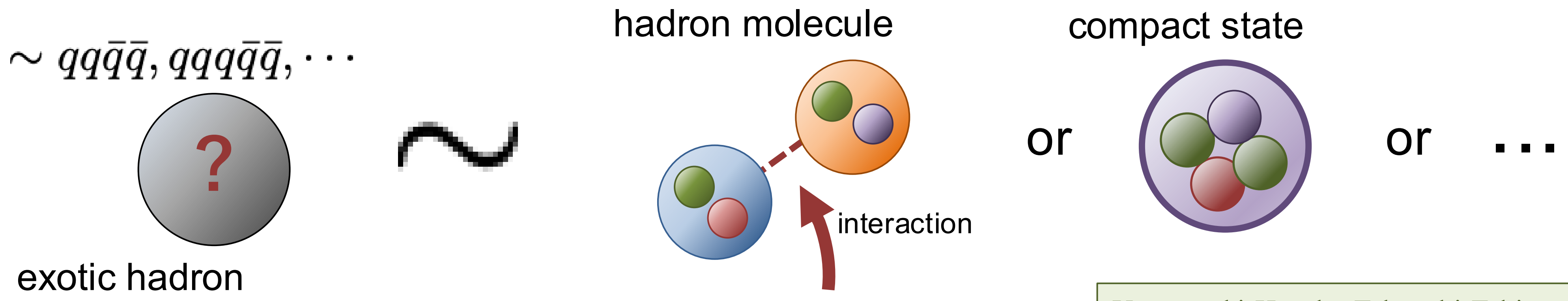
Two-pion exchange effects in DD* potential

Daiki Suenaga
(KMI/Nagoya U., Japan)

Introduction

- **Hadron-hadron interaction**

  - Understanding hadron-hadron interaction plays a key role in delineating exotic hadrons



How does it look like?

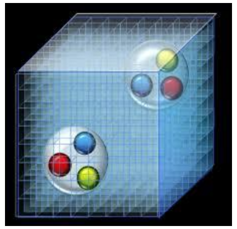
Yamaguchi-Hosaka-Takeuchi-Takizawa, J.Phys.G (2020)
Chen-Chen-Liu-Liu-Zhu, Rept. Prog. Phys. (2023)

  - Particularly interactions with heavy flavors are important

    → observation of (possibly) $T_{cc}, T_{c\bar{c}}, P_c. \cdots$ at KEK, LHC, BESIII, etc.

  - Lattice QCD is also powerful tool to examine hadron-hadron interaction in exotic hadrons

    → eg, HAL QCD collaboration, HadSpec collaboration, etc.



Introduction

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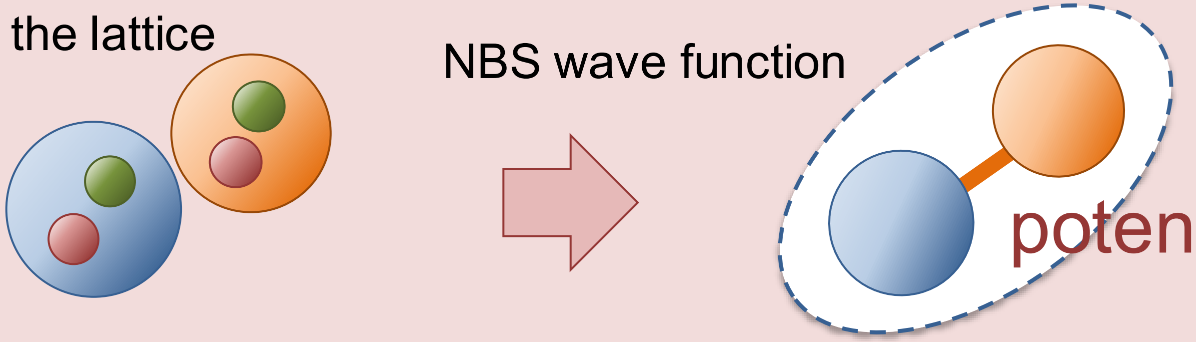
  - Understanding hadron-hadron interaction plays a key role in delineating exotic hadrons

hadron molecule compact state

correlator on the lattice

NBS wave function



(HAL: Hadrons to Atomic nuclei from Lattice QCD)

ys.G (2020)
023)

  - Particul

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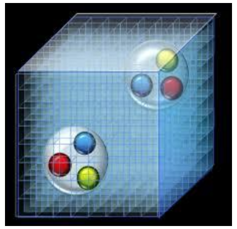
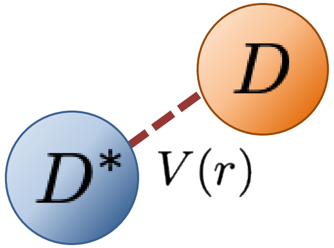


Introduction

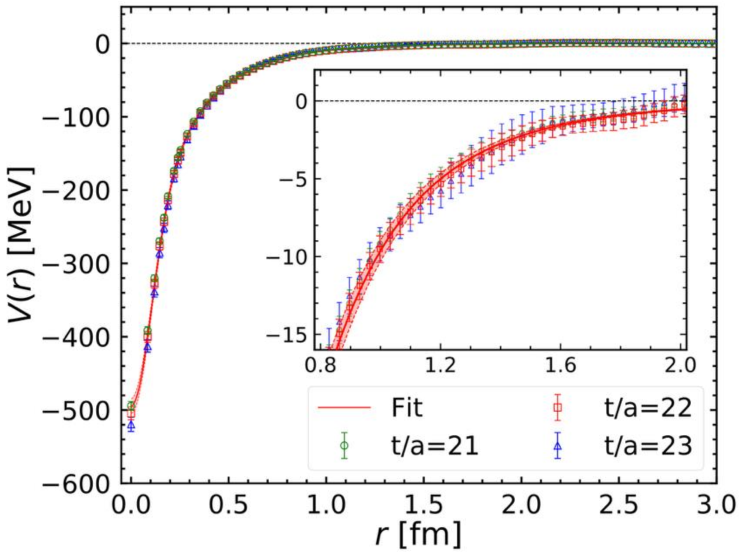
- **Two-pion exchange (TPE) potential found by HAL**

nearly physical: $m_\pi = 146.4\,\mathrm{MeV}$



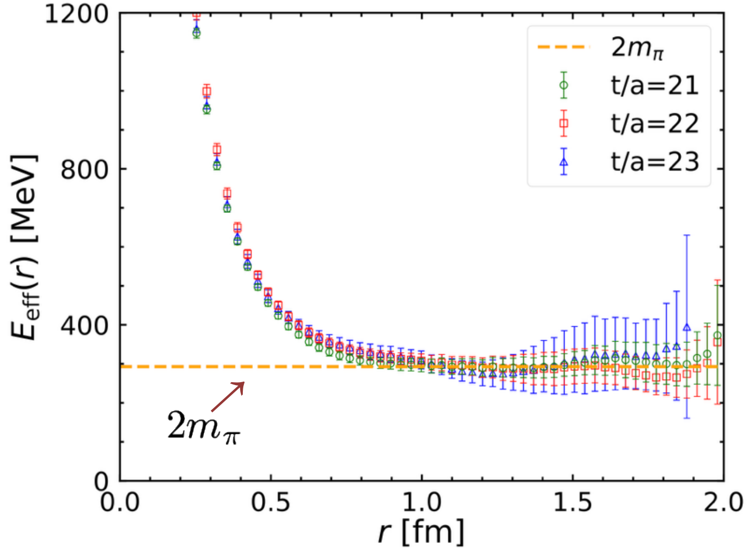
Lyu et al, PRL131, 161901 (2023)

DD^* potential



$(\alpha = -0.0011\ \mathrm{MeV}^{-1})$

$$E_{\mathrm{eff}}(r) = -\ln[V(r)r^2/\alpha]/r$$



If $V(r) \sim \alpha \left(\frac{e^{-m_\pi r}}{r}\right)^2$, then $E_{\mathrm{eff}}(r) \sim 2m_\pi$

