Current Challenges in Cosmology, Cali, Colombia

Cosmological Effects of Fluids with Microstructure (Hyperfluids)

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Outline

- Non-Riemannian Geometry: Conventions/Notation
- Conservation Laws of Metric-Affine Gravity
- Hyperfluids, Torsion and Non-metricity in Cosmology
- Including matter's microstructure in isotropic Cosmology
- A simple model with Spin and Shear charges
- Conclusions/Further Prospects

The talk is mostly based on the papers

- "Cosmological Hyperfluids, Torsion and Non-metricity" Published in: Eur.Phys.J.C 80 (2020) 11, 1042 • e-Print: 2003.07384 [gr-qc] (**DI**)
- "The Perfect Hyperfluid of Metric-Affine Gravity: The Foundation" Published in: JCAP 04 (2021) 072 • e-Print: 2101.07289 [gr-qc] (**DI**)
- Friedmann cosmology with hyperfluids Published in: Phys.Rev.D 111 (2025) 6, 064063 • e-Print: 2411.19127 [gr-qc] (Ilaria Andrei,**DI**, Laur Jarv, Margus Saal)

Metric-Affine Gravity

Metric Gravity

- $\Gamma^{\alpha}{}_{\mu\nu} \rightarrow$ *torsionless* , metric compatibility $\nabla_{\sigma} g_{\mu\nu} = 0$
- $S = S_{Gravity} + S_{Matter} = \int d^n x \sqrt{-g} \left[\mathcal{L}_G(g_{\mu\nu}) + \mathcal{L}_M(g_{\mu\nu}, \Phi) \right]$

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Teleparallel/Symmetric Teleparallel Gravity

- $R^{\alpha}{}_{\beta\mu\nu} = 0$, $\nabla_{\sigma} g_{\mu\nu} = 0$ but $S_{\mu\nu}{}^{\alpha} = \Gamma^{\alpha}{}_{[\mu\nu]} \neq 0$
- $R^{\alpha}{}_{\beta\mu\nu} = 0$, $S_{\mu\nu}{}^{\alpha} = 0$ but $Q_{\alpha\mu\nu} = -\nabla_{\alpha} g_{\mu\nu} \neq 0$

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Metric-Affine Gravity (MAG)

- $S = \int d^n x \sqrt{-g} \left[\mathcal{L}_G(g_{\mu\nu}, \Gamma^{\alpha}{}_{\mu\nu}) + \mathcal{L}_M(g_{\mu\nu}, \Gamma^{\alpha}{}_{\mu\nu}, \Phi) \right] \Rightarrow$ No a priori constraints on the geometry.

Geometrical Objects

Two distinctively different notions on a manifold

- Metric Tensor $g_{\mu\nu}$: Defines distances,lengths and dot products

$$||\alpha||^2 := \alpha^\mu \alpha^\nu g_{\mu\nu}\ , \ \ (\alpha \cdot \beta) := \alpha^\mu \beta^\nu g_{\mu\nu}$$

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- Affine-Connection ${\Gamma^\lambda}_{\mu\nu}$: Defines parallel transport of tensor fields on the manifold

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The two need not be related a priori! Their relation may be found after solving the field equations!

Geometrical Objects

Torsion

- $\nabla_{[\mu}\nabla_{\nu]}\phi = S_{\mu\nu}{}^{\lambda}\nabla_{\lambda}\phi$, Torsion Tensor $\boxed{S_{\mu\nu}{}^{\lambda} := \Gamma^{\lambda}{}_{[\mu\nu]}}$

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Curvature

- $[\nabla_{\alpha}, \nabla_{\beta}]u^{\mu} = R^{\mu}{}_{\nu\alpha\beta}u^{\nu} + 2S_{\alpha\beta}{}^{\nu}\nabla_{\nu}u^{\mu}$

  Curvature Tensor: $\boxed{R^{\mu}{}_{\nu\alpha\beta} := 2\partial_{[\alpha}\Gamma^{\mu}{}_{|\nu|\beta]} + 2\Gamma^{\mu}{}_{\rho[\alpha}\Gamma^{\rho}{}_{|\nu|\beta]}}$

