

Monopoles, Domain Walls, Cosmic Strings, and Their Implications for Gravitational Waves

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Cosmological Phase Transition



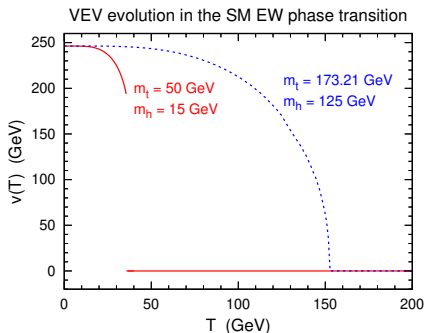
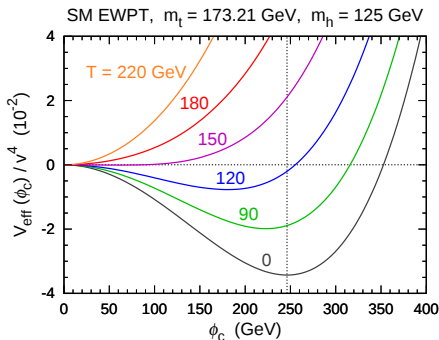
Spontaneously broken symmetries in field theories can be **restored** at **sufficiently high temperatures** due to **thermal corrections** to the **effective potential**



In the history of the Universe, **spontaneous symmetry breaking** manifests itself as a **cosmological phase transition**



If the **vacuum manifold** has **nontrivial topological structures**, **topological defects** would be formed after the phase transition



Topological Defects



Consider that **some scalar fields** acquire nonzero **vacuum expectation values** (VEVs), which **break** a **symmetry group** G to a **subgroup** H



The **manifold** consisting of all **degenerate vacua** is the **coset space** G/H



The **topology** of the **vacuum manifold** G/H can be characterized by its **n -th homotopy group** $\pi_n(G/H)$, which are formed by the homotopy classes of the mappings from an **n -dimensional sphere** S^n into G/H



Nontrivial $\pi_n(G/H)$ leads to **topological defects** [Kibble, J. Phys. A9 (1976) 1387]

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Nontrivial $\pi_0(G/H)$: two or more disconnected components



Domain walls (2-dim topological defects)



Nontrivial $\pi_1(G/H)$: incontractable closed paths



Cosmic strings (1-dim topological defects)



Nontrivial $\pi_2(G/H)$: incontractable spheres



Monopoles (0-dim topological defects)



$$G/H \cong Z_2$$



$$\pi_0(G/H) = Z_2$$



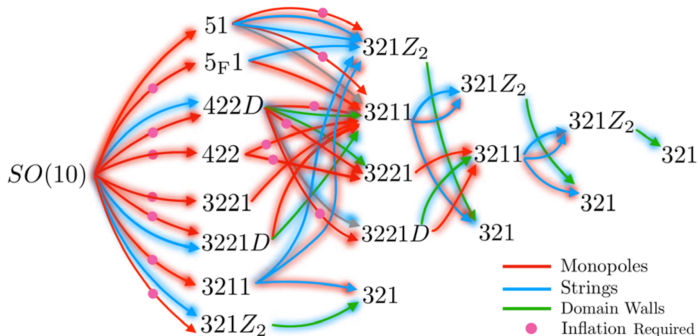
$$G/H \cong S^1$$

$$\pi_1(G/H) = \mathbb{Z}$$

Topological Defects in GUTs



Monopoles, **cosmic strings**, and **domain walls** are predicted by many **new physics theories**, particularly, **grand unified theories (GUTs)**



$$51 = \text{SU}(5) \times \text{U}(1)_X / Z_5, \quad 5_F1 = \text{SU}(5)_{\text{flipped}} \times \text{U}(1)_{\text{flipped}} / Z_5$$

$$422 = \text{SU}(4)_C \times \text{SU}(2)_L \times \text{SU}(2)_R / Z_2, \quad 3221 = \text{SU}(3)_C \times \text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_{B-L} / Z_6$$

$$3211 = \text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y \times \text{U}(1)_X / Z_6, \quad 321 = \text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y / Z_6$$

[Dunskey, Ghoshal, Murayama, Sakakihara, White, 2111.08750, PRD]

't Hooft-Polyakov Monopole



The **'t Hooft-Polyakov monopole** is a **static solution** with **finite energy** in a **nonabelian gauge theory** ['t Hooft, NPB **79** (1974) 276; Polyakov, JETP Lett. **20** (1974) 194]



Consider a **SU(2) gauge theory**, **spontaneously broken** to **U(1)_{EM}** by a **SU(2)-triplet Higgs field ϕ^a** with $|\langle\phi\rangle| = v$ ($a = 1, 2, 3$; e is **elementary electric charge**)

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + \frac{1}{2}(D_\mu\phi)^a (D^\mu\phi)^a - V(\phi), \quad V(\phi) = \frac{\lambda}{4}(|\phi|^2 - v^2)^2$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + e\epsilon^{abc}A_\mu^b A_\nu^c, \quad D_\mu\phi^a = \partial_\mu\phi^a + e\epsilon^{abc}A_\mu^b\phi^c$$



Defining $r \equiv |\mathbf{x}|$, the **'t Hooft-Polyakov monopole** corresponds to the **ansatz**

$$\phi^a(\mathbf{x}) = \frac{vf(r)x^a}{r}, \quad A^{ai}(\mathbf{x}) = \frac{a(r)\epsilon^{aij}x^j}{er^2}, \quad f(\infty) = a(\infty) = 1, \quad f(0) = a(0) = 0$$



Its **mass** (the total energy of the solution) is given by

$$M = \int d^3x \left[\frac{1}{2}B^{ai}B^{ai} + \frac{1}{2}(D^i\phi)^a(D^i\phi)^a + V(\phi) \right], \quad B^{ai} = \frac{1}{2}\epsilon^{ijk}F^{ajk}$$

Magnetic Charge and Mass

 Rewrite $\frac{1}{2}B^{ai}B^{ai} + \frac{1}{2}(D^i\phi)^a(D^i\phi)^a = \frac{1}{2}[B^{ai} + (D^i\phi)^a]^2 - \nabla \cdot (\mathbf{B}^a\phi^a)$

 For vacuum field configuration with **winding number** $n = 1$, Gauss's theorem gives

$$\int d^3x \nabla \cdot (\mathbf{B}^a\phi^a) = \int d\sigma \cdot \mathbf{B}^a\phi^a = Q_M v$$

[Srednicki, *Quantum Field Theory*, Chapter 92]

 $Q_M = -\frac{4\pi}{e}$ is the **magnetic charge** of the '**t Hooft-Polyakov monopole** with

$$M = |Q_M|v + \int d^3x \left\{ \frac{1}{2}[B^{ai} + (D^i\phi)^a]^2 + V(\phi) \right\}$$

 **Bogomolny bound** $M \geq |Q_M|v$ [Bogomolny, Sov.J.Nucl.Phys. 24 (1976) 449]

 In the limit $\lambda \rightarrow 0$ with $B^{ai} = -(D^i\phi)^a$, the **Bogomolny bound** is **saturated**, leading to **explicit solution** for the '**t Hooft-Polyakov monopole** with $M = |Q_M|v$:

$$\phi^a = \frac{x^a}{er^2} \left(\frac{evr}{\tanh evr} - 1 \right), \quad A^{ai} = \frac{\varepsilon^{aij}x^j}{er^2} \left(1 - \frac{evr}{\sinh evr} \right)$$

