\mu \partial_\mu \psi_{1r} + \bar{\psi}_{1l} i\gamma^\mu \partial_\mu \psi_{1l} + \bar{\psi}_{2r} i\gamma^\mu \partial_\mu \psi_{2r} + \bar{\psi}_{2l} i\gamma^\mu \partial_\mu \psi_{2l} \\
& - m_0 \left[\bar{\psi}_{1l}\psi_{2r} - \bar{\psi}_{1r}\psi_{2l} - \bar{\psi}_{2l}\psi_{1r} + \bar{\psi}_{2r}\psi_{1l} \right] \\
& - g_1 \left[\bar{\psi}_{1r} M^\dagger \psi_{1l} + \bar{\psi}_{1l} M \psi_{1r} \right] - g_2 \left[\bar{\psi}_{2r} M \psi_{2l} + \bar{\psi}_{2l} M^\dagger \psi_{2r} \right]
\end{aligned}
$$

- chiral invariant mas m_0 : baryons can have masses even without spontaneous chiral symmetry breaking.
- When this VEV σ_0 is zero, masses of two baryons are degenerated.
- Two masses are separated with each other by the effect of spontaneous chiral symmetry breaking.

Schematic density dependence of masses in PDM

- $m_\pm = \frac{1}{2}\left[\sqrt{(g_1+g_2)^2\sigma^2 + 4m_0^2} \mp (g_1-g_2)\sigma\right]$
  - $m_+ = m(N(939))$, $m_- = m(N(1535))$



- Both masses decrease with density and approach m_0 associated with chiral symmetry restoration.
- In the ordinary linear sigma model, the nucleon mass approaches 0.

Thermodynamic Potential

Y.Motohiro, Y.Kim, M.Harada, Phys. Rev. C 92, 025201 (2015)
Erratum: Phys. Rev. C 95, 059903 (2017).

- ρ and ω mesons are added based on the hidden local symmetry.
- σ^6 term is added to the scalar potential
- Mean field approximation

$$\Omega_h(\mu_B,\mu_Q) = \Omega_B(\mu_B,\mu_Q;\varphi) + V(\varphi) - V(\varphi_{\text{vac}}).\ ;$$

$$\varphi = \{\sigma,\omega,\rho\}\,;\ \varphi_{\text{vac}} = \{\sigma = f_\pi, \omega = 0, \rho = 0\}$$

$$V(\varphi) = -\frac{1}{2}\bar{\mu}^2\sigma^2 + \frac{1}{4}\lambda_4\sigma^4 - \frac{1}{6}\lambda_6\sigma^6 - m_\pi^2 f_\pi\sigma - \frac{1}{2}m_\omega^2\omega^2 - \frac{1}{2}m_\rho^2\rho^2,$$

$$\Omega_B(\mu_B,\mu_Q;\varphi) = -2\sum_{i=\pm}\sum_{\alpha=p,n}\int_{\mathbf{p}}(\mu_\alpha^* - E_{\mathbf{p}}^i)\Theta(\mu_\alpha^* - E_{\mathbf{p}}^i)$$

$$\mu_p^* = \mu_B - g_{\omega NN}\omega - \frac{1}{2}g_{\rho NN}\rho + \mu_Q, \qquad E_{\mathbf{p}}^i = \sqrt{\mathbf{p}^2 + m_i^2}$$

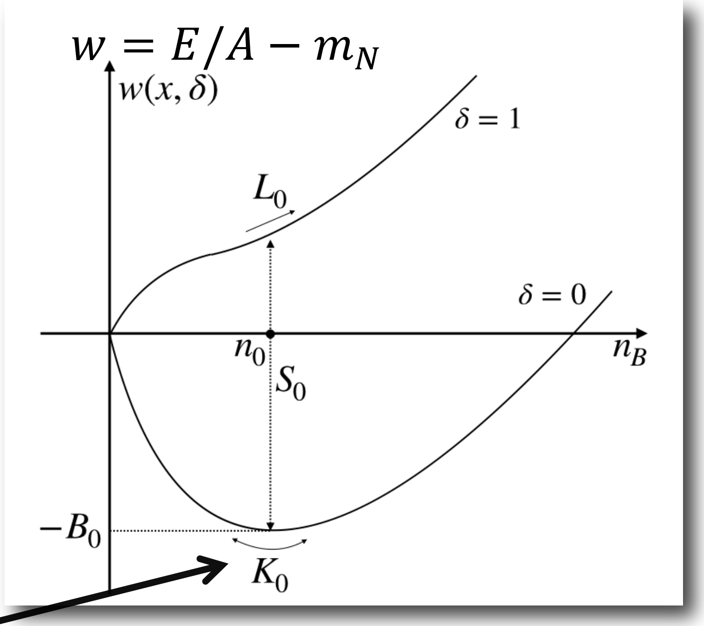
$$\mu_n^* = \mu_B - g_{\omega NN}\omega + \frac{1}{2}g_{\rho NN}\rho.$$

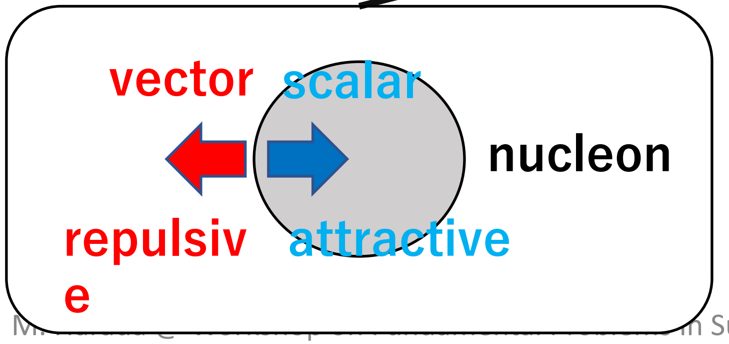
Nuclear Matter at normal nuclear density

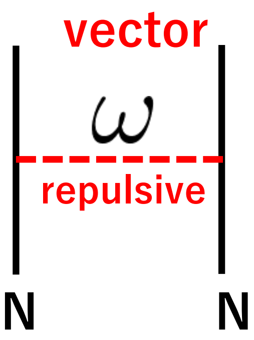
Y.-K. Kong, T. Minamikawa and M. Harada, Phys. Rev. C 108, 055206 (2023).

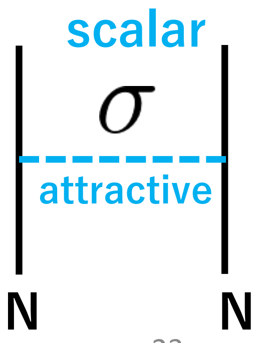
We determine the 5 parameters from the saturation properties as inputs for a given value of the chiral invariant mass m_0.

- Nuclear saturation density
  - $\rho(\mu_B^* = 923\text{MeV}) = n_0 = 0.16\text{fm}^{-3}$
- Binding energy at normal nuclear density
  - $w = \left[\frac{E}{A} - m(939)\right]_{n_0} = -16\text{MeV}$
- Incompressibility
  - $K_0 = 9\rho_0^2 \frac{\partial^2(E/A)}{\partial\rho^2}\Big|_{n_0} = 240\text{MeV}$
- Symmetry energy
  - $S_0 = 31$ MeV
- Slope parameter
  - $L_0 = 40, 60, 80$ MeV