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Non-Metricity

- $\boxed{Q_{\alpha\mu\nu} := -\nabla_{\alpha}g_{\mu\nu}} = -\partial_{\alpha}g_{\mu\nu} + \Gamma^{\lambda}{}_{\mu\alpha}g_{\lambda\nu} + \Gamma^{\lambda}{}_{\nu\alpha}g_{\lambda\mu}$

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Torsion/Non-metricity related vectors

$$S_{\mu} = S_{\mu\lambda}{}^{\lambda}\ , \qquad \tilde{S}^{\mu} = \epsilon^{\mu\nu\rho\sigma}S_{\nu\rho\sigma}\ \ (only\ \ for\ \ n=4)$$

$$Q_{\mu} = g^{\alpha\beta}Q_{\mu\alpha\beta}\ , \qquad q_{\mu} = g^{\rho\alpha}Q_{\rho\alpha\mu}$$

Affine Connection

Affine connection decomposition

$$\Gamma^{\lambda}{}_{\mu\nu} = \tilde{\Gamma}^{\lambda}{}_{\mu\nu} + \frac{1}{2} g^{\alpha\lambda}(Q_{\mu\nu\alpha} + Q_{\nu\alpha\mu} - Q_{\alpha\mu\nu}) - g^{\alpha\lambda}(S_{\alpha\mu\nu} + S_{\alpha\nu\mu} - S_{\mu\nu\alpha})$$

where $\tilde{\Gamma}^{\lambda}{}_{\mu\nu} := \frac{1}{2} g^{\alpha\lambda}(\partial_{\mu} g_{\nu\alpha} + \partial_{\nu} g_{\alpha\mu} - \partial_{\alpha} g_{\mu\nu})$ is the Levi-Civita part of the connection. Distortion: $N^{\lambda}{}_{\mu\nu} := \Gamma^{\lambda}{}_{\mu\nu} - \tilde{\Gamma}^{\lambda}{}_{\mu\nu}$

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Post-Riemannian expansions

Each quantity $\Rightarrow$ decomposed into Riemannian and non-Riemannian counterparts. Example:

$$R = \tilde{R} + S_{\mu\nu\alpha}S^{\mu\nu\alpha} - 2S_{\mu\nu\alpha}S^{\alpha\mu\nu} - 4S_{\mu}S^{\mu} - 4\tilde{\nabla}_{\mu}S^{\mu}$$
$$+\frac{1}{4}Q_{\alpha\mu\nu}Q^{\alpha\mu\nu} - \frac{1}{2}Q_{\alpha\mu\nu}Q^{\mu\nu\alpha} - \frac{1}{4}Q_{\mu}Q^{\mu} + \frac{1}{2}Q_{\mu}q^{\mu}$$
$$+2Q_{\alpha\mu\nu}S^{\alpha\mu\nu} + 2S_{\mu}(q^{\mu} - Q^{\mu}) + \tilde{\nabla}_{\mu}(q^{\mu} - Q^{\mu} - 4S^{\mu})$$

Hypermomentum, Canonical and Metrical Energy Momentum Tensors

Metrical and Canonical Energy Momentum Tensor

Metrical: $T_{\alpha\beta} := -\frac{2}{\sqrt{-g}}\frac{\delta S_M}{\delta g^{\alpha\beta}}$. Canonical: $t^{\mu}{}_{c} = \frac{1}{\sqrt{-g}}\frac{\delta S_M}{\delta e_{\mu}{}^{c}}$

Hypermomentum Tensor

Hypermomentum: $\Delta_{\lambda}{}^{\mu\nu} := -\frac{2}{\sqrt{-g}}\frac{\delta S_M}{\delta \Gamma^{\lambda}{}_{\mu\nu}}$

Relation Between Energy Tensors

$$t^{\mu}{}_{\lambda} = T^{\mu}{}_{\lambda} - \frac{1}{2\sqrt{-g}}\hat{\nabla}_{\nu}(\sqrt{-g}\Delta_{\lambda}{}^{\mu\nu})$$

where $\hat{\nabla}_{\nu} = 2S_{\nu} - \nabla_{\nu}$.