[Prasad & Sommerfield, PRL 35, 760 (1975)]

Generic Monopoles



The **magnetic charge** of a **generic monopole** with a **winding number** n is

$$Q_M = -\frac{4\pi n}{e}, \quad n \in \mathbb{Z}$$



Its **mass** is given by $M = |Q_M|v + \int d^3x \left\{ \frac{1}{2} [B^{ai} + \text{sign}(n)(D^i\phi)^a]^2 + V(\phi) \right\}$



Bogomolny bound becomes $M \geq |Q_M|v = \frac{4\pi|n|v}{e}$



$n = +1$ and $n = -1$ corresponds to the '**t Hooft-Polyakov monopole** and its **antimonopole**, respectively



For $\lambda > 0$, a **monopole** with **winding number** $n \neq \pm 1$ is **unstable** against breaking up into $|n|$ **monopoles** with **winding number** ± 1 , which are **stable**

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If a **SU(2)-doublet** field is added, then its components have **electric charges** $\pm \frac{e}{2}$, which is the smallest charges; all **possible electric charges** is $Q_E = \frac{je}{2}$, $j \in \mathbb{Z}$



Therefore, the possible **electric** and **magnetic** charges obey $Q_E Q_M = 2\pi k$, $k \in \mathbb{Z}$



This is the **Dirac charge quantization condition** from general considerations

Magnetic Monopoles in GUTs

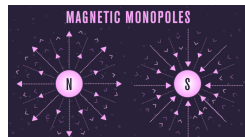
🛹 The **homotopy group** relevant to **monopoles** is $\pi_2(G/H)$

🛴 If $\pi_1(G) = \pi_2(G) = 1$, then $\pi_2(G/H) = \pi_1(H)$

[Vachaspati, hep-ph/0101270]

🚲 Because of $\pi_1[U(1)] = \mathbb{Z}$, $H = U(1)$ leads to **monopoles**

🛵 For a **GUT** with $G \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times U(1)_{EM}$, generation of **stable magnetic monopoles** with a typical mass $M \sim 10^{15}$ GeV is **inevitable**



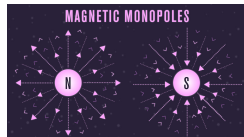
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🛴 Such magnetic monopoles would be **copiously produced** in the early universe

[Guth & Tye, PRL **44**, 631 (1980)]

🚚 They should remain to the **present day** with a **large number density** against the **null observation** [Zel'dovich & Khlopov, PLB **79** (1978) 239; Preskill, PRL **43**, 1365 (1979)]

🚚 This **magnetic-monopole problem** can be solved by assuming that the **cosmic inflation** occurs below the temperature where magnetic monopoles can be produced

Domain Walls



Domain walls (DWs) are **two-dimensional topological defects** which could be formed when a **discrete symmetry** of the **scalar potential** is **spontaneously broken** in the early Universe



Consider a **real scalar field** $\phi(x)$ with a **spontaneously broken** Z_2 **symmetry** $\phi \rightarrow -\phi$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)\partial^\mu \phi - V_0, \quad V_0 = \frac{\lambda}{4}(\phi^2 - v_\phi^2)^2$$



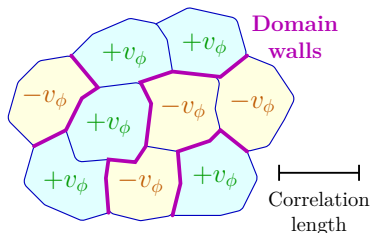
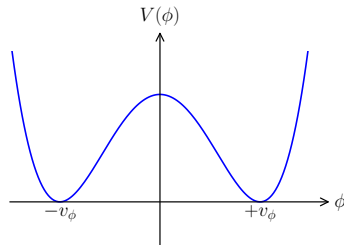
The Z_2 -**conserving potential** V_0 has **two degenerate minima** at $\phi = \pm v_\phi$



After the **spontaneous symmetry breaking**, $\phi(x)$ takes either $+v_\phi$ or $-v_\phi$, and **two different domains** can appear



DWs are produced around the **boundary** of the **two domains**



Domain Wall Configuration



Consider a **static planar DW configuration** lying perpendicular to the z -axis



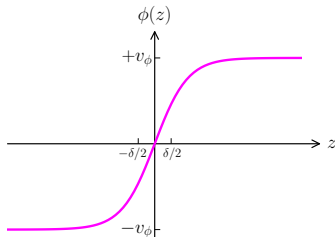
Solving the **equation of motion** $\frac{d^2\phi}{dz^2} - \frac{dV_0}{d\phi} = 0$ with **boundary conditions**

$\lim_{z \rightarrow \pm\infty} \phi(z) = \pm v_\phi$, we obtain a **kink solution**

$$\phi(z) = v_\phi \tanh \frac{z}{\delta}, \quad \delta \equiv \left(\sqrt{\frac{\lambda}{2}} v_\phi \right)^{-1}$$



The **DW** locates at $z = 0$ with a **thickness** δ , separating **two domains** with $\phi(z) > 0$ and $\phi(z) < 0$



The **energy-momentum tensor** for **this static solution** is

$$T^{\mu\nu}(z) = \left[\frac{d^2\phi(z)}{dz^2} \right]^2 \text{diag}(+1, -1, -1, 0)$$



The **DW tension** (surface energy density) is $\sigma = \int_{-\infty}^{+\infty} dz T^{00}(z) = \frac{4}{3} \sqrt{\frac{\lambda}{2}} v_\phi^3$

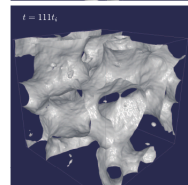
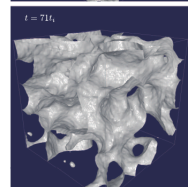
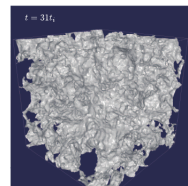
[Saikawa, 1703.02576, Universe]

Evolution of Domain Walls

 After **DWs** are created, the **tension** σ acts to **stretch** them up to the **Hubble horizon size** if the **friction** F_f is **small**, and they would enter the **scaling regime** with **energy density** $\rho_{\text{DW}} = \frac{\mathcal{A}\sigma}{t}$

[Press, Ryden, Spergel, ApJ **347**, 590 (1989)]

 $\mathcal{A} \approx 0.8 \pm 0.1$ is a numerical factor given by lattice simulation
[Hiramatsu, Kawasaki, Saikawa, 1309.5001, JCAP]



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 This implies that **DWs** are **diluted more slowly** than **radiation** ($\rho_r \propto t^{-2}$) and **matter** ($\rho_m \propto t^{-3/2}$) as the Universe expands

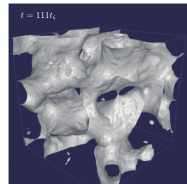
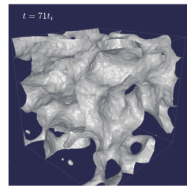
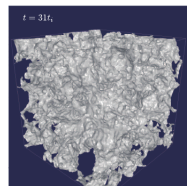
 If DWs are **stable**, they would soon **dominate** the Universe with a **state parameter** $w = \frac{p_{\text{DW}}}{\rho_{\text{DW}}} = -\frac{2}{3}$

