Current Challenges in Cosmology, Cali, Colombia

Cosmological Effects of Fluids with Microstructure (Hyperfluids)

Damianos Iosifidis

May 21, 2026

Scuola Superiore Meridionale (SSM), Napoli, Italy
INFN– Sezione di Napoli, Italy

Outline

- Non-Riemannian Geometry: Conventions/Notation

Outline

- Non-Riemannian Geometry: Conventions/Notation
- Conservation Laws of Metric-Affine Gravity

Outline

- Non-Riemannian Geometry: Conventions/Notation
- Conservation Laws of Metric-Affine Gravity
- Hyperfluids, Torsion and Non-metricity in Cosmology
- Including matter's microstructure in isotropic Cosmology

Outline

- Non-Riemannian Geometry: Conventions/Notation
- Conservation Laws of Metric-Affine Gravity
- Hyperfluids, Torsion and Non-metricity in Cosmology
- Including matter's microstructure in isotropic Cosmology
- A simple model with Spin and Shear charges

Outline

- Non-Riemannian Geometry: Conventions/Notation
- Conservation Laws of Metric-Affine Gravity
- Hyperfluids, Torsion and Non-metricity in Cosmology
- Including matter's microstructure in isotropic Cosmology
- A simple model with Spin and Shear charges
- Conclusions/Further Prospects

The talk is mostly based on the papers

- "Cosmological Hyperfluids, Torsion and Non-metricity" Published in: Eur.Phys.J.C 80 (2020) 11, 1042 • e-Print: 2003.07384 [gr-qc] (**DI**)
- "The Perfect Hyperfluid of Metric-Affine Gravity: The Foundation" Published in: JCAP 04 (2021) 072 • e-Print: 2101.07289 [gr-qc] (**DI**)
- Friedmann cosmology with hyperfluids Published in: Phys.Rev.D 111 (2025) 6, 064063 • e-Print: 2411.19127 [gr-qc] (Ilaria Andrei,**DI**, Laur Jarv, Margus Saal)

Metric-Affine Gravity

Metric Gravity

- $\Gamma^{\alpha}{}_{\mu\nu} \rightarrow$ *torsionless* , metric compatibility $\nabla_{\sigma} g_{\mu\nu} = 0$
- $S = S_{Gravity} + S_{Matter} = \int d^n x \sqrt{-g}\, [\mathcal{L}_G(g_{\mu\nu}) + \mathcal{L}_M(g_{\mu\nu}, \Phi)]$

Metric-Affine Gravity

Metric Gravity

- $\Gamma^{\alpha}{}_{\mu\nu} \to$ *torsionless* , metric compatibility $\nabla_{\sigma} g_{\mu\nu} = 0$
- $S = S_{Gravity} + S_{Matter} = \int d^n x \sqrt{-g} \left[\mathcal{L}_G(g_{\mu\nu}) + \mathcal{L}_M(g_{\mu\nu}, \Phi) \right]$

Teleparallel/Symmetric Teleparallel Gravity

- $R^{\alpha}{}_{\beta\mu\nu} = 0$, $\nabla_{\sigma} g_{\mu\nu} = 0$ but $S_{\mu\nu}{}^{\alpha} = \Gamma^{\alpha}{}_{[\mu\nu]} \neq 0$
- $R^{\alpha}{}_{\beta\mu\nu} = 0$, $S_{\mu\nu}{}^{\alpha} = 0$ but $Q_{\alpha\mu\nu} = -\nabla_{\alpha} g_{\mu\nu} \neq 0$

Metric-Affine Gravity

Metric Gravity

- $\Gamma^{\alpha}{}_{\mu\nu} \rightarrow$ *torsionless* , metric compatibility $\nabla_{\sigma} g_{\mu\nu} = 0$
- $S = S_{Gravity} + S_{Matter} = \int d^n x \sqrt{-g} \left[\mathcal{L}_G(g_{\mu\nu}) + \mathcal{L}_M(g_{\mu\nu}, \Phi)\right]$

Teleparallel/Symmetric Teleparallel Gravity

- $R^{\alpha}{}_{\beta\mu\nu} = 0$, $\nabla_{\sigma} g_{\mu\nu} = 0$ but $S_{\mu\nu}{}^{\alpha} = \Gamma^{\alpha}{}_{[\mu\nu]} \neq 0$
- $R^{\alpha}{}_{\beta\mu\nu} = 0$, $S_{\mu\nu}{}^{\alpha} = 0$ but $Q_{\alpha\mu\nu} = -\nabla_{\alpha} g_{\mu\nu} \neq 0$

Metric-Affine Gravity (MAG)

- $S = \int d^n x \sqrt{-g} \left[\mathcal{L}_G(g_{\mu\nu}, \Gamma^{\alpha}{}_{\mu\nu}) + \mathcal{L}_M(g_{\mu\nu}, \Gamma^{\alpha}{}_{\mu\nu}, \Phi)\right] \Rightarrow$ No a priori constraints on the geometry.

Geometrical Objects

Two distinctively different notions on a manifold

- Metric Tensor $g_{\mu\nu}$: Defines distances,lengths and dot products

$$||\alpha||^2 := \alpha^\mu \alpha^\nu g_{\mu\nu} \ , \ \ (\alpha \cdot \beta) := \alpha^\mu \beta^\nu g_{\mu\nu}$$

Geometrical Objects

Two distinctively different notions on a manifold

- Metric Tensor $g_{\mu\nu}$: Defines distances,lengths and dot products

$$||\alpha||^2 := \alpha^\mu \alpha^\nu g_{\mu\nu} \, , \;\; (\alpha \cdot \beta) := \alpha^\mu \beta^\nu g_{\mu\nu}$$

- Affine-Connection $\Gamma^\lambda{}_{\mu\nu}$: Defines parallel transport of tensor fields on the manifold

$$\nabla_\lambda u^\mu = \partial_\lambda u^\mu + \Gamma^\mu{}_{\nu\lambda} u^\nu$$

Geometrical Objects

Two distinctively different notions on a manifold

- Metric Tensor $g_{\mu\nu}$: Defines distances,lengths and dot products

$$||\alpha||^2 := \alpha^\mu \alpha^\nu g_{\mu\nu} \ , \ \ (\alpha \cdot \beta) := \alpha^\mu \beta^\nu g_{\mu\nu}$$

- Affine-Connection $\Gamma^\lambda{}_{\mu\nu}$: Defines parallel transport of tensor fields on the manifold

$$\nabla_\lambda u^\mu = \partial_\lambda u^\mu + \Gamma^\mu{}_{\nu\lambda} u^\nu$$

The two need not be related a priori! Their relation may be found after solving the field equations!