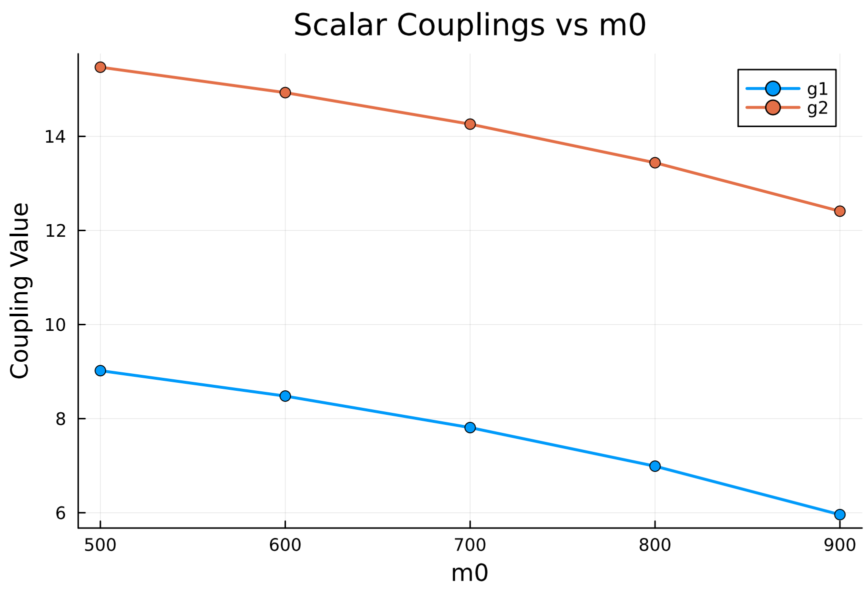


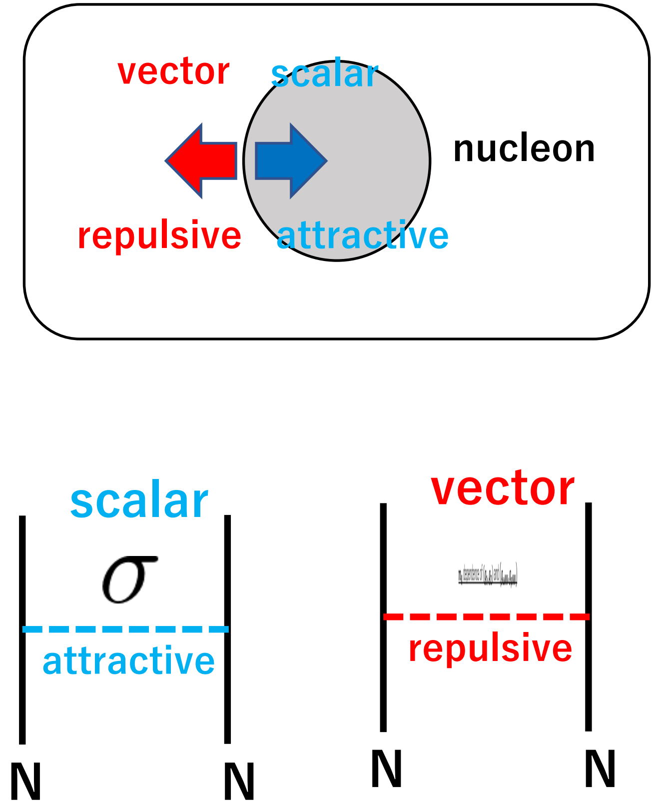


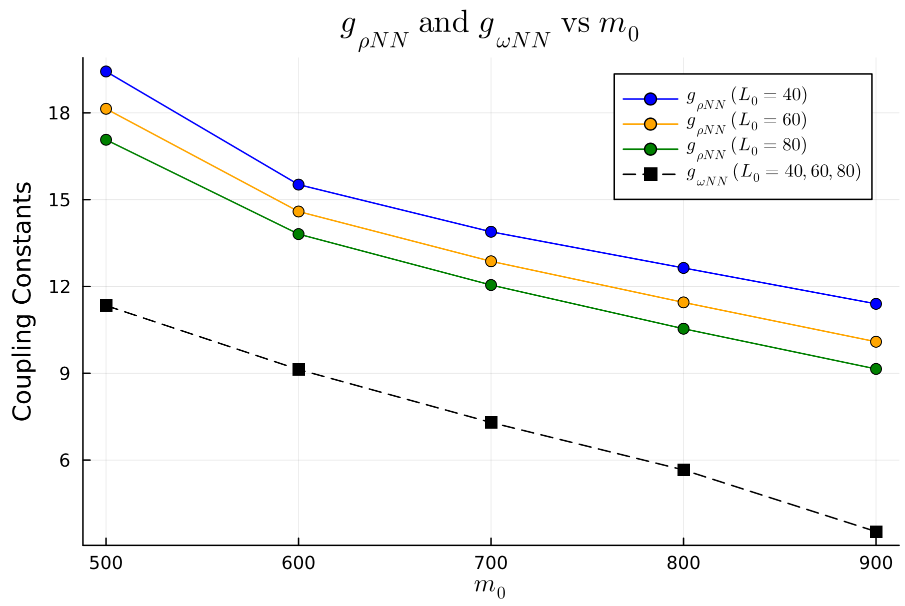
m_0 dependence of (g_1, g_2) and $(g_{\omega NN}, g_{\rho NN})$

Mass formula

$$m_\pm = \sqrt{m_0^2 + \left(\frac{g_1 + g_2}{2}\right)^2 \sigma^2} \mp \frac{g_1 - g_2}{2}\sigma$$







larger m_0 → smaller sigma coupling → smaller vector meson coupling

4. Constraint to m_0 from NS observations

T. Minamikawa, T. Kojo and M. Harada, Phys. Rev. C 103, 045205 (2021).

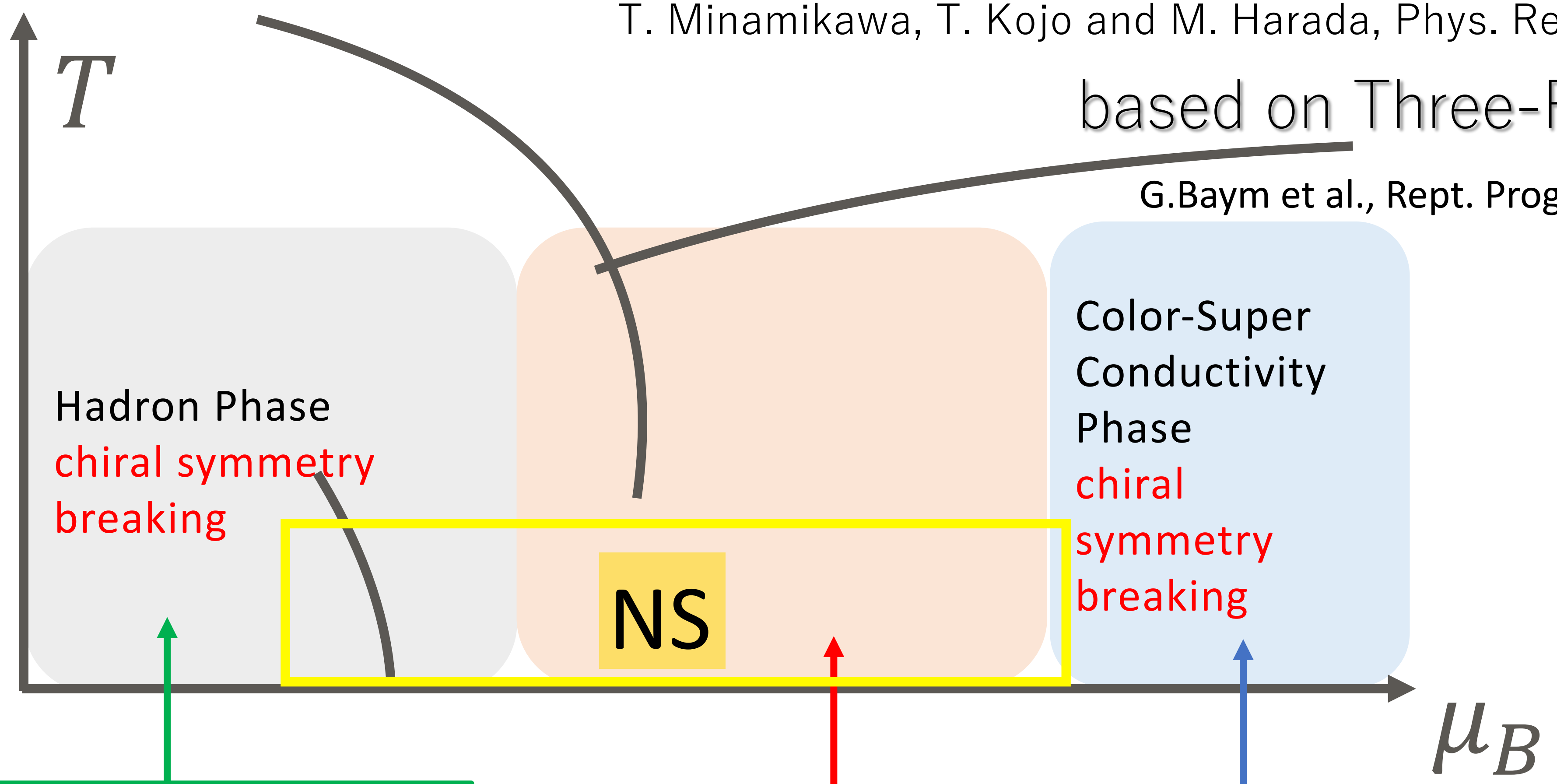
Y.-K. Kong, T. Minamikawa and M. Harada, Phys. Rev. C 108, 055206 (2023).

