Phys. Rev. D 113, 063518 (2026) · DOI: 10.1103/n46p-zd3j

Inconsistencies of Tsallis Cosmology

within horizon thermodynamics and holographic scenarios

Cai-Kim thermodynamic gravity · Holographic dark energy · BBN & CMB viability tests

Pedro. M. Ibarbo-Perlaza, J. Bayron. Orjuela-Quintana, Jose. L. Palacios-Córdoba, César. A. Valenzuela-Toledo

Department of Physics, Universidad del Valle · Cali, Colombia

GRECO Group · 4º WorkShop CCC· 2026

Overview

Motivation

Nonextensive entropy formalisms have been widely proposed to go beyond ΛCDM. **Tsallis entropy** introduces a nonextensivity parameter δ, recovering Bekenstein-Hawking at $\delta = 1$.

Key question: Are Tsallis-based cosmologies actually viable across the full expansion history?

Central finding

Even **infinitesimally small** departures from $\delta = 1$ produce **pathological behavior** in the early Universe:

- Negative or complex dark energy density
- Divergent equation of state w_{DE}
- Excessive early dark energy fraction

Violating BBN & CMB constraints.

Two frameworks studied

- **Cai-Kim:** first law of thermodynamics on the apparent horizon
- **Tsallis HDE:** holographic dark energy with Hubble or GO cutoff

Viable cosmology emerges **only** in the limit $\delta = 1$, collapsing both frameworks to ΛCDM.

Tsallis entropy on the apparent horizon

Clausius relation

Apparent horizon: $R_A = 1/H$ (flat FLRW)

↓

Hawking temperature: $T_h = 1/2\pi R_A$

↓

Tsallis entropy: $S_T = \frac{\tilde{\alpha}}{4G} A^{\delta}$

↓

First law: $\delta Q = T_h\, dS_T$

↓ *integrate*

Modified Friedmann equations

Modified Friedmann equations

$$H^2 = \frac{8\pi G}{3}(\rho + \rho_{\rm DE})$$

Effective dark energy density:

$$\rho_{\rm DE} = \frac{3}{8\pi G}\left[H^2\left(1 - \frac{\alpha\delta}{2-\delta} H^{2(1-\delta)}\right) + \frac{\Lambda}{3}\right]$$

Tsallis correction $\propto H^{2(1-\delta)}$: grows toward the past!

Equation of state:

$$w_{\rm DE} = -1 - \frac{2\dot{H}[1 - \delta\alpha H^{2(1-\delta)}]}{3H^2\left[1 - \frac{\delta\alpha}{2-\delta} H^{2(1-\delta)}\right] + \Lambda}$$

$\delta = 1 \Rightarrow \rho_{\rm DE} = \Lambda/8\pi G$ and $w_{\rm DE} = -1$: exact ΛCDM.

Cai-Kim Framework

Analytical solution and viability conditions

Closed-form dark energy parameter

$$1-\Omega_{\rm DE} = H^2_{(mr)}\left\{\frac{2-\delta}{\alpha\delta}\left[\frac{\Lambda}{3}+H^2_{(mr)}\right]\right\}^{\frac{1}{\delta-2}}$$

where $H^2_{(mr)} \equiv H_0^2\{\Omega_{m0}(1+z)^3+\Omega_{r0}(1+z)^4\}$

Acceleration condition ($q<0$ today):

$$\delta \gtrsim 0.45 \quad (\text{for } \hat{\alpha}=1,\ \Omega_{m0}=0.3)$$

But late-time acceleration $\neq$ early-Universe viability!

Pathologies at $z \approx 3200$ (radiation-matter equality)

$\delta < 1$:

Radiation overshoots: $\Omega_r \gtrsim 1$

Compensated by negative $\Omega_{\rm DE}$ $\Rightarrow$ unphysical.

$\delta > 1$:

Dark energy prematurely dominates.

Standard radiation $\to$ matter transition erased.

