

# Dark Energy Dynamics, Spatial Curvature, Neither, or Both?

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Please view & share:    [HowGlobalWarmingWorks.org](https://HowGlobalWarmingWorks.org)    (please forgive me for this P&A)

# Main contributors to the present cosmological energy budget :

about 5% baryonic matter (mostly atoms in gas clouds, stars, planets, dust , ... ), first clearly measured in the 1960's (Gamow, Alpher, Herman, Penzias & Wilson, Dicke et al.)

about 25% non-baryonic non-relativistic cold dark matter (probably a WIMP), first seen in the 1930's (Zwicky, Smith, Babcock,...) and first clearly measured in the 1970's (Rubin & Ford, Ostriker & Peebles, Einasto et al., Ostriker et al.)

about 70% non-baryonic relativistic dark energy (not clear what this is), first real suggestion in the 1980's (Peebles, Peebles & Ratra) and first clearly measured in the 1990's (Riess et al., Perlmutter et al.)

We do not understand 95% of the current cosmological energy budget, but we do have a “standard” model of cosmology!

# Outline

motivate dark energy

two illustrative dark energy models ( $\Lambda$ CDM,  $\phi$ CDM), two popular but incomplete parameterizations ( $w$ /XCDM,  $w_0w_a$ CDM) (parameterizations are arbitrary or physically inconsistent, usually have more free parameters than models)

compare to observations (neoclassical cosmological tests), derive model-parameter constraints, test consistency of different data

observational evidence for deceleration-acceleration transition

Hubble constant value

spatial curvature (Planck mildly favors mildly closed geometry)

excess CMB weak lensing in Planck PR3 data

$w_0w_a$ CDM parametrization and DESI 2024 dark energy dynamics

# The general motivation

Cosmological data not yet good enough to allow tight model-independent conclusions.

Analyzing observational data in the context of a physically-consistent model allows for tighter, but model-dependent, constraints.

Comparing observational constraints for various models gives an indication of the generality of the conclusions.

Comparing different observational constraints on a model might help uncover hidden systematic errors.

Models also allow us to combine constraints from different data sets.

Fact: farther apart the galaxies, the greater the redshift, and the faster the separation.

$$v = H_0 r$$

$v$  = recession speed of galaxy,  $r$  = distance to galaxy

$H_0$  = Hubble constant =  $(68 \pm 2.8) \text{ km s}^{-1} \text{ Mpc}^{-1}$

=  $100 h \text{ km s}^{-1} \text{ Mpc}^{-1}$

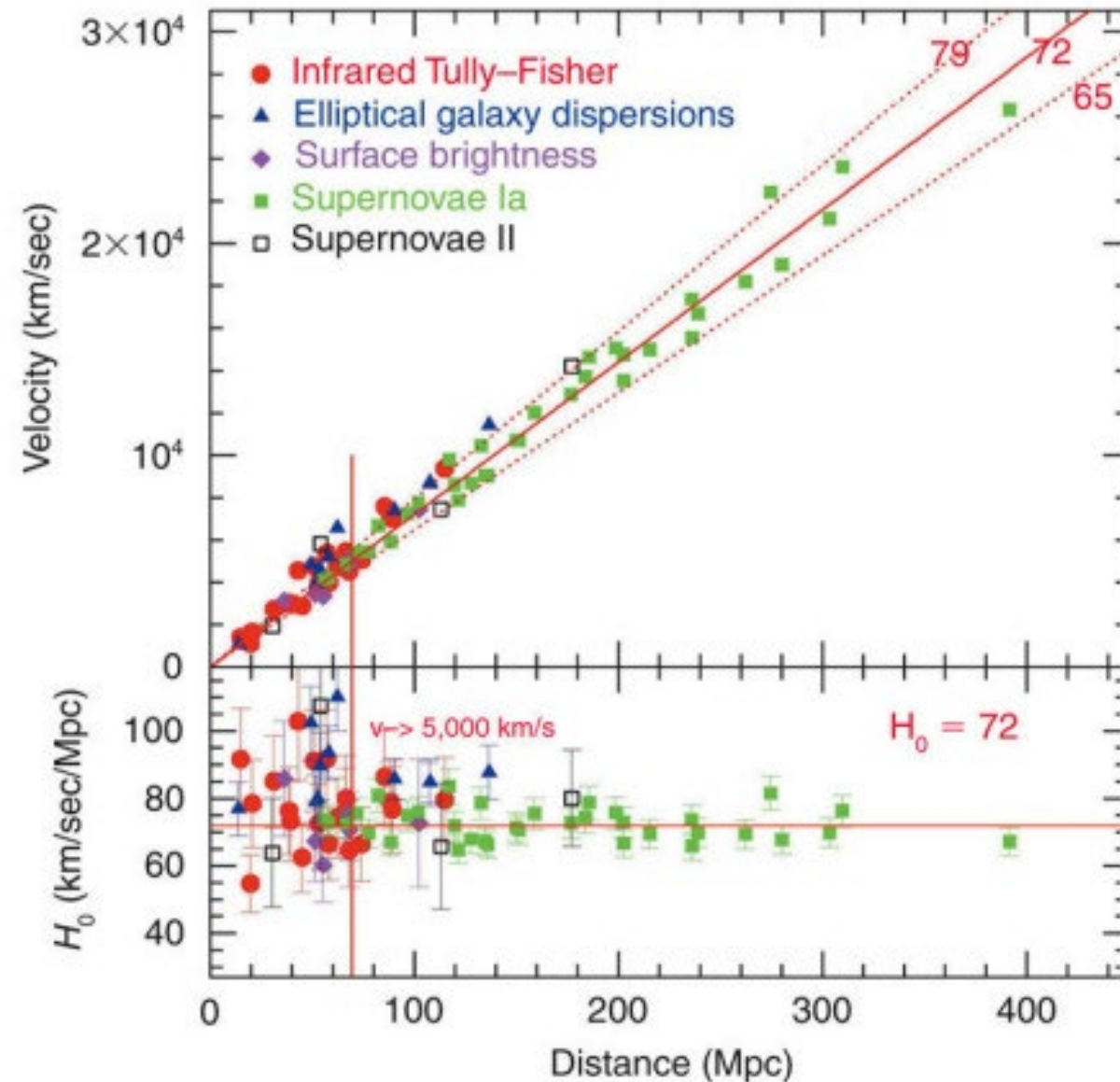
Chen & Ratra PASP123,1127 (2011)

$H_0$  is the present value of the Hubble parameter.

This is the Hubble (1929) law, discovered by Hubble and Humason.\*

\*Middle school dropout and one time muleskinner and janitor.

# Hubble law



HST Key Project final result  
Freedman+ ApJ553, 47 (2001)

Gott+ ApJ549, 1 (2001)  
median statistics for  $H_0$

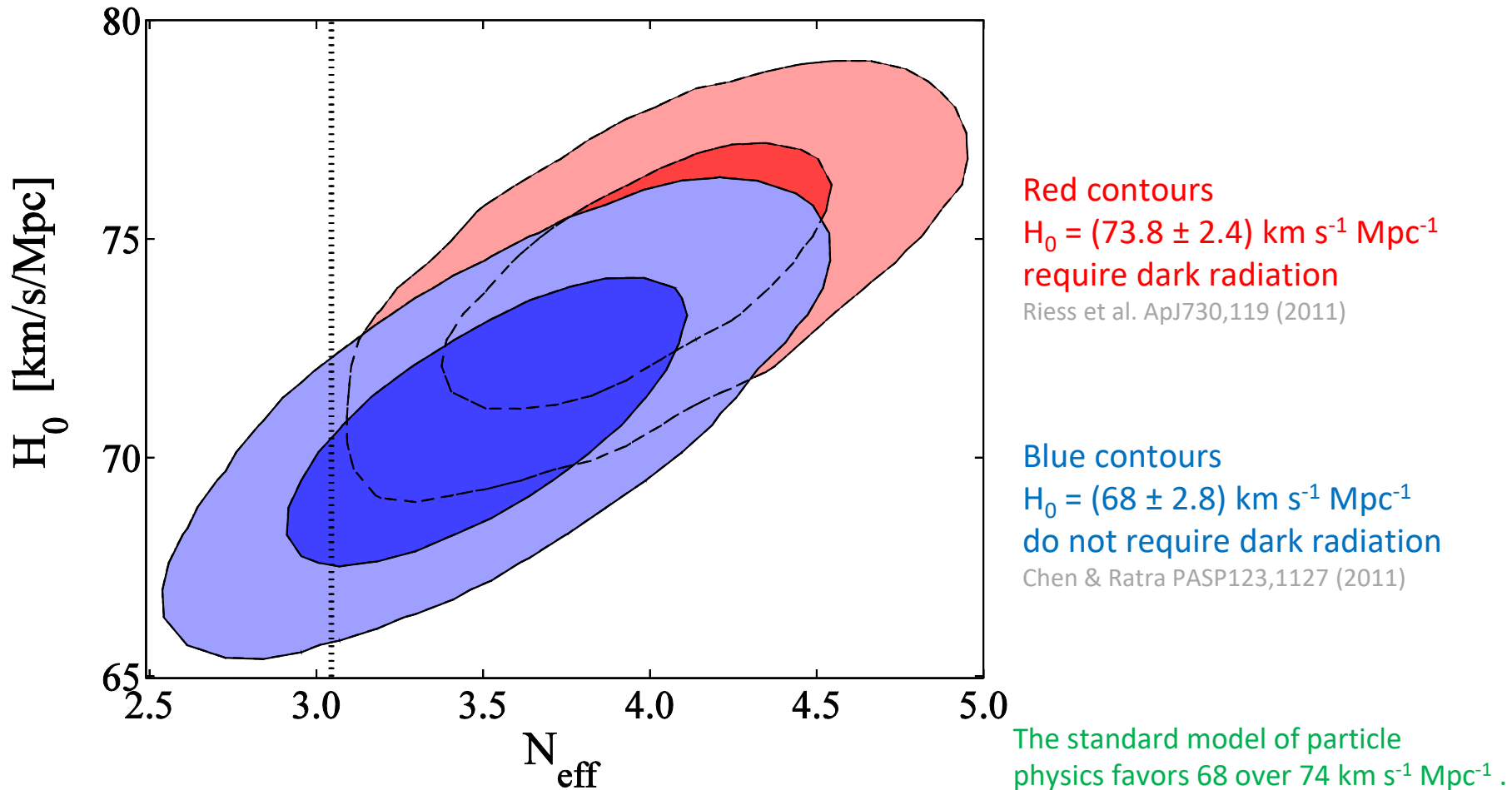
$$H_0 = (68 \pm 2.8) \text{ km s}^{-1} \text{ Mpc}^{-1}$$

from 553  $H_0$  measurements (Huchra)  
Chen & BR PASP123, 1127 (2011)

Most recent  $H_0$  estimates agree  
with this 2011 measurement.

**B**  
(Wendy L. Freedman, Observatories of the Carnegie Institution of Washington, and NASA)

## An aside: Large $H_0$ value might force consideration of dark radiation



From WMAP7, ACBAR, ACT, SPT & SDSS-DR7

Calabrese et al. PRD86, 043520 (2012)

Cosmology thus re-introduces preferred observers, cosmological observers, locally at rest w.r.t. the expansion.

**Cosmological Principle** (assumption): the universe is (statistically) spatially isotropic for all cosmological observers.

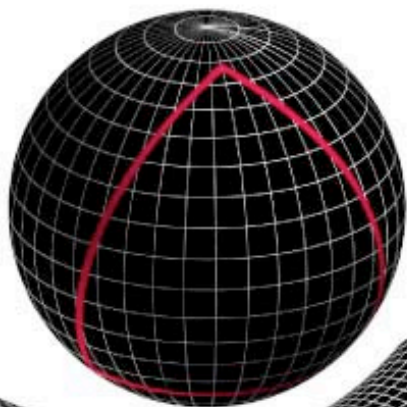
This implies (statistical) spatial homogeneity.

Ignoring global topology, there are then only three possible spatial geometries: the flat, open and closed Friedmann-Lemaitre-Robertson-Walker models.

$$ds^2 = dt^2 - a^2(t) [dr^2 + S_K^2(r) \{d\theta^2 + \sin^2(\theta)d\phi^2\}]$$

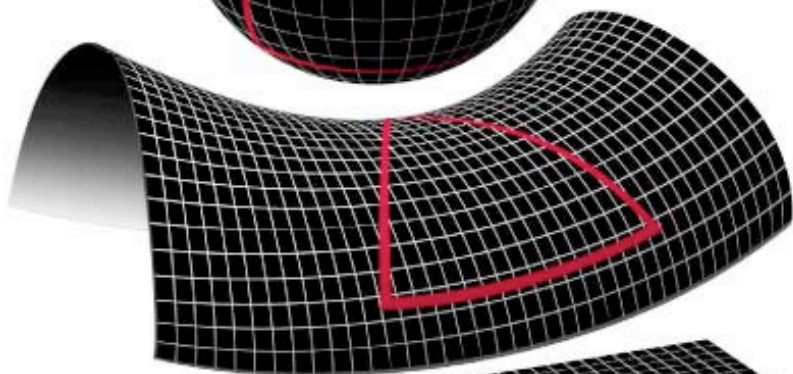
(2 dimensional analogs)

$\Omega_0 > 1$



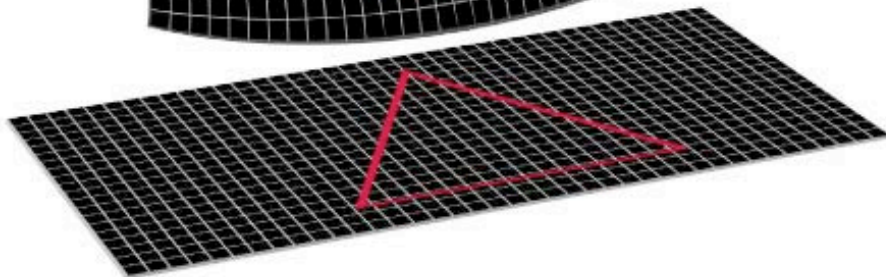
	$S_K(r)$	$K^2$
closed	$\sin(r)$	$> 0$

$\Omega_0 < 1$



open	$\sinh(r)$	$< 0$
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$\Omega_0 = 1$



flat	$r$	$= 0$
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## equations of motion (ideal fluid matter):

$$H^2 = (\dot{a}/a)^2 = 8\pi G\rho/3 - K^2/a^2 + \Lambda/3 \quad \text{Einstein-Friedmann}$$

$$\dot{\rho} = -3 (\dot{a}/a) (\rho + p) \quad \text{stress-energy conservation}$$

$$p = p(\rho) \quad \text{equation of state}$$

$H(t) = \dot{a}/a$  is the expansion rate

Is this increasing or decreasing with time?

$$\text{also, } \ddot{a}/a = -(4\pi G/3)[(\rho_m + 3p_m) + (\rho_\Lambda + 3p_\Lambda)]$$

matter and radiation with  $p > 0$  ( $\Lambda$  e.o.s. is  $p_\Lambda = -\rho_\Lambda$ )

$\Rightarrow \ddot{a} < 0$  decelerated expansion

Einstein-de Sitter mass density

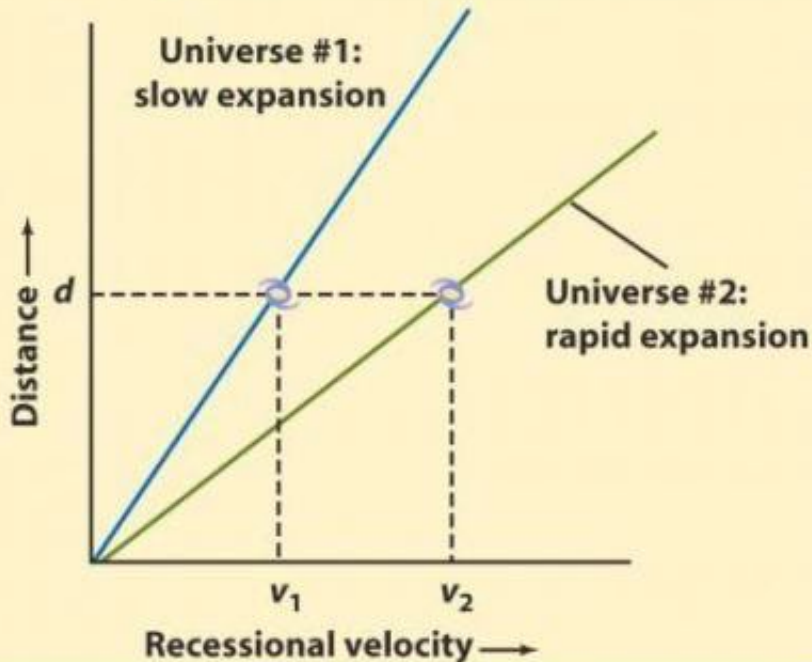
$$\rho_c = 3H^2/8\pi G = 1.9 \times 10^{-29} h^2 \text{ g cm}^{-3}$$

Density parameter  $\Omega = \rho/\rho_c$

# Dark Energy

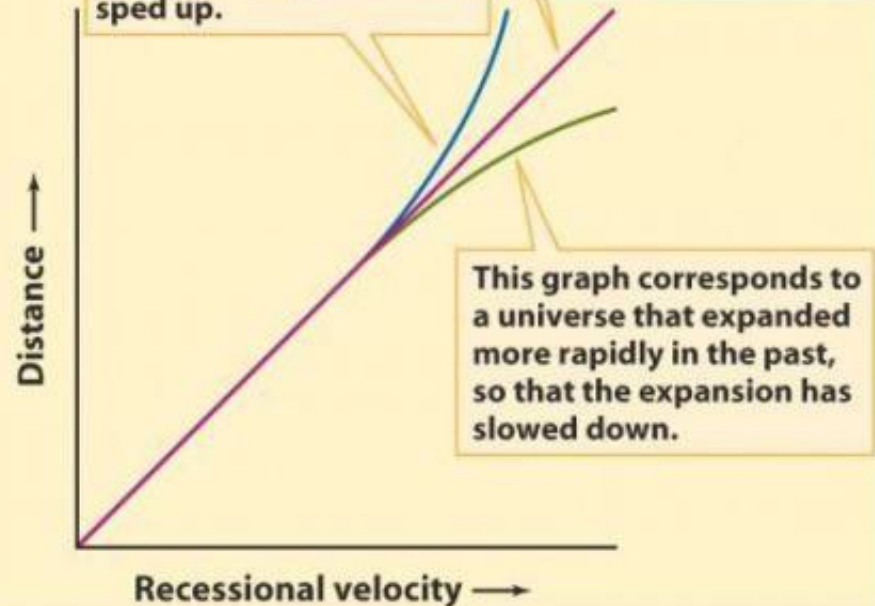
The general idea (more correctly discussed in terms of the m-z diagram).

