Fundamental Reactor Physics: Kinetics and Feedback Mechanisms

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Basics of Fission

Reactor Kinetics

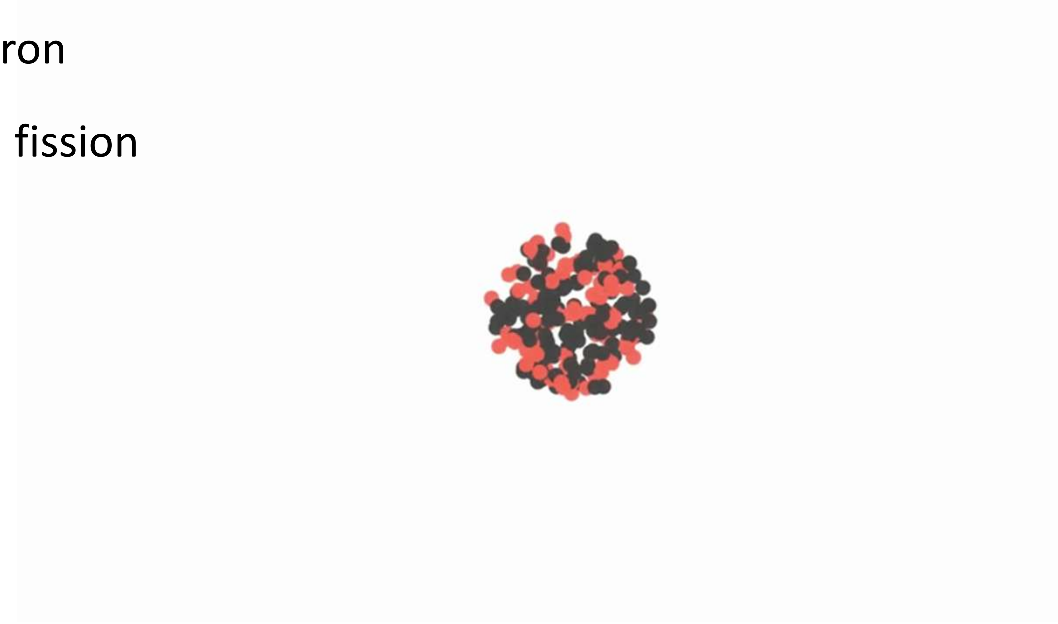
Feedback Mechanisms

Reactor Control

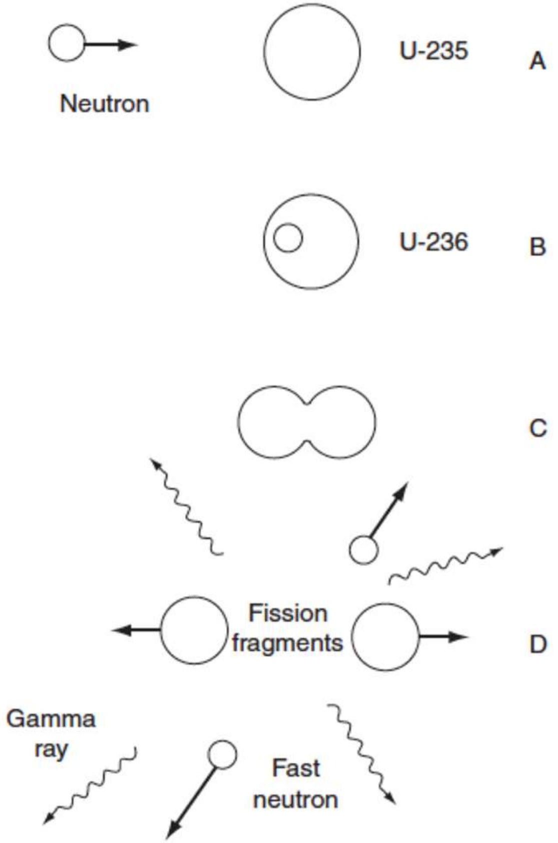
Basics of Fission - Fission Process

Fission is a nuclear reaction that occurs when a neutron interacts with a nucleus causing it to split into fission products and releasing a few neutrons.

- Fission (generally) requires a neutron
- Several neutrons are released in a fission reaction
- Energy is released in the process



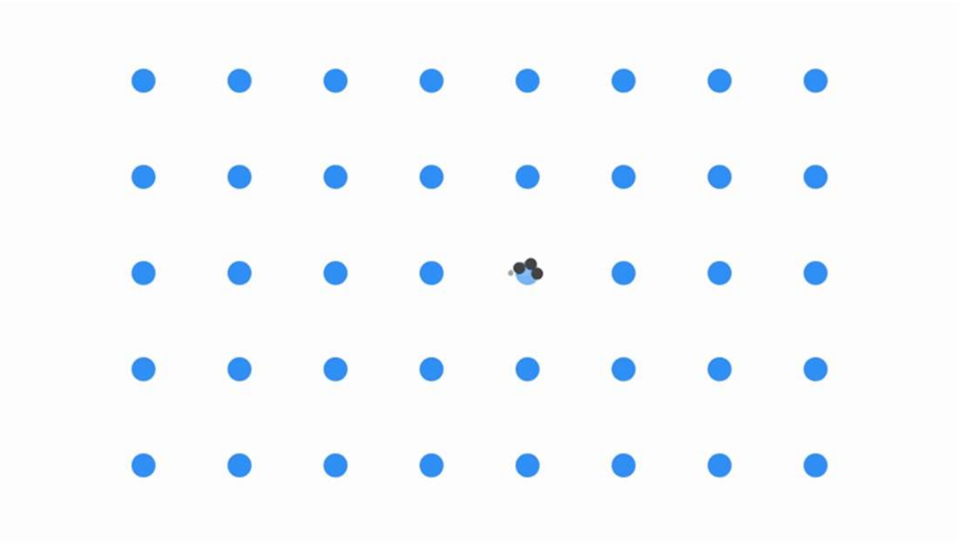
Basics of Fission - Fission Process



- Neutron is absorbed by U-235
- U-236 is formed in excited state
- Nucleus forms into a dumbbell shape
- Electrostatic forces separate two parts of the dumbbell

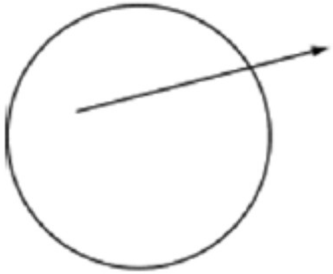
Basics of Fission - The Chain Reaction

- When neutrons from one fission reaction cause at least one subsequent fission reaction, it is called a **chain reaction**

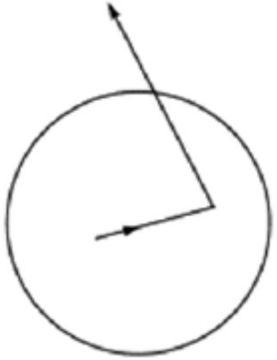


- When one fission causes exactly one subsequent fission, then the chain reaction is stable and referred to as **critical**
- When one fission causes more than one subsequent fission, the chain reaction is growing and is referred to as supercritical
- When one fission causes less than one subsequent fission

Basics of Fission - Absorption, Leakage, and Fission

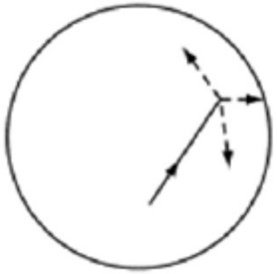


Escape in the first flight
A



Escape after one collision
B

Absorption in U-235 to form U-236
C



Absorption with neutron production
D

- If a small amount of fissile material is present, neutrons are likely to escape the fissile material
- For large enough fissile material, neutrons are likely to cause further fission reactions
- A minimum critical mass is required to sustain a neutron chain reaction

Basics of Fission - Neutron Multiplication Factor

- Nuclear fuel assembly is characterized by a multiplication factor:

$$k = \frac{neutron\,production}{neutron\,loss}$$

- k accounts for losses and production by fission in an arrangement of fuel and supporting materials

k > 1, supercritical

k = 1, critical

k < 1, subcritical

Basics of Fission - Neutron Multiplication Factor

- Infinite medium multiplication factor
  - Defined for a fuel composition
  - Assumes infinite dimension for fuel assembly

