WIMP-Nucleus Scattering: The Mean-Field Approach and Beyond

Dr. Navneet Krishnan, Australian National University

Fundamental Problems in Subatomic Physics, Cairns, September 2026

My Team

Dr. Raghda Abdel Khaleq

Prof. Cedric Simenel

Dark Matter

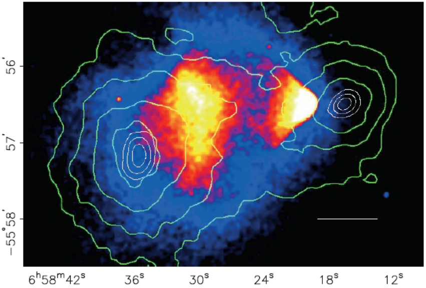


Figure 1: Mass distribution of the Bullet Cluster from gravitational lensing.
Image credit: Clowe, D. et al. (2006).

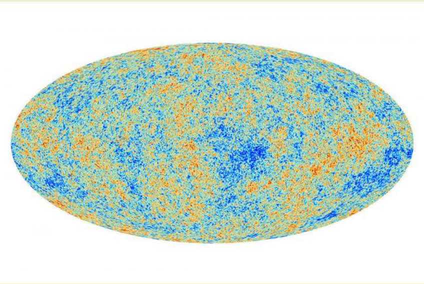
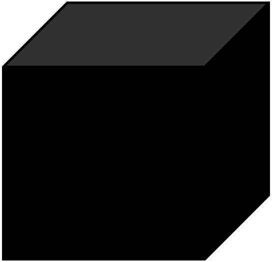


Figure 2: The cosmic microwave background.
Image credit: ESA and the Planck Collaboration.

Weakly Interacting Massive Particles

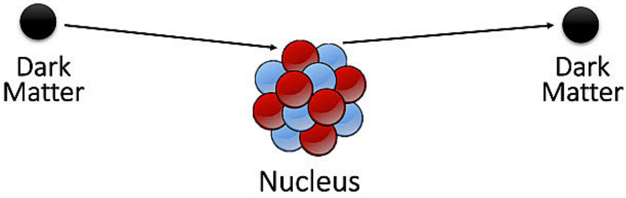
- WIMP: High-mass, spin-1/2 particle.
- Interactions below the electroweak scale.
- For this talk: we don't care about interaction specifics.



Direct Detection

$$\frac{\mathrm{d}R}{\mathrm{d}E_R} = \frac{N_T \rho_\chi}{m_\chi} \int v f(\vec{v}) \, \frac{\mathrm{d}\sigma_T}{\mathrm{d}E_R}(\vec{v}) \, \mathrm{d}^3 v.$$

- Particle/nuclear physics input: differential cross-section.
- Focus: elastic scattering.



Differential Cross-Section

$$\frac{\mathrm{d}\sigma_T}{\mathrm{d}E_R} = \frac{m_T}{2\pi v^2} \sum_{i,j} \sum_{N,N'=p,n} \mathfrak{c}_i^{(N)} \mathfrak{c}_j^{(N')} F_{i,j}^{(N,N')}(q^2).$$

- Effective Field Theory Formalism [1]: $\mathcal{L} = \sum_i \overline{\chi} \mathcal{O}_i^{\chi} \chi \overline{N} \mathcal{O}_i^N N$.
- 15 possible WIMP-nucleon operators:
  $\mathcal{O}_1 = \mathbb{I}, \mathcal{O}_3 = i\vec{S}_N \cdot (\vec{q} \times \vec{v}^{\perp}), \mathcal{O}_4 = \vec{S}_N \cdot \vec{S}_{\chi}, \ldots$
- $\Rightarrow$ 6 nuclear responses $M, \Delta, \Sigma', \Sigma'', \tilde{\Phi}', \Phi''$.
- Our focus: form factors
  $F_{X,Y}^{(N,N')} = \frac{4\pi}{2J_i+1} \sum_{J=0}^{2J_i} \langle J_i || X_J^{(N)} || J_i \rangle \langle J_i || Y_J^{(N')} || J_i \rangle$.

