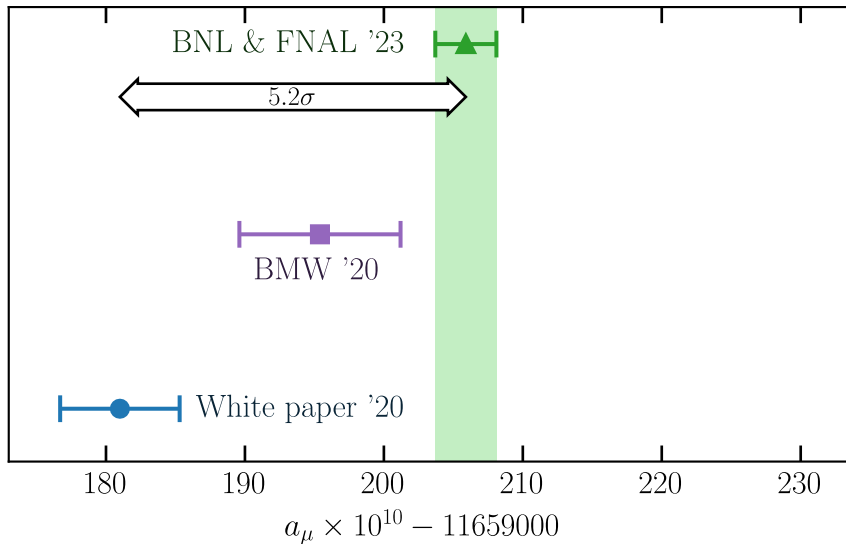


Resolving the muon magnetic moment mystery

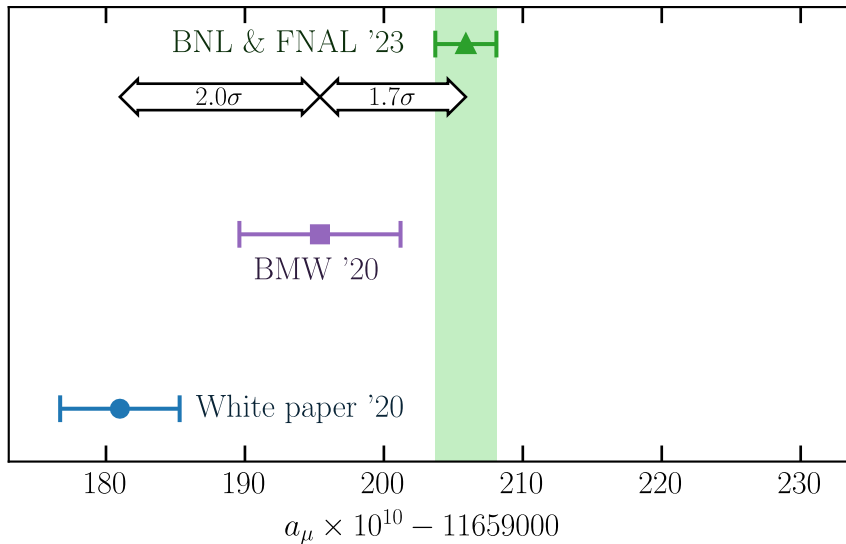
Finn M. Stokes (they/them)

