Entanglement suppression for $K\bar{K}$ scattering

Tokyo Metropolitan University

Katsuyoshi Sone

Tetsuo Hyodo

Background

S.R. Beane, D.B. Kaplan, N. Klco, and M.J. Savage, PRL122, 102001 (2019)

I. Low, T. Mehen, PRD104, 074014 (2021);

Hadron interactions are classified by the symmetry: SU(2) or SU(3)

⇨ NN scattering: two spin channels $J = 0,1$

Scattering length in $J = 0,1$ channels : $1/|a_{J=0}| \simeq 1/|a_{J=1}|$

・Wigner-$\mathrm{SU}(4)$ **emergent symmetry** for NN scattering

E. Wigner, PR51, 106 (1937), …, D.B. Kaplan and M.J. Savage, PLB365, 244 (1996), …

・Guiding principle : **Entanglement suppression**

Imposing a condition to suppress entanglement

⇨ **Emergent symmetries**

・This work : Applying this framework to $K\bar{K}$ scattering

Exotic candidates : $f_0(980)$ $(I = 0)$, $a_0(980)$ $(I = 1)$

Entanglement entropy

Two-body hadron scattering $\Longleftrightarrow$ bipartite system

A state $|\psi\rangle \in \mathscr{H}_{12}$ has the two subspaces $\mathscr{H}_1$ and $\mathscr{H}_2$: $\mathscr{H}_{12} = \mathscr{H}_1 \otimes \mathscr{H}_2$

$|\psi\rangle$ is **not entangled** $\Longleftrightarrow$ $|\psi\rangle$ is the **product state** : $|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$

Entanglement entropy (EE) measures how entangled the state $|\psi\rangle$ is

Finite EE $\Rightarrow$ **Entangled state**

Zero EE $\Rightarrow$ **Non-entangling state**

Entanglement measure in this work

Linear entropy: $\mathscr{E}(|\psi\rangle) = 1 - \mathrm{Tr}_1 \rho_1^2$

density matrix: $\rho = |\psi\rangle\langle\psi|$

Reduced density matrix: $\rho_1 = \mathrm{Tr}_2 |\psi\rangle\langle\psi|$

Entanglement entropy

EX.) two-qubit system: Two spin-$1/2$ particles

$$|\psi\rangle = \alpha|\uparrow\uparrow\rangle + \beta|\uparrow\downarrow\rangle + \gamma|\downarrow\uparrow\rangle + \delta|\downarrow\downarrow\rangle \qquad |\uparrow\downarrow\rangle = |\uparrow\rangle \otimes |\downarrow\rangle$$

・Maximally entangled state

Bell state: $\mathscr{E}(|\psi\rangle) = 1/2$

$$|\phi^{\pm}\rangle = (|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle)/\sqrt{2}$$

$$|\psi^{\pm}\rangle = (|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle)/\sqrt{2} \quad \Leftarrow \quad J_3 = 0$$

・Non-entangling state

Product state: $|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$

$$= (u_1 \quad u_2)^T \otimes (v_1 \quad v_2)^T$$

$$= u_1 v_1|\uparrow\uparrow\rangle + u_1 v_2|\uparrow\downarrow\rangle + u_2 v_1|\downarrow\uparrow\rangle + u_2 v_2|\downarrow\downarrow\rangle$$

$$\alpha = u_1 v_1,\ \beta = u_1 v_2,\ \gamma = u_2 v_1,\ \delta = u_2 v_2$$

$$\Rightarrow \quad \mathscr{E}(|\psi\rangle) = 2|u_1 u_2 v_1 v_2 - u_1 u_2 v_1 v_2|^2 = 0$$

Unitary transformation

Changes in entanglement under unitary transformations by **quantum gates**

- **Local unitary transformation** $U_{\mathrm{tot}} = U_1 \otimes U_2$

  EE does not change : $U_{\mathrm{tot}}(|\psi_1\rangle \otimes |\psi_2\rangle) = U_1|\psi_1\rangle \otimes U_2|\psi_2\rangle$

- **Global unitary transformation** $U_{\mathrm{tot}} \neq U_1 \otimes U_2$

  EE may change : $U_{\mathrm{tot}}(|\psi_1\rangle \otimes |\psi_2\rangle) = |\phi\rangle$ $|\phi\rangle$: entangled state

Quantum gate in hadron scattering

Scattering process is described by the **S-matrix** ⇦ **Quantum gate**

S-matrix is given by the linear combination of $\sigma_i \otimes \sigma_i$

⇨ **S-matrix may change the entanglement in the system**

Entanglement power

Entanglement power (EP) for the S-matrix $\hat{S}|\psi_{\rm in}\rangle = |\psi_{\rm out}\rangle$