 This implies that the **scale factor** evolves as $a(t) \propto t^2$; such a **rapid expansion** is **incompatible** with **standard cosmology**

 Therefore, **stable DWs** results in a **cosmological problem**

[Zeldovich, Kobzarev, Okun, Zh.Eksp.Teor.Fiz. **67** (1974) 3]

[Hiramatsu et al., 1002.1555, JCAP]



Biased Domain Walls



It is **allowed** if **DWs collapse** at a very early epoch [Vilenkin, PRD **23** (1981) 852; Gelmini, Gleiser, Kolb, PRD **39** (1989) 1558; Larsson, Sarkar, White, hep-ph/9608319, PRD]



Such **unstable DWs** can be realized if the Z_2 **symmetry** is **explicitly broken** by a **small potential term**

$$V_1 = \epsilon v_\phi \phi \left(\frac{\phi^2}{3} - v_\phi^2 \right)$$

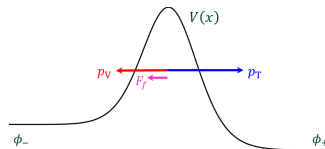
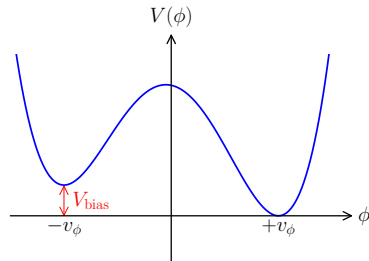


This gives an **energy bias** among the two minima of the potential:

$$V_{\text{bias}} \equiv V(-v_\phi) - V(+v_\phi) = \frac{4}{3} \epsilon v_\phi^4$$



The potential bias provides a **pressure**
 $p_V \sim V_{\text{bias}}$ acting on the DWs, against the
tension force per unit area $p_T \sim \rho_{\text{DW}} \propto T^2$



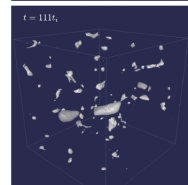
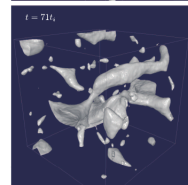
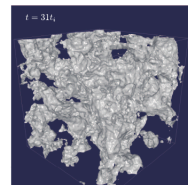
Collapsing Domain Walls and Gravitational Waves

💪 Since $p_T \propto T^2$, the **pressure** p_V would eventually **surpass** the **tension force** at **sufficient low temperatures**

🌹 This makes **DWs collapse** and **false vacuum domains shrink**

🌀 The **annihilation temperature** T_{ann} , at which **DWs collapse**, can be estimated by solving $p_V(T_{\text{ann}}) \simeq p_T(T_{\text{ann}})$:

$$T_{\text{ann}} = \frac{34.1 \text{ MeV}}{\sqrt{\mathcal{A}}} \left[\frac{g_*(T_{\text{ann}})}{10} \right]^{-1/4} \left(\frac{\sigma}{\text{TeV}^3} \right)^{-1/2} \left(\frac{V_{\text{bias}}}{\text{MeV}^4} \right)^{1/2}$$



[Hiramatsu et al., 1002.1555, JCAP]

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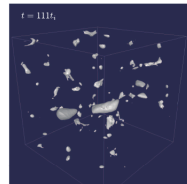
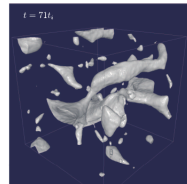
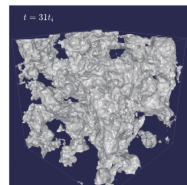
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📡 It is expected that such **collapsing domain walls** produce **gravitational waves** (GWs) [Preskill *et al.*, NPB 363 (1991) 207; Gleiser, Roberts, astro-ph/9807260, PRL]

☁️ A **stochastic gravitational wave background (SGWB)** could be formed and remain to the present time



[Hiramatsu *et al.*, 1002.1555, JCAP]

SGWB Spectrum from Collapsing DWs



The **SGWB spectrum** is commonly characterized by $\Omega_{\text{GW}}(f) = \frac{f}{\rho_c} \frac{d\rho_{\text{GW}}}{df}$



ρ_{GW} is the **GW energy density**, and ρ_c is the critical energy density



The SGWB from **collapsing DWs** can be estimated by **numerical simulations**

[Hiramatsu, Kawasaki, Saikawa, 1002.1555, 1309.5001, JCAP]



The **present SGWB spectrum** induced by collapsing DWs can be evaluated by

$$\Omega_{\text{GW}}(f)h^2 = \Omega_{\text{GW}}^{\text{peak}}h^2 \times \begin{cases} \left(\frac{f}{f_{\text{peak}}}\right)^3, & f < f_{\text{peak}} \\ \frac{f_{\text{peak}}}{f}, & f > f_{\text{peak}} \end{cases}$$

$$\Omega_{\text{GW}}^{\text{peak}}h^2 = 7.2 \times 10^{-18} \tilde{\epsilon}_{\text{GW}} \mathcal{A}^2 \left[\frac{g_{*s}(T_{\text{ann}})}{10} \right]^{-4/3} \left[\frac{\sigma(T_{\text{ann}})}{\text{TeV}^3} \right]^2 \left(\frac{T_{\text{ann}}}{10 \text{ MeV}} \right)^{-4}$$

$$f_{\text{peak}} = 1.1 \times 10^{-9} \text{ Hz} \left[\frac{g_{*}(T_{\text{ann}})}{10} \right]^{1/2} \left[\frac{g_{*s}(T_{\text{ann}})}{10} \right]^{-1/3} \frac{T_{\text{ann}}}{10 \text{ MeV}}$$



$\tilde{\epsilon}_{\text{GW}} = 0.7 \pm 0.4$ is derived from numerical simulation

SGWB Spectra Compared to Sensitivity Curves



By adjusting the **DW tension** σ and the **potential bias** V_{bias} , the **SGWB spectra** from **collapsing DWs** could fall in the sensitivity bands of various GW experiments



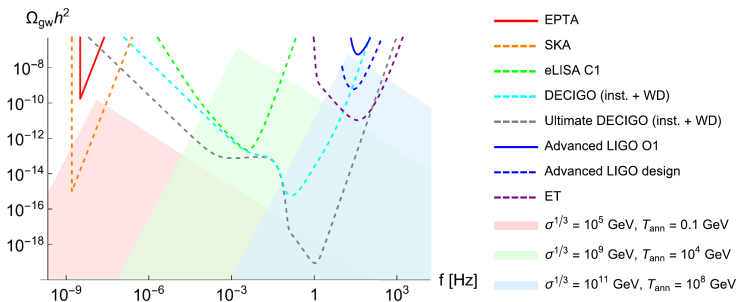
Pulsar timing arrays (PTAs) in 10^{-9} – 10^{-7} Hz: NANOGrav, PPTA, EPTA, CPTA, IPTA, SKA, ...



Space-borne interferometers in 10^{-4} – 10^0 Hz: LISA, TianQin, Taiji, BBO, DECIGO, ...



Ground-based interferometers in 10^0 – 10^4 Hz: LIGO, Virgo, KAGRA, CE, ET, ...

