

# Final state interactions and photon-induced charm production



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# Core message

- Near-threshold  $D_{(s)}^{(*)}\bar{D}_{(s)}^{(*)}$  production is strongly shaped by coupled-channel final-state interactions.
- FSI produces a characteristic charged/neutral pattern and changes line shapes and expected meson yields;
- Photon-induced processes provide clean environments to probe molecular charm dynamics
- FSI can describe the dynamical generation of meson molecules

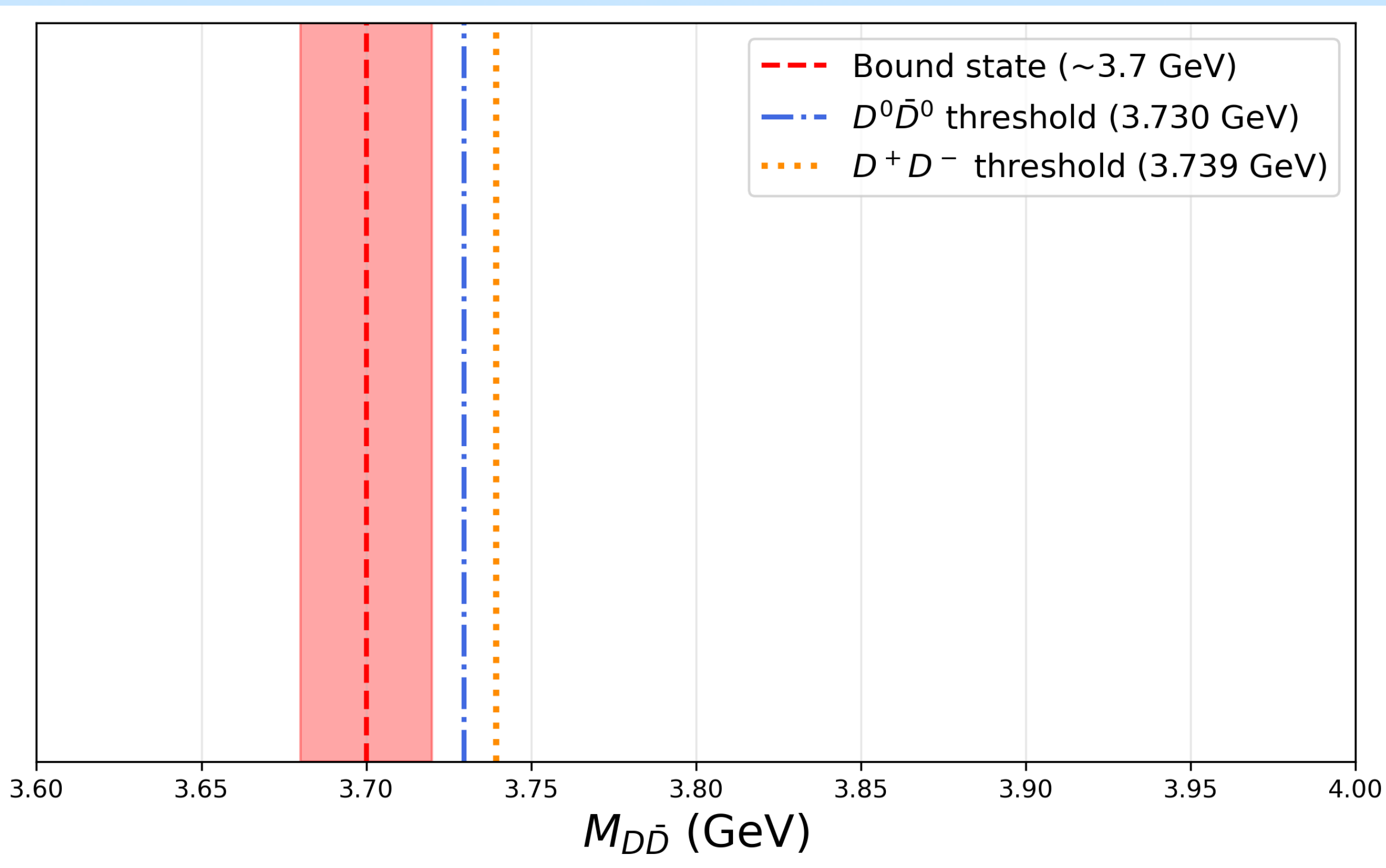
$$\gamma\gamma \rightarrow D_{(s)}^{(*)}\bar{D}_{(s)}^{(*)} + D_{(s)}^{(*)}\bar{D}_{(s)}^{(*)} \text{ FSI} \longrightarrow d\sigma/dW, d\sigma/dY, N_{events}, \dots$$

# Near-threshold charm dynamics

Gamermann, et al. Phys.Rev.D 76 (2007) 074016

Prelovsek, et al. JHEP 06 (2021) 035

Nieves, Pavon Valderrama. Phys.Rev.D 86 (2012) 056004



- Many charmonium-like states lie close to open-charm thresholds.
- A scalar  $D\bar{D}$  molecule candidate, often associated with  $X(3700)$ , appears in coupled-channel approaches.
- Production observables can reveal how this dynamics affects line shapes and yields.

# Why $\gamma\gamma$ production?

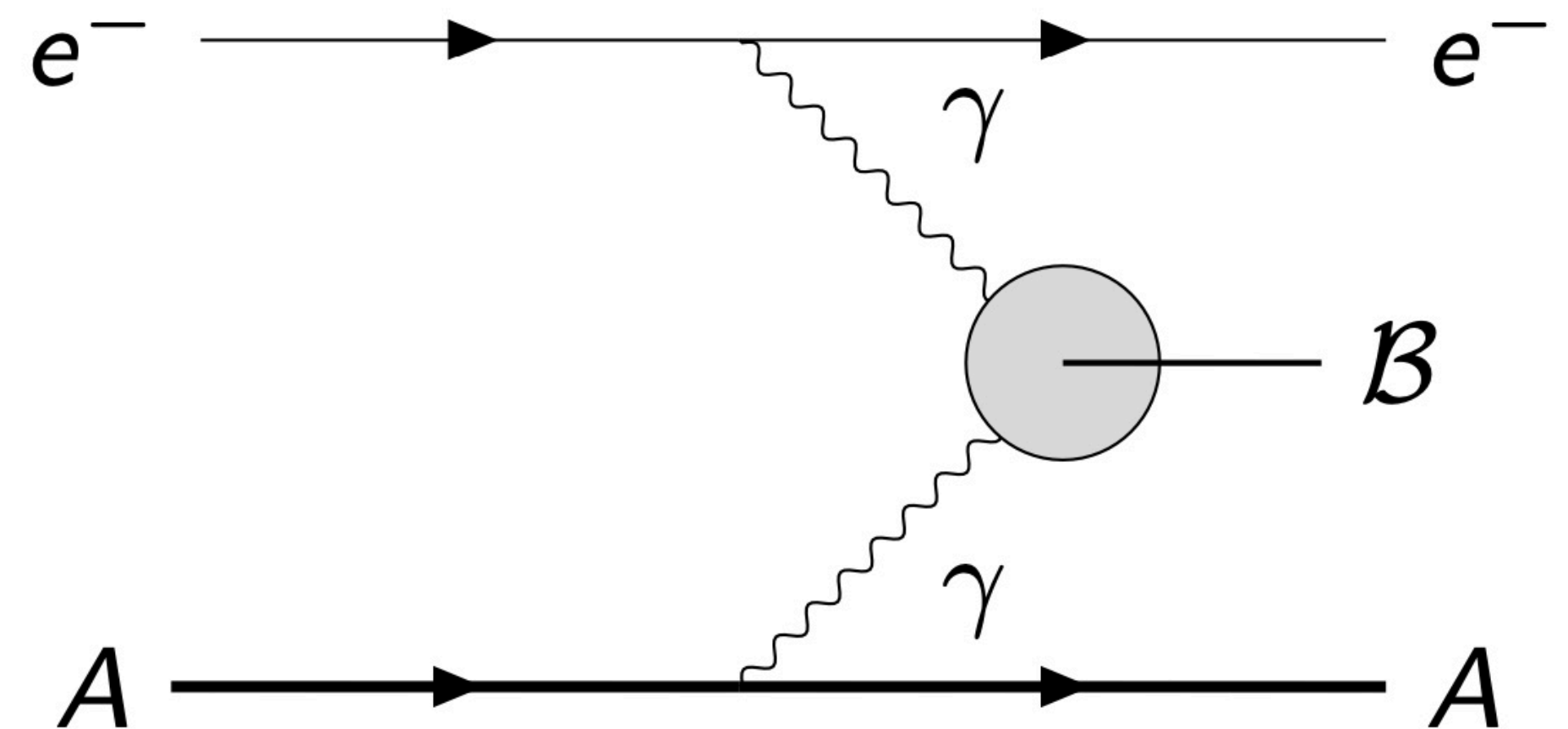
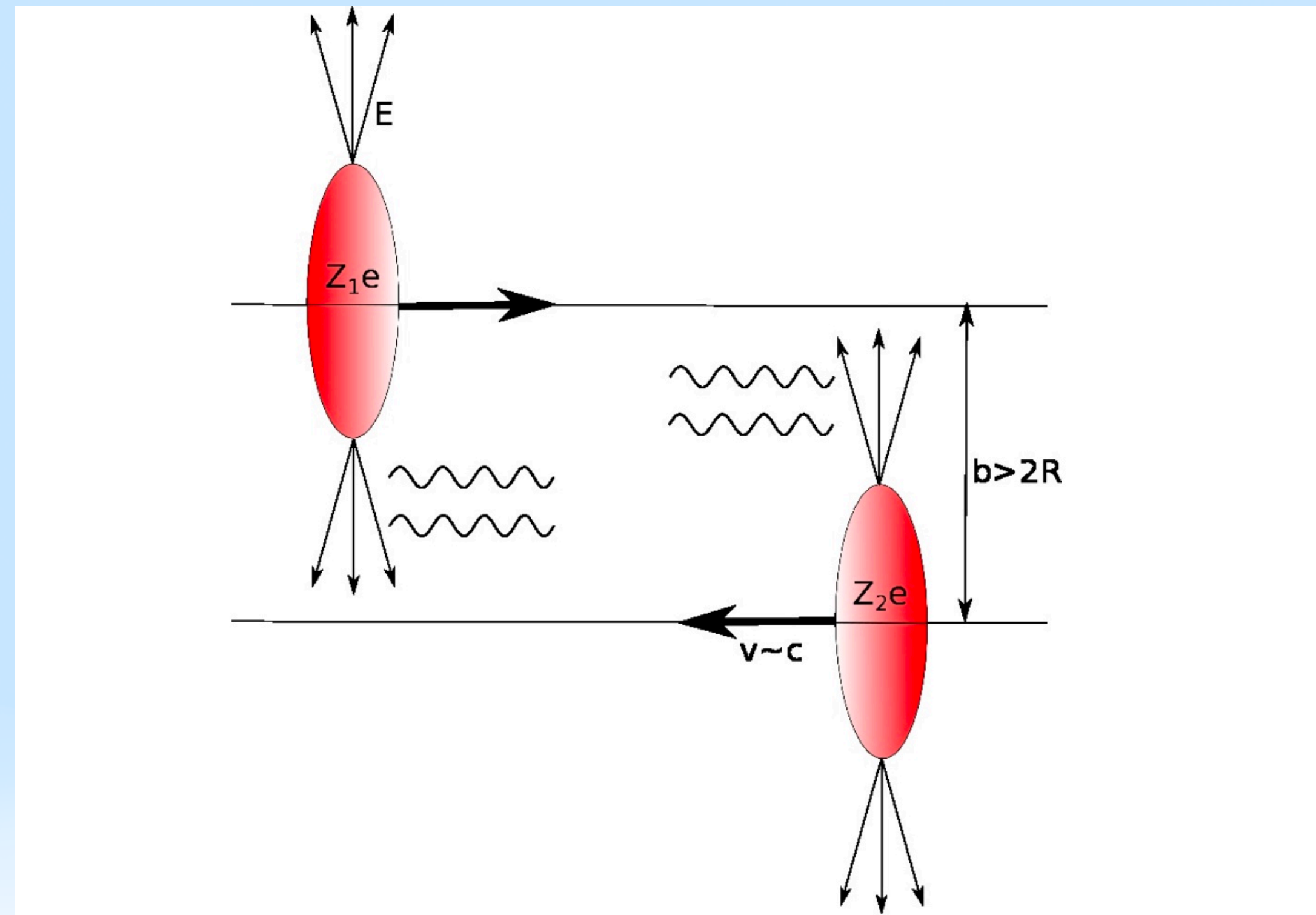
$$A \rightarrow A + \gamma, \quad e/A \rightarrow e/A + \gamma$$

$$\gamma\gamma \rightarrow D\bar{D}$$

- Electromagnetic initial state
- Relatively clean background
- Same subprocess can be studied in UPC and EIC environments