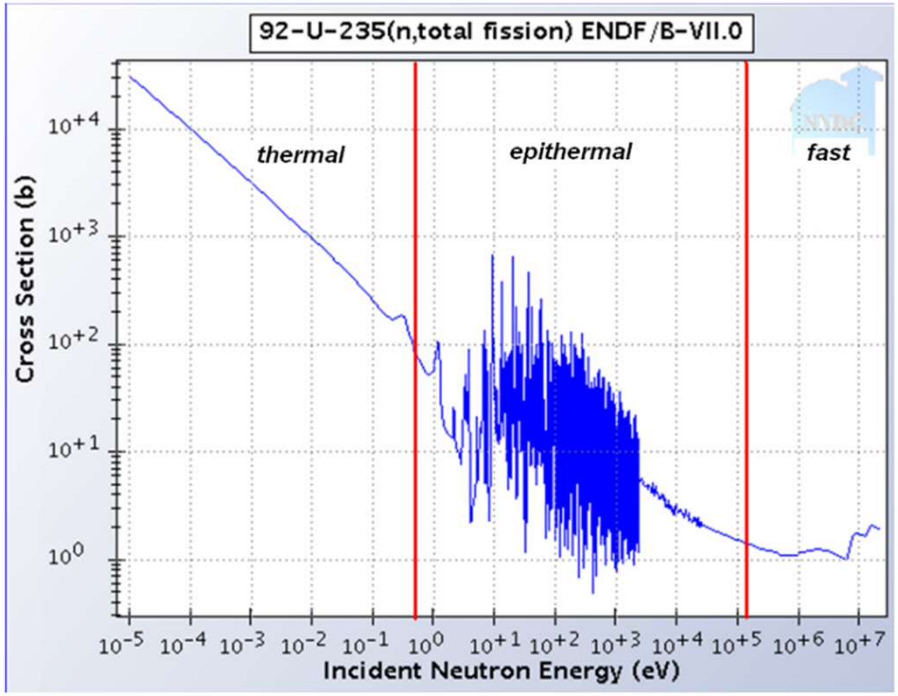
$$k_{\infty} = \frac{rate\ of\ neutron\ production}{rate\ of\ neutron\ absorption}$$

- Effective multiplication factor
  - Accounts for finite dimension of the fuel assembly

$$k_{eff} = \frac{rate\ of\ neutron\ production}{rate\ of\ neutron\ absorption + rate\ of\ neutron\ leakage}$$

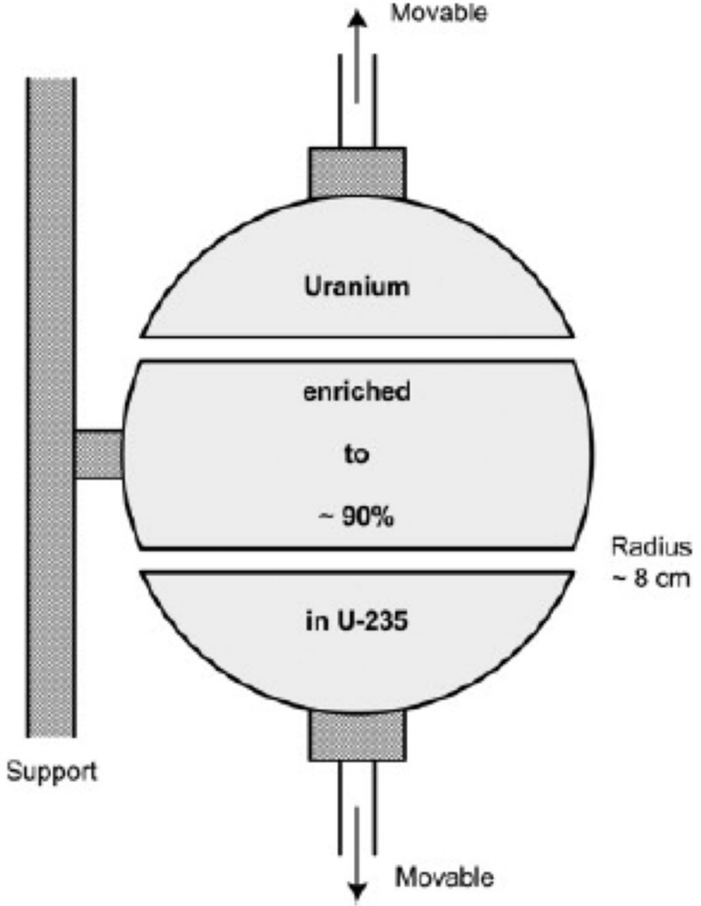
Basics of Fission - Fast vs. Thermal Neutrons

- Fast neutrons are emitted directly from fission with high kinetic energy.
- Moderation slows neutrons through repeated collisions with light nuclei (typically hydrogen in water).
- Thermal neutrons are in equilibrium with the surrounding material and have much lower energy.
- U-235 is much more likely to undergo fission when struck by a thermal neutron than by a fast neutron.



Basics of Fission - Lady Godiva

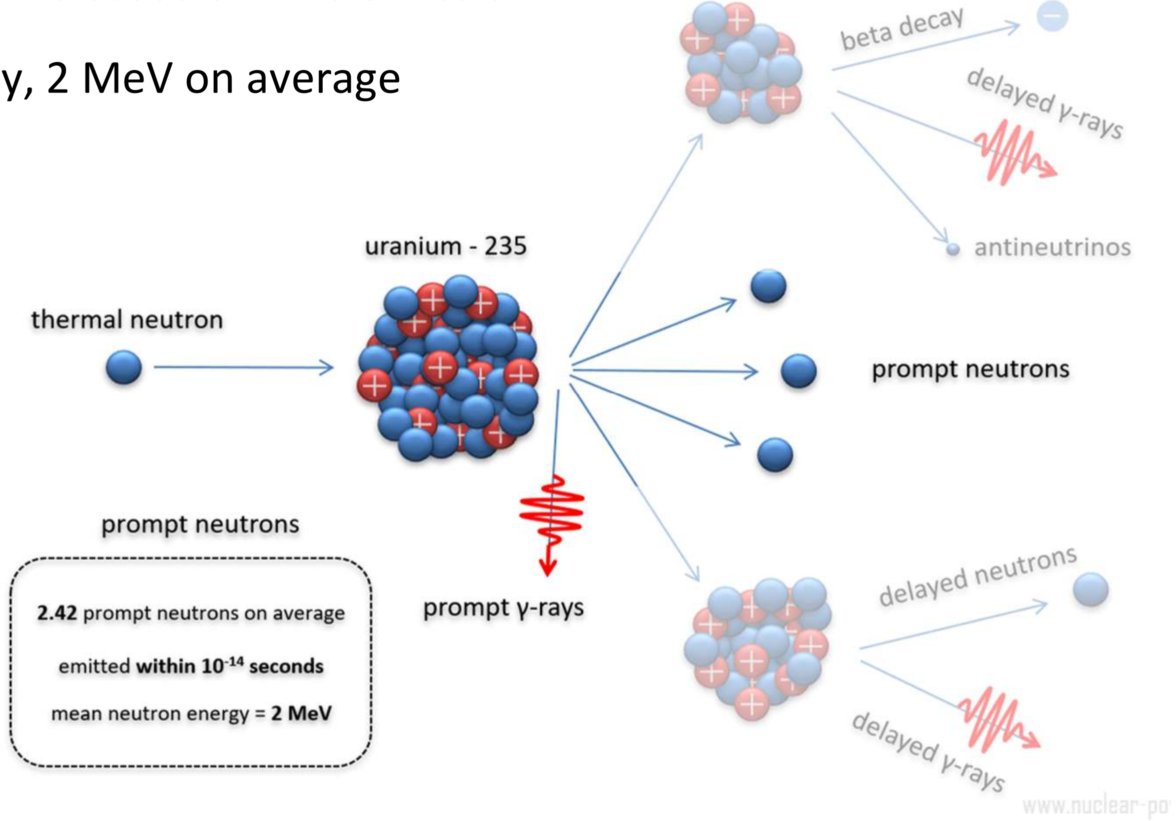
- Criticality is controlled by introduction of neutron absorbing materials that are easily moved in and out of fuel material
- In early experiments however, fuel assemblies were brought close to each other
- Lady Godiva fuel assembly
    - When pieces are separated, k < 1
    - When pieces are brought together, k ~ 1
    - Surrounding assembly in neutron reflector, k > 1



Basics of Fission - Lady Godiva

Reactor Kinetics - Prompt Neutrons

- 99% of neutrons are prompt: released at time of fission
- They release at a high energy, 2 MeV on average
- If reactors relied only on prompt neutrons, power would change in milliseconds, making the reactor uncontrollable



