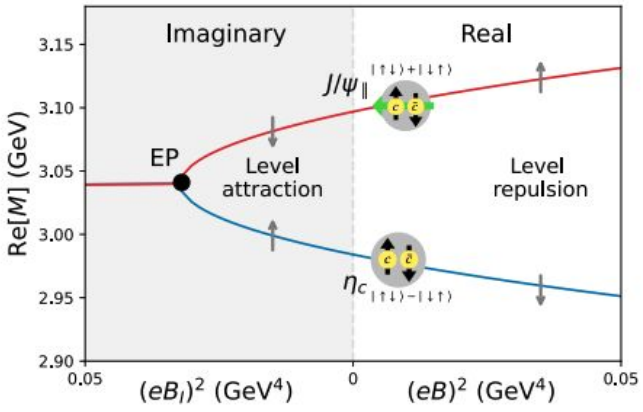
Hadronic Exceptional Points

arXiv: 2606.31697 [hep-ph]

Ahmad Jafar Arifi & Kei Suzuki
(Japan Atomic Energy Agency)

Contents



Exceptional points (EPs)

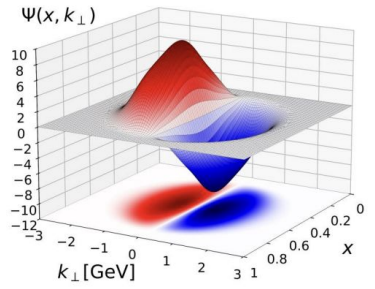
1. Non-hermitian physics
2. Exceptional points
3. Strong-field physics
4. Summary & Outlook

My research background

Main interest: **#hadron-physics**

Webpage: **https://ajarifi.github.io/**

#light-front

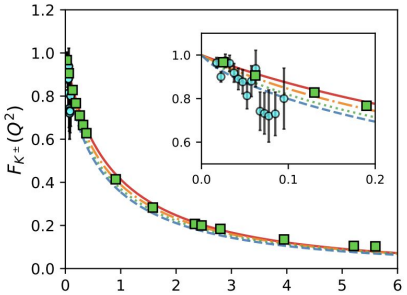


Quarkonium LFWFs

PRD 111, 016011 (2025)

Ho-meoyng Choi, Cheung-Ryong Ji,

#hadron-in-medium

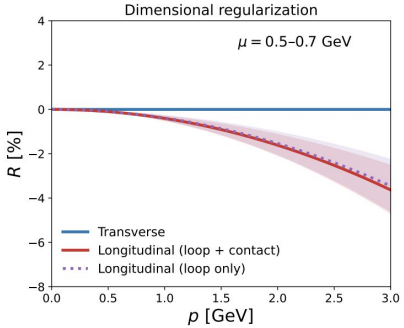


LFQM & QMC model

PRD 111, 074004 (2025)

Parada Hutauruk Kazuo Tsushima

#J-PARC

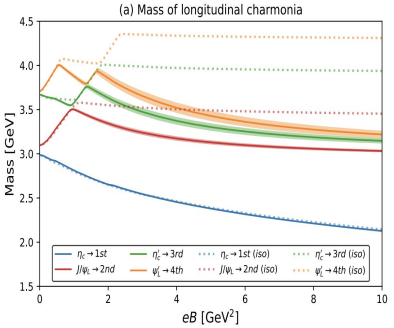


Phi meson in matter

PRD 114, 034041 (2026)

Philipp Gubler

#strong-field



Quarkonium in magnetic fields

PRD 114, 014067 (2026)

Kei Suzuki

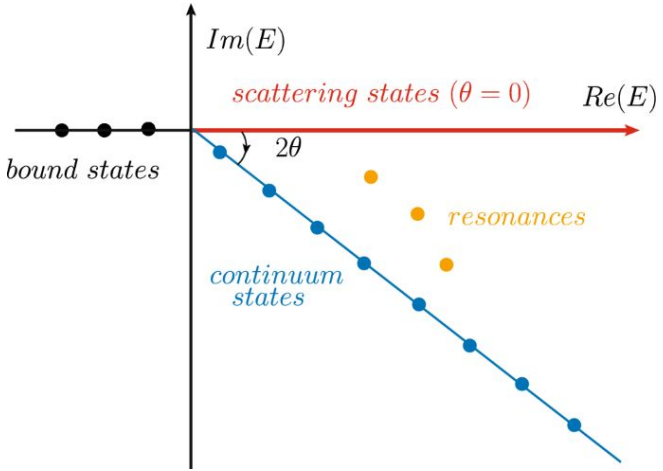
What are Exceptional Points (EPs)?

