Structure of the $\Omega(2012)^-$ with Hamiltonian effective field theory

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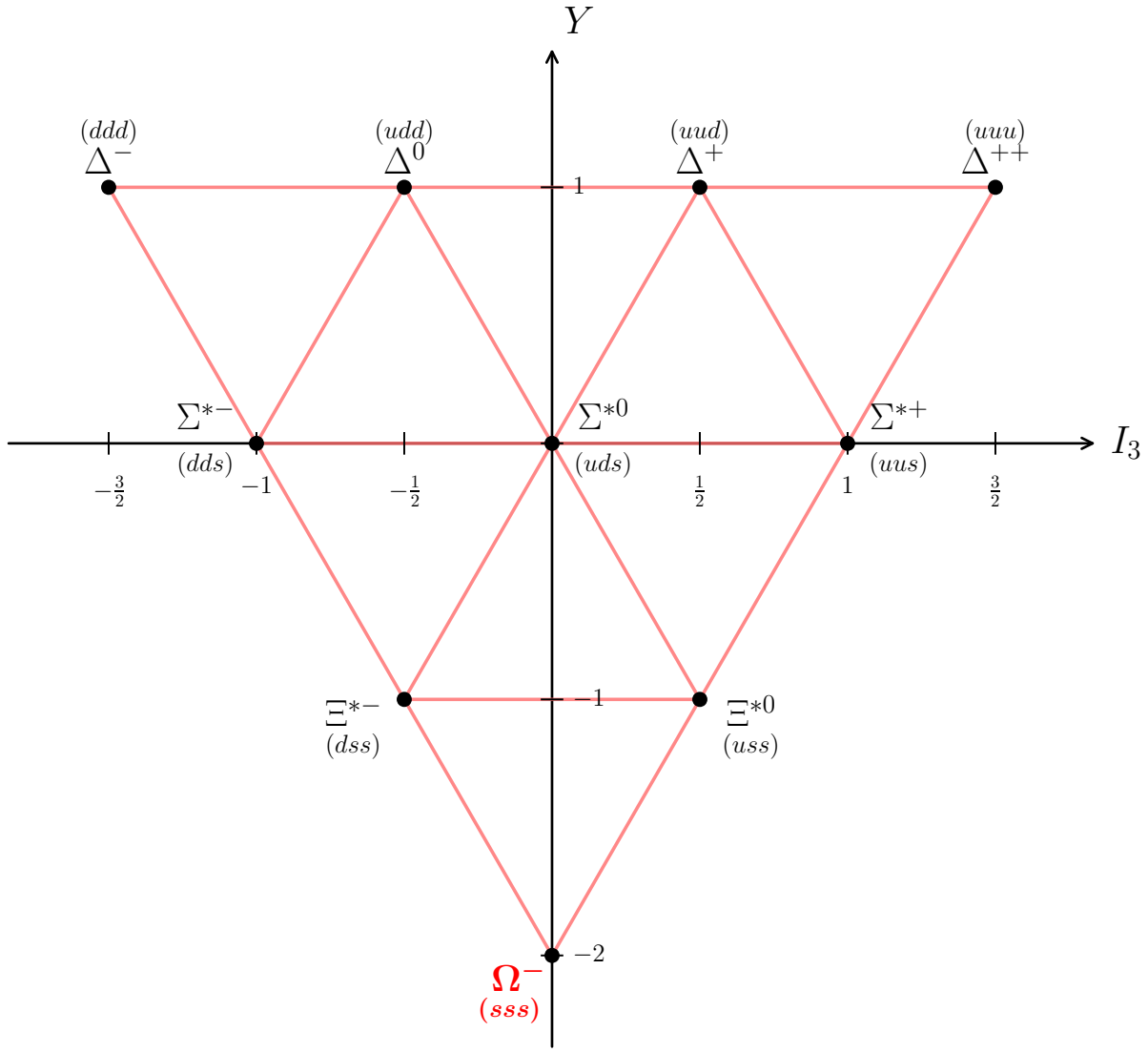
Based on Phys.Rev.D 112 (2025) 5, L051503,



Fundamental Problems in Subatomic Physics 2026

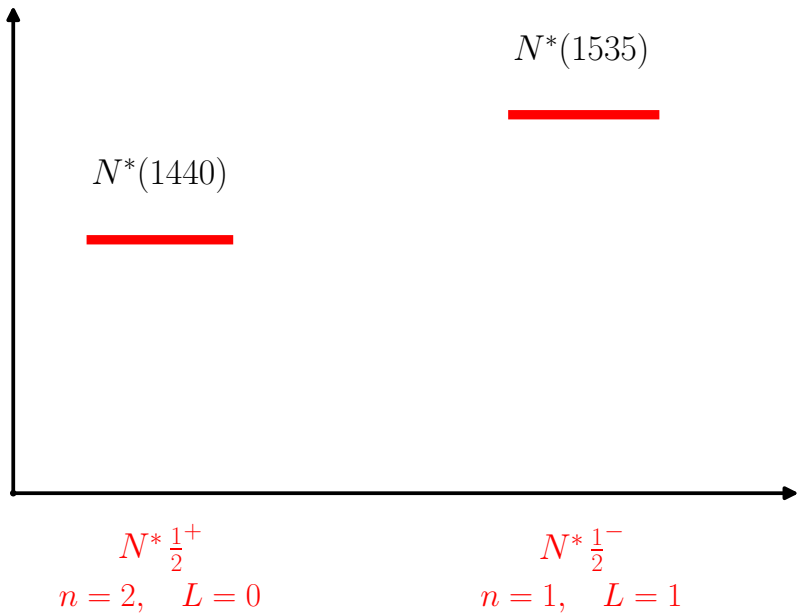
The challenge of hadron spectroscopy

- In 1964, the Ω^- baryon was experimentally discovered, providing a striking early success for the emerging quark picture of hadrons.