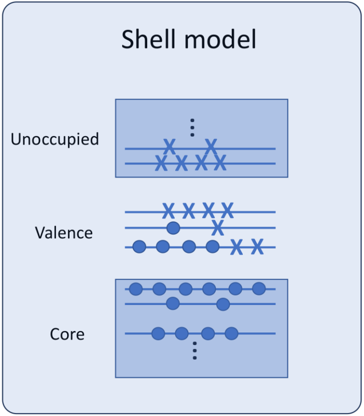
[1] A. Liam Fitzpatrick *et al.* (2013). JCAP02(2013)004.

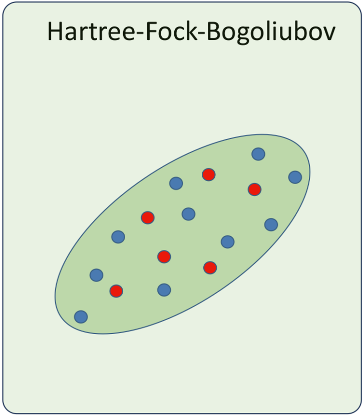
Test Case: ^{40}Ar

- Spherically symmetric, even-even nucleus $\Rightarrow J^{\pi} = 0^{+}$.
- Two relevant nuclear responses, M and Φ''.
- $M_{JM}(q\vec{x}) = j_J(qx) Y_{JM}(\Omega_x)$.
  Leading order: $\mathbb{I}$, counts nucleons.
- $\Phi''_{JM}(q\vec{x}) = i \left(\frac{\vec{\nabla}}{q} M_{JM}(q\vec{x}) \right) \cdot \left(\vec{\sigma}_N \times \frac{1}{q} \vec{\nabla} \right)$.
  Leading order: $\sigma \cdot \ell$, nucleon spin-orbit coupling.
- Computing form factors from nuclear ground state density matrix $\langle \Phi | c^{\dagger}_{\alpha} c_{\beta} | \Phi \rangle$ using DMFormFactor [2].

[2]: Anand, N., Fitzpatrick, A.L. & Haxton, W.C. (2014). Phys. Rev. C., 89, 065501.

Nuclear Structure: Shell Model vs. Mean-Field





Nucleon Pairing

- Nucleon pairs can form energetically-favoured bound states.
- Evidence: nuclear odd-even mass staggering, different excitation spectra patterns between adjacent odd and even nuclei.

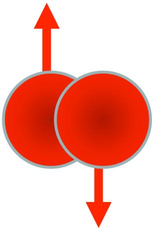


Image credit: Frauendorf, S. & Macchiavelli, A. O. (2014).

Skyrme HFB Theory

- Hartree-Fock: solve self-consistent Schrödinger equation.
- Skyrme force: phenomenological density-dependent force.
- Bogoliubov theory: add pairing interaction.
- Using code HFBTHO [3].

$$\begin{aligned}\hat{V}(1,2) &= t_0(1 + x_o\hat{P}_\sigma)\hat{\delta}(x_1 - x_2) \\ &+ t_3(1 + x_3\hat{P}_\sigma)\rho^\alpha\hat{\delta}(x_1 - x_2) \\ &+ \cdots\end{aligned}$$

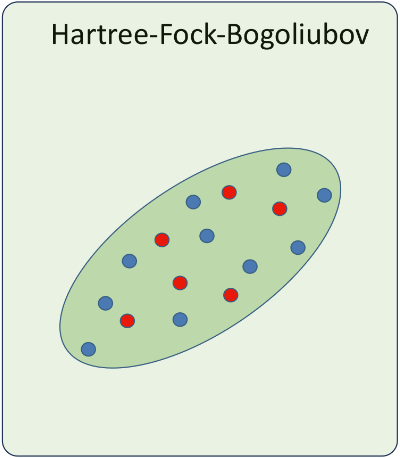
$$\hat{V}(1,2) \to E[\rho] \to h_{ij}[\rho] = \frac{\delta E[\rho]}{\delta \rho_{ij}}.$$

$$\to h[\rho]\varphi_i = e_i\varphi_i.$$

[3]: Marević, P., Schunck, N., Ney, E. M., Navarro Pérez, R., Verriere, M., & O'Neal, J. (2022). Computer Physics Communications, 276, 108367.

