

# Energy-energy correlators at strong coupling

Congkao Wen  
Queen Mary University of London

2601.05312, 2607.21139

w/ Lecheng Ren (QMUL), Bo Wang (Zhejiang U. → Yale)  
+ Max Jackson (QMUL)

UK-Japan Workshop on Quantum Gravity 2026  
August 24 - 27 @ Durham University

# Motivation

## Why energy correlators

- measured in experiments (e+e- Colliders, ep Colliders, pp Colliders, ...)
- Infrared safe; bridge formal theory and experiments; ... [Moult, Zhu 2025 — review](#)

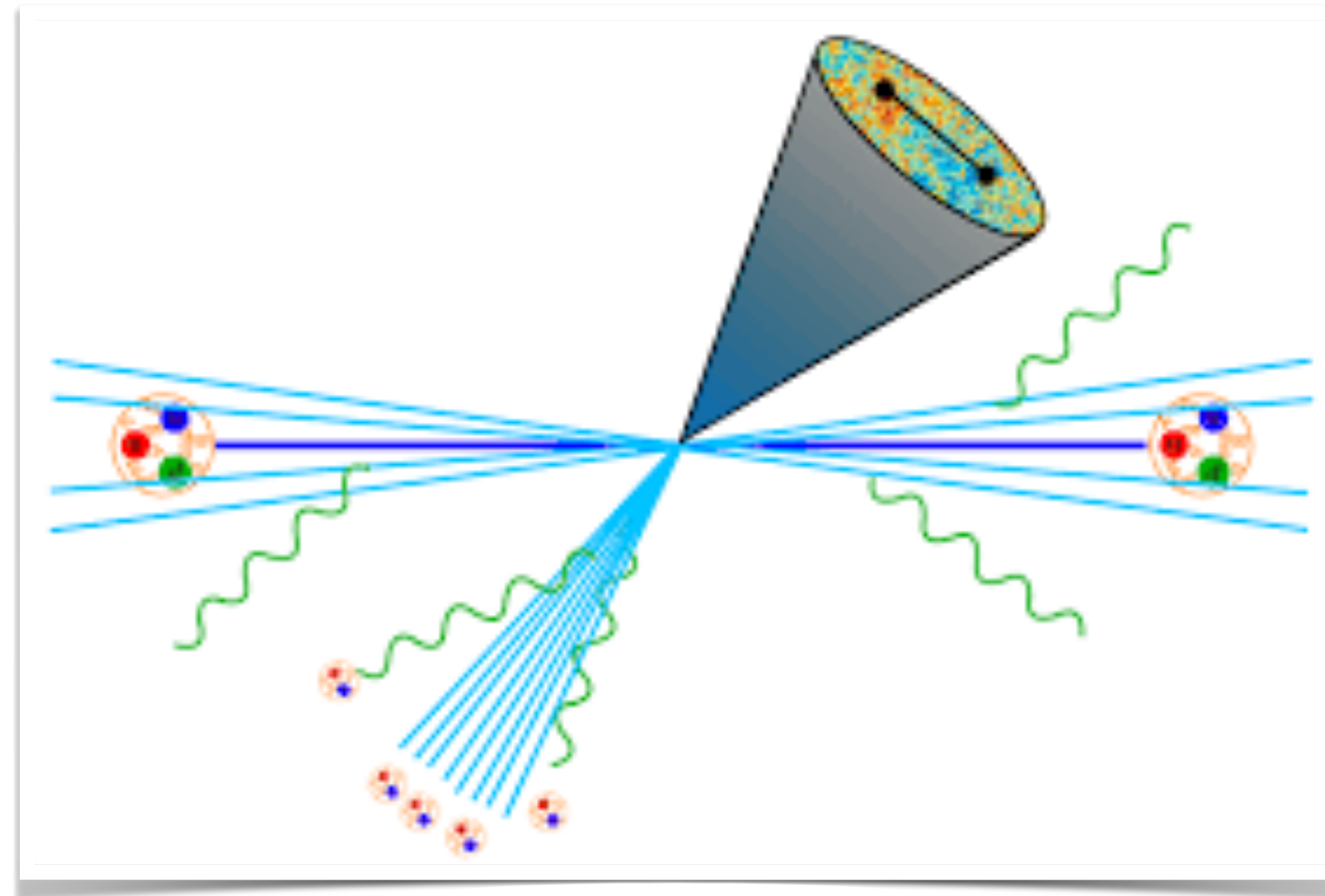
## Why $\mathcal{N}=4$ super Yang-Mills (SYM)

- AdS/CFT, allow for analytic computation at strong coupling, exact results

# Outline

- Brief introduction to energy correlators
- Energy-energy correlators (EECs) in  $\mathcal{N}=4$  SYM
- Strong coupling expansion of EECs in  $\mathcal{N}=4$  SYM
- Comments on the results
- Summary and outlook

# Energy correlators



# Introduction to energy correlators

- Energy flux operator [Sterman, 1975](#)

$$\mathcal{E}(\hat{n})|X\rangle = \sum_{k \in X} k^0 \delta^2(\Omega_n - \Omega_k) |X\rangle$$

- Later, an **operator definition** was introduced

[Sveshnikov, Tkachov; Korchemsky, Sterman](#)

[Hofman, Maldacena; Kologlu, Kravchuk, Simmons-Duffin, Zhiboedov](#)

$$\mathcal{E}(n) = \lim_{r \rightarrow \infty} r^2 \int_0^\infty dt n^i T_{0i}(t, r\hat{n})$$

where  $n = (1, \hat{n})$  is null

- Energy correlators in terms of local correlation functions

$$\frac{8\pi^2}{q^2 \sigma_{\text{tot}}(q)} \int d^4x e^{iq \cdot x} \langle \mathcal{O}(x) \mathcal{E}(n_1) \dots \mathcal{E}(n_i) \mathcal{O}(0) \rangle$$

# Introduction to energy correlators

Energy correlators provide a link between measurements of energy flux at macroscopic scales and local correlators underlying microscopic dynamics