Geometrical Objects

Torsion

- $\nabla_{[\mu}\nabla_{\nu]}\phi = S_{\mu\nu}{}^{\lambda}\nabla_{\lambda}\phi$, Torsion Tensor $\boxed{S_{\mu\nu}{}^{\lambda} := \Gamma^{\lambda}{}_{[\mu\nu]}}$

Geometrical Objects

Torsion

- $\nabla_{[\mu}\nabla_{\nu]}\phi = S_{\mu\nu}{}^{\lambda}\nabla_{\lambda}\phi$, Torsion Tensor $\boxed{S_{\mu\nu}{}^{\lambda} := \Gamma^{\lambda}{}_{[\mu\nu]}}$

Curvature

- $[\nabla_\alpha, \nabla_\beta]u^\mu = R^\mu{}_{\nu\alpha\beta}u^\nu + 2S_{\alpha\beta}{}^{\nu}\nabla_\nu u^\mu$

  Curvature Tensor: $\boxed{R^\mu{}_{\nu\alpha\beta} := 2\partial_{[\alpha}\Gamma^\mu{}_{|\nu|\beta]} + 2\Gamma^\mu{}_{\rho[\alpha}\Gamma^\rho{}_{|\nu|\beta]}}$

Geometrical Objects

Torsion

- $\nabla_{[\mu}\nabla_{\nu]}\phi = S_{\mu\nu}{}^{\lambda}\nabla_{\lambda}\phi$, Torsion Tensor $\boxed{S_{\mu\nu}{}^{\lambda} := \Gamma^{\lambda}{}_{[\mu\nu]}}$

Curvature

- $[\nabla_{\alpha}, \nabla_{\beta}]u^{\mu} = R^{\mu}{}_{\nu\alpha\beta}u^{\nu} + 2S_{\alpha\beta}{}^{\nu}\nabla_{\nu}u^{\mu}$

  Curvature Tensor: $\boxed{R^{\mu}{}_{\nu\alpha\beta} := 2\partial_{[\alpha}\Gamma^{\mu}{}_{|\nu|\beta]} + 2\Gamma^{\mu}{}_{\rho[\alpha}\Gamma^{\rho}{}_{|\nu|\beta]}}$

Non-Metricity

- $\boxed{Q_{\alpha\mu\nu} := -\nabla_{\alpha}g_{\mu\nu}} = -\partial_{\alpha}g_{\mu\nu} + \Gamma^{\lambda}{}_{\mu\alpha}g_{\lambda\nu} + \Gamma^{\lambda}{}_{\nu\alpha}g_{\lambda\mu}$

Contractions

Contractions of Curvature

- Ricci Tensor: $R_{\mu\nu} := R^{\alpha}{}_{\mu\alpha\nu}$

Contractions

Contractions of Curvature

- Ricci Tensor: $R_{\mu\nu} := R^{\alpha}{}_{\mu\alpha\nu}$
- Homothetic Curvature: $\widehat{R}_{\alpha\beta} := R^{\mu}{}_{\mu\alpha\beta}$

Contractions

Contractions of Curvature

- Ricci Tensor: $R_{\mu\nu} := R^{\alpha}{}_{\mu\alpha\nu}$
- Homothetic Curvature: $\widehat{R}_{\alpha\beta} := R^{\mu}{}_{\mu\alpha\beta}$
- Co-Ricci Tensor: $\check{R}_{\mu\nu} := R_{\mu\alpha\beta\nu} g^{\alpha\beta}$

Contractions

Contractions of Curvature

- Ricci Tensor: $R_{\mu\nu} := R^{\alpha}{}_{\mu\alpha\nu}$
- Homothetic Curvature: $\widehat{R}_{\alpha\beta} := R^{\mu}{}_{\mu\alpha\beta}$
- Co-Ricci Tensor: $\check{R}_{\mu\nu} := R_{\mu\alpha\beta\nu}g^{\alpha\beta}$
- Ricci Scalar: $R := R_{\mu\nu}g^{\mu\nu} = -\check{R}_{\mu\nu}g^{\mu\nu}$

Contractions

Contractions of Curvature

- Ricci Tensor: $R_{\mu\nu} := R^{\alpha}{}_{\mu\alpha\nu}$
- Homothetic Curvature: $\widehat{R}_{\alpha\beta} := R^{\mu}{}_{\mu\alpha\beta}$
- Co-Ricci Tensor: $\check{R}_{\mu\nu} := R_{\mu\alpha\beta\nu}g^{\alpha\beta}$
- Ricci Scalar: $R := R_{\mu\nu}g^{\mu\nu} = -\check{R}_{\mu\nu}g^{\mu\nu}$

Torsion/Non-metricity related vectors

$$S_{\mu} = S_{\mu\lambda}{}^{\lambda}\,, \qquad \tilde{S}^{\mu} = \epsilon^{\mu\nu\rho\sigma}S_{\nu\rho\sigma} \;\; (only\;\; for\;\; n = 4)$$

$$Q_{\mu} = g^{\alpha\beta}Q_{\mu\alpha\beta}\,, \qquad q_{\mu} = g^{\rho\alpha}Q_{\rho\alpha\mu}$$

Affine Connection

Affine connection decomposition

$$\Gamma^{\lambda}{}_{\mu\nu} = \tilde{\Gamma}^{\lambda}{}_{\mu\nu} + \frac{1}{2} g^{\alpha\lambda}(Q_{\mu\nu\alpha} + Q_{\nu\alpha\mu} - Q_{\alpha\mu\nu}) - g^{\alpha\lambda}(S_{\alpha\mu\nu} + S_{\alpha\nu\mu} - S_{\mu\nu\alpha})$$

where $\tilde{\Gamma}^{\lambda}{}_{\mu\nu} := \frac{1}{2} g^{\alpha\lambda}(\partial_\mu g_{\nu\alpha} + \partial_\nu g_{\alpha\mu} - \partial_\alpha g_{\mu\nu})$ is the Levi-Civita part of the connection. Distortion: $N^{\lambda}{}_{\mu\nu} := \Gamma^{\lambda}{}_{\mu\nu} - \tilde{\Gamma}^{\lambda}{}_{\mu\nu}$

Affine Connection

Affine connection decomposition

$$\Gamma^{\lambda}{}_{\mu\nu} = \tilde{\Gamma}^{\lambda}{}_{\mu\nu} + \frac{1}{2} g^{\alpha\lambda}(Q_{\mu\nu\alpha} + Q_{\nu\alpha\mu} - Q_{\alpha\mu\nu}) - g^{\alpha\lambda}(S_{\alpha\mu\nu} + S_{\alpha\nu\mu} - S_{\mu\nu\alpha})$$

where $\tilde{\Gamma}^{\lambda}{}_{\mu\nu} := \frac{1}{2} g^{\alpha\lambda}(\partial_\mu g_{\nu\alpha} + \partial_\nu g_{\alpha\mu} - \partial_\alpha g_{\mu\nu})$ is the Levi-Civita part of the connection. Distortion: $N^{\lambda}{}_{\mu\nu} := \Gamma^{\lambda}{}_{\mu\nu} - \tilde{\Gamma}^{\lambda}{}_{\mu\nu}$