Universe #2 expands at a faster constant rate than Universe #1, so a galaxy at a given distance  $d$  has a greater recessional velocity in Universe #2 than in Universe #1.



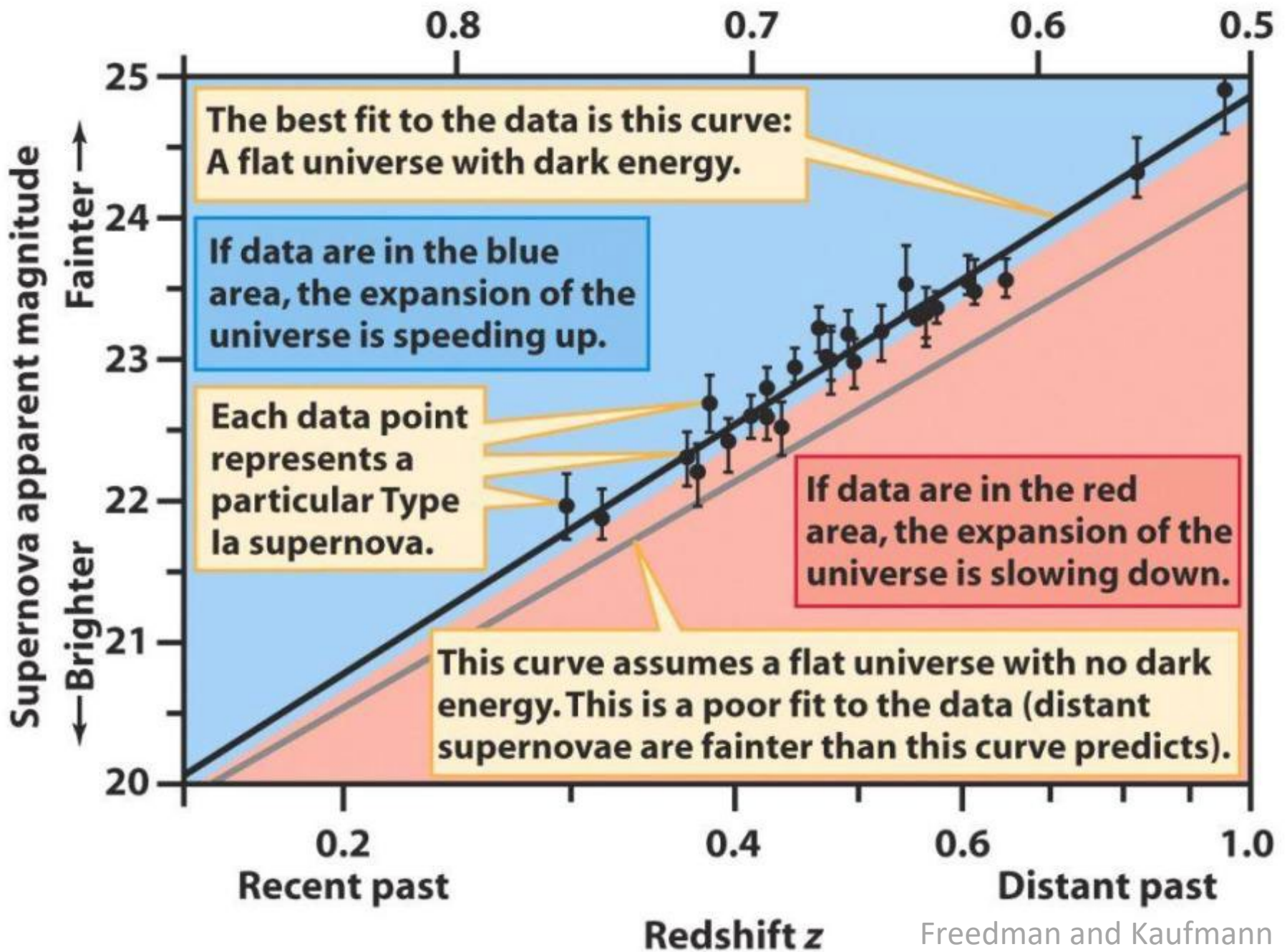
This graph corresponds to a universe that expanded more slowly in the past, so that the expansion has sped up.

This graph corresponds to a universe that expands at a constant rate.

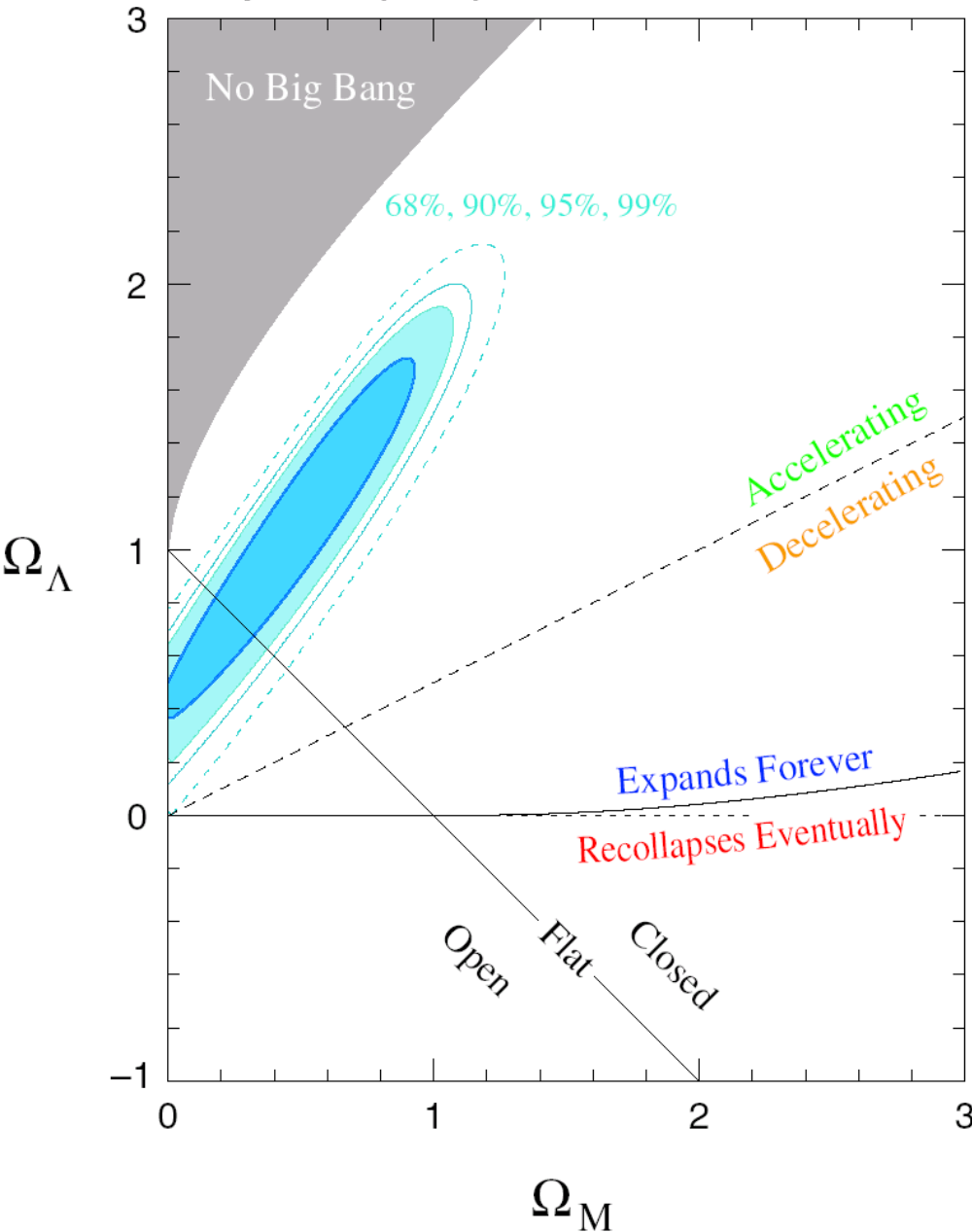


This graph corresponds to a universe that expanded more rapidly in the past, so that the expansion has slowed down.

# Scale of the universe relative to today



Supernova Cosmology Project  
Knop et al. (2003)



accelerated expansion

$$\ddot{a}/a = -(4\pi G/3)\sum_i(\rho_i + 3p_i)$$

$$p \leq -\rho/3$$

dark energy

What do we know about dark energy, the major contributor to the energy budget?

E.g., is it a cosmological constant, or does it vary with space and in time?

The fine print: The general theory of relativity is valid on cosmological length scales and astronomical evidence for dark energy is secure.

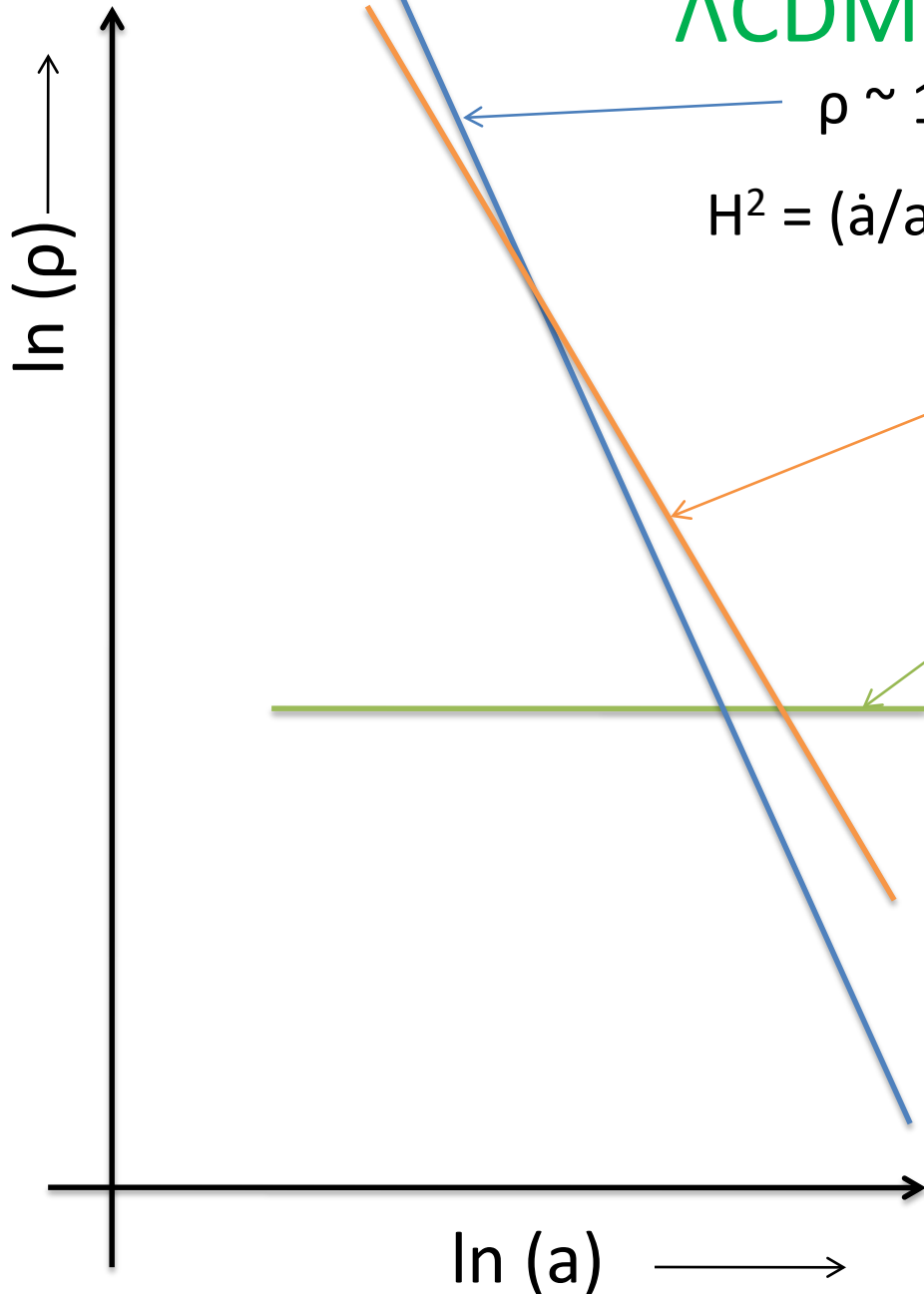
Simplest way to approach such questions is to compare predictions of different dark energy models to observational data. First look at models...

# $\Lambda$ CDM model

(Peebles 1984)

$$\rho \sim 1/a^3$$

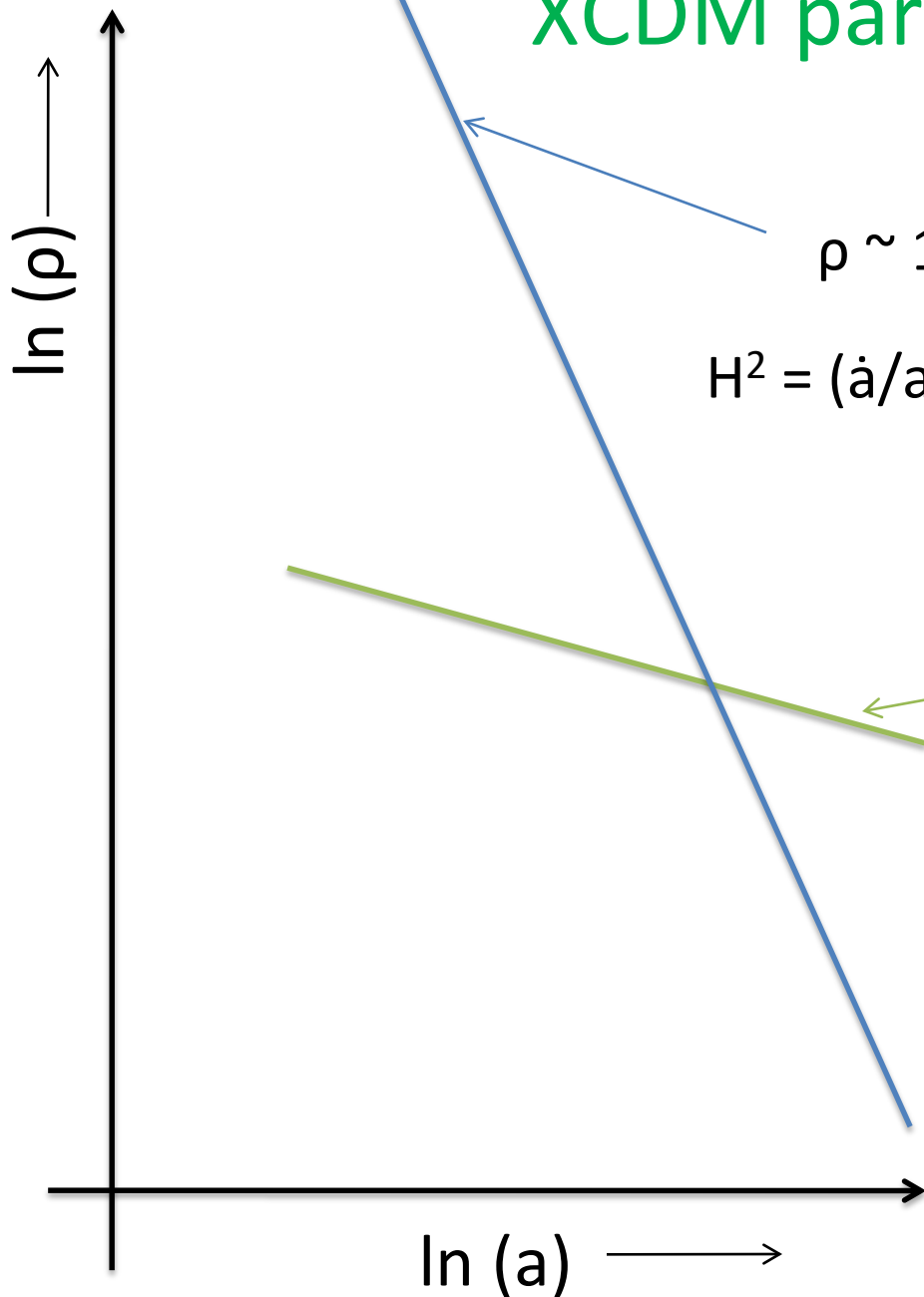
$$H^2 = (\dot{a}/a)^2 = 8\pi G\rho/3 - K^2/a^2 + \Lambda/3$$



Constraint  $\sum \Omega_0 = 1$ , so two free parameters specify  $\Lambda$ CDM:  $\Omega_{M0}, \Omega_\Lambda$

Non zero  $\Omega_\Lambda$  introduces a new “fundamental” energy scale of order an meV.  
(Neutrino mass?)