Budapest-Marseille-Wuppertal Collaboration
CSSM, Adelaide, Australia

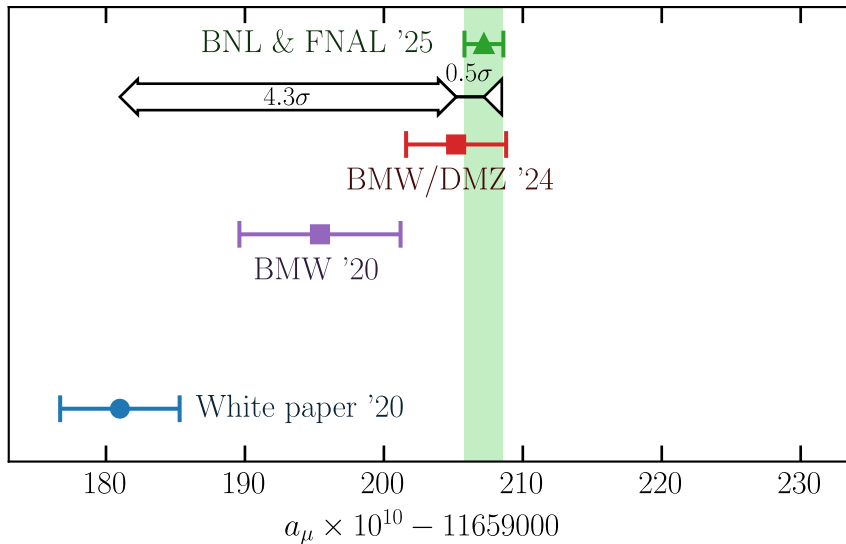
Overview



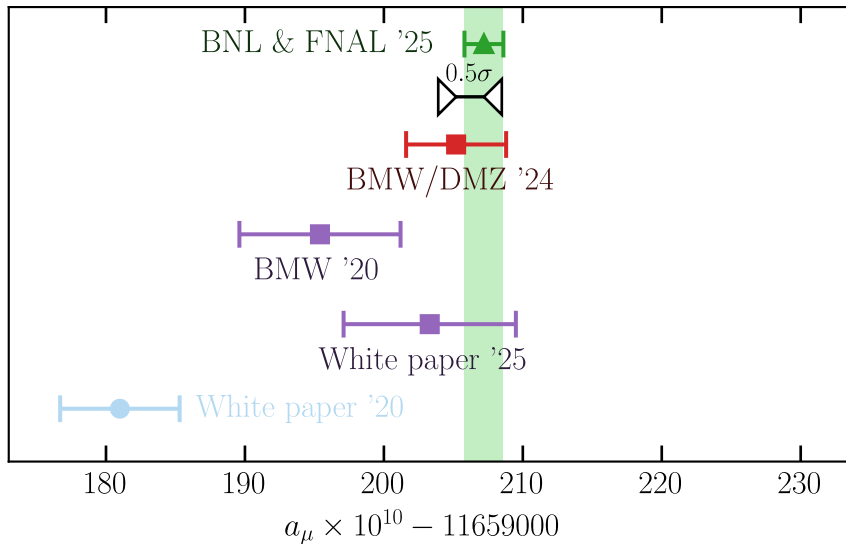
Overview



Overview



Overview



Anomalous magnetic moment

- Anomalous magnetic moment

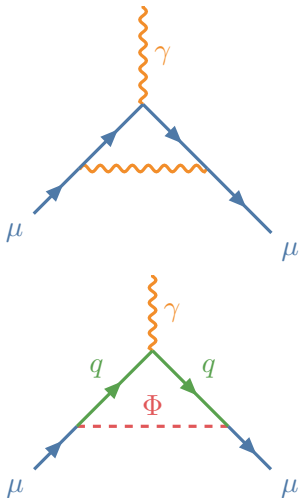
$$\vec{\mu} = g \left(\frac{q}{2m} \right) \vec{S}$$

$$a = (g - 2)/2$$

- Quantum corrections to magnetic moment

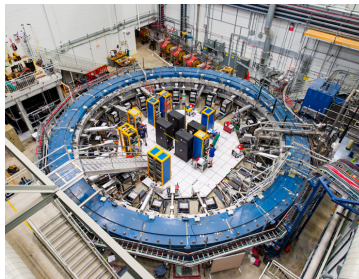
- New physics probe:

- Loop in a is sensitive to many types of BSM physics

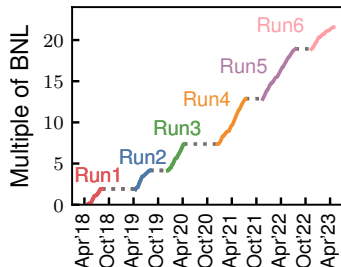


Experimental measurement

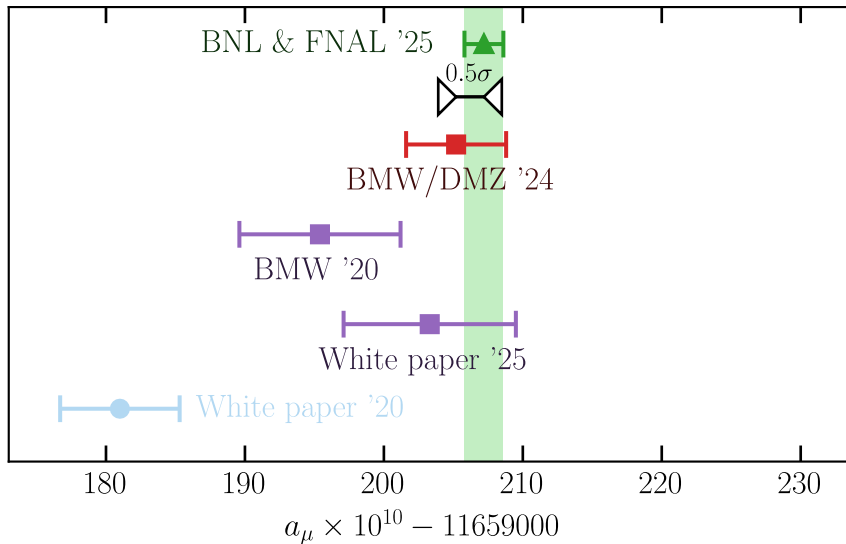
- Direct polarised beam of muons into large storage ring
- Precisely measure **rate of precession** of muon spins
- Latest update from Fermilab in August 2023: **0.20 ppm**
- **4×** statistics on the way