PDM-NJL crossover model

T. Minamikawa, T. Kojo and M. Harada, Phys. Rev. C 103, 045205 (2021).

based on Three-Region Structure

G.Baym et al., Rept. Prog. Phys. 81, 056902 (2018)



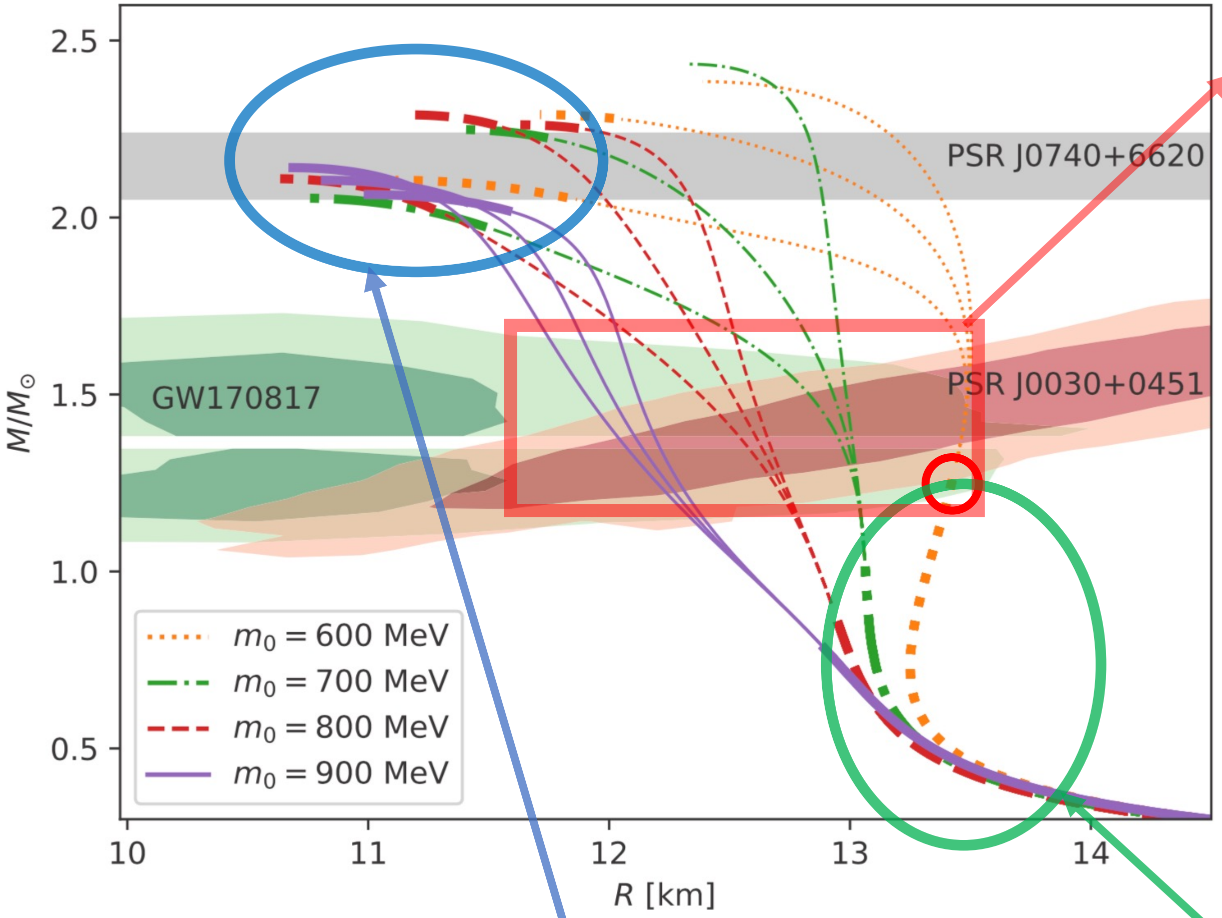
Hadronic Matter
(Parity doublet model)

Interpolation

Quark Matter
(3-flavor NJL-type quark model)

M-R relation

T. Minamikawa, T. Kojo and M.H., Phys. Rev. C 103, 045205 (2021).



$\boxed{600\text{MeV} \le m_0 \le 900\text{MeV}}$

Mass formula

$$m_\pm = \sqrt{m_0^2 + \left(\frac{g_1 + g_2}{2}\right)^2 \sigma^2} \mp \frac{g_1 - g_2}{2}\sigma$$

m_0	interaction	Attractive force by σ	Repulsive force by ω	EOS
small	strong	strong	strong	stiff
large	weak	weak	weak	soft

NSs include quark matter inside.

NSs are made from only hadronic matter.

Inclusion of $a_0(980)$ meson

Y.-K. Kong, T. Minamikawa and M. Harada, Phys. Rev. C 108, 055206 (2023).

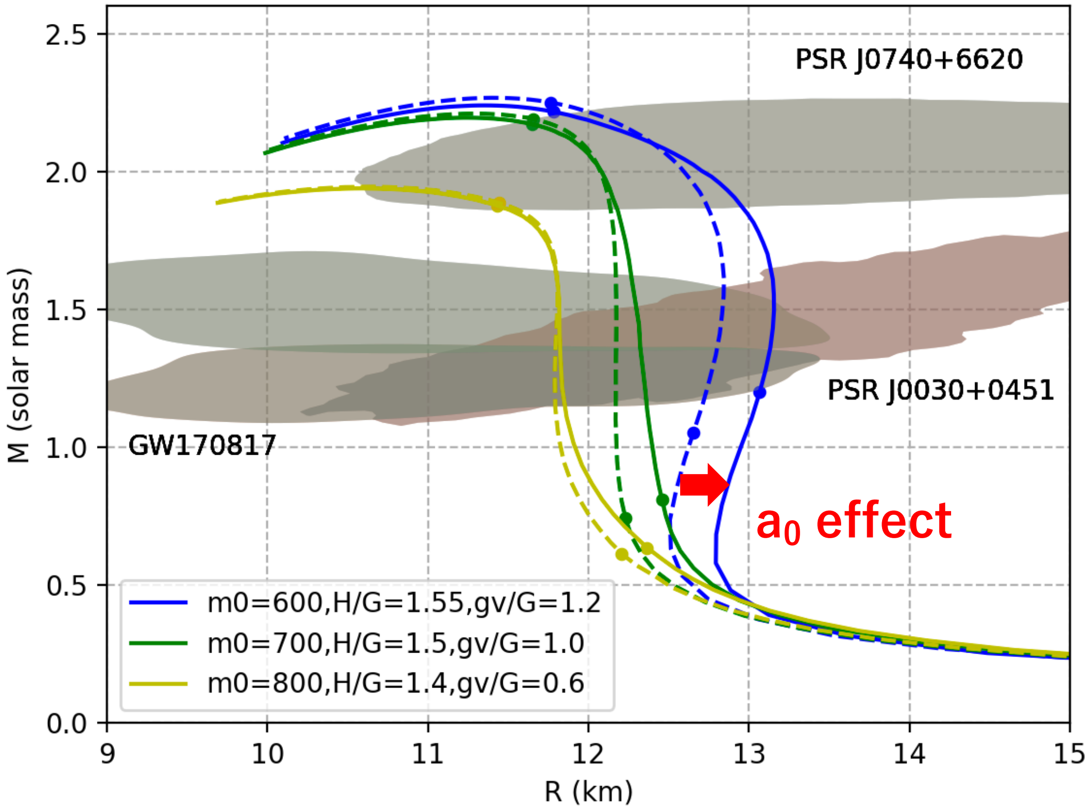
- We construct a PDM with a0(980) meson and study its effects in neutron stars.

- Effect of $a_0(980)$ **increases the radius of intermediate mass NS**

$$520\ \mathrm{MeV} \lesssim m_0 \lesssim 850\ \mathrm{MeV}$$

$a_0(980)$

$$\boxed{560\ \mathrm{MeV} \lesssim m_0 \lesssim 840\ \mathrm{MeV}}$$



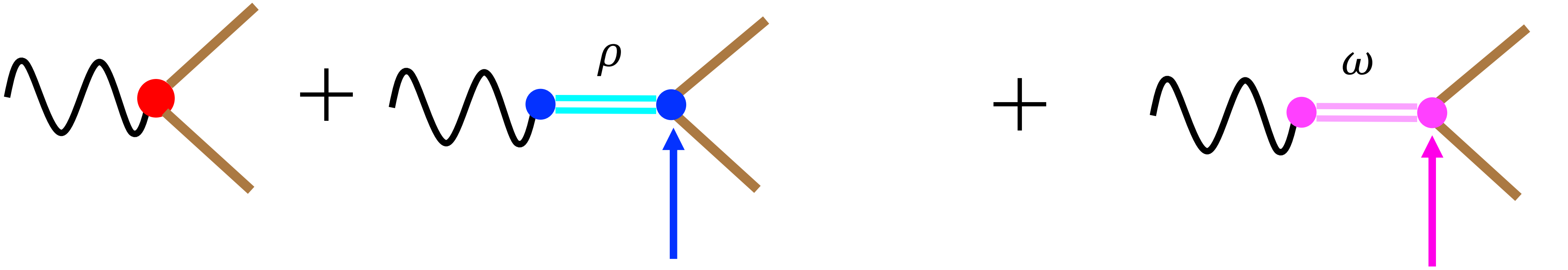
— PDM with a_0
--- PDM without a_0

$L_0 = 50$ MeV

5. Electric Form Factor of Proton in the PDM

M. Harada, Journal of Subatomic Particles and Cosmology 6, 100512 (2026).

Electric Form Factor



$$\frac{1}{2}\left(g_{\rho NN} + z_{\rho NN}\frac{Q^2}{m_\rho^2}\right) \qquad \left(g_{\omega NN} + z_{\omega NN}\frac{Q^2}{m_\omega^2}\right)$$

$$G_E^p(Q^2)$$

contact
$$= 1 - \frac{g_{\rho NN}}{2g_\rho} - \frac{z_{\rho NN}}{2g_\rho}\frac{Q^2}{m_\rho^2}$$