Post-Riemannian expansions

Each quantity $\Rightarrow$ decomposed into Riemannian and non-Riemannian counterparts. Example:

$$R = \tilde{R} + S_{\mu\nu\alpha}S^{\mu\nu\alpha} - 2S_{\mu\nu\alpha}S^{\alpha\mu\nu} - 4S_\mu S^\mu - 4\tilde{\nabla}_\mu S^\mu$$
$$+ \frac{1}{4}Q_{\alpha\mu\nu}Q^{\alpha\mu\nu} - \frac{1}{2}Q_{\alpha\mu\nu}Q^{\mu\nu\alpha} - \frac{1}{4}Q_\mu Q^\mu + \frac{1}{2}Q_\mu q^\mu$$
$$+ 2Q_{\alpha\mu\nu}S^{\alpha\mu\nu} + 2S_\mu(q^\mu - Q^\mu) + \tilde{\nabla}_\mu(q^\mu - Q^\mu - 4S^\mu)$$

Hypermomentum, Canonical and Metrical Energy Momentum Tensors

Metrical and Canonical Energy Momentum Tensor

Metrical: $T_{\alpha\beta} := -\frac{2}{\sqrt{-g}}\frac{\delta S_M}{\delta g^{\alpha\beta}}$. Canonical: $t^{\mu}{}_{c} = \frac{1}{\sqrt{-g}}\frac{\delta S_M}{\delta e_{\mu}{}^{c}}$

Hypermomentum, Canonical and Metrical Energy Momentum Tensors

Metrical and Canonical Energy Momentum Tensor

Metrical: $T_{\alpha\beta} := -\frac{2}{\sqrt{-g}}\frac{\delta S_M}{\delta g^{\alpha\beta}}$. Canonical: $t^{\mu}{}_{c} = \frac{1}{\sqrt{-g}}\frac{\delta S_M}{\delta e_{\mu}{}^{c}}$

Hypermomentum Tensor

Hypermomentum: $\Delta_{\lambda}{}^{\mu\nu} := -\frac{2}{\sqrt{-g}}\frac{\delta S_M}{\delta \Gamma^{\lambda}{}_{\mu\nu}}$

Hypermomentum, Canonical and Metrical Energy Momentum Tensors

Metrical and Canonical Energy Momentum Tensor

Metrical: $T_{\alpha\beta} := -\frac{2}{\sqrt{-g}}\frac{\delta S_M}{\delta g^{\alpha\beta}}$. Canonical: $t^{\mu}{}_{c} = \frac{1}{\sqrt{-g}}\frac{\delta S_M}{\delta e_{\mu}{}^{c}}$

Hypermomentum Tensor

Hypermomentum: $\Delta_{\lambda}{}^{\mu\nu} := -\frac{2}{\sqrt{-g}}\frac{\delta S_M}{\delta \Gamma^{\lambda}{}_{\mu\nu}}$

Relation Between Energy Tensors

$$t^{\mu}{}_{\lambda} = T^{\mu}{}_{\lambda} - \frac{1}{2\sqrt{-g}}\hat{\nabla}_{\nu}(\sqrt{-g}\Delta_{\lambda}{}^{\mu\nu})$$

where $\hat{\nabla}_{\nu} = 2S_{\nu} - \nabla_{\nu}$.

Physical Role of Hypermomentum

Hypermomentum Split

The Hypermomentum tensor is a generalization of the spin tensor (including also a symm+etric part) that fully describes the micro-properties of matter. It splits into its 3 physical parts of spin, dilation and shear according to:

Physical Role of Hypermomentum

Hypermomentum Split

The Hypermomentum tensor is a generalization of the spin tensor (including also a symm+etric part) that fully describes the micro-properties of matter. It splits into its 3 physical parts of spin, dilation and shear according to:

$$\tau_{\mu\nu\alpha} := \Delta_{[\mu\nu]\alpha} \tag{1}$$

$$\Delta_{\alpha} := \Delta_{\mu\nu\alpha} g^{\mu\nu} \tag{2}$$

$$\Sigma_{\mu\nu\alpha} := \Delta_{\mu\nu\alpha} - \frac{\Delta_{\alpha}}{n} g_{\mu\nu} \tag{3}$$

Physical Role of Hypermomentum

Hypermomentum Split

The Hypermomentum tensor is a generalization of the spin tensor (including also a symm+etric part) that fully describes the micro-properties of matter. It splits into its 3 physical parts of spin, dilation and shear according to:

$$\tau_{\mu\nu\alpha} := \Delta_{[\mu\nu]\alpha} \quad (1)$$

$$\Delta_{\alpha} := \Delta_{\mu\nu\alpha} g^{\mu\nu} \quad (2)$$

$$\Sigma_{\mu\nu\alpha} := \Delta_{\mu\nu\alpha} - \frac{\Delta_{\alpha}}{n} g_{\mu\nu} \quad (3)$$

Dilation and Shear are manifestations of the hadronic properties of matter.

Conservation Laws

Working in exterior calculus from the GL and diff invariance we get

From GL

$$t^{\mu}{}_{\lambda} = T^{\mu}{}_{\lambda} - \frac{1}{2\sqrt{-g}} \hat{\nabla}_{\nu}(\sqrt{-g}\Delta_{\lambda}{}^{\mu\nu})$$

From Diff

$$\frac{1}{\sqrt{-g}} \hat{\nabla}_{\mu}(\sqrt{-g}\, t^{\mu}{}_{\alpha}) = -\frac{1}{2}\Delta^{\lambda\mu\nu} R_{\lambda\mu\nu\alpha} + \frac{1}{2} Q_{\alpha\mu\nu} T^{\mu\nu} + 2S_{\alpha\mu\nu} t^{\mu\nu}$$

Conservation Laws

Working in exterior calculus from the GL and diff invariance we get

From GL

$$t^{\mu}{}_{\lambda} = T^{\mu}{}_{\lambda} - \frac{1}{2\sqrt{-g}}\hat{\nabla}_{\nu}(\sqrt{-g}\Delta_{\lambda}{}^{\mu\nu})$$