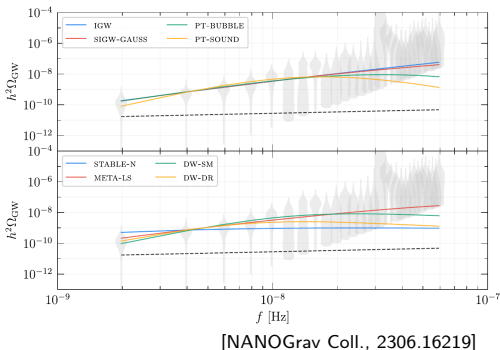
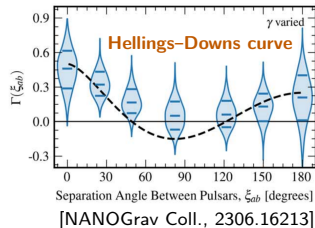


[Saikawa, 1703.02576, Universe]

Strong Evidence for a nHz SGWB from PTAs

On June 29, 2023, four **PTA collaborations**
NANOGrav [2306.16213, ApJL; 2306.16219, ApJL],
CPTA [2306.16216, RAA], **PPTA** [2306.16215, ApJL],
 and **EPTA** [2306.16214, 2306.16227, A&A] reported
strong evidence for a **nHz SGWB** with expected
Hellings–Downs correlations

-  Potential **GW sources** include
-  **Supermassive black hole binaries**
-  **Inflation**
-  **Scalar-induced GWs**
-  **First-order phase transitions**
-  **Cosmic strings**
-  **Collapsing domain walls**



nHz SGWB from DWs and Freeze-in Dark Matter

🏠 For interpreting the **nHz SGWB observation**, we assume that it comes from **collapsing DWs** arising from the **spontaneous breaking** of a Z_2 **symmetry** in a scalar field theory

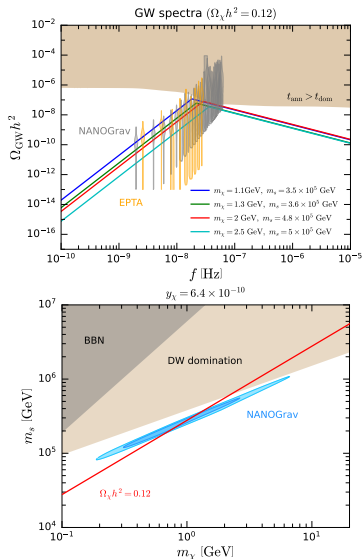
🏀 A **tiny Z_2 -violating potential term** with $\epsilon \sim \mathcal{O}(10^{-26})$ is required to reproduce the result

🏀 We propose that this Z_2 -violating potential is **radiatively induced** by a **feeble Yukawa coupling** y_χ between the scalar field and a **fermion field** χ

🏀 It is also responsible for **dark matter (DM)** production via the **freeze-in mechanism**

🏀 Combining the **PTA data** and the **observed DM relic density**, the model parameters can be narrowed down to small ranges

[Z Zhang, CF Cai, YH Su, SY Wang, **ZHY**, HH Zhang, 2307.11495, PRD]



Z_2 -violating Coupling to Thermalized Fermions



We study **DWs** formed through **spontaneous breaking** of an **approximate Z_2 symmetry**



Dynamics of DWs is influenced by **quantum** and **thermal corrections** induced by a **Z_2 -violating coupling** to **thermalized fermions**



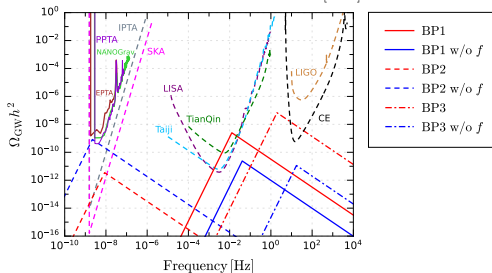
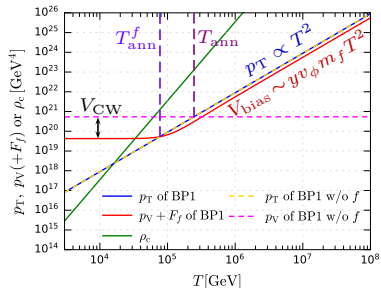
The thermal effects make the **potential bias**

V_{bias} dependent on the **temperature** and may lead to notable variations in the **DW**

annihilation temperature T_{ann} , in addition to the shift caused by **Coleman-Weinberg corrections**



This could substantially alter the **SGWB spectrum** produced by DWs, providing observable signatures for future GW detection experiments



[QQ Zeng, X He, **ZHY**, JM Zheng, 2501.10059, PRD]

Cosmic Strings from U(1) Gauge Symmetry Breaking

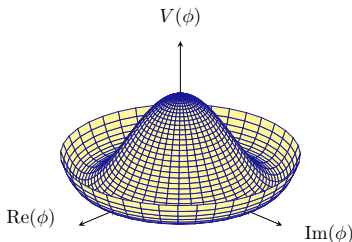
🧙 Consider the **Abelian Higgs model** with a **complex scalar field** ϕ

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + (D^\mu\phi)^\dagger D_\mu\phi - V(\phi), \quad V(\phi) = \frac{\lambda\phi}{4}(|\phi|^2 - v_\phi^2)^2$$

🌍 The covariant derivative of ϕ is $D_\mu\phi = (\partial_\mu - igA_\mu)\phi$

🧙 The field strength tensor of the **U(1) gauge field** A^μ is $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

🧸 The **Mexican-hat potential** $V(\phi)$ leads to **degenerate vacua** $\langle\phi\rangle = v_\phi e^{i\theta}/\sqrt{2}$



Cosmic Strings from U(1) Gauge Symmetry Breaking

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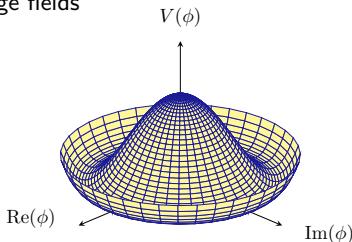
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The **spontaneous breaking** of the **U(1) gauge symmetry** in the early Universe would induce **cosmic strings (CSs)**, which are concentrated with energies of scalar and gauge fields



Degenerate vacua

$$v_\phi e^{i\theta}/\sqrt{2}$$

$$\theta = \theta + 2\pi n$$

$n \neq 0$ leads to
cosmic strings

Soliton Configuration



The **ansatz** for a **static solution (soliton)** representing a **string** along the **z axis** is

$$\phi(\rho, \varphi) = v_\phi f(\rho) U(\varphi), \quad \mathbf{A}(\rho, \varphi) = \frac{i}{g} a(\rho) U(\varphi) \nabla U^\dagger(\varphi)$$

$$U(\varphi) \equiv e^{in\varphi}, \quad f(\infty) = a(\infty) = 1, \quad f(0) = a(0) = 0$$



(ρ, φ) are polar coordinates in the xy plane



$n \in \mathbb{Z}$ is the **winding number**



The **soliton** with $n = 1$ is called a **Nielsen-Olesen vortex**

[Nielsen & Olesen, NPB **61** (1973) 45]



The **energy per unit length** of the **soliton** is [Srednicki, *Quantum Field Theory*]