[Fermi National Accelerator Laboratory 2017]



Overview

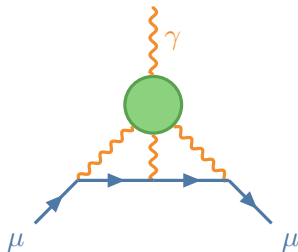
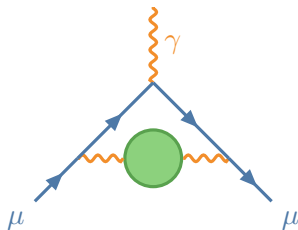


Standard Model (2020)

$a_{\mu}^{QED} \times 10^{10}$	11 658 471.8931	$\pm$	0.0104
$a_{\mu}^{EW} \times 10^{10}$	15.36	$\pm$	0.10
$a_{\mu}^{HVP} \times 10^{10}$	684.5	$\pm$	4.0
$a_{\mu}^{HLbL} \times 10^{10}$	9.2	$\pm$	1.8
$a_{\mu} \times 10^{10}$	11 659 208.9	$\pm$	6.3

[White Paper 2006.04822]

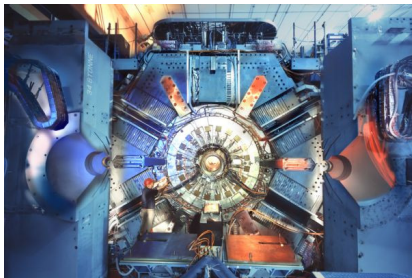
- Value is dominated by electromagnetic part (QED)
- Errors dominated by strong part (QCD)
 - Hadronic vacuum polarisation
 - Hadronic light-by-light scattering



QCD contributions

Data driven

- Input: Experimental cross-sections, form factors
- Optical theorem and dispersion relations



[BaBar at SLAC National Accelerator Laboratory]

Lattice QCD

- Input: QCD+QED Lagrangian with no additional parameters
- Non-perturbative numerical simulation

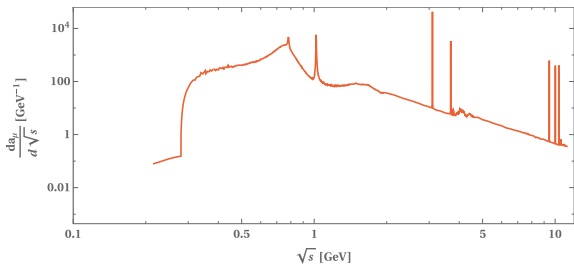


[Forschungszentrum Jülich]

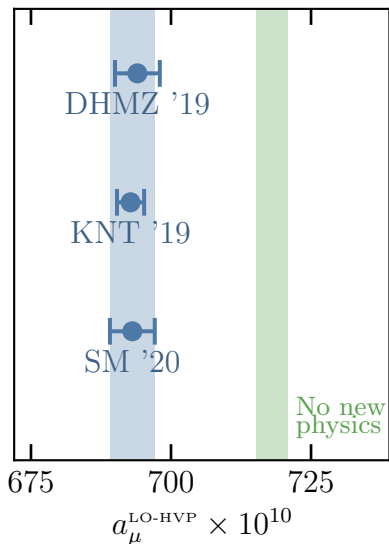
LO-HVP: data-driven (2020)

$$\text{Im} \left[\text{---} \text{---} \text{---} \right] \sim \left| \text{---} \text{---} \text{---} \right\} \text{hadrons} \Big|^2$$

- $e^+e^- \rightarrow \text{hadrons}$ integrated with appropriate kernel
- Major contributions from BaBar, KLOE and other experiments
- Benchmark SM result from [\[WP 2006.04822\]](#): KNT [\[1802.02995\]](#), DHMZ [\[1706.09436\]](#) & CHHKS [\[1810.00007, 1907.01556\]](#)
- 5σ tension with latest experimental avg.