From the Roper puzzle to the Ω^- family

- However, the Roper puzzle suggests that the internal structure of hadrons is governed by rich dynamical effects, rather than by a simple static three-quark picture.



Spectrum of the Ω^- Family

Resonance	m (MeV)	Γ (MeV)	J^P	Class.
$\Omega^-(1672)$	1672.4 ± 0.3	–	$3/2^+$	****
$\Omega^-(2012)$	2012.9 ± 0.4	6.3 ± 2.0	$?^-$	***
$\Omega^-(2109)$	2108 ± 5	18 ± 17	$?^?$	*
$\Omega^-(2250)$	2252 ± 9	55 ± 18	$?^?$	***
$\Omega^-(2380)$	2380 ± 17	26 ± 23	$?^?$	**
$\Omega^-(2470)$	2474 ± 12	72 ± 33	$?^?$	**

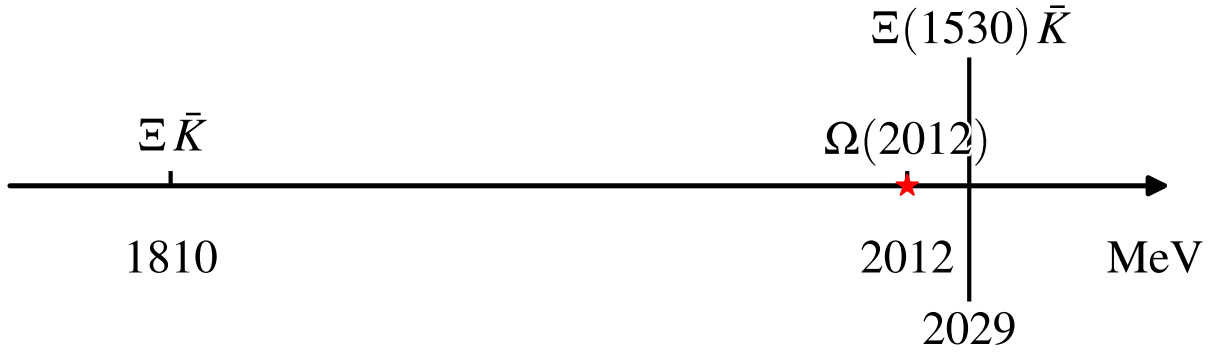
$$M_{\Omega(2012)} = 2012.4 \pm 0.7_{\text{stat}} \pm 0.6_{\text{syst}}\ \text{MeV},$$

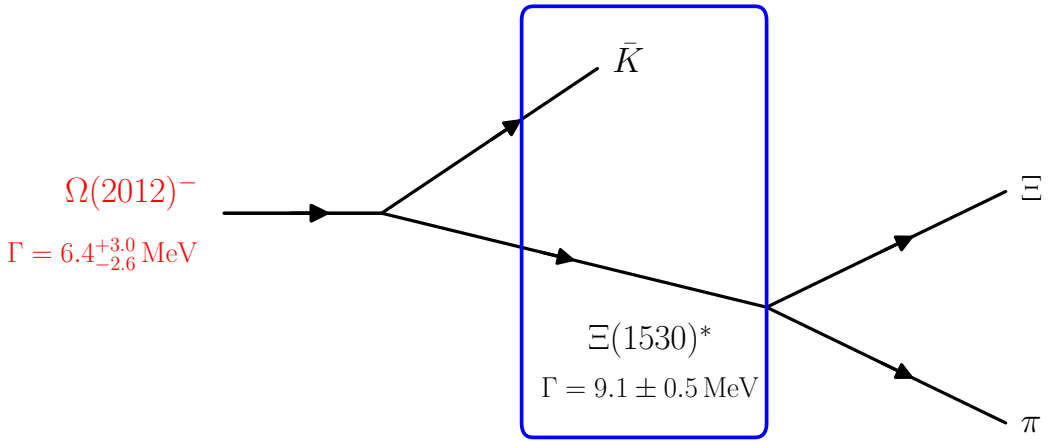
$$\Gamma_{\Omega(2012)} = 6.4^{+2.5}_{-2.0\text{stat}} \pm 1.6_{\text{syst}}\ \text{MeV}.$$

J. Yelton *et al.* (Belle),
Observation of an Excited Ω^- Baryon,
Phys. Rev. Lett. **121**, 052003 (2018).

Near-threshold channels of $\Omega(2012)^-$

A **traditional three-quark state** or a **molecular state**?





- If $\Omega(2012)$ is a $\Xi(1530)\bar{K}$ **molecular state**, the three-body decay is not expected to suffer from a strong phase-space suppression, implying

$$\Gamma\left[\Omega(2012) \to \Xi\pi\bar{K}\right] \sim \Gamma\left[\Omega(2012) \to \Xi\bar{K}\right]$$

- Therefore, the **branching fraction of** $\Xi\pi\bar{K}$ provides a **key experimental probe** of the internal structure of $\Omega(2012)$.