**UPC:** No hadronic overlap, rapidity gaps

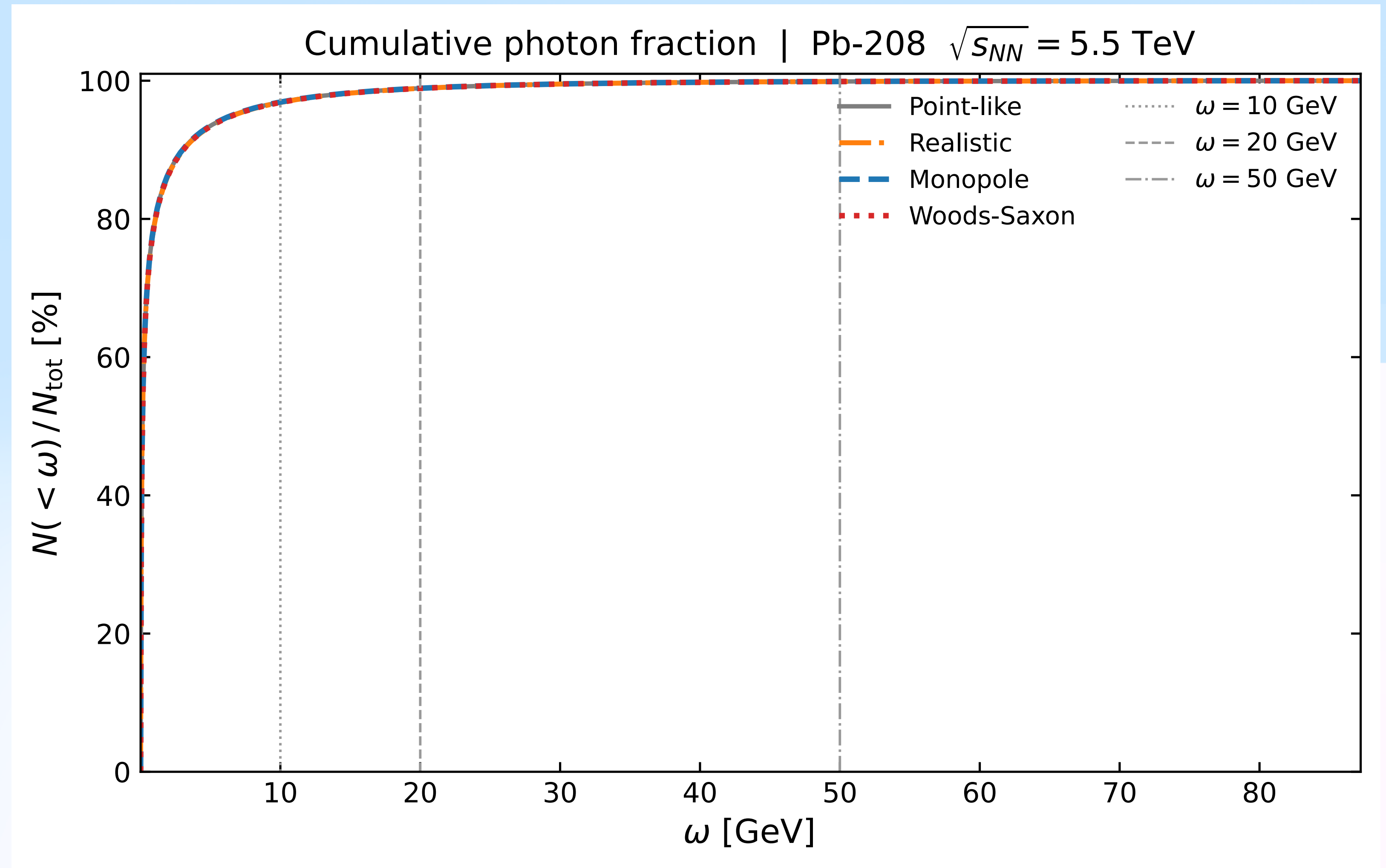
**EIC:** No Pomeron or hadronic contribution from the electron side



# Why $\gamma\gamma$ production?

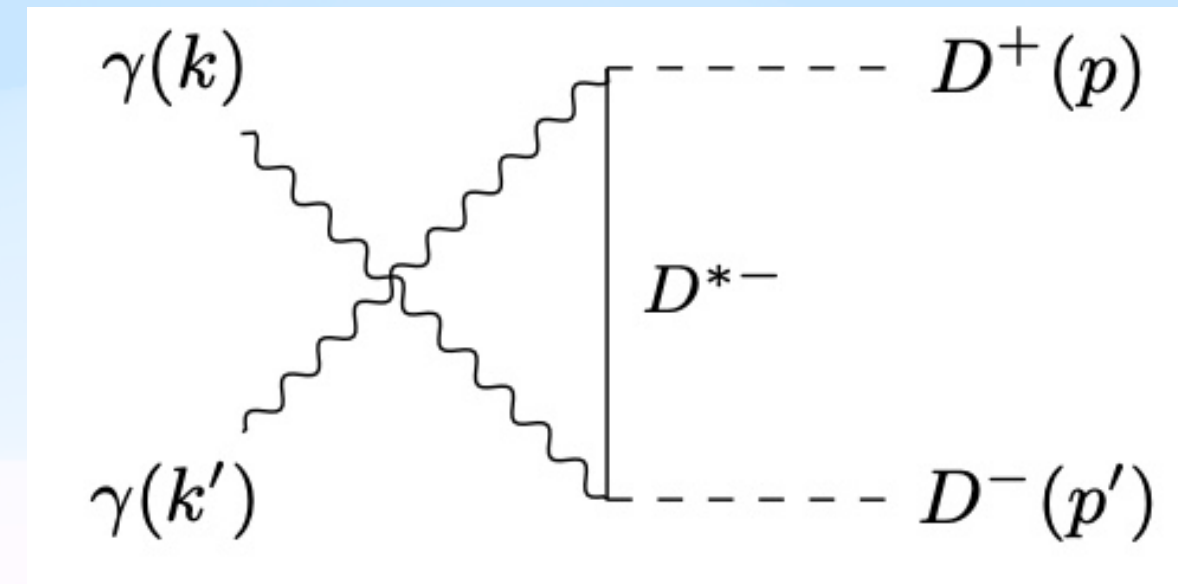
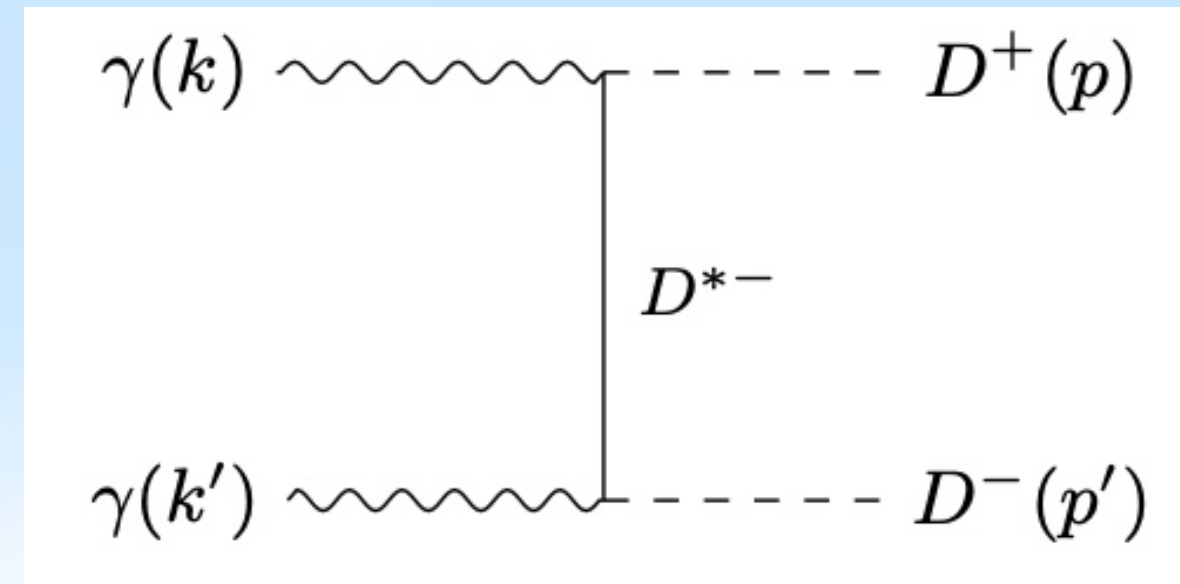
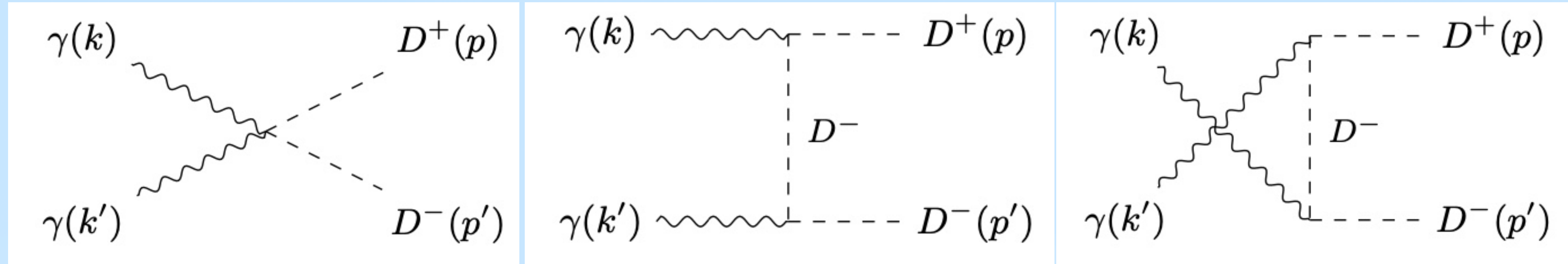
Useful for near  
threshold physics!

$n(\omega)$  is larger at lower  $\omega$

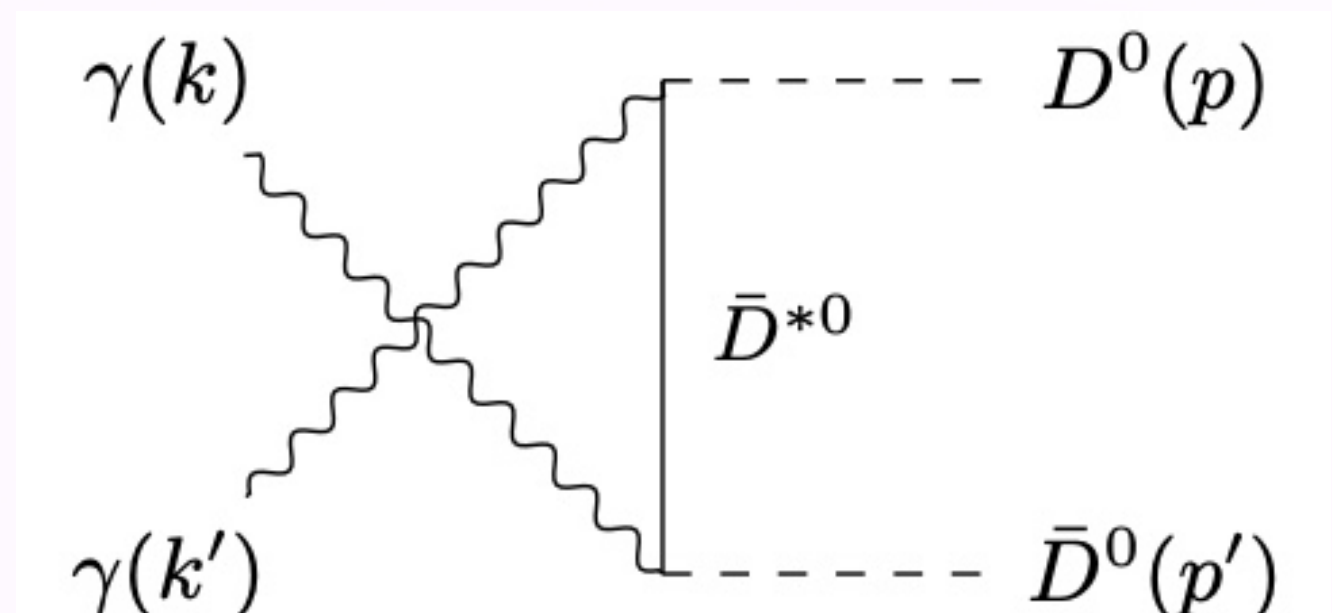
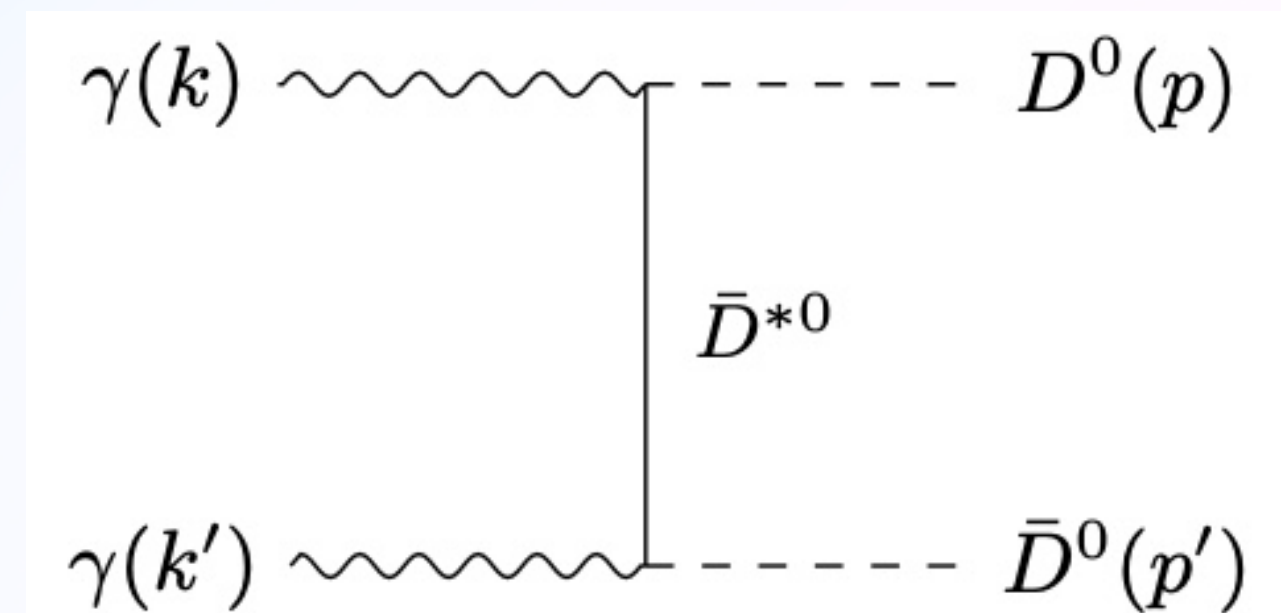


# Bare $\gamma\gamma \rightarrow D\bar{D}$ production

Charged channel:  $\gamma\gamma \rightarrow D^+D^-$   
 Scalar QED +  $D^{*\pm}$  exchange



Neutral channel:  $\gamma\gamma \rightarrow D^0\bar{D}^0$   
 Only  $D^{*0}$  exchange



The two charge channels already have different bare production mechanisms before FSI

**Since photons distinguish charge states, production is computed in the charge basis instead of isospin.**

# Near-threshold S-wave projection

$$|\vec{q}| \rightarrow 0 \text{ near threshold}$$

$l > 0$  suppressed by powers of  $|\vec{q}|^l$

$$\text{S-wave} \rightarrow J^{PC} = 0^{++}$$

Near threshold, we focus on the scalar  $0^{++}$  component.

$$M_0 = \frac{1}{2} \int_{-1}^1 d(\cos \theta) M(s, \cos \theta)$$

$$\hat{\sigma}(\gamma\gamma \rightarrow D\bar{D}) = \frac{\beta_D}{32\pi s} |M_0(s)|^2$$

# Coupled-channel FSI

Oller, Oset, Pelaez. Phys.Rev.D 59 (1999) 074001

$$D^+D^- \leftrightarrow D^0\bar{D}^0$$

$$T(s) = V(s) + V(s)G(s)T(s)$$

$$T(s) = [1 - V(s)G(s)]^{-1}V(s)$$

The produced pair can rescatter before being observed.