2.42 prompt neutrons on average

emitted **within 10^{-14} seconds**

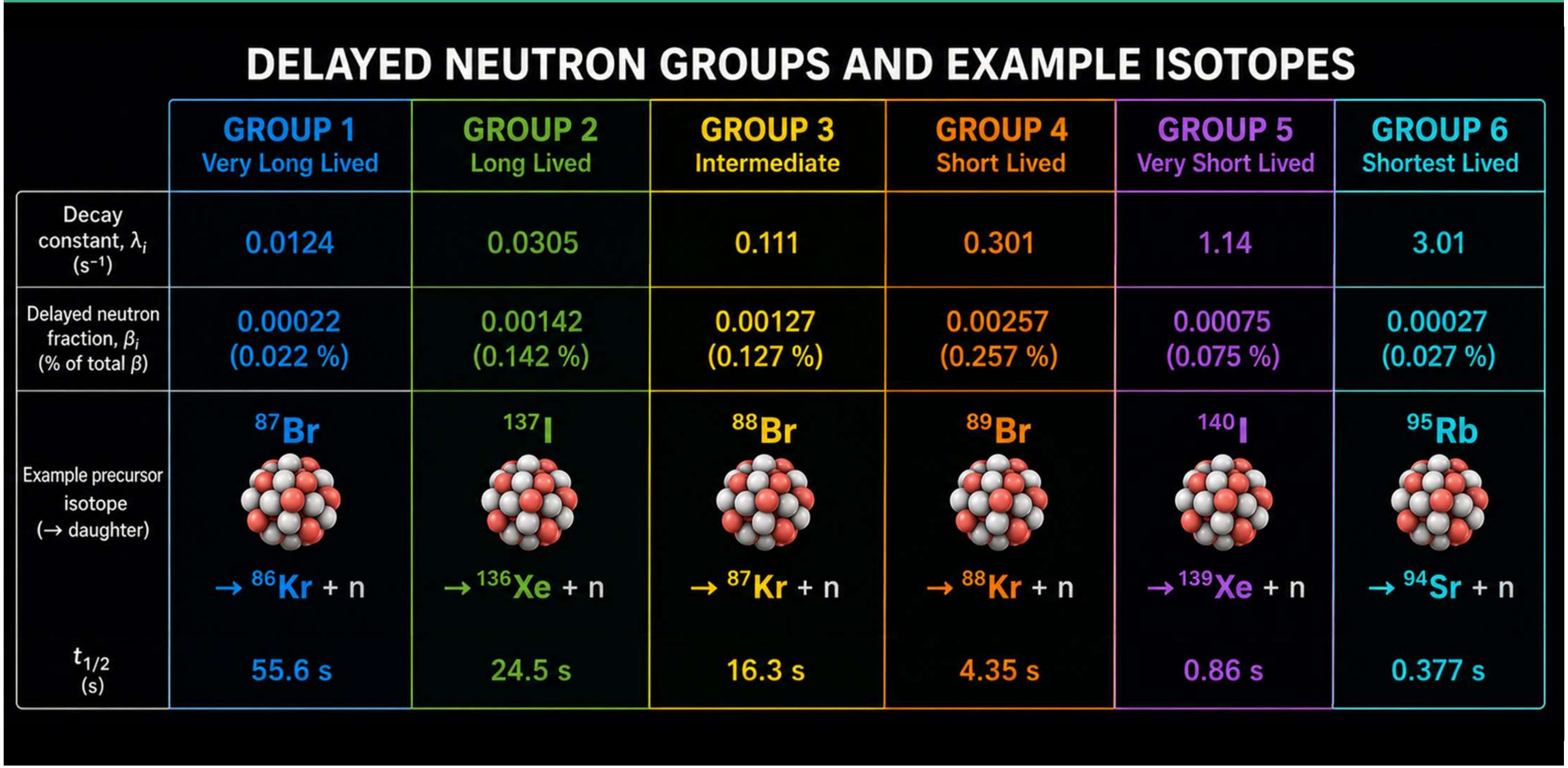
mean neutron energy = **2 MeV**

Reactor Kinetics - Delayed Neutrons

- Some fission products release neutrons with a time delay, based on their half life (precursors)
- 1% of neutrons are emitted as delayed neutrons
- Precursors are grouped into 6 groups with half lives ranging from 0.2 sec to 1 minute
- The average number depends strongly on:
  - Fissioning nucleus
  - Energy of the inducing neutron

Delayed neutrons are essential to the control of a nuclear reaction.

Reactor Kinetics - Delayed Neutrons



DELAYED NEUTRON GROUPS AND EXAMPLE ISOTOPES

	GROUP 1 Very Long Lived	GROUP 2 Long Lived	GROUP 3 Intermediate	GROUP 4 Short Lived	GROUP 5 Very Short Lived	GROUP 6 Shortest Lived
Decay constant, λ_i (s^{-1})	0.0124	0.0305	0.111	0.301	1.14	3.01
Delayed neutron fraction, β_i (% of total β)	0.00022 (0.022 %)	0.00142 (0.142 %)	0.00127 (0.127 %)	0.00257 (0.257 %)	0.00075 (0.075 %)	0.00027 (0.027 %)
Example precursor isotope (→ daughter)	^{87}Br → ^{86}Kr + n	^{137}I → ^{136}Xe + n	^{88}Br → ^{87}Kr + n	^{89}Br → ^{88}Kr + n	^{140}I → ^{139}Xe + n	^{95}Rb → ^{94}Sr + n
$t_{1/2}$ (s)	55.6 s	24.5 s	16.3 s	4.35 s	0.86 s	0.377 s

Reactor Kinetics - Reactivity

Reactivity is the departure of a reactor system from criticality

$$\rho = \frac{\delta k}{k} = \frac{k-1}{k}$$

$\rho > 0$, power rising

$\rho = 0$, power stable

$\rho < 0$, power falling

<u>Operator Units</u>
pcm (per cent mille)

1 pcm = $0.00001=10^{-5}$

“Increase the reactivity
by 100 pcm.”

Factors that change reactivity:

- Control rod movement
  poisoning
- Fuel temperature changes
- Coolant void formation
  buildup

- Xenon
- Fuel depletion
- Fission product

Reactor Kinetics - Inhour Equation

The inhour equation relates reactor reactivity to the rate at which reactor power changes

$$\rho = \underbrace{s\Lambda}_{\text{prompt contribution}} + \underbrace{\sum_{i=1}^{G} \frac{\beta_i s}{s+\lambda_i}}_{\text{delayed contribution}}$$

s: Power growth / decay rate

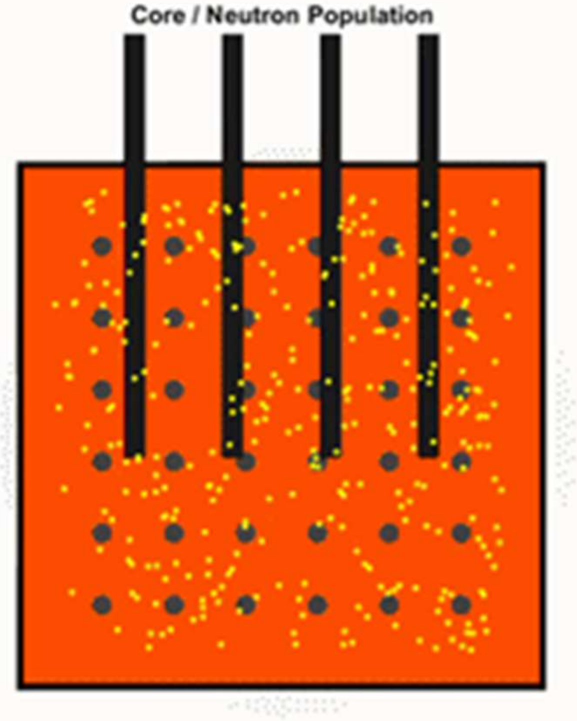
Λ: Prompt neutron generation time

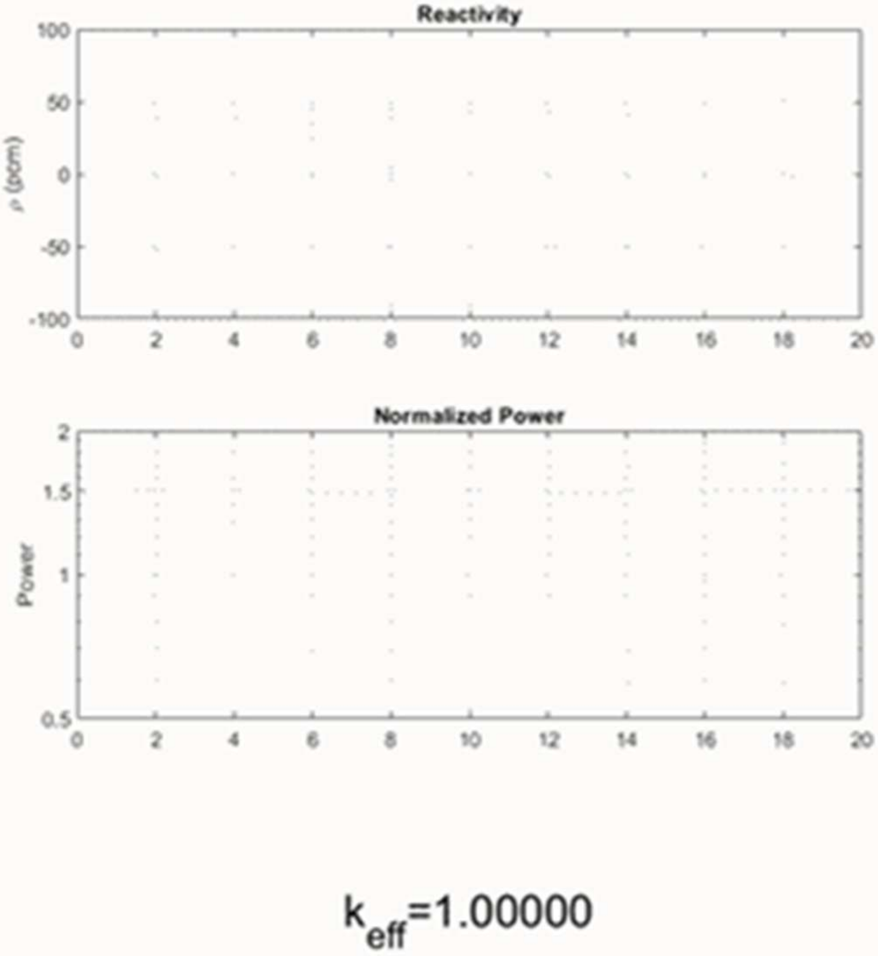
β_i: Delayed neutron fraction (by group)