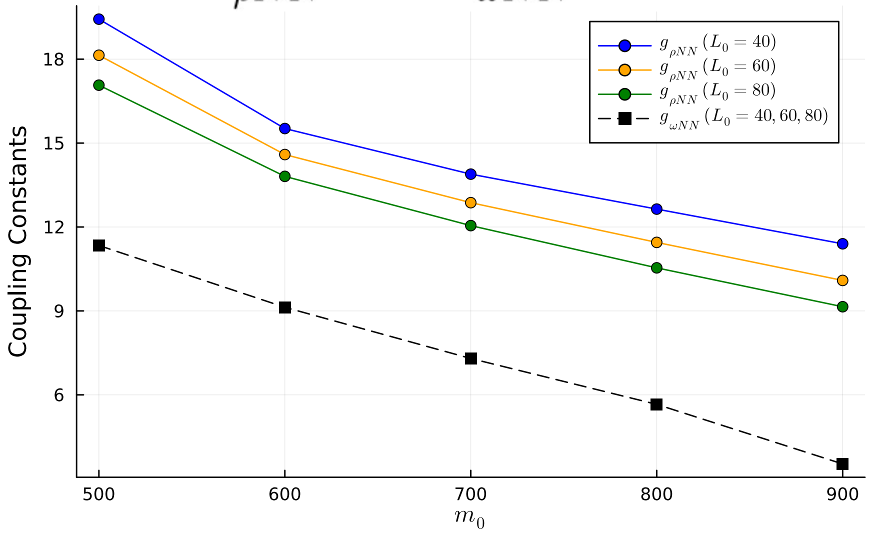
ρ meson
$$+ \frac{1}{2}\left(g_{\rho NN} + z_{\rho NN}\frac{Q^2}{m_\rho^2}\right)\frac{1}{m_\rho^2 + Q^2}\left(\frac{m_\rho^2}{g_\rho} + g_\rho z_3 \frac{Q^2}{m_\rho^2}\right)$$

ω meson
$$+ \left(g_{\omega NN} + z_{\omega NN}\frac{Q^2}{m_\omega^2}\right)\frac{1}{m_\omega^2 + Q^2}\left(z_{\omega\gamma}Q^2\right)$$

- $g_\rho, z_3, z_{\omega\gamma}$ are fixed from meson properties at vacuum
- $g_{\rho NN}$ and $g_{\omega NN}$ are determined to reproduce the nuclear saturation properties.
- $z_{\rho NN}$ and $z_{\omega NN}$ are fitted to the experimental data of electric form factor of the proton.

Result from fitting

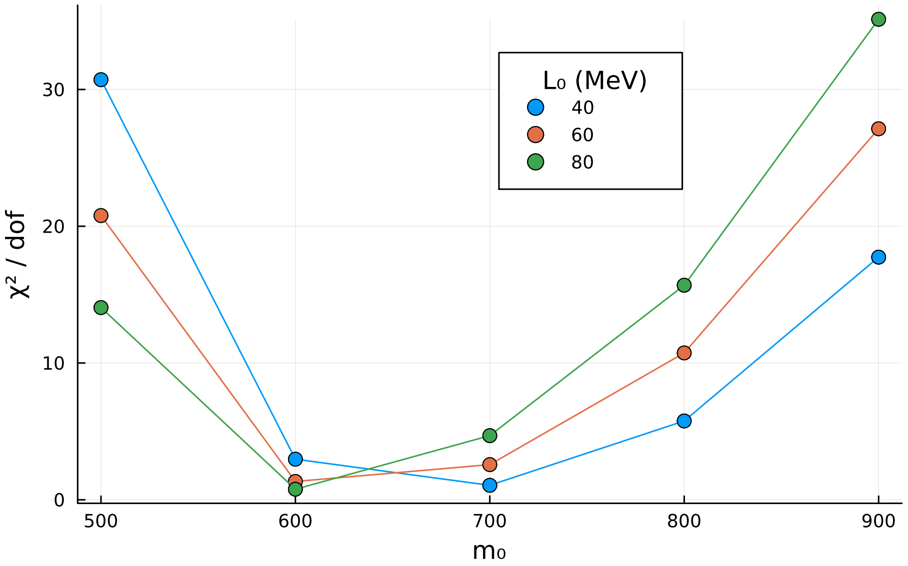
$g_{\rho NN}$ and $g_{\omega NN}$ vs m_0



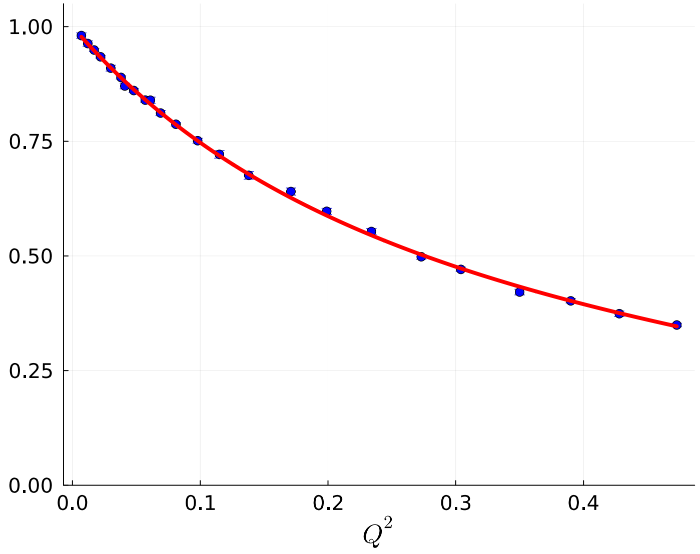
determined from nuclear saturation properties

L_0: Slope parameter

χ² / dof vs m₀



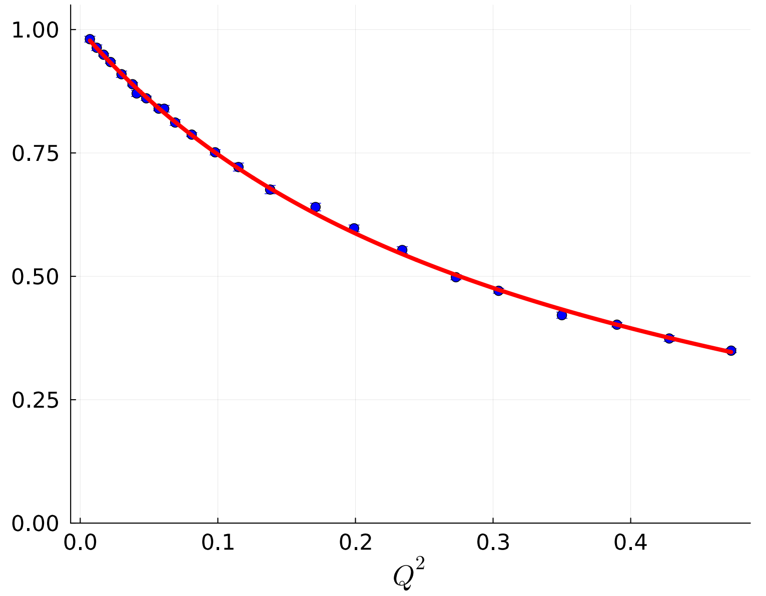
G_E^p for $m_0 = 600$ MeV & $L_0 = 80$ MeV

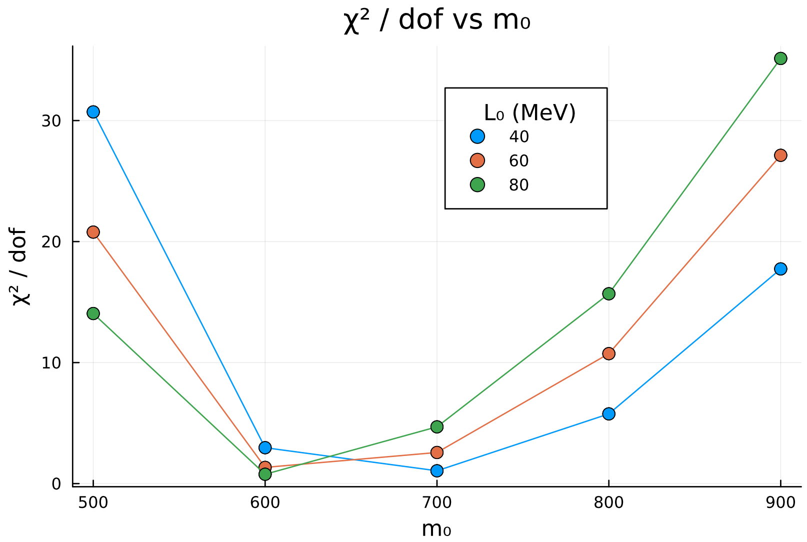


$m_0 \sim 600 - 700$ MeV is preferred by electric form factor

6. Summary

- I studied the electric form factor of proton constructed within a parity doublet model based on the two-parameter formula.
- Non-derivative couplings of rho and omega mesons ($g_{\rho NN}$ and $g_{\omega NN}$) are determined to reproduce the saturation properties of normal nuclear matter.
- Derivative couplings ($z_{\rho NN}$ and $z_{\omega NN}$) are fitted to experimental data.
- My result shows that $m_0 \sim 600\text{–}700$ MeV is preferred by electric form factor.
- This implies that we can obtain some constraint to m_0 from the electric form factor.





Thank you very much for your attention !