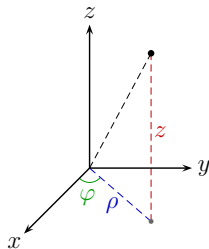
$$\mu = 2\pi v_\phi^2 \int_0^\infty d\rho \rho \left[f'^2 + \frac{n^2}{\rho^2} (a-1)^2 f^2 + \frac{\lambda v_\phi^2}{4} (f^2 - 1)^2 + \frac{n^2 a'^2}{g^2 v_\phi^2 \rho^2} \right]$$



For $\lambda_\phi > g^2$, one can prove a **Bogomolny bound** $\mu > 2\pi v_\phi^2 |n|$



A **soliton** with **winding number** $n \neq \pm 1$ is **unstable** against breaking up into $|n|$ **stable solitons**, each with **winding number** ± 1



Cosmic Strings

🎮 We can **translate** and **boost** a **stable soliton** with $n = \pm 1$

📍 It behaves like a **particle** in **two space dimensions**

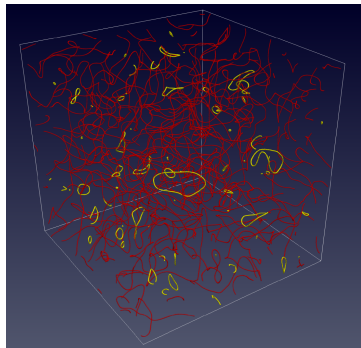
🎲 In **three space dimensions**, the soliton becomes a **Nielsen–Olesen string**

🌲 It is a structure **localized** in **two directions**,
but **extended** in the **third**

🌿 Such strings can **bend**, and even form
closed loops

🌌 When they are formed in the early universe,
we call them **cosmic strings**

🕸 Therefore, a **network** of **cosmic strings**
would be formed after the **spontaneous
breaking** of the **U(1) gauge symmetry**



[Kitajima, Nakayama, 2212.13573, JHEP]

Cosmic String Tension



The **tension** of **cosmic string** (energy per unit length) can be estimated as

$$\mu \simeq \begin{cases} 1.19\pi v_\phi^2 b^{-0.195}, & 0.01 < b < 100, \\ \frac{2.4\pi v_\phi^2}{\ln b}, & b > 100, \end{cases} \quad b \equiv \frac{2g^2}{\lambda_\phi}$$

[Hill, Hodges, Turner, PRD **37**, 263 (1988)]



As $\mu \propto v_\phi^2$, a **high symmetry-breaking scale** v_ϕ would lead to cosmic strings with **high tension**



Denoting G as the **Newtonian constant of gravitation**, the **dimensionless quantity** $G\mu$ is commonly used to describe the **CS tension**

Gravitational Waves from Cosmic Strings



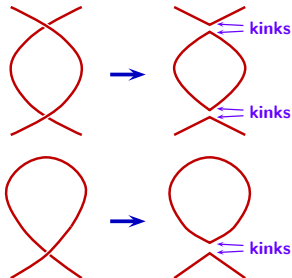
According to the analysis of string dynamics, the **intersections** of **long strings** could produce **closed loops**, whose size is smaller than the Hubble radius



Cosmic string loops could further fragment into **smaller loops** or reconnect to **long strings**



Loops typically have localized features called “**cusps**” and “**kinks**”



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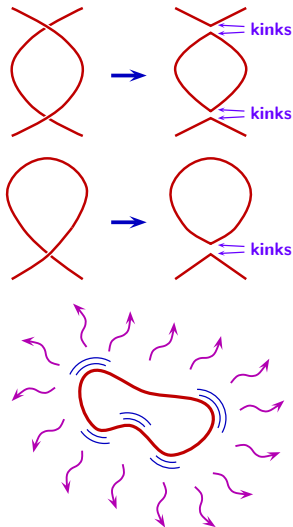
Loops typically have localized features called **“cusps”** and **“kinks”**



The **relativistic oscillations** of the **loops** due to their **tension** emit **Gravitational Waves (GWs)**, and the loops would **shrink** because of **energy loss**



Moreover, the **cusps** and **kinks** propagating along the loops could produce **GW bursts** [Damour & Vilenkin, gr-qc/0004075, PRL]



Power of Gravitational Radiation



At the **emission time** t_e , a **cosmic string loop** of **length** l emits GWs with **frequencies** $f_e = \frac{2n}{l}$



$n = 1, 2, 3, \dots$ denotes the **harmonic modes** of the loop oscillation



Denoting P_n as the **power** of **gravitational radiation** for the harmonic mode n in units of $G\mu^2$, the total power is given by $P = G\mu^2 \sum_n P_n$



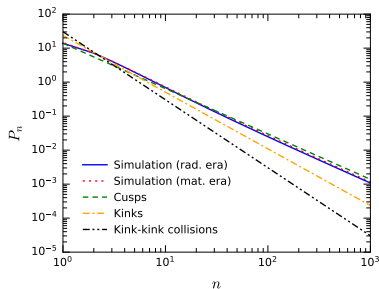
According to the **simulation** of **smoothed cosmic string loops** [Blanco-Pillado & Olum, 1709.02693, PRD], P_n for loops in the **radiation** and **matter** eras are obtained



The **total dimensionless power** $\Gamma = \sum_n P_n$ is estimated to be ~ 50



For comparison, analytic studies imply $P_n \simeq \frac{\Gamma}{\zeta(q)n^q}$ with $q = \frac{4}{3}, \frac{5}{3}, 2$ for **cusps**, **kinks**, and **kink-kink collisions**



Stochastic GW Background Induced by Cosmic Strings

🔌 The **energy** of **cosmic strings** is converted into the **energy** of **GWs**, and an **SGWB** is formed due to **incoherent superposition**

💡 The **SGWB energy density** ρ_{GW} per unit frequency at the present is

$$\frac{d\rho_{\text{GW}}}{df} = G\mu^2 \int_{t_{\text{ini}}}^{t_0} a^5(t) \sum_n \frac{2nP_n}{f^2} n_{\text{CS}} \left(\frac{2na(t)}{f}, t \right) dt$$

🕯️ $n_{\text{CS}}(l, t)$ is the **number density per unit length** of **CS loops** with length l at cosmic time t

🕯️ $a(t)$ is the **scale factor** satisfying $\frac{da(t)}{dt} = a(t)H(t)$ and $a(t_0) = 1$

🔭 $H(t)$ is the **Hubble rate** and t_{ini} is the cosmic time when the GW emissions start

💡 The **SGWB spectrum** is commonly represented by

$$\Omega_{\text{GW}}(f) = \frac{f}{\rho_c} \frac{d\rho_{\text{GW}}}{df}, \quad \rho_c \equiv \frac{3H_0^2}{8\pi G}$$

Velocity-dependent One-scale Model



The evolution of the **CS network** can be described using the **velocity-dependent one-scale (VOS) model** [Martins & Shellard, hep-ph/9507335, PRD]



The parameters are the **correlation length** L and the **root-mean-square velocity** v of string segments; the **energy density** of **long strings** is expressed as $\rho = \mu/L^2$