λ_i: Delayed neutron precursor decay constant (by group)

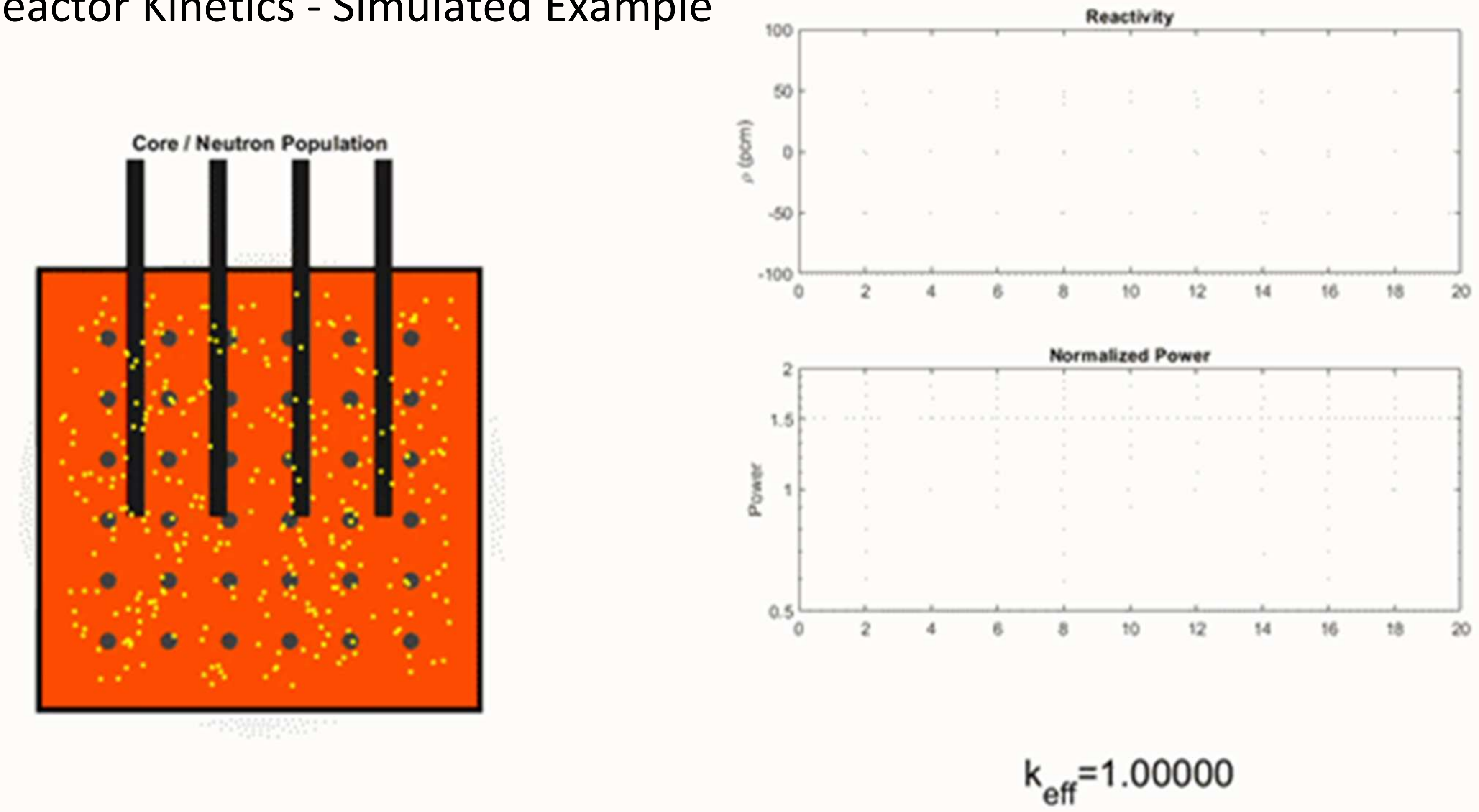
For typical thermal reactors, Λ is extremely small, so the prompt term is small compared to the delayed term. This is why **delayed neutrons dominate kinetics.**