# XCDM parameterization



$$\rho \sim 1/a^3$$

$$H^2 = (\dot{a}/a)^2 = 8\pi G\rho/3 + 8\pi G\rho_X/3$$

$$p_X = \omega_X \rho_X$$

$$\rho_X \sim 1/a^{3(1+\omega_X)}$$

Spatially flat  
 $K^2 = 0$

$$\omega_X < -1/3$$

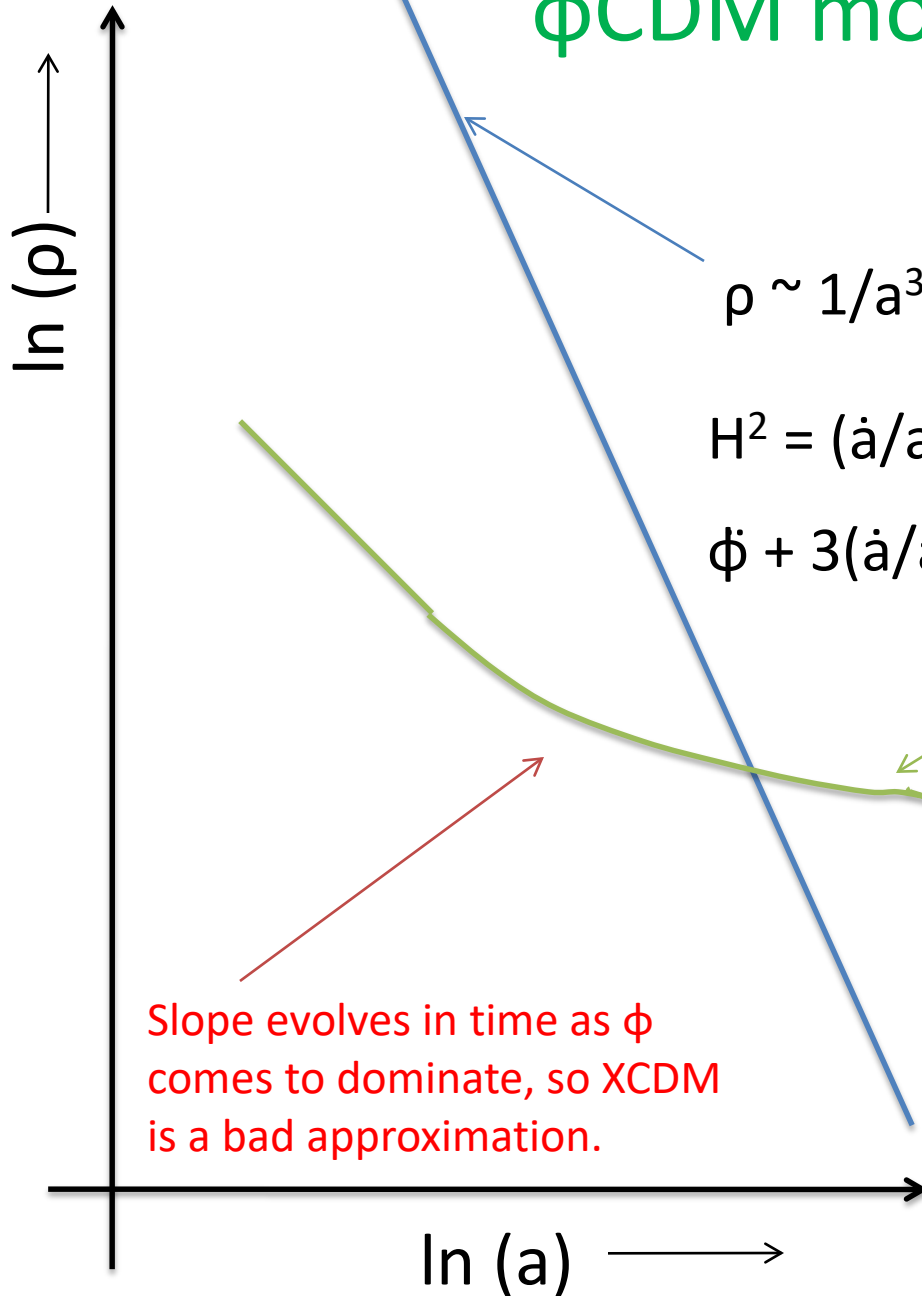
Spatially flat,  $K^2 = 0$ , but now dark energy evolves in time so again two free parameters specify XCDM parameterization:  $\Omega_{M0}, \omega_X$

Widely used parameterization is incomplete; arbitrarily specify  $c_{sX}^2 = dp_X/d\rho_X > 0$ , usually = 1.

# $\phi$ CDM model

Spatially flat  
 $K^2 = 0$

(Peebles and Ratra 1988)



$$\rho_\phi = (\dot{\phi}^2 + \kappa\phi^{-\alpha}/G)/2$$

$$\rho \sim 1/a^3$$

$$H^2 = (\dot{a}/a)^2 = 8\pi G\rho/3 + 8\pi G\rho_\phi/3$$

$$\ddot{\phi} + 3(\dot{a}/a)\dot{\phi} - \kappa\alpha\phi^{-(\alpha+1)}/(2G) = 0$$

numerically integrate

Spatially flat,  $K^2 = 0$ , but now dark energy evolves in time so again two free parameters specify  $\phi$ CDM:  $\Omega_{M0}, \alpha$

$\phi$ CDM model is special for some  $V(\phi)$ : the  $\phi$  solution is an attractor,  $\rho_\phi$  decreases less rapidly than  $\rho_M$  and comes to dominate. This helps to partially resolve the coincidence problem and makes  $\Lambda$  small because the universe is old.

Slope evolves in time as  $\phi$  comes to dominate, so XCDM is a bad approximation.

The new energy scale can be much higher; time evolution decreases it to of order an meV<sup>17</sup> now.

# Cosmological Tests

Type Ia supernova apparent magnitude vs. redshift

Baryon acoustic oscillations peak

Hubble parameter vs. redshift

Growth factor vs. redshift

There are many others but these 4, using older data, suffice for introductory illustrative purposes.

# Procedure

Compute model-parameter-dependent predictions for the lookback time, the luminosity distance, etc., as functions of redshift  $z$ :  $1+z = \lambda_{\text{obs}}/\lambda_{\text{em}} = a(t_0)/a(t)$

$$H^2 = (\dot{a}/a)^2 = H_0^2 [\Omega_{M0}(1+z)^3 + \Omega_{K0}(1+z)^2 + \Omega_{\Lambda}] = H_0^2 E^2(z, p)$$

(Einstein-Friedmann equation for  $\Lambda$ CDM model)

at  $z = 0$ :  $\Omega_{M0} + \Omega_{K0} + \Omega_{\Lambda} = 1$  so  $p = (\Omega_{M0}, \Omega_{\Lambda})$

E.g., Hubble parameter vs. redshift:

$$H(z, p, H_0) = H_0 E(z, p)$$

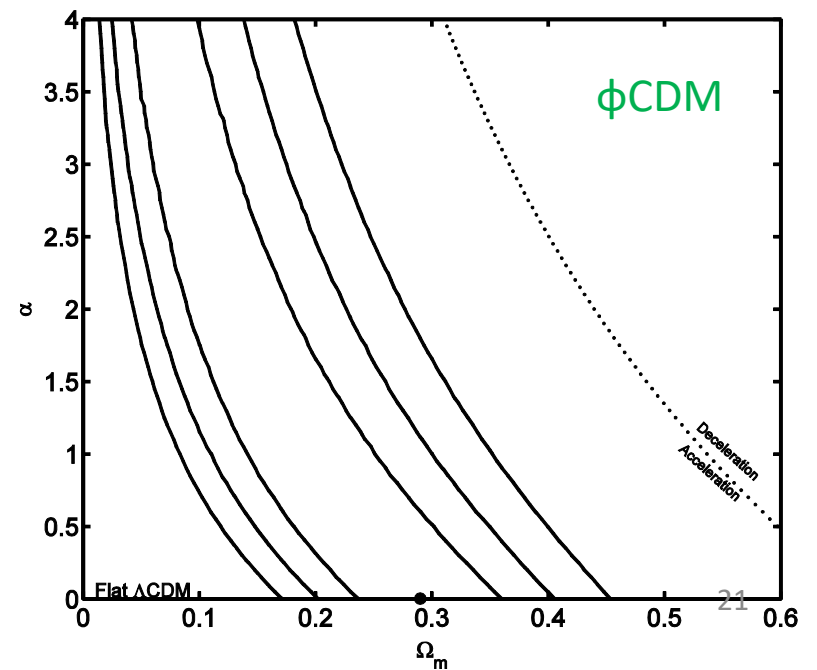
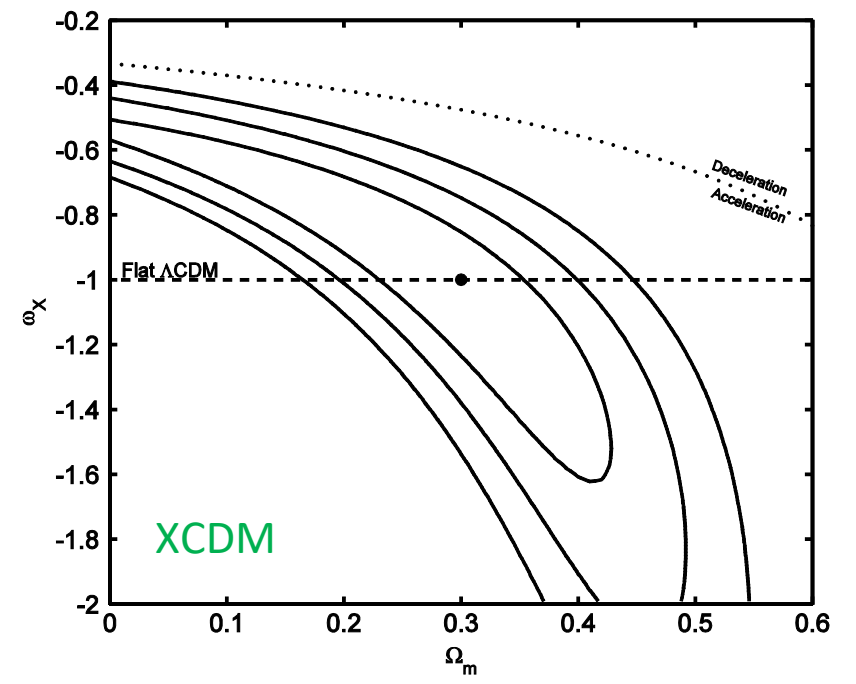
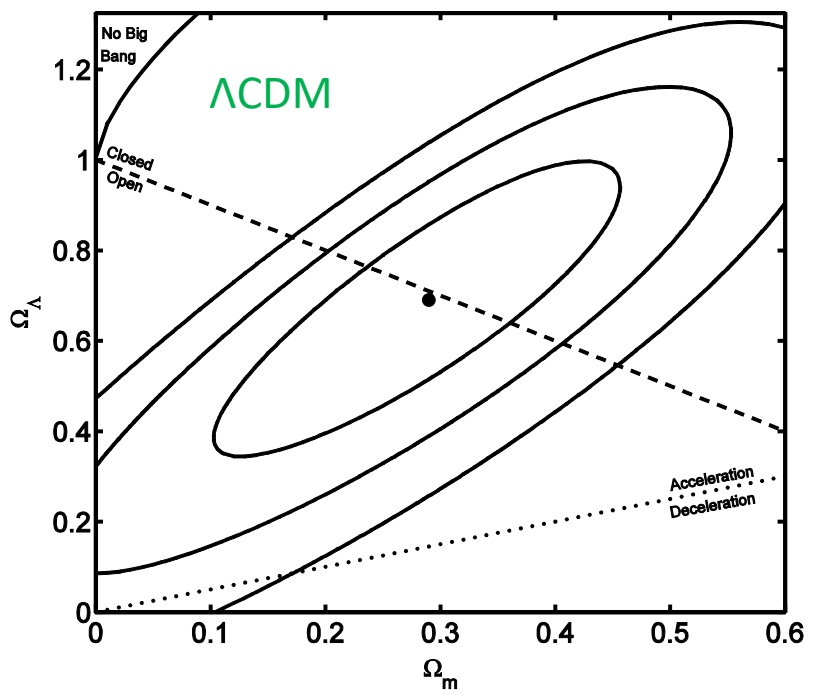
Use such **model-parameter-dependent predictions** and **observational data** on these quantities and a technique such as least squares or maximum likelihood to constrain the cosmological parameters of these models.

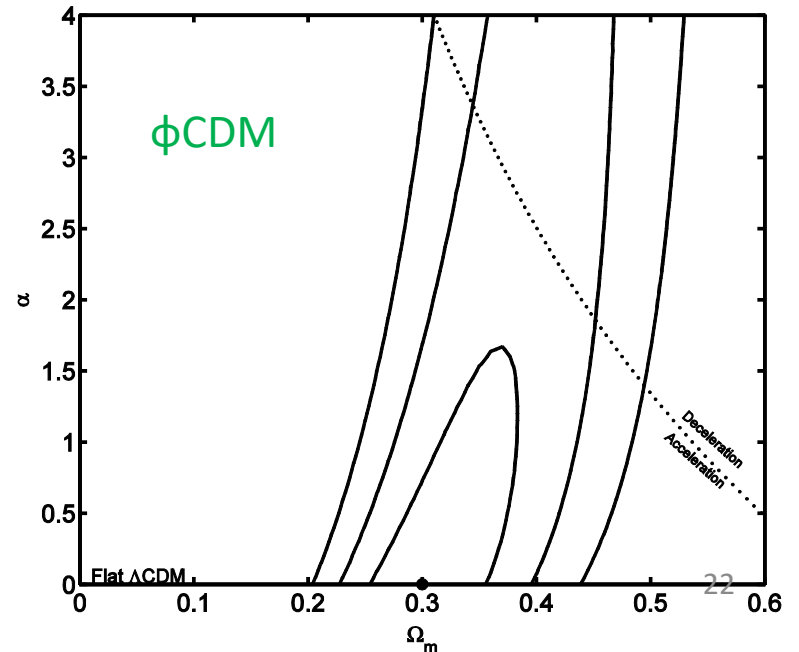
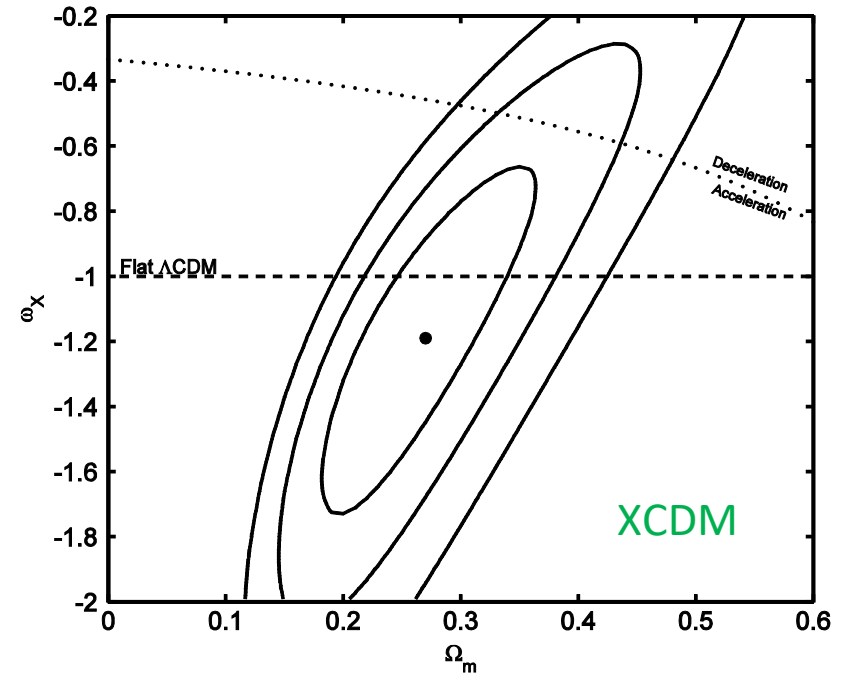
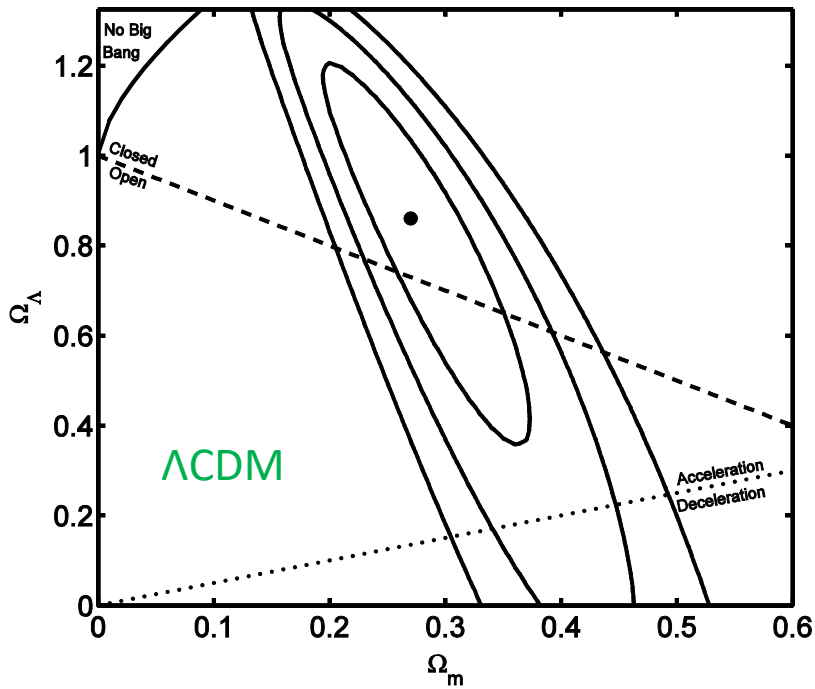
For nice reviews see the Ph.D. theses of Farooq 1309.3710 and Ryan 2104.10354.

Type Ia SN magnitude-redshift test.  
 Union2.1, with systematic errors.

Suzuki et al. *ApJ*746, 85 (2012) 580 SNe.  
 Marginalize over  $h$  with flat prior.