- The averaged null energy condition (ANEC)

$$\langle X | \mathcal{E} | X \rangle \geq 0$$

Related to three-point functions of local operators (with one stress tensor): conformal collider bounds in the case of CFTs [Hofman, Maldacena; ...](#)

We'll be interested in two-point functions: **energy-energy correlators**

# Introduction to energy correlators

The energy-energy correlators (EEC)

$$\text{EEC}(\xi) = \frac{8\pi^2}{q^2 \sigma_{\text{tot}}(q)} \int d^4x e^{iq \cdot x} \langle 0 | \mathcal{O}(x) \mathcal{E}(n_1) \mathcal{E}(n_2) \mathcal{O}(0) | 0 \rangle$$

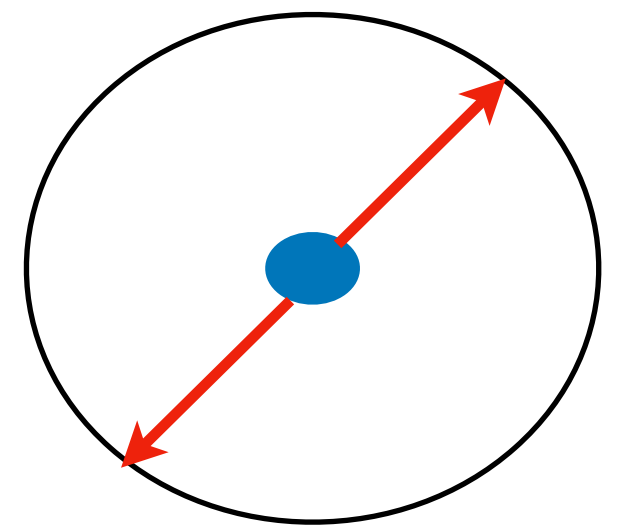
In the rest frame  $\xi = \frac{1}{2}(1 - \cos \theta)$

- Ward identities:  $\int_0^1 d\xi \text{EEC}(\xi) = 1$ ,  $\int_0^1 d\xi (1 - 2\xi) \text{EEC}(\xi) = 0$   
  
energy conservation                      momentum conservation

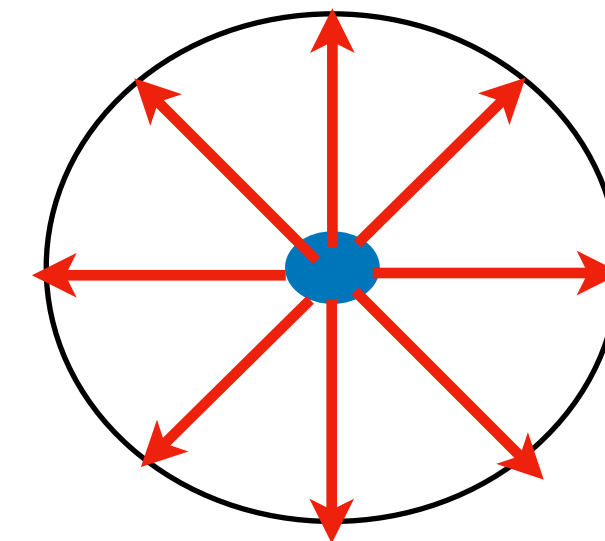
# Introduction to energy correlators

## Examples of EEC

- Weak coupling:  $\text{EEC}(\xi) = 1/2 (\delta(\xi) + \delta(1 - \xi)) + O(\lambda)$



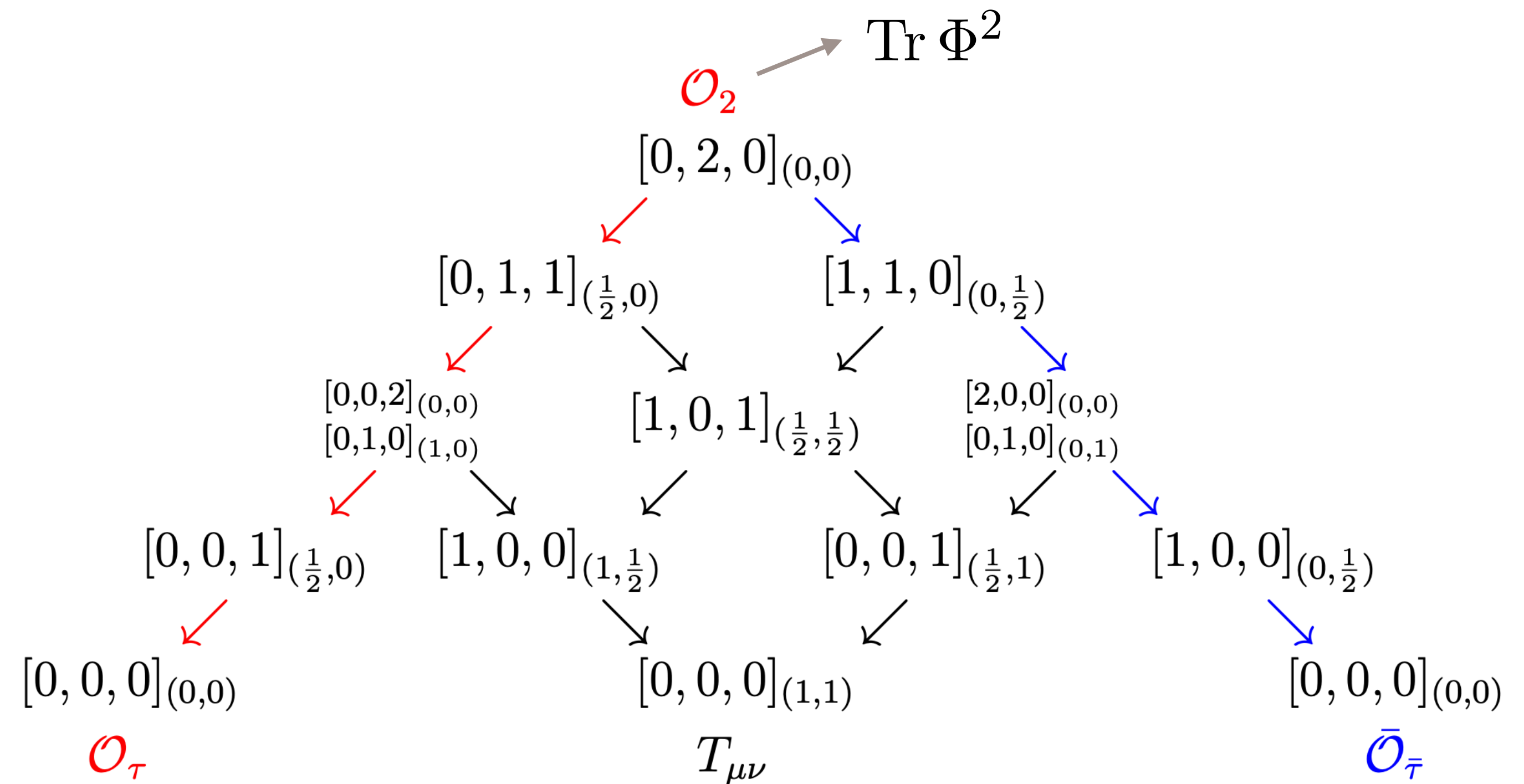
- Strong coupling:  $\text{EEC}(\xi) = 1 + O(1/\lambda)$