Introducing a **dimensionless quantity** $\xi \equiv L/t$, the evolution equations are

$$t\dot{\xi} = H(1 + v^2)t\xi - \xi + \frac{1}{2}\tilde{c}v, \quad t\dot{v} = (1 - v^2) \left[\frac{k(v)}{\xi} - 2Htv \right]$$

$$\tilde{c} \simeq 0.23, \quad k(v) = \frac{2\sqrt{2}}{\pi}(1 - v^2)(1 + 2\sqrt{2}v^3) \frac{1 - 8v^6}{1 + 8v^6}$$



The solutions converge to **constant values** [Marfatia & YL Zhou, 2312.10455, JHEP]:

$$\xi_r = 0.271, \quad v_r = 0.662, \quad \text{radiation-dominated (RD) era}$$

$$\xi_m = 0.625, \quad v_m = 0.582, \quad \text{matter-dominated (MD) era}$$



This implies that the CS network quickly evolves into a **linear scaling regime** characterized by $L \propto t$

Loop Production Functions

🍊 The **CS loop number density** is given by $n_{\text{CS}}(l, t) = \frac{1}{a^3(t)} \int_{t_{\text{ini}}}^t \mathcal{P}(l', t') a^3(t') dt'$

🍏 Motivated by **numerical simulations** [Blanco-Pillado, Olum & Shlaer, 1309.6637, PRD], the **loop production functions** can be approximated as

$$\mathcal{P}_r(l, t) = \frac{\mathcal{F}_r \tilde{c} v \delta(\alpha_r \xi - l/t)}{\gamma_v \alpha_r \xi^4 t^5}, \quad \text{RD era}$$

$$\mathcal{P}_m(l, t) = \frac{\mathcal{F}_m \tilde{c} v \Theta(\alpha_m \xi - l/t)}{\gamma_v (l/t)^{1.69} \xi^3 t^5}, \quad \text{MD era}$$

🍏 $\gamma_v = (1 - v^2)^{-1/2}$ is the Lorentz factor

🍒 At the **loop production time** t_* , we have

$$l_* = l + \Gamma G \mu (t - t_*), \quad \alpha_r \xi_* \simeq 0.1 \text{ and } \alpha_m \xi_* \simeq 0.18$$

🍏 Adopting $\mathcal{F}_r = 0.1$ and $\mathcal{F}_m = 0.36$, the obtained **loop number densities** in the **RD** and **MD eras** **agrees** with the **simulation results** in the **scaling regime**

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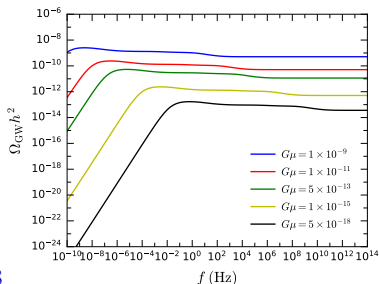
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🍏 The **SGWB spectra** in the Λ CDM cosmological model can be calculated



Scaling Loop Number Density: BOS model



There are other approaches for modeling the $n_{CS}(l, t)$ in the **scaling regime**



The **BOS model** [Blanco-Pillado, Olum & Shlaer, 1309.6637, PRD] extrapolates the loop production function found in simulations of Nambu-Goto strings



The loop number densities produced in the **radiation** and **matter** era, and that **produced in the radiation era and still surviving in the matter era** are given by

$$n_{CS}^r(l, t) \simeq \frac{0.18 \theta(0.1t - l)}{t^4 (\gamma + \gamma_d)^{5/2}}$$

$$n_{CS}^m(l, t) \simeq \frac{(0.27 - 0.45\gamma^{0.31}) \theta(0.18t - l)}{t^4 (\gamma + \gamma_d)^2}$$

$$n_{CS}^{r \rightarrow m}(l, t) \simeq \frac{0.18 t_{eq}^{1/2} \theta(0.09 t_{eq} - \gamma_d t - l)}{t^{9/2} (\gamma + \gamma_d)^{5/2}}$$



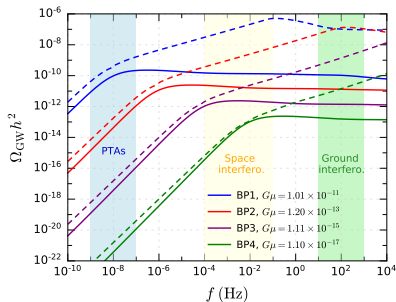
$\gamma \equiv \frac{l}{t}$ is a **dimensionless variable**



$\gamma_d = -\frac{dl}{dt} \simeq \Gamma G\mu$ is the **loop shrinking rate**



$t_{eq} = 51.1 \pm 0.8$ kyr is the cosmic time at the **matter-radiation equality**



BOS model: solid lines

Scaling Loop Number Density: LRS model



The **LRS model** [Lorenz, Ringeval & Sakellariadou, 1006.0931, JCAP] takes into account the **gravitational backreaction effect**, which prevents loop production below a certain scale $\gamma_c \simeq 20(G\mu)^{1+2\chi}$ [Polchinski & Rocha, gr-qc/0702055, PRD]

$$n_{CS}(l, t) \simeq \begin{cases} \frac{C}{t^4(\gamma + \gamma_d)^{3-2\chi}}, & \gamma_d < \gamma \\ \frac{(3\nu - 2\chi - 1)C}{2t^4(1 - \chi)\gamma_d\gamma^{2(1-\chi)}}, & \gamma_c < \gamma < \gamma_d \\ \frac{(3\nu - 2\chi - 1)C}{2t^4(1 - \chi)\gamma_d\gamma_c^{2(1-\chi)}}, & \gamma < \gamma_c \end{cases}$$



RD era: $\nu = 1/2$, $C \simeq 0.0796$, $\chi \simeq 0.2$



MD era: $\nu = 3/2$, $C \simeq 0.0157$, $\chi \simeq 0.295$

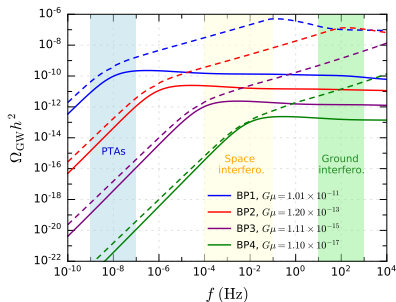


Smaller $G\mu$ means smaller GW emission power,

and loops could survive longer, leading to **more smaller loops** radiating at **higher f**



The **LRS model** gives a **very high number density** of **small loops** in the $\gamma < \gamma_c$ regime, which significantly contribute to **high frequency GWs**



LRS model: dashed lines

GWs from Cosmic Strings Associated with pNGB Dark Matter



We study the **SGWB** from **cosmic strings** generated in a UV-complete model for **pNGB DM** with a **spontaneously broken $U(1)_X$ gauge symmetry** [DY Liu, CF Cai, XM Jiang, ZHY, HH Zhang, 2208.06653, JHEP]



The DM candidate in this model can **naturally evade direct detection bounds**



The **bound** on the **DM lifetime** implies a symmetry-breaking scale $v_\Phi > 10^9$ GeV