Three body decay branching fraction

- At present, only the Belle Collaboration has measured the three-body decay branching fraction.

$$\mathcal{R}^{\Xi\pi\bar{K}}_{\Xi\bar{K}} = \frac{\mathcal{B}\left(\Omega(2012) \to \Xi(1530)(\to \Xi\pi)\,\bar{K}\right)}{\mathcal{B}\left(\Omega(2012) \to \Xi\bar{K}\right)} = (6.0 \pm 3.7\,(\text{stat.}) \pm 1.3\,(\text{syst.}))\%.$$

S. Jia *et al.* (Belle),
"Search for $\Omega(2012) \to K\Xi(1530) \to K\pi\Xi$ at Belle",
Phys. Rev. D **100**, 032006 (2019).

Supports the traditional three quark configuration

- In 2022, the same Belle data were reanalyzed with a revised mass window for the $\Xi(1530)$ fit.

$$\mathcal{R}^{\Xi\pi\bar{K}}_{\Xi\bar{K}} = 0.99 \pm 0.26 \pm 0.06.$$

S. Jia *et al.* (Belle),
Observation of $\Omega(2012)^- \to \Xi(1530)\bar{K}$ and measurement of the effective couplings of $\Omega(2012)^-$ to $\Xi(1530)\bar{K}$ and $\Xi\bar{K}$,
Phys. Lett. B **860**, 139224 (2025).

Supports the molecular configuration

- Only one experiment is available, yet the extracted results are rather different. The internal structure of $\Omega(2012)$ should be interpreted carefully.

Main challenges in constructing

- Unfortunately, our previous model could not include the three-body decay channel.

  However, recent developments of HEFT provide a general framework to include interactions among multi-hadron channels.

  K. Yu *et al.*, "General Hamiltonian Approach to the **N**-Body Finite-Volume Formalism: Extracting the ω Resonance Parameters from Lattice QCD", arXiv:2603.07205 [hep-lat] (2026).

- The conventional $\bar{K}N$ scattering cannot generate the Ω baryon resonance.
  Therefore, experimental constraints are insufficient.

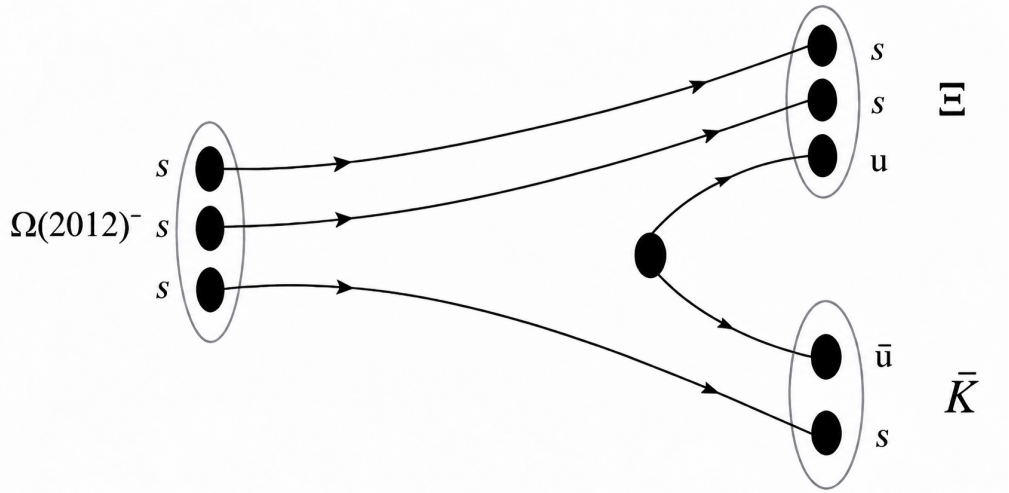
  So we follow the strategy developed for near-threshold D_s states and their bottom analogs, where the quark model is connected with lattice QCD and experimental data through the HEFT.

  Z. Yang *et al.*, "Novel Coupled Channel Framework Connecting the Quark Model and Lattice QCD for the Near-threshold D_s States", *Phys. Rev. Lett.* **128**, 112001 (2022).

  Z. Yang *et al.*, "The Investigations of the P-Wave B_s States Combining Quark Model and Lattice QCD in the Coupled Channel Framework", *JHEP* **01**, 058 (2023).

Goal: Connect the quark-model picture, lattice QCD, and the experimental pole within the Hamiltonian effective field theory framework to predict the internal structure and determine the spin-parity assignment.

The quark-pair-creation model (QPC)



- In the quark model, the hadron wave function is represented as

$$|\Psi\rangle = |\text{Color}\rangle \otimes |\text{Flavor}\rangle \otimes |\text{Spin}\rangle \otimes |\text{Spatial}\rangle\,.$$

The decay amplitude in the QPC model is determined by the overlap between the initial and final hadron wave functions.