Farooq et al *ApJ*764,139 (2013)

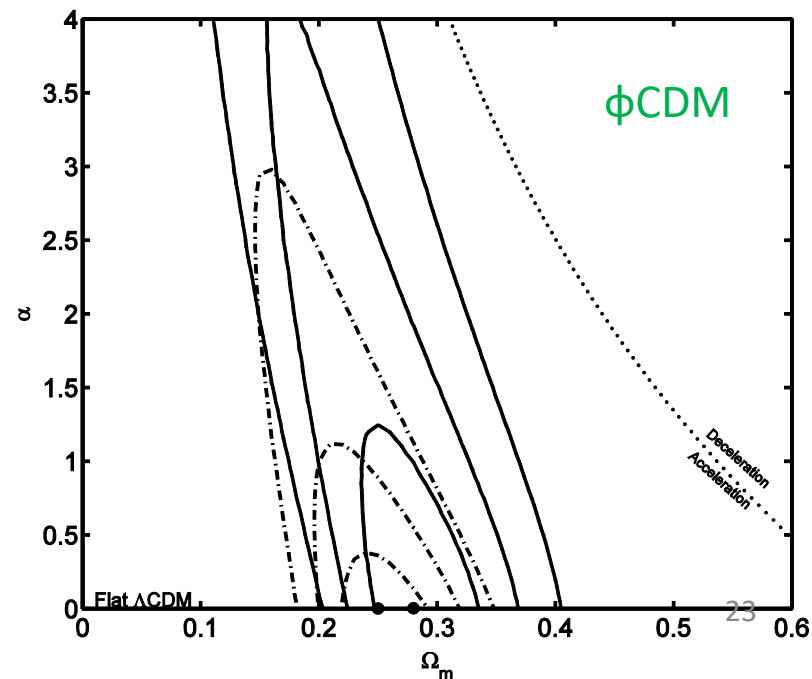
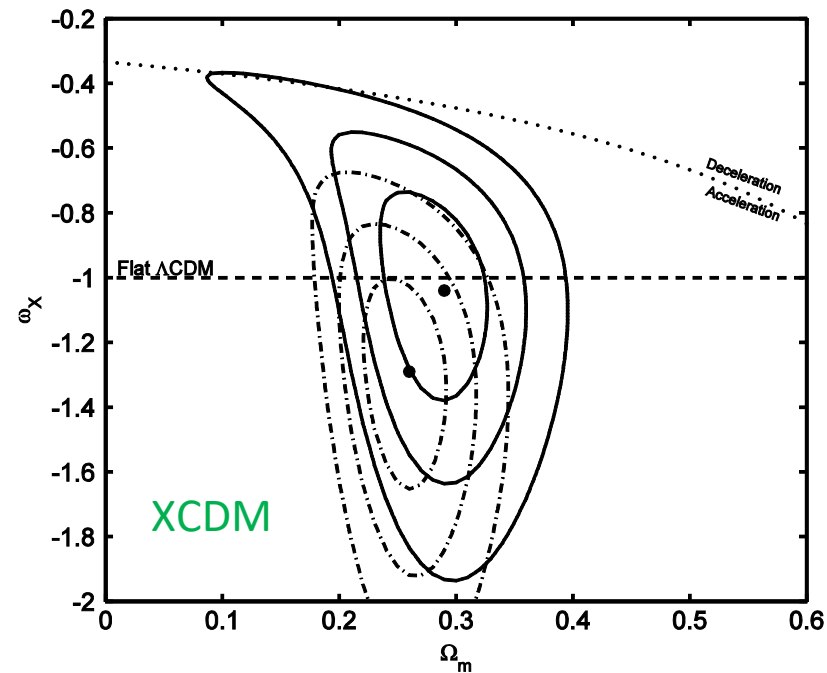
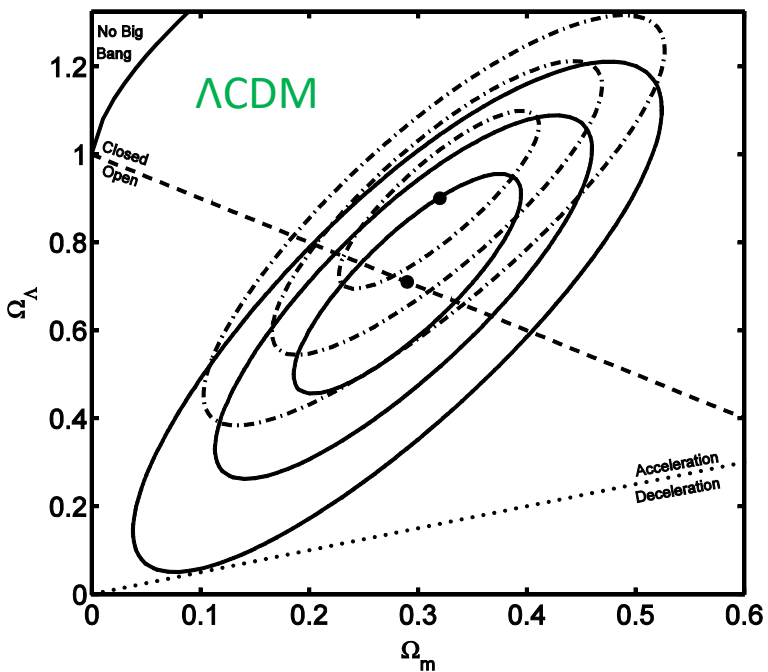




## Baryon acoustic peak test.

- 3 WiggleZ Blake et al MNRAS418, 1707 (2011)
- 1 6dFGS Beutler et al MNRAS416, 3017 (2011)
- 2 SDSS Percival et al MNRAS410, 2148 (2010)

Farooq et al ApJ764,139 (2013)

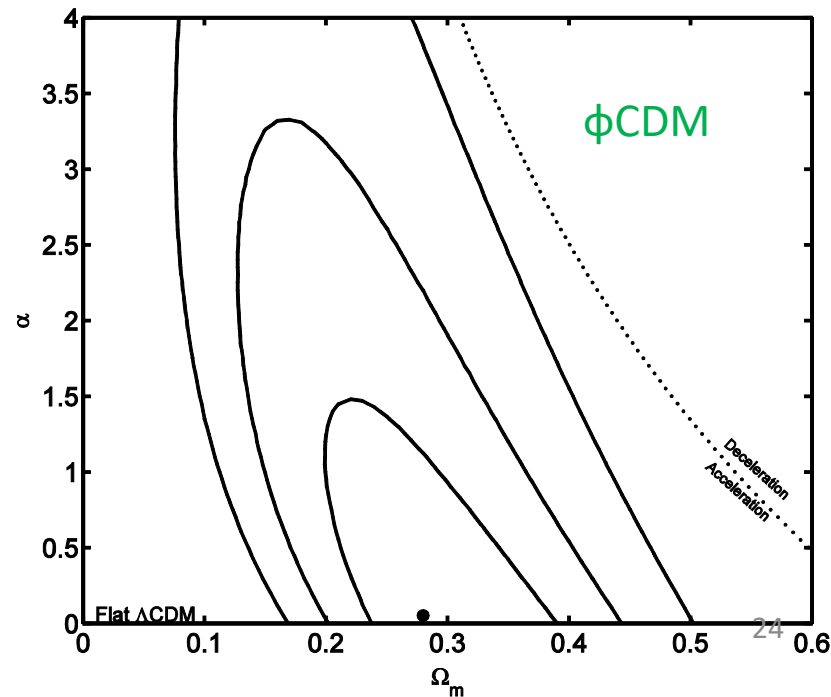
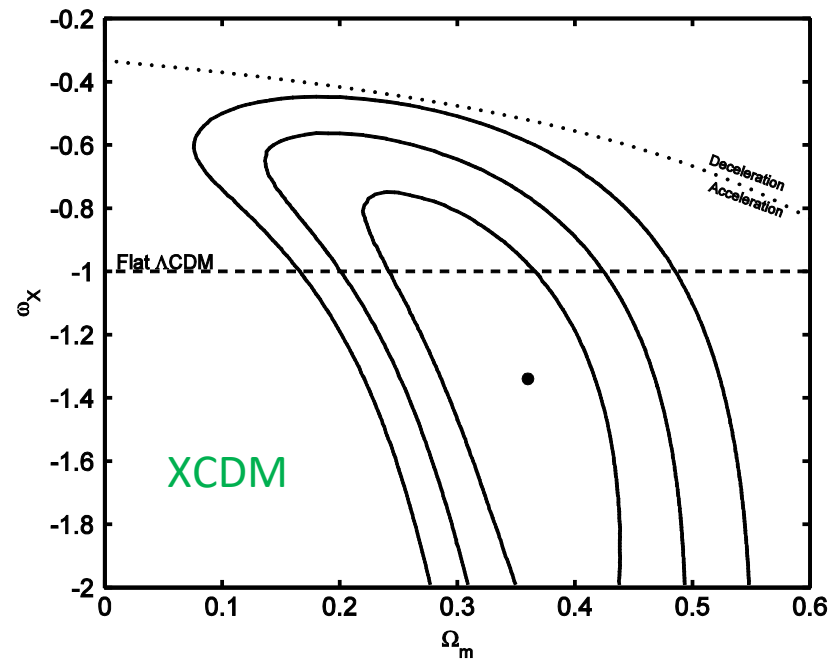
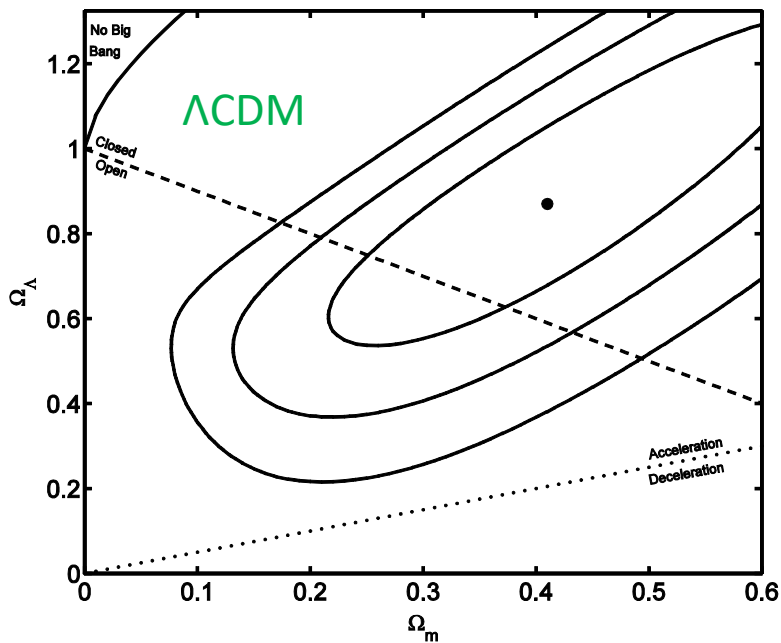


## Hubble parameter vs. redshift test.

Farooq & Ratra ApJ766, L7 (2013) 28 points.

Marginalize over:

- 1)  $h = 0.68 \pm 0.028$  solid lines
- 2)  $h = 0.738 \pm 0.024$  dash-dotted lines



## Growth rate vs. redshift test.

Pavlov et al PRD90, 023006 (2014)

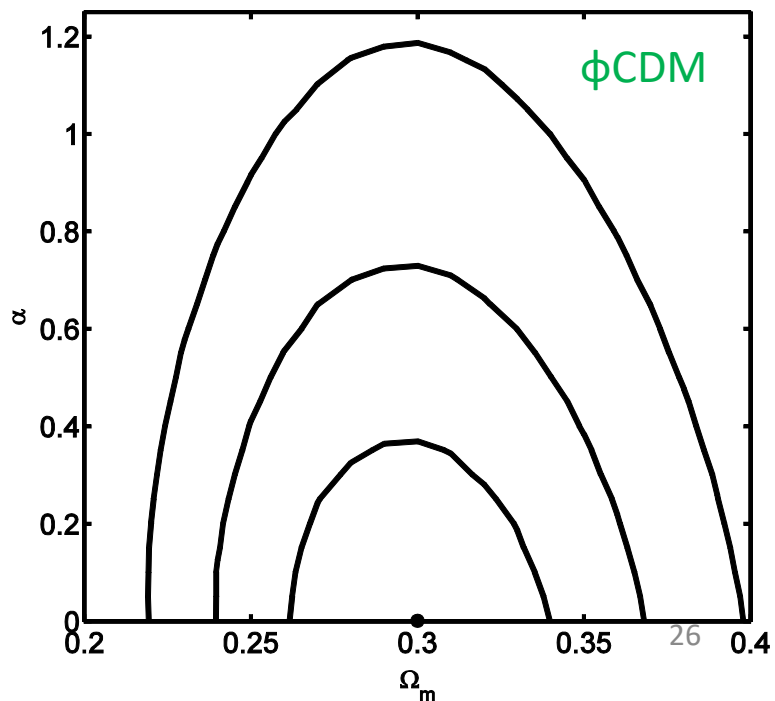
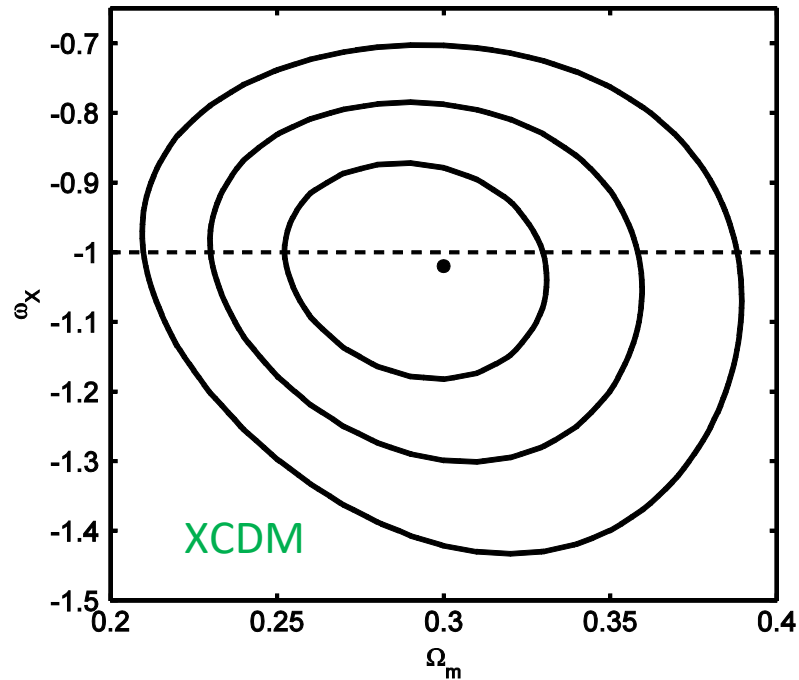
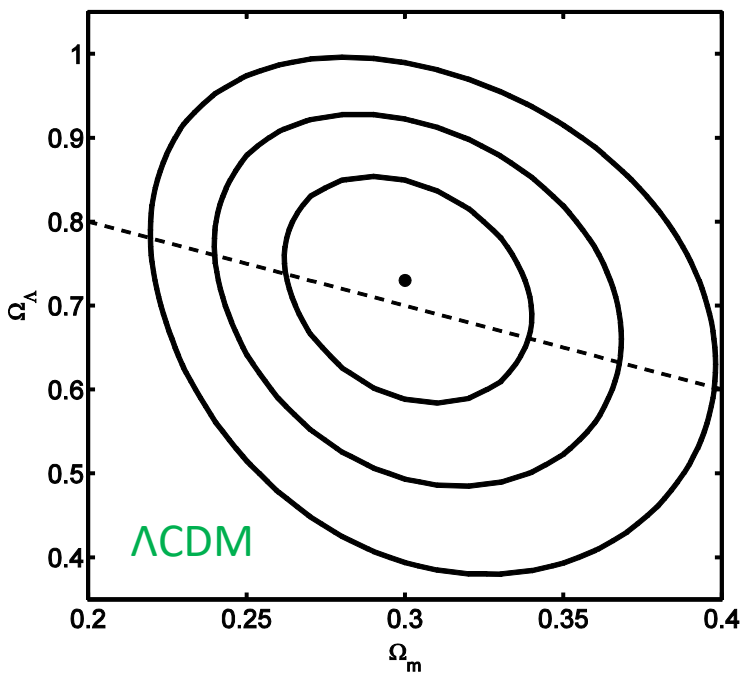
14 measurements

Constraints from different data are not inconsistent.

Individual data sets are consistent with a spatially-flat  $\Lambda$ CDM model with  $\Omega_\Lambda$  of order 0.7 and  $\Omega_{\text{matter}}$  of order 0.3, but do not yet rule out time-evolving dark energy or some spatial curvature.