#Non-Hermitian-Physics

Non-hermitian & hadron resonances

Non-hermitian physics

- **Resonance → complex spectrum**
- **Complex scaling method**



Complex scaling: Physics of unbound light nuclei and perspective

Takayuki Myo[1,2,*] and Kiyoshi Katō[3]

[1]*General Education, Faculty of Engineering, Osaka Institute of Technology, Osaka 535-8585, Japan*
[2]*Research Center for Nuclear Physics (RCNP), Osaka University, Ibaraki, Osaka 567-0047, Japan*
[3]*Nuclear Reaction Data Centre, Faculty of Science, Hokkaido University, Sapporo 060-0810, Japan*
*E-mail: takayuki.myo@oit.ac.jp

Received March 29, 2020; Revised May 11, 2020; Accepted June 29, 2020; Published September 13, 2020

The complex scaling method (CSM) is one of the most powerful methods of describing the resonances with complex energy eigenstates based on non-Hermitian quantum mechanics. We present the basic application of CSM to the properties of the unbound phenomena of light nuclei. In particular, we focus on many-body resonant and non-resonant continuum states observed in unstable nuclei. We also investigate the continuum level density (CLD) in the scattering problem in terms of the Green's function with CSM. We discuss the explicit effects of resonant and non-resonant contributions in CLD and transition strength functions.

Subject Index D10, D11, D13

Hunting hadron resonances

Real spectrum and non-hermiticity

Hermitian $H = H^\dagger$ **A condition for real spectra in quantum mechanics**

PT-symmetric

Non-Hermitian $H = (PT)^{-1} H (PT)$ **Can produce real spectra.**

What is the condition for real spectra?

C. M. Bender and S. Boettcher, Phys. Rev. Lett. 80, 5243 (1998), Real spectra in non-Hermitian Hamiltonians having PT symmetry,

Non-Hermitian Physics

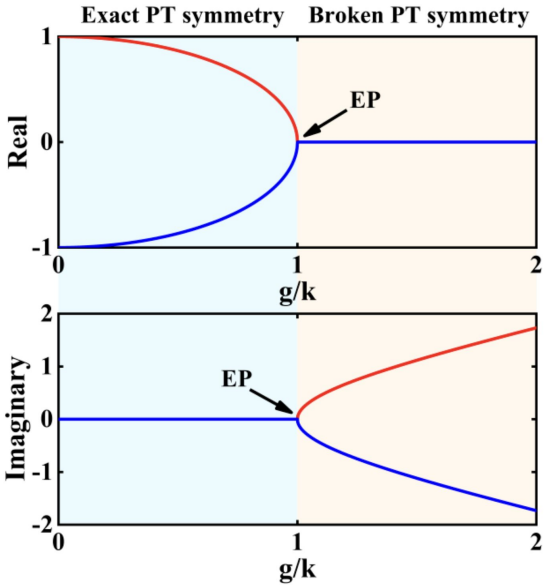
Yuto Ashida[a*], Zongping Gong[b], and Masahito Ueda[b,c]

[a]*Department of Applied Physics, University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-8656, Japan*
[b]*Department of Physics, University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan*
[c]*RIKEN Center for Emergent Matter Science (CEMS), Wako, Saitama 351-0198, Japan*

Non-hermitian becomes a new trend recently.

Exceptional points (EPs)

An EP is a point where two eigenvalues and eigenvectors become identical.



Src: [10.1109/ACCESS.2023.3251572](https://doi.org/10.1109/ACCESS.2023.3251572)

To mention a few. **#non-hermitian #interdisiplinary**

Experimental Observation of the Topological Structure of Exceptional Points

C. Dembowski,[1] H.-D. Gräf,[1] H. L. Harney,[2] A. Heine,[1] W. D. Heiss,[3] H. Rehfeld,[1] and A. Richter[1]
[1]*Institut für Kernphysik, Technische Universität Darmstadt, D-64289 Darmstadt, Germany*
[2]*Max-Planck-Institut für Kernphysik, D-69029 Heidelberg, Germany*
[3]*Department of Physics, University of Witwatersrand, P.O. Wits 2050, Johannesburg, South Africa*
(Received 26 April 2000)

Non-Hermitian Casimir effect of magnons

Kouki Nakata[1,*] and Kei Suzuki[1,†]
[1]*Advanced Science Research Center, Japan Atomic Energy Agency, Tokai, Ibaraki 319-1195, Japan*
(Dated: June 5, 2024)

Exceptional point in a trimer chain of oscillators with a quadratic driving

M Shoufie Ukhtary,[1] Albert Andersen,[1,2] Donny Dwiputra,[1] and M. Jauhar Kholili[1]
[1]*Research Center for Quantum Physics, National Research and Innovation Agency (BRIN), South Tangerang 15314, Indonesia**
[2]*Department of Physics, Faculty of Mathematics and Natural Sciences, Universitas Indonesia, Depok 16424, Indonesia*
(Dated: January 3, 2025)