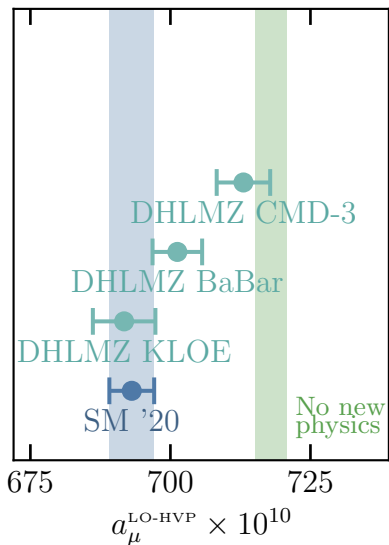


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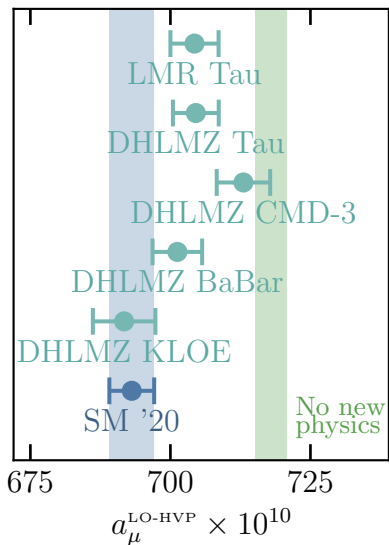
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LO-HVP: data-driven discrepancies



- Tensions between experiments near ρ peak
- Since WP, new result from CMD-3 [2302.08834] shows even greater tension
- BaBar: issues in PHOKHARA Monte Carlo: BaBar insensitive [2308.05233]
- Motivate reevaluation of dispersive calculations and how experiments combined [DHLMZ 2312.02053]
- New efforts to include determinations from τ decays [DHLMZ 2312.02053, LMR 2411.07696]

LO-HVP: data-driven discrepancies



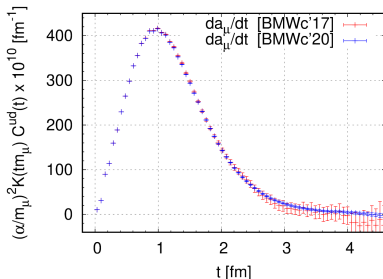
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LO-HVP: lattice QCD

- Formulate QCD+QED
Lagrangian on discrete
four-dimensional grid
- Fix physical scale and
parameters of theory using
physical input
 - $\text{Scale} \leftarrow f_\pi$
 - $(m_u + m_d)/2 \leftarrow M_{\pi_0}^2$
 - $m_u - m_d \leftarrow M_{K_0}^2 - M_{K_+}^2$
 - $m_s \leftarrow M_{K_0}^2 + M_{K_+}^2 - M_{\pi_+}^2$
 - $m_c \leftarrow M_{D_s}^2$
 - QED $\leftarrow \alpha$
- a_μ^{LO-HVP} obtained from integral
in Euclidean time

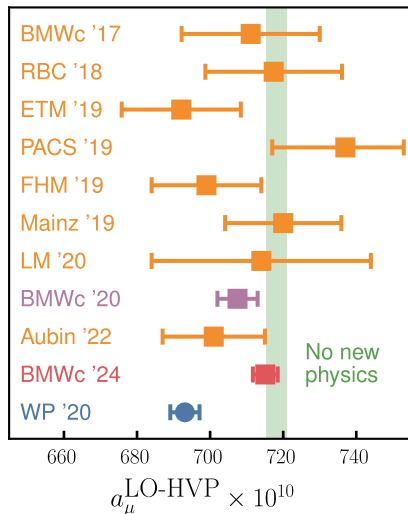


[High-Performance Computing Center Stuttgart]



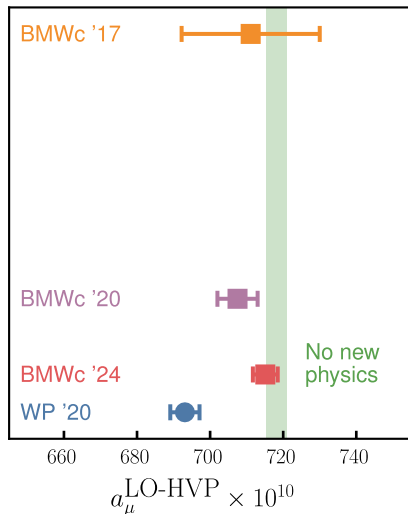
LO-HVP: lattice QCD (2024)