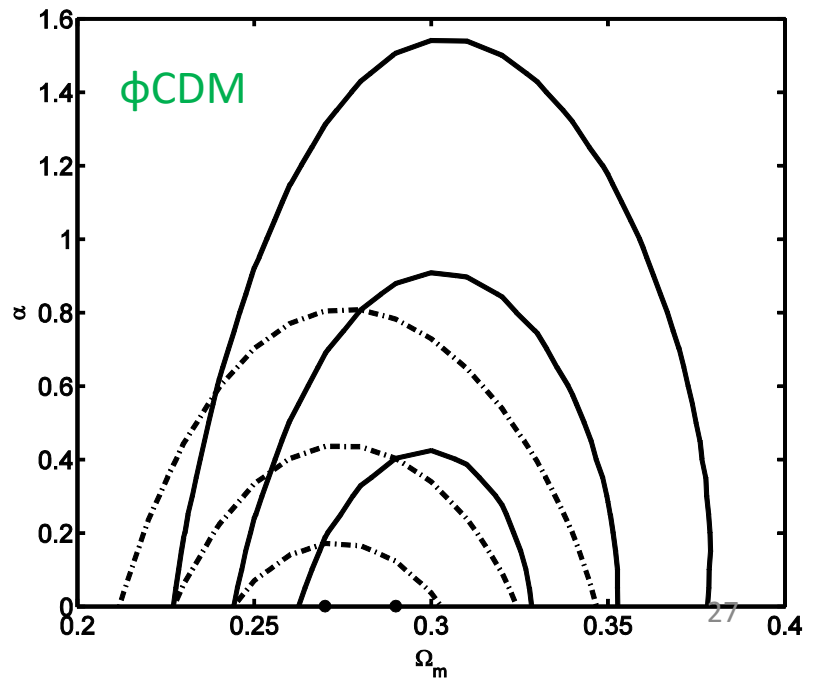
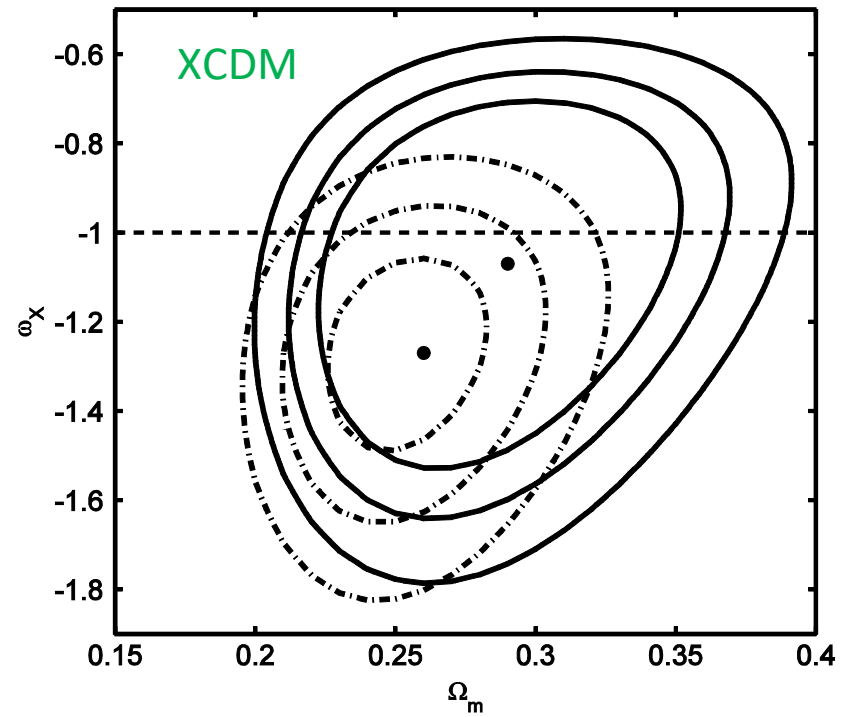
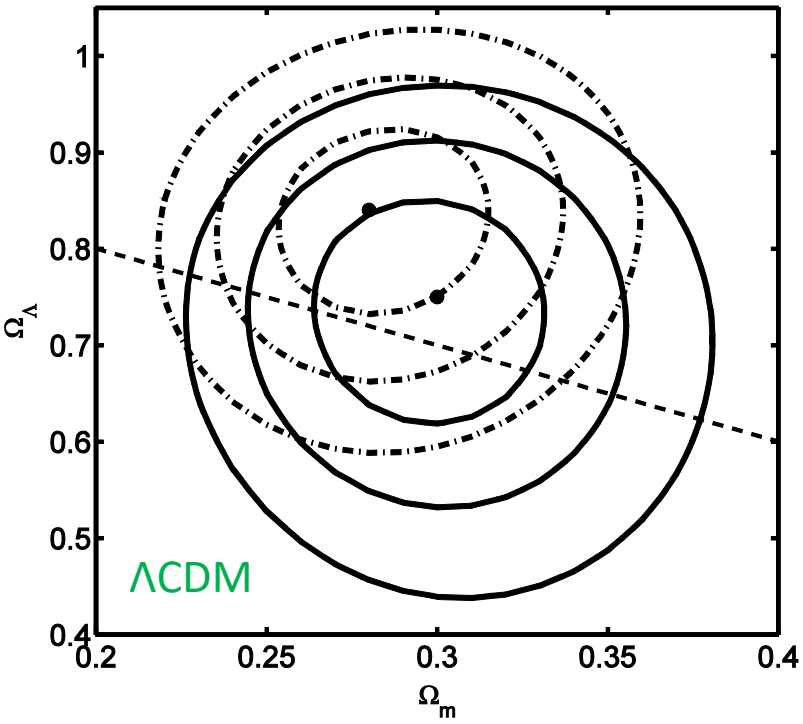
What about combinations of data sets?

Two data sets at a time, except for growth + Hubble parameter, since there is some correlation.



Supernovae and BAO.

A. Pavlov

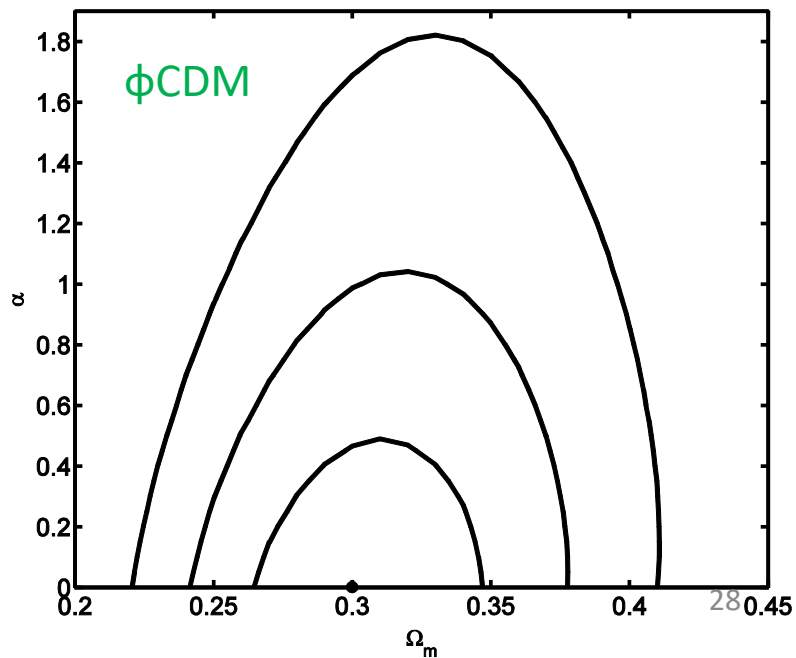
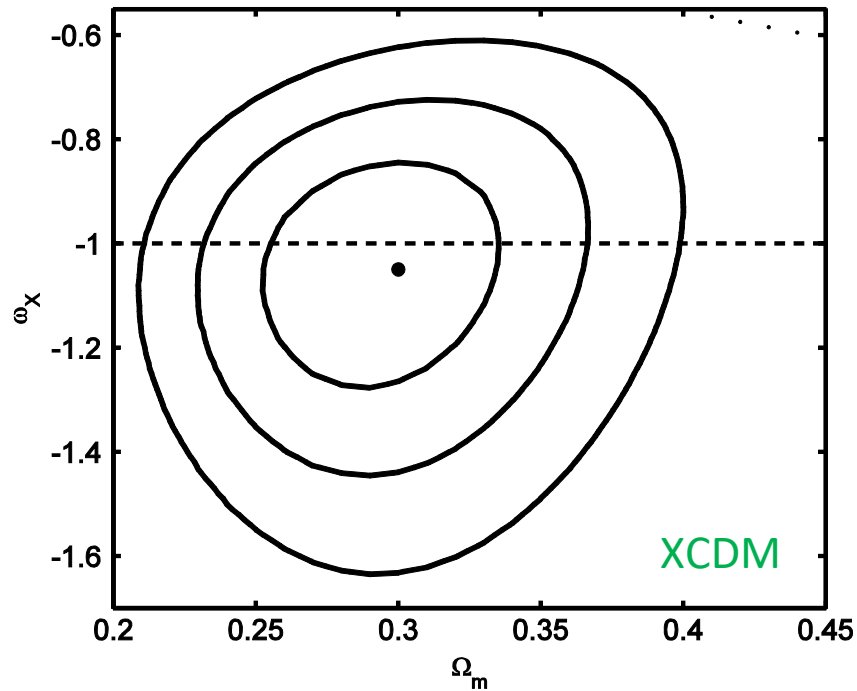
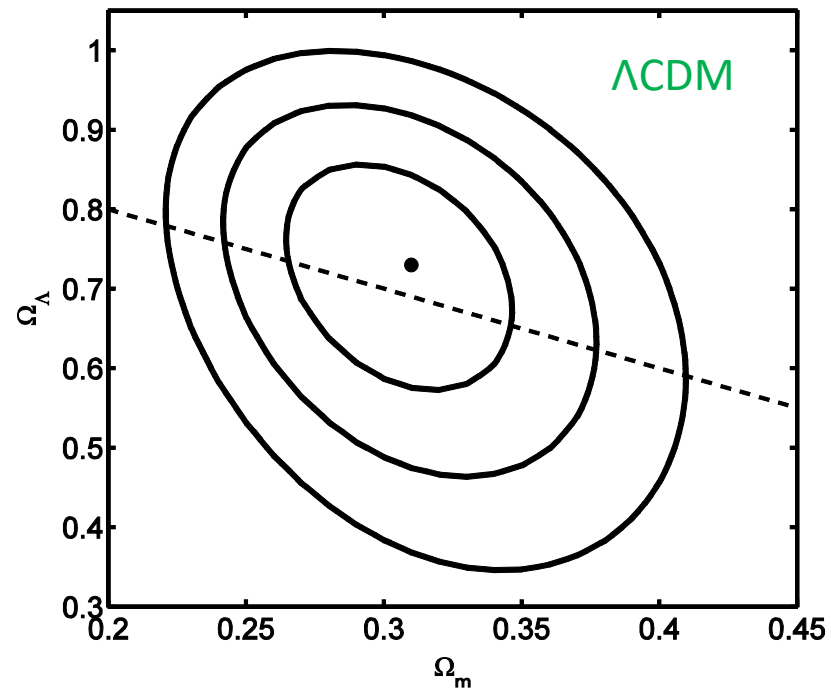


## Hubble parameter and BAO.

Marginalize over:

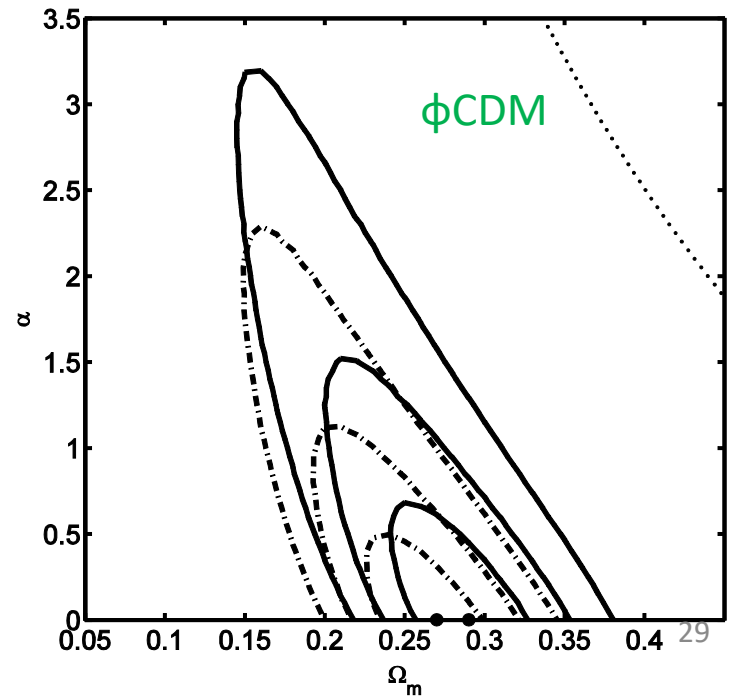
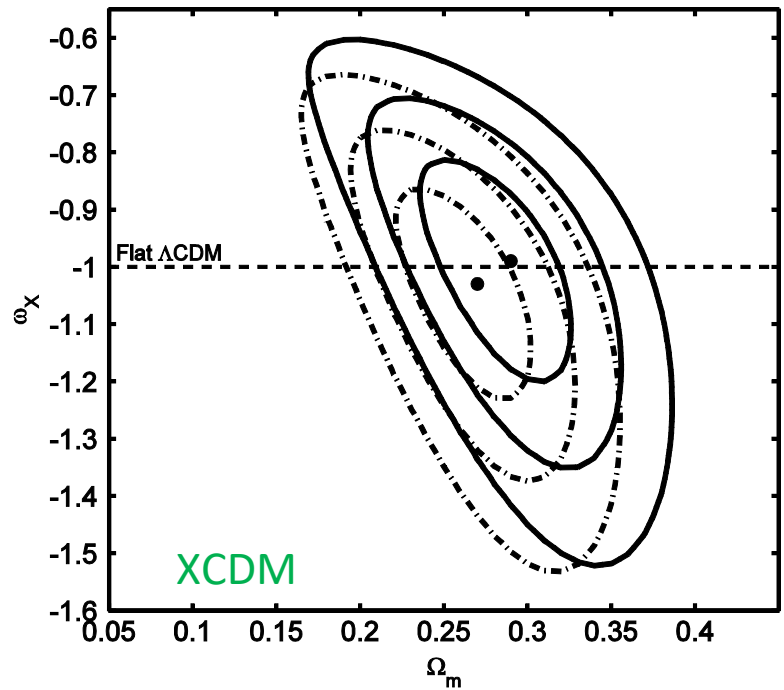
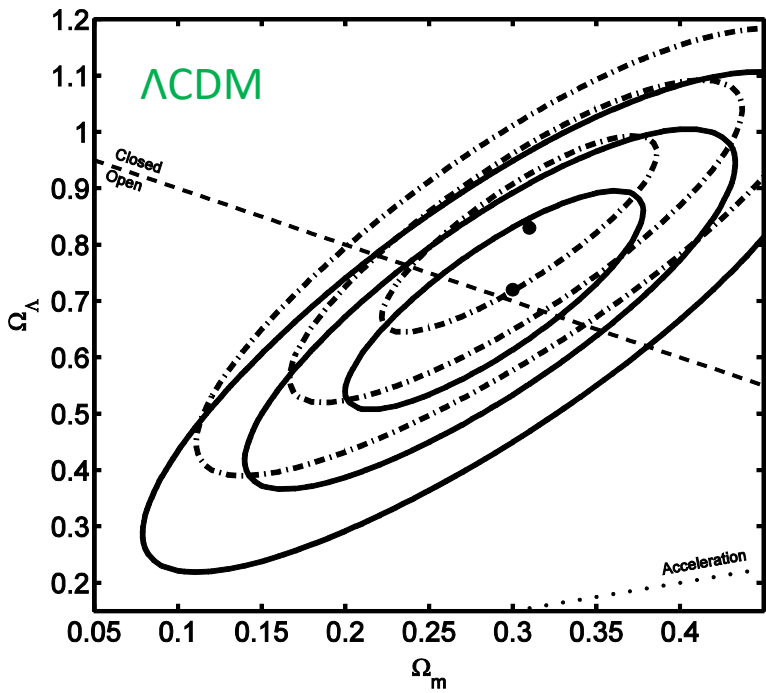
- 1)  $h = 0.68 \pm 0.028$  solid lines
- 2)  $h = 0.738 \pm 0.024$  dash-dotted lines

A. Pavlov



BAO and growth factor.

A. Pavlov

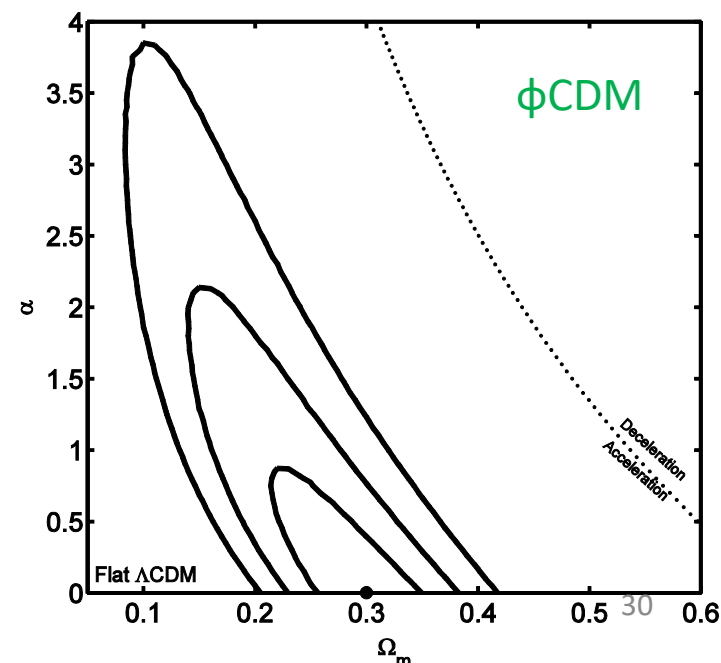
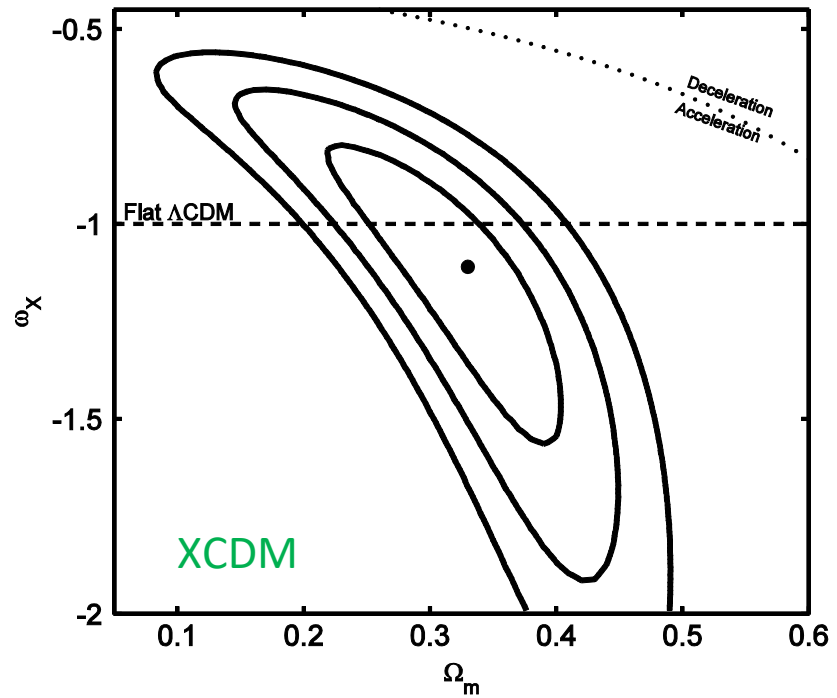
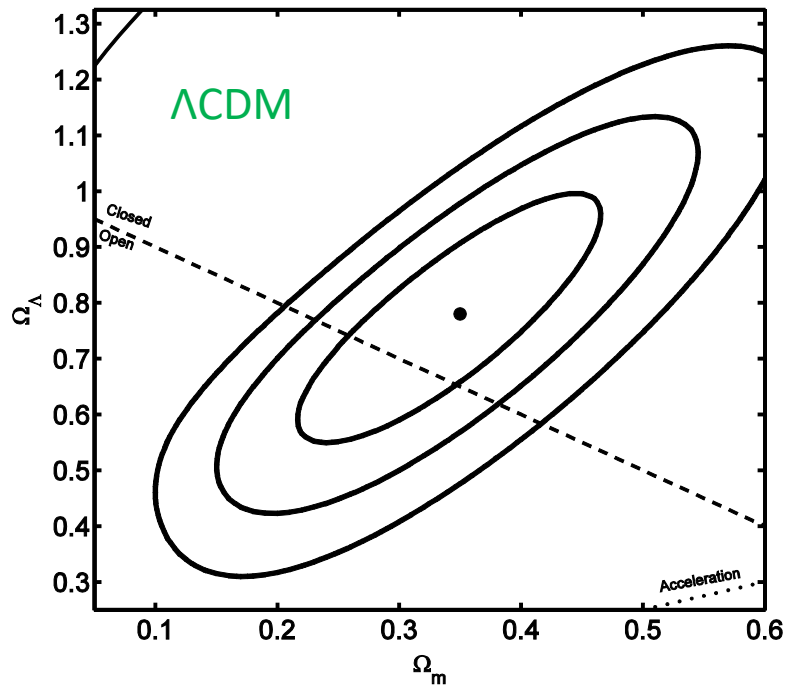


## Hubble parameter and supernovae.

Marginalize over:

- 1)  $h = 0.68 \pm 0.028$  solid lines
- 2)  $h = 0.738 \pm 0.024$  dash-dotted lines

A. Pavlov



# Supernovae and growth factor.

A. Pavlov

Data sets combined two at a time result in tighter constraints which are consistent with a spatially-flat  $\Lambda$ CDM model with  $\Omega_\Lambda$  of order 0.7 and  $\Omega_{\text{matter}}$  of order 0.3, but do not yet strongly rule out time-evolving dark energy or some spatial curvature.