Reactor Kinetics - Simulated Example



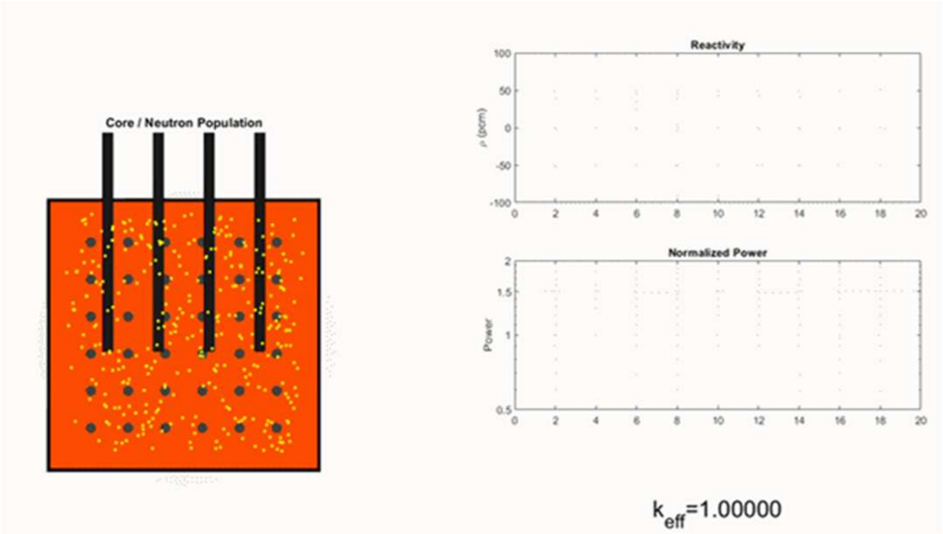


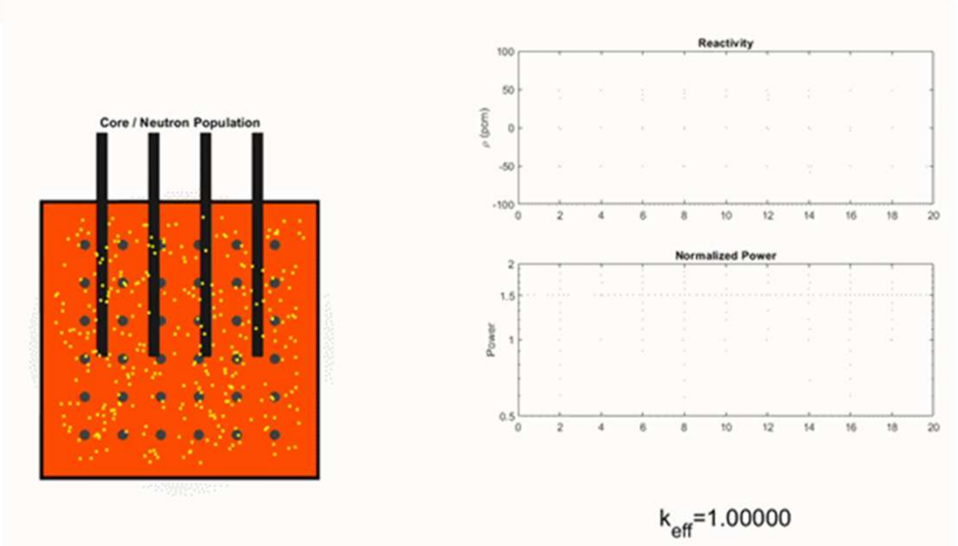
Reactor Kinetics - Simulated Example



Reactor Kinetics - Simulated Example

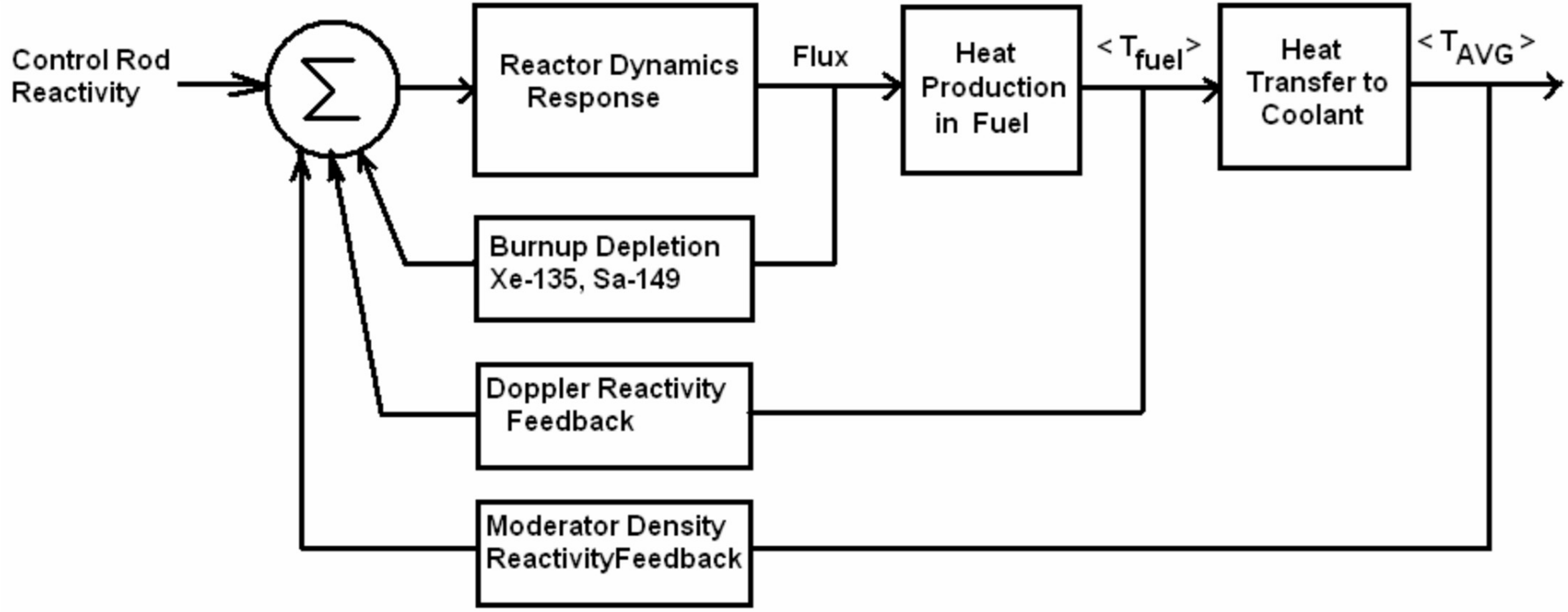
1) Initial steady state
    - Critical reactor
1) Prompt jump or drop
    - Prompt neutrons adjust almost instantaneously
1) Delayed neutron buildup or decay
    - Precursors continue emitting delayed neutrons
    - New precursor production is higher/lower because fission rate has changed



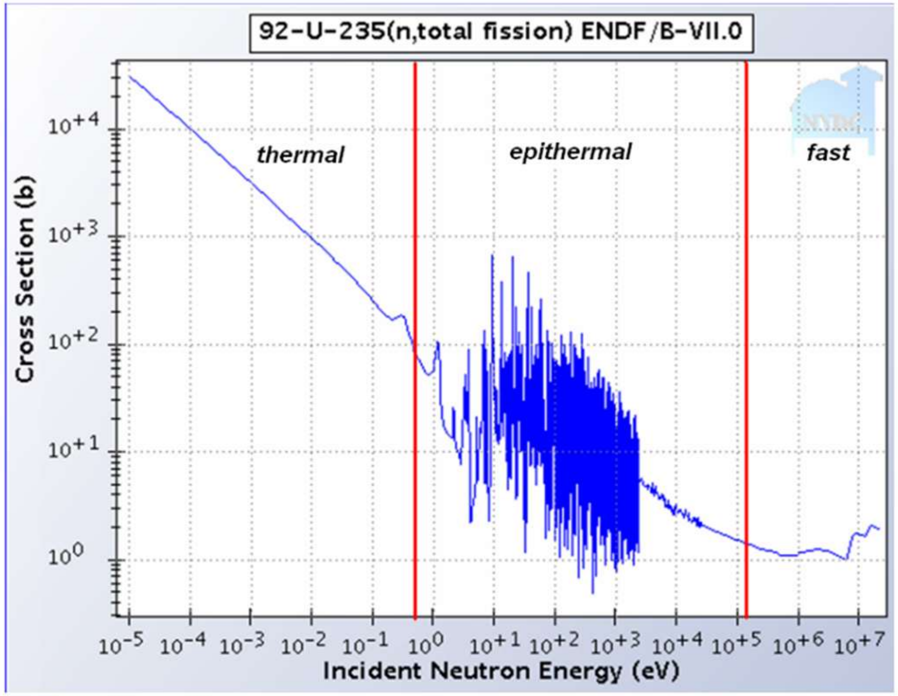


Feedback Mechanisms - Overall Reactivity Feedback Paths

Reactivity changes the reactor conditions, and in turn changes the reactivity



Feedback Mechanisms - Doppler Feedback



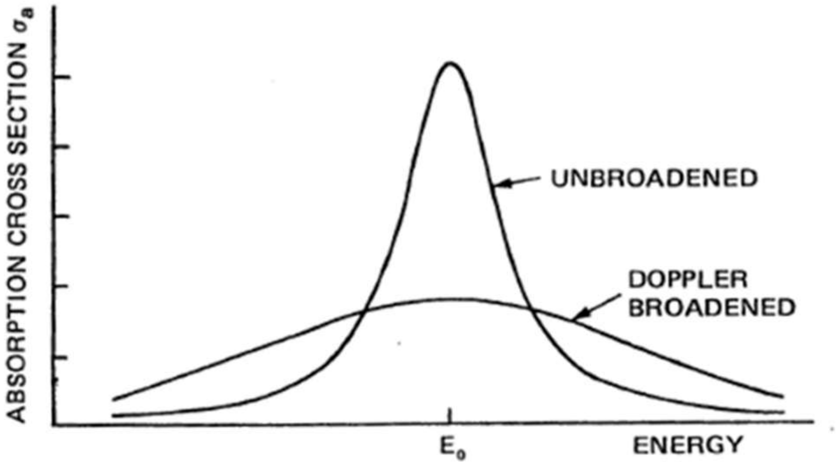


FIGURE 5-2
Effect of temperature on the effective shape of a resonance absorption cross section.

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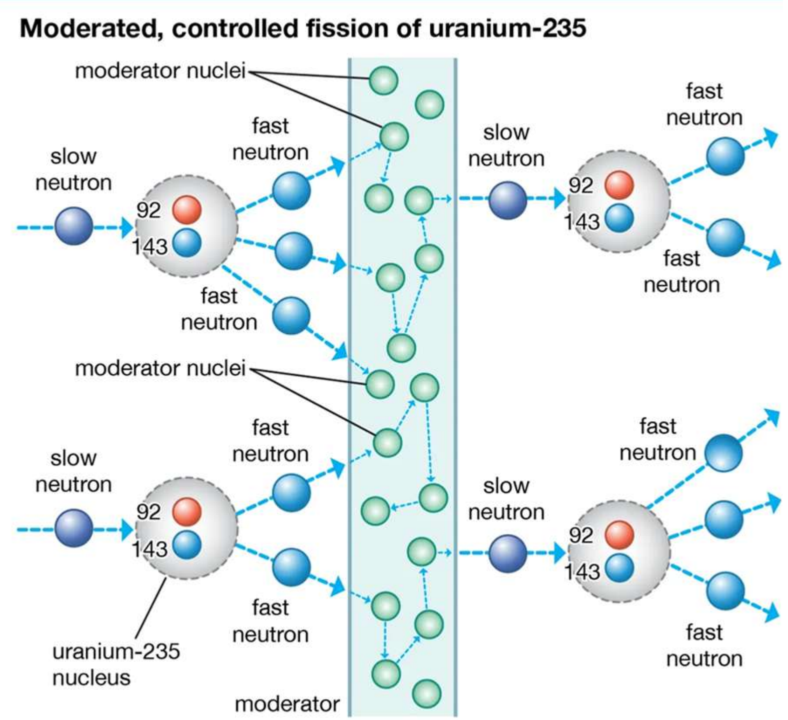
Power ↑ → Fuel temp ↑ → Resonance absorption ↑ → Reactivity ↓

Feedback Mechanisms - Moderator Density Feedback

- Moderator slows neutrons
- Temperature affects moderator density
- Density changes alter neutron moderation
- Often negative in light-water reactors

Lower water density reduces the probability of neutron collisions, making moderation less effective.