Exceptional points in optics and photonics

Mohammad-Ali Miri and Andrea Alù*

Experimental observation of exceptional points in coupled pendulums

Nicolas Even[a,b], Benoit Nennig[b], Gautier Lefebvre[a], Emmanuel Perrey-Debain[a]

[a]*Université de technologie de Compiègne, Roberval (Mechanics, energy and electricity), Centre de recherche Royallieu, CS 60319 - 60203, Compiègne, France*
[b]*Laboratoire Quartz (EA7393), ISAE-Supméca – Institut supérieur de mécanique de Paris, 3 rue Fernand Hainaut, 93407, Saint-Ouen, France*

Exceptional Points in Atomic Spectra

Holger Cartarius, Jörg Main, and Günter Wunner
Institut für Theoretische Physik 1, Universität Stuttgart, 70550 Stuttgart, Germany
(Dated: January 29, 2018)

EPs have been observed in many quantum systems.

Can we find EPs in hadronic systems?

Hadronic Exceptional Points

Complex 2D Matrix Model and Geometrical Map on Complex-N_c Plane

Kanabu Nawa[1,*], Sho Ozaki[1,2], Hideko Nagahiro[3,4], Daisuke Jido[5,6], and Atsushi Hosaka[4]

Spectral function of the η' meson in nuclear medium based on phenomenological models

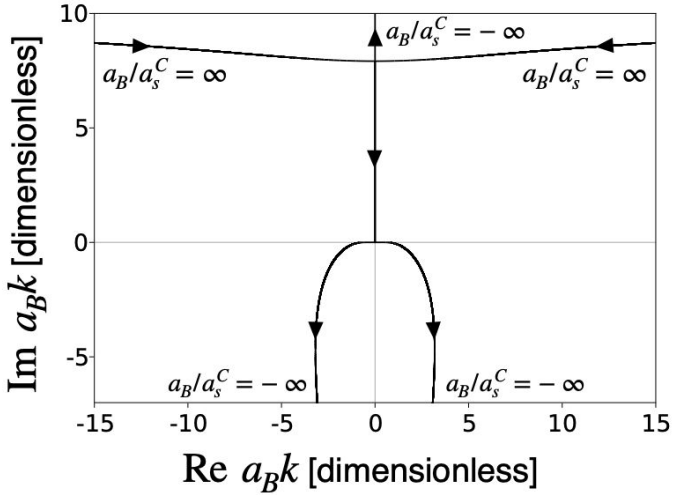
Shuntaro Sakai*
Research Center for Nuclear Physics (RCNP), Osaka University, Ibaraki, Osaka 567-0047, Japan

Daisuke Jido
Department of Physics, Tokyo Institute of Technology, Meguro, Tokyo 152-8551, Japan
(Dated: February 1, 2023)

Compositeness of near-threshold eigenstates with Coulomb plus short-range interactions

Tomona Kinugawa*
Nishina Center for Accelerator-Based Science, RIKEN, Wako 351-0198, Japan

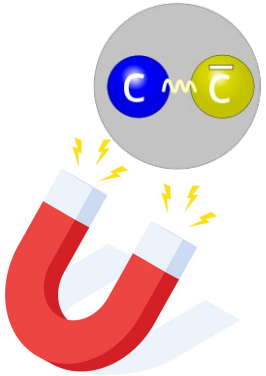
Tetsuo Hyodo†
Research Center for Nuclear Physics, The University of Osaka, Ibaraki, Osaka 567-0047, Japan
(Dated: April 21, 2026)



Origin: complex Nc, complex density, Interaction strength, etc

Not yet systematically studied.

Hadrons in magnetic fields



1. **Strong magnetic fields** produced in HIC.

   ❌ *Extremely difficult in experiments.*

2. Mostly studied in **lattice QCD** and effective models.

   ✅ *Lattice-motivated studies.*

3. **Interdisciplinary works** & relevance to other fields.

   ✅ *External magnetic fields are everywhere*

Strong-Field Physics in QED and QCD:
From Fundamentals to Applications

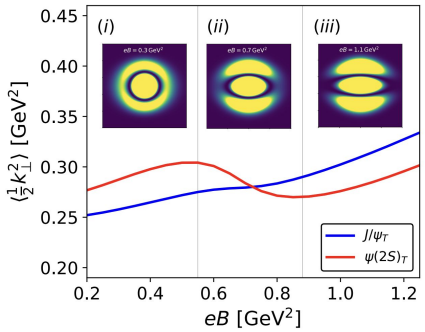
Koichi Hattori,[1,2*] Kazunori Itakura,[3,4†] and Sho Ozaki[5‡]

On Gauge Invariance and Vacuum Polarization

JULIAN SCHWINGER
Harvard University, Cambridge, Massachusetts
(Received December 22, 1950)