Analytic calculation in the strong coupling regime made possible due to AdS/CFT correspondence [Hofman, Maldacena](#)

# Energy-energy correlators in $\mathcal{N}=4$ SYM

$\mathcal{N}=4$  stress tensor multiplet



EECs in  $\mathcal{N}=4$  SYM are related to  $\langle \mathcal{O}_p \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_p \rangle$

# Four-point correlators in $\mathcal{N}=4$ SYM

# Four-point correlators in $\mathcal{N}=4$ SYM

Superconformal primary operators

$$\mathcal{O}_p(x, Y) = \text{Tr} \left( \Phi^{I_1}(x) \dots \Phi^{I_p}(x) \right) Y_{I_1} \dots Y_{I_p}$$

$$\mathcal{O}_{p_1, \dots, p_n}(x, Y) = \frac{p_1 \dots p_n}{p} \mathcal{O}_{p_1}(x, Y) \dots \mathcal{O}_{p_n}(x, Y)$$

$\Phi^I(x)$  are scalars, and  $Y_I$  are  $\text{SO}(6)$  R-symmetry null vectors.

They are 1/2 BPS operators.

# Four-point correlators in $\mathcal{N}=4$ SYM

- We will be interested in correlators in planar  $\mathcal{N}=4$  SYM of the form

$$\langle \mathcal{O}_p(x_1, Y_1) \mathcal{O}_2(x_2, Y_2) \mathcal{O}_2(x_3, Y_3) \mathcal{O}_p(x_4, Y_4) \rangle$$