From Diff

$$\frac{1}{\sqrt{-g}}\hat{\nabla}_{\mu}(\sqrt{-g}t^{\mu}{}_{\alpha}) = -\frac{1}{2}\Delta^{\lambda\mu\nu}R_{\lambda\mu\nu\alpha} + \frac{1}{2}Q_{\alpha\mu\nu}T^{\mu\nu} + 2S_{\alpha\mu\nu}t^{\mu\nu}$$

From Diff using coordinates

$$\sqrt{-g}(2\tilde{\nabla}_{\mu}T^{\mu}{}_{\alpha} - \Delta^{\lambda\mu\nu}R_{\lambda\mu\nu\alpha}) + \hat{\nabla}_{\mu}\hat{\nabla}_{\nu}(\sqrt{-g}\Delta_{\alpha}{}^{\mu\nu}) + 2S_{\mu\alpha}{}^{\lambda}\hat{\nabla}_{\nu}(\sqrt{-g}\Delta_{\lambda}{}^{\mu\nu}) = 0 \quad (4)$$

Homogeneous Cosmology with Torsion and non-metricity

- Applying Cosmological Principle to Torsion [Tsamparlis,1979]:
  $S_{01}{}^{1} = S_{02}{}^{2} = S_{03}{}^{3} = ... = S_{0m}{}^{m} \neq 0 \quad (no\ sum)$
  $S_{ijk} \propto \epsilon_{ijk} \neq 0$ (only for $n = 4$)

Homogeneous Cosmology with Torsion and non-metricity

- Applying Cosmological Principle to Torsion [Tsamparlis,1979]:
  $S_{01}{}^{1} = S_{02}{}^{2} = S_{03}{}^{3} = \ldots = S_{0m}{}^{m} \neq 0 \quad (no\ sum)$
  $S_{ijk} \propto \epsilon_{ijk} \neq 0$ (only for $n = 4$)
- Applying it to Non-Metricity[Minkevich,1998]:
  $Q_{011} = \ldots = Q_{0mm} \neq 0,\ Q_{110} = \ldots = Q_{mm0} \neq 0,$
  $Q_{000} \neq 0$ Here $m = n - 1$=spatial space dim
  $\Longrightarrow$ The rest vanish!

Homogeneous Cosmology with Torsion and non-metricity

- Applying Cosmological Principle to Torsion [Tsamparlis,1979]:
  $S_{01}{}^{1} = S_{02}{}^{2} = S_{03}{}^{3} = ... = S_{0m}{}^{m} \neq 0 \quad (\textit{no sum})$
  $S_{ijk} \propto \epsilon_{ijk} \neq 0$ (only for $n = 4$)
- Applying it to Non-Metricity[Minkevich,1998]:
  $Q_{011} = ... = Q_{0mm} \neq 0,\ Q_{110} = ... = Q_{mm0} \neq 0,$
  $Q_{000} \neq 0$ Here $m = n - 1$=spatial space dim
  $\Longrightarrow$ The rest vanish!

Covariant Forms

The covariant forms of the above read [D.I,2020]

- $S^{(n)}_{\mu\nu\alpha} = 2u_{[\mu}h_{\nu]\alpha}\Phi(t) + \epsilon_{\mu\nu\alpha\rho}u^{\rho}P(t)\delta_{n,4}$
- $Q_{\alpha\mu\nu} = A(t)u_{\alpha}h_{\mu\nu} + B(t)h_{\alpha(\mu}u_{\nu)} + C(t)u_{\alpha}u_{\mu}u_{\nu}, \quad \forall$ n

$N^{(n)}_{\alpha\mu\nu} = X(t)u_{\alpha}h_{\mu\nu} + Y(t)u_{\mu}h_{\alpha\nu} + Z(t)u_{\nu}h_{\alpha\mu} + V(t)u_{\alpha}u_{\mu}u_{\nu} + \epsilon_{\alpha\mu\nu\lambda}u^{\lambda}W(t)\delta_{n,4}$ for the distortion.

A note on the distortion variables

Quite generally, it is a simple matter to show that given a distortion tensor, torsion and non-metricity are computed through

$$S_{\mu\nu\alpha} = N_{\alpha[\mu\nu]} \ , \ \ Q_{\alpha\mu\nu} = 2N_{(\mu\nu)\alpha} \tag{5}$$

A note on the distortion variables

Quite generally, it is a simple matter to show that given a distortion tensor, torsion and non-metricity are computed through

$$S_{\mu\nu\alpha} = N_{\alpha[\mu\nu]} \ , \quad Q_{\alpha\mu\nu} = 2N_{(\mu\nu)\alpha} \tag{5}$$

For the previous cosmological expressions, these imply the identifications:

Relations between Cosmological variables

$$2(X+Y) = B \ , \ \ 2Z = A \ , \ \ 2V = C \ , \ \ 2\Phi = Y - Z \ , \ \ P = W \tag{6}$$

Whichever set one uses is then totally irrelevant to the actual physics. The set $A, B, ...$ has more transparent geometric meaning whereas the set $X, Y, ...$ is more convenient for calculations.

Isotropic Hypermomentum [**D.I**,2020, EPJC]

Imposing Cosm. Principle to Hypermomentum($\mathcal{L}_{\xi^i}\Delta_{\alpha\mu\nu}=0$)

$\Delta_{i00}=\Delta_{0i0}=\Delta_{00i}=0$,

$\Delta_{110}=...=\Delta_{mm0}$,$\Delta_{011}=...=\Delta_{0mm}$(no sum)

Isotropic Hypermomentum [**D.I**,2020, EPJC]

Imposing Cosm. Principle to Hypermomentum($\mathcal{L}_{\xi^i}\Delta_{\alpha\mu\nu}=0$)

$\Delta_{i00}=\Delta_{0i0}=\Delta_{00i}=0$,

$\Delta_{110}=...=\Delta_{mm0}$,$\Delta_{011}=...=\Delta_{0mm}$(no sum)

Covariant Form of Hypermomentum

Using an $1+(n-1)$ split we get the covariant form:

- $\Delta^{(n)}_{\alpha\mu\nu}=\phi h_{\mu\alpha}u_{\nu}+\chi h_{\nu\alpha}u_{\mu}+\psi u_{\alpha}h_{\mu\nu}+\omega u_{\alpha}u_{\mu}u_{\nu}+\delta_{n,4}\epsilon_{\alpha\mu\nu\kappa}u^{\kappa}\zeta$

Most General form of Hypermomentum respecting isotropy!