Some data issues: different SNe Ia data result in different constraints (systematics?); different GRB data analysis techniques result in different constraints (not yet standard candles?), improve  $h$  determination, improve  $\Omega_b h^2$  determination (is simplest BBN model adequate?), ...

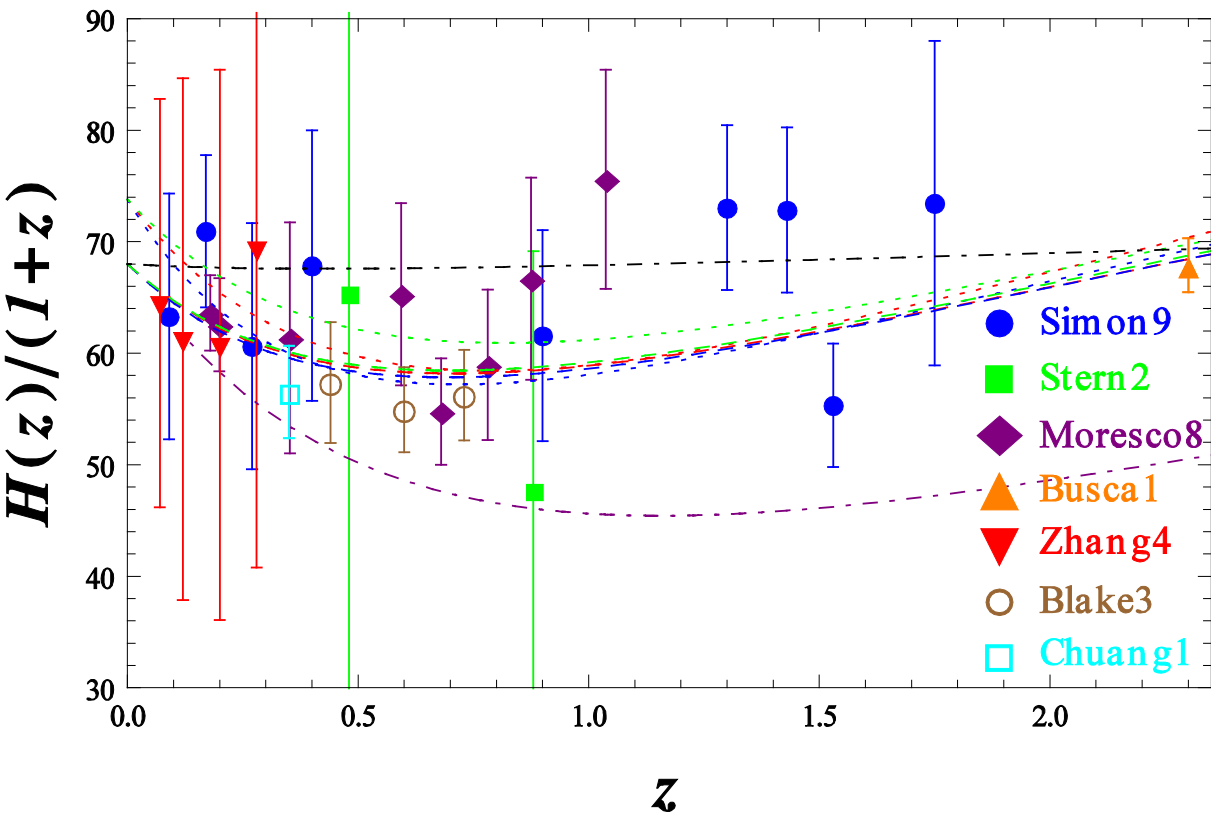
Will look at more and better-quality data soon and draw stronger conclusions.

# H(z) data & deceleration-acceleration transition

It is now possible to measure H(z) by using cosmic chronometers or radial BAO data (e.g., Moresco JCAP1208, 006; Busca A&A552, A96 (2013))

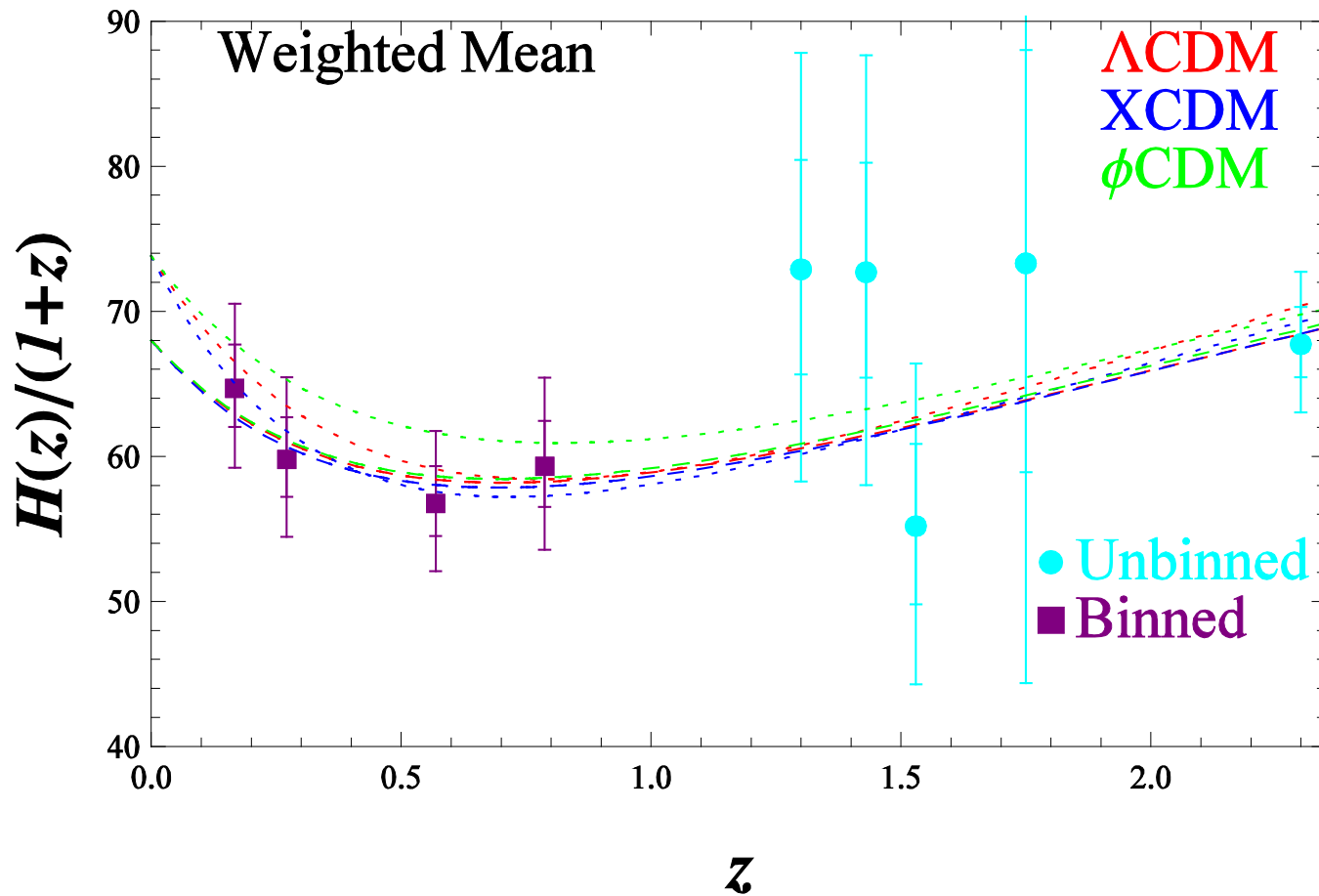
Combining 28 independent measurements over  $0.07 < z < 2.3$

(Farooq & BR ApJ766, L7 (2013); Farooq, Crandall & BR PLB726, 72 (2013)) shows a transition:



Six best-fit models and two  $3\sigma$  deviant models

Data are noisy, so let's bin them



For  $\Lambda$ CDM:  $H/(1+z) = H_0 [\Omega_{M0}(1+z) + \Omega_{K0} + \Omega_{\Lambda}/(1+z)^2]^{1/2}$

Averaging over models and  $H_0$  priors, transition redshift  $z = 0.74 \pm 0.04$

(This is the first real measurement of the deceleration-acceleration transition redshift.)

# Hubble constant $H_0$ from consistent non-CMB low-z data

Measure  $H_0$  from  $z < 8.2$  BAO +  $H(z)$  + SN-Pantheon+ + QSO-AS + Mg II QSO + C IV QSO + GRB data by using cosmological models (Cao & BR PRD107,

103521 (2023)). No DESI data. Independent of local calibration and CMB, since these data are also used to measure sound horizon at drag epoch  $r_s$  (i.e.,  $\Omega_b h^2$  and  $\Omega_c h^2$  instead of  $\Omega_{m0} h^2$ ). Independent of  $P(k)$ .

Flat  $\Lambda$ CDM:  $(69.5 \pm 2.4) \text{ km s}^{-1} \text{ Mpc}^{-1}$

Non-flat  $\Lambda$ CDM:  $(68.9 \pm 2.4) \text{ km s}^{-1} \text{ Mpc}^{-1}$

Flat XCDM:  $(69.5 \pm 2.4) \text{ km s}^{-1} \text{ Mpc}^{-1}$

Non-flat XCDM:  $(69.3 \pm 2.4) \text{ km s}^{-1} \text{ Mpc}^{-1}$

Flat  $\phi$ CDM:  $(68.9 \pm 2.4) \text{ km s}^{-1} \text{ Mpc}^{-1}$

Non-flat  $\phi$ CDM:  $(69.2 \pm 2.4) \text{ km s}^{-1} \text{ Mpc}^{-1}$

“Independent” of  
cosmological model.

$(69.25 \pm 2.42) \text{ km s}^{-1} \text{ Mpc}^{-1}$

Closer to  $(68 \pm 2.8) \text{ km s}^{-1} \text{ Mpc}^{-1}$  MS (Chen & BR PASP123, 1127 (2011))

and  $(70.39 \pm 1.94) \text{ km s}^{-1} \text{ Mpc}^{-1}$  JWST+HST TRGB+SN Ia (Freedman+ 2408.06153)

than to  $(73.04 \pm 1.04) \text{ km s}^{-1} \text{ Mpc}^{-1}$  Cepheids+SN Ia (Riess+ ApJ934, L7 (2022))

1.2 $\sigma$  difference

our values are 1.3—1.6 $\sigma$  lower

and  $(67.36 \pm 0.54) \text{ km s}^{-1} \text{ Mpc}^{-1}$  CMB Planck (Planck A&A641, A6 (2020))

our Flat  $\Lambda$ CDM value is 0.86 $\sigma$  higher

which might be interesting. Also,

$(68.3 \pm 1.1) \text{ km s}^{-1} \text{ Mpc}^{-1}$  CMB ACT (Madhavacheril+ ApJ962, 113 (2024))

and  $(68.3 \pm 1.5) \text{ km s}^{-1} \text{ Mpc}^{-1}$  CMB SPT (Dutcher+ PRD104, 022003 (2021))

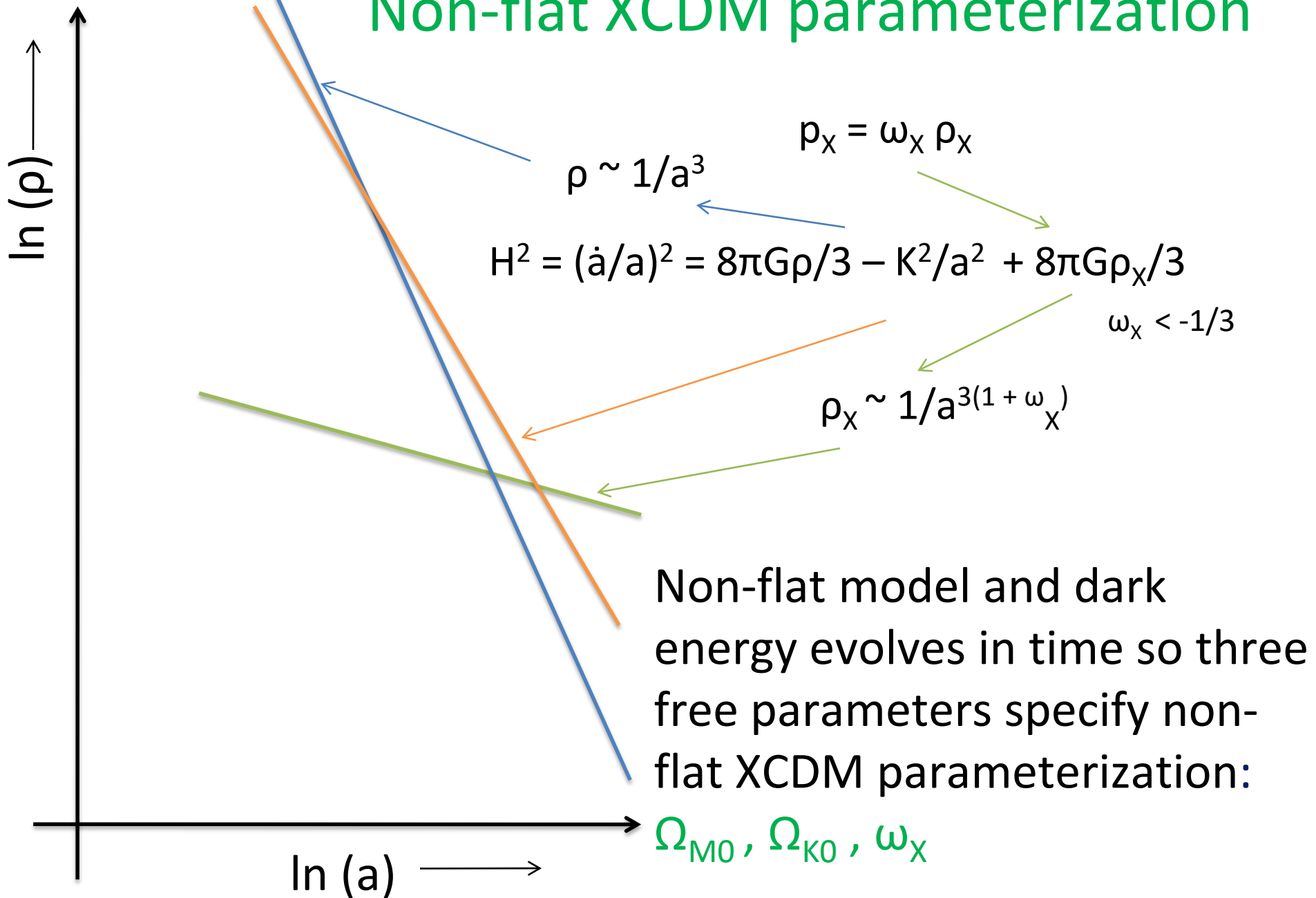
## Do observations really require close to zero space curvature?

NO, Planck 2018 CMB anisotropy data require a mildly closed geometry, but, YES when combined with non-CMB data CMB anisotropy data require an almost flat geometry, IF dark energy density is time-independent as in  $\Lambda$ CDM, but possibly not if the dark energy density varies in time as in the XCDM parameterization or the  $\phi$ CDM model.

And in non-flat models data do not as strongly demand time-independent dark energy density.

Consider 2 options, non-flat XCDM parameterization and non-flat  $\phi$ CDM.