- Superconformal Ward identity

Eden, Petkou, Schubert, Sokatchev; Nirschl, Osborn

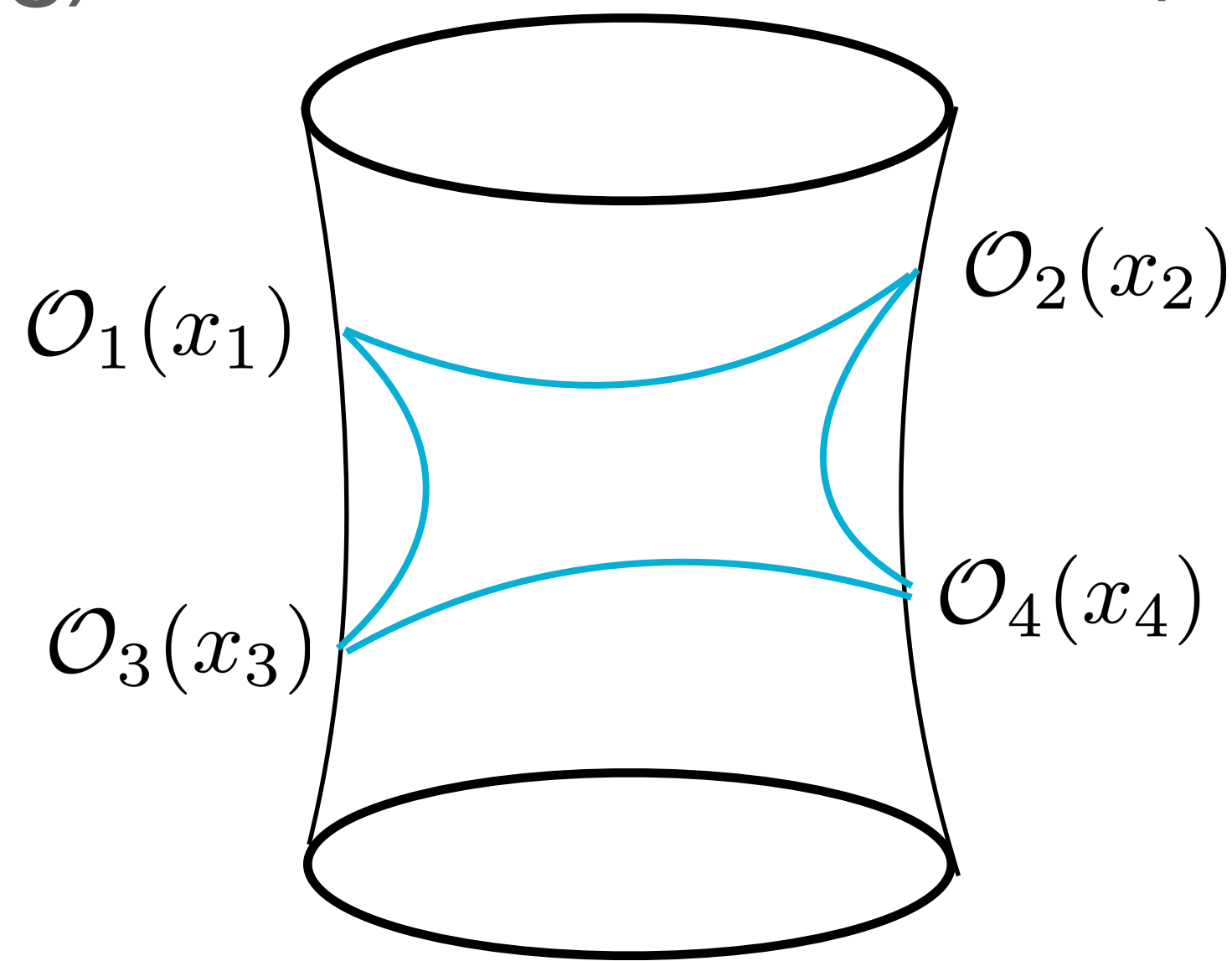
Fixed by  
symmetry

Our focus

$$\langle \mathcal{O}_p(x_1, Y_1) \mathcal{O}_2(x_2, Y_2) \mathcal{O}_2(x_3, Y_3) \mathcal{O}_p(x_4, Y_4) \rangle = \mathcal{G}_{\text{free}}(x_i, Y_i) + \mathcal{I}_4(x_i, Y_i) (d_{14})^{p-2} \mathcal{T}_p(u, v)$$

# Four-point correlators in $\mathcal{N}=4$ SYM

String amplitudes in AdS  $\longleftrightarrow$  Correlators in the dual CFT  
(Type IIB superstring)  $(\mathcal{N}=4$  super Yang-Mills)



$$A(s, t; \alpha' / R^2) \leftrightarrow \langle \mathcal{O}_1(x_1) \mathcal{O}_2(x_2) \mathcal{O}_3(x_3) \mathcal{O}_4(x_4) \rangle \quad 1/\sqrt{\lambda} = \alpha' / R^2$$

AdS radius 

# Mellin amplitudes in $\mathcal{N}=4$ SYM

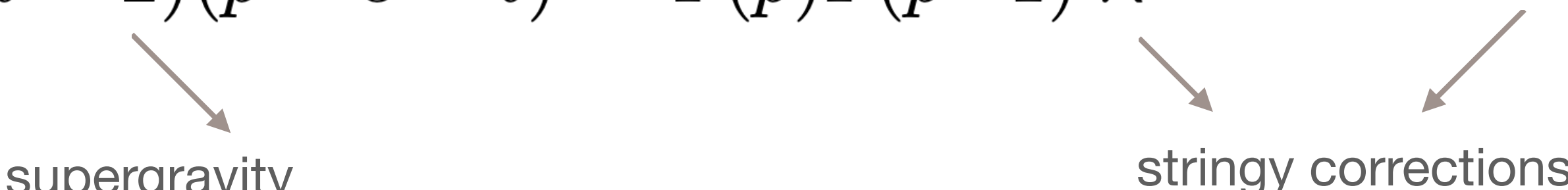
Correlators in Mellin space Mack; Penedones

$$\mathcal{T}_p(u, v) = \frac{1}{4p} \int_{-i\infty}^{i\infty} \frac{ds dt}{(2\pi i)^2} u^{\frac{1}{2}(\tilde{u}-p)} v^{\frac{1}{2}(s-p)} \Gamma^2\left(\frac{p+2-s}{2}\right) \Gamma\left(2-\frac{t}{2}\right) \Gamma\left(p-\frac{t}{2}\right) \Gamma^2\left(\frac{p+2-\tilde{u}}{2}\right) \mathcal{M}_p(s, t)$$

with Mellin variables  $s + t + \tilde{u} = 2p$

at strong coupling (or low-energy) expansion

$$\mathcal{M}_p(s, t) = \frac{4p}{\Gamma(p-1)} \frac{1}{(s-p)(t-2)(p-s-t)} + \frac{\Gamma(4+p)}{\Gamma(p)\Gamma(p-1)} \frac{\zeta(3)}{\lambda^{3/2}} + \dots \text{polynomials}$$



supergravity                      stringy corrections

# EECs from Mellin amplitudes

- The EECs are Wightman functions: Euclidean to Wightman [Mack, 2009](#)

$$x_{ij}^2 \rightarrow -x_{ij}^2 + i\epsilon x_{ij}^0 \theta(j - i)$$

Mellin amplitudes make analytic continuation relatively easier

- After the Wick rotation and Fourier transform, the EECs can be expressed as [Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov](#)

$$\text{EEC}_p(\xi) = \int_{-i\infty}^{i\infty} \frac{ds dt}{(2\pi i)^2} \mathcal{M}_p(s, t) \mathcal{K}_p(t; \xi)$$

# EECs from Mellin amplitudes

The detector kernel

$$\mathcal{K}_p(t; \xi) = \sum_{k=0}^2 \binom{2}{k} \frac{(-1)^k \xi^{\frac{t}{2}-k} \pi^2 t (t-2) \Gamma(k-\frac{t}{2}) \Gamma(p-1) \Gamma(p) \Gamma(p-\frac{t}{2})}{32 \sin^2(\frac{\pi t}{2}) \Gamma(p+k-2-\frac{t}{2}) \Gamma(k-2-\frac{t}{2}) \Gamma(-2-\frac{t}{2}) \Gamma(p-k+3+\frac{t}{2})} \\ \times {}_2F_1\left(3-k+\frac{t}{2}, 3-k+\frac{t}{2}, p-k+3+\frac{t}{2}; \xi\right),$$

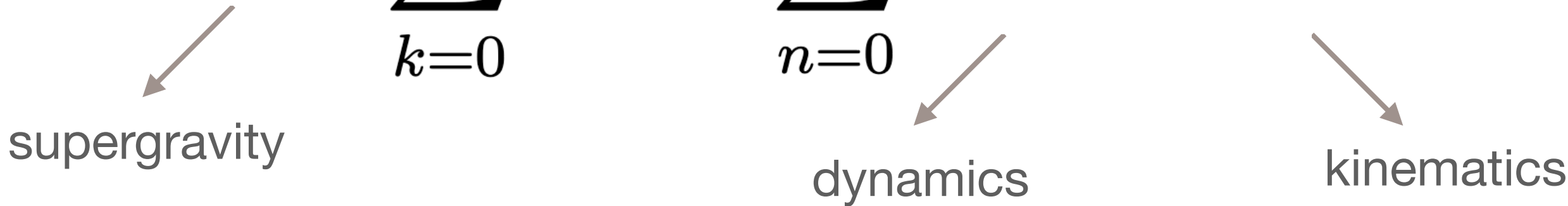
for the simplest case  $p=2$ ,

$$\mathcal{K}_2(t; \xi) = -\frac{\pi t^2 (t-2)^2}{128 \xi^3 \sin(\pi t/2)} \left(\frac{\xi}{1-\xi}\right)^{1+\frac{t}{2}}$$