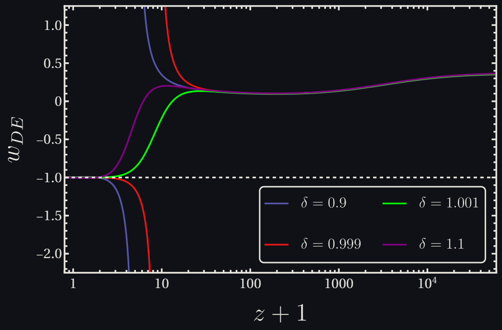
BBN bound ($\Omega_{\rm DE}|_{z\sim 3200} < 0.045$):

$$1.00 \leq \delta < 1.00038$$

Essentially indistinguishable from $\delta = 1$!

Dark energy equation of state

$w_{\mathrm{DE}}(z)$ for various δ



Late-time convergence (misleading)

All solutions converge to $w_{\mathrm{DE}} \to -1$ (de Sitter) at low z. This is why earlier studies *(Lymperis & Saridakis 2018, Nojiri et al. 2019)* only working up to $z \sim 2$–3 **missed** the divergence entirely.

High-z catastrophe

For $\delta < 1$: denominator of w_{DE} vanishes at some intermediate z, causing a **pole**.

For $\delta > 1$: Ω_{DE} grows excessively, already excluded by BBN.

Lesson: Supernovae-only fits ($z \lesssim 2$) cannot diagnose these pathologies. Full expansion history analysis is essential.

Tsallis cosmology as a perturbative deformation of ΛCDM

Modified expansion law

$$\frac{\alpha\delta}{2-\delta}H^{2(2-\delta)} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} \equiv H_\Lambda^2$$

Expanding around $\delta = 1$ (with $\hat{\alpha} = 1$):

$$H^2 \approx H_\Lambda^2 + \delta H_{\Lambda,1}^2(\delta-1) + \delta H_{\Lambda,2}^2(\delta-1)^2$$

where the correction coefficients grow as:

$$\delta H_{\Lambda,1}^2 \sim H_\Lambda^2 \ln\frac{H_\Lambda^2}{H_0^2}$$

At high z: $\ln(H_\Lambda^2/H_0^2) \propto \ln(1+z)^4 \to \infty$

Impact on radiation era

$$\Omega_r \propto \frac{\rho_r}{H_\Lambda^2}\left[1 + \frac{\delta H_{\Lambda,1}^2}{H_\Lambda^2}(\delta-1) + \cdots\right]^{-1}$$

Correction factor $\propto \ln(1+z)^4$:

- $\delta < 1$: $\Omega_r > 1$ at high $z \Rightarrow \Omega_{\rm DE} < 0$
- $\delta > 1$: excess early dark energy, violates BBN

Key insight: Tsallis corrections are *not* small perturbations around ΛCDM. They grow **non-perturbatively** toward the past, invalidating the model for any $\delta \neq 1$.

Tsallis HDE with Hubble horizon cutoff

Setup

Holographic principle with Tsallis entropy $S_T = \gamma A^{\delta}$ and IR cutoff $L_{\rm IR} = 1/H$:

$$\rho_{\rm HDE} = BH^{2(2-\delta)}$$

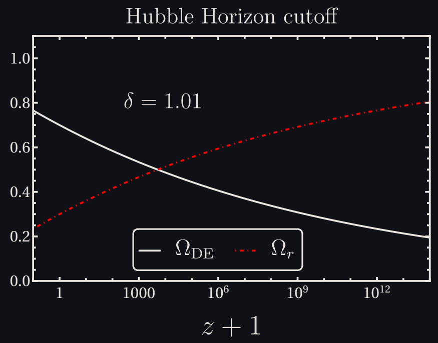
Continuity equation for the HDE component:

$$\dot{\rho}_{\rm HDE} + 3H\rho_{\rm HDE}(1 + w_{\rm HDE}) = 0$$

Evolution of $\Omega_{\rm HDE}$:

$$\frac{d\Omega_{\rm HDE}}{d\ln a} = \frac{4(\delta-1)\Omega_{\rm HDE}(1-\Omega_{\rm HDE})}{1-(2-\delta)\Omega_{\rm HDE}}$$

Radiation-era pathology



For $\delta = 1.01$, even at $z = 10^{14}$: $\Omega_{\rm HDE} \sim 20\%$

Standard radiation era is **impossible**.