New developments in “hadrons in magnetic fields”

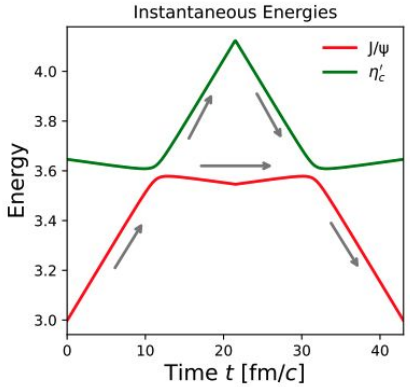
A. J. Arifi & K. Suzuki (2025-present)



LFWFs in magnetic fields,

PRD 112, 094013 (2025).

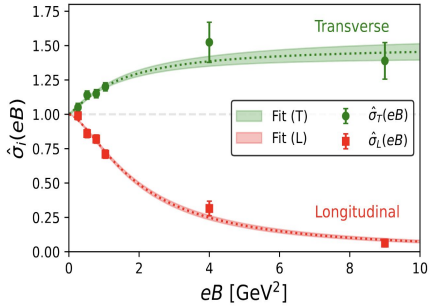
#light-front



Landau-Zener transition,

PRD 113, 054047 (2026).

#quantum-physics



Anisotropic confinements,

PRD 114, 014067 (2026)

#lattice-QCD



Exceptional points,

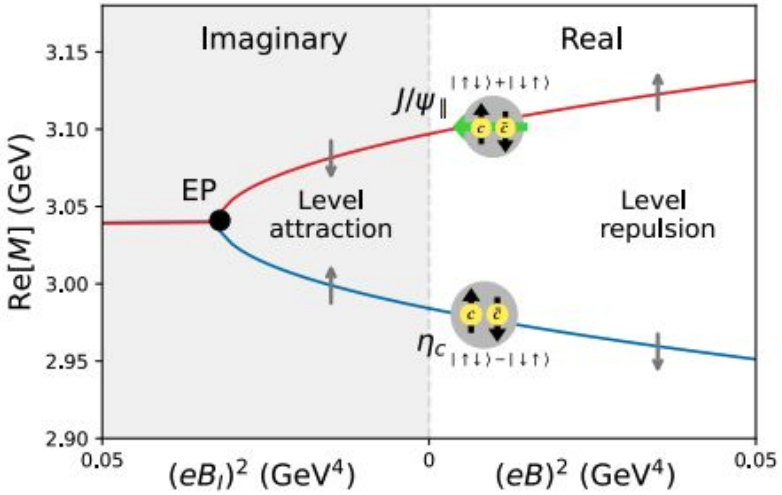
arXiv: 2606.31697

#non-hermitian

Hadronic EPs

Imaginary field is often used in lattice QCD to overcome a technical problem.

$$B \to iB_I$$



Arifi & Suzuki. 2606.31697

Non-hermitian matrix elements

$$\begin{pmatrix} m_{\eta_c}^2 - \omega^2 & \frac{g_{J/\psi\eta_c\gamma}}{m_0}\omega e B_I \\ -\frac{g_{J/\psi\eta_c\gamma}}{m_0}\omega e B_I & m_{J/\psi}^2 - \omega^2 \end{pmatrix} \begin{pmatrix} \eta_c \\ J/\psi_\parallel \end{pmatrix} = 0,$$

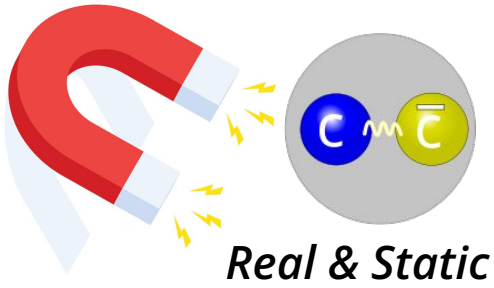
S. Cho et al. PRL 113, 172301 (2014).

Exceptional point

$$eB_I^{\mathrm{EP}} = \frac{m_{J/\psi}^2 - m_{\eta_c}^2}{2g_{J/\psi\eta_c\gamma}}.$$

EP is a hallmark of non-hermitian quantum system.