# EECs at strong coupling

The t-integral can be done by expanding Mellin amplitudes order by order, leading to general structure of EECs at strong coupling

$$\text{EEC}_p(\xi) = 1 + \sum_{k=0}^{\infty} \lambda^{-\frac{k}{2}-1} \sum_{n=0}^k c_{k,n}(p) Q_{k-n,p}(\xi) \quad Q_{n,p}(\xi) = \int_{-i\infty}^{i\infty} \frac{dt}{2\pi i} t^n \mathcal{K}_p(t; \xi)$$



- $Q_{n,p}(\xi)$  are degree-(n+2) polynomials obeying

$$\int_0^1 d\xi Q_{n,p}(\xi) = 0, \quad \int_0^1 d\xi (1-2\xi) Q_{n,p}(\xi) = 0$$

# EECs at strong coupling

- Some examples:

$$Q_{0,p}(\xi) = \frac{24\Gamma(p-1)}{(p)_3} (6\xi^2 - 6\xi + 1) ,$$

$$Q_{1,p}(\xi) = \frac{48\Gamma(p-1)}{(p)_4} (-150\xi^3 + 216\xi^2 - 81\xi + 12\xi^2 p - 12\xi p + 2p + 6)$$

- They are explicitly determined through recursion relations.

- Convert them into Legendre polynomials Dempsey, Karlsson, Pufu, Zahraee, Zhiboedov

$$\text{EEC}_2(\xi) = \sum_{s=0}^{\infty} c_s P_s(1 - 2\xi) \quad \text{with } c_0 = 1, \quad c_1 = 0$$



dynamics



kinematics

# EECs at strong coupling

- The s-integral, which determines coefficients  $c_{k,n}(p)$ , however is not well-defined for low-energy expansion of Mellin amplitudes
- The light transform and low-energy (strong coupling) expansion **do not commute**
- It requires an alternative organising principle: **AdS Virasoro-Shapiro amplitudes** that resum low-energy expansions

e.g. flat space

$$A^{(0)}(S, T) = -\frac{\Gamma(-S)\Gamma(-T)\Gamma(S+T)}{\Gamma(S+1)\Gamma(T+1)\Gamma(1-S-T)}$$

Virasoro; Shapiro

# EECs at strong coupling

- AdS curvature expansion [Alday, Hansen; ...](#)

$$A(S, T) = \sum_{k=0}^{\infty} \lambda^{-k/2} A^{(k)}(S, T) \quad \text{we've used} \quad 1/\sqrt{\lambda} = \alpha'/R^2$$

- Relate to Mellin amplitudes via a Borel transform [Penedones](#)

$$\mathcal{M}_p(\hat{s}, \hat{t}) = \frac{1}{2\lambda^{3/2}\Gamma(p-1)\Gamma(p)} \int_0^\infty d\beta \beta^{p+3} e^{-\beta} A(\beta\hat{s}, \beta\hat{t})$$

with

$$\hat{s} = \frac{s_1}{2\sqrt{\lambda}} = \frac{s - 2p/3}{2\sqrt{\lambda}}, \quad \hat{t} = \frac{s_2}{2\sqrt{\lambda}} = \frac{t - 2p/3}{2\sqrt{\lambda}}$$

EECs from AdS VS amplitudes

# AdS Virasoro-Shapiro amplitudes

World-sheet formulation for AdS VS amplitudes Alday, Hansen, + Silva; ...

- The flat space amplitude

$$A^{(0)}(S, T) = \int d^2 z |z|^{-2S-2} |1 - z|^{-2T-2}$$

- AdS curvature corrections

$$A^{(k)}(S, T) = \int d^2 z |z|^{-2S-2} |1 - z|^{-2T-2} G_{\text{tot}}^{(k)}(z, \bar{z}; S, T)$$

$G_{\text{tot}}^{(k)}(z, \bar{z}; S, T)$  : **single-valued** multiple polylogarithms (SVMP) with **weight**  $3k$ .

# EECs from VS amplitudes

In terms of AdS VS amplitudes, what required to compute is

$$I^{(k)}(T) = \int_{-i\infty}^{+i\infty} \frac{dS}{2\pi i} A^{(k)}(S, T)$$

- Flat space:

$$I^{(0)}(T) = \frac{1}{T^2} \left( \frac{\Gamma(1-2T)}{\Gamma(1-T)^2} - 1 \right)$$

this determines the EEC coefficients of the form  $c_{k,0}$

$$c_{0,0} = \frac{\Gamma(p+3)}{\Gamma(p-1)\Gamma(p)} \zeta(2), \quad c_{1,0} = \frac{\Gamma(p+4)}{\Gamma(p-1)\Gamma(p)} \zeta(3), \quad c_{2,0} = \frac{19 \Gamma(p+5)}{16 \Gamma(p-1)\Gamma(p)} \zeta(4), \dots$$

# EECs from VS amplitudes

For the curvature corrections, using world-sheet representation, we deal with integrals of the form

$$\mathcal{J}_j(T) = \int_{-i\infty}^{i\infty} \frac{dS}{2\pi i} \int d^2z S^j |z|^{-2S-2} J(z, \bar{z}; T)$$

perform the s-integral first:

- for  $j < 0$ , the  $z$ -integral restricts to a unit disk
- for  $j \geq 0$ , the  $z$ -integral restricts to a unit circle