P. Zanardi, C. Zalka, L. Faoro, PRA62, 030301 (2000)

⇨ Quantifying the **ability of $\hat{S}$ to generate the entanglement**

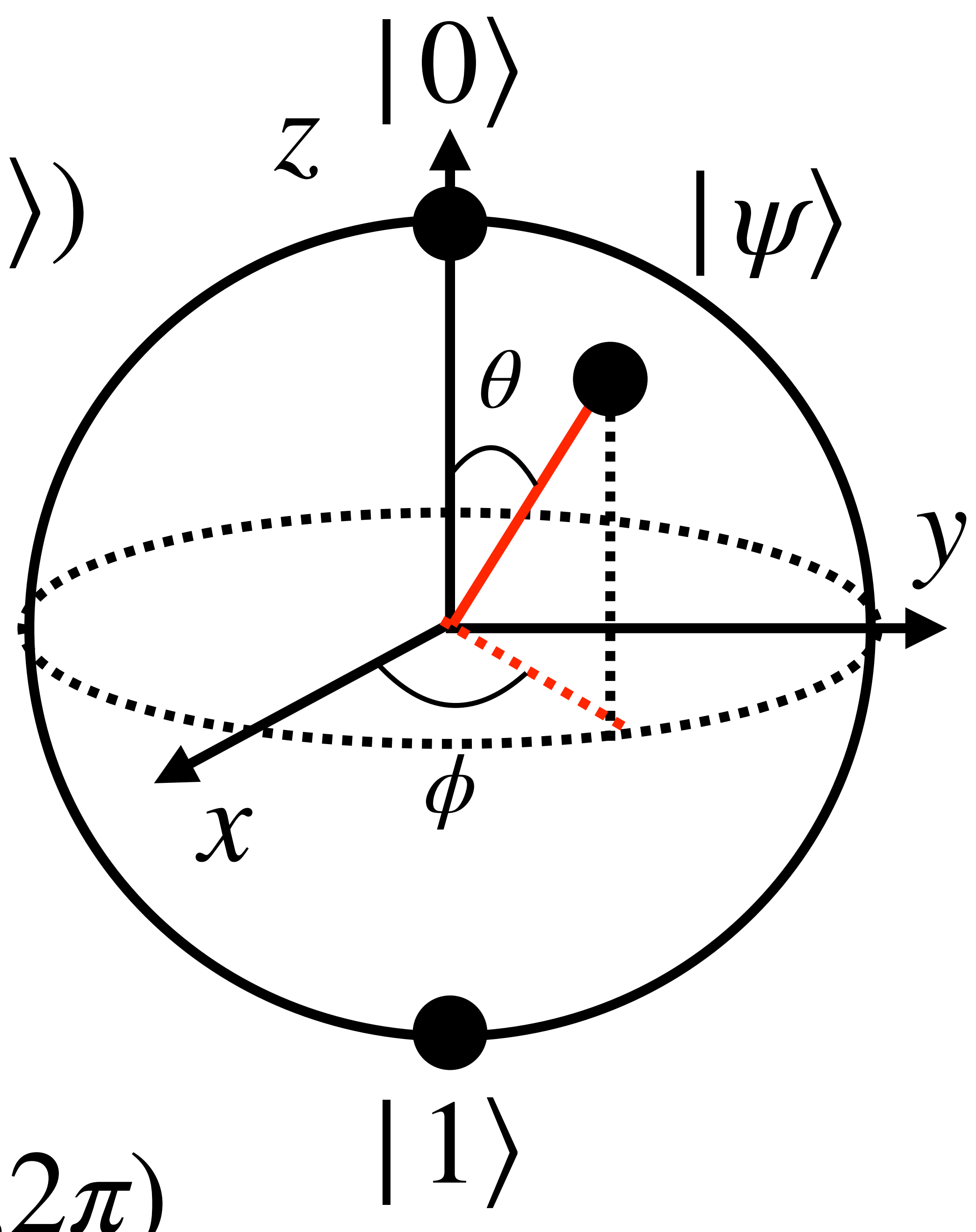
Initial state : $|\psi_{\rm in}\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \quad \mathcal{E}(|\psi_{\rm out}\rangle) \geq \mathcal{E}(|\psi_{\rm in}\rangle) = 0$

Definition : $E(\hat{S}) = \overline{\mathcal{E}(\hat{S}|\psi_1\rangle \otimes |\psi_2\rangle)} = \int d\omega_1 d\omega_2 \mathcal{E}(\hat{S}|\psi_1\rangle \otimes |\psi_2\rangle)$

⇨ **Integrating over all possible** $|\psi_{\rm in}\rangle$

EX.) spin-1/2

Bloch sphere:



one-particle state: $|\psi\rangle = \begin{pmatrix}\cos\theta & e^{i\phi}\sin\theta\end{pmatrix}^T$

$\theta \in [0,\pi), \quad \phi \in [0,2\pi)$

Integration measure : $d\omega = \dfrac{\sin\theta d\theta d\phi}{4\pi}$

NN scattering

Nucleon : $\mathrm{SU}(2)_{\mathrm{spin}} \otimes \mathrm{SU}(2)_{\mathrm{isospin}}$ ⇨ $N = \begin{pmatrix} \uparrow \\ \downarrow \end{pmatrix} \otimes \begin{pmatrix} p \\ n \end{pmatrix}$

NN scattering in low-energy region s-wave

Isospin is automatically determined by the spin

Fermi-Dirac(FD) statistics ⇨ Only the spin space

Irreducible decomposition $\quad \frac{1}{2} \otimes \frac{1}{2} = \underset{\text{A}}{0} \oplus \underset{\text{S}}{1}$

S-matrix for NN scattering

$$\boldsymbol{\sigma} \cdot \boldsymbol{\sigma} = \sum_i \sigma_i \otimes \sigma_i$$

$$\hat{S} = e^{2i\delta_0} \frac{1 - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2}{4} + e^{2i\delta_1} \frac{3 + \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2}{4}$$