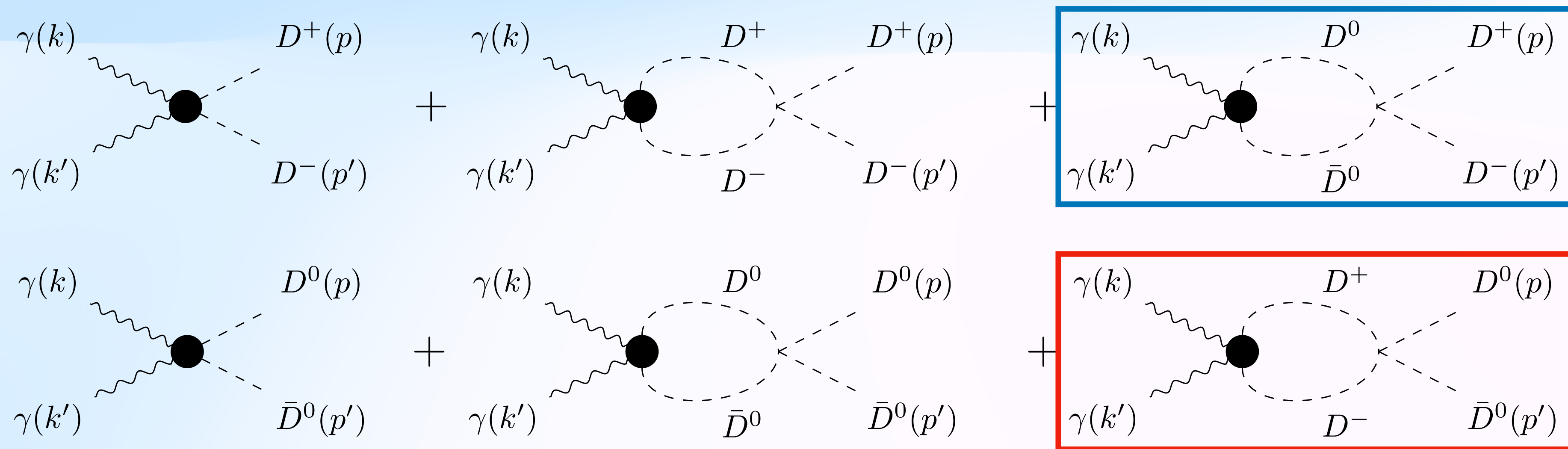
$V$ : Interaction kernel       $G$ : Two meson loop       $T$ : Rescattering amplitude.

# Dressed production amplitude

$$M_i^{\text{dressed}} = M_i^{\text{bare}} + \sum_{j=1}^2 M_j^{\text{bare}} G_j T_{ji}.$$

$$M_{ch}^{\text{dressed}} = M_{ch}^{\text{bare}} + M_{ch}^{\text{bare}} G_{ch} T_{11} + M_n^{\text{bare}} G_n T_{12}$$

$$M_n^{\text{dressed}} = M_n^{\text{bare}} + M_{ch}^{\text{bare}} G_{ch} T_{21} + M_n^{\text{bare}} G_n T_{22}$$



**Our result: Neutral feeds charged; Charged suppresses neutral**

# From $\gamma\gamma$ subprocess to UPC/EIC observables

$$PbPb \text{ UPC: } \gamma_{Pb}\gamma_{Pb} \rightarrow D\bar{D}$$

$$eAu \text{ EIC: } \gamma_e\gamma_{Au} \rightarrow D\bar{D}$$

**Equivalent Photon Approximation:**

$$\sigma = \int d\omega_1 d\omega_2 n_1(\omega_1) n_2(\omega_2) \hat{\sigma}_{\gamma\gamma \rightarrow D\bar{D}}(\omega_1, \omega_2)$$

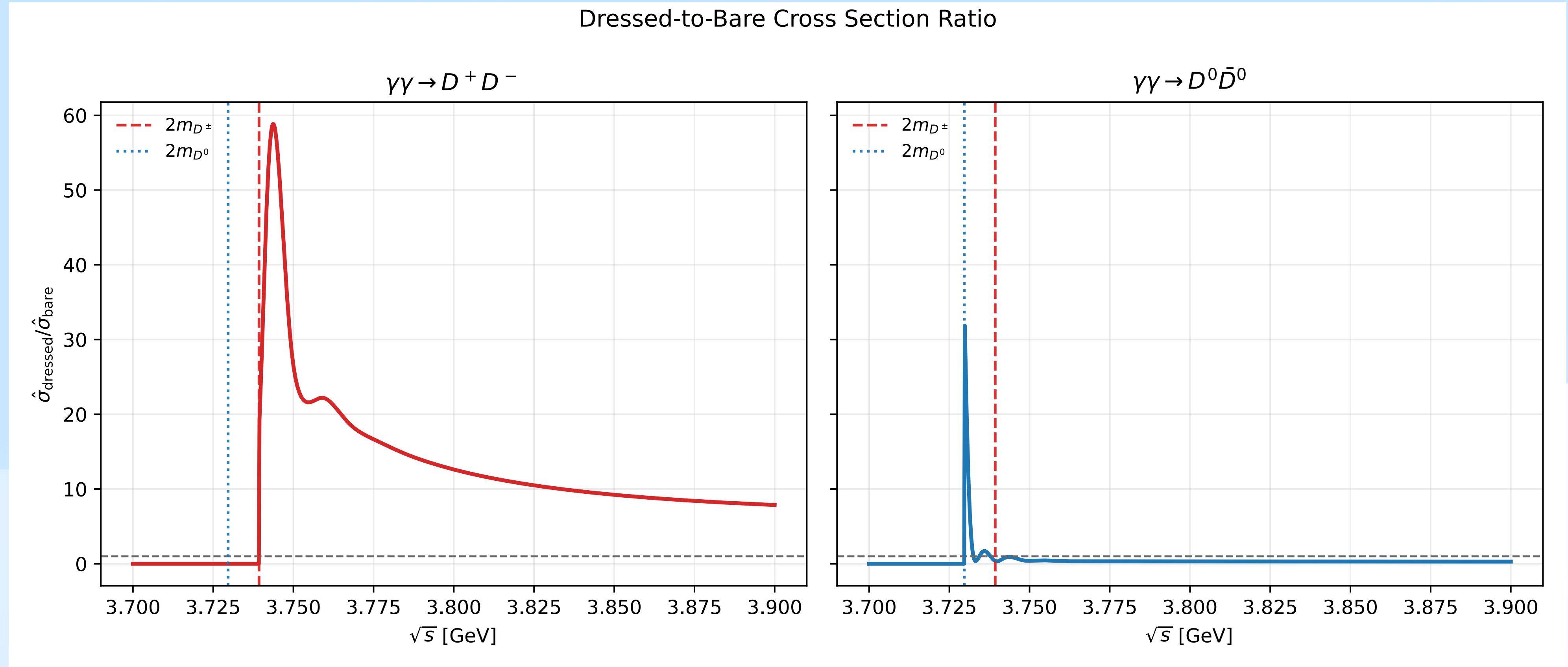
$$\downarrow \quad \omega_{1,2} = \frac{W}{2} e^{\pm Y}$$

$$\frac{d^2\sigma}{dWdY} = \frac{2}{W} f_1(\omega_1) f_2(\omega_2) \hat{\sigma}_{\gamma\gamma \rightarrow D\bar{D}}(W).$$

UPC and EIC: Different photon fluxes, same  $\gamma\gamma \rightarrow D\bar{D}$  subprocess

# Subprocess result: FSI signature

$$R_{FSI} = \frac{\hat{\sigma}_{dressed}}{\hat{\sigma}_{bare}}$$

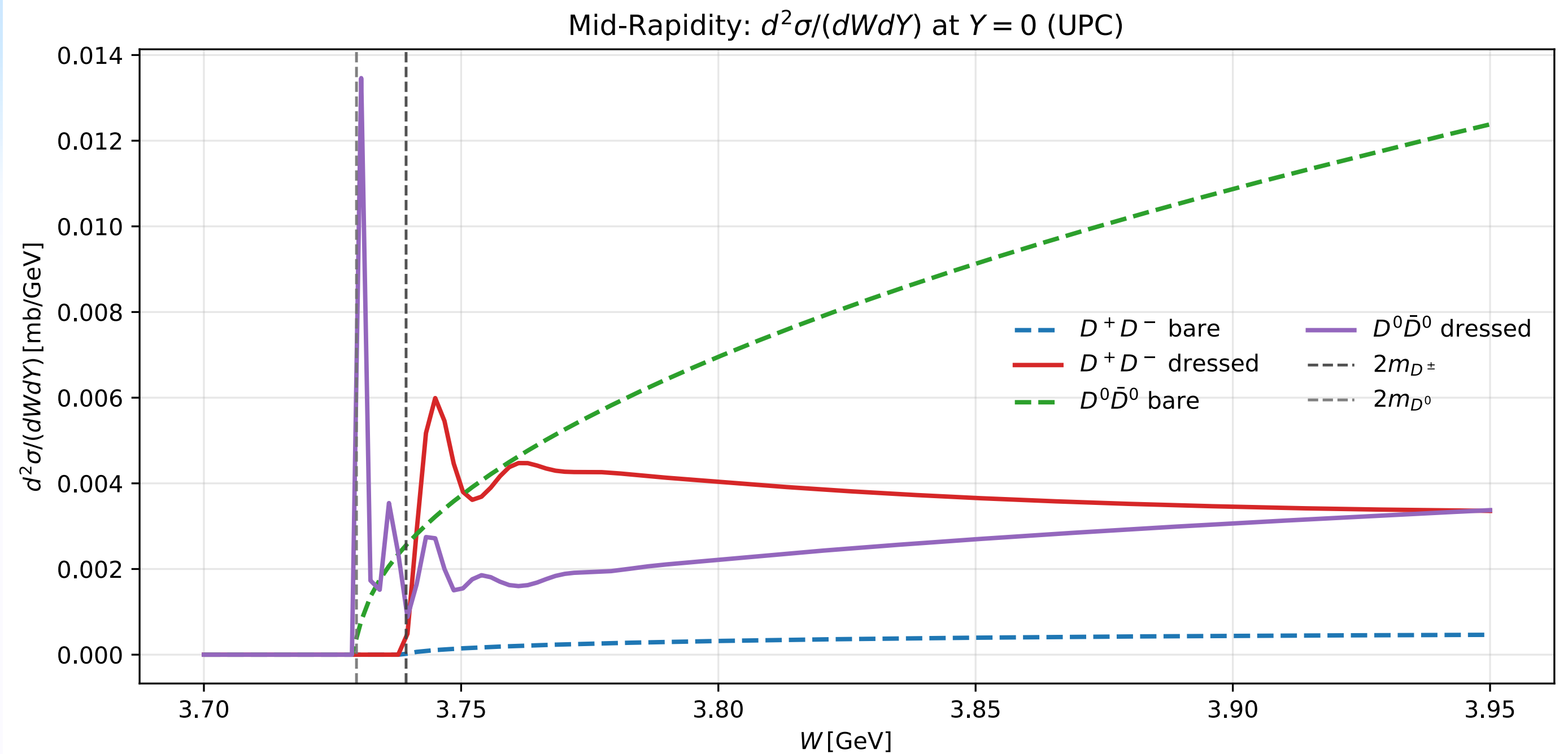
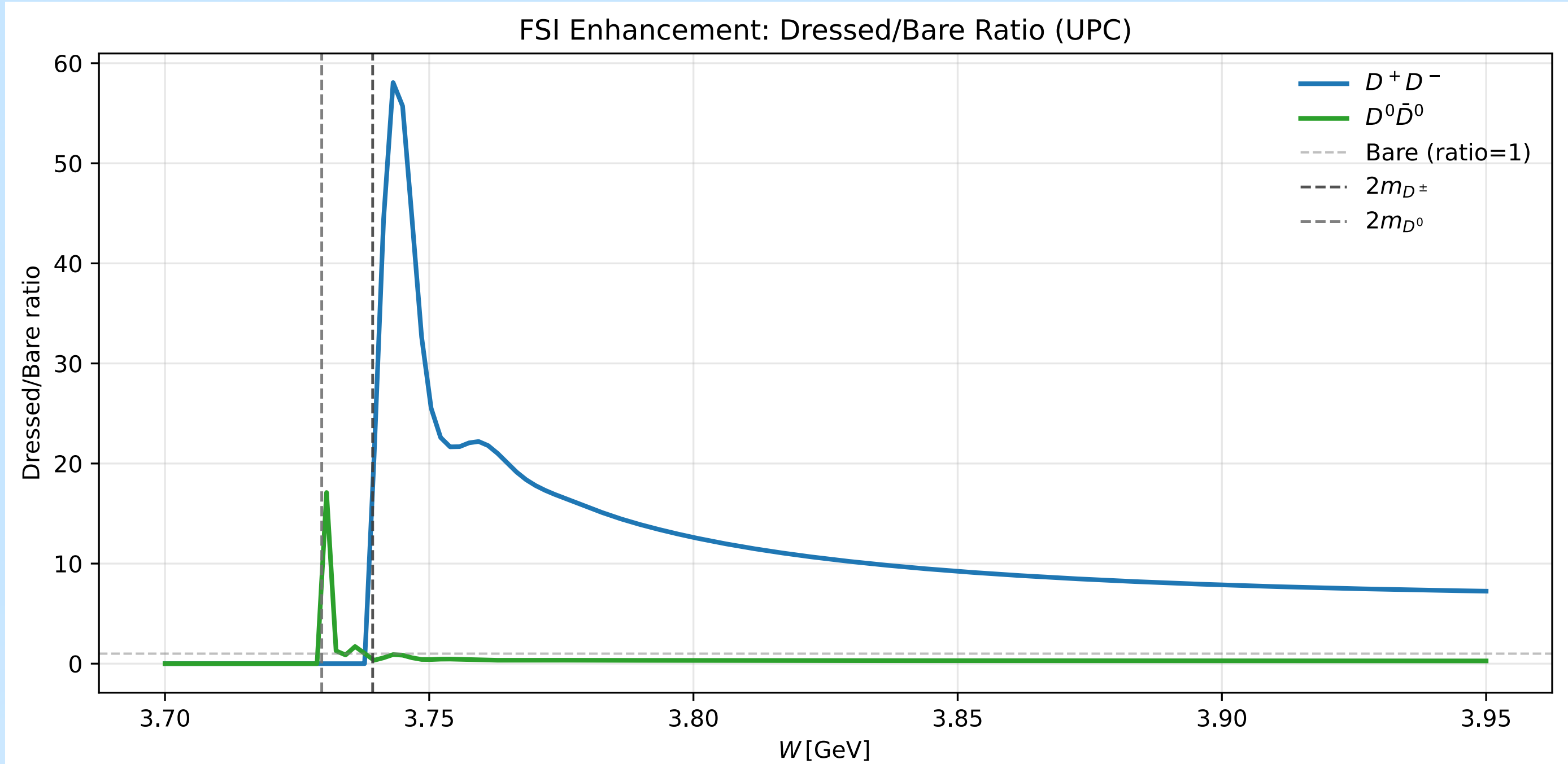