Isotropic Hypermomentum [**D.I**,2020, EPJC]

Imposing Cosm. Principle to Hypermomentum($\mathcal{L}_{\xi^i}\Delta_{\alpha\mu\nu}=0$)

$\Delta_{i00}=\Delta_{0i0}=\Delta_{00i}=0$,

$\Delta_{110}=...=\Delta_{mm0}$,$\Delta_{011}=...=\Delta_{0mm}$(no sum)

Covariant Form of Hypermomentum

Using an $1+(n-1)$ split we get the covariant form:

- $\Delta^{(n)}_{\alpha\mu\nu}=\phi h_{\mu\alpha}u_{\nu}+\chi h_{\nu\alpha}u_{\mu}+\psi u_{\alpha}h_{\mu\nu}+\omega u_{\alpha}u_{\mu}u_{\nu}+\delta_{n,4}\epsilon_{\alpha\mu\nu\kappa}u^{\kappa}\zeta$

Most General form of Hypermomentum respecting isotropy!

Comments

1. In an FLRW $\phi,\chi,...$ depend only on time t. If homogeneity is relaxed $\phi=\phi(t,x^i)$ etc. (more about it later)
2. Hypermomentum generally contributes 5 dof in a Cosmological setting($n=4$). (and 4 dof for $n\neq 4$).

Hypermomentum Decomposition (Matter with Microstructure)

- Spin Part: $\Delta_{[\alpha\mu]\nu} = (\psi - \chi)u_{[\alpha}h_{\mu]\nu} + \delta_{n,4}\epsilon_{\alpha\mu\nu\kappa}u^{\kappa}\zeta$
- Dilation Part: $\Delta_{\nu} := \Delta_{\alpha\mu\nu}g^{\alpha\mu} = \Big[(n-1)\phi - \omega\Big]u_{\nu}$
- Shear Part: $\breve{\Delta}_{\alpha\mu\nu} = \Delta_{(\alpha\mu)\nu} - \frac{1}{n}g_{\alpha\mu}\Delta_{\nu} = \frac{(\phi+\omega)}{n}\Big[h_{\alpha\mu} + (n-1)u_{\alpha}u_{\mu}\Big]u_{\nu} + (\psi+\chi)u_{(\mu}h_{\alpha)\nu}$

Hypermomentum Decomposition (Matter with Microstructure)

- Spin Part: $\Delta_{[\alpha\mu]\nu} = (\psi - \chi)u_{[\alpha}h_{\mu]\nu} + \delta_{n,4}\epsilon_{\alpha\mu\nu\kappa}u^{\kappa}\zeta$
- Dilation Part: $\Delta_{\nu} := \Delta_{\alpha\mu\nu}g^{\alpha\mu} = \Big[(n-1)\phi - \omega\Big]u_{\nu}$
- Shear Part: $\breve{\Delta}_{\alpha\mu\nu} = \Delta_{(\alpha\mu)\nu} - \frac{1}{n}g_{\alpha\mu}\Delta_{\nu} = \frac{(\phi+\omega)}{n}\Big[h_{\alpha\mu} + (n-1)u_{\alpha}u_{\mu}\Big]u_{\nu} + (\psi+\chi)u_{(\mu}h_{\alpha)\nu}$

Sourcing Torsion and Non-Metricity ($5 = 2 + 3$)

By means of the connection field eqs, the above parts act as sources producing spacetime torsion and non-metricity (see example later).

Physical variables

The variables $\phi, \chi, ...$ themselves don't have a physical meaning but rather the combinations

$$\sigma = \frac{(\psi - \chi)}{2} \qquad \zeta = \zeta \qquad (spin) \tag{7}$$

$$\Delta = 3\phi - \omega \,, \qquad (dilation) \tag{8}$$

$$\Sigma_1 = \frac{(\psi + \chi)}{2} \,, \qquad \Sigma_2 = \frac{(\phi + \omega)}{4} \qquad (shear) \tag{9}$$

describe properly the cosmological parts of spin, dilation and shear.

Physical variables

The variables $\phi, \chi, ...$ themselves don't have a physical meaning but rather the combinations

$$\sigma = \frac{(\psi - \chi)}{2} \qquad \zeta = \zeta \qquad (\textit{spin}) \tag{7}$$

$$\Delta = 3\phi - \omega \,, \qquad (\textit{dilation}) \tag{8}$$

$$\Sigma_1 = \frac{(\psi + \chi)}{2} \,, \qquad \Sigma_2 = \frac{(\phi + \omega)}{4} \qquad (\textit{shear}) \tag{9}$$

describe properly the cosmological parts of spin, dilation and shear.

- Therefore in a cosmological setting the spin part of hypermomentum contributes 2 dof, the dilation 1 and the shear 2 (2+1+2=5 dof of hypermomentum).

The Perfect Hypermomentum Preserving Hyperfluid [D.I, 2020]

Energy Momentum:

$T_{\mu\nu} = t_{\mu\nu} = \rho u_\mu u_\nu + p h_{\mu\nu}$

Hypermomentum :

$\Delta^{(n)}_{\alpha\mu\nu} = \phi h_{\mu\alpha} u_\nu + \chi h_{\nu\alpha} u_\mu + \psi u_\alpha h_{\mu\nu} + \omega u_\alpha u_\mu u_\nu + \delta_{n,4} \epsilon_{\alpha\mu\nu\kappa} u^\kappa \zeta$

The Perfect Hypermomentum Preserving Hyperfluid [D.I, 2020]

Energy Momentum:

$T_{\mu\nu} = t_{\mu\nu} = \rho u_\mu u_\nu + p h_{\mu\nu}$

Hypermomentum :

$\Delta^{(n)}_{\alpha\mu\nu} = \phi h_{\mu\alpha} u_\nu + \chi h_{\nu\alpha} u_\mu + \psi u_\alpha h_{\mu\nu} + \omega u_\alpha u_\mu u_\nu + \delta_{n,4} \epsilon_{\alpha\mu\nu\kappa} u^\kappa \zeta$

Conservation laws (obtained from diff invariance)

$\tilde{\nabla}_\mu T^\mu{}_\nu = \frac{1}{2} \Delta^{\alpha\beta\gamma} R_{\alpha\beta\gamma\nu}. \quad \hat{\nabla}_\nu \left(\sqrt{-g} \Delta_\lambda{}^{\mu\nu} \right) = 0$

We call it hypermomentum preserving.

The Perfect Hypermomentum Preserving Hyperfluid [D.I, 2020]

Energy Momentum:
$T_{\mu\nu} = t_{\mu\nu} = \rho u_\mu u_\nu + p h_{\mu\nu}$
Hypermomentum :
$\Delta^{(n)}_{\alpha\mu\nu} = \phi h_{\mu\alpha} u_\nu + \chi h_{\nu\alpha} u_\mu + \psi u_\alpha h_{\mu\nu} + \omega u_\alpha u_\mu u_\nu + \delta_{n,4} \epsilon_{\alpha\mu\nu\kappa} u^\kappa \zeta$

Conservation laws (obtained from diff invariance)

$\tilde{\nabla}_\mu T^\mu{}_\nu = \frac{1}{2} \Delta^{\alpha\beta\gamma} R_{\alpha\beta\gamma\nu}$. $\quad \hat{\nabla}_\nu \left(\sqrt{-g} \Delta_\lambda{}^{\mu\nu} \right) = 0$
We call it hypermomentum preserving.