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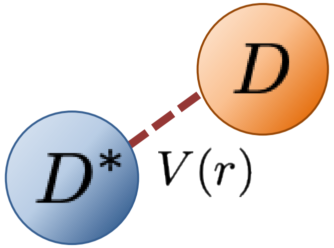
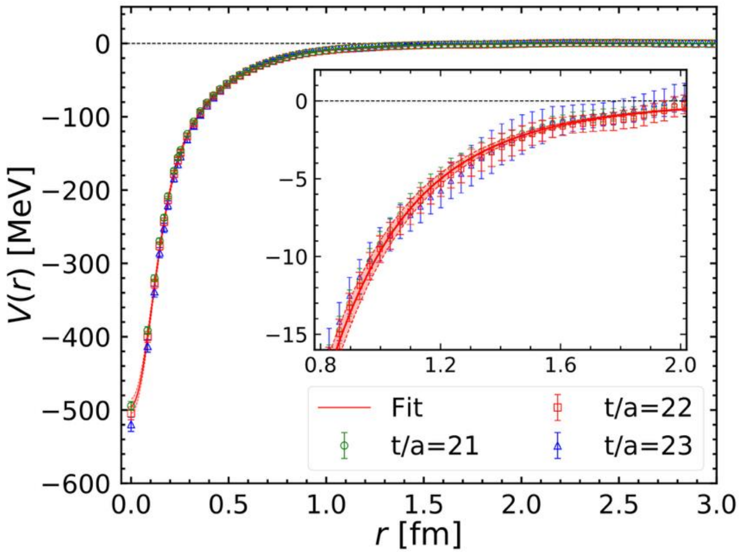


DD^* potential



$(\alpha = -0.0011\ \mathrm{MeV}^{-1})$

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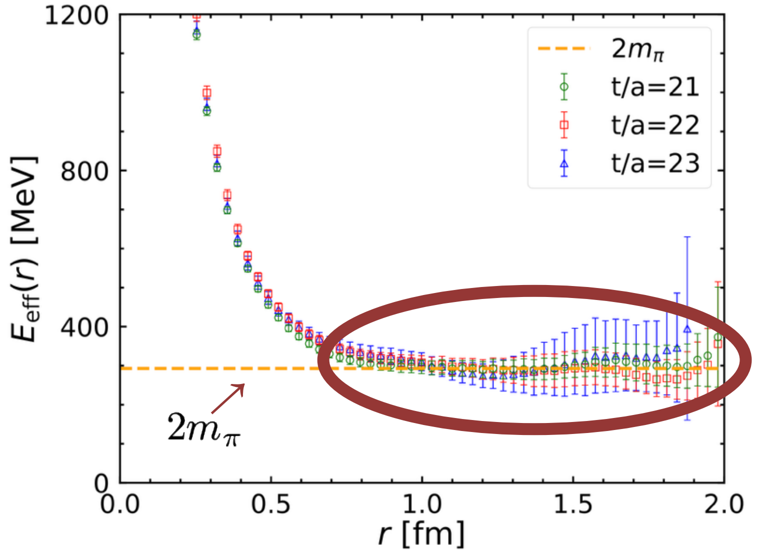

this is the case!

Lyu et al, PRL131, 161901 (2023)

If $V(r) \sim \alpha \left(\frac{\mathrm{e}^{-m_\pi r}}{r}\right)^2$, then $E_{\mathrm{eff}}(r) \sim 2m_\pi$

$\Rightarrow$ DD^* potential has a $V(r) \sim \alpha \left(\frac{\mathrm{e}^{-m_\pi r}}{r}\right)^2$ tail for $1\,\mathrm{fm} \lesssim r \lesssim 2\,\mathrm{fm}$

- Similar TPE tail is also found in $N-\phi$ $(\phi = \bar{s}s, \bar{c}c, \cdots)$ systems by HAL

Lyu et al, PRD106, 074507 (2022)
Lyu et al, PLB 860, 139178 (2025)

Introduction

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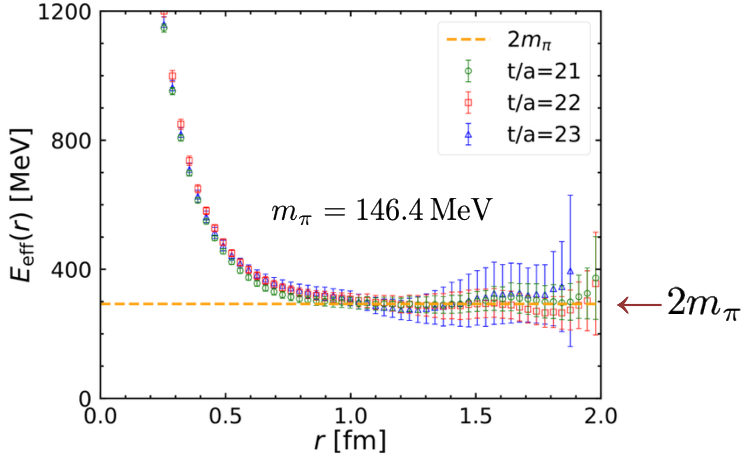
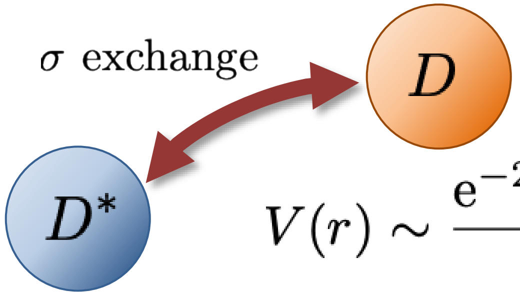
$$E_{\rm eff}(r) = -\ln[V(r)r^2/\alpha]/r$$



<u>The TPE tail</u>

$$V(r) \sim \alpha \left(\frac{\mathrm{e}^{-m_\pi r}}{r} \right)^2 \neq$$



$$V(r) \sim \frac{\mathrm{e}^{-2m_\pi r}}{r}$$

$$(m_\sigma \sim 2m_\pi)$$

- Naively speaking, the TPE tail $V(r) \sim \alpha \left(\frac{\mathrm{e}^{-m_\pi r}}{r} \right)^2$ cannot be understood as a simple σ exchange

Q: What is the origin of this TPE tail?

→ I'll study within the heavy-meson chiral perturbation theory (HMChPT)

Model

- **Heavy-meson chiral perturbation theory (HMChPT)**
  - The leading-order HMChPT Lagrangian with HQSS reads

$$\mathcal{L}_{\mathrm{HMChPT}} = \mathcal{L}_\pi + \mathcal{L}_{\mathcal{O}(p^1)}$$

with
$$\begin{cases} \mathcal{L}_\pi = f_\pi^2 \left(\mathrm{tr}[\alpha_{\perp\mu}\alpha_\perp^\mu]_f + \mathrm{tr}[\hat{\chi} + \hat{\chi}^\dagger]_f \right) \\ \mathcal{L}_{\mathcal{O}(p^1)} = -\mathrm{tr}[H(iv\cdot D)\bar{H}]_D + g\,\mathrm{tr}_D[H\gamma_5\gamma^\mu \alpha_{\perp,\mu}\bar{H}]_D \end{cases}$$

$$D_\mu H = (\partial_\mu - i\alpha_{\parallel\mu})H$$

Maurer–Cartan 1 form

$$\begin{cases} \alpha_\perp^\mu = \dfrac{1}{2i}(\partial^\mu \xi_R \xi_R^\dagger - \partial^\mu \xi_L \xi_L^\dagger) \ \in \mathcal{G} - \mathcal{H} \\ \alpha_\parallel^\mu = \dfrac{1}{2i}(\partial^\mu \xi_R \xi_R^\dagger + \partial^\mu \xi_L \xi_L^\dagger) \ \in \mathcal{H} \end{cases}$$

with $\xi_R = \xi_L^\dagger = \exp(i\pi^a\tau^a/2f_\pi)$

Spurion: $\hat{\chi} = \xi_L^\dagger \chi \xi_R \ \ (\chi \to m_\pi^2/4)$

HQS-doublet: $H = \dfrac{1+\not{v}}{2}(i\gamma_5 D_v + \not{D}_v^*)$

<u>chiral transformation law</u>

$$\begin{cases} \alpha_\perp^\mu \to h\alpha_\perp^\mu h^\dagger \\ \alpha_\parallel^\mu \to h\alpha_\parallel^\mu h^\dagger - i\partial^\mu h h^\dagger \end{cases}$$

$\hat{\chi} \to h\hat{\chi}h^\dagger \quad H \to Hh^\dagger$

Model

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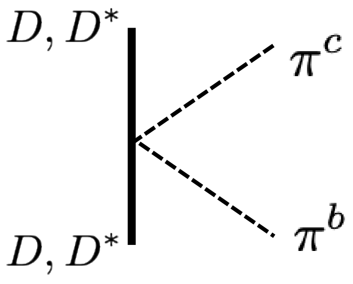
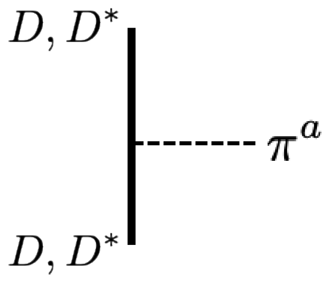
  - $\mathcal{L}_{\mathcal{O}(p^1)}$ generate two types of vertices