- $D^+ D^-$  enhanced by FSI:  $R_{FSI}^{ch} \sim 10$
- $D^0 \bar{D}^0$  suppressed by FSI:  $R_{FSI}^n \sim 0.3$
- Strongest structure close to the  $D\bar{D}$  thresholds

# UPC observables

The FSI pattern survives after convolution with the UPC photon flux

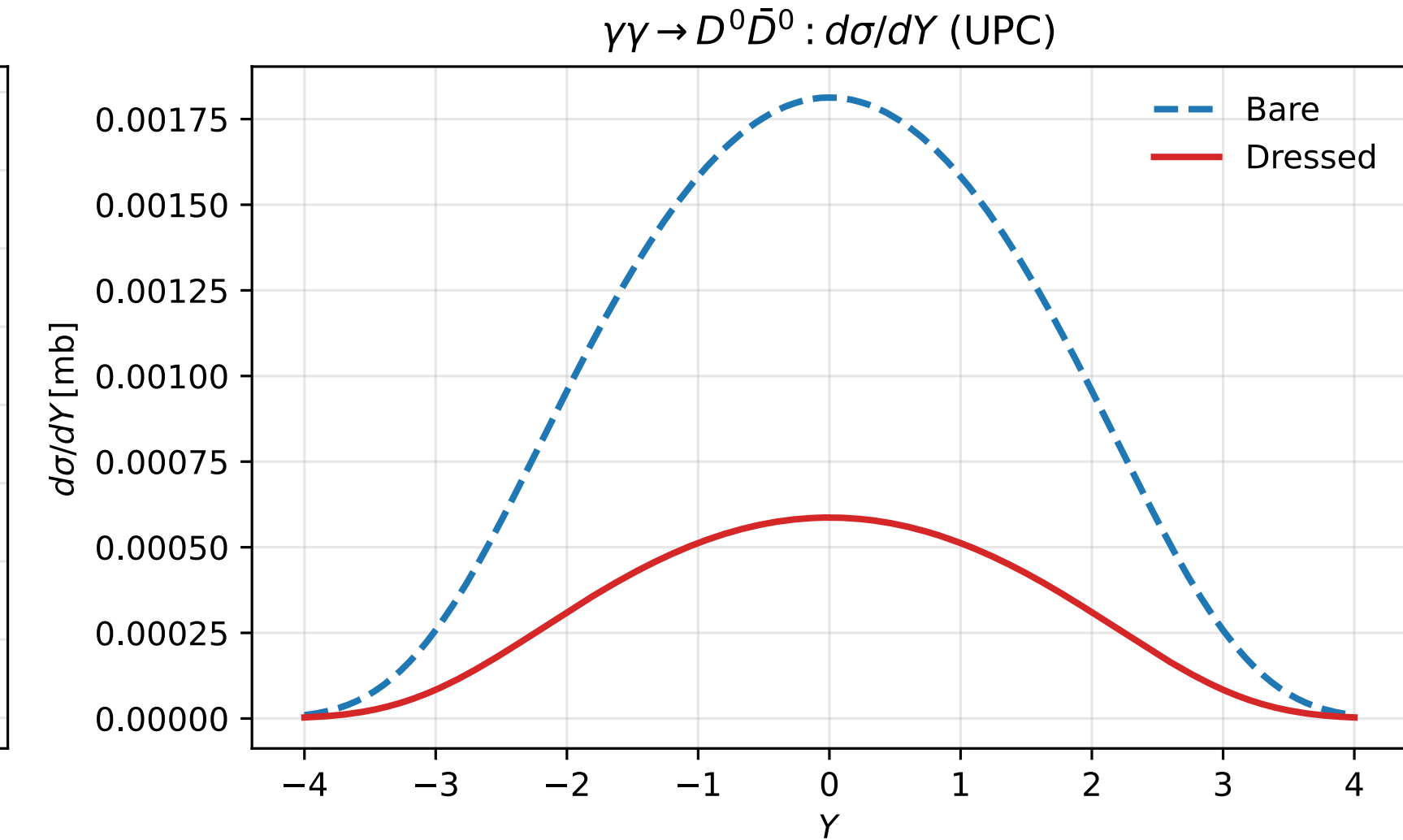
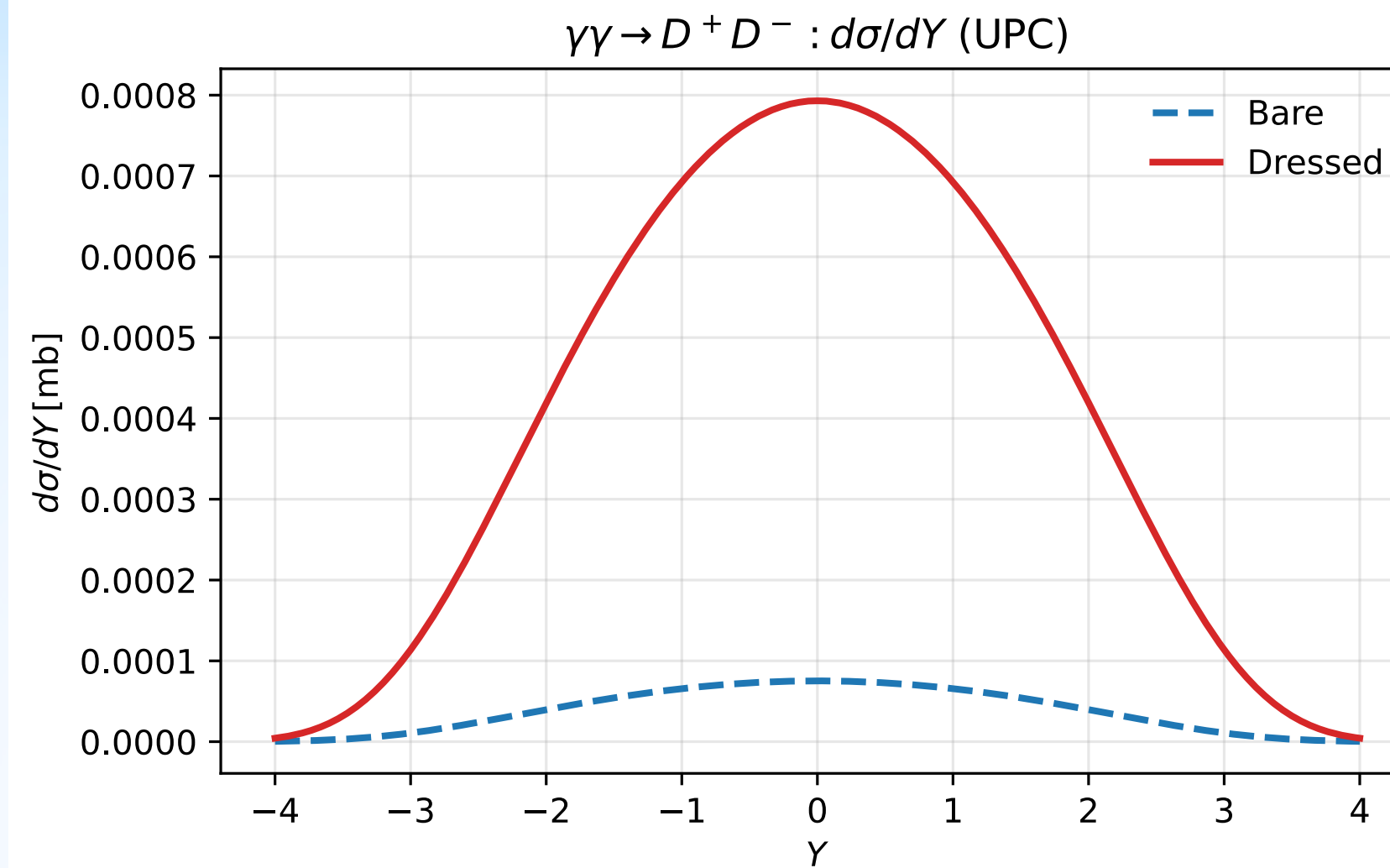
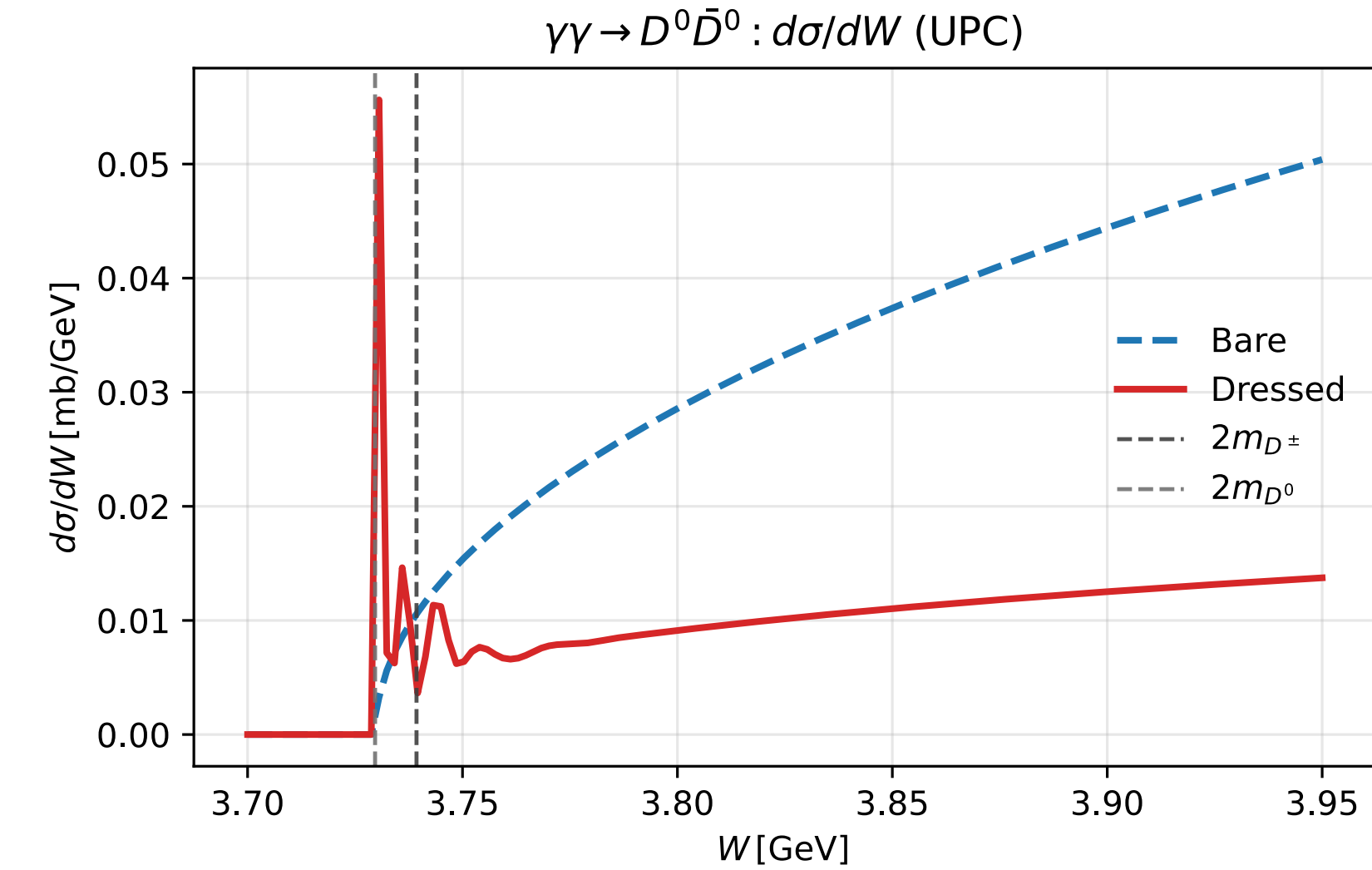
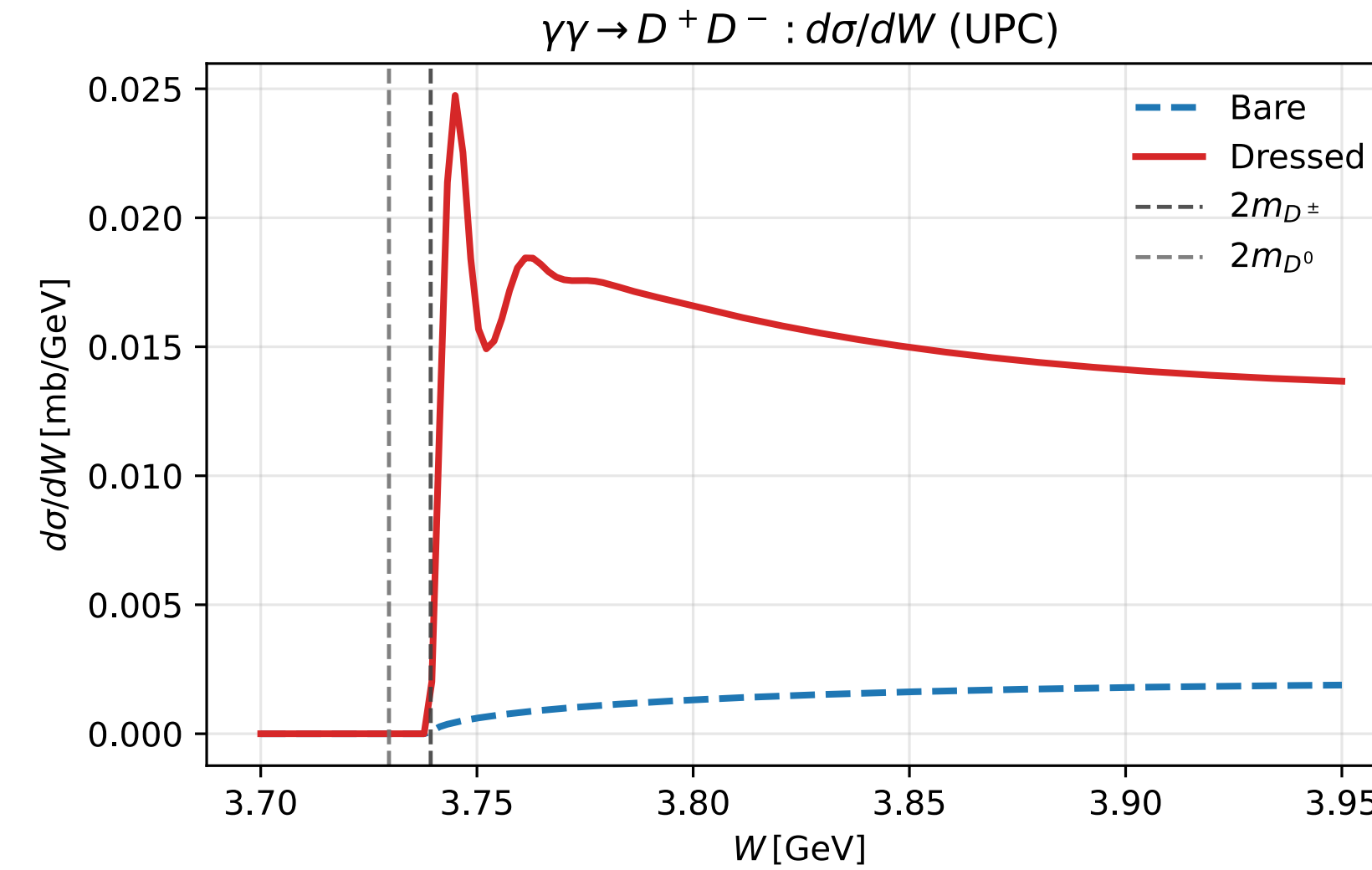
$$R_{FSI}^{UPC} \approx R_{FSI}^{\gamma\gamma}$$