Quark model in magnetic fields



Real & Static

$$H_{q\bar{q}} = \sum_{i=q,\bar{q}} \left[m_i + \frac{(\boldsymbol{p}_i - q_i \boldsymbol{A})^2}{2m_i} - \boldsymbol{\mu}_i \cdot \boldsymbol{B} \right] + V_{q\bar{q}}(r).$$

$$\boldsymbol{K} = \sum_{i=q,\bar{q}} \left[\boldsymbol{p}_i + \frac{1}{2} q_i \boldsymbol{B} \times \boldsymbol{r}_i \right],$$

Pseudo-momentum

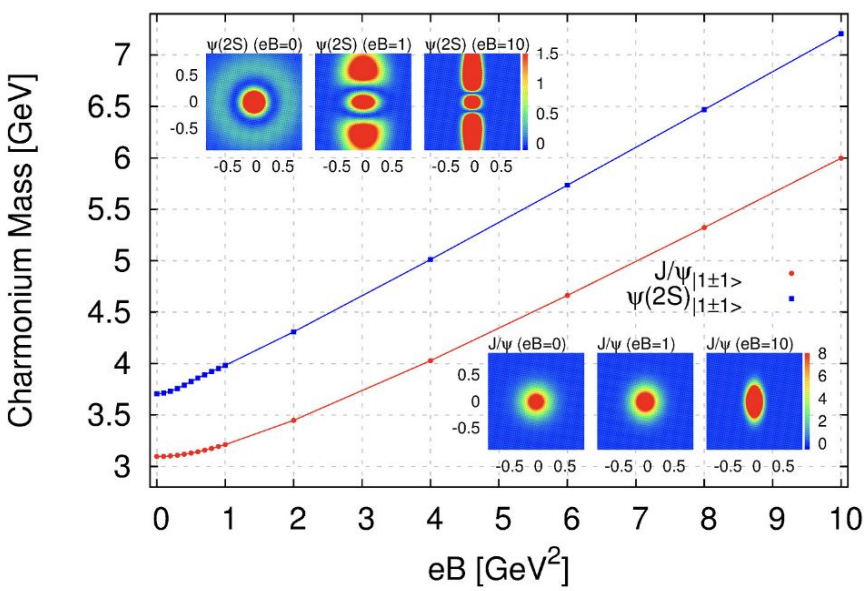
Diagonalization using
Cylindrical GEM

E. Hiyama, Y. Kino, and M. Kamimura,
Prog. Part. Nucl. Phys. 51, 223 (2003).

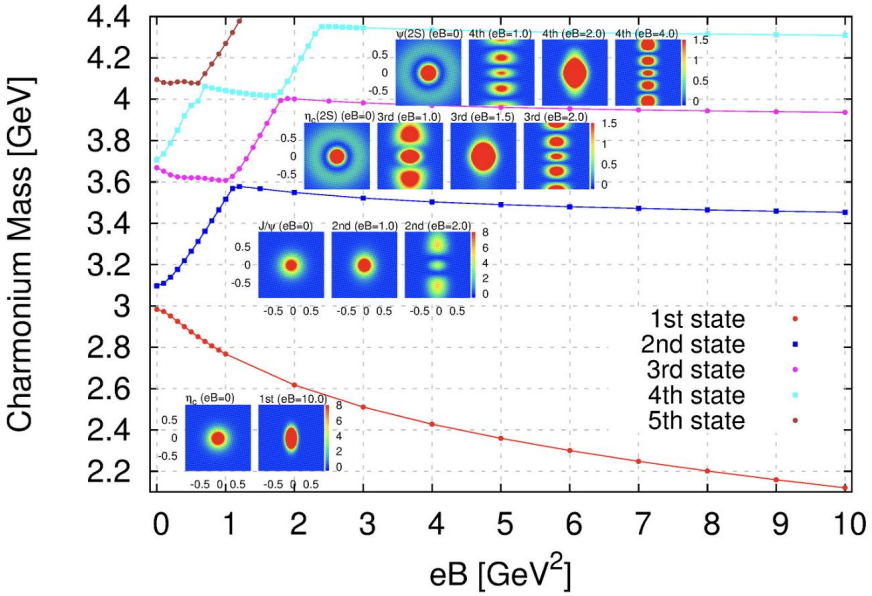
$$H = -\frac{\boldsymbol{\nabla}^2}{2\mu} + \frac{q^2 B^2}{8\mu} r_\perp^2 - \sum_{i=q,\bar{q}} \boldsymbol{\mu}_i \cdot \boldsymbol{B} + V(r).$$