$$\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}(\vec{k}) = \left\langle B(n_B^{2S_B+1}L_B(J_B,M_{J_B})), C(n_C^{2S_C+1}L_C(J_C,M_{J_C}))\middle|\hat{T}\middle|A(n_A^{2S_A+1}L_A(J_A,M_{J_A}))\right\rangle \tag{1}$$

$$= \underbrace{-3}_{\text{Color and Rearrangement}}\ \underbrace{\gamma}_{\text{Strength}} \sum_{M_{L_A},M_{S_A}} \sum_{M_{L_B},M_{S_B}} \sum_{M_{L_C},M_{S_C},m} \underbrace{\langle L_A M_{L_A}; S_A M_{S_A}|J_A M_{J_A}\rangle\langle 1m; 1-m|00\rangle\langle L_B M_{L_B}; S_B M_{S_B}|J_B M_{J_B}\rangle\langle L_C M_{L_C}; S_C M_{S_C}|J_C M_{J_C}\rangle}_{\text{Orbital-spin}}$$

$$\times \underbrace{\langle \chi^{124}_{S_B M_{S_B}} \chi^{35}_{S_C M_{S_C}} | \chi^{123}_{S_A M_{S_A}} \chi^{45}_{1-m}\rangle}_{\text{Spin}} \underbrace{\langle \phi_B^{124}\phi_C^{35}|\phi_A^{123}\phi_0^{45}\rangle}_{\text{Flavor}} \sum_{M_{L_A},M_{L_B},M_{L_C}} \underbrace{I^{n_A L_A,M_{L_A},n_B L_B,M_{L_B},n_C L_C,M_{L_C}}(\vec{P}_B)}_{\text{Spatial}}.$$

- Using Eq. (1), the partial-wave amplitude is obtained as

$$\mathcal{M}^{JL}(k) = \frac{\sqrt{4\pi(2L+1)}}{2J_A+1} \sum_{M_B,M_C} \langle L0; J_{BC}M|JM_A\rangle \langle J_BM_B; J_CM_C|J_{BC}M\rangle \mathcal{M}^{MM_BM_C}(k\hat{z}) \qquad (2)$$

Theoretical $\Xi\bar{K}$ decay widths and $\Xi^0K^-/\Xi^-\bar{K}^0$ branching ratios

States	$\Gamma^{\rm th}_{\rm total}$/MeV	$\Gamma^{\rm th}_{\rm Ref}$/MeV	$\mathfrak{B}$	$\mathfrak{B}_{\rm Ref}$
$\|1P,1/2^-\rangle$	42.9	43.0	0.96	0.96
$\|1P,3/2^-\rangle$	8.18	8.19	1.11	1.11

$$\Gamma = 6.4^{+2.5}_{-2.0}(\text{stat}) \pm 1.6(\text{syst})\ \text{MeV}$$

Z. Y. Wang *et al.*, "Newly observed Ω(2012) state and strong decays of the low-lying Ω excitations", *Phys. Rev. D* **98**, 114023 (2018).

Hamiltonian Effective Field Theory (HEFT)

$$H = H_0 + H_{\text{int}}, \tag{3}$$

$$H_0 = \underbrace{|\Omega_0\rangle\, m_\Omega^0\, \langle\Omega_0|}_{\text{bare core}} + \underbrace{\sum_\alpha \int d^3\vec{k}\; |\alpha(\vec{k})\rangle [\omega_{\alpha_M}(k) + \omega_{\alpha_B}(k)] \langle\alpha(\vec{k})|}_{\text{The energy of channel } \alpha}, \tag{4}$$

$$\omega_{\alpha_{B(M)}}(k) = \sqrt{m^2_{\alpha_{B(M)}} + k^2}\,, \tag{5}$$

- The leading-order WT diagonal interaction vanishes for $\Xi\bar{K}$ and $\Xi^*\bar{K}$. Motivated by this, we only consider the vertex between the bare core and the channels.($\alpha = \Xi\bar{K}, \Xi(1530)\bar{K}$)

$$H_{\text{int}} = \sum_\alpha \int d^3\vec{k}\, \{|\alpha(\vec{k})\rangle\, g^J_{\alpha,\Omega_0}(\vec{k})\, \langle\Omega_0| + |\Omega_0\rangle\, g^{J\dagger}_{\alpha,\Omega_0}(\vec{k})\, \langle\alpha(\vec{k})|\}. \tag{6}$$

- We construct the vertex using the QPC partial wave amplitude in Eq. (2).

$$g^J_{\alpha,\Omega_0}(k) = \mathcal{M}^{JL}(k)\, e^{-k^2/(2\Lambda^2)}\,. \tag{7}$$

Finite Volume

$$\int_0^\infty dk\, k^2 = \frac{1}{4\pi}\int d^3\vec{k} \longrightarrow \frac{1}{4\pi}\left(\frac{2\pi}{L}\right)^3 \sum_{n=0}^{\infty} C_3(n). \tag{8}$$

- $C_3(n)$ denotes the degeneracy: The number of integer triplets (n_x, n_y, n_z) satisfying

$$n = n_x^2 + n_y^2 + n_z^2.$$

- After discretization, the discrete momenta are given by

$$k_n^2 = \left(\frac{2\pi}{L}\right)^2 n. \qquad \tilde{g}^J_{\alpha,B_0}(k_n) = \sqrt{\frac{C_3(n)}{4\pi}}\left(\frac{2\pi}{L}\right)^{3/2} g^J_{\alpha,B_0}(k_n). \tag{9}$$

$$\mathcal{H}_0 = \mathrm{diag}\left\{m_B^0,\, \omega_{\Xi(1530)\bar{K}}(k_0),\, \omega_{\Xi\bar{K}}(k_1),\, \omega_{\Xi(1530)\bar{K}}(k_1),\, \cdots\right\}. \tag{10}$$

$$\mathcal{H}_{\mathrm{int}} = \begin{pmatrix} 0 & \mathbf{g}^\dagger \\ \mathbf{g} & \mathbf{0} \end{pmatrix}, \qquad \mathbf{g} = \begin{pmatrix} \tilde{g}^J_{\Xi(1530)\bar{K},B_0}(k_0) \\ \tilde{g}^J_{\Xi\bar{K},B_0}(k_1) \\ \tilde{g}^J_{\Xi(1530)\bar{K},B_0}(k_1) \\ \tilde{g}^J_{\Xi\bar{K},B_0}(k_2) \\ \vdots \end{pmatrix}. \tag{11}$$