Note

The conservation law for hypermomentum (2nd eq. above) in an FLRW Universe really contains 2 independent eqs for the 5 fields.
$\Rightarrow$ 3 eqs of state must be provided.

Continuity equation and hypermomentum evolution

Evolution Equations of the Perfect (Hypermomentum preserving) Hyperfluid

$$\dot{\rho}+(n-1)H(\rho+p)=-\frac{1}{2}u^{\mu}u^{\nu}(\chi R_{\mu\nu}+\psi\check{R}_{\mu\nu}) \tag{10}$$

$$\dot{\phi}+(n-1)H\phi+H(\chi+\psi)+\psi X-\chi Y=0 \tag{11}$$

$$\dot{\omega}+(n-1)H(\chi+\psi+\omega)+(n-1)(\psi X-\chi Y)=0 \tag{12}$$

Continuity equation and hypermomentum evolution

Evolution Equations of the Perfect (Hypermomentum preserving) Hyperfluid

$$\dot{\rho} + (n-1)H(\rho + p) = -\frac{1}{2}u^{\mu}u^{\nu}(\chi R_{\mu\nu} + \psi \breve{R}_{\mu\nu}) \tag{10}$$

$$\dot{\phi} + (n-1)H\phi + H(\chi + \psi) + \psi X - \chi Y = 0 \tag{11}$$

$$\dot{\omega} + (n-1)H(\chi + \psi + \omega) + (n-1)(\psi X - \chi Y) = 0 \tag{12}$$

Equations of state

As mentioned earlier, the above system must be supplemented with appropriate equations of state among the energy density, pressure and hypermomentum variables.

Generalization: There exists a Perfect Hyperfluid, generalizing the Perfect Fluid notion of GR, for which: (D.I. 2021, JCAP)

$$t_{\mu\nu} = \rho_c u_\mu u_\nu + p_c h_{\mu\nu} \quad , \quad T_{\mu\nu} = \rho u_\mu u_\nu + p h_{\mu\nu} \tag{13}$$

$$\Delta^{(n)}_{\alpha\mu\nu} = \phi h_{\mu\alpha} u_\nu + \chi h_{\nu\alpha} u_\mu + \psi u_\alpha h_{\mu\nu} + \omega u_\alpha u_\mu u_\nu + \delta_{n,4}\epsilon_{\alpha\mu\nu\kappa} u^\kappa \zeta \tag{14}$$

These sources are subject to the conservation laws:

$$\tilde{\nabla}_\mu t^\mu{}_\alpha = \frac{1}{2}\Delta^{\lambda\mu\nu} R_{\lambda\mu\nu\alpha} + \frac{1}{2} Q_{\alpha\mu\nu}(t^{\mu\nu} - T^{\mu\nu}) \tag{15}$$

$$t^\mu{}_\lambda = T^\mu{}_\lambda - \frac{1}{2\sqrt{-g}} \hat{\nabla}_\nu (\sqrt{-g}\Delta_\lambda{}^{\mu\nu}) \tag{16}$$

Generalization: There exists a Perfect Hyperfluid, generalizing the Perfect Fluid notion of GR, for which: (D.I. 2021, JCAP)

$$t_{\mu\nu} = \rho_c u_\mu u_\nu + p_c h_{\mu\nu} \quad , \quad T_{\mu\nu} = \rho u_\mu u_\nu + p h_{\mu\nu} \tag{13}$$

$$\Delta^{(n)}_{\alpha\mu\nu} = \phi h_{\mu\alpha} u_\nu + \chi h_{\nu\alpha} u_\mu + \psi u_\alpha h_{\mu\nu} + \omega u_\alpha u_\mu u_\nu + \delta_{n,4} \epsilon_{\alpha\mu\nu\kappa} u^\kappa \zeta \tag{14}$$

These sources are subject to the conservation laws:

$$\tilde{\nabla}_\mu t^\mu{}_\alpha = \frac{1}{2} \Delta^{\lambda\mu\nu} R_{\lambda\mu\nu\alpha} + \frac{1}{2} Q_{\alpha\mu\nu} (t^{\mu\nu} - T^{\mu\nu}) \tag{15}$$

$$t^\mu{}_\lambda = T^\mu{}_\lambda - \frac{1}{2\sqrt{-g}} \hat{\nabla}_\nu (\sqrt{-g} \Delta_\lambda{}^{\mu\nu}) \tag{16}$$

The Perfect Hyperfluid is a direct generalization of the Perfect Fluid description where now the microscopic characteristics of matter are also taken into account.

Lagrangian Formulation of Hyperfluids

Hyperhydrodynamics

Before we jump into applications let us comment on the Lagrangian formulation of Hyperfluids as developed in [**D.I**, Tomi Koivisto, JCAP, 2024].

Lagrangian Formulation of Hyperfluids

Hyperhydrodynamics

Before we jump into applications let us comment on the Lagrangian formulation of Hyperfluids as developed in [**D.I**, Tomi Koivisto, JCAP, 2024].

Key Aspects

- The Lagrangian that produces this isotropic hypermomentum was given there.

Lagrangian Formulation of Hyperfluids

Hyperhydrodynamics

Before we jump into applications let us comment on the Lagrangian formulation of Hyperfluids as developed in [**D.I**, Tomi Koivisto, JCAP, 2024].

Key Aspects

- The Lagrangian that produces this isotropic hypermomentum was given there.
- In FLRW the energy-momentum is of course of perfect fluid form but for generic backgrounds we have an imperfect fluid!

Lagrangian Formulation of Hyperfluids

Hyperhydrodynamics

Before we jump into applications let us comment on the Lagrangian formulation of Hyperfluids as developed in [**D.I**, Tomi Koivisto, JCAP, 2024].

Key Aspects

- The Lagrangian that produces this isotropic hypermomentum was given there.
- In FLRW the energy-momentum is of course of perfect fluid form but for generic backgrounds we have an imperfect fluid!
- The viscous fluid characteristics, such as heat flux, anisotropic stresses etc. have an origin from hypermomentum.

Lagrangian Formulation of Hyperfluids

Hyperhydrodynamics

Before we jump into applications let us comment on the Lagrangian formulation of Hyperfluids as developed in [**D.I**, Tomi Koivisto, JCAP, 2024].