Moderated, controlled fission of uranium-235

Power ↑ → Moderator temp ↑ → Moderator density ↓ → Resonance absorption ↑ → Reactivity (usually) ↓

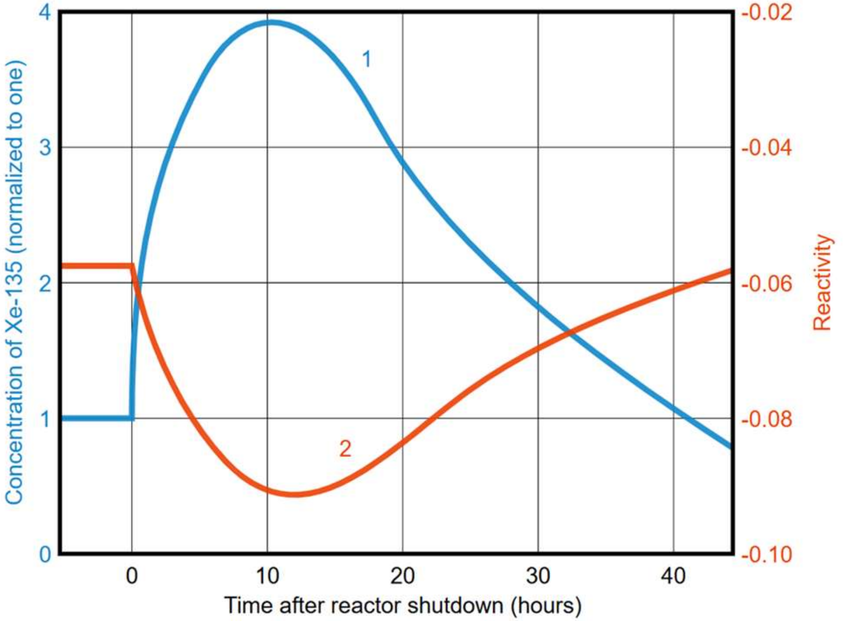
Feedback Mechanisms - Xenon Feedback

Xe-135 is a fission product and it is the most powerful known neutron-absorbing nuclear poison

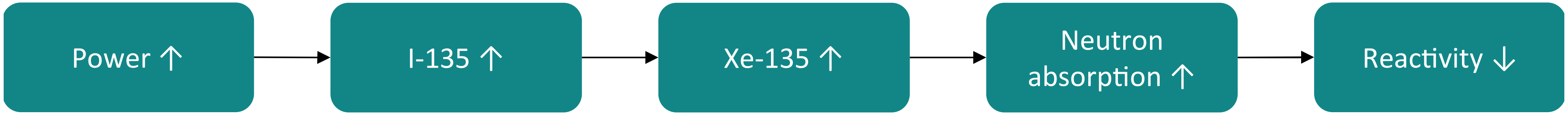
The yield of Xe-135 from fission is about 6.6% (uranium) or 7.4% (plutonium)

The xenon will continue to lower the reactivity until it naturally decays

($T_{1/2}$ = 9.14 hours)



By Wikkiwonkk (translation and touch-ups) - Own work based on: Reactor shutdown xe chart.svg by Д.Ильин, CC0, https://commons.wikimedia.org/w/index.php?curid=159443257



Power ↑ → I-135 ↑ → Xe-135 ↑ → Neutron absorption ↑ → Reactivity ↓

Reactor Control - Managing Reactivity

To properly control a reactor, we need to be able to:

- Maintain criticality during steady operation
- Increase power safely
- Reduce power safely
- Shut the reactor down when required

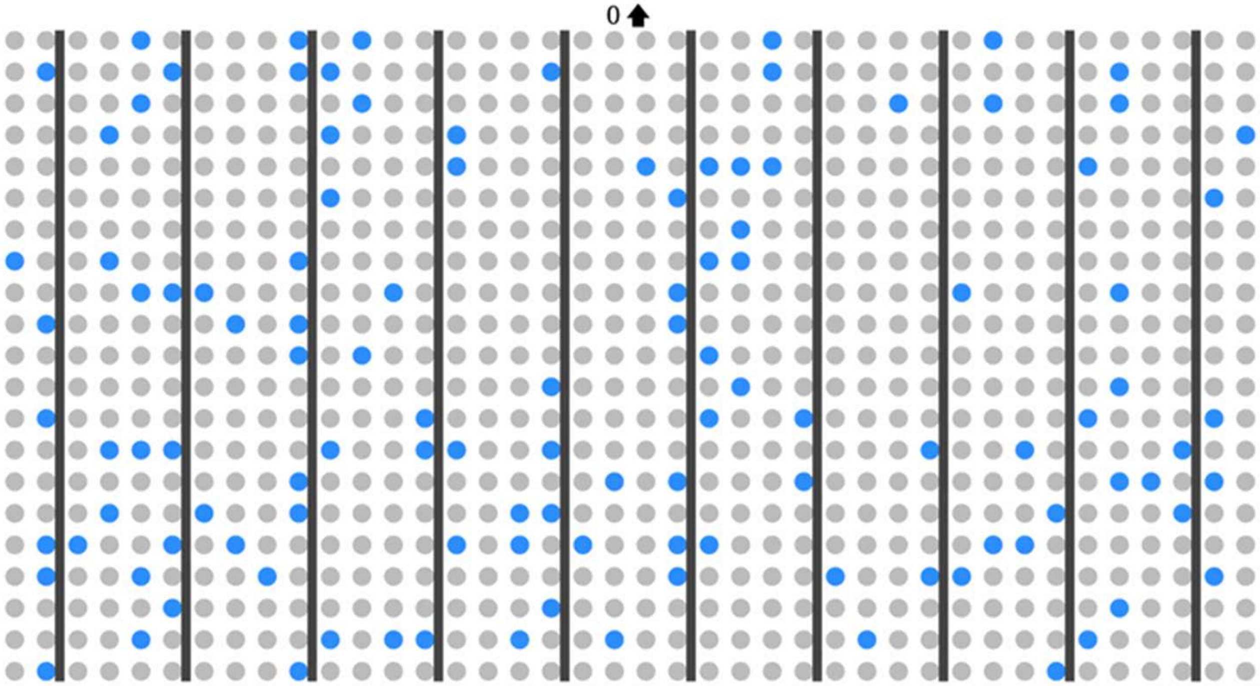
Reactor Control - Methods

Method	Purpose	How It Works	Time Scale
Control rods	Rapid reactivity adjustment	Mechanically inserted and withdrawn from core	Seconds - minutes
Soluble boron (PWR)	Controllable long-term reactivity control	Dissolved into water coolant, can be diluted	Hours - months
Burnable poisons	Fixed long-term reactivity control	Placed inside fuel assembly	Months - years
Natural feedback	Automatic stabilization	Negative feedback mechanisms force ρ toward steady state	Seconds - hours

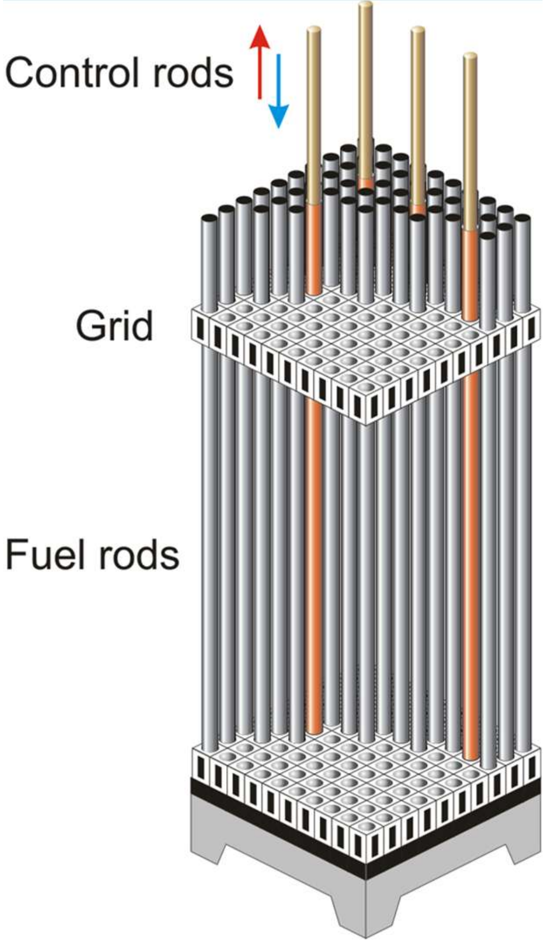
Reactor Control - Control Rods

When a neutron is born in a reactor, one of three things can happen:

- It escapes the reactor
- It is absorbed without causing a fission
- It causes a fission



Reactor Control - Control Rods



Control rods are made of neutron-absorbing materials like boron or cadmium, which allows for rapid reactivity adjustments. **They are the primary method for raising and lowering power level.**

Insertion → reduced reactivity

Withdrawal → increased reactivity

Black rods: stronger absorbers, made from boron carbide, hafnium, or gadolinium

Grey rods: weaker absorbers, made from stainless steel

Reactor Control - Soluble Boron

Nuclear reactors start with extra uranium, called "excess reactivity".