Lattice QCD Data

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IOP Publishing
Journal of Physics G: Nuclear and Particle Physics
J. Phys. G: Nucl. Part. Phys. **52** (2025) 065103 (28pp)
https://doi.org/10.1088/1361-6471/add551

Exploring the Ω^- spectrum in lattice QCD

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$$m_\Omega^{3/2^-} = 2051 \pm 45\,\text{MeV},$$
$$m_\Omega^{1/2^-} = 1994 \pm 55\,\text{MeV}, \qquad m_\Omega^{1/2^-} = 2041 \pm 55\,\text{MeV}.$$
(with all five points) (with only first four points)

- All five points are calculated using three-quark interpolating operators.

Energy Spectrum and Eigenvectors $J^P = \frac{3}{2}^-$

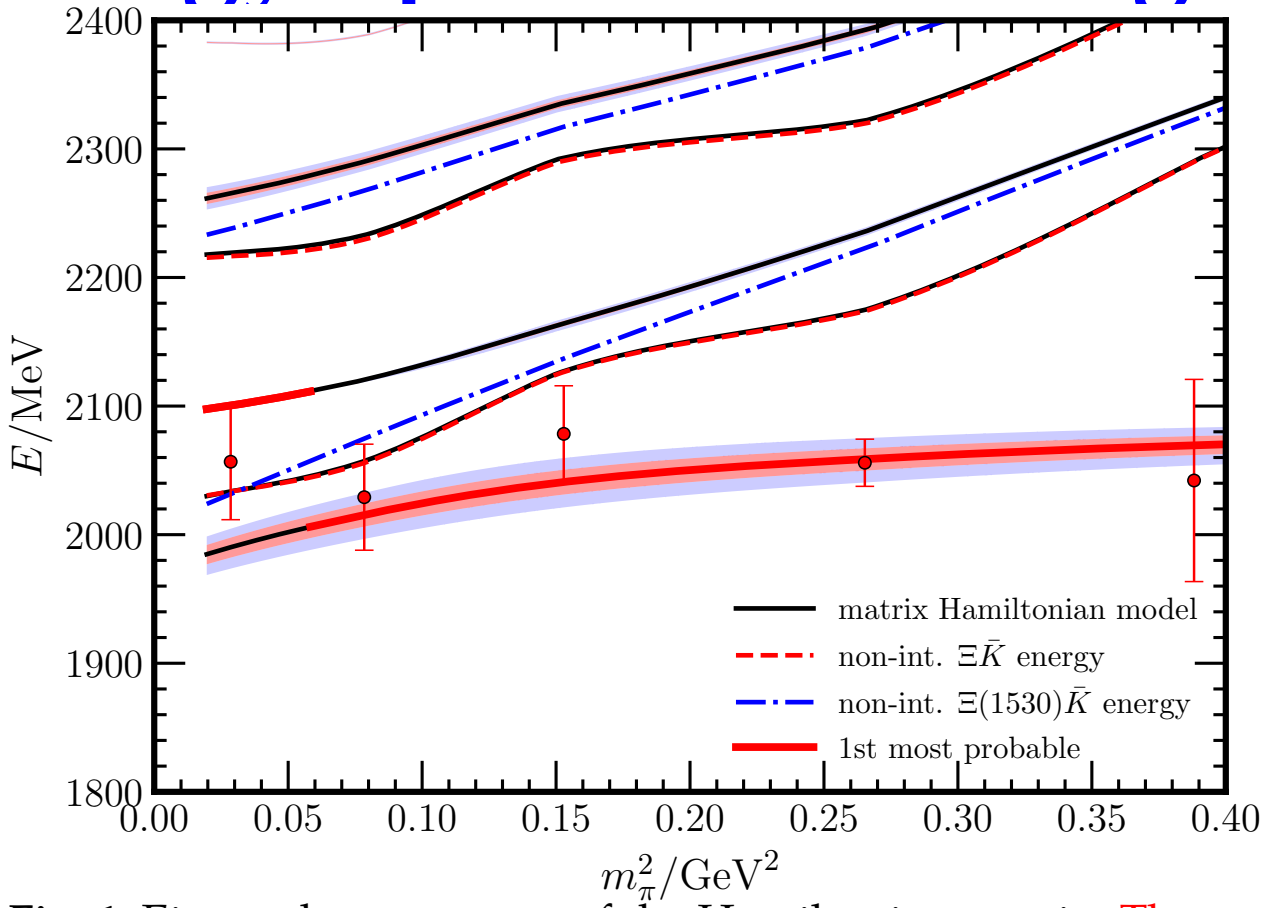


Fig. 1: Eigenvalue spectrum of the Hamiltonian matrix. The red solid line denotes the state with the largest bare-state component, while the dashed lines denote the noninteracting channel energy levels. The energy levels of the $\Xi\bar{K}$ channel start from $n = 0$. The brown and blue shaded regions indicate the uncertainties in γ within 10% and 20%, respectively.