Tsallis HDE with Granda-Oliveros cutoff

GO cutoff (avoids causality problem)

$$L_{\rm IR}^{\rm (GO)} = \left(\tilde{\alpha} H^2 + \tilde{\beta}\dot{H}\right)^{-1/2}$$

$$\rho_{\rm HDE} = 3m_P^2(\alpha H^2 + \beta\dot{H})^{2-\delta}$$

Perturbative expansion around $\delta = 1$:

$$\frac{\Omega_{\rm HDE}}{H^2_{(r)}} = (\alpha - 2\beta) - (\alpha - 2\beta)(\delta - 1)\ln\frac{H_0^2\Omega_{r0}}{a^4(\alpha - 2\beta)} + \mathcal{O}[(\delta - 1)^2]$$

Problems

Always large at high z

Correction $\propto \ln(1+z)^4$ for $\delta \neq 1$. Only $\delta = 1$ keeps it constant — i.e. no dynamics.

Fine-tuning dilemma

$\alpha \gtrsim 2\beta$: radiation era OK, but **significant dark energy during matter domination.**

$2\alpha \gtrsim 3\beta$: matter era OK, but HDE non-negligible during radiation.

No choice of α, β avoids **both** pathologies.

This coincidence-like fine-tuning adds an extra theoretical burden beyond what Tsallis entropy was meant to solve.

Comparison of pathologies across all frameworks

Framework	$\Omega_{\rm DE} < 0$	$w_{\rm DE}$ diverges	Excess early DE
Cai-Kim, $\delta < 1$	✓	✓	—
Cai-Kim, $\delta > 1$	—	—	✓
Tsallis HDE (Hubble), $\delta \neq 1$	—	—	✓ (always)
Tsallis HDE (GO), $\delta \neq 1$	—	—	✓ (unavoidable)
Viable range (Cai-Kim)			$1.00 \leq \delta < 1.00038$

Common mechanism: all pathologies trace to the same root. The Tsallis modification introduces $H^{2(2-\delta)}$ terms. As H grows toward the past, these corrections **dominate** regardless of how small $|\delta - 1|$ is.

Unlike ΛCDM where dark energy is negligible at high z, Tsallis corrections are **IR safe but UV dangerous**: they vanish today but diverge in the past.

Summary and implications

What we showed

- Cai-Kim: $\delta < 1 \Rightarrow$ negative DE density; $\delta > 1 \Rightarrow$ excess early DE
- HDE (Hubble): $\sim 20\%$ dark energy at $z = 10^{14}$ even for $\delta \approx 1$
- HDE (GO): no choice of parameters avoids all pathologies
- Perturbative analysis reveals logarithmic growth: not a small perturbation!

Viable Cai-Kim cosmology requires:

$$\boxed{1.00 \leq \delta < 1.00038}$$

Indistinguishable from $\delta = 1$ (ΛCDM)

Physical interpretation

Tsallis corrections $\propto H^{2(2-\delta)}$ are negligible at low z (where H is small) but grow **uncontrollably** toward the past. This is not a failure of the fit — it is a **fundamental structural incompatibility** with the standard thermal history.

Broader lesson

Dynamical consistency and cosmological viability tests (BBN, CMB, radiation era) are **essential** when assessing nonextensive entropy formalisms.

Future work: Apply same tests to Kaniadakis, Rényi, Barrow, and related proposals.

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Tsallis cosmology is viable *only* at $\delta = 1 \implies$ **exact ΛCDM**

2 Frameworks

Cai-Kim · HDE

3 Pathologies

$\rho < 0$ · $w \to \infty$ · EDE

1 Conclusion

$\delta \to 1$ in all cases

Pedro. M. Ibarbo-Perlaza, J. Bayron. Orjuela-Quintana, Jose. L. Palacios-Córdoba, César. A. Valenzuela-Toledo

Universidad del Valle, Cali, Colombia

Thank you!

pedro.ibarbo@correounivalle.edu.co