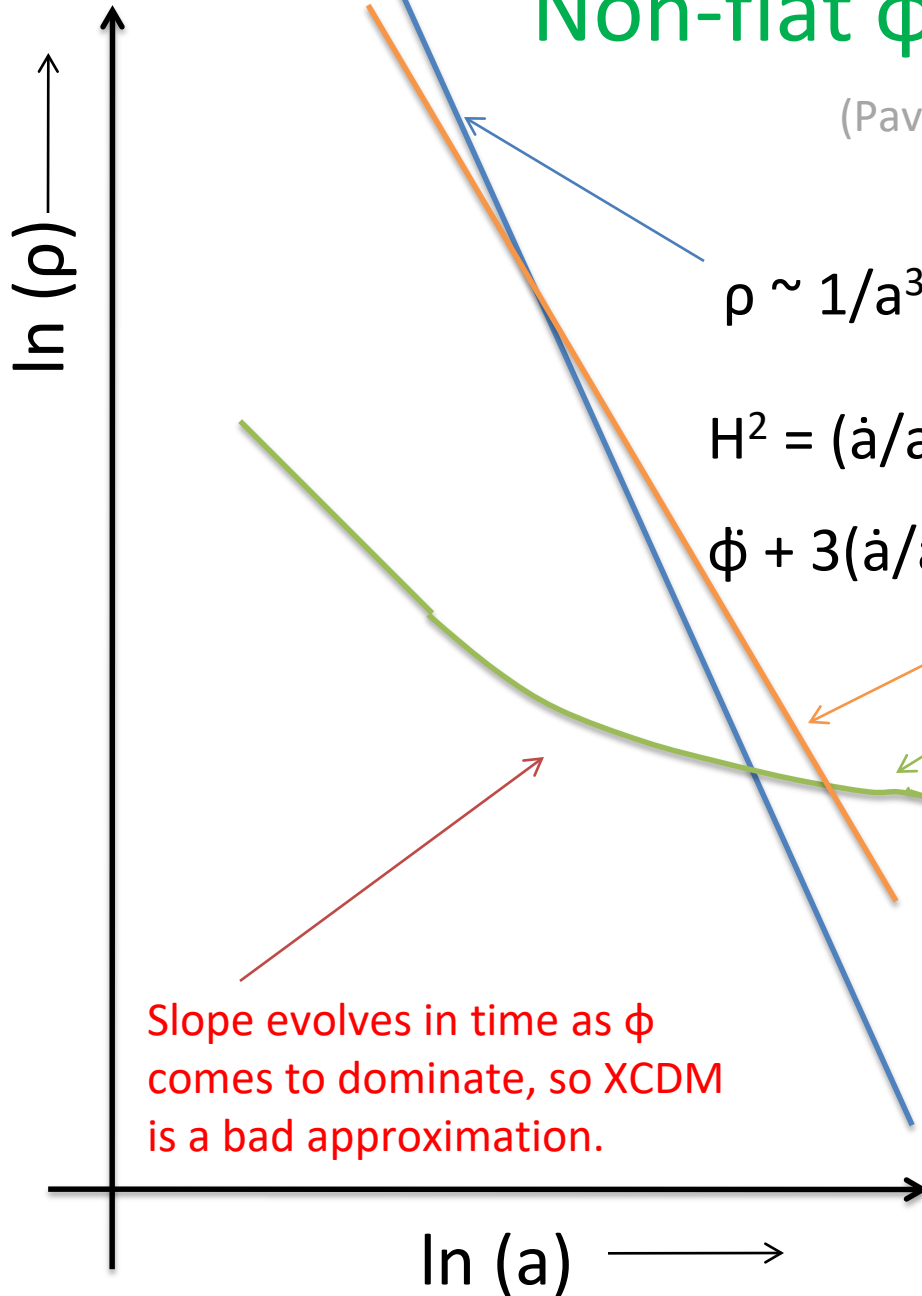
# Non-flat XCDM parameterization



Widely used parameterization is incomplete; arbitrarily specify  $c_{sX}^2 = dp_X/d\rho_X > 0$ , usually = 1. <sup>36</sup>

# Non-flat $\phi$ CDM model

(Pavlov et al. PRD88, 123513 (2013))



$$\rho_\phi = (\dot{\phi}^2 + \kappa\phi^{-\alpha}/G)/2$$

$$\rho \sim 1/a^3$$

$$H^2 = (\dot{a}/a)^2 = 8\pi G\rho/3 - K^2/a^2 + 8\pi G\rho_\phi/3$$

$$\ddot{\phi} + 3(\dot{a}/a)\dot{\phi} - \kappa\alpha\phi^{-(\alpha+1)}/(2G) = 0$$

← numerically integrate

Non-flat and dark energy evolves in time so three free parameters specify non-flat  $\phi$ CDM:  $\Omega_{M0}$ ,  $\Omega_{K0}$ ,  $\alpha$

$\phi$ CDM model is special for some  $V(\phi)$ : the  $\phi$  solution is an attractor,  $\rho_\phi$  decreases less rapidly than  $\rho_M$  and comes to dominate. This helps to partially resolve the coincidence problem and makes  $\Lambda$  small because the universe is old.

Slope evolves in time as  $\phi$  comes to dominate, so XCDM is a bad approximation.

Also constrain other parameters of these six models using  $z < 8.2$  BAO + H(z) + SN-Pantheon+ + QSO-AS + Mg II QSO + C IV QSO + GRB data (Cao + BR PRD107, 103521 (2023)).  
No DESI data.

BAO + H(z) + SN-Pantheon+ are most restrictive.

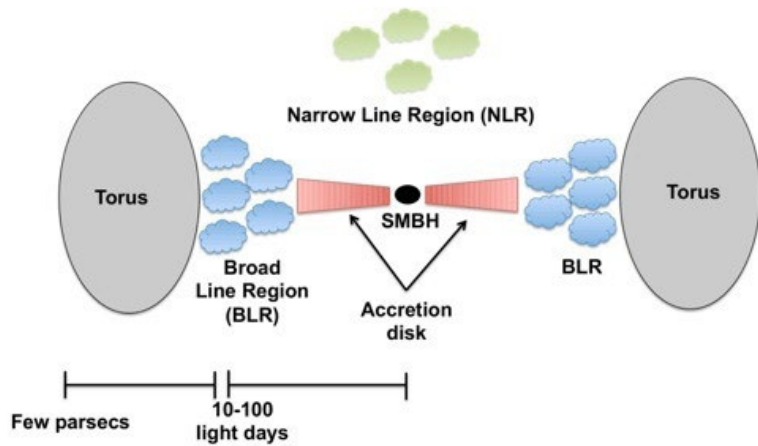
These data give mutually consistent constraints, so can be used jointly to constrain parameters.

Do not include  $L_x$ - $L_{UV}$  QSOs (Lusso+ A&A642, A150 (2020)) which are not standard candles (Khadka + BR MNRAS510, 2753 (2022)), nor R-L  $H\beta$  QSOs (Khadka+ MNRAS513, 1985 (2022)), nor H II starburst galaxies (Cao + BR PRD109, 123527 (2024)), for the same reason.

Consistent with flat geometry, deviating by only  $0.38$  --  $1.2 \sigma$ .

Dark energy dynamics is mildly favored in all cases at  $1.8$  --  $2.3 \sigma$ .

# Reverberation mapped QSOs as potential standard candles



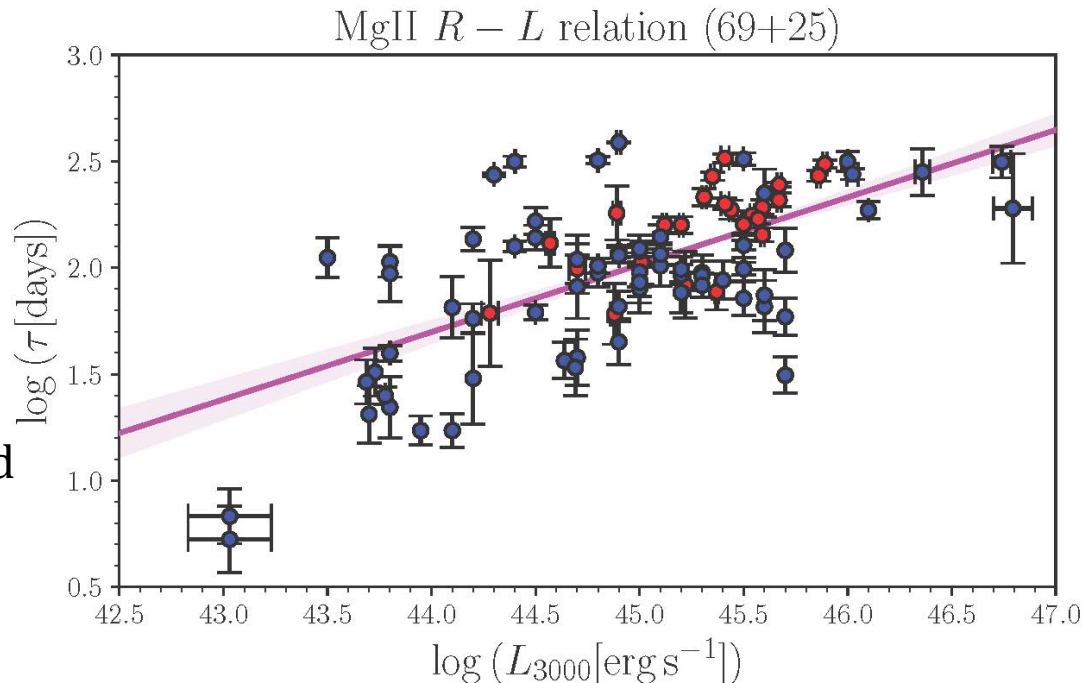
Mg II 1350 Å  
C IV 3000 Å

time-lag vs absolute luminosity:  
 $\log \tau = \beta + \gamma \log L$

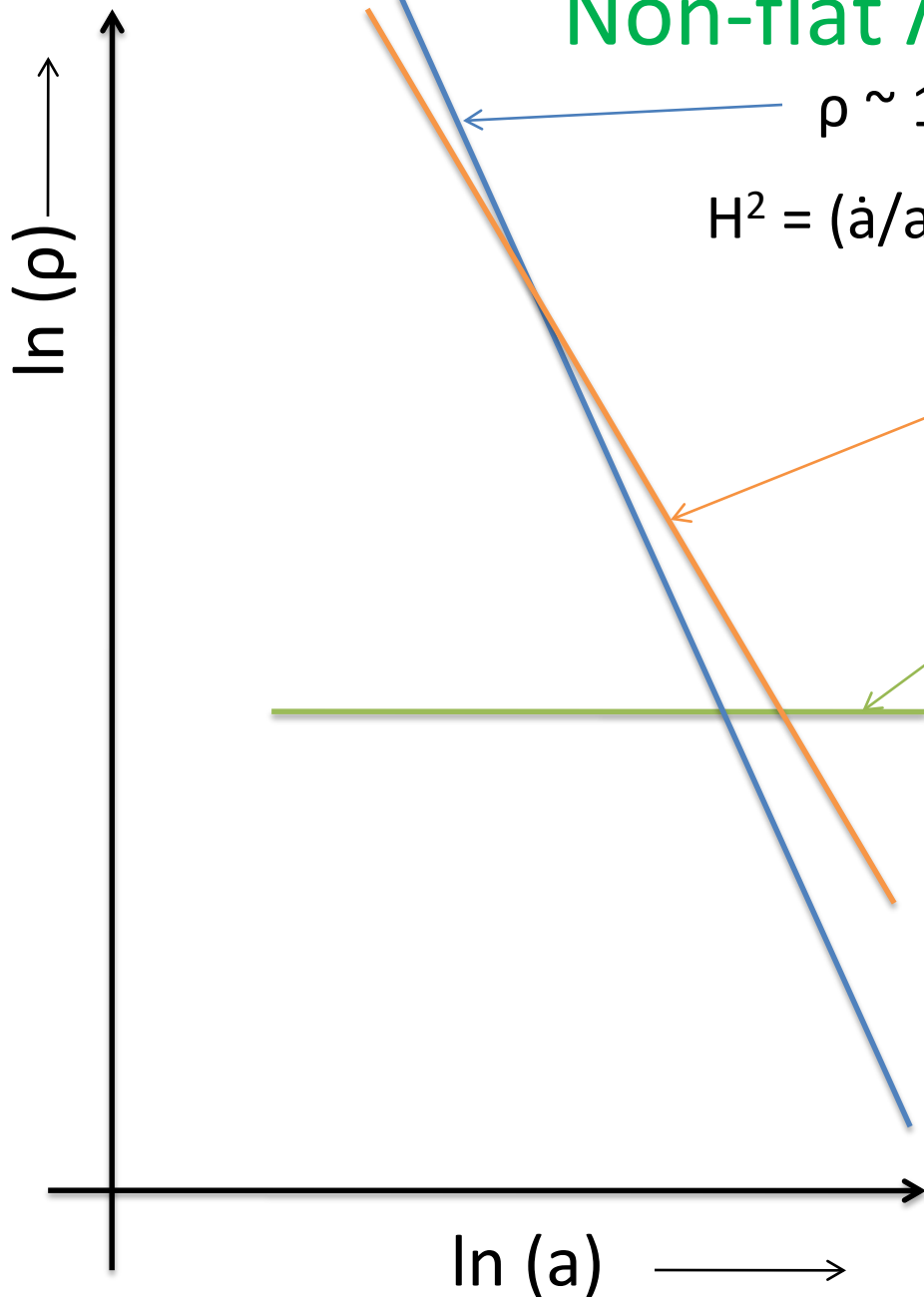
Cao+ MNRAS528, 6444 (2024)

[https://www.isdc.unige.ch/~ricci/Website/Active\\_Galactic\\_Nuclei.html](https://www.isdc.unige.ch/~ricci/Website/Active_Galactic_Nuclei.html)

Simultaneously determine  $\beta$ ,  $\gamma$  & cosmological parameters from  $\tau$  & flux in multiple cosmological models. If  $\beta$  &  $\gamma$  are independent of cosmological model then QSOs are standard candles. This is the case in some  $O(50)$  QSO compilations now, and might remain true with better data. Rubin LSST should get  $10^{4-5}$  such QSOs to  $z = 4$  or higher.



# Non-flat $\Lambda$ CDM & CMB anisotropy



Cannot use HPYZ  $P(k)$ , must use open and closed inflation model  $P(k)$  (BR + Peebles PRD52, 1837 (1995), BR PRD96, 103534 (2017), PRD106, 123524 (2022)). Planck 2018 did not verify that the  $P(k)$  they used satisfies this!

In **spatially-flat** case  $P(k) \sim k^n$  where  $n$  is spectral index.

In closed model, eigenvalue of spatial Laplacian =  $-A(A+2)$  where  $A = 2, 3, 4, \dots$  and  $q \sim A + 1$ . (Open is somewhat similar.)

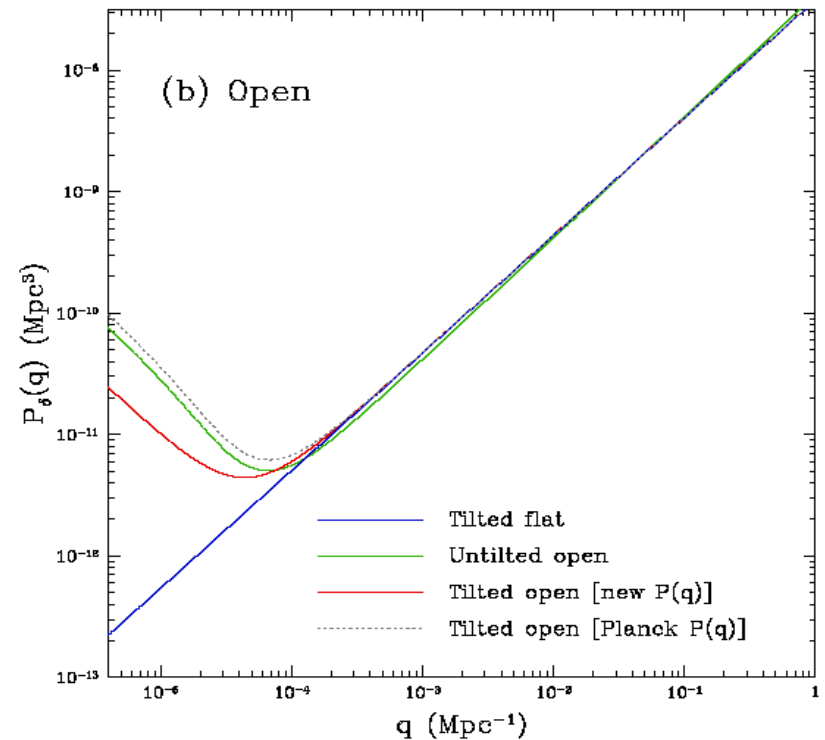
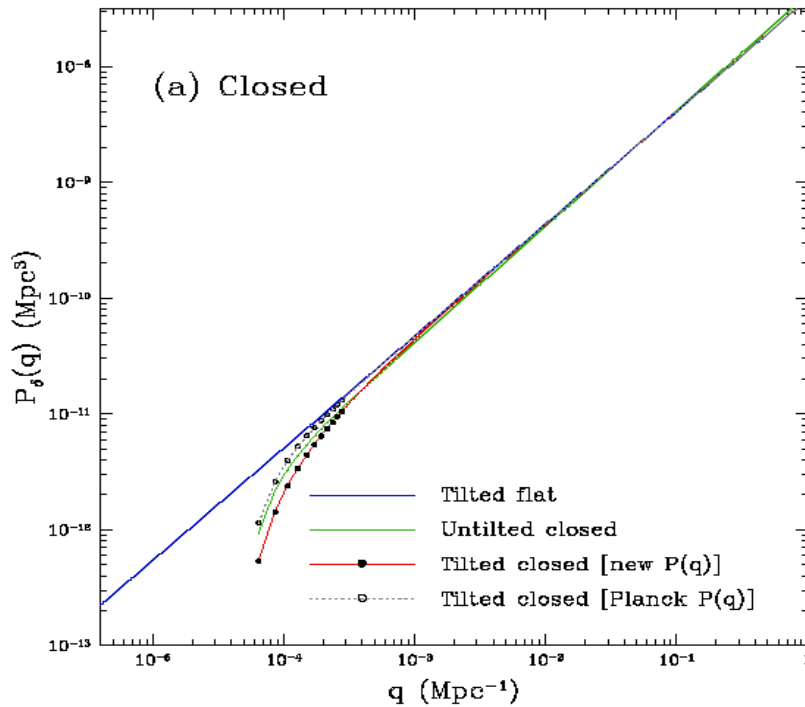
Slow roll inflation gives in **non-flat models** (BR & Peebles PRD52, 1837 (1995), BR PRD96, 103534 (2017))  $P(q) \sim (q^2 - 4K)^2 / [q (q^2 - K)]$  where spatial curvature  $K = -H_0^2 \Omega_{K0}$ . **This was the only known physically consistent  $P(k)$  in a non-flat model. It is un-tilted and is a bad fit to Planck CMB data.**

In the non-flat case Planck 2018 and others have added an **arbitrary tilt prescription** to the **un-tilted non-flat** case, “Planck  $P(q)$ ” :  $P(q) \sim (q^2 - 4K)^2 / [q (q^2 - K)] k^{n-1}$  with  $q^2 = k^2 + K$ . Can find closed inflation models that give  $P(k)$  that are numerically similar to this (Guth, Namjoo + BR, in preparation).