Backup

Lagrangian of the PDM (scalar-nucleon sector)

- A scalar-pseudoscalar field : $M = \sigma + i\vec{\pi}\cdot\vec{\tau} - (\vec{a_0}\cdot\vec{\tau} + i\eta)$
  - Transforms as $M \to g_L M g_R^\dagger$; VEV $\langle M \rangle = \sigma_0 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
- A linear sigma model with 2 baryons

$$
\begin{aligned}
\mathcal{L}_N =& \bar{\psi}_{1r} i\gamma^\mu \partial_\mu \psi_{1r} + \bar{\psi}_{1l} i\gamma^\mu \partial_\mu \psi_{1l} + \bar{\psi}_{2r} i\gamma^\mu \partial_\mu \psi_{2r} + \bar{\psi}_{2l} i\gamma^\mu \partial_\mu \psi_{2l} \\
& - m_0 \left[\bar{\psi}_{1l}\psi_{2r} - \bar{\psi}_{1r}\psi_{2l} - \bar{\psi}_{2l}\psi_{1r} + \bar{\psi}_{2r}\psi_{1l} \right] \\
& - g_1 \left[\bar{\psi}_{1r} M^\dagger \psi_{1l} + \bar{\psi}_{1l} M \psi_{1r} \right] - g_2 \left[\bar{\psi}_{2r} M \psi_{2l} + \bar{\psi}_{2l} M^\dagger \psi_{2r} \right]
\end{aligned}
$$

- chiral invariant mas m_0 : baryons can have masses even without spontaneous chiral symmetry breaking.
- Two masses are separated with each other by the effect of spontaneous chiral symmetry breaking.

Lagrangian of the PDM (vector-nucleon sector)

- rho and omega mesons are included based on the hidden local symmetry.

$$\mathcal{L}_{vN(1)} = -\frac{g_{\rho NN}}{g_\rho}\left[\bar{N}_{1l}\gamma^\mu\left(\xi_{\mathrm{L}}^\dagger \hat{\alpha}_{(\rho)\mu}\xi_{\mathrm{L}}\right)N_{1l} + \bar{N}_{1r}\gamma^\mu\left(\xi_{\mathrm{R}}^\dagger \hat{\alpha}_{(\rho)\mu}\xi_{\mathrm{R}}\right)N_{1r}\right]$$
$$-\frac{g_{\rho NN}}{g_\rho}\left[\bar{N}_{2l}\gamma^\mu\left(\xi_{\mathrm{R}}^\dagger \hat{\alpha}_{(\rho)\mu}\xi_{\mathrm{R}}\right)N_{2l} + \bar{N}_{2r}\gamma^\mu\left(\xi_{\mathrm{L}}^\dagger \hat{\alpha}_{(\rho)\mu}\xi_{\mathrm{L}}\right)N_{2r}\right]$$
$$-\frac{g_{\omega NN}}{g_\omega}\hat{\alpha}_{(\omega)\mu}\left[\bar{N}_{1l}\gamma^\mu N_{1l} + \bar{N}_{1r}\gamma^\mu N_{1r} + \bar{N}_{2l}\gamma^\mu N_{2l} + \bar{N}_{2r}\gamma^\mu N_{2r}\right]$$

$$\mathcal{L}_{vN(2)} = -\frac{z_{\rho NN}}{g_\rho m_\rho^2}\left[\bar{N}_{1l}\gamma^\mu\left(\xi_{\mathrm{L}}^\dagger D_\nu D^\nu \hat{\alpha}_{(\rho)\mu}\xi_{\mathrm{L}}\right)N_{1l} + \bar{N}_{1r}\gamma^\mu\left(\xi_{\mathrm{R}}^\dagger D_\nu D^\nu \hat{\alpha}_{(\rho)\mu}\xi_{\mathrm{R}}\right)N_{1r}\right]$$
$$-\frac{z_{\rho NN}}{g_\rho m_\rho^2}\left[\bar{N}_{2l}\gamma^\mu\left(\xi_{\mathrm{R}}^\dagger D_\nu D^\nu \hat{\alpha}_{(\rho)\mu}\xi_{\mathrm{R}}\right)N_{2l} + \bar{N}_{2r}\gamma^\mu\left(\xi_{\mathrm{L}}^\dagger D_\nu D^\nu \hat{\alpha}_{(\rho)\mu}\xi_{\mathrm{L}}\right)N_{2r}\right]$$
$$-\frac{z_{\omega NN}}{g_\omega m_\omega^2}\left(\partial_\nu\partial^\nu \hat{\alpha}_{(\omega)\mu}\right)\left[\bar{N}_{1l}\gamma^\mu N_{1l} + \bar{N}_{1r}\gamma^\mu N_{1r} + \bar{N}_{2l}\gamma^\mu N_{2l} + \bar{N}_{2r}\gamma^\mu N_{2r}\right]$$

two-parameter formula

$$G_E^p(Q^2) = 1 - \frac{\tilde{a}_\rho + \tilde{a}_\omega}{2} + \left(\tilde{z}_\rho \frac{Q^2}{m_\rho^2} + \tilde{z}_\omega \frac{Q^2}{m_\omega^2}\right) + \frac{\tilde{a}_\rho}{2} \frac{m_\rho^2}{m_\rho^2 + Q^2} + \frac{\tilde{a}_\omega}{2} \frac{m_\omega^2}{m_\omega^2 + Q^2}$$

- $\tilde{a}_\rho = \frac{(1-g_\rho^2 z_3)(g_{\rho NN} - z_{\rho NN})}{g_\rho}$
- $\tilde{a}_\omega = 2(g_{\omega NN} - z_{\omega NN}) z_{\omega\gamma}$
- $\tilde{z}_\rho = -\frac{z_{\rho NN}}{2} \frac{(1-g_\rho^2 z_3)}{g_\rho}$
- $\tilde{z}_\omega = z_{\omega NN} z_{\omega\gamma}$

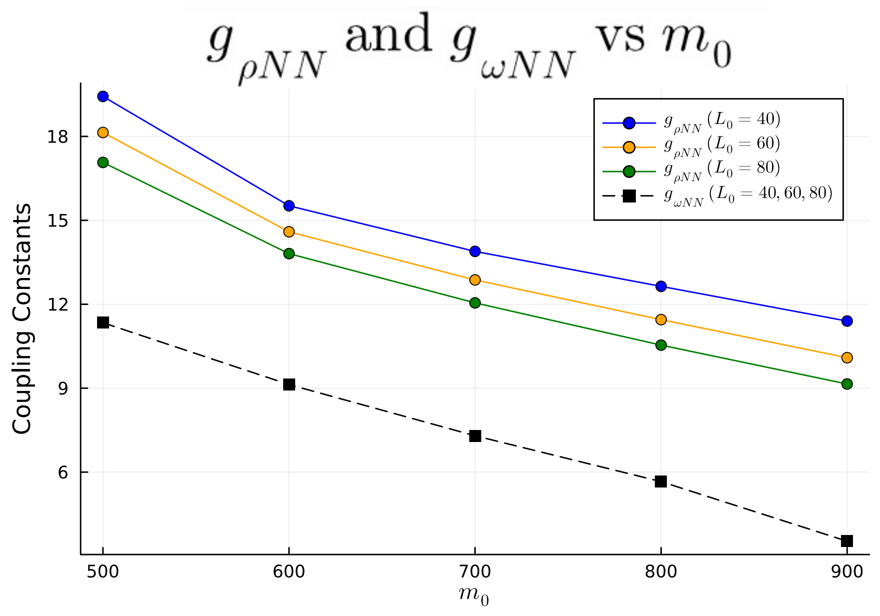
When $m_\rho = m_\omega = m_V$

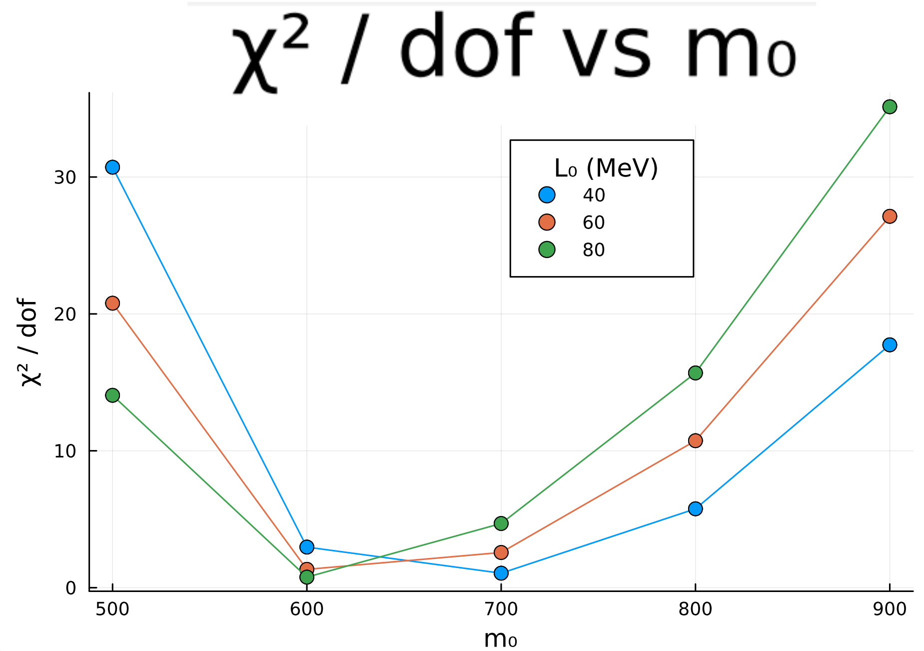
$$G_E^p = 1 - \frac{\tilde{a}}{2} + \tilde{z} \frac{Q^2}{m_V^2} + \frac{\tilde{a}}{2} \frac{m_\rho^2}{m_V^2 + Q^2}$$