- Until 2020, **lattice** uncertainty larger than **data-driven**
- Lattice results mostly consistent with both data-driven and **experiment**
- **Sub-percent** lattice determination [2002.12347]: First lattice calculation with errors comparable to data-driven determinations
- New BMWc result [2407.10913]: **More precise** than data-driven determinations

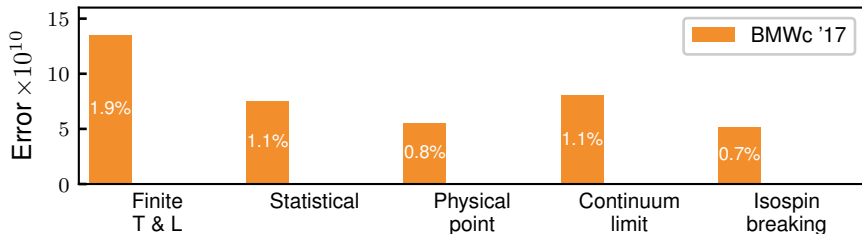


Seven years of progress

- 2020 publication: $3.4\times$ increase in precision over our earlier work [BMWc '17]
- Update this year: further $1.7\times$ increase in precision
- Many improvements needed to attain this precision
- Made possible thanks to the work of many groups around the world

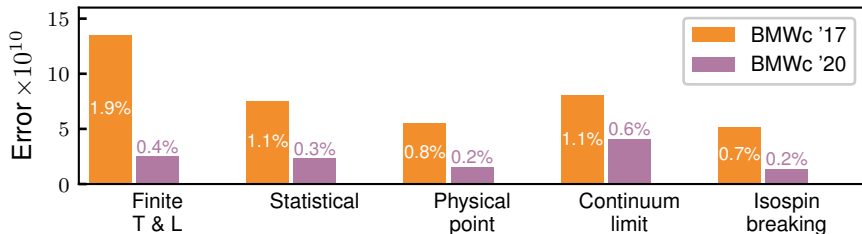


Statistical and systematic errors



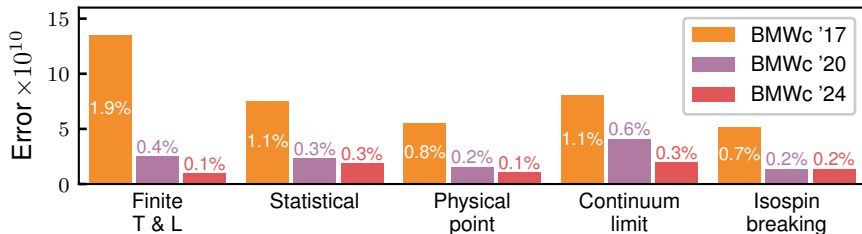
- Five major sources of uncertainty in our **first work**
- Dominant error from finite-size effects
- Sub-leading uncertainties from
 - Statistical Monte-Carlo sampling of path integral
 - Physical inputs to set the scale and parameters
 - Uncertainty in the continuum limit extrapolation
 - Isospin-breaking effects from $\alpha \neq 0$ and $\delta m \equiv m_d - m_u \neq 0$

Statistical and systematic errors



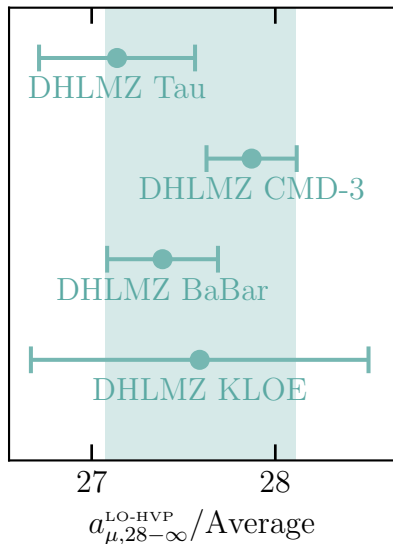
- In **2020 update**, finite-size effects addressed using dedicated **large-volume simulations** (6 fm $\rightarrow$ 11 fm)
- Further improvement from algorithmic improvements, extra calculations and updated analysis