Key Aspects

- The Lagrangian that produces this isotropic hypermomentum was given there.
- In FLRW the energy-momentum is of course of perfect fluid form but for generic backgrounds we have an imperfect fluid!
- The viscous fluid characteristics, such as heat flux, anisotropic stresses etc. have an origin from hypermomentum.
- Quite remarkably first order hydrodynamics naturally derives from our fluid action!

A Simple extension of Einstein-Cartan Theory

Our model

We consider the Metric-Affine version of the Einstein-Hilbert Action with the matter sector being a Perfect Hyperfluid:

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}R + S_{hyp}[g,\Gamma] \tag{17}$$

A Simple extension of Einstein-Cartan Theory

Our model

We consider the Metric-Affine version of the Einstein-Hilbert Action with the matter sector being a Perfect Hyperfluid:

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}R + S_{hyp}[g,\Gamma] \tag{17}$$

Field Equations

$$R_{(\mu\nu)} - \frac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu}\,, \tag{18}$$

$$\left(\frac{Q_\lambda}{2} + 2S_\lambda\right)g^{\mu\nu} - ({Q_\lambda}^{\mu\nu} + 2{S_\lambda}^{\mu\nu}) + \left(q^\mu - \frac{Q^\mu}{2} - 2S^\mu\right)\delta^\nu_\lambda = \kappa {\Delta_\lambda}^{\mu\nu}\,. \tag{19}$$

Torsion and Non-metricity in terms of Hypermomentum

The connection field equations yield in this case

$$\frac{A}{2} + 4\Phi - \frac{C}{2} = \kappa\psi\,, \tag{20}$$

$$B - \frac{3A}{2} - 4\Phi - \frac{C}{2} = \kappa\chi\,, \tag{21}$$

$$2P = -\kappa\zeta\,, B = -2\kappa\phi\,, 3B = -2\kappa\omega\,. \tag{22}$$

This can be solved for A, B, C etc in terms of the sources $\phi, \chi, ...$

Torsion and Non-metricity in terms of Hypermomentum

The connection field equations yield in this case

$$\frac{A}{2} + 4\Phi - \frac{C}{2} = \kappa\psi\,, \tag{20}$$

$$B - \frac{3A}{2} - 4\Phi - \frac{C}{2} = \kappa\chi\,, \tag{21}$$

$$2P = -\kappa\zeta\,, B = -2\kappa\phi\,, 3B = -2\kappa\omega\,. \tag{22}$$

This can be solved for A, B, C etc in terms of the sources $\phi, \chi, ...$

Important Note

Due to projective invariance we have a vanishing dilation current. Then we may fix the projective gauge such that $C = 3/2B$.

Friedmann equations with spin and shear (hyperfluid)

$$H^2 = \frac{\kappa}{2}\Big(\dot{\phi} + (3\phi+\psi+\chi)H - \frac{\kappa}{4}(\psi-\chi)(\psi+\chi+2\phi)\Big) + \frac{\kappa^2}{4}\phi^2 + \frac{\kappa^2}{4}\zeta^2 - \frac{\kappa^2}{16}(\psi-\chi)^2 + \frac{\kappa}{2}H(\psi-\chi) + \frac{\kappa\rho}{3}\,. \tag{23}$$

$$\frac{\ddot{a}}{a} = -\frac{\kappa}{6}(\rho+3p) + \frac{\kappa}{4}(\dot{\psi}-\dot{\chi}) + \frac{\kappa}{4}H(\psi-\chi) - \frac{\kappa^2}{4}\phi(\psi+\chi+2\phi) - \frac{\kappa}{2}\Big[\dot{\phi} + (3\phi+\chi+\psi)H - \frac{\kappa}{4}(\psi-\chi)(\psi+\chi+2\phi)\Big]\,. \tag{24}$$

Friedmann equations with spin and shear (hyperfluid)

$$H^2 = \frac{\kappa}{2}\Big(\dot{\phi} + (3\phi + \psi + \chi)H - \frac{\kappa}{4}(\psi - \chi)(\psi + \chi + 2\phi)\Big) + \frac{\kappa^2}{4}\phi^2 + \frac{\kappa^2}{4}\zeta^2 - \frac{\kappa^2}{16}(\psi - \chi)^2 + \frac{\kappa}{2}H(\psi - \chi) + \frac{\kappa\rho}{3}. \tag{23}$$

$$\frac{\ddot{a}}{a} = -\frac{\kappa}{6}(\rho + 3p) + \frac{\kappa}{4}(\dot{\psi} - \dot{\chi}) + \frac{\kappa}{4}H(\psi - \chi) - \frac{\kappa^2}{4}\phi(\psi + \chi + 2\phi) - \frac{\kappa}{2}\Big[\dot{\phi} + (3\phi + \chi + \psi)H - \frac{\kappa}{4}(\psi - \chi)(\psi + \chi + 2\phi)\Big]. \tag{24}$$

Note: The ζ spin mode

Consistency of the conservation laws and the Friedmann equations demands that $\zeta \propto \frac{1}{a}$ namely it behaves exactly as a curvature contribution!

Alternative form

We may recast the system of equations along with the conservation laws as (for $\zeta = 0$)

$$3H^2 = \kappa\rho + \kappa\rho_h \,, \tag{25a}$$

$$2\dot{H} + 3H^2 = -\kappa p - \kappa p_h \,, \tag{25b}$$

$$\dot{\rho} + 3H(\rho + p) = -\dot{\rho}_h - 3H(\rho_h + p_h) \,, \tag{25c}$$

where the effective density of the hyperfluid is given by

$$\rho_h := \frac{3\dot{\Sigma}_2}{2} + \kappa\left(-\frac{3\Sigma_1\sigma}{2} + \frac{3\Sigma_2^2}{4} - \frac{3\Sigma_2\sigma}{2} - \frac{3\sigma^2}{4}\right) + H\left(3\Sigma_1 + \frac{9\Sigma_2}{2} + 3\sigma\right) \tag{26}$$

and a similar expression for p_h.

Distinguished Indexes

A series of different indexes

$$w := \frac{p}{\rho}\,, \tag{27a}$$

$$w_h := \frac{p_h}{\rho_h}\,, \tag{27b}$$

$$w_{\rm eff} := \frac{p + p_h}{\rho + \rho_h} = \frac{w\,\rho + w_h\,\rho_h}{\rho + \rho_h} \neq w + w_h\,. \tag{27c}$$

It is also a simple matter to show that

$$w_{eff} = -1 - \frac{2\dot{H}}{3H^2}\,. \tag{28}$$

We call w just the barotropic index of matter eos, w_h the hypermomentum index, and w_{eff} as effective (expansion) index.