# EECs from VS amplitudes

Examples from 1st curvature correction Alday, Hansen; Fardelli, Hansen, Silva

- disc integral
 
$$I_1^{(1)}(T) = \int \frac{dS}{2\pi i} \int d^2z |z|^{-2S-2} |1-z|^{-2T-2} \frac{p^2 T}{6(S+T)} \left( \mathcal{L}_{000} - \mathcal{L}_{001} - \mathcal{L}_{010} + \mathcal{L}_{011} \right. \\ \left. - \mathcal{L}_{100} + \mathcal{L}_{101} + \mathcal{L}_{110} - \mathcal{L}_{111} \right)$$

weight-3 SVMPs

$$= \frac{p^2 T}{36} \int_{|z|<1} d^2z |z|^{2T-2} |1-z|^{-2T-2} \log^3 \frac{|z|^2}{|1-z|^2} = -\frac{p^2 T}{36} \frac{\partial^3}{\partial T^3} \frac{\Gamma(-2T)}{\Gamma(1-T)^2},$$
- circle integral
 
$$I_3^{(1)}(T) = \oint_{|z|=1} \frac{d\theta}{2\pi} |1-z|^{-2T-2} (2\zeta(3) - \text{Li}_3(z) - \text{Li}_3(\bar{z}))$$

$$= \frac{2\zeta(3)\Gamma(-2T-1)}{\Gamma(-T)^2} + F(T),$$

# EECs from VS amplitudes

First curvature contribution to EECs

$$I^{(1)}(T) = \frac{2\zeta(3)\Gamma(-2T-1)}{\Gamma(-T)^2} - \frac{p^2 T}{18} \frac{\partial^3}{\partial T^3} \frac{\Gamma(-2T)}{\Gamma(1-T)^2} + \frac{p(p-2)T}{12} \frac{\partial^2}{\partial T^2} \frac{\Gamma(1-2T)}{T^2 \Gamma(1-T)^2} \\ - \frac{(p^2 + 6)}{36} \frac{\partial^3}{\partial T^3} \frac{\Gamma(1-2T)}{\Gamma(1-T)^2} + F(T) - F(T-1),$$

$$F(T) = \sum_{n=1}^{\infty} \frac{2 \sin(\pi T) \Gamma(-2T-1) \Gamma(n+T+1)}{\pi n^3 \Gamma(n-T)}$$

Combing with  $I^{(0)}(T)$ , it determines EEC coefficients of the form  $C_{k,1}$

# EECs from VS amplitudes

These results determine EECs up to

$$\text{EEC}_p(\xi) = 1 + \frac{c_{0,0}}{\lambda} Q_{0,p}(\xi) + \frac{c_{1,1} Q_{0,p}(\xi) + c_{1,0} Q_{1,p}(\xi)}{\lambda^{3/2}} + \mathcal{O}(\lambda^{-2})$$

$c_{0,0}$  ,  $c_{1,0}$  are fixed by **flat-space VS amplitude**, and

 Hofman, Maldacena; Gonçalves

Chicherin, Korchemsky, Sokatcheva, Zhiboedov

$$c_{1,1} = \frac{\Gamma(p+3)}{\Gamma(p-1)\Gamma(p)} (\zeta(2) - (p^2 + 2p + 2)\zeta(3))$$

which requires both **flat-space and first-curvature** contributions

# EECs from VS amplitudes

2nd curvature involves **weight-6 SVMP** and is only known for  $p=2$  Alday, Hansen

To determine the EEC to next order, we actually only need  $I^{(2)}(T)|_{T^0}$

- This term receives contribution only from the **unit circle integral**, which greatly simplifies the calculation (even though still not easy)

$$I^{(2)}(T) = 6 \zeta(2) - \frac{13}{2} \zeta(3) + \frac{263}{6} \zeta(4) + \zeta(2) \zeta(3) - \frac{5}{3} \zeta(5) + \mathcal{O}(T)$$

- Putting together flat-space, 1st and 2nd curvatures, we determine the EEC up to  $\lambda^{-2}$  (cont'd)

# EECs from VS amplitudes

The EEC to the highest order we have so far

$$\begin{aligned} \text{EEC}_2(\xi) = 1 &+ \frac{c_{0,0}}{\lambda} Q_{0,2}(\xi) + \frac{c_{1,1} Q_{0,2}(\xi) + c_{1,0} Q_{1,2}(\xi)}{\lambda^{3/2}} \\ &+ \frac{c_{2,2} Q_{0,2}(\xi) + c_{2,1} Q_{1,2}(\xi) + c_{2,0} Q_{2,2}(\xi)}{\lambda^2} + \mathcal{O}(\lambda^{-5/2}) \end{aligned}$$

Hofman, Maldacena

$$\begin{aligned} c_{0,0} &= 24\zeta(2), \quad c_{1,0} = 120\zeta(3), \quad c_{2,0} = 855\zeta(4), \\ c_{1,1} &= 24\zeta(2) - 240\zeta(3), \quad c_{2,1} = -120\zeta(2) + 240\zeta(3) - 3870\zeta(4) \\ c_{2,2} &= 304\zeta(2) - 476\zeta(3) + 24\zeta(2)\zeta(3) + 4692\zeta(4) - 40\zeta(5). \end{aligned}$$

It's instructive to reproduce the results from a “traditional” method:  
low-energy expansion

EECs from Wilson coefficients

# EECs from Wilson coefficients

Consider  $\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \rangle$  for simplicity, the Mellin amplitude

$$\mathcal{M}_2(s_1, s_2) = \mathcal{M}_{\text{SUGRA}} + \sum_{a,b=0}^{\infty} \frac{\Gamma(2a+3b+6)}{8^{a+b} \lambda^{\frac{3}{2}+a+\frac{3}{2}b}} (s_1^2 + s_2^2 + s_3^2)^a (s_1 s_2 s_3)^b \left( \alpha_{a,b}^{(0)} + \frac{\alpha_{a,b}^{(1)}}{\sqrt{\lambda}} + \dots \right)$$