Physical Role of Hypermomentum

Hypermomentum Split

The Hypermomentum tensor is a generalization of the spin tensor (including also a symmetric part) that fully describes the micro-properties of matter. It splits into its 3 physical parts of spin, dilation and shear according to:

$$\tau_{\mu\nu\alpha} := \Delta_{[\mu\nu]\alpha} \tag{1}$$

$$\Delta_{\alpha} := \Delta_{\mu\nu\alpha} g^{\mu\nu} \tag{2}$$

$$\Sigma_{\mu\nu\alpha} := \Delta_{\mu\nu\alpha} - \frac{\Delta_{\alpha}}{n} g_{\mu\nu} \tag{3}$$

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Dilation and Shear are manifestations of the hadronic properties of matter.

Conservation Laws

Working in exterior calculus from the GL and diff invariance we get

From GL

$$t^{\mu}{}_{\lambda} = T^{\mu}{}_{\lambda} - \frac{1}{2\sqrt{-g}}\hat{\nabla}_{\nu}(\sqrt{-g}\Delta_{\lambda}{}^{\mu\nu})$$

From Diff

$$\frac{1}{\sqrt{-g}}\hat{\nabla}_{\mu}(\sqrt{-g}t^{\mu}{}_{\alpha}) = -\frac{1}{2}\Delta^{\lambda\mu\nu}R_{\lambda\mu\nu\alpha} + \frac{1}{2}Q_{\alpha\mu\nu}T^{\mu\nu} + 2S_{\alpha\mu\nu}t^{\mu\nu}$$

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From Diff using coordinates

$$\sqrt{-g}(2\tilde{\nabla}_{\mu}T^{\mu}{}_{\alpha} - \Delta^{\lambda\mu\nu}R_{\lambda\mu\nu\alpha}) + \hat{\nabla}_{\mu}\hat{\nabla}_{\nu}(\sqrt{-g}\Delta_{\alpha}{}^{\mu\nu}) + 2S_{\mu\alpha}{}^{\lambda}\hat{\nabla}_{\nu}(\sqrt{-g}\Delta_{\lambda}{}^{\mu\nu}) = 0 \tag{4}$$

Homogeneous Cosmology with Torsion and non-metricity

- Applying Cosmological Principle to Torsion [Tsamparlis,1979]:
  $S_{01}{}^{1} = S_{02}{}^{2} = S_{03}{}^{3} = \dots = S_{0m}{}^{m} \neq 0 \quad (no\ sum)$
  $S_{ijk} \propto \epsilon_{ijk} \neq 0$ (only for $n = 4$)
- Applying it to Non-Metricity[Minkevich,1998]:
  $Q_{011} = \dots = Q_{0mm} \neq 0$, $Q_{110} = \dots = Q_{mm0} \neq 0$,
  $Q_{000} \neq 0$ Here $m = n - 1$=spatial space dim
  $\Longrightarrow$ The rest vanish!

Covariant Forms

The covariant forms of the above read [D.I,2020]

- $S^{(n)}_{\mu\nu\alpha} = 2u_{[\mu}h_{\nu]\alpha}\Phi(t) + \epsilon_{\mu\nu\alpha\rho}u^{\rho}P(t)\delta_{n,4}$
- $Q_{\alpha\mu\nu} = A(t)u_{\alpha}h_{\mu\nu} + B(t)h_{\alpha(\mu}u_{\nu)} + C(t)u_{\alpha}u_{\mu}u_{\nu}, \quad \forall$ n

$N^{(n)}_{\alpha\mu\nu} = X(t)u_{\alpha}h_{\mu\nu} + Y(t)u_{\mu}h_{\alpha\nu} + Z(t)u_{\nu}h_{\alpha\mu} + V(t)u_{\alpha}u_{\mu}u_{\nu} + \epsilon_{\alpha\mu\nu\lambda}u^{\lambda}W(t)\delta_{n,4}$ for the distortion.