Weinberg-Tomozawa type



g_A coupling type

$$D_{\mu}H = (\partial_{\mu} - i\alpha_{\parallel\mu})H$$

Maurer–Cartan 1 form

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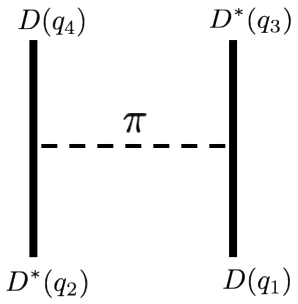
Model

- $DD^* \, (^3S_1; I = 0)$ potential at LO

LO:



→ only a one-pion exchange (OPE) contributes

- The wave function is $|DD^*(^3S_1)\rangle = \frac{1}{\sqrt{2}}\Big(|DD^*\rangle_{I=0} - |D^*D\rangle_{I=0}\Big)$ (bosonic system is symmetric in total)

$$\langle DD^*(^3S_1)|\hat{V}|DD^*(^3S_1)\rangle = -\frac{1}{2}\Big({}_{I=0}\langle DD^*|\hat{V}|D^*D\rangle_{I=0} + {}_{I=0}\langle D^*D|\hat{V}|DD^*\rangle_{I=0}\Big)$$

extra minus sign appears

- The potential with $\delta^3(\vec{r})$ term subtracted reads

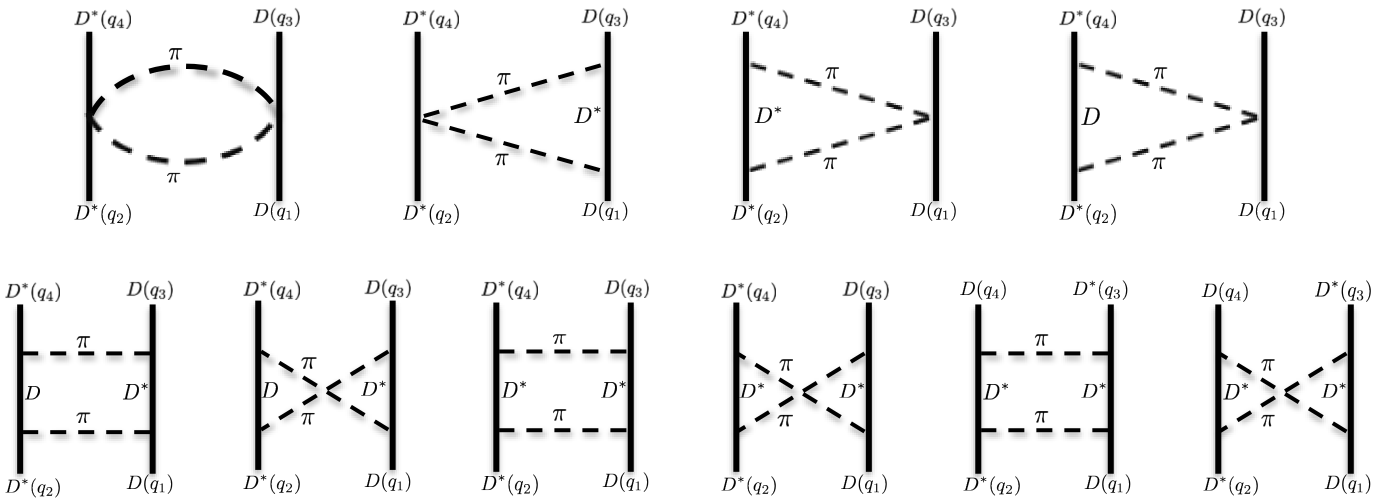
$$V_{\mathrm{LO}}^{1\pi} = -\mathcal{C}\,\frac{g^2 m_\pi^2}{48\pi f_\pi^2}\,\frac{\mathrm{e}^{-m_\pi r}}{r} \quad \text{with} \quad \mathcal{C} \equiv \vec{\tau}_f \cdot \vec{\tau}_f = -3 \ (I = 0)$$

$\boxed{\text{OPE for } DD^* \, (^3S_1, I = 0) \text{ is repulsive}}$

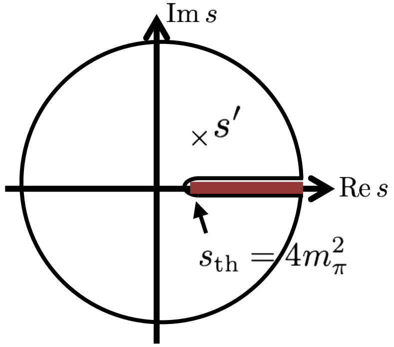
Model

- $DD^* \; ({}^3S_1; I=0)$ potential at NLO

NLO:



Kaiser-Brockmann-Weise, NPA625, 758 (1997)



$$\mathcal{M}(s) = \frac{1}{\pi}\int_{s_{\rm th}}^{\infty} ds' \frac{\mathrm{Im}\mathcal{M}(s')}{s'-s-i\epsilon}$$

- The potential is derived via the dispersion relation representation

$$V^{2\pi}(r) = -\frac{1}{4}\int \frac{d^3q}{(2\pi)^3}\mathcal{M}(\vec{q}^{\,2})\mathrm{e}^{-i\vec{q}\cdot\vec{r}} = -\frac{1}{4\pi}\int_{s_{\rm th}}^{\infty} ds'\mathrm{Im}\mathcal{M}(s')\int\frac{d^3q}{(2\pi)^3}\frac{\mathrm{e}^{i\vec{q}\cdot\vec{r}}}{s'+\vec{q}^{\,2}-i\epsilon} \quad (s=-\vec{q}^{\,2})$$

Model

- DD^* $({}^3S_1; I=0)$ **potential at NLO**

  - Summing up imaginary parts of all the NLO diagram reads (after S-wave projection)

$$\mathrm{Im}[\mathcal{M}_{\mathrm{NLO}}] = \frac{\mathcal{C}}{192\pi f_\pi^4}\left[-(s-4m_\pi^2)+2g^2(5s-8m_\pi^2)+g^4(23s-20m_\pi^2)\right]\sqrt{1-\frac{4m_\pi^2}{s}} + \frac{g^4}{16\pi f_\pi^4}s\sqrt{1-\frac{4m_\pi^2}{s}}$$

$$\begin{aligned} V_{\mathrm{NLO}}^{2\pi} &= \frac{\mathcal{C}m_\pi}{128\pi^3 f_\pi^4 r^4}\left[\left(1-10g^2-23g^4\right)m_\pi r K_0(2m_\pi r)+\left(1-10g^2-23g^4-4\left(g^2+3g^4\right)m_\pi^2r^2\right)K_1(2m_\pi r)\right] \\ &\quad -\frac{m_\pi}{128\pi^3 f_\pi^4 r^4}\left[12g^4 m_\pi r K_0(2m_\pi r)+\left(12g^4+8g^4m_\pi^2r^2\right)K_1(2m_\pi r)\right] \\ &\to \frac{g^2(3+7g^2)m_\pi^{5/2}}{64\pi^{5/2}f_\pi^4}\frac{\mathrm{e}^{-2m_\pi r}}{r^{5/2}} \quad (r \gg m_\pi^{-1}) \end{aligned}$$

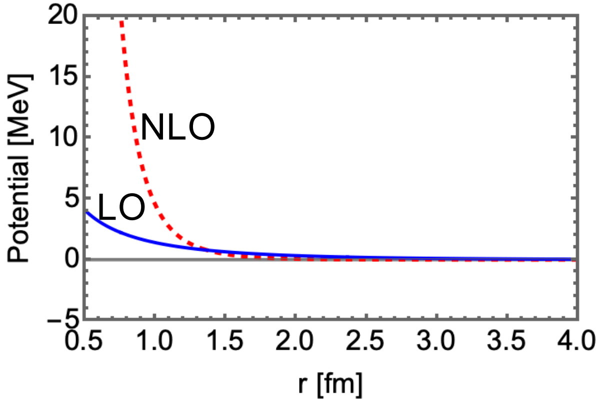
$K_n(x)$: mod. Bessel function

nearly physical point of HAL

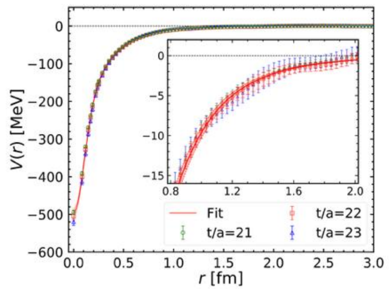
- With $g=0.55,\ f_\pi = 93\,\mathrm{MeV},\ m_\pi = 146.4\,\mathrm{MeV}$, this NLO DD^* potential is also repulsive!