$\boldsymbol{\sigma}$: Pauli matrix in spin space

δ_J : phase shift for J channel

E.S. for NN scattering

S.R. Beane *et al*., PRL122, 102001 (2019);
I. Low, T. Mehen, PRD104, 074014 (2021);

EP for NN scattering : $E(\hat{S}) = \frac{1}{6}\sin^2[2(\delta_0 - \delta_1)] \geq 0$

EP is determined only by the phase shifts

Minimizing EP : $E(\hat{S}) = 0$ **Entanglement suppression**

$\Rightarrow$ $|\delta_0 - \delta_1| = 0, \frac{\pi}{2}$ **Only two solutions**

Identity gate for $|\delta_0 - \delta_1| = 0$

$\hat{S} \propto 1$ $\Rightarrow$ $\hat{1}|\psi_1\rangle \otimes |\psi_2\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$

The final state remains separable

$\mathscr{E}(\hat{S}|\psi_{\rm in}\rangle) = 0$ **for all initial state**

NN scattering

SWAP gate for $|\delta_0 - \delta_1| = \pi/2$

$$\hat{S} \propto -\frac{1-\sigma_1\cdot\sigma_2}{4} + \frac{3+\sigma_1\cdot\sigma_2}{4} = \frac{1+\sigma_1\cdot\sigma_2}{2}$$ Spin-exchange operator

$$\mathrm{SWAP}|\psi_1\rangle \otimes |\psi_2\rangle = |\psi_2\rangle \otimes |\psi_1\rangle$$ Swapping of particle 1 and 2

The final state remains separable

The conditions of the scattering lengths

$|a_0| \to \infty,\ a_1 \to 0$ or $a_0 \to 0,\ |a_1| \to \infty$ **T. Mehen *et al*., PLB474, 145 (2000)**

$\Updownarrow$ $\Updownarrow$

$\delta_0 = \pi/2,\ \delta_1 = 0$ $\delta_0 = 0,\ \delta_1 = \pi/2$

Symmetries in NN scattering

- **Emergent symmetries:**

Identity for $|\delta_0 - \delta_1| = 0$ ⇨ Wigner-$\mathrm{SU}(4)$

Interactions of $^1S_0$ and $^3S_1$ are identical

Fundamental representation: $x = (p_\uparrow \quad p_\downarrow \quad n_\uparrow \quad n_\downarrow)$

SWAP for $|\delta_0 - \delta_1| = \dfrac{\pi}{2}$ ⇨ **NR Conformal symmetry :**

Unitary limit and non-interaction

$|a_0| \to \infty,\ a_1 \to 0$

or $a_0 \to 0,\ |a_1| \to \infty$

T. Mehen *et al*., PLB474, 145 (2000)

low-energy physics is universally governed by the scattering length

Entanglement suppression derives the emergent symmetries

Numerical calculation

We numerically calculate the EP for np scattering

The low-energy phase shift $\delta_J(p)$ is determined by **effective range expansion**

$$e^{2\delta_J(p_{\mathrm{CM}})} = \frac{-1/a_J + r_J p_{\mathrm{CM}}^2 + i p_{\mathrm{CM}}}{-1/a_J + r_J p_{\mathrm{CM}}^2 - i p_{\mathrm{CM}}}$$