- Boron is dissolved in the water to absorb these extra neutrons
- The boron is slowly removed over time to keep the reaction steady as fuel burns up

Pros	Cons
The solubility with water allows for even neutron absorption across core	Boric acid can corrode metal pipes and reactor vessels
Large reactors can run longer without refueling	Pumps and tanks needed to filter boron complicates plumbing
Borated water can be injected in an emergency to stop the reaction	Only used in water-moderated reactors

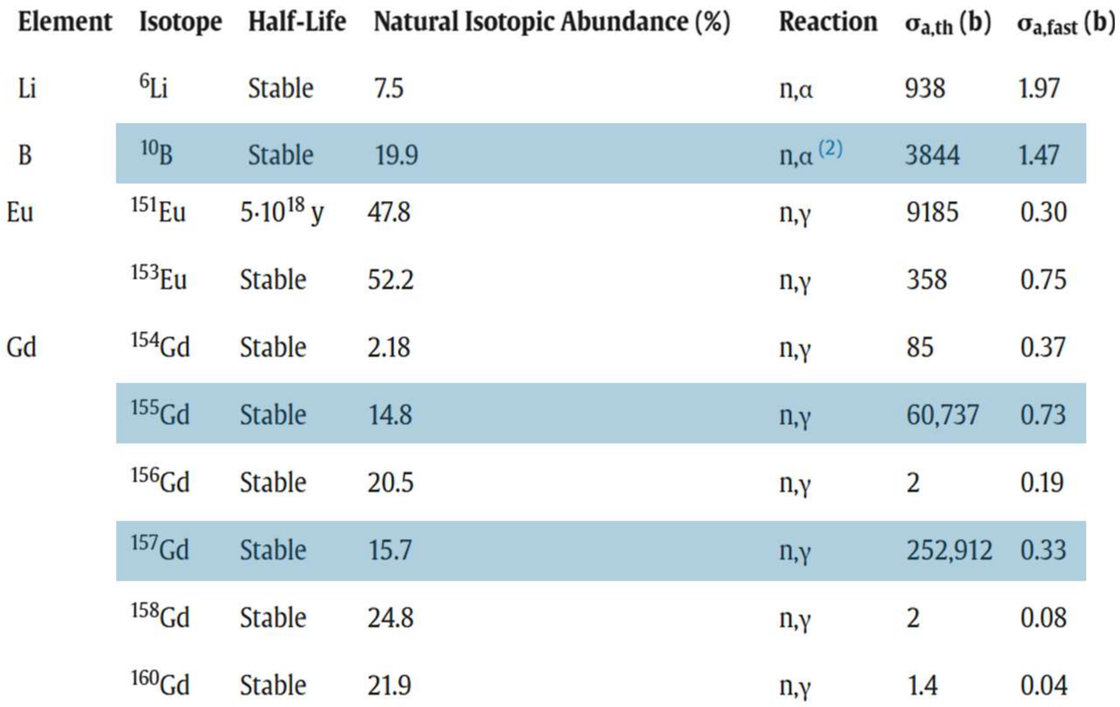
Reactor Control - Burnable Poisons

Unlike methods like soluble boron, burnable poisons are **built into the fuel**

The poison is ideally burned up at the same rate of nuclear fuel consumption to keep the core's power production steady over time

Qualities of a good burnable poison:

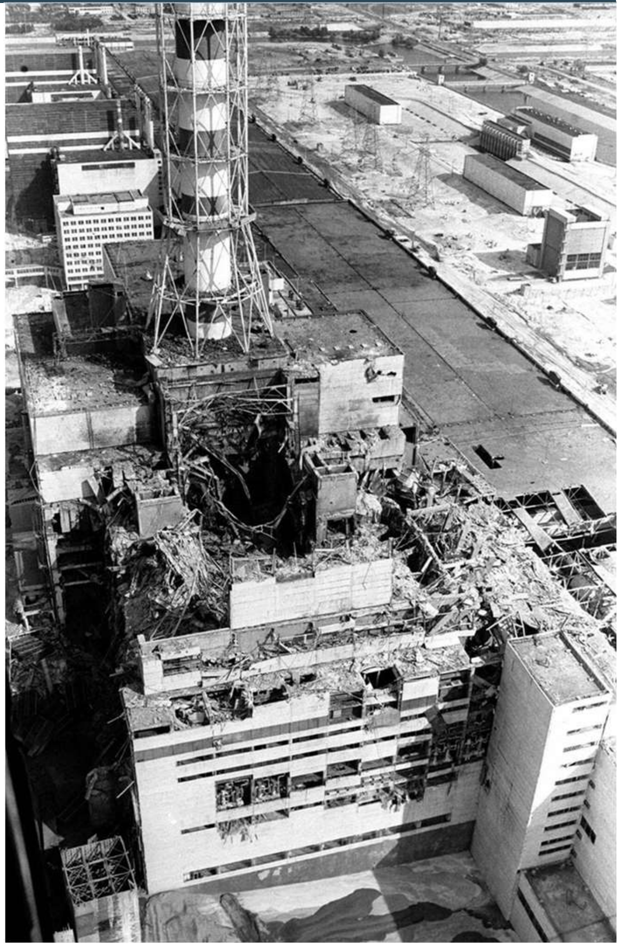
- High neutron absorption cross-section
- Converts to a low absorption material after initial absorption
- Minimal radioactive byproducts
- High melting point
- Abundant / low cost material



Element	Isotope	Half-Life	Natural Isotopic Abundance (%)	Reaction	$\sigma_{a,th}$ (b)	$\sigma_{a,fast}$ (b)
Li	^{6}Li	Stable	7.5	n,α	938	1.97
B	^{10}B	Stable	19.9	n,α (2)	3844	1.47
Eu	^{151}Eu	$5 \cdot 10^{18}$ y	47.8	n,γ	9185	0.30
	^{153}Eu	Stable	52.2	n,γ	358	0.75
Gd	^{154}Gd	Stable	2.18	n,γ	85	0.37
	^{155}Gd	Stable	14.8	n,γ	60,737	0.73
	^{156}Gd	Stable	20.5	n,γ	2	0.19
	^{157}Gd	Stable	15.7	n,γ	252,912	0.33
	^{158}Gd	Stable	24.8	n,γ	2	0.08
	^{160}Gd	Stable	21.9	n,γ	1.4	0.04

Evans *et al.*, "Burnable Absorbers in Nuclear Reactors – A Review," *Nuclear Engineering and Design*, 2022.

Case Study - Chernobyl



- On April 25th-26th, 1986 the World's worst nuclear power accident occurred at Chernobyl in the former USSR (now Ukraine).
- During a safety test on Reactor 4, the reactor was operated under unstable low-power conditions while multiple operating and safety procedures were violated. The reactor's design characteristics, combined with the operating conditions, created a highly unstable reactivity state.
- A rapid power excursion caused a series of steam explosions that severely damaged the reactor and breached the containment structures, releasing large quantities of radioactive material into the surrounding environment.

Case Study - Chernobyl

Event 1 - Test Delay

25 April 1986

01:06 - Reactor Unit 4 began a gradual reduction in power in preparation for a planned test

14:00 - Power was reduced to 50%. Another regional power station unexpectedly went offline, so the reactor staff were advised to postpone further power reduction in order to meet grid demands

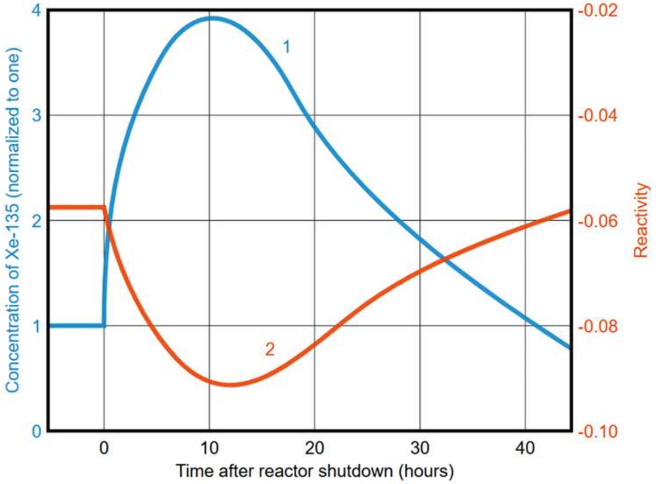
23:04 - Power reduction was allowed to resume