Statistical and systematic errors



- Update: posted in 2024, published in 2026
- Add a 7th, finer lattice spacing ($a = 0.0483$ fm)
- Break continuum limit into pieces with different systematics
 - Short-distance part with complicated discretisation effects
 - Long-distance part with larger statistical errors
 - Most of the value from intermediate-distance parts with small statistical errors and simple, well-constrained continuum limit

Combining Data-Driven & Lattice QCD



- Idea: Use data-driven result instead of lattice in tail
- Proposed in RBC '18
- What about problems with data-driven inputs?
- Only take data-driven above 2.8 fm
 - ρ peak strongly suppressed: dominated by low energies
 - Here, all experiments agree, and also agree with lattice
- Gives $< 5\%$ of final value, but significant reduction in error
- May be best way to match final experimental precision

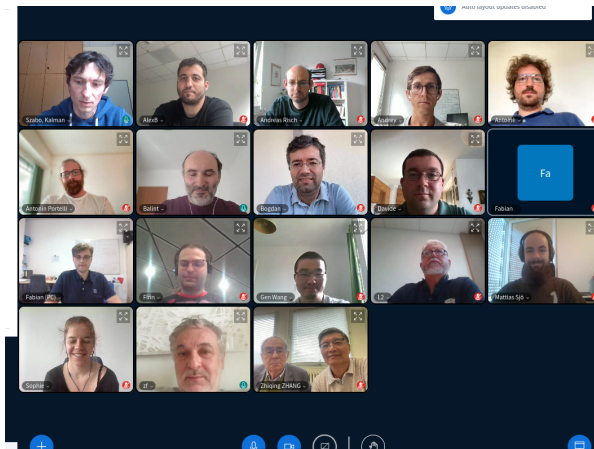
Combining Data-Driven & Lattice QCD



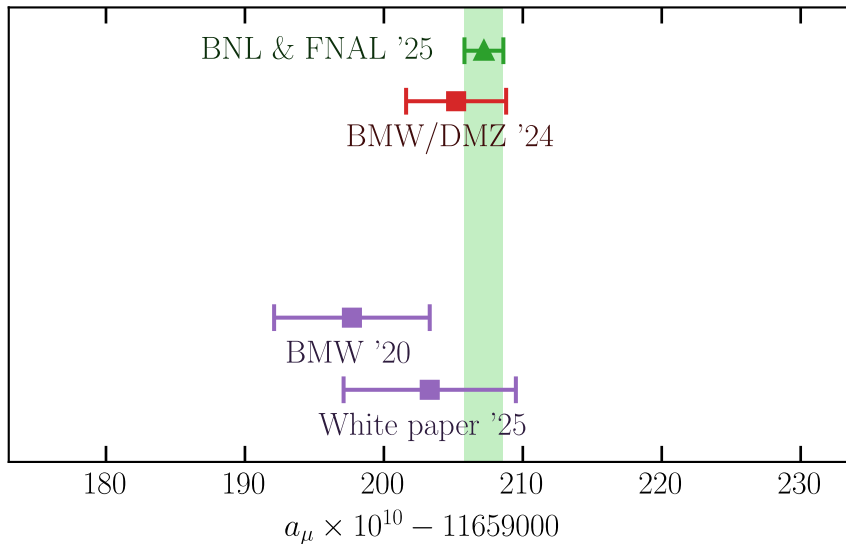
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Blinding

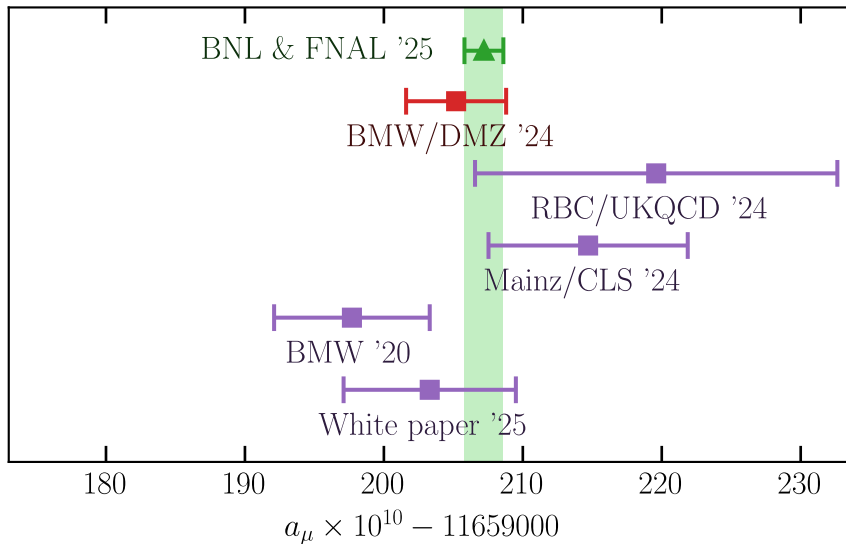
- **2024 study** was fully blinded
- All a_μ contributions were multiplied by some unknown blinding factor
- Blinding factor was only revealed when analysis was completely finalised and manuscript almost complete