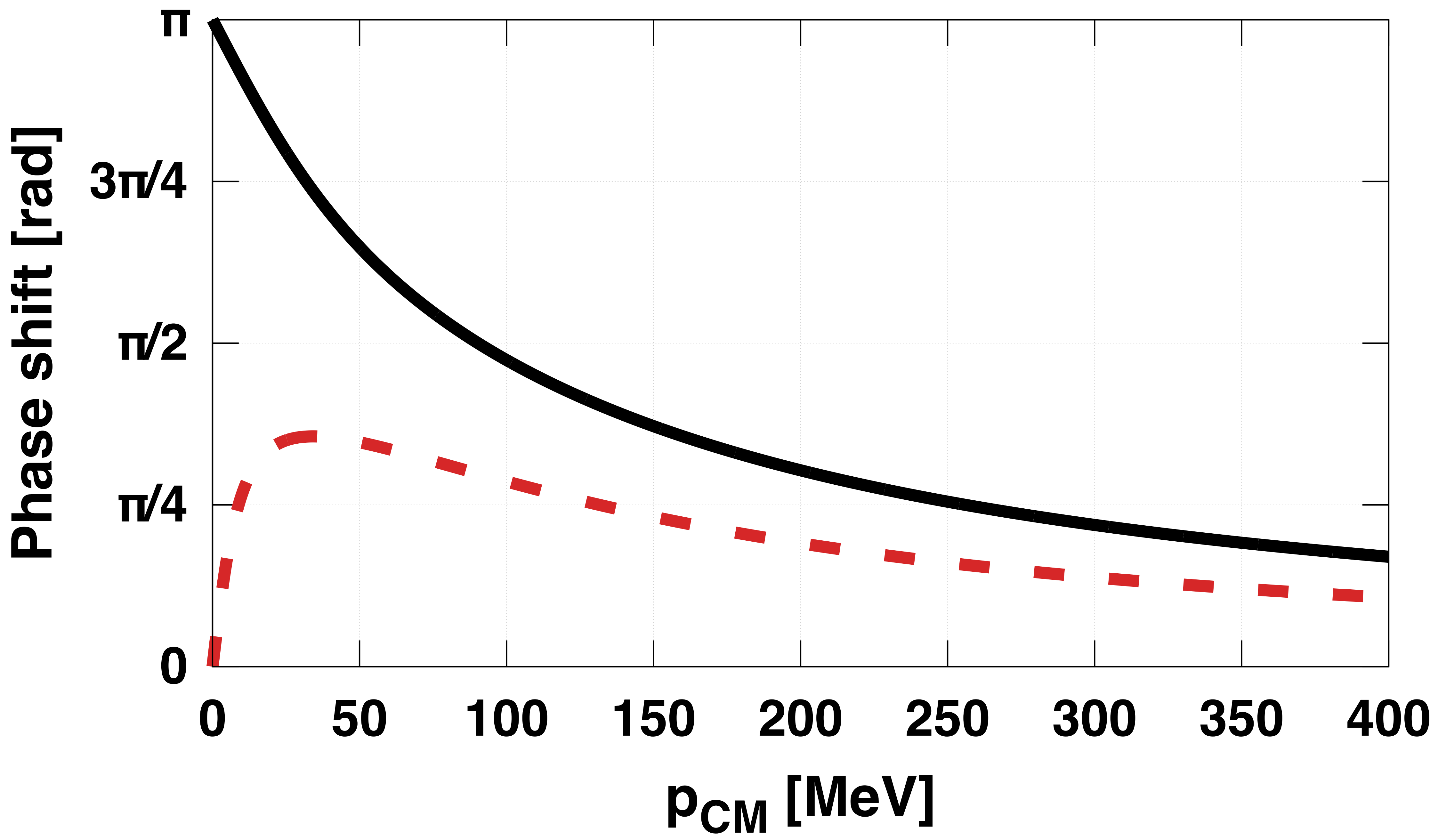
p_{CM} : relative momentum of np

a_J : scattering length for spin J

r_J : effective range for spin J

$J = 1 : a_0 = 5.42\ \mathrm{fm}, \quad r_0 = 1.75\ \mathrm{fm}$ Bound

$J = 0 : a_1 = -23.7\ \mathrm{fm}, \quad r_1 = 2.77\ \mathrm{fm}$ Virtual



Both the phase shifts change **rapidly** due to the **near-threshold poles**

Numerical calculation

The EP for np is calculated in Ref.[1] **[1] S.R. Beane *et al.*, PRL122, 102001 (2019)**

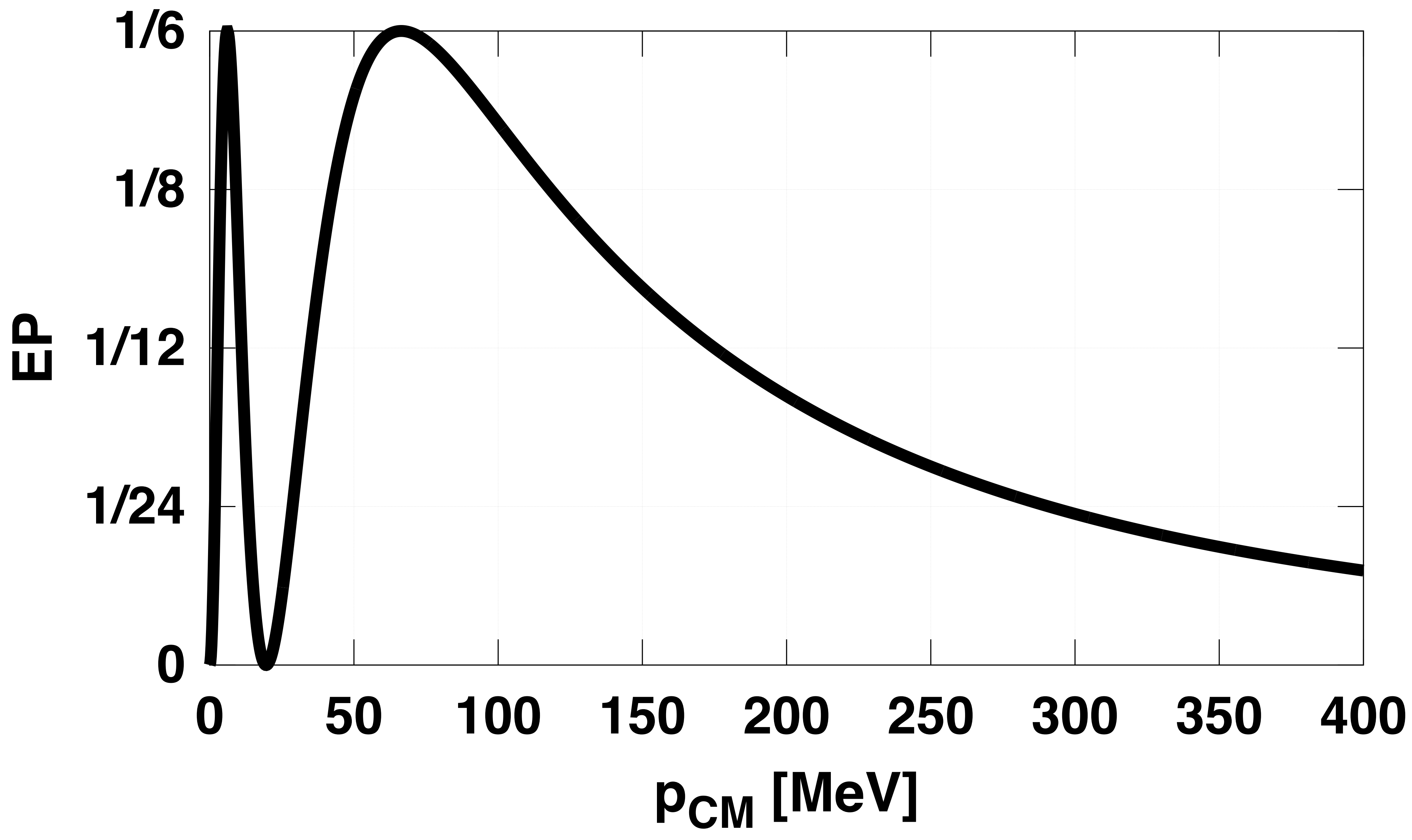
$J = 1 : a_0 = 5.42 \text{ fm}, \quad r_0 = 1.75 \text{ fm}$ Bound