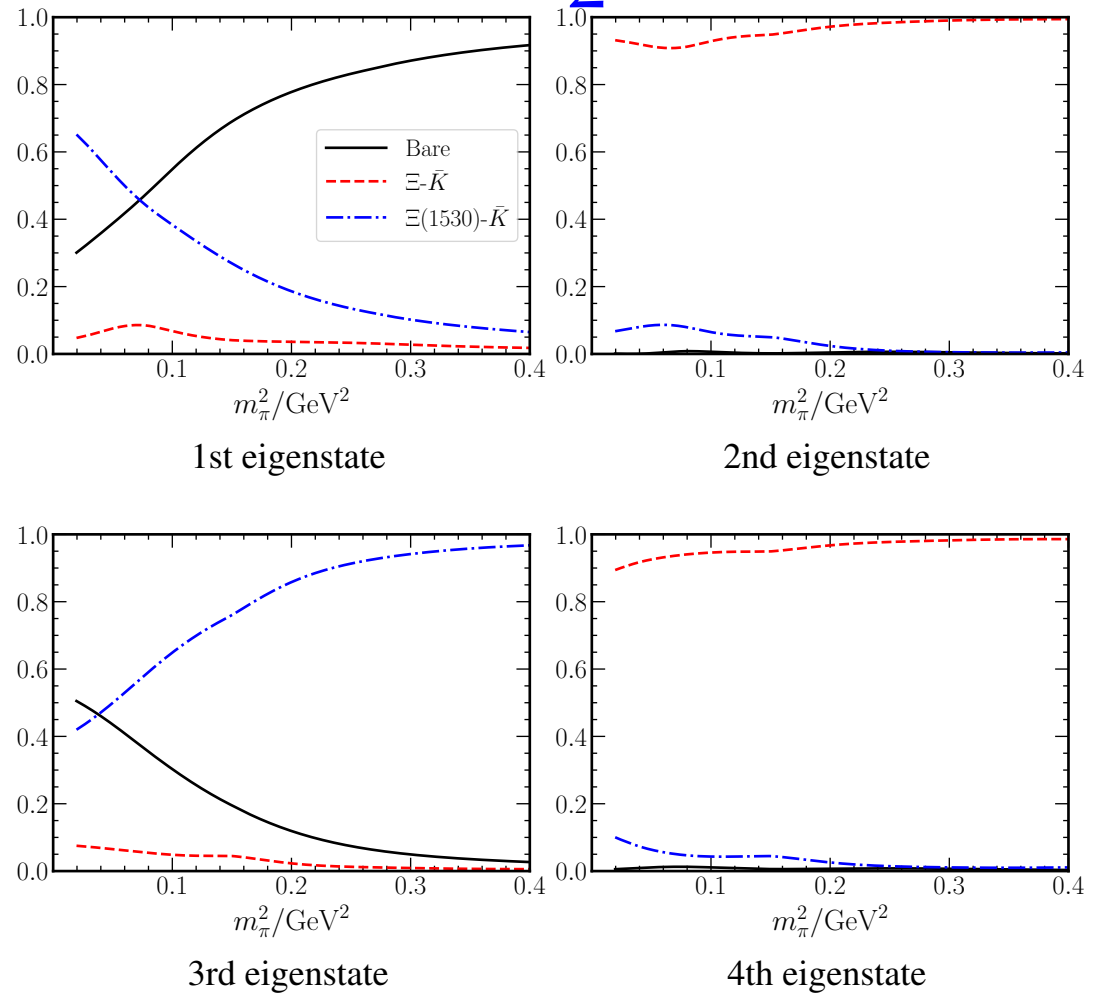


Fig. 2: Eigenvectors of the Hamiltonian matrix. The curves show the variation of different hadronic components with m_π^2.