LO-HVP: Lattice



LO-HVP: Lattice



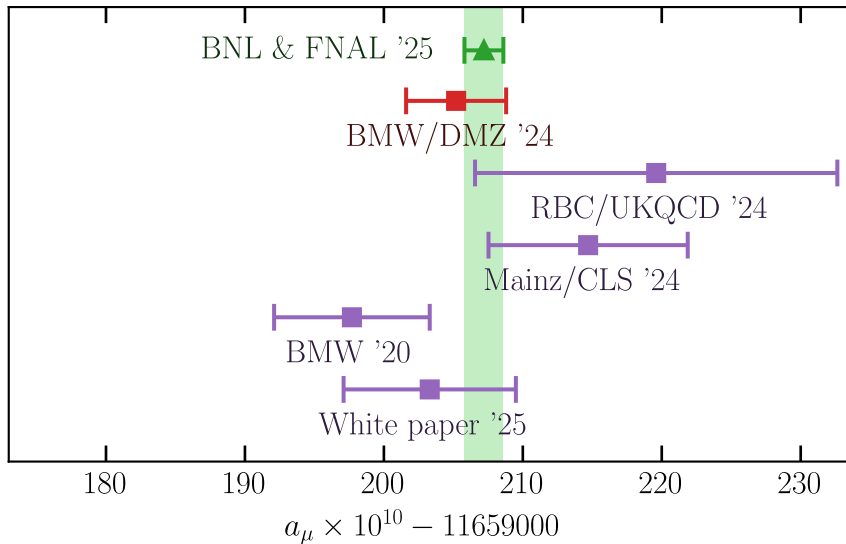
Data-Driven vs Lattice QCD

- Phenomenological estimate shows of IB contributions can't explain discrepancy [[Hoferichter et. al. 2307.02532](#)]
- GeV-scale NP might explain tension [[2212.03877](#)]

What **energies** do they disagree at?

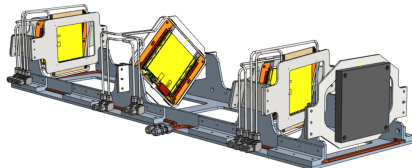
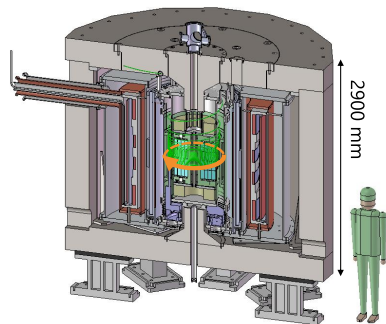
- To convert from lattice result to energy spectrum requires inverse Laplace transform: **ill-conditioned**
- Two possible workarounds:
 - What **modifications** to the data-driven inputs would give consistency [[BMWc/DMZ 2308.04221](#)]
 - **Smeared** version of data-driven inputs [[ETM 2212.12493](#)]
- Both approaches suggest an enhancement near the ρ peak
- Similar to what is seen in CMD-3

Conclusion

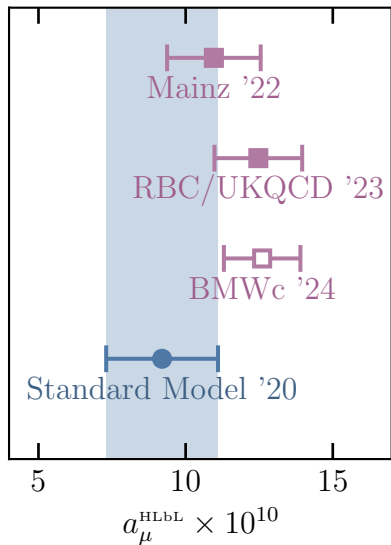


Experimental measurement: Future

- New experiment planned at J-PARC [Just now: K. Aoki]
 - Ultra-cold muon beam from muonium
 - Compact storage ring
 - Very different from BNL/FNAL
 - Expected to start in 2028
- MUonE proposal at CERN
 - Would measure LO-HVP contribution from muon-electron scattering
 - Proof of concept built and tested
 - First phase planned to be online in 2025
 - Full experiment in 2029