$J = 0 : a_1 = -23.7 \text{ fm}, \quad r_1 = 2.77 \text{ fm}$ Virtual

The result in the right panel **reproduces qualitative behavior** in Ref[1]

np scattering is **dominated by the large** a_J, r_J **over a wide energy range**

- $p_{\mathrm{CM}} \to 0$: $\mathrm{EP} \to 0$ (**threshold effects**)
- $p_{\mathrm{CM}} \sim 150$ MeV : EP is **rapidly changed**
  Effects of the **near-threshold poles**
- $p_{\mathrm{CM}} \geq 150$ MeV : EP is **suppressed**
  $\hat{S} \sim \hat{1}$



$K\bar{K}$ scattering

$K\bar{K}$ scattering in low-energy region

⇨ **Two isospin channels** : $I = 0{,}1$

distinguishable particles

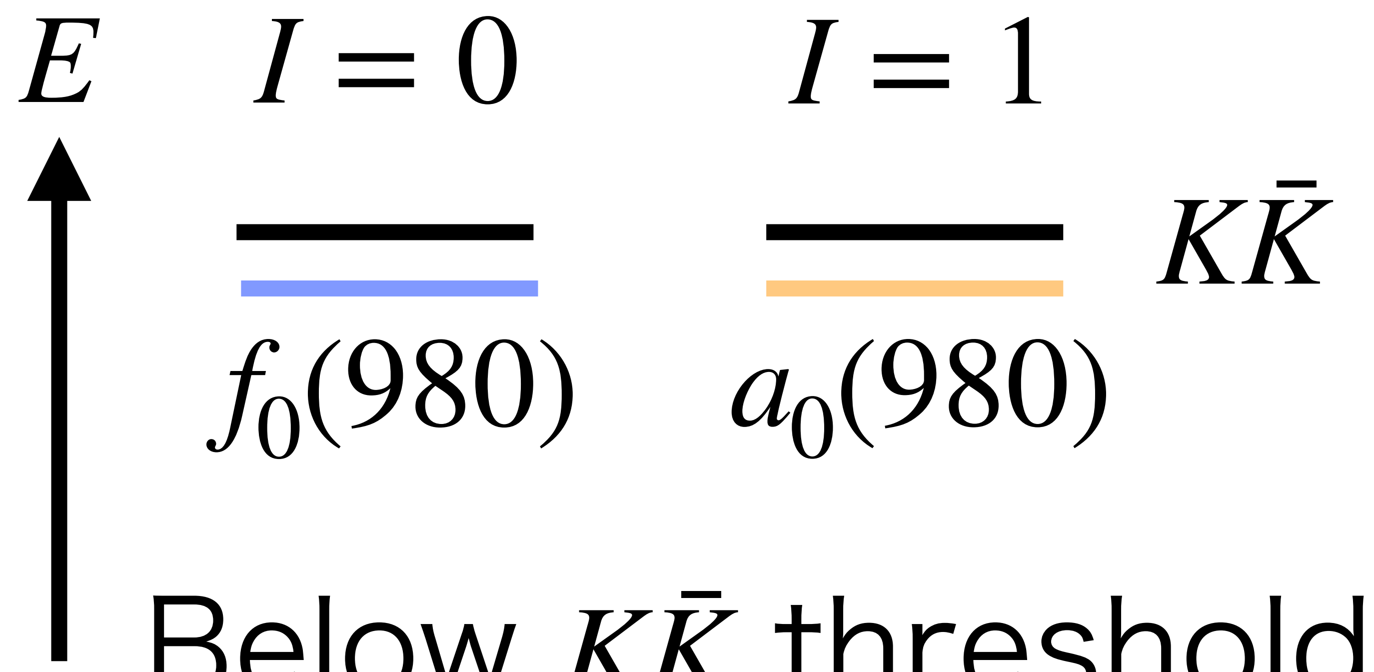
Exotic candidates : $f_0(980)$ $(I = 0)$, $a_0(980)$ $(I = 1)$

Nearly degenerate

S-matrix for $K\bar{K}$ scattering

$$\hat{S} = e^{2i\delta_0}\frac{1 - \tau_1 \cdot \tau_2}{4} + e^{2i\delta_1}\frac{3 + \tau_1 \cdot \tau_2}{4}$$