- One can show Gonalves

$$c_{0,0} = \int_{-i\infty}^{+i\infty} \frac{ds_1}{2\pi i} \sum_{a=0}^{\infty} \frac{\Gamma(2a+6)}{4^a} \alpha_{a,0}^{(0)} s_1^{2a}$$

$$\alpha_{a,0}^{(0)} = \zeta(2a+3)$$

# EECs from Wilson coefficients

- $\mathcal{C}_{0,0}$  can be determined by a standard Borel summation [Gonçalves](#) or better a modified Borel summation [Arutyunov, Dorigoni, Savin; Dorigoni, Green, C.W.](#)

$$\zeta(n) = \frac{2^{n-1}}{\Gamma(n+1)} \int_0^\infty d\omega \frac{\omega^n}{\sinh^2(\omega)}$$

- $\mathcal{C}_{0,0}$  can be more efficiently determined through

$$c_{0,0} = \frac{1}{2} \frac{\Gamma(2a+6)}{4^a} \alpha_{a,0}^{(0)} \Big|_{a \rightarrow -\frac{1}{2}} = 24\zeta(2)$$

# EECs from Wilson coefficients

In general we encounter integrals of the form

$$\mathcal{I} = \int_{-i\infty}^{+i\infty} \frac{ds_1}{2\pi i} \sum_{a=0}^{\infty} \underbrace{f(a)}_{\text{polynomials}} \frac{\Gamma(2a+p)}{4^a} \zeta(2a+q, \vec{w}) s_1^{2a+m}$$

- Using  $\lim_{s \rightarrow \pm i\infty} |M(s, t)| \leq |s|^{-2}$  the sum is equivalent to setting  $a = -\frac{m+1}{2}$
- This is in a similar spirit of [Dempsey, Karlsson, Pufu, Zahraee, Zhiboedov](#)
- Beyond flat space, the limit is **not always well-defined**, thus requires regularisation

# EECs from Wilson coefficients

- The first curvature contribution [Alday, Hansen, Silva](#)

$$\alpha_{a,0}^{(1)} = 2\zeta(2a+1)\zeta(3) - 2\zeta(2a+1, 3) - (2a+1)\zeta(2a+2, 2) \\ + \frac{1}{3}(6a^2 + 23a + 3)\zeta(2a+3, 1) - \frac{1}{3}(4a^3 + 16a^2 + 13a + 6)\zeta(2a+4)$$

- Setting  $a = -1/2$  is straightforward, **except one term**

$$\lim_{\varepsilon \rightarrow 0} -(2a+1)\zeta(2a+2, 2)|_{a \rightarrow -\frac{1}{2} + \frac{\varepsilon}{2}} = \lim_{\varepsilon \rightarrow 0} -\varepsilon \zeta(1+\varepsilon, 2) = -\zeta(2)$$

we included this term, but not by [Dempsey, Karlsson, Pufu, Zahraee, Zhiboedov](#)