  → **still contradicts HAL result!**

obtained potential shapes



HAL



attractive

Model

- $\mathcal{O}(p^2)$HMChPT Lagrangian (with HQSS)

$$
\begin{aligned}
\mathcal{L}_{\mathcal{O}(p^2)} \quad = \quad & h_1 \mathrm{tr}[H g_{\mu\nu} \alpha_\perp^\mu \alpha_\perp^\nu \bar{H}]_D + h_2 \mathrm{tr}[H v_\mu v_\nu \alpha_\perp^\mu \alpha_\perp^\nu \bar{H}]_D + h_3(\mathrm{tr}[H \gamma_\mu v_\nu \alpha_\perp^\mu \alpha_\perp^\nu \bar{H}]_D + \mathrm{tr}[H v_\mu \gamma_\nu \alpha_\perp^\mu \alpha_\perp^\nu \bar{H}]_D) \\
& + h_4 \mathrm{tr}[H(\hat{\chi}^\dagger + \hat{\chi})\bar{H}]_D + i h_5 \mathrm{tr}[H \sigma_{\mu\nu} \alpha_\perp^\mu \alpha_\perp^\nu \bar{H}]_D + k_1 \mathrm{tr}[H g_{\mu\nu} \overleftarrow{D}^\mu D^\nu \bar{H}]_D + k_2 \mathrm{tr}[H v_\mu v_\nu \overleftarrow{D}^\mu D^\nu \bar{H}]_D \\
& + k_3(\mathrm{tr}[H \gamma_\mu v_\nu \overleftarrow{D}^\mu D^\nu \bar{H}]_D + \mathrm{tr}[H v_\nu \gamma_\mu \overleftarrow{D}^\nu D^\mu \bar{H}]_D) \\
& + c(\mathrm{tr}[H \sigma_{\mu\nu} \gamma_5 \alpha_\perp^\mu D^\nu \bar{H}]_D - \mathrm{tr}[H \overleftarrow{D}^\nu \alpha_\perp^\mu \sigma_{\mu\nu} \gamma_5 \bar{H}]_D)
\end{aligned}
$$

$D_\mu H = (\partial_\mu - i\alpha_{\parallel\mu})H$

- We only consider $h_1 - h_5$ terms hereafter

- $k_1 - k_4$ terms have derivatives acting on external D, D^*

  $\rightarrow$ irrelevant to DD^* potential

Maurer–Cartan 1 form

$$
\begin{cases}
\alpha_\perp^\mu = \dfrac{1}{2i}(\partial^\mu \xi_R \xi_R^\dagger - \partial^\mu \xi_L \xi_L^\dagger) & \in \mathcal{G} - \mathcal{H} \\
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\end{cases}
$$

with $\xi_R = \xi_L^\dagger = \exp(i\pi^a \tau^a / 2f_\pi)$

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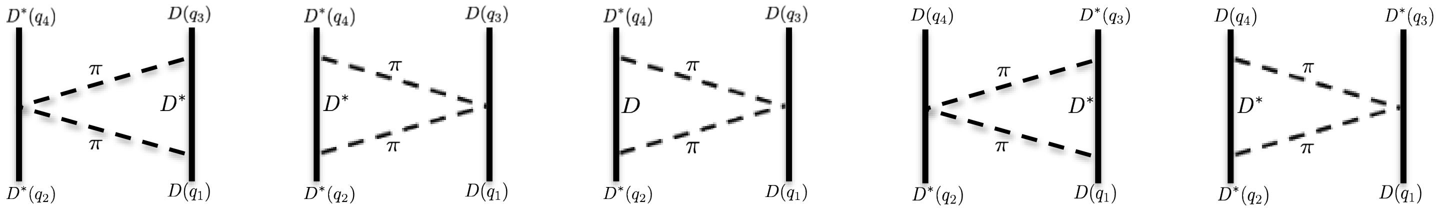
Model

- $DD^* \; (^3S_1; I = 0)$ potential at N^{2}LO

N^{2}LO:



$$\mathrm{Im}[\mathcal{M}^{2\pi}_{\mathrm{N^2LO}}] = \frac{3g^2}{64 f_\pi^4}\frac{h_1(s-2m_\pi^2)^2 + 2h_4 m_\pi^2 (s-2m_\pi^2)}{\sqrt{s}} + \frac{\mathcal{C}h_5 g^2}{96 f_\pi^4}\frac{s(s-4m_\pi^2)}{\sqrt{s}}$$

$$V^{2\pi}_{\mathrm{N^2LO}} = -\frac{g^2}{128\pi^2 f_\pi^4 r^6}\mathrm{e}^{-2m_\pi r}\left[3h_1(6+12m_\pi r+10m_\pi^2 r^2+4m_\pi^3 r^3+m_\pi^4 r^4)+3h_4 m_\pi^2 r^2(1+m_\pi r)^2\right]$$
$$-\frac{\mathcal{C}g^2 h_5}{96\pi^2 f_\pi^4 r^6}\mathrm{e}^{-2m_\pi r}\left(3+6m_\pi r+5m_\pi^2 r^2+2m_\pi^3 r^3\right)$$
$$\rightarrow \underline{-\frac{3g^2(h_1+h_4)m_\pi^4}{128\pi^2 f_\pi^4}\frac{\mathrm{e}^{-2m_\pi r}}{r^2}} \quad (r \gg m_\pi^{-1}) \;\; !$$

cf. $V^{2\pi}_{\mathrm{NLO}} \rightarrow \dfrac{g^2(3+7g^2)m_\pi^{5/2}}{64\pi^{5/2} f_\pi^4}\dfrac{\mathrm{e}^{-2m_\pi r}}{r^{5/2}} \quad (r \gg m_\pi^{-1})$

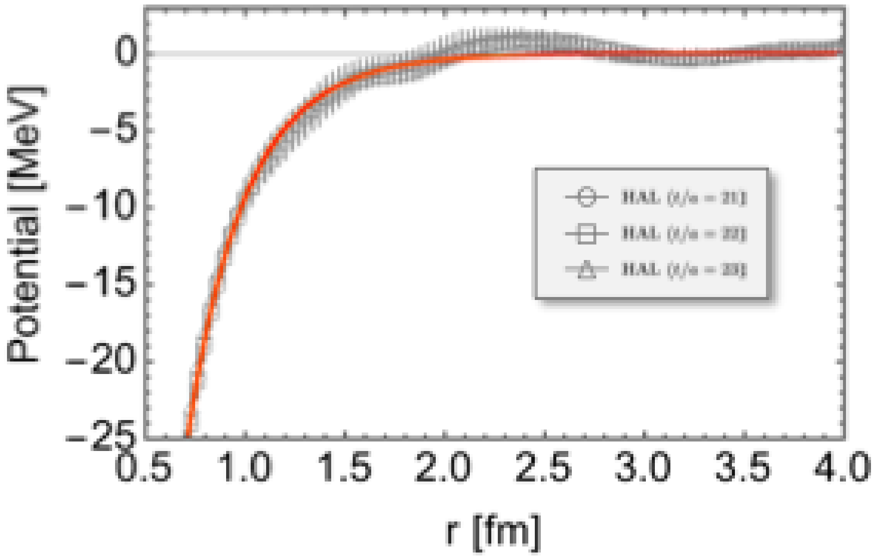
- The TPE tail of $V \sim \alpha\left(\mathrm{e}^{-2m_\pi r}/r^2\right)$ is derived at N^{2}LO

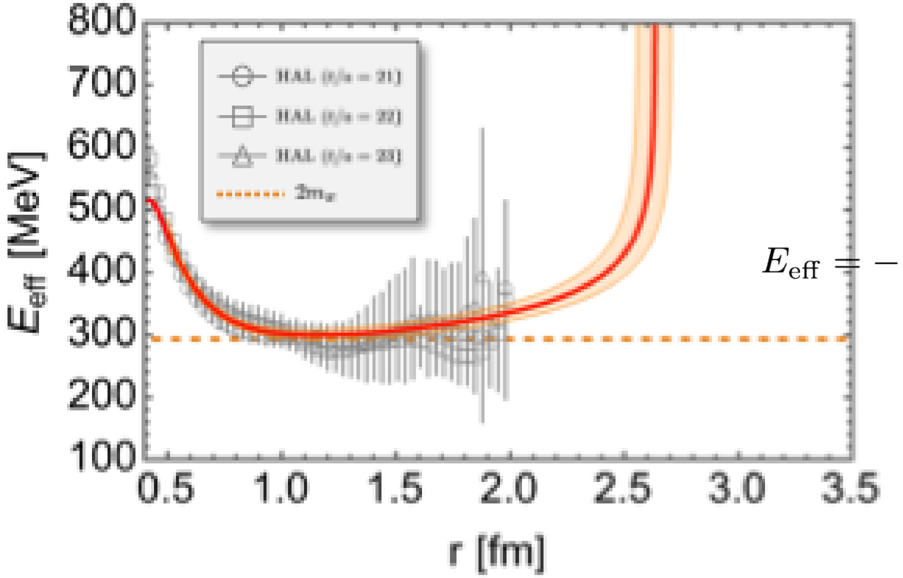
Numerical results

- The HAL QCD result is fitted very well with

f_π [MeV]	m_π [MeV]	g	h_1 [MeV^{-1}]	h_4 [MeV^{-1}]	h_5 [MeV^{-1}]
93	146.4	0.55	0.328	0.005	-0.491