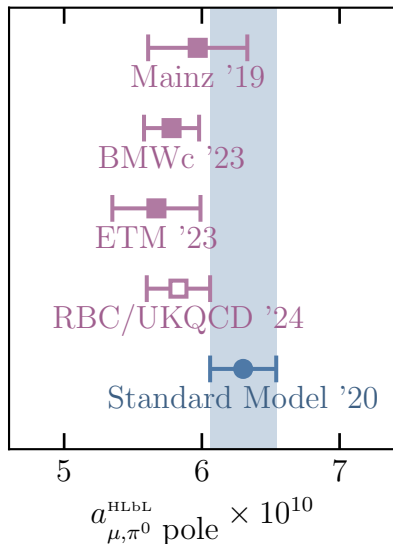


Hadronic Light-by-Light



- Good agreement between data-driven [WP 2006.04822] and lattice [2204.08844, 2304.04423]
- Ongoing efforts to improve precision
- Takeaway: HLbL is in good shape

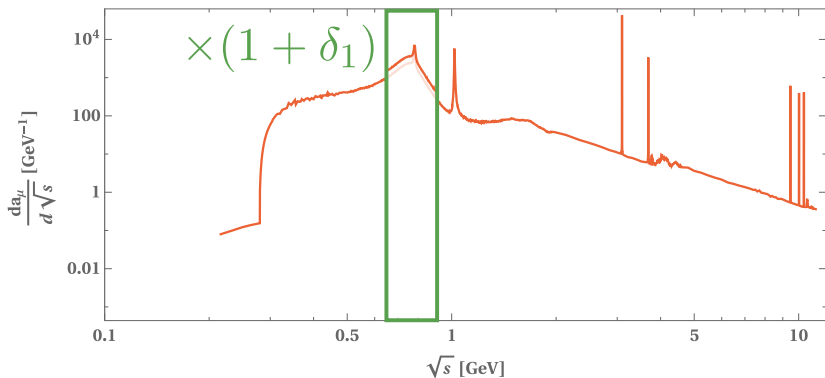
Hadronic Light-by-Light: Pion pole



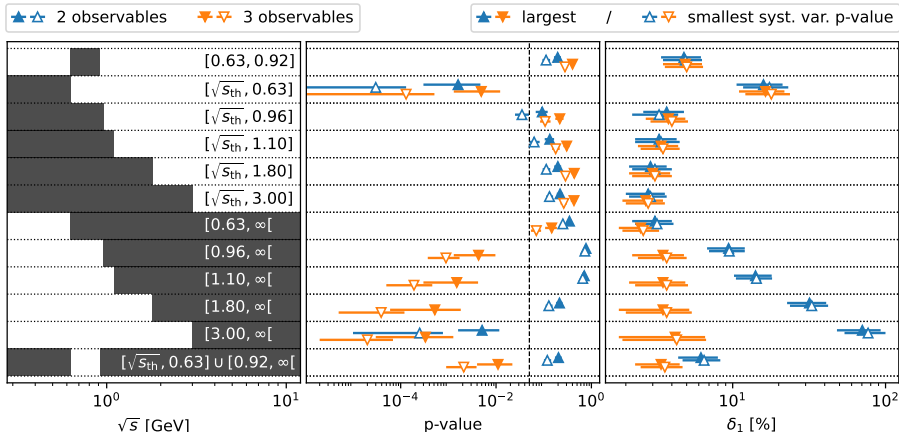
- Contribution from meson exchange: can use lattice QCD input to dispersive calculations [1903.09471, 2305.04570, 2308.12548]

Data-Driven vs Lattice QCD: Rescaling

- Idea: select an **energy range** and scale the data inside that region
- **Scaling factor** of $1 + \delta_1$ for some small δ_1
- Find δ_1 that **minimises tensions** in $a_\mu^{\text{LO-HVP}}$ and $a_{\mu,\text{win}}^{\text{LO-HVP}}$
- $a_\mu^{\text{LO-HVP}}$ and $a_{\mu,\text{win}}^{\text{LO-HVP}}$ don't constrain high energies
- Consider **third observable**, related to hadronic running of α



Data-Driven vs Lattice QCD: Rescaling



- If rho peak included, have good p-values with δ from 2–5%
- Below the rho peak, have bad p-values
- Above the rho peak, have good p-values for 2 observables and bad p values for 3