τ : Pauli matrix in isospin space



Below $K\bar{K}$ threshold

binding energies ~ 10 MeV

$K\bar{K}$ couples to $\pi\pi$ $(I = 0)$ and $\pi\eta$ $(I = 1)$ ⇨ **Complex scattering lengths**

We neglect the inelasticity parameter : $\eta = 1$

$K\bar{K}$ scattering

EP for $K\bar{K}$ scattering : $E(\hat{S}) = \frac{1}{6}\sin^2[2(\delta_0 - \delta_1)] \geq 0$

Minimizing EP : $E(\hat{S}) = 0$ $\Rightarrow$ $|\delta_0 - \delta_1| = 0, \frac{\pi}{2}$

- **Identity gate** for $|\delta_0 - \delta_1| = 0$ $\hat{S} \propto 1$ $\Rightarrow$ $a_0 = a_1$

We have **not yet discovered the corresponding emergent symmetry**

- **SWAP gate** for $|\delta_0 - \delta_1| = \pi/2$ $\hat{S} \propto \mathrm{SWAP}$ **NR Conformal symmetry**

$|a_0| \to \infty,\ a_1 \to 0$ or $a_0 \to 0,\ |a_1| \to \infty$

Experimental data suggests $a_0 \sim a_1$ $|a_0|, |a_1| \sim 1\ \mathrm{fm}$

a_0, a_1 are **not close to unitary limit** (Difference from np scattering)

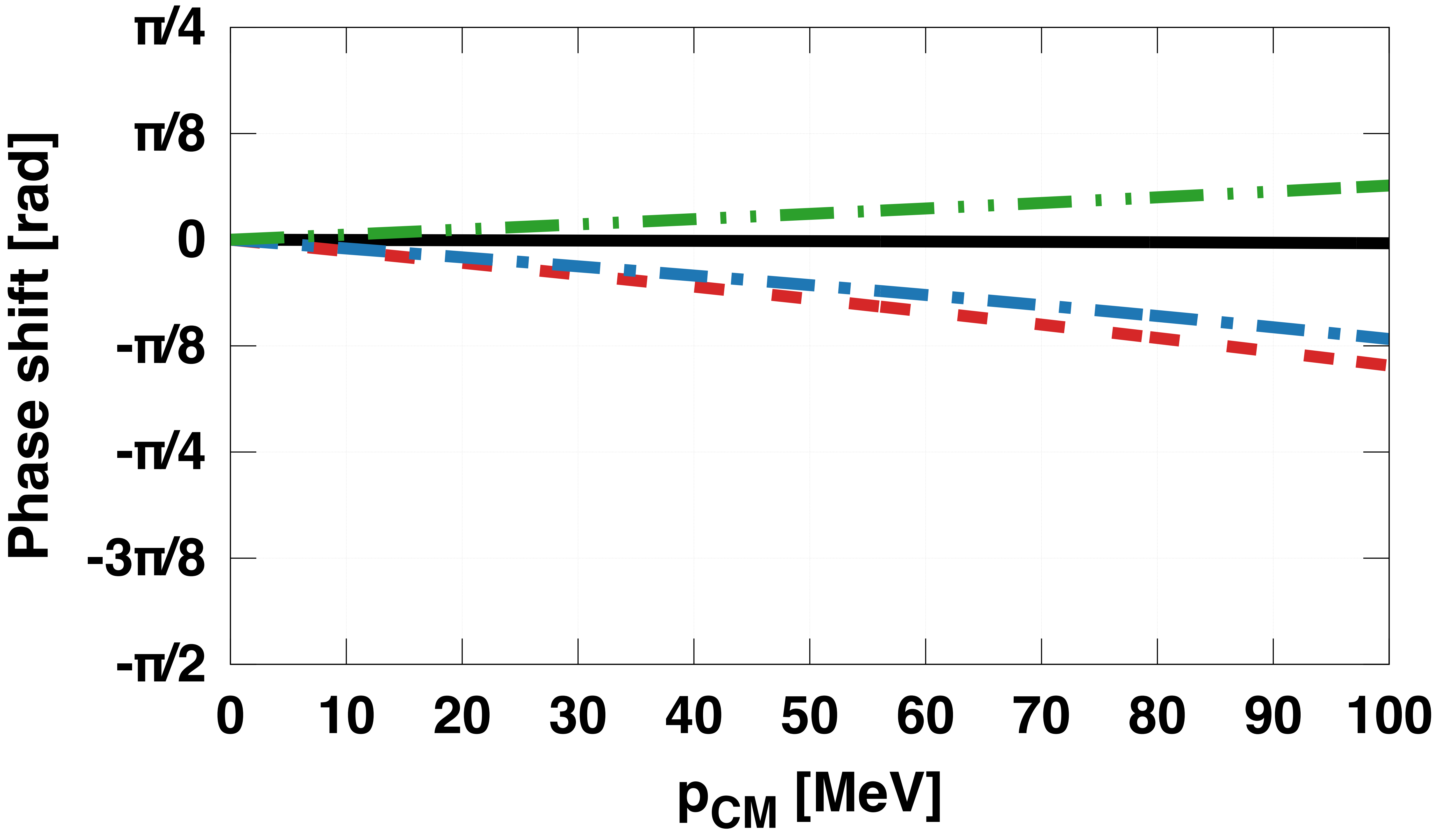
Numerical calculation

We numerically calculate the EP for $K\bar{K}$ scattering

Scattering length approximation $$e^{2\delta_I(p_{\mathrm{CM}})} = \frac{-1/a_I + ip_{\mathrm{CM}}}{-1/a_I - ip_{\mathrm{CM}}}$$

The phase shift for $I = 0,1$ up to $p_{\mathrm{CM}} = 100$ MeV

1 a_1 = -0.371 - i0.549 fm [2]
2 a_0 = 0.02 - i0.95 fm
3 a_0 = 0.84 - i0.85 fm [3]
4 a_0 = 0.64 - i0.83 fm



δ_0, δ_1 are small at low energy region

$$\delta_0, \delta_1 \sim 0$$

[2] H.-P. Li et al., Eur. Phys. J. A 61, 97 (2025).
[3] Y. Kamiya et al., Prog. Theor. Exp. Phys. 2017, 023D02 (2017).