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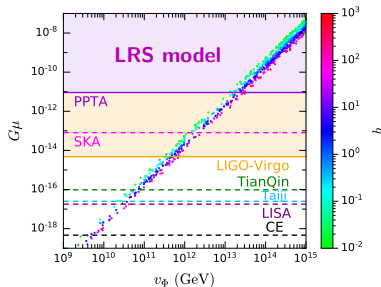
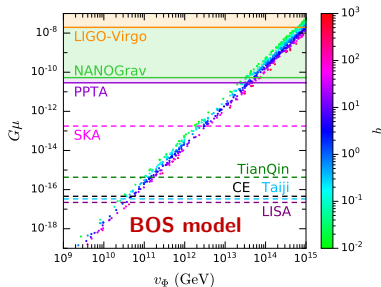
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🐮 The **bound** on the **DM lifetime** implies a symmetry-breaking scale $v_\Phi > 10^9$ GeV

🐎 Constraints from **LIGO-Virgo**, **NANOGrav**, and **PPTA** have excluded the parameter points with $v_\Phi \gtrsim 5 \times 10^{13}$ (7×10^{11}) GeV

🦄 The future experiment **LISA (CE)** can probe v_Φ down to $\sim 2 \times 10^{10}$ (5×10^9) GeV assuming the **BOS (LRS)** model for loop production

[ZY Qiu, ZHY, 2304.02506, CPC]



Early Matter-dominated Era and Cosmic String GWs



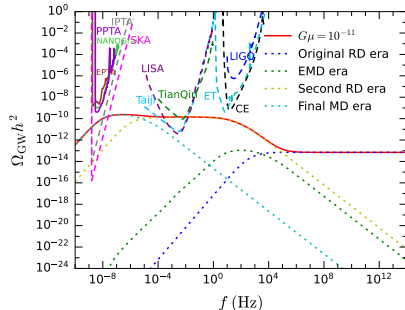
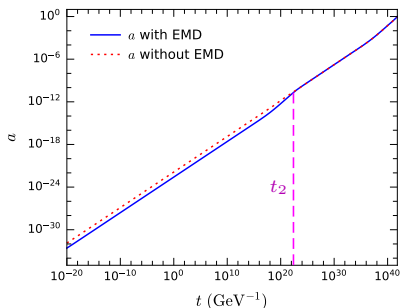
We investigate the influence of an **early matter-dominated (EMD)** era in cosmic history on the **dynamics** of **cosmic strings** based on the **VOS model**



For a particle model related to the **DM dilution mechanism**, we analyze the modifications to the **cosmological scale factor** and the **CS loop number density**



The **EMD era** causes a **characteristic suppression** in the **high-frequency regime** of the **SGWB spectrum**



[SQ Ling & ZHY, 2502.16576, CPC]

Summary

- In the early Universe, the **spontaneous breaking** of **symmetries** could lead to **topological defects**, such as **monopoles**, **domain walls** and **cosmic strings**
- **Collapsing domain walls** and **cosmic strings** may result in a **stochastic GW background**, which could be probed in future GW experiments

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Thanks for your attention!

Symmetry Breaking $G \rightarrow H$



Consider that **some scalar fields** acquire nonzero **vacuum expectation values** (VEVs), which **break** a **symmetry group** G to a **subgroup** H ($G \rightarrow H$)



For a **VEV** ϕ_0 , the action by any element $h \in H$ is **trivial**: $h\phi_0 = \phi_0$



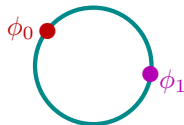
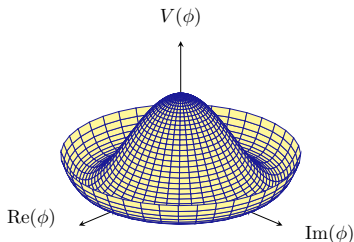
A **nontrivial** action on ϕ_0 : $g\phi_0 = \phi_1$, $g \in G$, $g \notin H$ 👉 $gh\phi_0 = \phi_1$



All the **nontrivial** transformations are given by the **left cosets** of H (e.g., gH), which constitute the **coset space** G/H



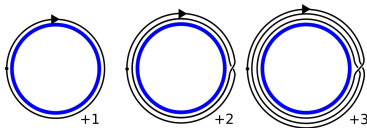
G/H is **isomorphic** to the **manifold** consisting of all **degenerate vacua**



Vacuum manifold

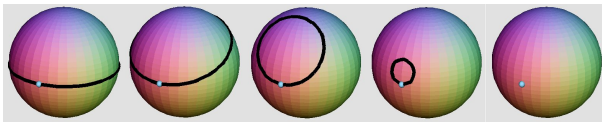
Homotopy Groups $\pi_n(G/H)$

 The **topology** of the **vacuum manifold** G/H can be characterized by its n -th **homotopy group** $\pi_n(G/H)$, which is constituted by the **homotopy classes** of the mappings from an n -dimensional **sphere** S^n into G/H



$$\pi_1(S^1) = \mathbb{Z}$$

A homotopy class is identified with a winding number n



$$\pi_1(S^2) = 1 \text{ (trivial)}$$

Any continuous mapping from S^1 to S^2 can be continuously deformed to a 1-point mapping



$$\pi_2(S^2) = \mathbb{Z}$$

Mappings from S^2 to S^2 can be visualized as wrapping a twisted plastic bag around a ball n times

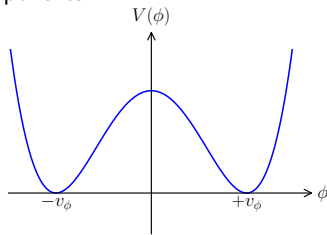
 How S^2 can be wrapped twice around another S^2

Topological Defects

🌀 A **nontrivial** $\pi_n(G/H)$ leads **topological defects** [Kibble, J. Phys. A9 (1976) 1387]

■ **Nontrivial** $\pi_0(G/H)$: two or more disconnected components

👉 **Domain walls** (2-dim topological defects)



- $G/H \cong Z_2$
- $\pi_0(G/H) = Z_2$

Topological Defects

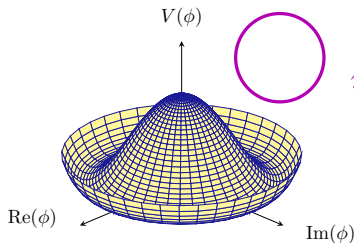
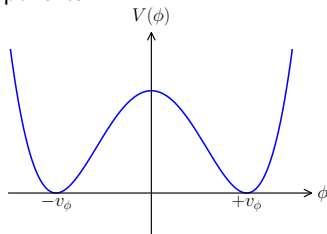
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👉 **Domain walls** (2-dim topological defects)

🌀 **Nontrivial** $\pi_1(G/H)$: incontractable closed paths

👉 **Cosmic strings** (1-dim topological defects)



$$G/H \cong S^1$$

$$\pi_1(G/H) = \mathbb{Z}$$

- $G/H \cong Z_2$
- $\pi_0(G/H) = Z_2$

Topological Defects

🌀 A **nontrivial** $\pi_n(G/H)$ leads **topological defects** [Kibble, J. Phys. A9 (1976) 1387]

■ **Nontrivial** $\pi_0(G/H)$: two or more disconnected components

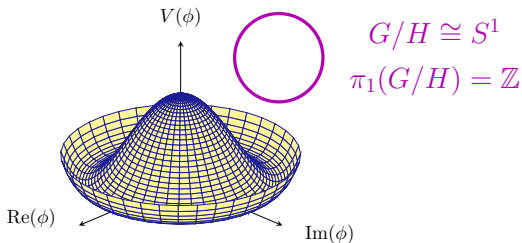
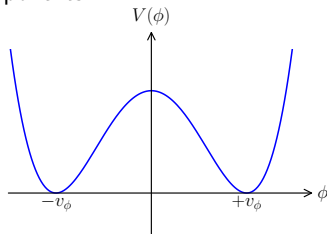
👉 **Domain walls** (2-dim topological defects)