Energy Spectrum and Eigenvectors $J^P = \frac{1}{2}^-$

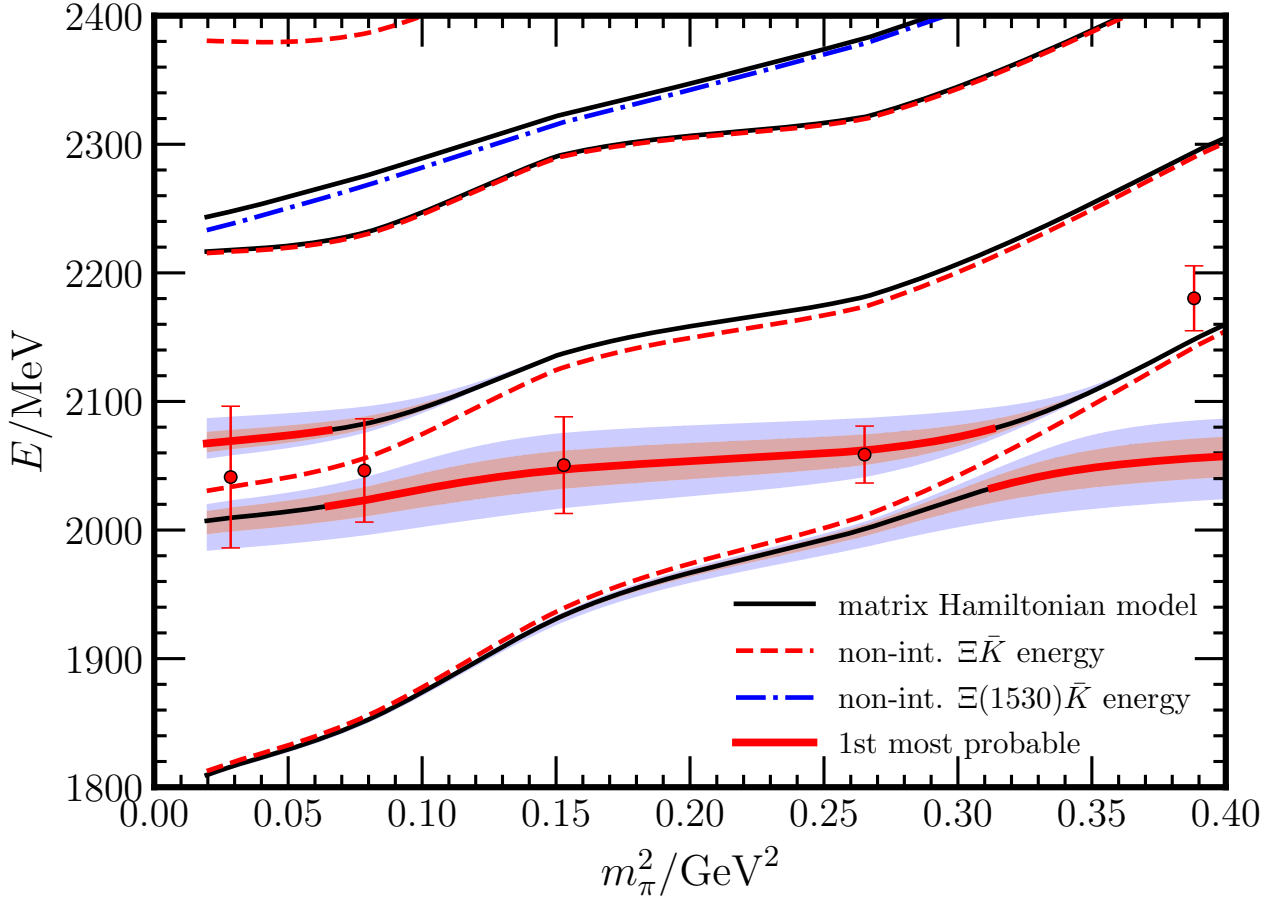


Fig. 3: Eigenvalue spectrum of the Hamiltonian matrix. The red solid line denotes the state with the largest bare-state component, while the dashed lines denote the noninteracting channel energy levels. The energy levels of the $\Xi(1530)\bar{K}$ channel start from $n = 0$. The brown and blue shaded regions indicate the uncertainties in γ within 10% and 20%,

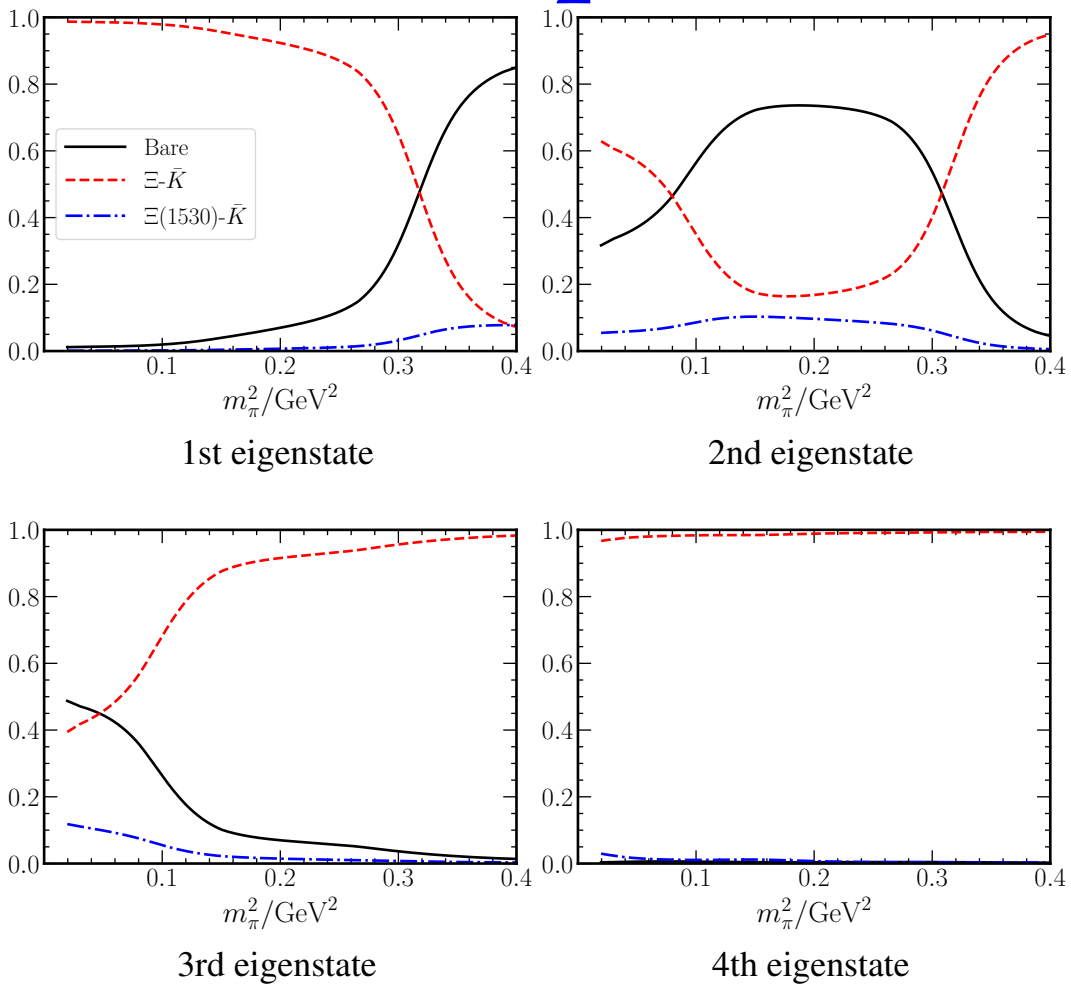


Fig. 4: Eigenvectors of the Hamiltonian matrix. The curves show the variation of different hadronic components with m_π^2.