A note on the distortion variables

Quite generally, it is a simple matter to show that given a distortion tensor, torsion and non-metricity are computed through

$$S_{\mu\nu\alpha} = N_{\alpha[\mu\nu]} \ , \ \ Q_{\alpha\mu\nu} = 2N_{(\mu\nu)\alpha} \tag{5}$$

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For the previous cosmological expressions, these imply the identifications:

Relations between Cosmological variables

$$2(X+Y) = B \;,\;\; 2Z = A \;,\;\; 2V = C \;,\;\; 2\Phi = Y-Z \;,\;\; P = W \tag{6}$$

Whichever set one uses is then totally irrelevant to the actual physics. The set $A, B, \ldots$ has more transparent geometric meaning whereas the set $X, Y, \ldots$ is more convenient for calculations.

Isotropic Hypermomentum [**D.I**,2020, EPJC]

Imposing Cosm. Principle to Hypermomentum($\mathcal{L}_{\xi^i}\Delta_{\alpha\mu\nu}=0$)

$\Delta_{i00}=\Delta_{0i0}=\Delta_{00i}=0$,

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Covariant Form of Hypermomentum

Using an $1+(n-1)$ split we get the covariant form:

- $\Delta^{(n)}_{\alpha\mu\nu}=\phi h_{\mu\alpha}u_{\nu}+\chi h_{\nu\alpha}u_{\mu}+\psi u_{\alpha}h_{\mu\nu}+\omega u_{\alpha}u_{\mu}u_{\nu}+\delta_{n,4}\epsilon_{\alpha\mu\nu\kappa}u^{\kappa}\zeta$

Most General form of Hypermomentum respecting isotropy!

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Comments

1. In an FLRW $\phi,\chi,...$ depend only on time t. If homogeneity is relaxed $\phi=\phi(t,x^i)$ etc. (more about it later)
2. Hypermomentum generally contributes 5 dof in a Cosmological setting($n=4$). (and 4 dof for $n\neq 4$).

Hypermomentum Decomposition (Matter with Microstructure)

- Spin Part: $\Delta_{[\alpha\mu]\nu} = (\psi - \chi) u_{[\alpha} h_{\mu]\nu} + \delta_{n,4} \epsilon_{\alpha\mu\nu\kappa} u^{\kappa} \zeta$
- Dilation Part: $\Delta_{\nu} := \Delta_{\alpha\mu\nu} g^{\alpha\mu} = \Big[(n-1)\phi - \omega \Big] u_{\nu}$
- Shear Part: $\breve{\Delta}_{\alpha\mu\nu} = \Delta_{(\alpha\mu)\nu} - \frac{1}{n} g_{\alpha\mu} \Delta_{\nu} = \frac{(\phi+\omega)}{n} \Big[h_{\alpha\mu} + (n-1) u_{\alpha} u_{\mu} \Big] u_{\nu} + (\psi + \chi) u_{(\mu} h_{\alpha)\nu}$

Sourcing Torsion and Non-Metricity $(5 = 2 + 3)$

By means of the connection field eqs, the above parts act as sources producing spacetime torsion and non-metricity (see example later).

Physical variables

The variables $\phi, \chi, \ldots$ themselves don't have a physical meaning but rather the combinations

$$\sigma = \frac{(\psi - \chi)}{2} \qquad \zeta = \zeta \qquad (\textit{spin}) \tag{7}$$

$$\Delta = 3\phi - \omega\,, \qquad (\textit{dilation}) \tag{8}$$

$$\Sigma_1 = \frac{(\psi + \chi)}{2}\,, \qquad \Sigma_2 = \frac{(\phi + \omega)}{4} \qquad (\textit{shear}) \tag{9}$$

describe properly the cosmological parts of spin, dilation and shear.