# UPC observables

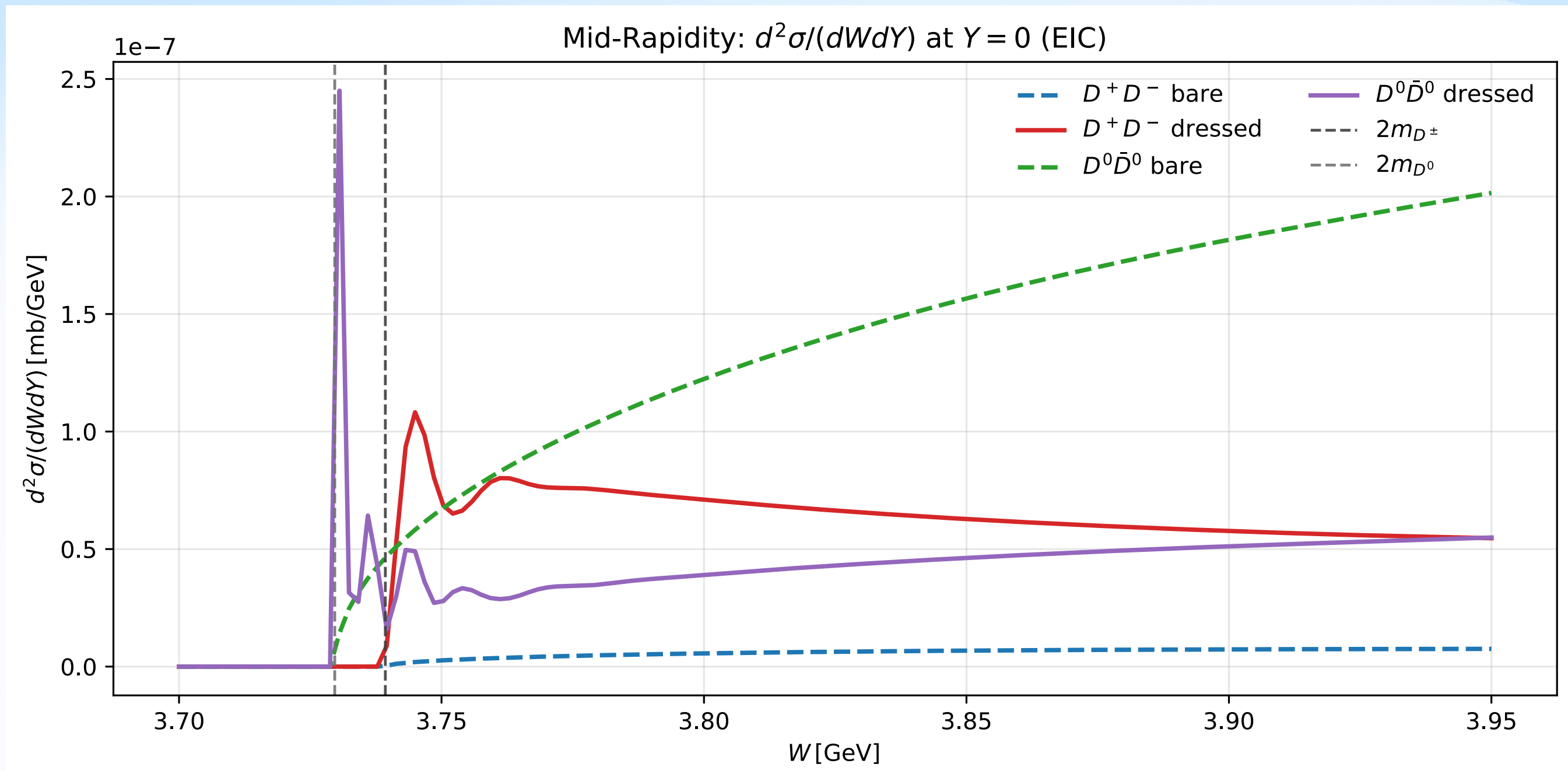
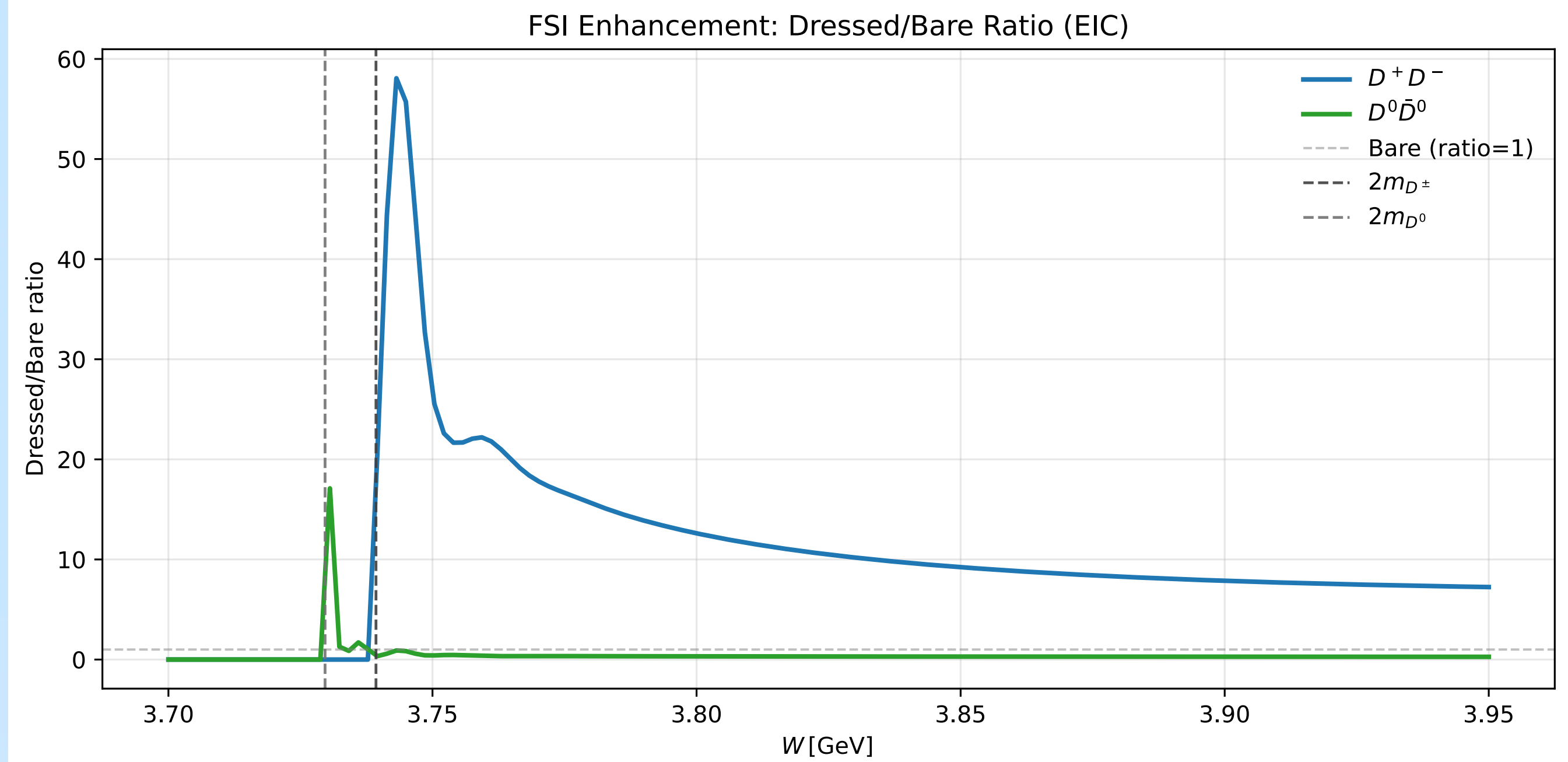
- Symmetric rapidity distribution
- Presence of bound state mostly seen on the  $d\sigma/dW$  line shapes

UPC Observables (Pb-Pb @ LHC, gamma=2750)



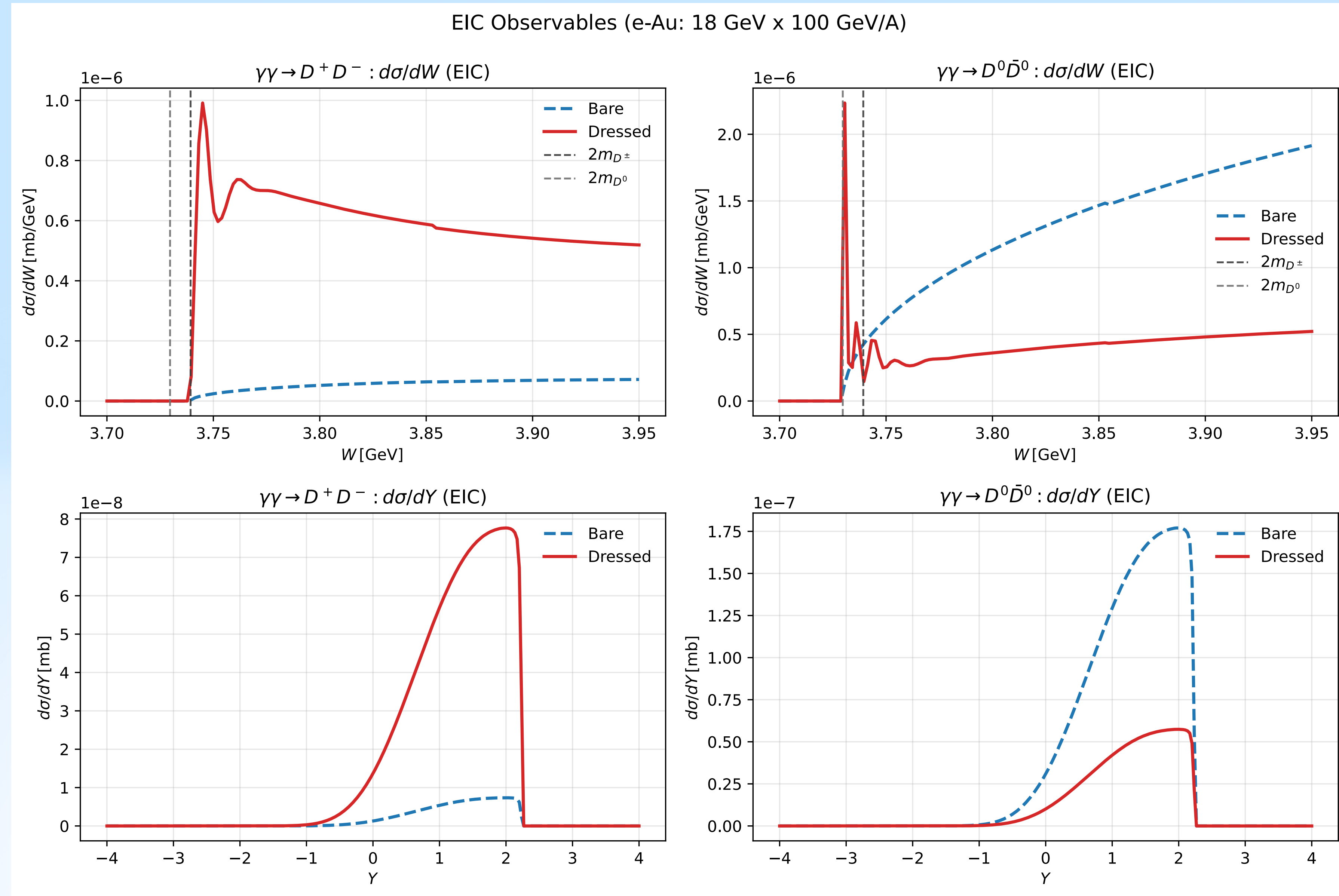
# EIC observables

Similar FSI pattern despite different photon fluxes.