Pure Spin Case

The full analysis of the Cosmological dynamics of this model was performed in [Ilaria Andrei, **D.I,**, Laur Jarv, Margus Saal,Phys.Rev.D 111 (2025) 6, 064063].

System of Eqns for pure spin

$$3H^2 = \kappa\rho + 3H\kappa\sigma - \frac{3\kappa^2\sigma^2}{4}\,, \tag{29}$$

$$2\dot{H} + 3H^2 = -\kappa p + 2H\kappa\sigma + \dot{\sigma}\kappa - \frac{\kappa^2\sigma^2}{4}\,, \tag{30}$$

$$\dot{\rho} + 3H(\rho + p) = \frac{\kappa\sigma}{2}(\rho + 3p)\,, \tag{31}$$

Pure Spin Case

The full analysis of the Cosmological dynamics of this model was performed in [Ilaria Andrei, **D.I.**, Laur Jarv, Margus Saal,Phys.Rev.D 111 (2025) 6, 064063].

System of Eqns for pure spin

$$3H^2 = \kappa\rho + 3H\kappa\sigma - \frac{3\kappa^2\sigma^2}{4}\,, \tag{29}$$

$$2\dot{H} + 3H^2 = -\kappa p + 2H\kappa\sigma + \dot{\sigma}\kappa - \frac{\kappa^2\sigma^2}{4}\,, \tag{30}$$

$$\dot{\rho} + 3H(\rho + p) = \frac{\kappa\sigma}{2}(\rho + 3p)\,, \tag{31}$$

Note

The continuity eqn is identical to that of an open thermodynamic system with particle production rate $\Gamma \propto \sigma$!

Spin proportional to (square root of) energy-density

From the Friedmann eqn one sees that σ^2 has the same dimensions as ρ. This suggests the ansatz:

$$\sigma = b\sqrt{\frac{3\rho}{\kappa}}\,, \tag{32}$$

where b is a dimensionless constant.

Spin proportional to (square root of) energy-density

From the Friedmann eqn one sees that σ^2 has the same dimensions as ρ. This suggests the ansatz:

$$\sigma = b\sqrt{\frac{3\rho}{\kappa}}\,, \tag{32}$$

where b is a dimensionless constant.

Using this the modified continuity eqn becomes

$$\dot{\rho} \pm \sqrt{3\kappa}\,(1 + w \pm b)\,\rho^{3/2} = 0\,, \tag{33}$$

namely

$$w_\rho = w \pm b\,. \tag{34}$$

is the barotropic index of matter monitoring how energy dilutes.

Effective Expansion Index

On the other hand, one finds the effective expansion index to be

$$w_{eff} = \frac{2w \mp b}{2 \pm 3b} \,. \tag{35}$$

The fact that $w_\rho \neq w_{eff}$ has dramatic consequences on the conclusions that can be drawn for the behaviour of ρ and H. For instance:

- $w = -1$ does not necessarily imply that $\rho = constant$ and $H = constant$.
- De Sitter expansion ($H = H_0$) does not demand that $\rho = constant$.

In preparation...

'Interacting fluids cosmologies from the metric affine framework'

The previous theory is extended to include perfect fluid+hyperfluid

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}R + S_{pf} + S_{hyper} \tag{36}$$

In preparation...

'Interacting fluids cosmologies from the metric affine framework'

The previous theory is extended to include perfect fluid+hyperfluid

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}R + S_{pf} + S_{hyper} \tag{36}$$

Preliminary results

- The model resembles interacting fluid cosmologies where the creation/annihilation rate is due to the interaction of spin with spacetime torsion.

In preparation...

'Interacting fluids cosmologies from the metric affine framework'

The previous theory is extended to include perfect fluid+hyperfluid

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}R + S_{pf} + S_{hyper} \tag{36}$$

Preliminary results

- The model resembles interacting fluid cosmologies where the creation/annihilation rate is due to the interaction of spin with spacetime torsion.
- Model $\rightarrow$ statistically equivalent to the standard ΛCDM scenario across most datasets. Considering the most complete dataset selection, it provides a significantly improved fit according to Information Criteria.

Conclusions/Further Prospects

- The Perfect Hyperfluid (Perfect Fluid with microstructure) is the Cosmological Fluid of MAG.

Conclusions/Further Prospects

- The Perfect Hyperfluid (Perfect Fluid with microstructure) is the Cosmological Fluid of MAG.
- In isotropic Cosmologies the hypermomentum contributes 5 degrees of freedom $(\phi, \chi, ...)$.

Conclusions/Further Prospects

- The Perfect Hyperfluid (Perfect Fluid with microstructure) is the Cosmological Fluid of MAG.
- In isotropic Cosmologies the hypermomentum contributes 5 degrees of freedom $(\phi, \chi, ...)$.
- The modifications coming from hypermomentum offer rich phenomenology and explain rather naturally many scenarios that are frequently arbitrary imposed in standard theories.

Conclusions/Further Prospects

- The Perfect Hyperfluid (Perfect Fluid with microstructure) is the Cosmological Fluid of MAG.
- In isotropic Cosmologies the hypermomentum contributes 5 degrees of freedom $(\phi, \chi, ...)$.
- The modifications coming from hypermomentum offer rich phenomenology and explain rather naturally many scenarios that are frequently arbitrary imposed in standard theories.
- For instance a stiff matter equation of state arises naturally by the dilation part of the hyperfluid. The production rate of an open system related to the spin part, etc.

Conclusions/Further Prospects

- The Perfect Hyperfluid (Perfect Fluid with microstructure) is the Cosmological Fluid of MAG.
- In isotropic Cosmologies the hypermomentum contributes 5 degrees of freedom $(\phi, \chi, ...)$.
- The modifications coming from hypermomentum offer rich phenomenology and explain rather naturally many scenarios that are frequently arbitrary imposed in standard theories.
- For instance a stiff matter equation of state arises naturally by the dilation part of the hyperfluid. The production rate of an open system related to the spin part, etc.
- Different effective indexes describe the various cosmological quantities.

Conclusions/Further Prospects

- The Perfect Hyperfluid (Perfect Fluid with microstructure) is the Cosmological Fluid of MAG.
- In isotropic Cosmologies the hypermomentum contributes 5 degrees of freedom $(\phi, \chi, ...)$.
- The modifications coming from hypermomentum offer rich phenomenology and explain rather naturally many scenarios that are frequently arbitrary imposed in standard theories.
- For instance a stiff matter equation of state arises naturally by the dilation part of the hyperfluid. The production rate of an open system related to the spin part, etc.
- Different effective indexes describe the various cosmological quantities.
- Connection to observations and bounds on hypermomentum variables?

...Thank you!!!
Gracias!!!