Quark Landau level ***Zeeman effect***

Quarkonium spectra in real magnetic fields



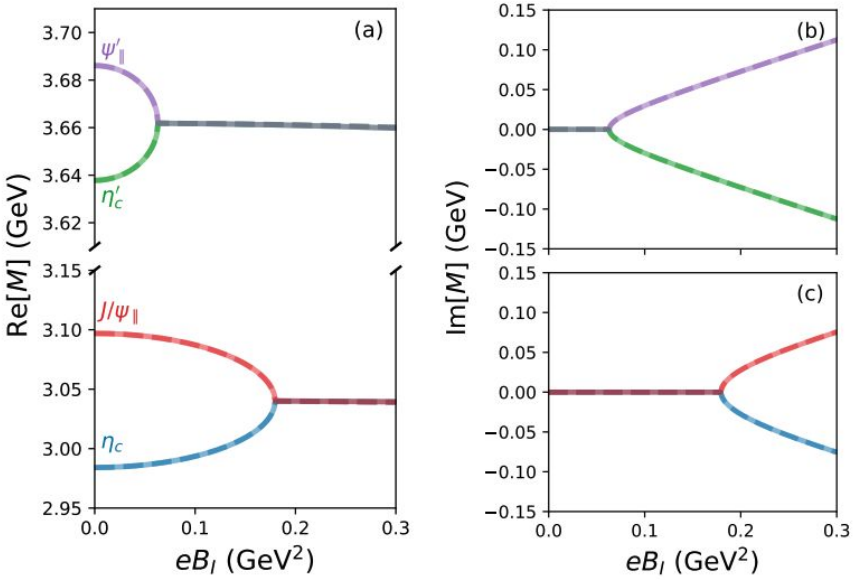
Transverse charmonia



Longitudinal charmonia (mixed)

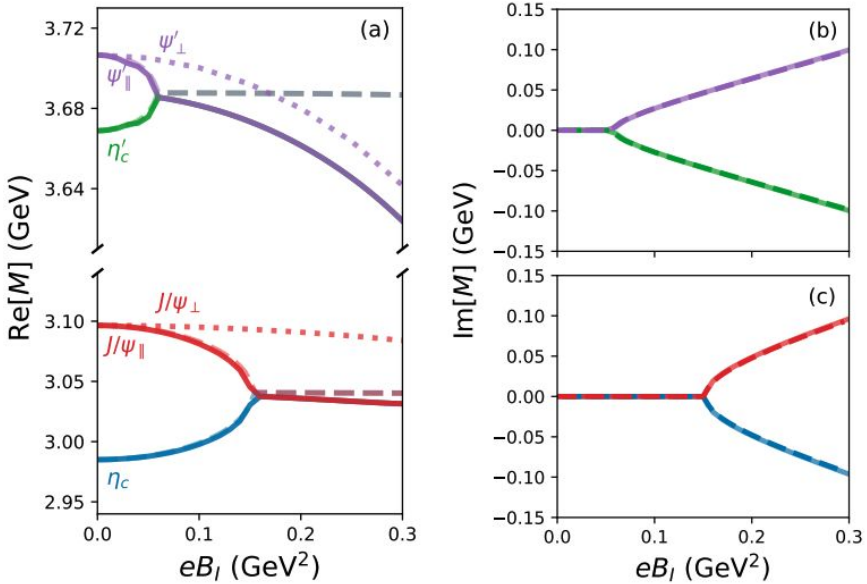
Yoshida & Suzuki. ***PRD94, 074043 (2016).***