Fit range: 0.5 fm < r < 2.0 fm
$\chi^2/(\mathrm{d.o.f}) = 0.034$





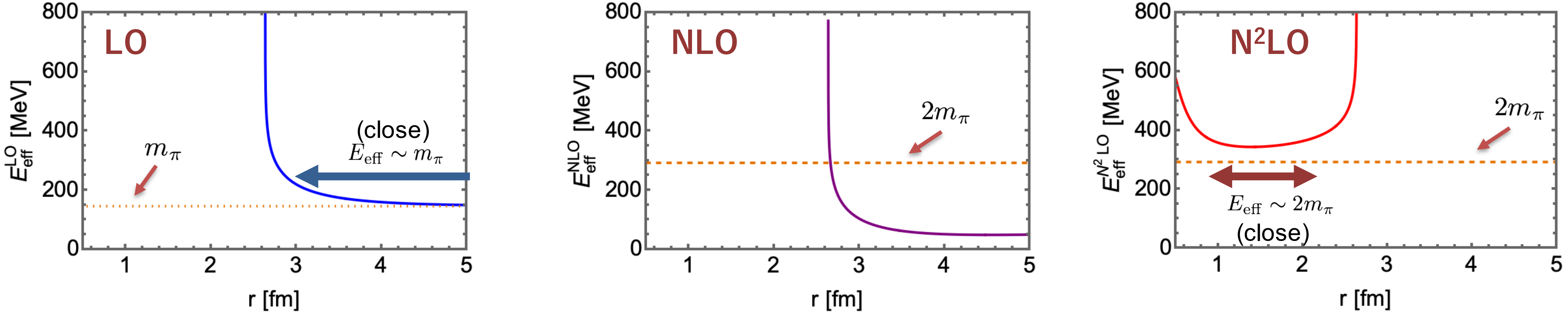
$$E_{\rm eff} = -\frac{1}{r}\ln\left(\frac{V_{DD^*}r^2}{\alpha}\right)$$

- HAL result at semi-long range is reproduced well, but with very large values of h_1, h_4, h_5

Numerical results

- Examination of the "dominance" of potential

$$E_{\rm eff} = -\frac{1}{r}\ln\left(Vr/\alpha^{1\pi}_{\rm LO}\right) \qquad E_{\rm eff} = -\frac{1}{r}\ln\left(Vr^{5/2}/\alpha^{2\pi}_{\rm NLO}\right) \qquad E_{\rm eff} = -\frac{1}{r}\ln\left(Vr^{2}/\alpha^{2\pi}_{\rm N^2LO}\right)$$



$$V^{1\pi}_{\rm LO} = \frac{g^2 m_\pi^2}{16\pi f_\pi^2}\frac{{\rm e}^{-m_\pi r}}{r} \equiv \alpha^{1\pi}_{\rm LO}\frac{{\rm e}^{-m_\pi r}}{r}$$

$$V^{2\pi}_{\rm NLO} \sim \frac{g^2(3+7g^2)m_\pi^{5/2}}{64\pi^{5/2} f_\pi^4}\frac{{\rm e}^{-2m_\pi r}}{r^{5/2}} \equiv \alpha^{2\pi}_{\rm NLO}\frac{{\rm e}^{-2m_\pi r}}{r^{5/2}}$$

$$V^{2\pi}_{\rm N^2LO} \sim -\frac{3g^2(h_1+h_4)m_\pi^4}{128\pi^2 f_\pi^4}\frac{{\rm e}^{-2m_\pi r}}{r^2} \equiv \alpha^{2\pi}_{\rm N^2LO}\frac{{\rm e}^{-2m_\pi r}}{r^2}$$

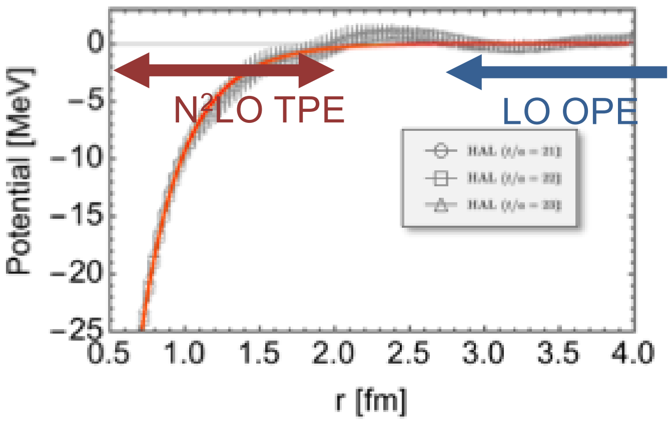
- LO OPE dominance for $3.0\,{\rm fm} \lesssim r$
- No NLO TPE dominance regime
- N²LO TPE dominance for $0.5\,{\rm fm} \lesssim r \lesssim 2.0\,{\rm fm}$

Short Summary

- HAL's TPE tail is reproduced within a simple HMChPT description with a few parameters, but very large $\mathcal{O}(p^2)$ couplings h_1, h_4, h_5 are necessary

$$h_1, h_4, h_5 \sim \mathcal{O}(10^{-2} - 10^{-1})\,\mathrm{MeV}^{-1} = \mathcal{O}(10^0 - 10^1)/100\,\mathrm{MeV}$$

cf, $\mathcal{O}(p^1)$ coupling is $g = 0.55$



- Resonance saturation model predicts $h_1, h_4, h_5 \sim \mathcal{O}(10^{-4} - 10^{-3})\,\mathrm{MeV}^{-1} = \mathcal{O}(10^{-2} - 10^{-1})/100\,\mathrm{MeV}$

  → N^{2}LO TPE is suppressed and the HAL's TPE tail is understood as just an *effective behavior*

  Wang-Meng, PRD(2023)

isospin independent

- We need more information on $\mathcal{O}(p^2)$ couplings: h_1, h_4, h_5 isospin dependent

$$V^{2\pi}_{\mathrm{N^2LO}} \sim -\frac{3g^2(h_1+h_4)m_\pi^4}{128\pi^2 f_\pi^4}\frac{\mathrm{e}^{-2m_\pi r}}{r^2}$$ is isospin independent

Q: How about other channels with $I = 1$?

Other channels

- **Potential for other channels**

  - When considering S-wave for $D^{(*)}D^{(*)}$ system, the followings are possible

$$\langle DD(^1S_0; I=1)|\hat{V}|DD(^1S_0; I=1)\rangle$$

$$\langle D^*D^*(^1S_0; I=1)|\hat{V}|D^*D^*(^1S_0; I=1)\rangle$$

$$\langle D^*D^*(^3S_1; I=0)|\hat{V}|D^*D^*(^3S_1; I=0)\rangle$$

$$\langle D^*D^*(^5S_2; I=1)|\hat{V}|D^*D^*(^5S_2; I=1)\rangle$$

$$\langle DD^*(^3S_1; I=0)|\hat{V}|DD^*(^3S_1; I=0)\rangle$$ ← I already showed (relevant to the observed Tcc)

$$\langle DD^*(^3S_1; I=1)|\hat{V}|DD^*(^3S_1; I=1)\rangle$$ ← DD* system with I=1

+ coupled channel (off-diagonal)