# EIC observables

- Asymmetric rapidity distribution
- Presence of bound state mostly seen on the  $d\sigma/dW$  line shapes



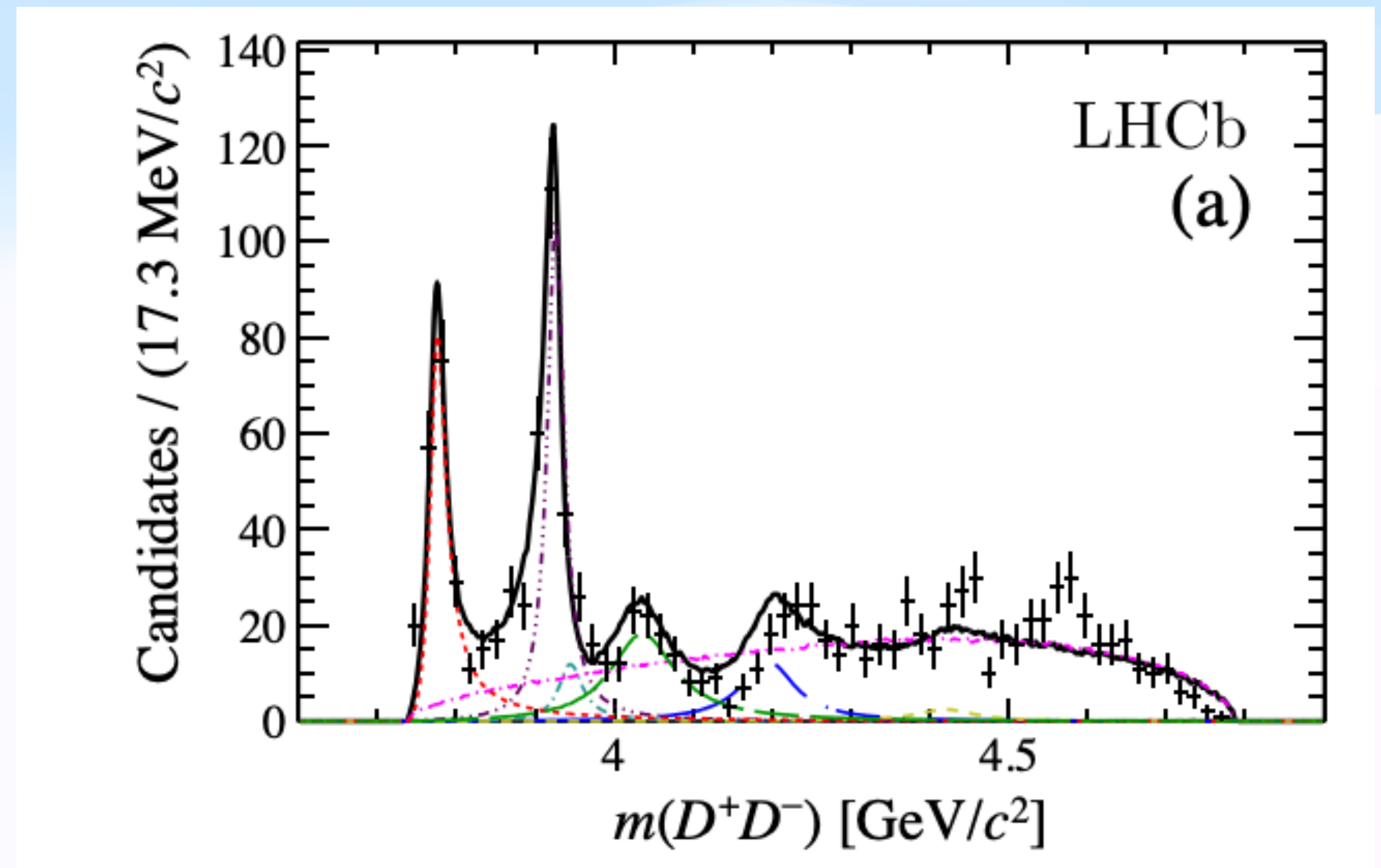
# Outlook: larger coupled-channel systems

$$D\bar{D}, \quad D_s\bar{D}_s, \quad D^*\bar{D}^*, \quad D_s^*\bar{D}_s^*$$

$PP$  sector  $\leftrightarrow X(3700)$

$VV$  and  $D_s^{(*)}\bar{D}_s^{(*)}$  sector  $\leftrightarrow X(3960)$

- Extend the coupled channel framework beyond  $D\bar{D}$
- Include  $D_s^{(*)}\bar{D}_s^{(*)}$  and vector channels
- Can describe both  $X(3700)$  and  $X(3960)$
- Connect scattering dynamics with production observables



# $PP$ Sector

- Describes a candidate  $X(3700)$  using only  $PP$  coupled-channels
- Describes both a candidate  $X(3700)$  and a  $0^{++}$  structure at around  $\sim 3930$  MeV using only  $PP$  coupled-channels, but changing the interaction and applying some approximations

## Dynamically generated open and hidden charm meson systems

D. Gamermann,<sup>1</sup> E. Oset,<sup>1</sup> D. Strottman,<sup>1</sup> and M. J. Vicente Vacas<sup>1</sup>

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(Received 18 December 2006; revised manuscript received 25 June 2007; published 16 October 2007)

## $X(3960)$ seen in $D_s^+ D_s^-$ as the $X(3930)$ state seen in $D^+ D^-$

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We perform a calculation of the interaction of the  $D\bar{D}$ ,  $D_s\bar{D}_s$  coupled channels and find two bound states, one coupling to  $D\bar{D}$  and another one at higher energies coupling mostly to  $D_s^+ D_s^-$ . We identify this latter state with the  $X_0(3930)$  seen in the  $D^+ D^-$  mass distribution in the  $B^+ \rightarrow D^+ D^- K^+$  decay, and also show that it produces an enhancement of the  $D_s^+ D_s^-$  mass distribution close to threshold which is compatible with the recent LHCb observation in the  $B^+ \rightarrow D_s^+ D_s^- K^+$  decay which has been identified as a new state,  $X_0(3960)$ .

# *VV* Sector

PHYSICAL REVIEW D **80**, 114013 (2009)

***Y*(3940), *Z*(3930), and the *X*(4160) as dynamically generated resonances  
from the vector-vector interaction**

R. Molina<sup>1</sup> and E. Oset<sup>1</sup>

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(Received 24 July 2009; revised manuscript received 28 October 2009; published 15 December 2009)

Describes several structures at around  $\sim 3950$  MeV,  
including  $0^{++}$  only using *VV* coupled-channels

**What if the *X*(3960) is a *VV* structure appearing in *PP* channels?**

# $PP$ and $VV$ coupled-channels

PHYSICAL REVIEW D **90**, 114023 (2014)

## $X(3872)$ production in high energy heavy ion collisions

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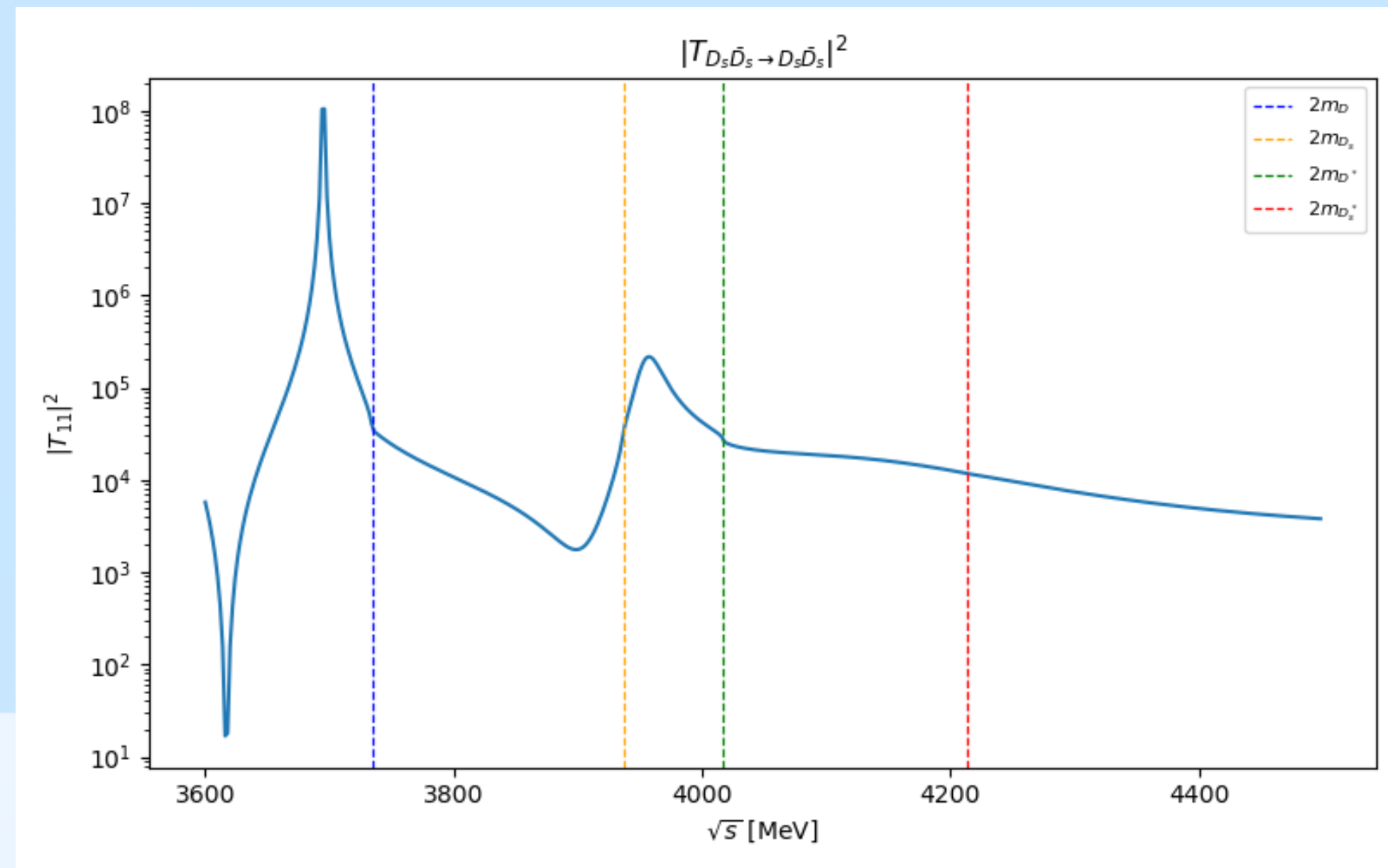
Luciano M. Abreu<sup>||</sup>

*Instituto de Física, Universidade Federal da Bahia, 40210-340 Salvador, BA, Brazil*

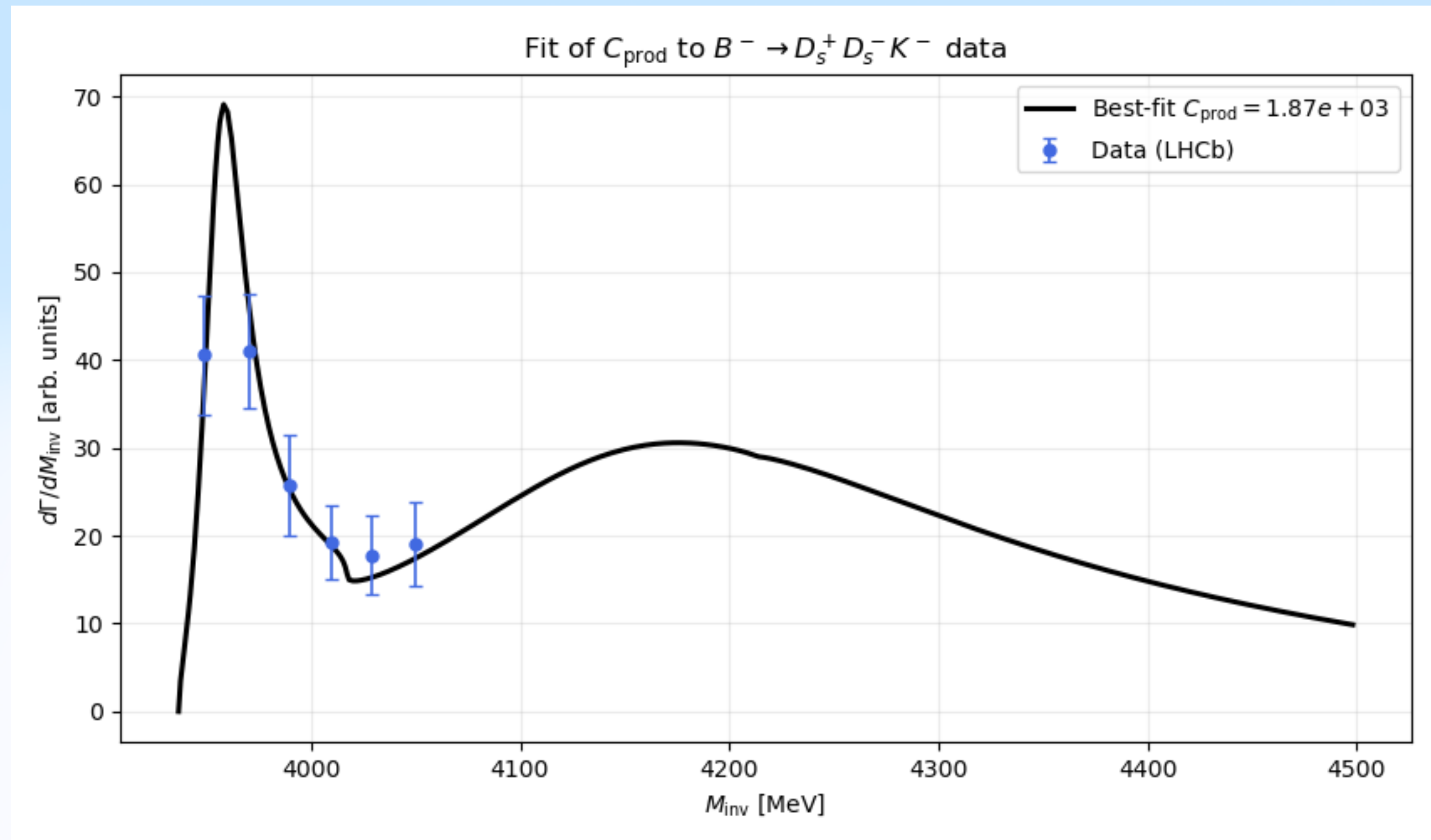
(Received 8 June 2014; published 18 December 2014)

**Interaction Lagrangians that couple both  $P$  and  $V$  mesons**

# Results



**The coupling between sectors is able to describe the data!**



# Conclusions

- Coupled-channel FSI strongly modifies near-threshold  $\gamma\gamma \rightarrow D\bar{D}$ ;
- The effect is channel dependent:  $D^+D^-$  enhancement and  $D^0\bar{D}^0$  suppression;
- UPC and EIC provide complementary environments, while the FSI ratio is mainly controlled by the subprocess;
- FSI affects both line shapes and expected yields.
- Extending the coupled channels can provide a description of structures at both  $PP$  and  $VV$  sectors!