Numerical calculation

We numerically calculate the EP for $K\bar{K}$ scattering

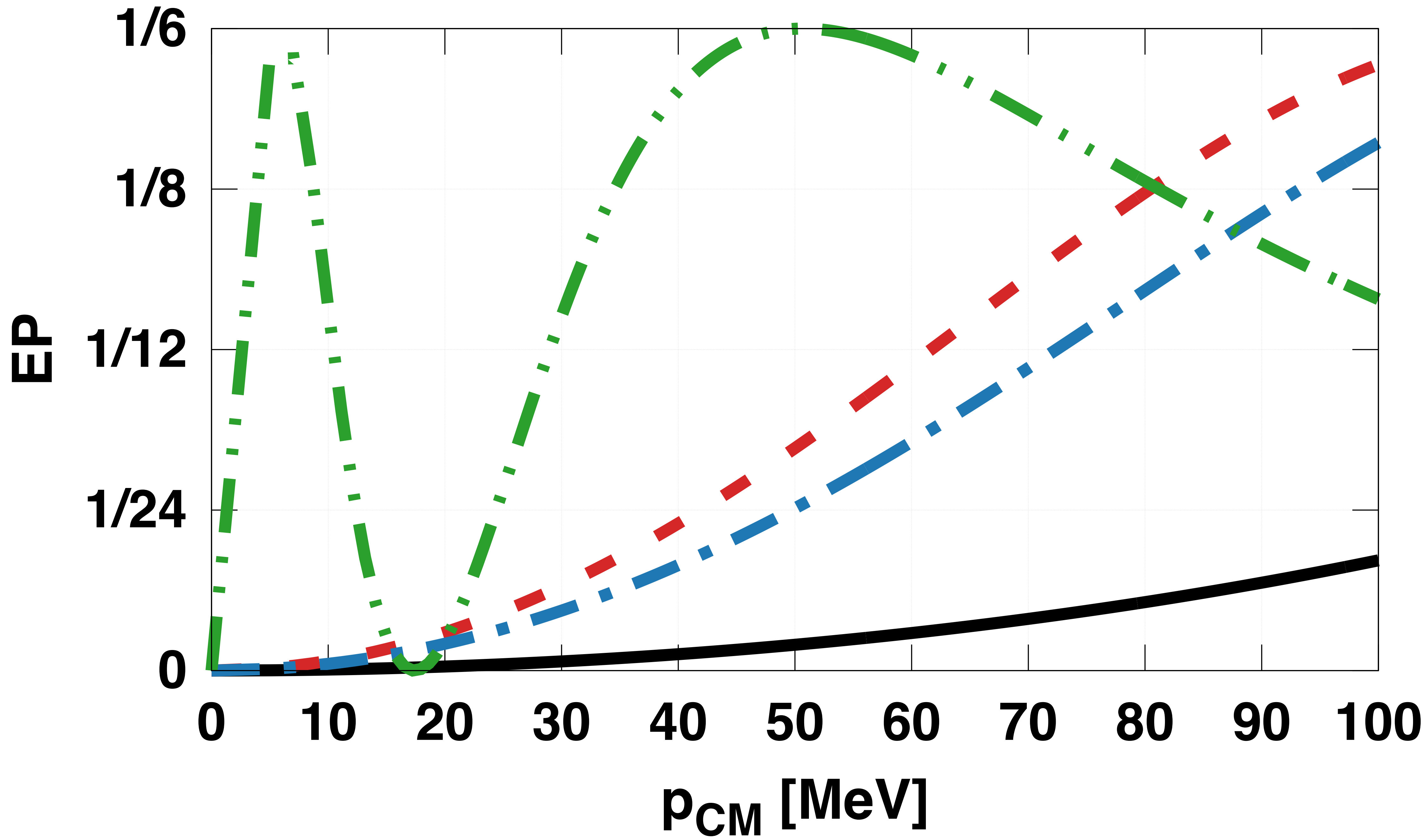
a_1 is fixed as that in [2]

1 K Kbar: a_0=0.02-i0.95 fm, a_1=-0.371-i0.549 fm

2 K Kbar: a_0=0.84-i0.85 fm, a_1=-0.371-i0.549 fm

3 K Kbar: a_0=0.64-i0.83 fm, a_1=-0.371-i0.549 fm

4 np: a_0=5.4194 fm, a_1=-23.740 fm



- $K\bar{K}$ entanglement power is **suppressed** below $p_{\mathrm{CM}} \sim 50\mathrm{MeV}$, because phase shifts are **small** near the threshold $\delta_0, \delta_1 \sim 0$ $\Rightarrow$ $\hat{S} \sim \hat{1}$
- **No clear pole effects in EP (Strong threshold effect)**

[2] H.-P. Li et al., Eur. Phys. J. A 61, 97 (2025).