$$\langle D^*D^*(^3S_1; I=0)|\hat{V}|DD^*(^3S_1; I=0)\rangle$$

$$\langle DD(^1S_0; I=1)|\hat{V}|D^*D^*(^1S_0; I=1)\rangle$$

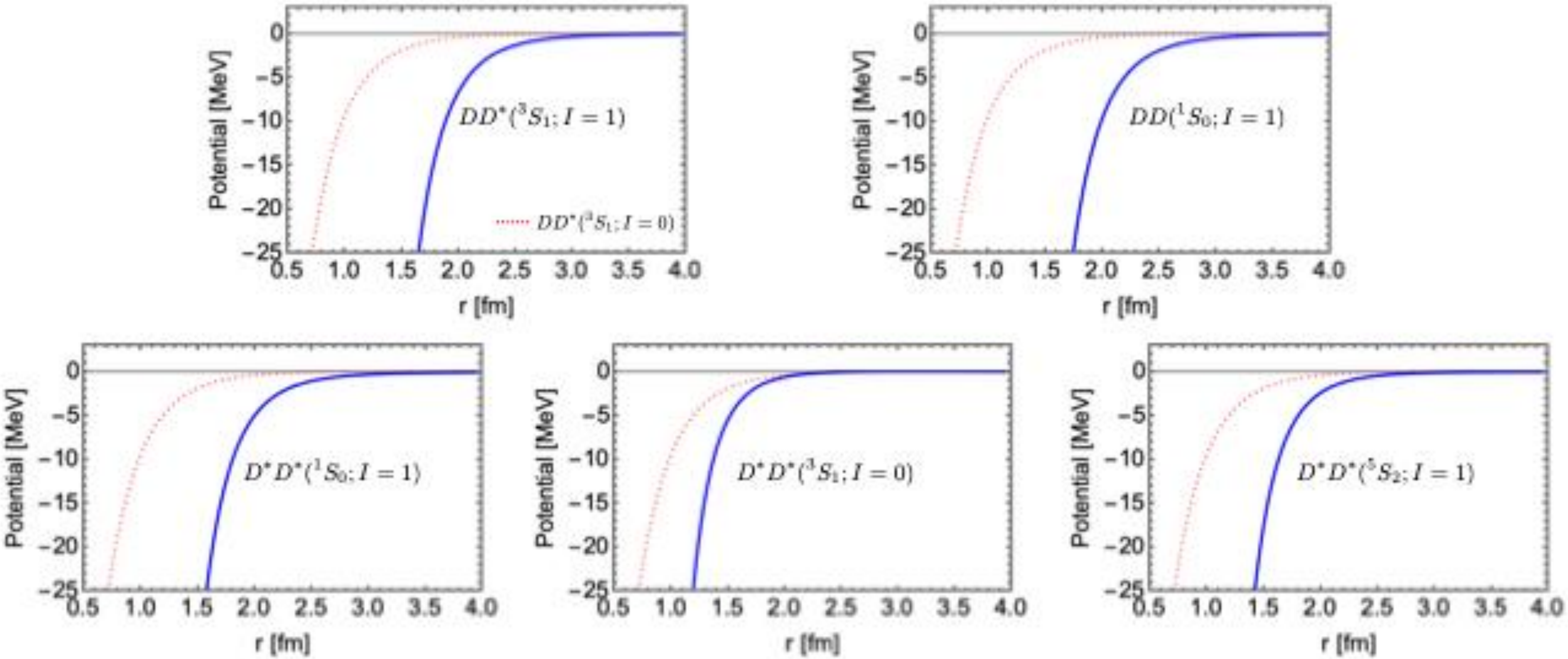
$D^{(*)}$ can be replaced by $\bar{B}^{(*)}$

(HQSS is more accurate)

Other channels

- **Potential for other channels (LO + NLO + N²LO)**

f_π [MeV]	m_π [MeV]	g	h_1 [MeV^{-1}]	h_4 [MeV^{-1}]	h_5 [MeV^{-1}]
93	146.4	0.55	0.328	0.005	-0.491



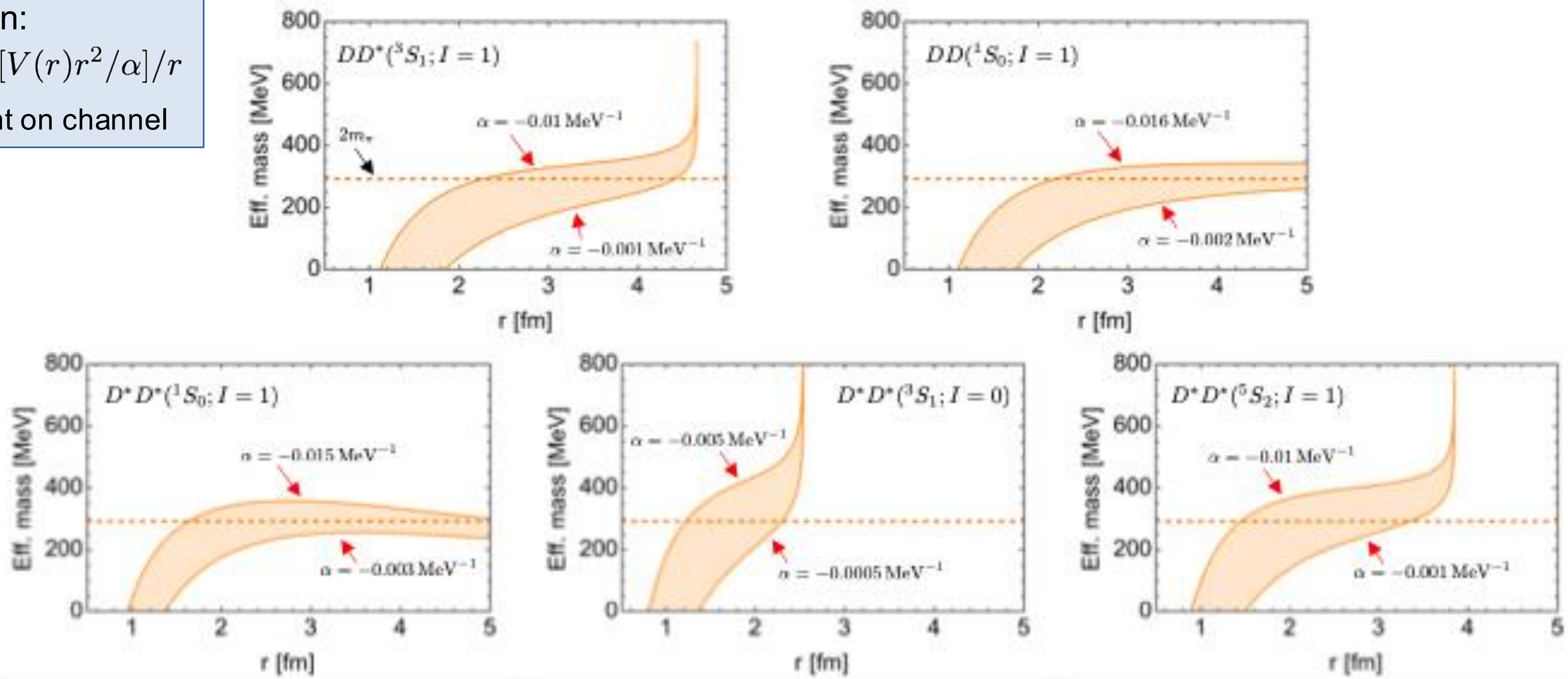
- The other channels have stronger attractive potential within the present simple description

Other channels

- $\mathbf{E_{eff}}$ **for other channels (LO + NLO + N²LO)**

f_π [MeV]	m_π [MeV]	g	h_1 [MeV^{-1}]	h_4 [MeV^{-1}]	h_5 [MeV^{-1}]
93	146.4	0.55	0.328	0.005	−0.491

Definition:
$$E_{\rm eff}(r) = -\ln[V(r)r^2/\alpha]/r$$
α is dependent on channel



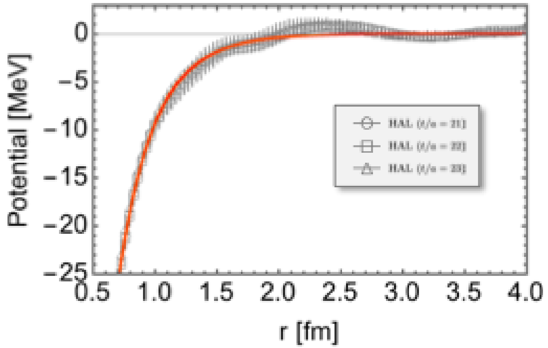
- Clear $E_{\rm eff} \sim 2m_\pi$ is observed in $DD(^1S_0; I=1)$ and $D^*D^*(^1S_0; I=1)$

Conclusions

- I investigated two-pion exchange effects in DD* system motivated by HAL QCD simulation

Hadrons to Atomic nuclei HAL from Lattice QCD

This talk: TPE tail of $V(r) \sim \alpha \left(\mathrm{e}^{-m_\pi r}/r\right)^2$ is derived at N^{2}LO diagram of HMChPT,
with very large $\mathcal{O}(p^2)$ couplings (higher-order couplings) needed



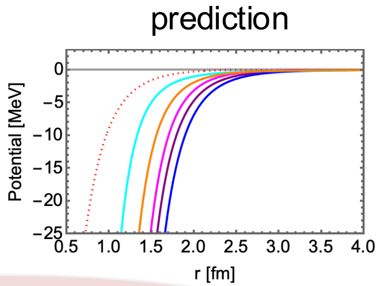
Distinct two scenarios

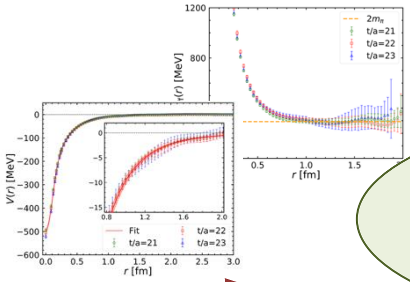
Previous work: N^{2}LO TPE is suppressed and the HAL's TPE tail is just an *effective behavior*

Wang-Meng, PRD(2023)

- I also presented predictions for other channels
- We need more HAL results with other channels to pin down this issue!