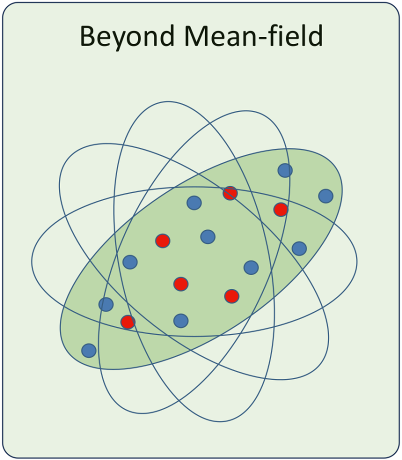
Mean-Field: Correlations and Breaking Symmetries

- Localised $\rightarrow$ break translational invariance.
- Deformed $\rightarrow$ break rotational invariance.
- Pairing $\rightarrow$ break particle number symmetry.



Restoring Symmetries

- Spin-dependent interactions couple WIMP and nuclear spin $\rightarrow$ need to project onto good angular momentum states.
- Pairing breaks particle number symmetry $\rightarrow$ need to project onto good particle number states.
- $P_N = \frac{1}{2\pi} \int_0^{2\pi} d\varphi e^{-i\varphi(\hat{N}-N)}$.
- $|\psi\rangle = \sum_{N,Z} c_{N,Z} |\psi_{N,Z}\rangle$.



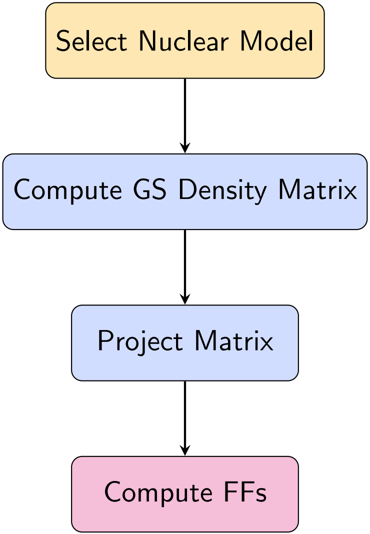
Particle Number Projection

$$P^N = \frac{1}{2\pi}\int_0^{2\pi} d\varphi e^{-i\varphi(\hat{N}-N)} \longrightarrow \frac{1}{N_\varphi}\sum_{\ell=1}^{N_\varphi} e^{i\varphi_\ell(\hat{N}-N_0)}$$

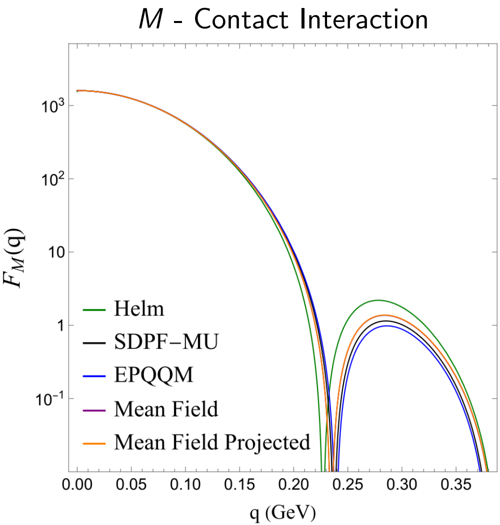
$$\rho_{\alpha\beta}^{N_0} = \frac{\langle\Phi|\,\hat{c}_\alpha^\dagger\hat{c}_\beta\hat{P}^{N_0}\,|\Phi\rangle}{\langle\Phi|\,\hat{P}^{N_0}\,|\Phi\rangle} \longrightarrow \frac{\sum_{\ell=1}^{N_\varphi} e^{-i\varphi_\ell N_0}\rho_{\alpha\beta}(\varphi_\ell)\,\langle\Phi|\Phi(\varphi_\ell)\rangle}{\sum_{\ell=1}^{N_\varphi} e^{-i\varphi_\ell N_0}\,\langle\Phi|\Phi(\varphi_\ell)\rangle}$$