For “Planck  $P(q)$ ”, P18 data:  $\Omega_{K0} = -0.04$  at  $2.5\sigma$  and P18 + lensing:  $\Omega_{K0} = -0.01$  at  $1.6\sigma$ .

Can also find non-flat inflation models that give “new  $P(q)$ ” that differs from what Planck 2018 assumed (BR PRD106, 123524 (2022)).

Inverse powers of  $\sinh(c\varphi)$  and  $\cosh(c\varphi)$  inflaton potential energy densities in open and closed models.  $\Omega_{k0} = \pm 0.0103$  and other parameters from P18+lensing Planck  $P(q)$  analysis (de Cruz+ PRD107, 063522 (2023); PRD110, 023506 (2024)).



Data: P18 = TT, TE, EE + low E

No DESI data.

(P18) **lensing** = lensing potential power spectrum

**Non-CMB** = BAO (16, including  $4 f\sigma_8$ ) +  $f\sigma_8$  (9)  
+ SNIa (1590 Pantheon+) +  $H(z)$  (32)

Models (twelve, 6  $\Lambda$ CDM, 6 XCDM):

Flat tilted  $P(k) \sim k^n$

Non-flat tilted Planck  $P(q)$

Non-flat tilted new  $P(q)$

without and with lensing consistency parameter  $A_L$  as  
there is degeneracy with  $\Omega_{K0}$  (di Valentino+ NatAst4, 196 (2019)).

# $\Lambda$ CDM

$A_L = 1$  no inconsistency

P18 vs  
non-CMB

P18+lens  
vs non-CMB

All data  
 $\Omega_{K0}$

Handley (+Lemos) PRD103,  
L041301 (2021)  
Suspiciousness gaussian  
approximation, qualitatively  
consistent with Joudaki et al.  
MNRAS465, 2033 (2017) DIC  
statistic.

Flat  $P(k) \sim k^n$

$1.2\sigma$

$1.2\sigma$

.....

Non-flat Planck  $P(q)$

$2.7\sigma$

$1.6\sigma$

$0.0009 \pm 0.0017$  Flat

Non-flat new  $P(q)$

$2.3\sigma$

$1.5\sigma$

$0.0008 \pm 0.0017$  Flat

$A_L \neq 1$  consistency

P18 vs  
non-CMB

All data  
 $\Omega_{K0}$

All data  
 $A_L$

Flat  $P(k) \sim k^n$

$0.16\sigma$

.....

$1.087 \pm 0.035$   $2.5\sigma$

Non-flat Planck  $P(q)$

$0.60\sigma$

$0.0004 \pm 0.0017$

$1.084 \pm 0.035$   $2.4\sigma$

Non-flat new  $P(q)$

$0.29\sigma$

$0.0004 \pm 0.0017$

$1.084 \pm 0.034$   $2.5\sigma$

Both flat

Consistent with flat geometry, but Planck PR3 (not ACT or SPT with bigger errors or Planck PR4) wants more lensing than standard  $\Lambda$ CDM predicts.

# XCDM

$A_L = 1$  inconsistencies

P18 vs  
non-CMB

P18+lens  
vs non-CMB

Flat $P(k) \sim k^n$	$3.4\sigma$	$3.6\sigma$	← Ruled out
Non-flat Planck $P(q)$	$4.3\sigma$	$3.4\sigma$	← Ruled out
Non-flat new $P(q)$	$4.0\sigma$	$3.2\sigma$	← Ruled out

$A_L \neq 1$  consistency

P18 vs  
non-CMB

All data  
 $\Omega_{K0}$

All data  
w

All data  
 $A_L$

Flat $P(k) \sim k^n$	$2.1\sigma$	.....	$-0.968 \pm 0.024$ $1.3\sigma$ DDE	$1.101 \pm 0.037$ $2.7\sigma$
Non-flat Planck $P(q)$	$2.6\sigma$	$0.0015 \pm 0.0019$ $0.79\sigma$ open	$-0.958 \pm 0.026$ $1.6\sigma$ DDE	$1.102 \pm 0.037$ $2.8\sigma$
Non-flat new $P(q)$	$2.7\sigma$	$0.0015 \pm 0.0019$ $0.79\sigma$ open mildly non-flat	$-0.959 \pm 0.027$ $1.5\sigma$ DDE moderate DDE	$1.101 \pm 0.038$ $2.7\sigma$

Consistent with flat geometry, mild DDE, but Planck PR3 wants more lensing than best-fit XCDM predicts. Need to better understand this CMB weak lensing issue.

# DESI 2024 and flat $w_0w_a$ CDM cosmological parametrization

(Park+ PRD110, 123533 (2024))

Use our previous (non-DESI) data to constrain

$p(z)/\rho(z) = w(z) = w_0 + w_a z/(1+z)$  parametrization. (Chevallier & Polarski IJMPD10, 213 (2001), Linder PRL90, 091301 (2003))

P18 vs non-CMB  $2.8\sigma$

P18+lens vs non-CMB  $2.7\sigma$

$w(z=0) = w_0$ ,  $w(z=\infty) = w_0 + w_a$  Behaves like constant  $w$ !

$\Lambda$ CDM is  $w_0 = -1$ ,  $w_a = 0$ .

We get

$w_0 = -0.850 \pm 0.059$ ,  $2.5\sigma$  from  $\Lambda$ CDM,

$w_a = -0.59^{+0.26}_{-0.22}$ ,

$w_0 + w_a = -1.44^{+0.20}_{-0.17}$ ,  $2.2\sigma$  from  $\Lambda$ CDM.

These are  $-0.27\sigma$  and  $0.44\sigma$  from DESI+CMB+PantheonPlus results,

$w_0 = -0.827 \pm 0.063$ ,  $w_a = -0.75^{+0.29}_{-0.25}$

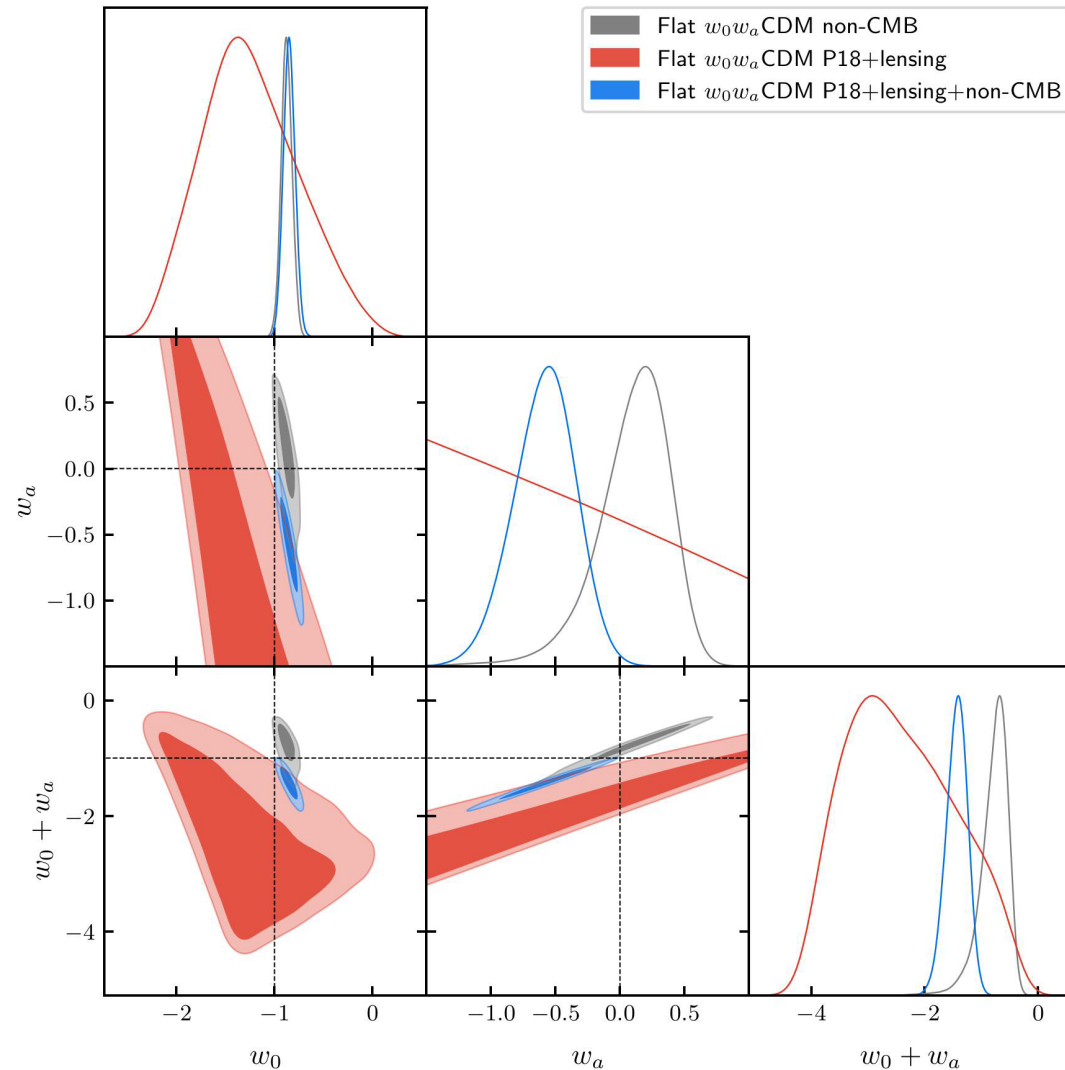
(Adame+ 2404.03002) and somewhat more

constraining and favor flat  $w_0w_a$ CDM

somewhat more than do DESI 2024

data.

In addition to not depending on DESI data, this  $\sim 2\sigma$  support for dark energy dynamics does not depend on SNIa data.



# DESI 2024 and flat $w_0w_a$ CDM+ $A_L$ cosmological parametrization

(Park+ 2410.13627, 2501.03480)

Use our previous (non-DESI) data to constrain

P18 vs non-CMB  $1.9\sigma$

P18+lens vs non-CMB  $2.1\sigma$

$p(z)/\rho(z) = w(z) = w_0 + w_a z/(1+z) + A_L$  parametrization.

$w(z=0) = w_0$ ,  $w(z=\infty) = w_0 + w_a$  Behaves like constant  $w$ !

$\Lambda$ CDM is  $w_0 = -1$ ,  $w_a = 0$ .

We get

$w_0 = -0.879 \pm 0.060$ ,  $2\sigma$  from  $\Lambda$ CDM,

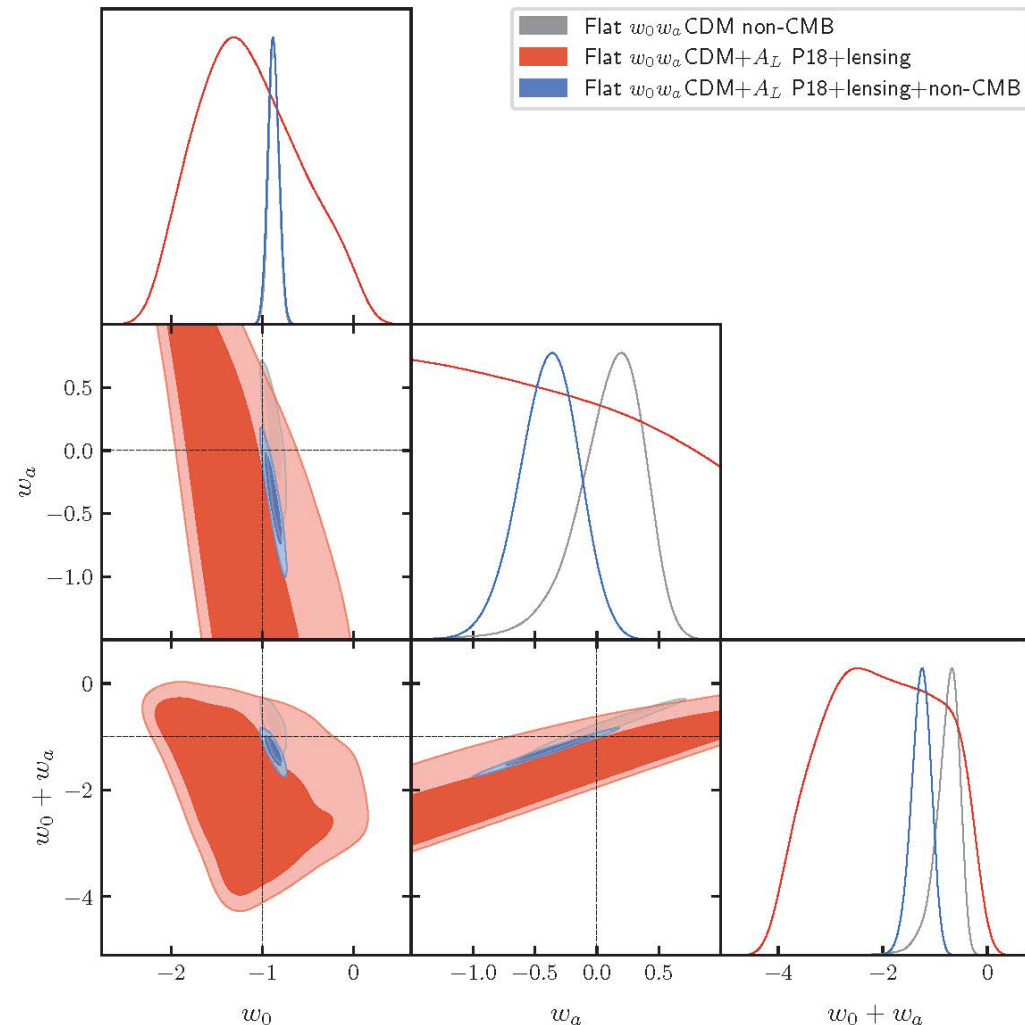
$w_a = -0.39^{+0.26}_{-0.22}$ ,

$w_0 + w_a = -1.27^{+0.20}_{-0.17}$ ,  $1.35\sigma$  from  $\Lambda$ CDM

$A_L = 1.078^{+0.036}_{-0.040}$ ,  $2.2\sigma$  from  $A_L = 1$ .

This is more consistent with flat  $\Lambda$ CDM  $w_0 = -1$  and  $w_a = 0$ , deviating by  $\sim 1.5\sigma$ , but wants more smoothing of CMB than is predicted by flat  $\Lambda$ CDM/ $w_0w_a$ CDM model

Is flat  $w_0w_a$ CDM evidence for dark energy dynamics partially caused by the excess smoothing of Planck PR3 CMB anisotropy data?



# Physically-consistent $\phi$ CDM model (Park+ 2509.25812)

(no phantom crossing)

No DESI data.

(w/XCDM,  $w_0w_a$ CDM are not physically-consistent DDE parameterizations)

Use our previous data compilation to get

$\alpha = 0.055 \pm 0.041$  (quintessence-like  $1.3\sigma$  from  $\Lambda$ CDM) for  $\phi$ CDM

and

$\alpha = 0.095 \pm 0.056$  (quintessence-like  $1.7\sigma$  from  $\Lambda$ CDM)

&  $A_L = 1.105 \pm 0.037$  ( $2.8\sigma$  from  $A_L = 1$ ) for  $\phi$ CDM+ $A_L$

Behavior differs from what happens in physically-incomplete  $w_0w_a$ CDM case.

Current data cannot rule out mild dark energy dynamics but Planck PR3 wants more lensing than best-fit  $\phi$ CDM model predicts.

# Open Questions, Missing Links

B.R. & M. Vogele, PASP120,235 (2008)

## What is dark energy?

Is it a cosmological constant, or does it vary with space and in time?

Current data cannot rule out mild dark energy dynamics; should use physical DDE models.

Is the general theory of relativity correct on large scales? Probably is.

Are the astronomy observations for dark energy secure? Probably are.

BAO data from DESI and Euclid will be important in the next five years.

Is it really decoupled (except gravitationally) from everything else?

## Is the universe closed?

It is probably flat.

Need to better understand non-flat inflation models.

## Is $A_L > 1$ a problem for standard $\Lambda$ CDM or Planck PR3?

## What is dark matter?

Supersymmetry? Axions?

Large Hadron Collider at CERN and (deep underground) laboratory searches for dark matter will be important in the next five years.

Dwarf galaxy abundances, galactic nuclear profiles might be problems for “pure” CDM<sub>49</sub>

What are the masses of neutrinos?

Are the constraints on baryon density consistent?

When and how was the baryon excess generated?

What is the topology of space?

What are the initial seeds for structure formation?

Did the early universe inflate and reheat?

When, how, and what were the first structures formed?

How do baryons light up galaxies and what is their connection to mass?

How do galaxies and black holes co-evolve?

Does the Gaussian, adiabatic CDM structure formation model have a real flaw?

Is the low quadrupole moment of the CMB anisotropy a problem for flat  $\Lambda$ CDM?

Are the largest observed structures a problem for flat  $\Lambda$ CDM?

Is there a cosmological magnetic field and what effects does it have?

...when you have eliminated the impossible,  
whatever remains, however improbable, must  
be the truth.



Sherlock Holmes (Arthur Conan Ignatius Doyle)