Infinite-Volume Scattering

- Returning to the vertex between the bare core and the coupled channel constructed from the QPC model in Eq. (7),

$$g^{J}_{\alpha,B_0}(k) = \mathcal{M}^{JL}(k)\, e^{-k^2/(2\Lambda^2)}. \tag{7}$$

$$V_{\alpha,\beta}(k,k';E) = g^{J\dagger}_{\alpha,B_0}(k)\, \frac{1}{E - m^0_\Omega}\, g^{J}_{\beta,B_0}(k'). \tag{13}$$

$$T_{\alpha,\beta}(k,k';E) = V_{\alpha,\beta}(k,k';E) + \sum_\lambda \int q^2\, dq\, \frac{V_{\alpha,\lambda}(k,q;E)\, T_{\lambda,\beta}(q,k';E)}{E - \omega_\lambda(q) + i\epsilon}. \tag{11}$$

$$\omega_\lambda(q) = \sqrt{m^2_{\lambda_B} + q^2} + \sqrt{m^2_{\lambda_M} + q^2}. \tag{12}$$

- Finally, all interaction effects are encoded in the matrix, and the three-dimensional reduced Bethe–Salpeter equation is solved by solving the corresponding matrix equation.

Pole Positions and Comparison with Experiment

$$E_{\text{pole}} = M - i\frac{\Gamma}{2}$$

$\Omega(sss) \quad J^P = 3/2^-$

$E_{\text{pole}} = (2012 \pm 8) - i(1.6 \pm 0.2)\,\text{MeV}$

$M = 2012 \pm 8\,\text{MeV}, \quad \Gamma = 3.2 \pm 0.4\,\text{MeV}$

ALICE(2025) Experiment: $\Omega(2012)$

$M_{\text{exp}} = 2013.35 \pm 0.57_{\text{stat}} \pm 0.27_{\text{syst}}\,\text{MeV}$

$\Gamma_{\text{exp}} = 6.2 \pm 2.1_{\text{stat}} \pm 2.0_{\text{syst}}\text{MeV}$

Consistent with the observed $\Omega(2012)$**.**

$\Omega(sss) \quad J^P = 1/2^-$

$E_{\text{pole}} = (2052 \pm 20) - i(13 \pm 2)\,\text{MeV}$

$M = 2052 \pm 20\,\text{MeV}, \quad \Gamma = 26 \pm 4\,\text{MeV}$

BESIII(2025) Experiment: $\Omega(2109)$

$M_{\text{exp}} = 2108.5 \pm 5.2_{\text{stat}} \pm 0.9_{\text{syst}}\,\text{MeV}$

$\Gamma_{\text{exp}} = 18.3 \pm 16.4_{\text{stat}} \pm 5.7_{\text{syst}}\,\text{MeV}$

A possible candidate for the reported $\Omega(2109)$**.**

T. Veloso, K. P. Khemchandani, A. Martinez Torres, H. Nagahiro, and A. Hosaka, Nature of the Newly Found Ω(2109), arXiv:2606.27572 [hep-ph] (2026).(1/2⁻ for Ω(2109).)
N. Su, H.-X. Chen, P. Gubler, and A. Hosaka, Improved Prediction of the Mass Splitting for *P*-Wave Ω Baryons, Phys. Rev. D 113, 016005 (2026).
S. Acharya *et al.* (ALICE Collaboration), Observation of the Ω(2012) baryon at the LHC, *Phys. Rev. D* **112**, 092002 (2025).
M. Ablikim *et al.* (BESIII), Evidence for Two Excited Ω⁻ Hyperons, *Phys. Rev. Lett.* **134**, 131903 (2025).

Summary

- In the energy region of $2.0 \sim 2.2$ GeV, two negative-parity Ω resonances are found in this work: a narrow $J^P = 3/2^-$ state and a broad $J^P = 1/2^-$ state.
- Both states decay into the $\Xi\bar{K}$ channel:
  - For the $J^P = 3/2^-$ state, the $\Xi\bar{K}$ decay proceeds through a D wave. The near-threshold effect and the angular-momentum jointly suppress its width.
  - For the $J^P = 1/2^-$ state, the $\Xi\bar{K}$ decay proceeds through an S wave, leading to a stronger decay and hence a broader width.
- By fitting the lattice QCD data, the pole position of the $J^P = 3/2^-$ state is obtained as

$$E_{\text{pole}}^{(3/2^-)} = (2012 \pm 8) - (1.6 \pm 0.2)i \text{ MeV},$$

  corresponding to $\Gamma \simeq 3.2 \pm 0.4$ MeV. This result is in agreement with the experimental mass and width of $\Omega(2012)^-$, supporting its assignment as a $J^P = 3/2^-$ state.
- Both channels, together with the bare core, are indispensable for achieving a consistent and realistic description of the $\Omega(2012)^-$ resonance.