[3] Y. Kamiya et al., Prog. Theor. Exp. Phys. 2017, 023D02 (2017).

Summary

Solutions for the entanglement suppression $\Rightarrow$ $E(\hat{S}) = 0$

- $K\bar{K}$ scattering

$\hat{S} \propto 1$ $\Rightarrow$ **Not yet found** **Experimentally favored**

$\hat{S} \propto \mathrm{SWAP}$ $\Rightarrow$ NR conformal symmetry

- Numerical results

$K\bar{K}$ entanglement power is **suppressed** at very low-energy region.

Phase shifts are **small** near the threshold $\delta_0, \delta_1 \sim 0$ $\Rightarrow$ $\hat{S} \sim \hat{1}$

$\Rightarrow$ **No clear pole effects in EP**

- Future work

Channel coupling effects $\Leftrightarrow$ **Inelasticity** parameter

Exploration of the **high-momentum region** with **effective range**

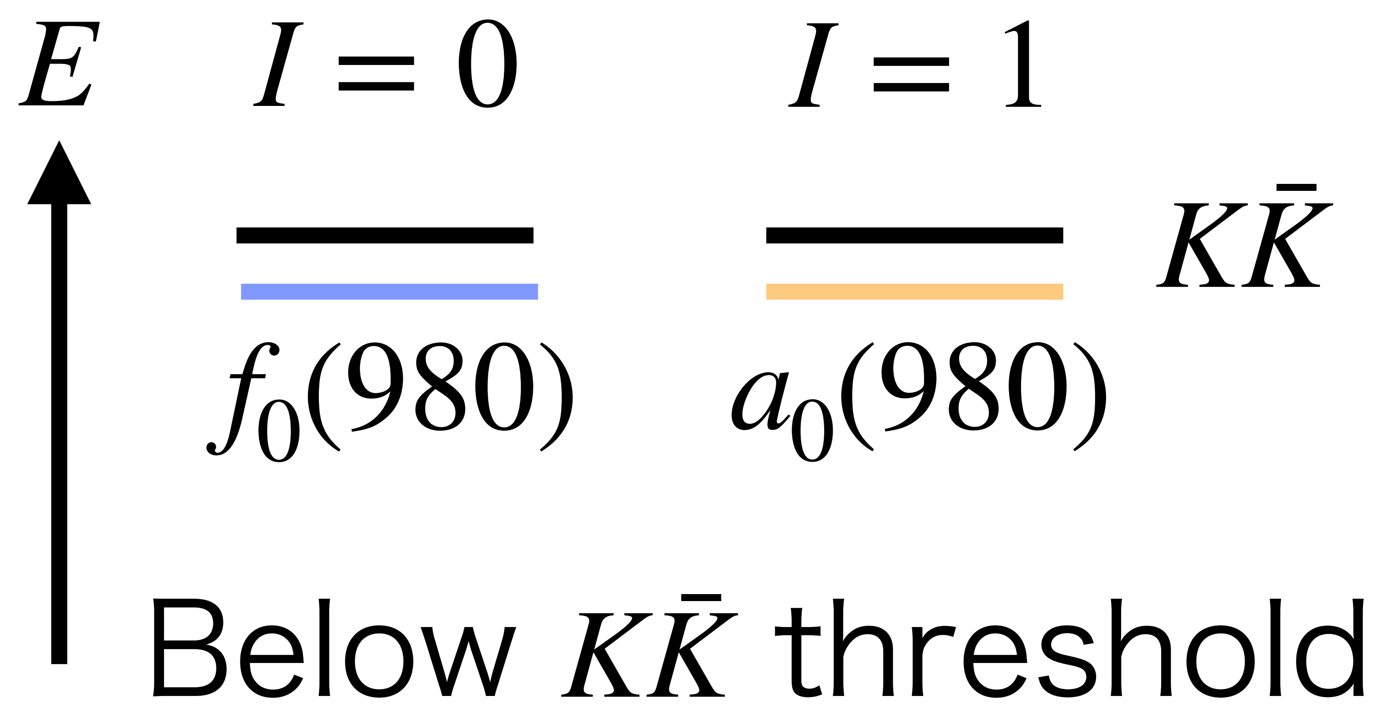
This work

We apply the entanglement suppression to $K\bar{K}$ **scattering**

Two isospin channels : $I = 0,1$ ⇨ **Entanglement for isospin space**

Exotic candidate : $f_0(980)$ $(I = 0)$, $a_0(980)$ $(I = 1)$ **Nearly degenerate**

A situation similar to the NN scattering



Below $K\bar{K}$ threshold

binding energies ~ 10 MeV

Objective :

Exploring the $K\bar{K}$ interaction through entanglement suppression

Numerical calculation

Scaling both complex scattering lengths

by a common factor λ

$$a_0 \to \lambda a_0 \qquad a_1 \to \lambda a_1$$

The choose the scattering lengths:

$$a_1 = -0.371 - i0.549 \text{ fm} \quad a_0 = 0.64 - i0.83 \text{ fm}$$

- As λ increases, **the EP approaches the EP for np scattering**
- **strong pole effect**
- EP is **suppressed** ≥ 70 MeV $\quad \lambda = 10$