- Therefore in a cosmological setting the spin part of hypermomentum contributes 2 dof, the dilation 1 and the shear 2 (2+1+2=5 dof of hypermomentum).

The Perfect Hypermomentum Preserving Hyperfluid [D.I, 2020]

Energy Momentum:

$$T_{\mu\nu} = t_{\mu\nu} = \rho u_\mu u_\nu + p h_{\mu\nu}$$

Hypermomentum :

$$\Delta^{(n)}_{\alpha\mu\nu} = \phi h_{\mu\alpha} u_\nu + \chi h_{\nu\alpha} u_\mu + \psi u_\alpha h_{\mu\nu} + \omega u_\alpha u_\mu u_\nu + \delta_{n,4} \epsilon_{\alpha\mu\nu\kappa} u^\kappa \zeta$$

Conservation laws (obtained from diff invariance)

$$\tilde{\nabla}_\mu T^\mu{}_\nu = \frac{1}{2} \Delta^{\alpha\beta\gamma} R_{\alpha\beta\gamma\nu}. \quad \hat{\nabla}_\nu \left(\sqrt{-g} \Delta_\lambda{}^{\mu\nu} \right) = 0$$

We call it hypermomentum preserving.

Note

The conservation law for hypermomentum (2nd eq. above) in an FLRW Universe really contains 2 independent eqs for the 5 fields.
$\Rightarrow$ 3 eqs of state must be provided.

Continuity equation and hypermomentum evolution

Evolution Equations of the Perfect (Hypermomentum preserving) Hyperfluid

$$\dot{\rho} + (n-1)H(\rho + p) = -\frac{1}{2}u^{\mu}u^{\nu}(\chi R_{\mu\nu} + \psi \breve{R}_{\mu\nu}) \tag{10}$$

$$\dot{\phi} + (n-1)H\phi + H(\chi + \psi) + \psi X - \chi Y = 0 \tag{11}$$

$$\dot{\omega} + (n-1)H(\chi + \psi + \omega) + (n-1)(\psi X - \chi Y) = 0 \tag{12}$$

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Equations of state

As mentioned earlier, the above system must be supplemented with appropriate equations of state among the energy density, pressure and hypermomentum variables.

Generalization: There exists a Perfect Hyperfluid, generalizing the Perfect Fluid notion of GR, for which: (D.I. 2021, JCAP)

$$t_{\mu\nu} = \rho_c u_\mu u_\nu + p_c h_{\mu\nu} \quad , \quad T_{\mu\nu} = \rho u_\mu u_\nu + p h_{\mu\nu} \tag{13}$$

$$\Delta^{(n)}_{\alpha\mu\nu} = \phi h_{\mu\alpha} u_\nu + \chi h_{\nu\alpha} u_\mu + \psi u_\alpha h_{\mu\nu} + \omega u_\alpha u_\mu u_\nu + \delta_{n,4}\epsilon_{\alpha\mu\nu\kappa} u^\kappa \zeta \tag{14}$$

These sources are subject to the conservation laws:

$$\tilde{\nabla}_\mu t^\mu{}_\alpha = \frac{1}{2}\Delta^{\lambda\mu\nu} R_{\lambda\mu\nu\alpha} + \frac{1}{2} Q_{\alpha\mu\nu}(t^{\mu\nu} - T^{\mu\nu}) \tag{15}$$

$$t^\mu{}_\lambda = T^\mu{}_\lambda - \frac{1}{2\sqrt{-g}}\hat{\nabla}_\nu(\sqrt{-g}\Delta_\lambda{}^{\mu\nu}) \tag{16}$$

The Perfect Hyperfluid is a direct generalization of the Perfect Fluid description where now the microscopic characteristics of matter are also taken into account.

Lagrangian Formulation of Hyperfluids

Hyperhydrodynamics

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- The Lagrangian that produces this isotropic hypermomentum was given there.

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- In FLRW the energy-momentum is of course of perfect fluid form but for generic backgrounds we have an imperfect fluid!
- The viscous fluid characteristics, such as heat flux, anisotropic stresses etc. have an origin from hypermomentum.
- Quite remarkably first order hydrodynamics naturally derives from our fluid action!