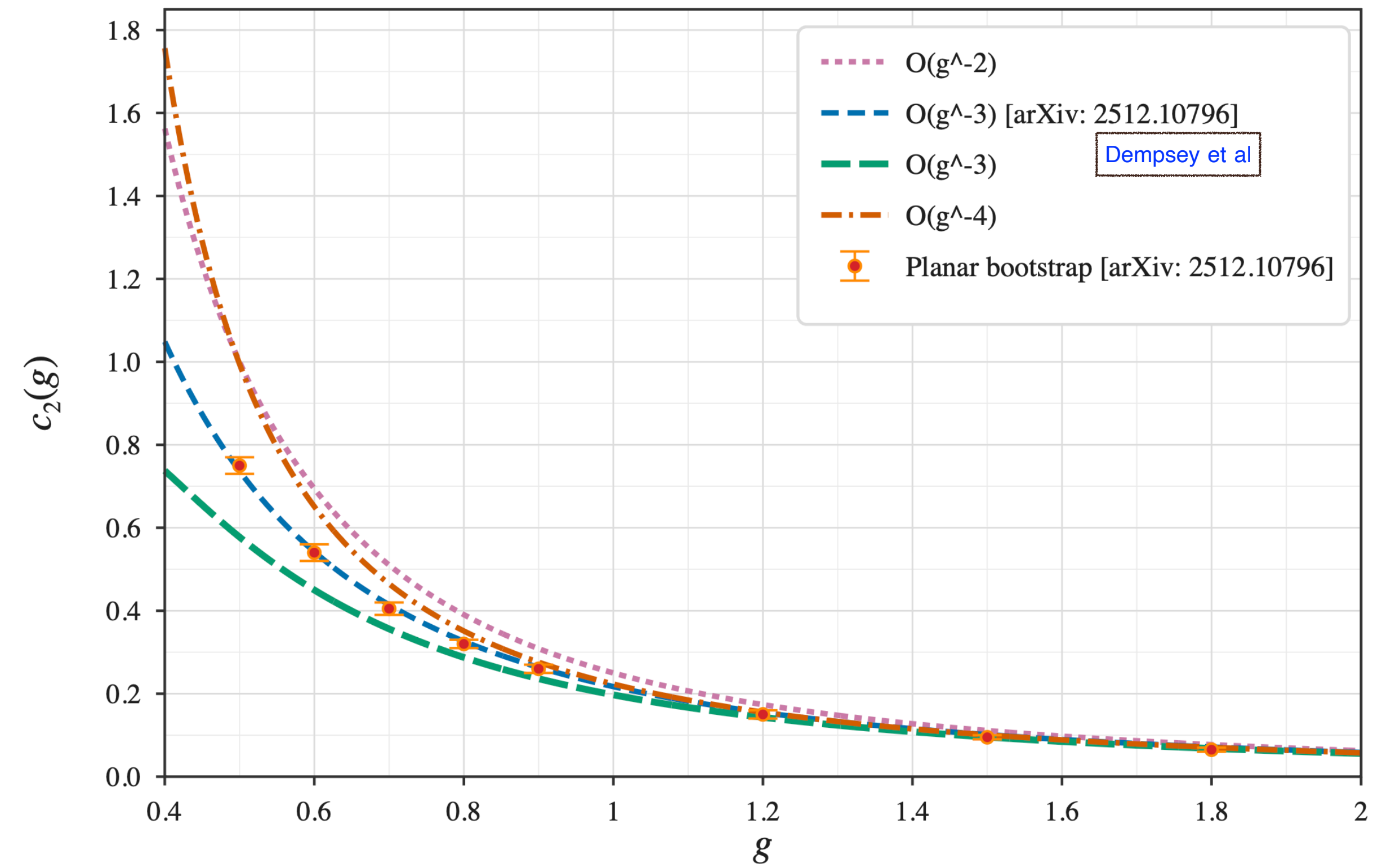
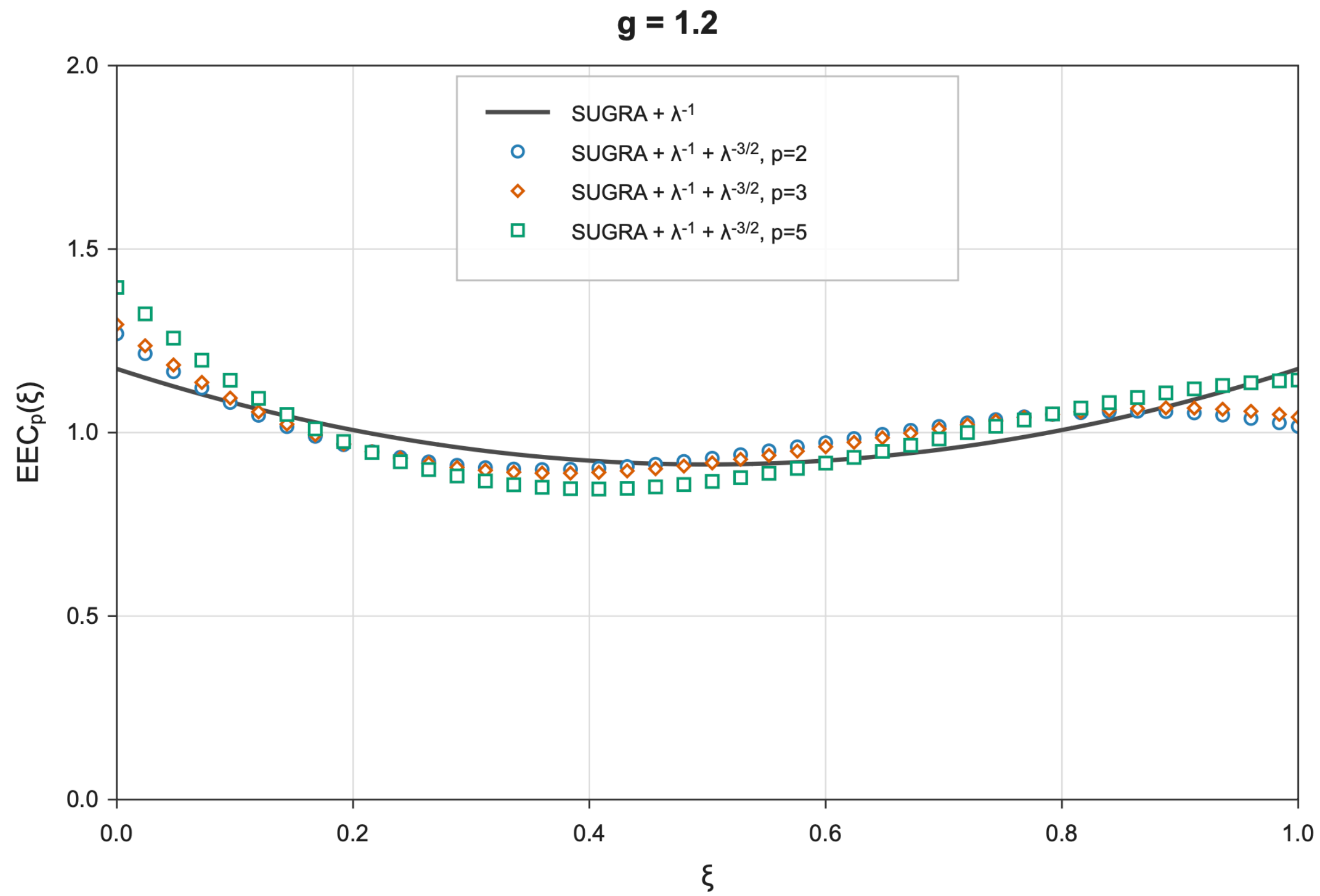
# EECs from Wilson coefficients

- Wilson coefficient for 2nd curvature is much more involved (not known in the literature)

$$\alpha_{a,0}^{(2)} = \dots + \zeta(-1 + 2a, 6) + \dots + \left(3 - \frac{a}{6} - a^2\right) \zeta(1 + 2a, 4) + \dots + \left(7 - \frac{5a}{6} + 2a^2 + \frac{2a^3}{3}\right) \zeta(2 + 2a, 3) \\ + \dots + (4 + 4a)\zeta(2a, 2, 3) + \dots + 4\zeta(-1 + 2a, 3, 3) + \dots$$

- Setting  $a = -1/2$  is **divergent term by term**, so we have to introduce a regulator
- All the  $1/\varepsilon$  **divergences cancel out**, and the finite part agrees with world-sheet calculation

# Plots for EECs



$$g^2 = \lambda / (16\pi^2)$$

# Summary and outlook

# Summary

- Light transform and strong coupling expansion of Mellin amplitude do not commute
- EECs in  $\mathcal{N}=4$  SYM are deeply related to AdS Virasoro-Shapiro amplitudes.
- Using this connection, we obtained the EECs in the strong coupling expansion for  $\mathcal{O}_p$  to 3rd order, for  $\mathcal{O}_2$  to 4th order.
- Combining with weak coupling expansion, the results can be used for Padé approximation of EECs [Dempsey, Karlsson, Pufu, Zahraee, Zhiboedov](#)

# Outlook

- Can we compute EECs at strong coupling directly without using AdS VS amplitudes? this may also help to resolve the discrepancy
- EECs with heavy states, exploiting results from heavy-state (or large charge) expansion.
- Modular completion due to S-duality: one possible starting point is the leading stringy correction
$$\frac{24\zeta(2)}{\lambda}(1 - 6\xi + 6\xi^2)$$
- Less supersymmetric theories

Thank you!