- $\tilde{a} = \frac{(1-g_\rho^2 z_3)}{g_\rho} g_{\rho NN} - 2 g_{\omega NN} + \left\{-\frac{(1-g_\rho^2 z_3)}{g_\rho} z_{\rho NN} + 2 z_{\omega\gamma} z_{\omega NN}\right\}$
- $\tilde{z} = \frac{1}{2}\left\{-\frac{(1-g_\rho^2 z_3)}{g_\rho} z_{\rho NN} + 2 z_{\omega\gamma} z_{\omega NN}\right\}$

Numerical results of fitting

L_0 (MeV)	m_0 (MeV)	$g_{\rho NN}$	$g_{\omega NN}$	$z_{\rho NN}$	$z_{\omega NN}$	χ^2	χ^2/dof
40	500	19.43	11.34	-2338	4231	675.8	30.72
	600	15.52	9.13	-902	1628	65.29	2.968
	700	13.89	7.3	-161.3	285.5	23.32	1.06
	800	12.64	5.66	443.8	-811.5	126.8	5.763
	900	11.4	3.53	1123	-2043	390.3	17.74
60	500	18.14	11.34	-1978	3579	457.2	20.78
	600	14.59	9.13	-642.9	1159	29.46	1.339
	700	12.87	7.3	123	-229.8	56.58	2.572
	800	11.45	5.66	775.5	-1413	236.2	10.74
	900	10.09	3.53	1488	-2704	596.8	27.13
80	500	17.07	11.34	-1680	3039	309.2	14.05
	600	13.81	9.13	-425.5	764.5	16.98	0.7717
	700	12.05	7.3	351.5	-644.2	103.2	4.69
	800	10.54	5.66	1029	-1873	345.1	15.69
	900	9.15	3.53	1750	-3179	772.9	35.13





Quark Matter (High density region)

- The Color-Super Conductivity is expected to occur in the high density limit of QCD, in which two quarks make a Cooper pair breaking the color symmetry and the chiral symmetry.
- In the present analysis, we use a model of NJL-type including the following 4-point interaction terms:
  - Attractive force between two quarks

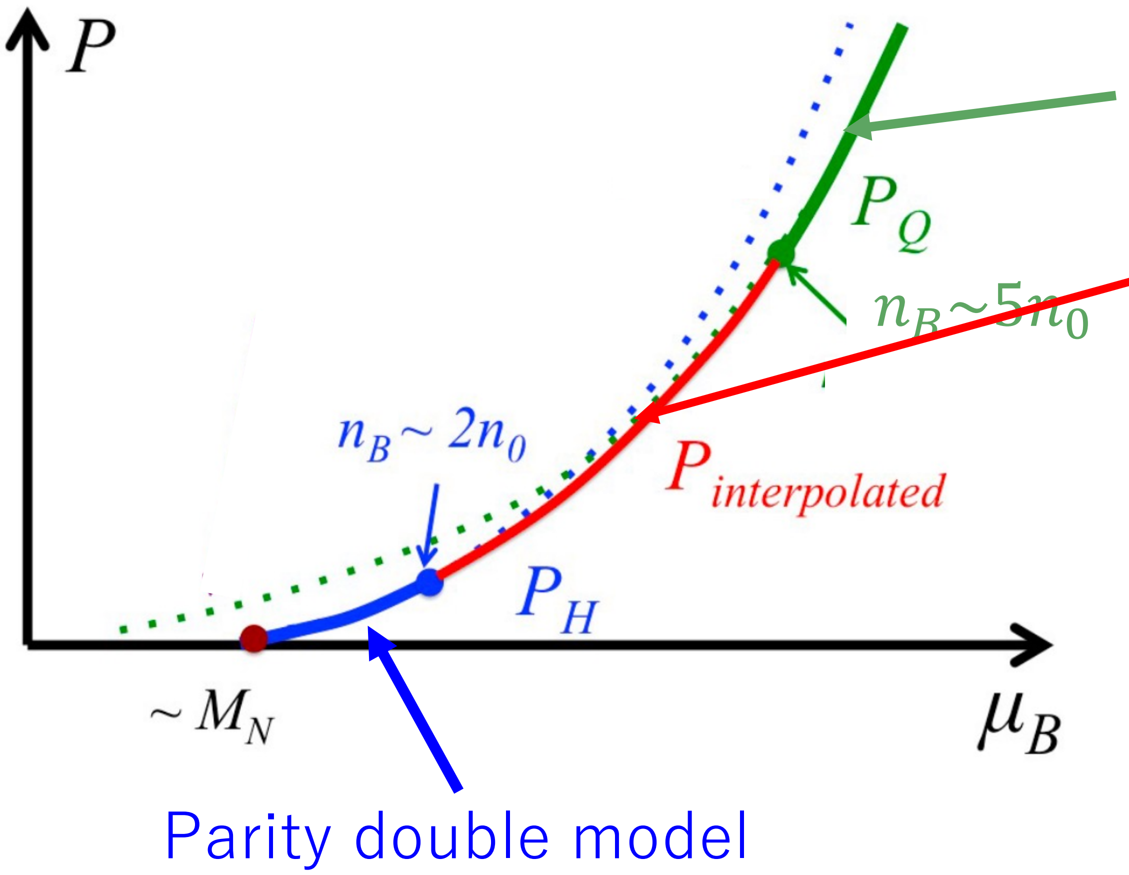
$$H \sum_{A,A'=2,5,7} \left[\left(\bar{q} i\gamma_5 \tau_A \lambda_{A'} C \bar{q}^T\right) \left(q^T C i\gamma_5 \tau_A \lambda_{A'} q\right) + \left(\bar{q} \tau_A \lambda_{A'} C \bar{q}^T\right) \left(q^T C \tau_A \lambda_{A'} q\right) \right]$$

  - Repulsive force between two quarks

$$-g_V \left(\bar{q} \gamma^\mu q\right)^2$$

Unified EOS for NS in the PDM-NJL crosssover model

G. Baym et al., Rept. Prog. Phys. 81, 056902 (2018).
T. Minamikawa, T. Kojo and M.H., Phys. Rev. C 103, 045205 (2021).



Interpolation

We use 5-th order polynomial of μ for the pressure P.

6-coefficients are determined by connecting P to the second order at $2n_0$ and $5n_0$.

$$P_{had} = P_{int},$$

$$\frac{\partial P_{had}}{\partial \mu} = \frac{\partial P_{int}}{\partial \mu}, \quad \frac{\partial^2 P_{had}}{\partial \mu^2} = \frac{\partial^2 P_{int}}{\partial \mu^2}$$

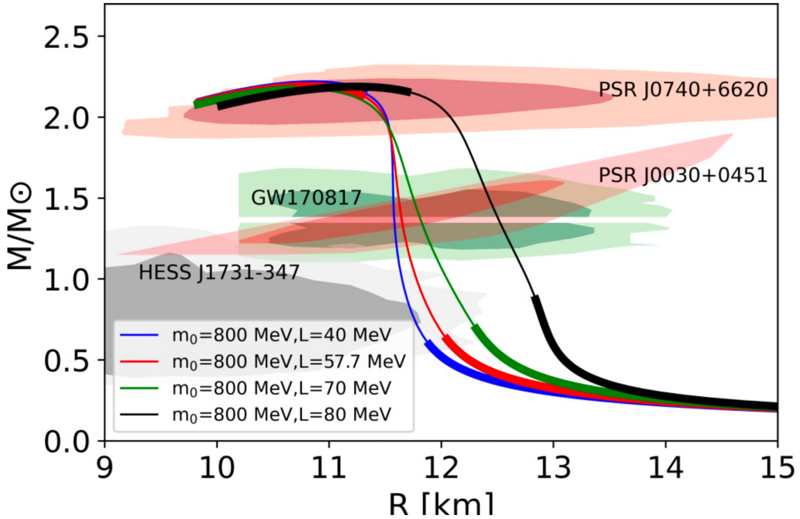
Constraints from supernova remnant HESS J1731-347

B. Gao, Y. Yan and M. Harada, Phys. Rev. C 109, 065807 (2024)

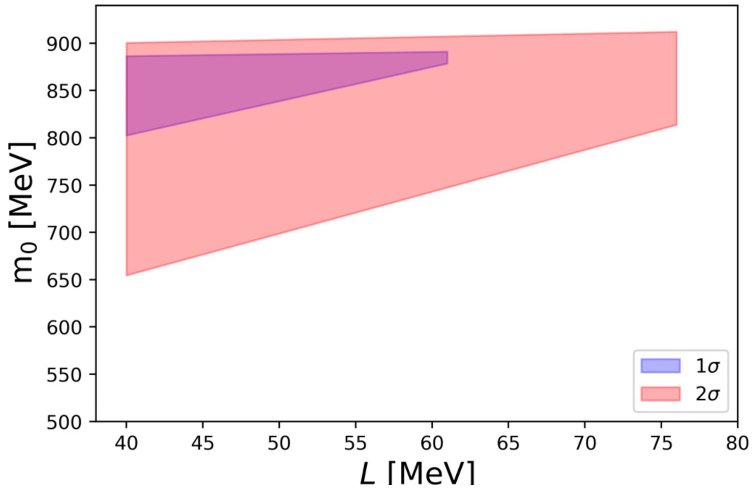
- We added the $\omega-\rho$ mixing term to the model for controlling the slope parameter L_0.

$$-\lambda_{\omega\rho}(g_\omega\omega)^2(g_\rho\rho)^2$$

$m_0 = 800\text{MeV}$ and
$L_0 = 40, 57.7, 60, 80\text{MeV}$



Constraints to m_0 for given L_0.