A Simple extension of Einstein-Cartan Theory

Our model

We consider the Metric-Affine version of the Einstein-Hilbert Action with the matter sector being a Perfect Hyperfluid:

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}R + S_{hyp}[g,\Gamma] \tag{17}$$

Field Equations

$$R_{(\mu\nu)} - \frac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu}\,, \tag{18}$$

$$\left(\frac{Q_\lambda}{2} + 2S_\lambda\right)g^{\mu\nu} - ({Q_\lambda}^{\mu\nu} + 2{S_\lambda}^{\mu\nu}) + \left(q^\mu - \frac{Q^\mu}{2} - 2S^\mu\right)\delta^\nu_\lambda = \kappa{\Delta_\lambda}^{\mu\nu}\,. \tag{19}$$

Torsion and Non-metricity in terms of Hypermomentum

The connection field equations yield in this case

$$\frac{A}{2}+4\Phi-\frac{C}{2}=\kappa\psi\,, \tag{20}$$

$$B-\frac{3A}{2}-4\Phi-\frac{C}{2}=\kappa\chi\,, \tag{21}$$

$$2P=-\kappa\zeta\,, B=-2\kappa\phi\,, 3B=-2\kappa\omega\,. \tag{22}$$

This can be solved for A, B, C etc in terms of the sources $\phi, \chi, \ldots$

Important Note

Due to projective invariance we have a vanishing dilation current. Then we may fix the projective gauge such that $C=3/2B$.

Friedmann equations with spin and shear (hyperfluid)

$$H^2 = \frac{\kappa}{2}\Big(\dot{\phi} + (3\phi + \psi + \chi)H - \frac{\kappa}{4}(\psi - \chi)(\psi + \chi + 2\phi)\Big)$$
$$+\frac{\kappa^2}{4}\phi^2 + \frac{\kappa^2}{4}\zeta^2 - \frac{\kappa^2}{16}(\psi - \chi)^2 + \frac{\kappa}{2}H(\psi - \chi) + \frac{\kappa\rho}{3}\,. \qquad (23)$$

$$\frac{\ddot{a}}{a} = -\frac{\kappa}{6}(\rho + 3p) + \frac{\kappa}{4}(\dot{\psi} - \dot{\chi}) + \frac{\kappa}{4}H(\psi - \chi) - \frac{\kappa^2}{4}\phi(\psi + \chi + 2\phi)$$
$$-\frac{\kappa}{2}\Big[\dot{\phi} + (3\phi + \chi + \psi)H - \frac{\kappa}{4}(\psi - \chi)(\psi + \chi + 2\phi)\Big]\,. \qquad (24)$$

Friedmann equations with spin and shear (hyperfluid)

$$H^2 = \frac{\kappa}{2}\left(\dot{\phi} + (3\phi + \psi + \chi)H - \frac{\kappa}{4}(\psi - \chi)(\psi + \chi + 2\phi)\right) + \frac{\kappa^2}{4}\phi^2 + \frac{\kappa^2}{4}\zeta^2 - \frac{\kappa^2}{16}(\psi - \chi)^2 + \frac{\kappa}{2}H(\psi - \chi) + \frac{\kappa\rho}{3}. \tag{23}$$

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Note: The ζ spin mode

Consistency of the conservation laws and the Friedmann equations demands that $\zeta \propto \frac{1}{a}$ namely it behaves exactly as a curvature contribution!

Alternative form

We may recast the system of equations along with the conservation laws as (for $\zeta = 0$)

$$3H^2 = \kappa\rho + \kappa\rho_h \,, \tag{25a}$$

$$2\dot{H} + 3H^2 = -\kappa p - \kappa p_h \,, \tag{25b}$$

$$\dot{\rho} + 3H(\rho + p) = -\dot{\rho}_h - 3H(\rho_h + p_h) \,, \tag{25c}$$

where the effective density of the hyperfluid is given by

$$\rho_h := \frac{3\dot{\Sigma}_2}{2} + \kappa\left(-\frac{3\Sigma_1\sigma}{2} + \frac{3\Sigma_2^2}{4} - \frac{3\Sigma_2\sigma}{2} - \frac{3\sigma^2}{4}\right) + H\left(3\Sigma_1 + \frac{9\Sigma_2}{2} + 3\sigma\right) \tag{26}$$

and a similar expression for p_h.