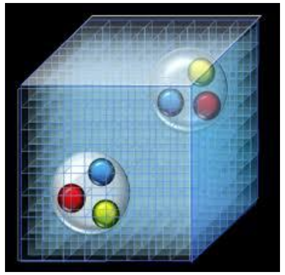
Conclusions





numerical experiment

lattice QCD

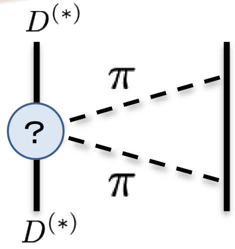


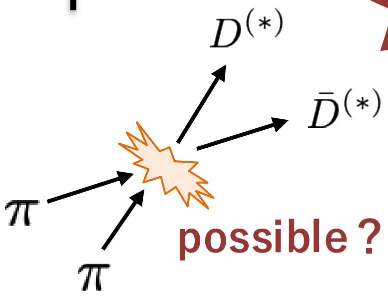
effective/QCD theory

ChPT, pQCD, etc.

$\pi\pi D^{(*)}\bar{D}^{(*)}$ coupling

This is what we want to know

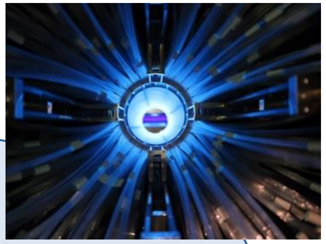




possible ?

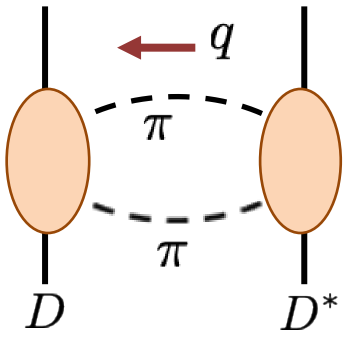
hadron experiment

J-PARC, Belle II, BES III, J-Lab, etc.



Back up

TPE calculation



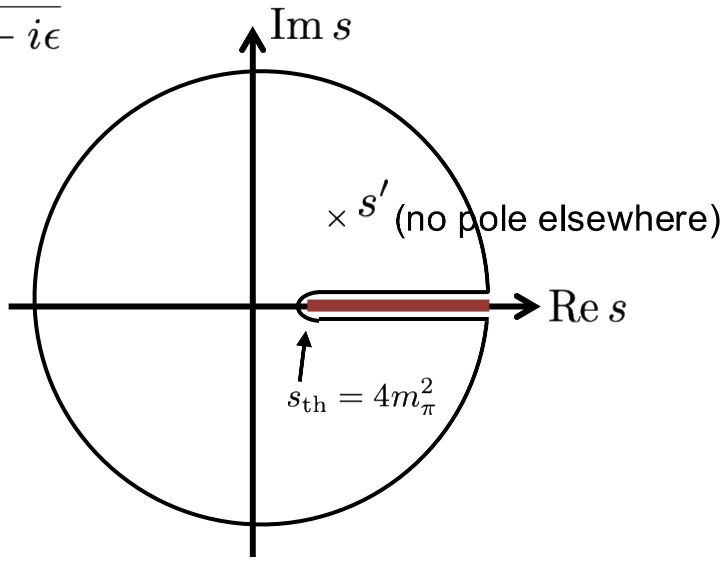
- Dispersion relation method
  - Potential $V(r)$ is derived from Fourier transformation of amplitude $\mathcal{M}(\boldsymbol{q}^2)$ (in NR limit)

    $\rightarrow$ not easy to see its analytic expression

  - Dispersion relation (DR) is useful $(s \equiv q^2 \text{ with } q_0 = 0)$

$$\mathcal{M}(s) = \frac{1}{2\pi i}\int_C ds' \frac{\mathcal{M}(s')}{s'-s} = \frac{1}{2\pi i}\int_{s_{\rm th}}^{\infty} ds' \frac{\mathcal{M}(s'+i\epsilon)-\mathcal{M}(s'-i\epsilon)}{s'-s} = \frac{1}{\pi}\int_{s_{\rm th}}^{\infty} ds' \frac{\mathrm{Im}\mathcal{M}(s')}{s'-s-i\epsilon}$$

when sufficiently suppressed at $|s'| \rightarrow \infty$



simple Yukawa F.T.

then

$$V(r) = \int \frac{d^3q}{(2\pi)^3}\mathcal{M}(-\vec{q}^{\,2})\mathrm{e}^{-i\vec{q}\cdot\vec{r}} = \frac{1}{\pi}\int_{s_{\rm th}}^{\infty} ds' \mathrm{Im}\mathcal{M}(s') \int \frac{d^3q}{(2\pi)^3}\frac{\mathrm{e}^{i\vec{q}\cdot\vec{r}}}{s'+\vec{q}^{\,2}-i\epsilon} \quad (s = -\vec{q}^{\,2})$$

$$= \frac{1}{4\pi^2 r}\int_{s_{\rm th}}^{\infty} ds' \mathrm{e}^{-\sqrt{s'}r}\mathrm{Im}\mathcal{M}(s')$$

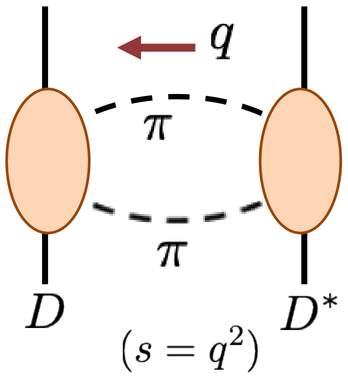
looks handleable!

TPE calculation

- Renormalization

  - The integrand may not go zero fast: $\mathcal{M}(s) = \frac{1}{2\pi i}\int_C ds' \frac{\mathcal{M}(s')}{s'-s} \overset{\times}{=} \frac{1}{2\pi i}\int_{s_{\rm th}}^{\infty} ds' \frac{\mathcal{M}(s'+i\epsilon)-\mathcal{M}(s'-i\epsilon)}{s'-s}$

doesn't hold



  - When $\mathcal{M}(s') \sim s'^0$ $\rightarrow$ logarithmic divergences

once-subtracted DR: $\mathcal{M}(s) = C_1 + \frac{s-\bar{s}}{\pi}\int_{s_{\rm th}}^{\infty} ds' \frac{{\rm Im}\mathcal{M}(s')}{(s'-\bar{s})(s'-s-i\epsilon)}$

$$= C_1 - \frac{1}{\pi}\int_{s_{\rm th}}^{\infty} ds' \frac{{\rm Im}\mathcal{M}(s')}{s'-\bar{s}} + \frac{1}{\pi}\int_{s_{\rm th}}^{\infty} ds' \frac{{\rm Im}\mathcal{M}(s')}{s'-s-i\epsilon}$$

Renormalization condition:
$\mathcal{M}(\bar{s}) = C_1$

  - When $\mathcal{M}(s') \sim s'^1$ $\rightarrow$ quadratic divergences

Renormalization condition:
$\mathcal{M}(\bar{s}) = C_1 \quad \mathcal{M}'(\bar{s}) = C_2$

twice-subtracted DR: $\mathcal{M}(s) = C_1 + C_2(s-\bar{s}) + \frac{(s-\bar{s})^2}{\pi}\int_{s_{\rm th}}^{\infty} ds' \frac{{\rm Im}\mathcal{M}(s')}{(s'-\bar{s})^2(s'-s-i\epsilon)}$

$$= C_1 - \frac{1}{\pi}\int_{s_{\rm th}}^{\infty} ds' \frac{{\rm Im}\mathcal{M}(s')}{s'-\bar{s}} + C_2(s-\bar{s}) - \frac{s-\bar{s}}{\pi}\int_{s_{\rm th}}^{\infty} ds' \frac{{\rm Im}\mathcal{M}(s')}{(s'-\bar{s})^2} + \frac{1}{\pi}\int_{s_{\rm th}}^{\infty} ds' \frac{{\rm Im}\mathcal{M}(s')}{s'-s-i\epsilon}$$

TPE calculation

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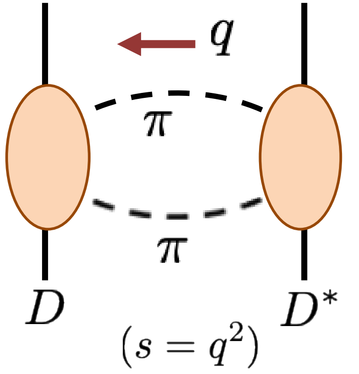