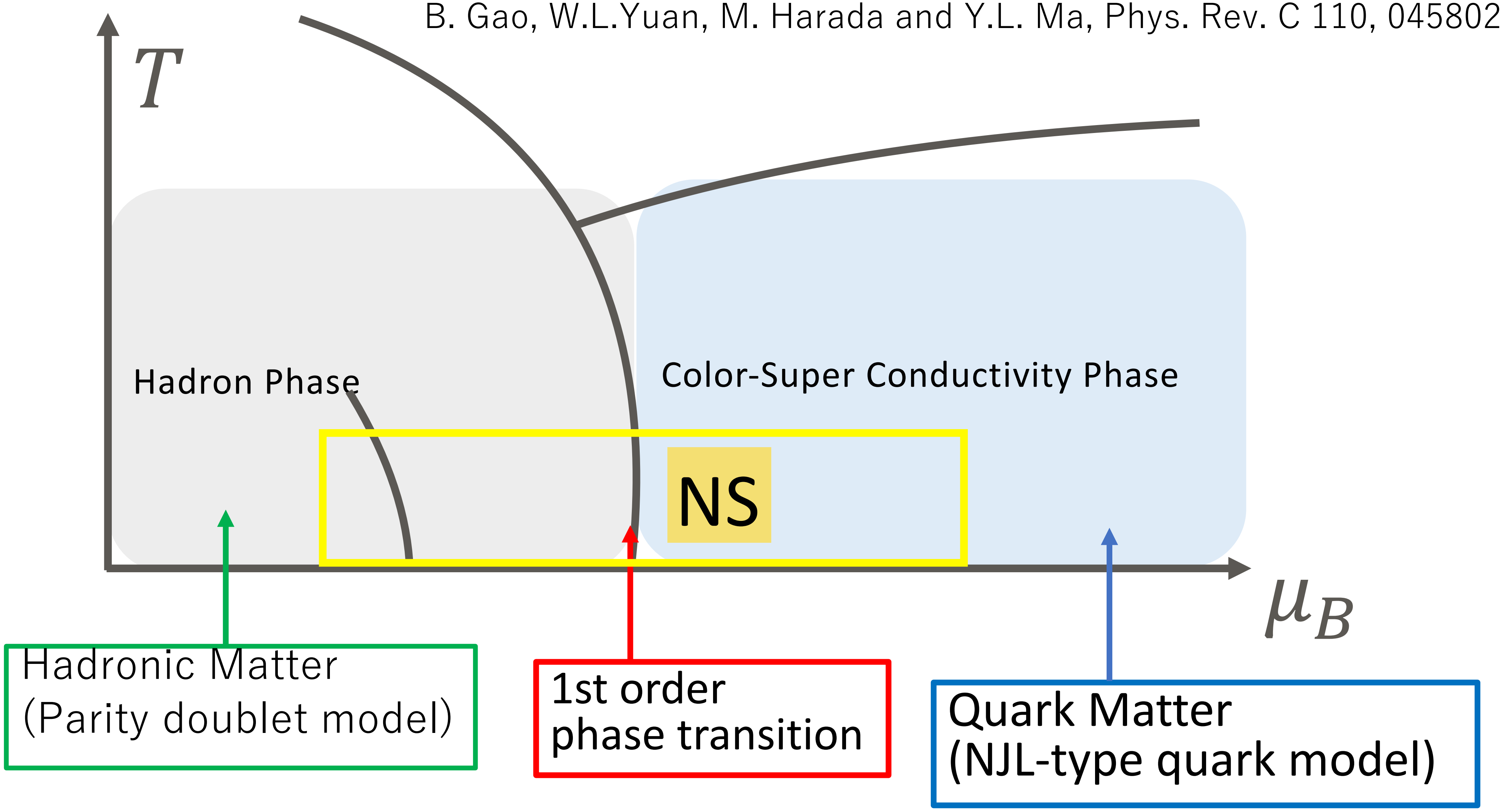
- For $m_0 = 800\text{MeV}$ and $L_0 = 40\text{MeV}$ (blue curve), it is possible to explain the HESS as a neutron star with satisfying constraints from LIGO/VIRGO and NICER.

- If HESS is a neutron star, HESS data is used to obtain constraint to m_0:

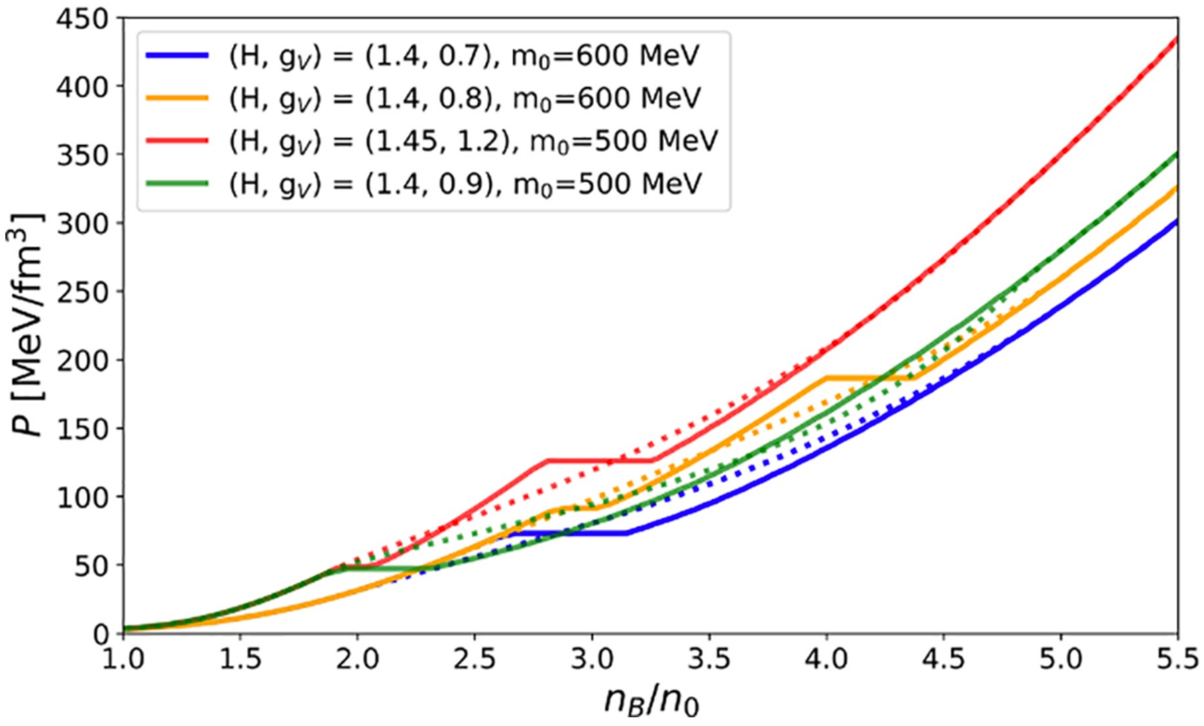
  e.g. $m_0 \gtrsim 850$ MeV for $L_0 = 57.7$ MeV