# Obrigado!



# Backup



## $D^*(2010)^\pm$ DECAY MODES

$D^*(2010)^-$  modes are charge conjugates of the modes below.

Mode	Fraction ( $\Gamma_i / \Gamma$ )	Scale Factor/ Conf. Level	$P(\text{MeV}/c)$
$\Gamma_1$ $D^0\pi^+$	$(67.7 \pm 0.5) \%$		39 
$\Gamma_2$ $D^+\pi^0$	$(30.7 \pm 0.5) \%$		38 
$\Gamma_3$ $D^+\gamma$	$(1.6 \pm 0.4) \%$		136 

## $D^*(2007)^0$ DECAY MODES

$\bar{D}^*(2007)^0$  modes are charge conjugates of modes below.

Mode	Fraction ( $\Gamma_i / \Gamma$ )	Scale Factor/ Conf. Level	$P(\text{MeV}/c)$
$\Gamma_1$ $D^0\pi^0$	$(64.7 \pm 0.9) \%$		43 
$\Gamma_2$ $D^0\gamma$	$(35.3 \pm 0.9) \%$		137 

$$Q^2 \leq \frac{1}{R^2} \quad \omega < \omega_{max} \approx \frac{\gamma}{R}, \quad q_{\perp} \leq \frac{1}{R}$$

$$f_{\gamma|e}(\omega_e) = \frac{\alpha_{em}}{\pi \omega_e} \int \frac{dQ^2}{Q^2} \left[ \left( 1 - \frac{\omega_e}{E_e} + \frac{\omega_e^2}{2E_e^2} \right) - \left( 1 - \frac{\omega_e}{E_e} \right) \frac{Q_{min}^2}{Q^2} \right]$$

$$f_{\gamma|A}(\omega_A) = \frac{Z^2 \alpha}{\pi^2} \int d^2b \frac{1}{b^2 v^2 \omega_A} \left[ \int du u^2 F_A \left( \sqrt{\frac{\left( \frac{b\omega_A}{\gamma_L} \right)^2 + u^2}{b^2}} \right) \frac{1}{\left( \frac{b\omega_A}{\gamma_L} \right)^2 + u^2} J_1(u) \right]^2 ,$$

# Interaction Lagrangians

Cao, Du, Guo. J.Phys.G 51 (2024) 10, 105002

- For the charged sector we have the following interaction Lagrangians:

$$\mathcal{L}_{DD\gamma} = (\partial_\mu D^+ + ieA_\mu D^+)(\partial^\mu D^- - ieA^\mu D^-)$$

$$\mathcal{L}_{DD^*\gamma} = -ig_{\gamma D^+ D^{*-}} F_{\mu\nu} \epsilon^{\mu\nu\alpha\beta} (D_\alpha^{*-} \overleftrightarrow{\partial}_\beta D^+ + D^- \overleftrightarrow{\partial}_\beta D_\alpha^{*+})$$

Where  $g_{\gamma D^+ D^{*-}} = -0.035 \text{ GeV}^{-1}$ .

- For the neutral sector we have:

$$\mathcal{L}_{D^0 \bar{D}^{*0} \gamma} = -ig_{\gamma D^0 \bar{D}^{*0}} F_{\mu\nu} \epsilon^{\mu\nu\alpha\beta} (\bar{D}_\alpha^{*0} \overleftrightarrow{\partial}_\beta D^0 + \bar{D}^0 \overleftrightarrow{\partial}_\beta D_\alpha^{*0})$$

Where  $g_{\gamma D^0 \bar{D}^{*0}} = 0.142 \text{ GeV}^{-1}$ .

# Backup

$$G_{ii} = \frac{1}{16\pi^2} \left\{ \alpha_i + \log\left(\frac{m_1^2}{\mu^2}\right) + \frac{m_2^2 - m_1^2 + s}{2s} \log\left(\frac{m_2^2}{m_1^2}\right) + \frac{b}{\sqrt{s}} \left[ \log\left(\frac{s - m_2^2 + m_1^2 + 2b\sqrt{s}}{-s + m_2^2 - m_1^2 + 2b\sqrt{s}}\right) + \log\left(\frac{s + m_2^2 - m_1^2 + 2b\sqrt{s}}{-s - m_2^2 + m_1^2 + 2b\sqrt{s}}\right) \right] \right\}.$$

$$\alpha_H(P) = -1.3, \quad \alpha_H(V) = -2.07$$

# Form factors

Lebiedowicz, Nachtmann, Szczurek. Phys. Rev. D 98, 014001 (2018).

- We include form factors at the vertices
- Monopole form factors:

$$F(q^2) = \frac{\Lambda^2 - m_{D^{(*)}}^2}{\Lambda^2 - q^2}$$

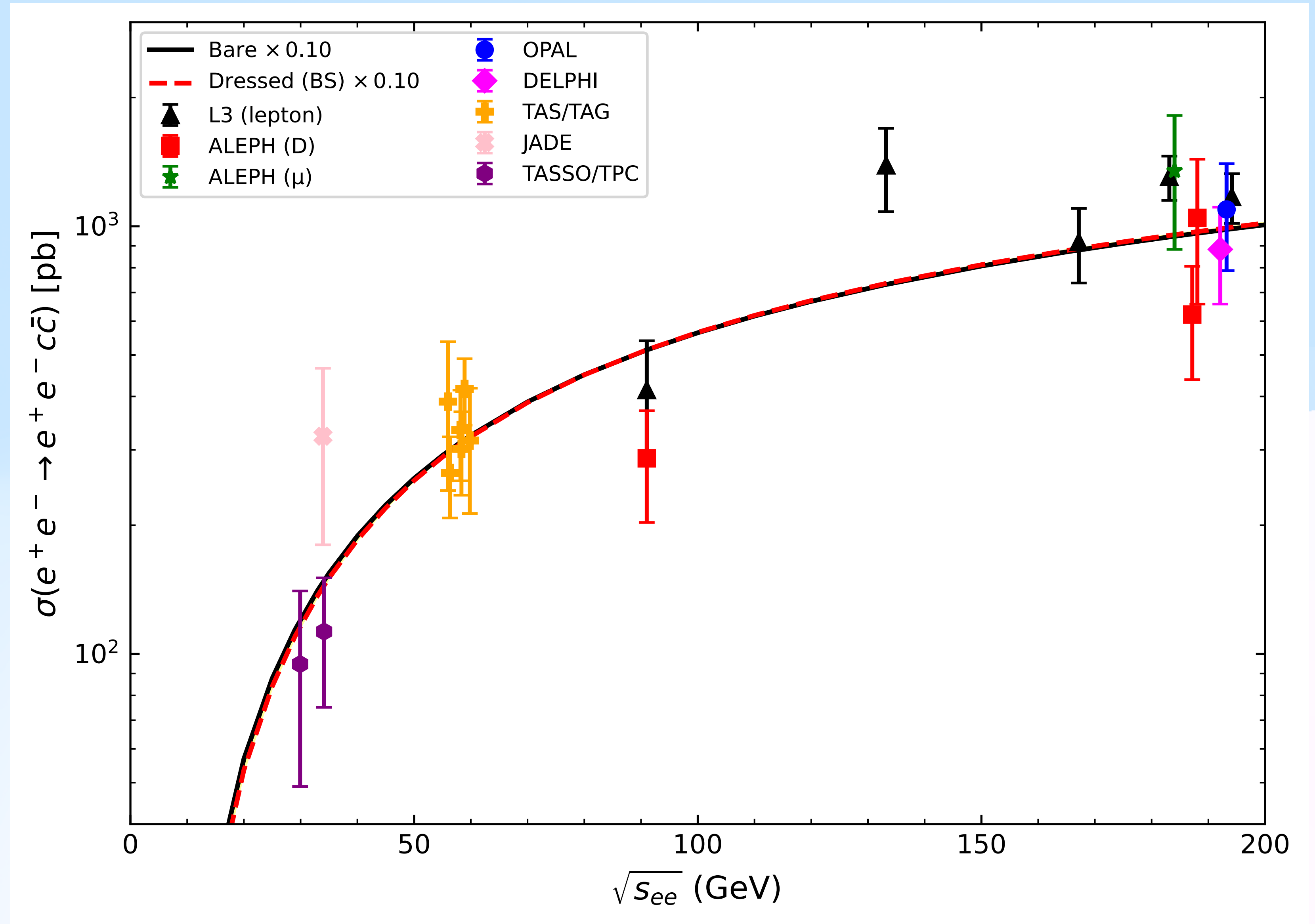
- $q$  is the exchange meson four momentum.  $\Lambda$  is the cutoff parameter. This choice guarantees that  $F(m_{D^{(*)}}^2) = 1$  when the exchanged meson is on-shell.
- We use a similar form factor for the neutral channel

$$F(q^2) = \frac{\Lambda^2 - m_{D^{0*}}^2}{\Lambda^2 - q^2}$$

$\Lambda \in [1.0, 1.2]$  GeV value obtained similarly  
as in *Phys.Rev.D* 110 (2024) 3, 034037

# LEP data fitting

We normalize the overall cross sections form factor to LEP data

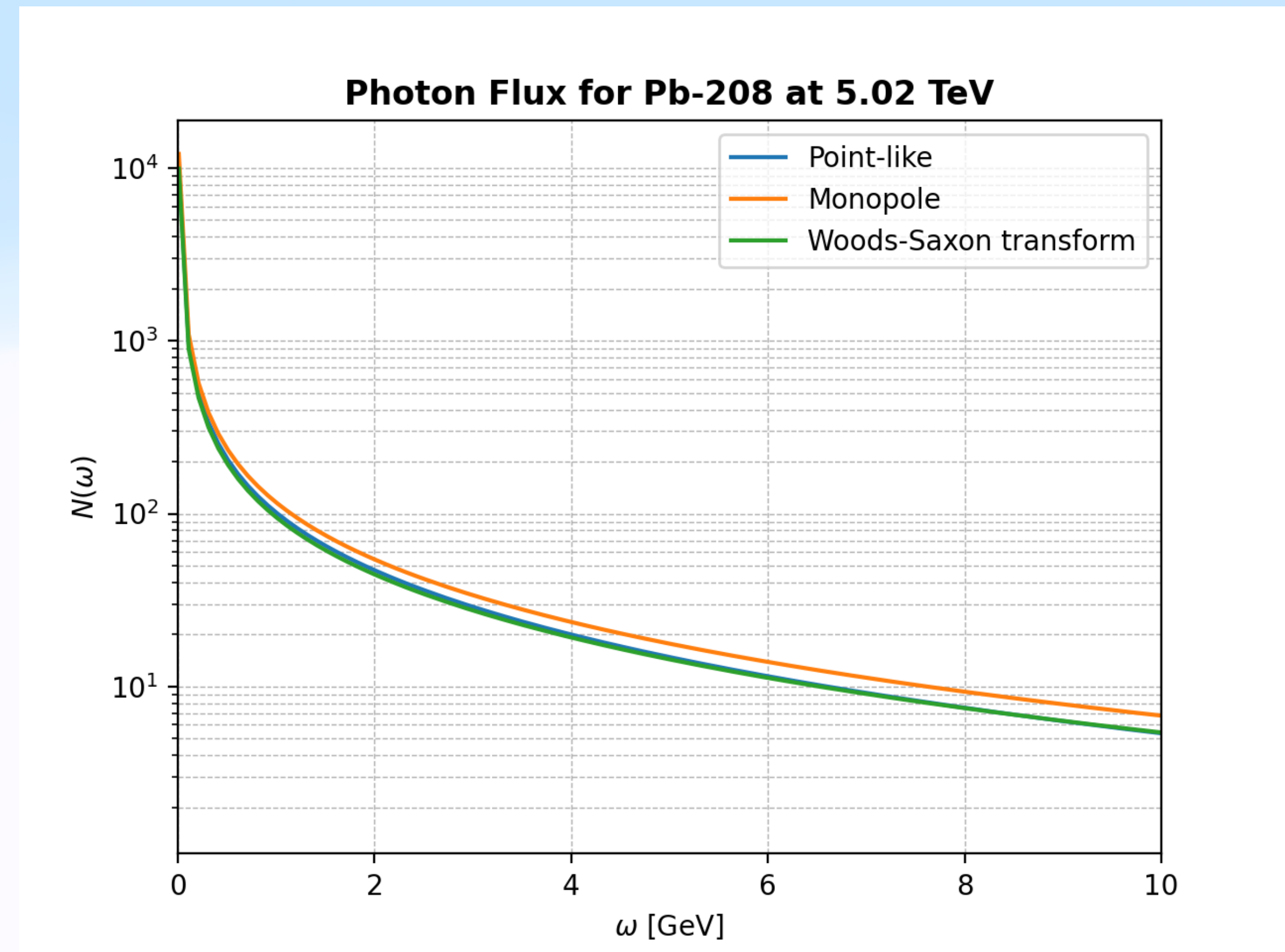


# Equivalent photon approximation

$$\sigma(s_1 s_2 \rightarrow s_1 + X + s_2) = \int d\omega_1 d\omega_2 n_1(\omega_1) n_2(\omega_2) \sigma_{\gamma\gamma \rightarrow X}(W)$$