  - When $\mathcal{M}(s') \sim s'^0$ $\rightarrow$ logarithmic divergences

once-subtracted DR: $\mathcal{M}(s) = C_1 + \frac{s-\bar{s}}{\pi}\int_{s_{\rm th}}^{\infty} ds' \frac{{\rm Im}\mathcal{M}(s')}{(s'-\bar{s})(s'-s-i\epsilon)}$

$$= \underbrace{C_1 - \frac{1}{\pi}\int_{s_{\rm th}}^{\infty} ds' \frac{{\rm Im}\mathcal{M}(s')}{s'-\bar{s}}}_{\delta^3(r)\text{ after F.T.}} + \boxed{\frac{1}{\pi}\int_{s_{\rm th}}^{\infty} ds' \frac{{\rm Im}\mathcal{M}(s')}{s'-s-i\epsilon}}$$

(ren. independent)

information on large is derived by those terms
→ **Let's focus!**

  - When $\mathcal{M}(s') \sim s'^1$ $\rightarrow$ quadratic divergences

twice-subtracted DR: $\mathcal{M}(s) = C_1 + C_2(s-\bar{s}) + \frac{(s-\bar{s})^2}{\pi}\int_{s_{\rm th}}^{\infty} ds' \frac{{\rm Im}\mathcal{M}(s')}{(s'-\bar{s})^2(s'-s-i\epsilon)}$

$$= \underbrace{C_1 - \frac{1}{\pi}\int_{s_{\rm th}}^{\infty} ds' \frac{{\rm Im}\mathcal{M}(s')}{s'-\bar{s}}}_{\delta^3(r)\text{ after F.T.}} + \underbrace{C_2(s-\bar{s}) - \frac{s-\bar{s}}{\pi}\int_{s_{\rm th}}^{\infty} ds' \frac{{\rm Im}\mathcal{M}(s')}{(s'-\bar{s})^2}}_{\vec{\nabla}^2\delta^3(r)\text{ after F.T.}} + \boxed{\frac{1}{\pi}\int_{s_{\rm th}}^{\infty} ds' \frac{{\rm Im}\mathcal{M}(s')}{s'-s-i\epsilon}}$$

TPE calculation

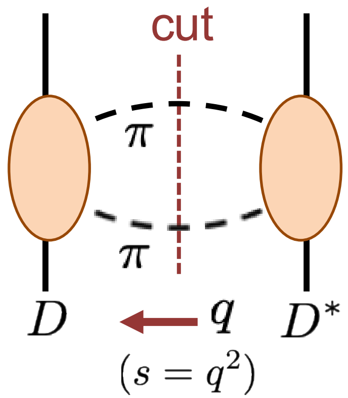
- TPE potential

$$V^{2\pi}(r) = \int \frac{d^3q}{(2\pi)^3}\mathcal{M}(\vec{q}^{\,2})e^{-i\vec{q}\cdot\vec{r}} = \frac{1}{\pi}\int_{s_{\rm th}}^{\infty} ds'\,\mathrm{Im}\mathcal{M}(s')\int\frac{d^3q}{(2\pi)^3}\frac{e^{i\vec{q}\cdot\vec{r}}}{s'+\vec{q}^{\,2}-i\epsilon} \quad (s=-\vec{q}^{\,2})$$

$$= \frac{1}{4\pi^2 r}\int_{4m_\pi^2}^{\infty} ds'\,e^{-\sqrt{s'}r}\,\mathrm{Im}\mathcal{M}(s')$$

$$V^{2\pi}(r) = \frac{1}{2\pi^2 r}\int_{2m_\pi}^{\infty} d\mu\,\mu\,e^{-\mu r}\,\mathrm{Im}\mathcal{M}(\mu^2) \quad (s'=\mu^2)$$

origin of $2m_\pi$ in the exponent



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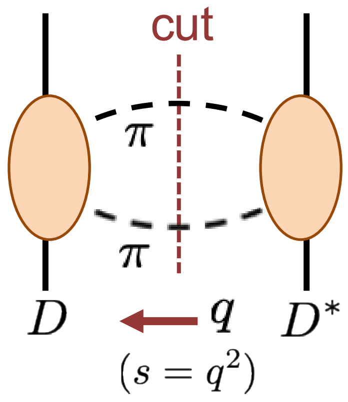
origin of $2m_\pi$ in the exponent



potential is defined at which $q_0 = 0$

NOTE

- Since $s=-\vec{q}^{\,2}<0$ $(q_0=0)$ *within the HQET formalism*, it looks always $\mathrm{Im}\mathcal{M}(s)=0$ ..

however

- There must be remnant contribution of $\mathcal{O}(\Lambda_{\mathrm{QCD}}/m_D)$ which could exceed 2π threshold

The “analytic continuation” is justified in this sense

Model

- Computation of NLO $({}^3S_1)$

  - For instance, the box diagram reads

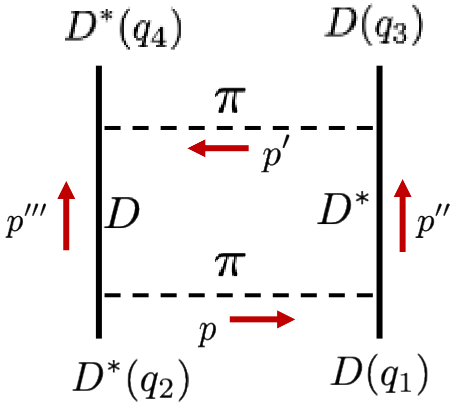
ϵ : polarization of D^*

$$\mathcal{M}_{\text{box}} = -\frac{g^4}{f_\pi^4}\mathcal{D}_1 \epsilon^{*j}\epsilon^i L^{ij}_{\text{box}}$$

with $L^{ij}_{\text{box}} \equiv i^{-1}\int \frac{d^dp}{(2\pi)^d}\frac{[p\cdot p' - (v\cdot p)(v\cdot p')]p^i p'^j}{(p^2-m_\pi^2)(p'^2-m_\pi^2)(2v\cdot p'')(2v\cdot p''')}$

$$= 6\int_0^1 dx \int_0^\infty d\lambda \int \frac{d^d l_E}{(2\pi)^d}\frac{\lambda[\vec{p}\cdot(\vec{p}+\vec{q})]p^i p'^j}{(l_E^2+\Delta')^4}$$

$$= 6\int_0^1 dx \int_0^\infty d\lambda \frac{\lambda}{(4\pi)^2}\left[-\frac{5}{24}\delta^{ij}\ln\Delta' + \left(\frac{7x^2-7x+1}{4}q^i q^j + \frac{x(1-x)s}{4}\delta^{ij}\right)\frac{1}{3\Delta'}\right] + \cdots$$



$$q \equiv q_1 - q_3 = q_4 - q_2 \quad (s = q^2)$$

Definitions

$$\mathcal{D}_1 \equiv \langle I=0|(\tau^b\tau^a)\otimes(\tau^b\tau^a)|I=0\rangle$$
$$\Delta' \equiv m_\pi^2 - x(1-x)s + \lambda^2 - i0$$

Formulas

$$\frac{1}{A^\alpha B^\beta} = \frac{\Gamma[\alpha+\beta]}{\Gamma[\alpha]\Gamma[\beta]}\int_0^1 dx \frac{x^{\alpha-1}(1-x)^{\beta-1}}{[xA+(1-x)B]^{\alpha+\beta}}$$
$$\frac{1}{A^r B^s} = 2^s\frac{\Gamma[r+s]}{\Gamma[r]\Gamma[s]}\int_0^\infty d\lambda \frac{\lambda^{s-1}}{(A+2\lambda B)^{r+s}}$$

Model

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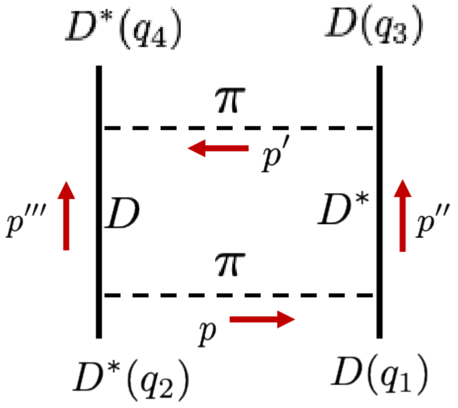
Im part

Im part

$$\mathrm{Im}[L_{\rm box}^{ij}] = \frac{6\pi}{(4\pi)^2}\int_{x_-}^{x_+} dx \left[\int_0^{\lambda_0} d\lambda \frac{5}{24}\lambda\delta^{ij} + \left(\frac{7x^2-7x+1}{24}q^i q^j + \frac{x(1-x)s}{24}\delta^{ij}\right)\right]$$

$$= \frac{1}{768\pi}\sqrt{1-\frac{4m_\pi^2}{s}}\left[(7s-16m_\pi^2)\delta^{ij} - 2(s+14m_\pi^2)\frac{q^i q^j}{s}\right]$$



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