🌀 **Nontrivial** $\pi_1(G/H)$: incontractable closed paths

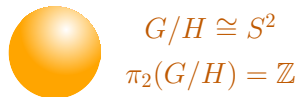
👉 **Cosmic strings** (1-dim topological defects)

🟠 **Nontrivial** $\pi_2(G/H)$: incontractable spheres

👉 **Monopoles** (0-dim topological defects)



- $G/H \cong Z_2$
- $\pi_0(G/H) = Z_2$



Hybrid Topological Defects



For a field theory, such as a **GUT**, where **multiple stages** of **symmetry breaking** give rise to **topological defects**, **hybrid defects** may form [Vilenkin & Shellard, *Cosmic Strings and Other Topological Defects*; Dunskey, et al., 2111.08750, PRD]



Monopoles attach to **cosmic strings** [Vilenkin, NPB **196** (1982) 240]

$$G \xrightarrow{\text{monopoles}} H \times \text{U}(1) \xrightarrow{\text{strings}} H, \quad \pi_1(G/H) = 1$$



1 Monopoles form when G breaks to a subgroup containing a **U(1) symmetry**



2 Strings form and connect to **monopoles** when this **U(1) symmetry** is later broken



Cosmic strings attach to **domain walls** [Kibble, Lazarides, Shafi, PRD **26** (1982) 435]

$$G \xrightarrow{\text{strings}} H \times \text{Z}_2 \xrightarrow{\text{walls}} H, \quad \pi_0(G/H) = 1$$



1 Strings form when G breaks to a subgroup containing a **discrete symmetry** with $\pi_1[G/(H \times \text{Z}_2)] \supset \pi_0(H \times \text{Z}_2) \neq 1$



2 Walls form and connect to **strings** when the same **discrete symmetry** associated with the strings is broken

Strings Eating Monopoles

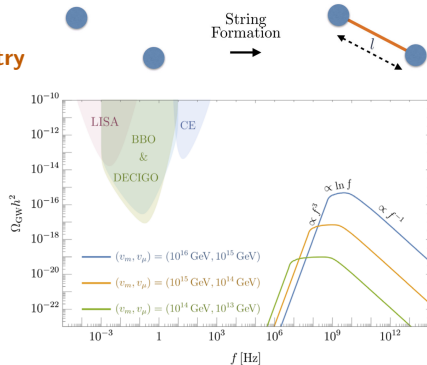
Such **hybrid defects** are **unstable**, with **one defect “eating” the other** via the conversion of the **rest mass** of **the latter** into the **kinetic energy** of **the former**, and subsequently, **decaying** via **gravitational waves**

Strings eating monopoles [Lazarides, Shafi, Walsh, NPB **195** (1982) 157]

A **monopole network** form at a **scale** v_m

At temperatures below the **string symmetry breaking scale** v_μ , the magnetic field of the **monopoles** squeezes into **flux tubes (cosmic strings)** connecting each monopole-antimonopole pair

GW emission occurs in a burst, **peaking** at **high frequencies** corresponding to the monopole-antimonopole separation distance at $T \simeq v_\mu$



[Dunskey, *et al.*, 2111.08750, PRD]

Monopoles Eating Strings



Monopoles eating strings (metastable cosmic strings)



The **symmetry breaking chains** are the **same** as in the previous case



Inflation occurs after **monopole formation** but before **string formation**



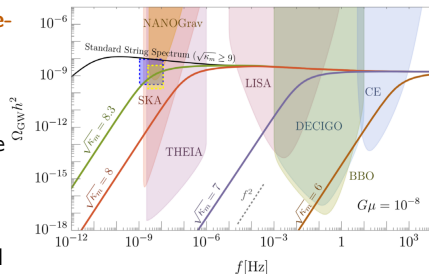
Because of the **absence** of **monopoles**, a **normal string network** forms



Nevertheless, the **strings** of **tension** μ can later become **bounded** by **monopoles** of **mass** m by the **Schwinger nucleation** of **monopole-antimonopole pairs**, which cut the strings into pieces bounded by monopoles



Conversion of the string rest mass into the monopole kinetic energy leads to **relativistic oscillations** of the monopoles before the system decays via **gravitational radiation** and monopole annihilation ($\kappa_m = m^2/\mu$)



[Dunskey, et al., 2111.08750, PRD]

Domain Walls Eating Strings

Domain walls eating strings

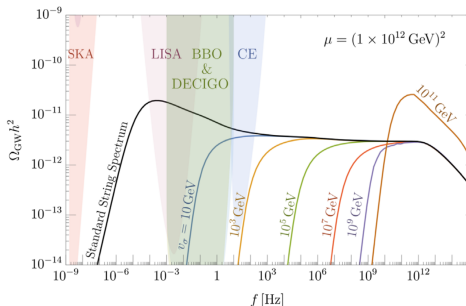
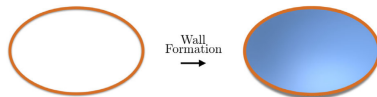
🍩 A **string network** form at a **scale** v_μ

🥟 At temperatures below the **wall symmetry breaking scale** v_σ , **walls** fill in the space between **strings**

👉 A **wall-string network** forms, evolves, and finally **collapses** and **decays**

🔨 Prior to wall domination at t_* , the **wall-string network** behaves similarly to a **pure string network**, resulting the **GW spectrum** $\Omega_{\text{GW}} \propto f^0$ at **high frequencies**

🍝 After the network collapses and the largest string-bounded walls decay, Ω_{GW} drops as f^3 at **low frequencies**



[Dunskey, et al., 2111.08750, PRD]

Strings Eating Domain Walls



Strings eating domain walls



The **symmetry breaking chains** are the **same** as in the previous case



Inflation occurs after **string formation** but before **domain wall formation**



Because of the **absence** of **strings**, a **normal wall network** forms

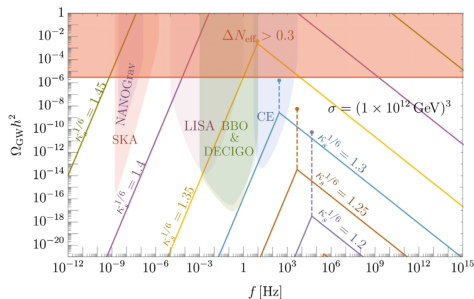
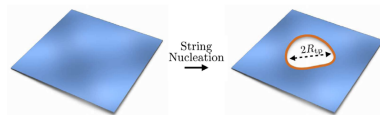


Nevertheless, the **walls** can become **bounded** by **strings** later by the **Schwinger nucleation** of **string holes**



Conversion of wall rest mass into string kinetic energy causes the **string** of **tension** μ to **rapidly expand** and **eat** the **wall** of **tension** σ , causing the **wall network** to **decay** with **GW**

emissions ($\kappa_s = \mu^3/\sigma^2$)



[Dunskey, *et al.*, 2111.08750, PRD]