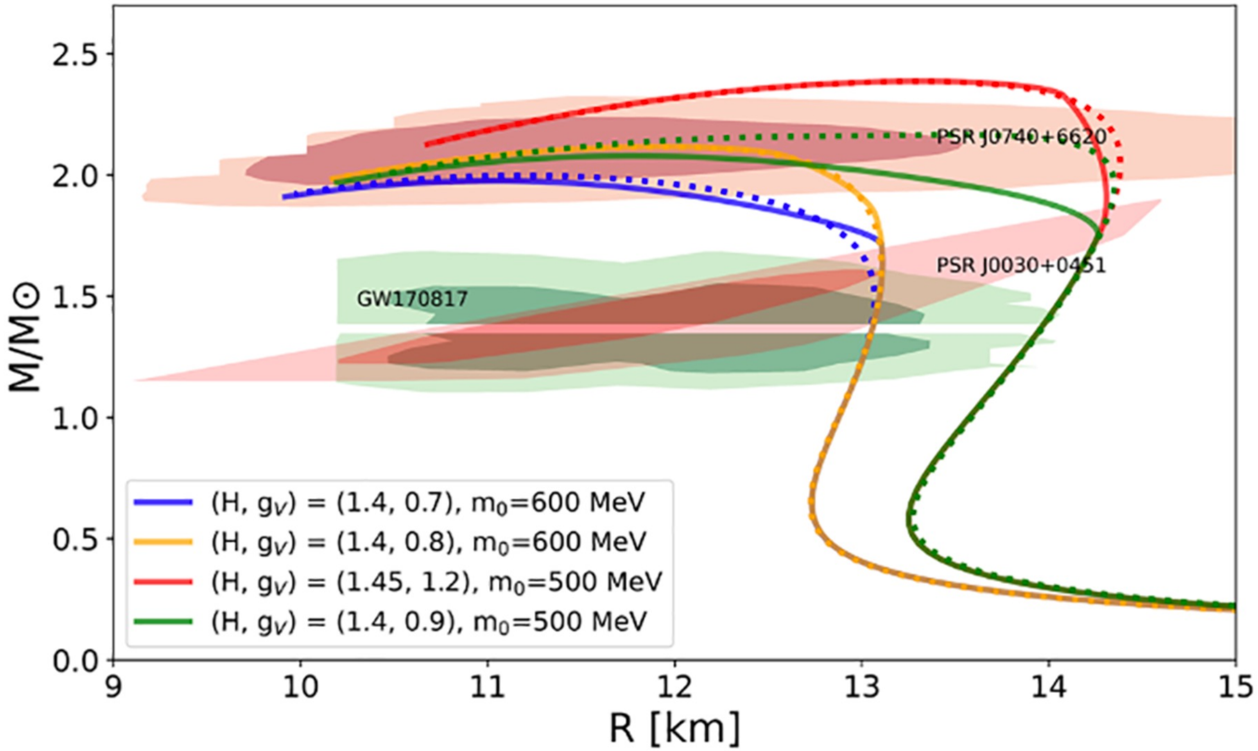
5. Study of 1st order phase transition from PDM to NJL

B. Gao, W.L.Yuan, M. Harada and Y.L. Ma, Phys. Rev. C 110, 045802 (2024).



Comparison with crossover





- Difference
  - ➢ M-R relation: sudden change for 1st order
  - ➢ maximum mass of NS: smaller for 1st order

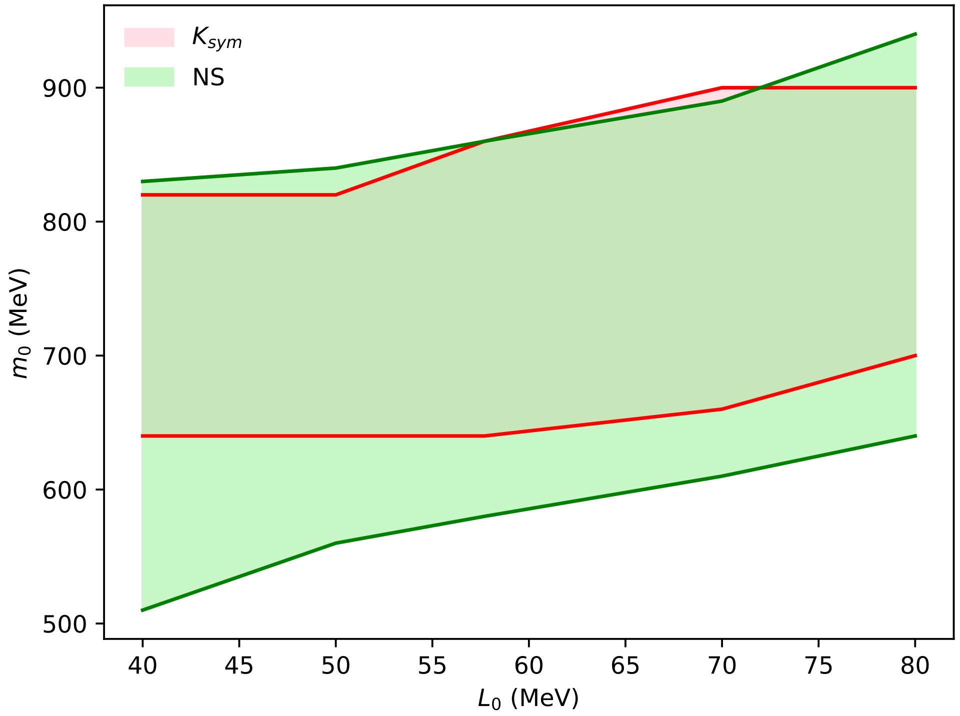
➢ Constraint to m_0 is not affected.

Constraints to $\boldsymbol{m_0}$ from $\boldsymbol{K_{sym}}$

Y.K. Kong and Masayasu. Harada, Nuclear Physics Review, 2024, 41(3): 787-793

$$S(n_B) = S_0 + \left(\frac{n_B - n_0}{n_0}\right)\frac{L_0}{3} + \left(\frac{n_B - n_0}{n_0}\right)^2 \frac{K_{\rm sym}}{18} + \cdots$$

m_0



L_0

$K_{sym} = -107 \pm 88$ MeV

Empirical value from
LI B A, CAI B J, XIE W J, et al. Universe, 2021, 7(6).

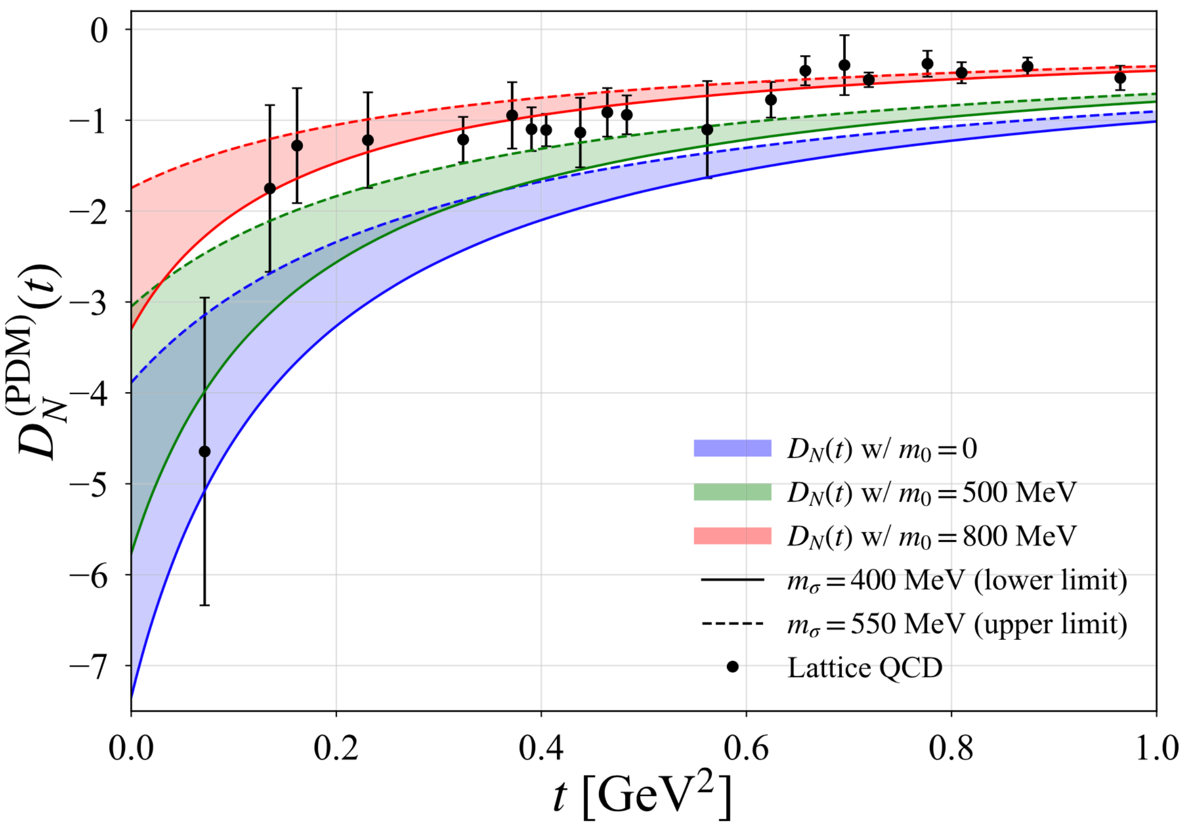
Gree area : constraint from NS
Pink area : constraint from K_{sym}

- K_{sym} seems to provide stronger constraint to m_0.

Analysis of Gravitational Form Factor

M. Kawaguchi, M.Harada and Y.L. Ma, Phys. Lett. B 876, 140400 (2026)

- We constructed a Gravitational Form Factor of Nucleon assuming the sigma meson dominance.
- We compared our prediction with the lattice data.
- We concluded that the preferred value of m_0 lies within the range $m_0 = 500$ - 900 MeV.



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