EPs in two different approaches



Effective Lagrangian approach

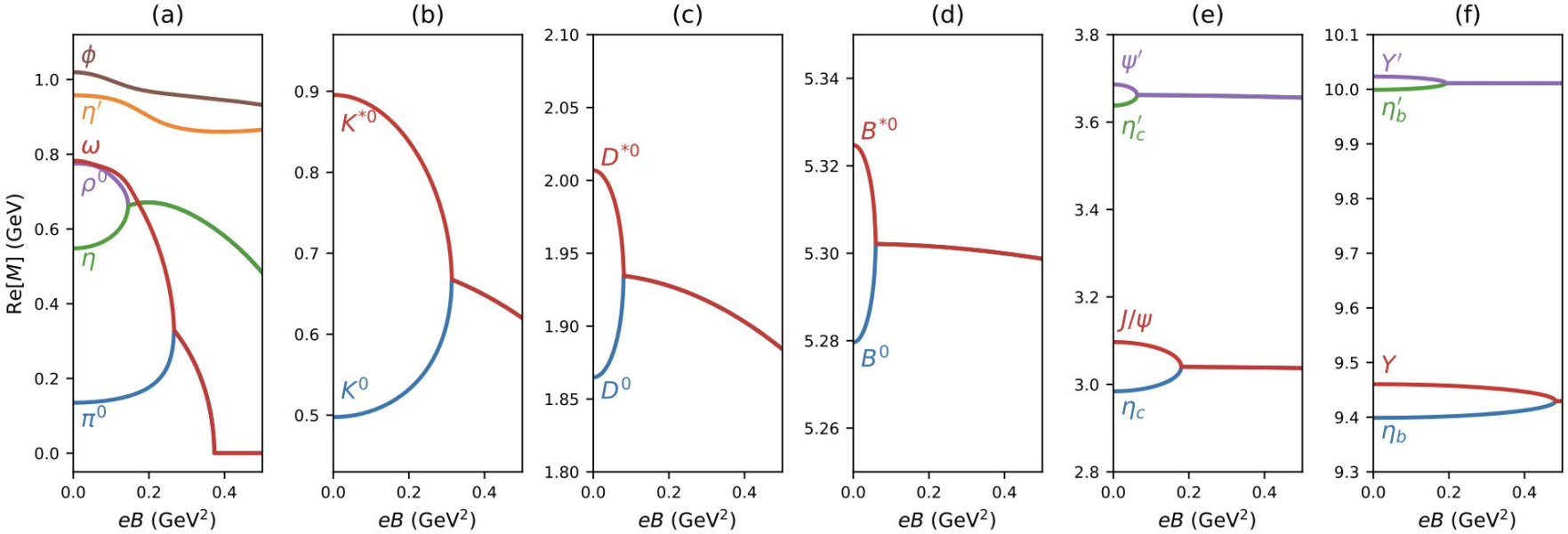
Coupled-channel Klein-Gordon equation



Constituent quark model

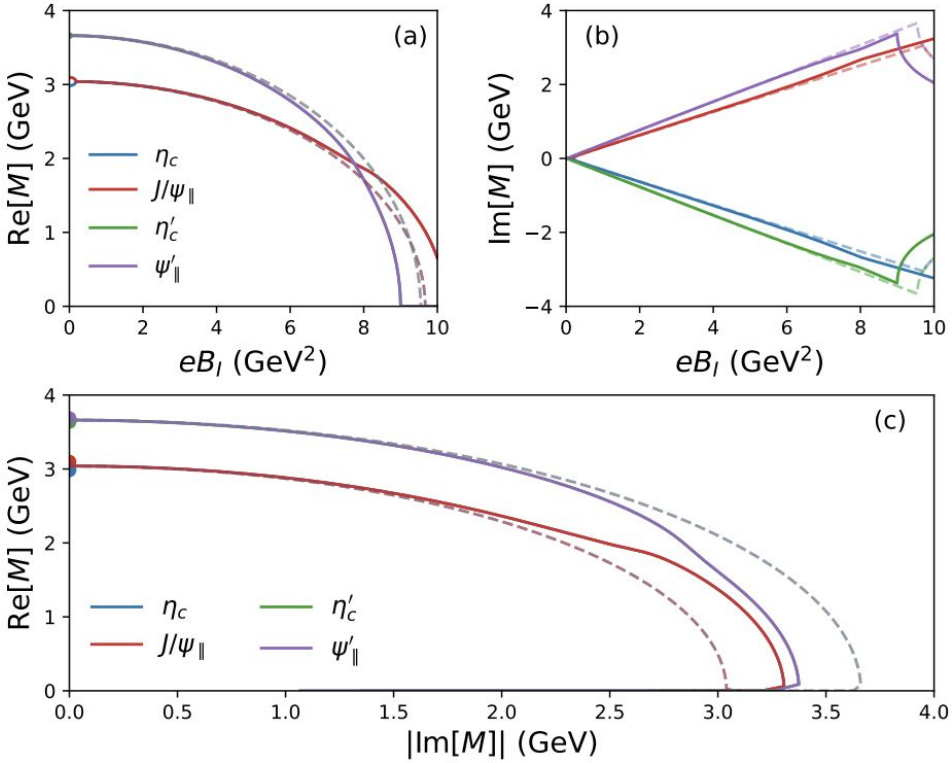
Imaginary Zeeman & deconfinement

Various neutral meson EPs



EP locations: eB = 0.1-0.5 GeV^2

Strong-field Regime



Effective interaction Lagrangian

$$\mathcal{L}_{VP\gamma} = \frac{g_{VP\gamma}}{m_0} ie \tilde{F}^{[\mathrm{Im}]}_{\mu\nu} (\partial^\mu P) V^\nu .$$

Coupled-channel Klein-Gordon equation

$$\begin{pmatrix} m^2_{\eta_c} - \omega^2 & \frac{g_{J/\psi\eta_c\gamma}}{m_0}\omega e B_I \\ -\frac{g_{J/\psi\eta_c\gamma}}{m_0}\omega e B_I & m^2_{J/\psi} - \omega^2 \end{pmatrix} \begin{pmatrix} \eta_c \\ J/\psi_\parallel \end{pmatrix} = 0,$$