- $\rho(\varphi_\ell)$: function of ρ, φ_ℓ
- $\langle\Phi|\Phi(\varphi_\ell)\rangle = \langle\Phi|\,e^{i\varphi_\ell\hat{N}}\,|\Phi\rangle$

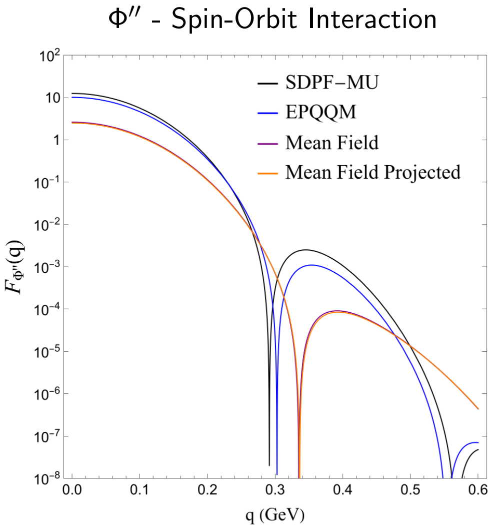
Workflow


Select Nuclear Model
Compute GS Density Matrix
Project Matrix
Compute FFs

Results: ^{40}Ar Form Factors

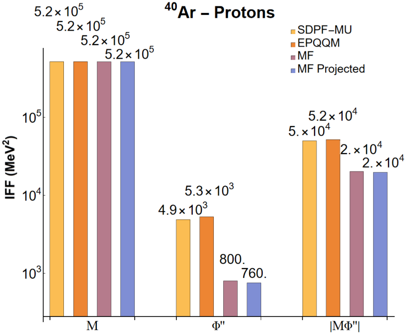


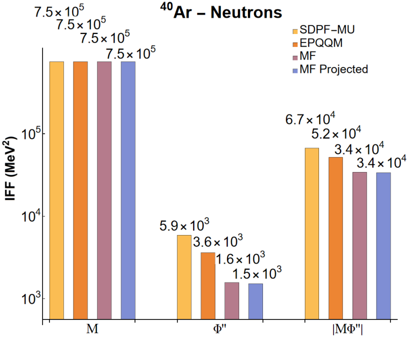
Results: ^{40}Ar Form Factors



Results: ^{40}Ar Integrated Form Factors

$$\int_0^{100\ \mathrm{MeV}} \frac{q\,dq}{2} F^{(N,N)}_{X(,Y)}(q^2)$$





Final Notes

- Need a mean-field approach to compliment existing work in WIMP-nucleus scattering.
- Demonstrated proof-of-principle of this approach in ^{40}Ar.
- Ongoing work: applying beyond mean-field techniques for odd and deformed nuclei.

Paper

Application of the Skyrme Hartree-Fock-Bogoliubov Theory to WIMP-Nucleus Interactions in ^{40}Ar

N. Krishnan,[1,2,3,*] R. Abdel Khaleq,[1,2,3,†] and C. Simenel[1,2,3,‡]

[1]*Department of Fundamental and Theoretical Physics, Research School of Physics, Australian National University, ACT, 2601, Australia*

[2]*Department of Nuclear Physics and Accelerator Applications, Research School of Physics, Australian National University, ACT, 2601, Australia*

[3]*ARC Centre of Excellence for Dark Matter Particle Physics, Australia*

(Dated: September 23, 2026)

WIMP scattering from ^{40}Ar is investigated using a self-consistent Skyrme Hartree–Fock–Bogoliubov (HFB) approach. Nuclear form factors relevant to dark matter direct detection are calculated from the resulting one-body density matrix elements and compared with shell-model predictions. Good agreement is found for the spin-independent response, while significant differences are observed for the spin-orbit response due to variations in single-particle occupancies. The effects of particle-number projection are shown to be small for ^{40}Ar. These results demonstrate the sensitivity of certain dark matter response channels to the underlying nuclear structure model and establish a framework for extending mean-field calculations to nuclei beyond the reach of large-scale shell-model studies.

arXiv: 2606.11668

Results: ^{40}Ar Form Factors

Φ'' - Spin-Orbit Interaction