Distinguished Indexes

A series of different indexes

$$w := \frac{p}{\rho}\,, \tag{27a}$$

$$w_h := \frac{p_h}{\rho_h}\,, \tag{27b}$$

$$w_{\text{eff}} := \frac{p + p_h}{\rho + \rho_h} = \frac{w\,\rho + w_h\,\rho_h}{\rho + \rho_h} \neq w + w_h\,. \tag{27c}$$

It is also a simple matter to show that

$$w_{eff} = -1 - \frac{2\dot{H}}{3H^2}\,. \tag{28}$$

We call w just the barotropic index of matter eos, w_h the hypermomentum index, and w_{eff} as effective (expansion) index.

Pure Spin Case

The full analysis of the Cosmological dynamics of this model was performed in [Ilaria Andrei, **D.I,**, Laur Jarv, Margus Saal,Phys.Rev.D 111 (2025) 6, 064063].

System of Eqns for pure spin

$$3H^2 = \kappa\rho + 3H\kappa\sigma - \frac{3\kappa^2\sigma^2}{4}\,, \tag{29}$$

$$2\dot{H} + 3H^2 = -\kappa p + 2H\kappa\sigma + \dot{\sigma}\kappa - \frac{\kappa^2\sigma^2}{4}\,, \tag{30}$$

$$\dot{\rho} + 3H(\rho + p) = \frac{\kappa\sigma}{2}(\rho + 3p)\,, \tag{31}$$

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Note

The continuity eqn is identical to that of an open thermodynamic system with particle production rate $\Gamma \propto \sigma$!

Spin proportional to (square root of) energy-density

From the Friedmann eqn one sees that σ^2 has the same dimensions as ρ. This suggests the ansatz:

$$\sigma = b\sqrt{\frac{3\rho}{\kappa}}\,, \tag{32}$$

where b is a dimensionless constant.

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where b is a dimensionless constant.

Using this the modified continuity eqn becomes

$$\dot{\rho} \pm \sqrt{3\kappa}\,(1 + w \pm b)\,\rho^{3/2} = 0\,, \tag{33}$$

namely

$$w_\rho = w \pm b\,. \tag{34}$$

is the barotropic index of matter monitoring how energy dilutes.

Effective Expansion Index

On the other hand, one finds the effective expansion index to be

$$w_{eff} = \frac{2w \mp b}{2 \pm 3b}\,. \tag{35}$$

The fact that $w_\rho \neq w_{eff}$ has dramatic consequences on the conclusions that can be drawn for the behaviour of ρ and H. For instance:

- $w = -1$ does not necessarily imply that $\rho = constant$ and $H = constant$.
- De Sitter expansion $(H = H_0)$ does not demand that $\rho = constant$.

In preparation...

'Interacting fluids cosmologies from the metric affine framework'

The previous theory is extended to include perfect fluid+hyperfluid

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}R + S_{pf} + S_{hyper} \tag{36}$$

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Preliminary results

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Preliminary results

- The model resembles interacting fluid cosmologies where the creation/annihilation rate is related to the interaction of spin with spacetime torsion.
- Model $\rightarrow$ statistically equivalent to the standard ΛCDM scenario across most datasets. Considering the most complete dataset selection, it provides a significantly improved fit according to Information Criteria

Conclusions/Further Prospects

- The Perfect Hyperfluid (Perfect Fluid with microstructure) is the Cosmological Fluid of MAG.

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- Different effective indexes describe the various cosmological quantities
- Connection to observations and bounds on hypermomentum variables?

...Thank you!!!
Gracias!!!