The underlying physics of EPs

Non-hermitian Hamiltonian

$$H = \begin{pmatrix} a & ic \\ ic & b \end{pmatrix},$$

Eigenvalues

$$\omega_{\pm} = \frac{1}{2}\left(a + b \pm \sqrt{(a-b)^2 - 4c^2}\right).$$

Determinant

$$\Delta = (a-b)^2 - 4c^2.$$

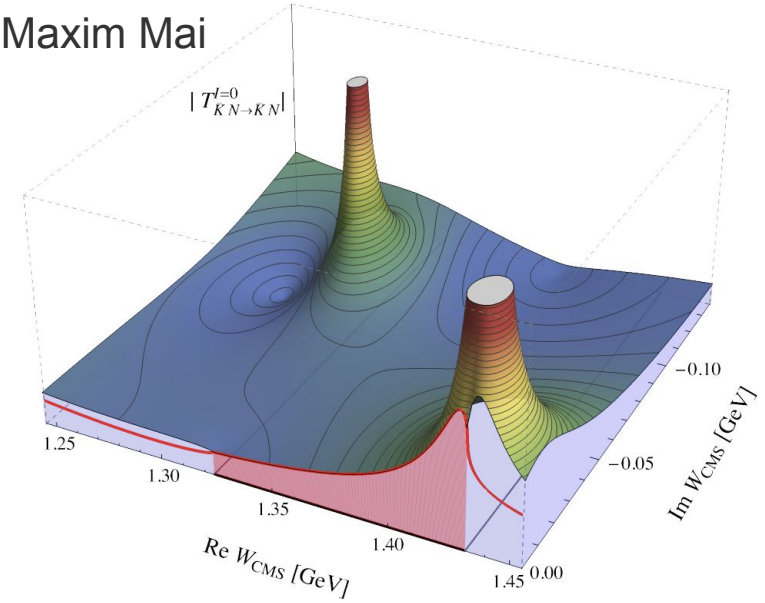
$(\Delta > 0)$ **Real spectrum**

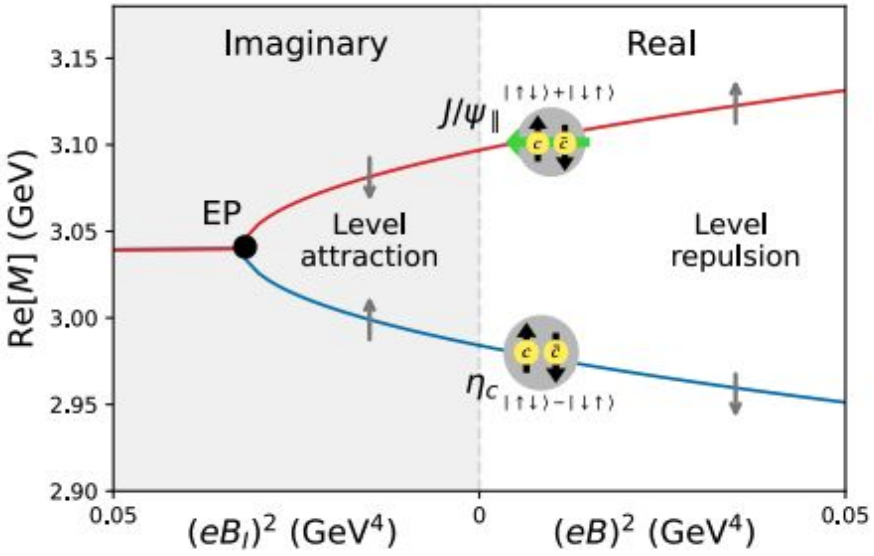
$(\Delta = 0)$ **Exceptional point**

$(\Delta < 0)$ **Complex spectrum**

Analogy: Poles & EPs

Maxim Mai





Resonance peak → Resonance pole

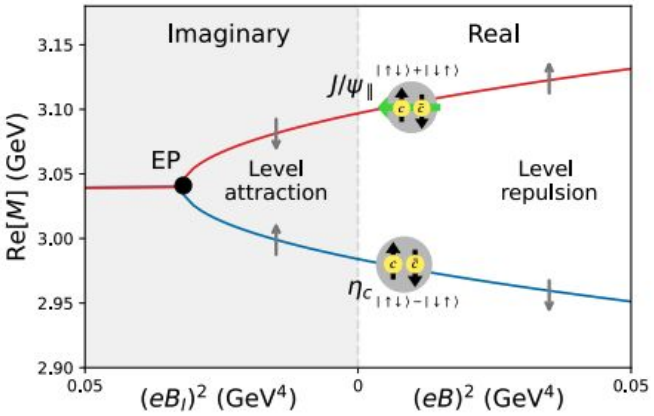
Real → Complex

Level repulsion → Exceptional point

Real → Imaginary

Summary & Outlook





arXiv: 2606.31697 [hep-ph]

Summary

- **EPs** are observed in many quantum systems, but **hadronic EPs** are rarely discussed.
- The emergence of EPs in imaginary magnetic fields.

Outlook

- **A survey of various mechanisms that generate EPs in hadronic systems.**

Can we find EPs in hadronic systems?

This trip is supported by RCNP, The University of Osaka

Thank you very much