Reactivity Effects

fuel temperature ↓

coolant temperature ↓

xenon-135 concentration ↑



Case Study - Chernobyl

Event 2 - Unexpected Power Drop

26 April 1986

00:05 - Power was reduced to 25%, which was in the goal range for the test to be conducted. However, power continued to decrease due to xenon being produced faster than the neutron flux could “burn it off”

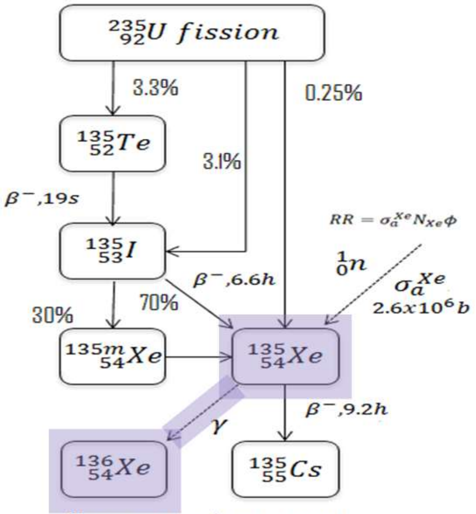
00:28 - A faulty low-power regulator cause the reactor power to quickly drop to < 1%

00:39 - Operators raised many control rods to their upper limits, at which power was restored to about 6%

Reactivity Effects

xenon-135 concentration ↑

control rod height ↑



Source: www.nuclear-power.net
Data: JANIS 4.0 / NEA

Case Study - Chernobyl

Event 3 - Test Preparation Continues

26 April 1986

00:39 - With power at 6% (still lower than prescribed testing range), preparation continued

01:05 - 2 additional circulating pumps are activated. Operators respond to the reactivity decrease by removing more control rods (nearly all 211 rods are withdrawn at this point)

- excessive coolant flow prevented the water from adequately cooling between cycles, causing it to re-enter the core at close to boiling point

Reactivity Effects

coolant temperature ↓

control rod height ↑

coolant temperature ↑

Unlike light-water reactor designs, the RBMK design had a **positive void coefficient**, so voids formed during boiling **increased reactivity**

Case Study - Chernobyl

Event 4 - Accident Occurs

26 April 1986

01:23:04 - Reactor appears to be stable. Test begins. Feed flow is sharply reduced per testing procedure.

01:23:40 - Experiment is wrapped up and an operator presses the button to scram the reactor, where rods begin inserting slowly (taking 7 minutes to reach full insertion).

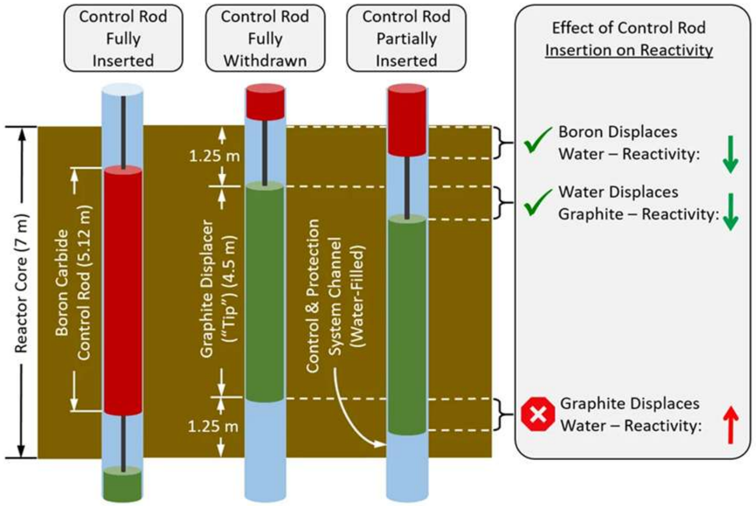
01:23:43 - Power spike overheats the core, causing some fuel rods to fracture. Control rods are jammed at 1/3 insertion. Fuel cladding failed and fuel elements leaked into coolant

Reactivity Effects

coolant temperature ↑

Rod height ↓

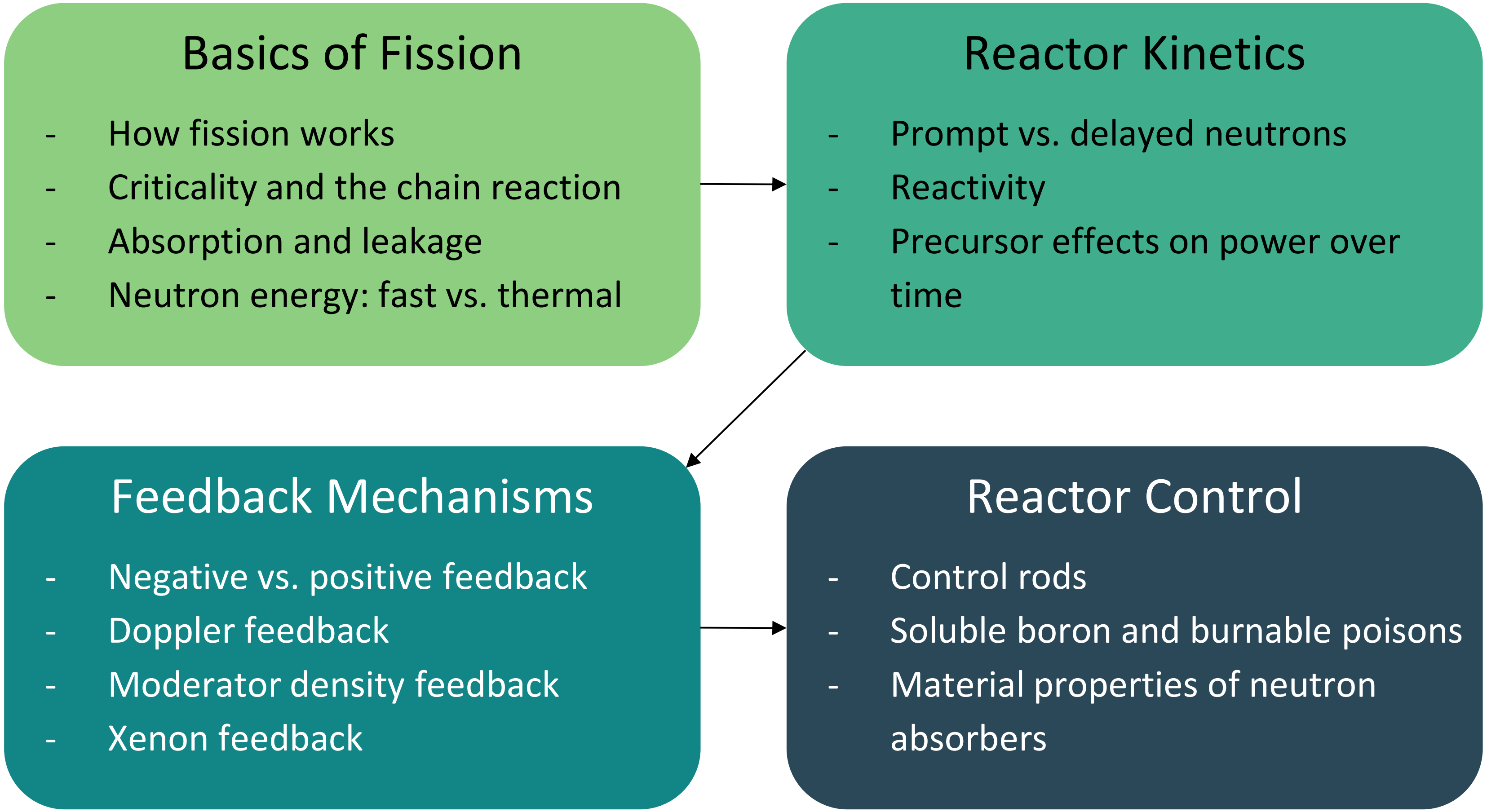
CHERNOBYL CONTROL ROD ILLUSTRATION



Case Study - Chernobyl

Event 5 - Consequences

- Reactor output power jumped to at least 1000%
- Steam explosion occurred
- More fuel channels are ruptured
- Coolant lines are ruptured, causing remaining coolant to flash evaporate
- Water loss combined with high positive void coefficient further increase reactor power
- Another, more powerful explosion occurs second later, which disperses the damaged core and terminates the chain reaction



Basics of Fission

- How fission works
- Criticality and the chain reaction
- Absorption and leakage
- Neutron energy: fast vs. thermal

Reactor Kinetics

- Prompt vs. delayed neutrons
- Reactivity
- Precursor effects on power over time

Feedback Mechanisms

- Negative vs. positive feedback
- Doppler feedback
- Moderator density feedback
- Xenon feedback

Reactor Control

- Control rods
- Soluble boron and burnable poisons
- Material properties of neutron absorbers

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