- $W^2 = 4\omega_1\omega_2$
- $n(\omega)$  depends on the source's features
- We can change variables from  $\omega_1, \omega_2$  to  $W, Y$  using  
 $\omega_1 = (W/2)e^Y, \omega_2 = (W/2)e^{-Y}$
- This gives the EPA as:

$$\sigma = \int dW dY \frac{W}{2} n_1(\omega_1) n_2(\omega_2) \sigma_{\gamma\gamma \rightarrow X}(W)$$



# Minimal coupled channels for $D\bar{D}$ production

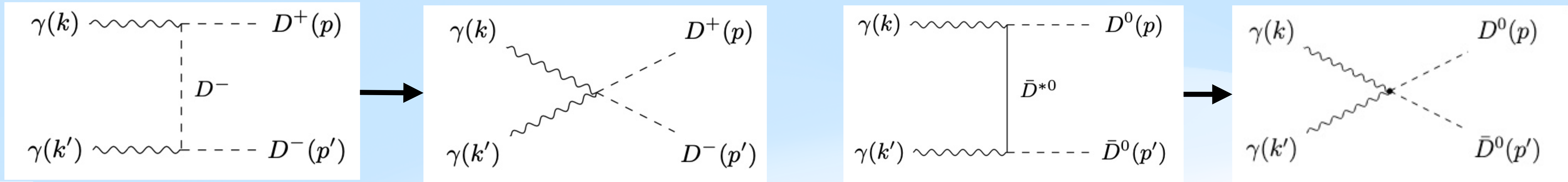
- We consider only the two channels  $|D^+D^-\rangle$  and  $|D^0\bar{D}^0\rangle$
- Isospin structure  $|D\bar{D}(I=0)\rangle \sim (|D^+D^-\rangle + |D^0\bar{D}^0\rangle)/\sqrt{2}$
- Not yet  $D_s\bar{D}_s, \dots$   
We are interested in computing observables and using effective Lagrangians coupled to photons:

Particle basis!

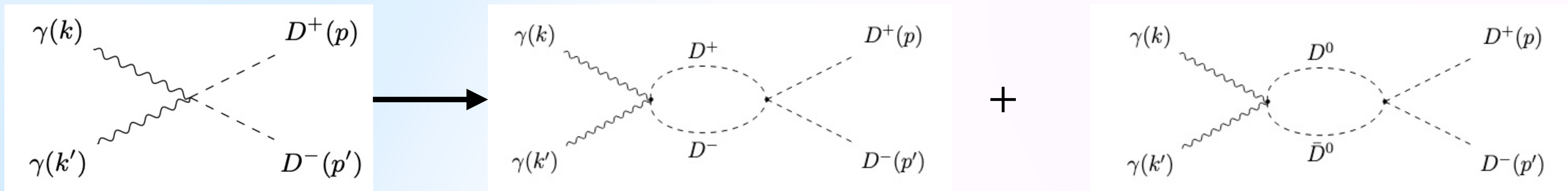
# Contact approximation

Wang, et al. Phys.Rev.D 103 (2021) 5, 054008

Near-threshold  $W$ ,  $|\vec{q}|^2 \ll m_D^2$  so that our exchange diagrams all approximate a contact term



We can then turn on FSI for contact interactions



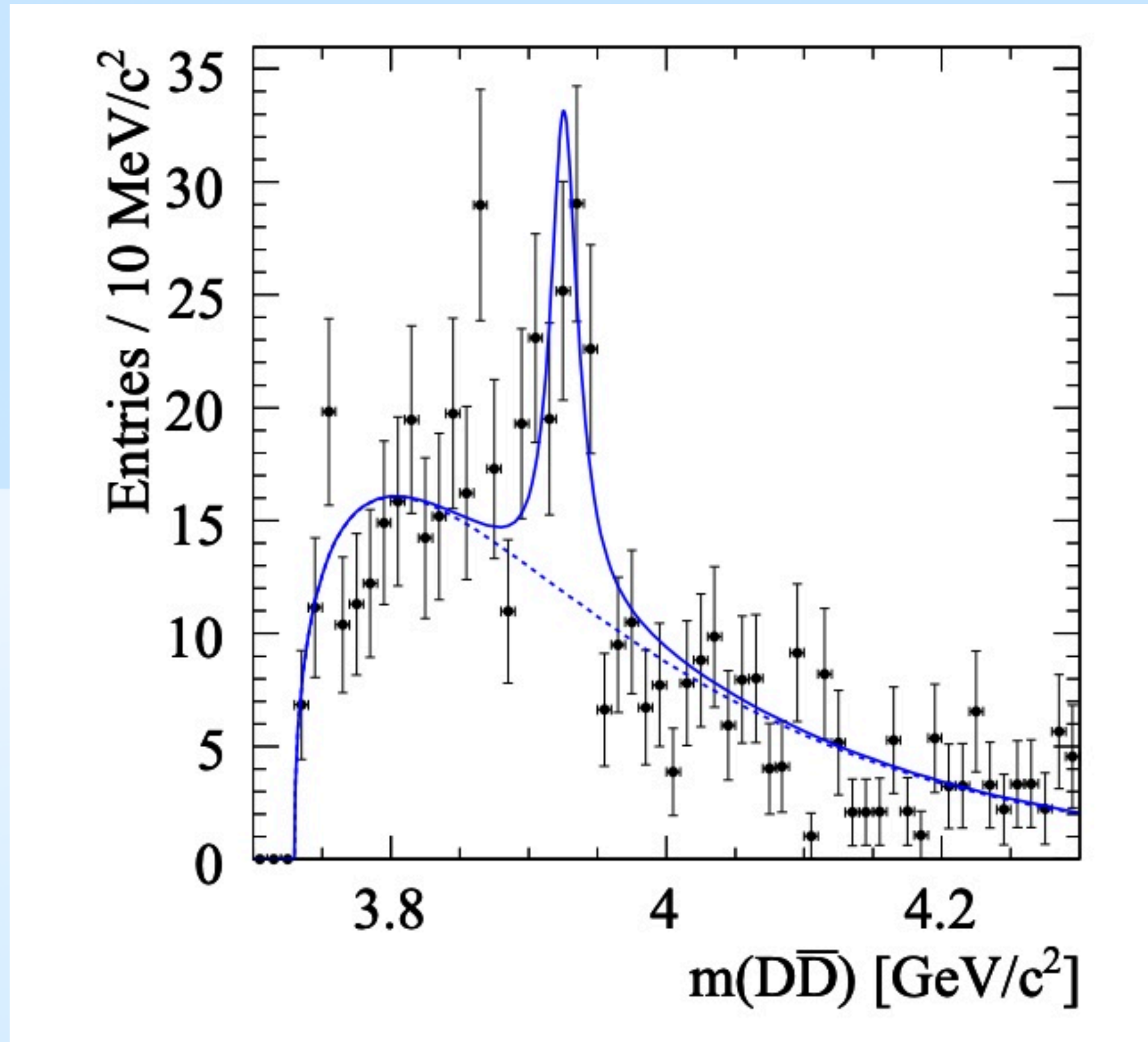
# Project structure

- Objective: To build a coupled-channel scattering framework for hidden charm  $0^{++}, 2^{++}$  sectors
- Processes of interest:  $\gamma\gamma \rightarrow D^{(*)}\bar{D}^{(*)}$ ,  $\gamma\gamma \rightarrow D_s^{(*)}\bar{D}_s^{(*)}$
- Strategy:
  - A. Compute tree-level amplitudes (kernels) for  $PP$ ,  $VV$  and  $PP \leftrightarrow VV$
  - B. Apply partial wave/ spin projection to obtain  $V_J(s)$
  - C. Unitarize to obtain  $T_J(s)$ , poles, couplings and line shapes
- Current status:
  - Main amplitudes obtained for all blocks
  - Bethe-Salpeter numerical solutions for  $PP \rightarrow PP$ ,  $VV \rightarrow VV$
  - Mixed  $PP \leftrightarrow VV$  still under development

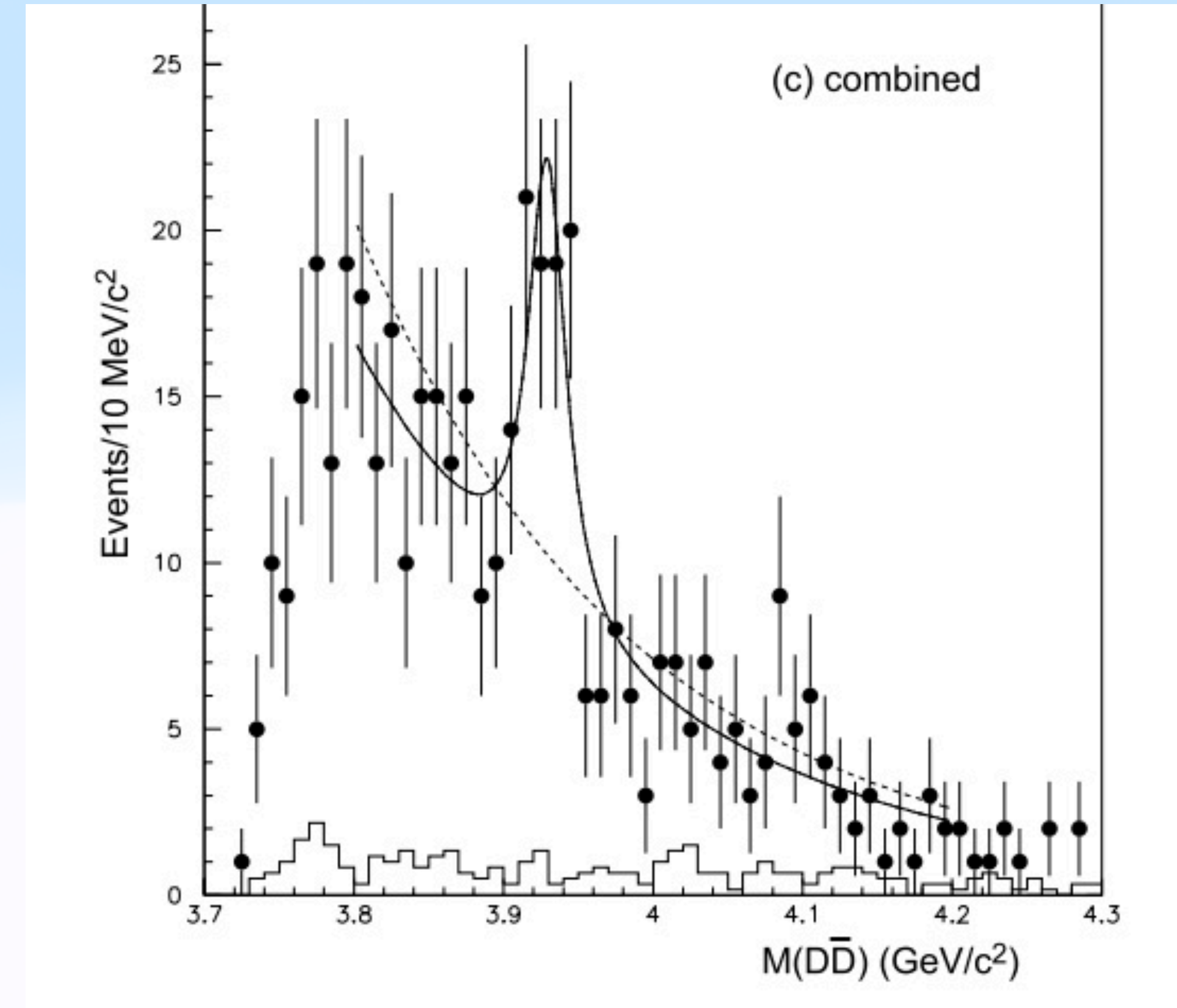
# Experiments and data available

BaBar Collaboration. Phys.Rev.D 81 (2010) 092003

Belle Collaboration. Phys.Rev.Lett. 96 (2006) 082003



BaBar



Belle

# From amplitudes to $T$ matrix using Bethe-Salpeter

- Fix quantum numbers ( $I = 0$ , s-wave) and build  $V(s)$  from projected amplitudes

$$T(s) = V(s) + V(s)G(s)T(s) \longrightarrow T(s) = [1 - V(s)G(s)]^{-1}V(s)$$

- Solve the coupled channel BSE
- $G(s)$  diagonal matrix of two-meson loop functions. In dimensional regularization it is written in closed form that depends on  $\alpha_i$  and scale  $\mu$ .
- Physics outputs:
  - Poles from  $\det[1 - VG] = 0$  (bound states/resonances)
  - Residues give channel couplings
  - The same physics that generate threshold effects relevant for  $\gamma\gamma$  production

# PP sector

Gamermann, et al. Phys.Rev.D 76 (2007) 074016

- We use the PP  $\rightarrow$  PP amplitudes given in Phys.Rev.D 76 (2007) 074016 for our pseudoscalar sector kernel  $V_{ij}(s)$ . They are obtained from the Lagrangian:

$$\mathcal{L} = \frac{1}{12f^2} \left( \text{Tr} \left( J_{88\mu} J_{88}^\mu + 2 J_{\bar{3}3\mu} J_{88}^\mu + J_{3\bar{3}\mu} J_{\bar{3}\bar{3}}^\mu \right) + \frac{8}{3} \gamma J_{\bar{3}1\mu} J_{13}^\mu + \frac{4}{\sqrt{3}} \gamma \left( J_{\bar{3}1\mu} J_{83}^\mu + J_{\bar{3}8\mu} J_{13}^\mu \right) + 2\gamma J_{\bar{3}8\mu} J_{83}^\mu + \psi_5 J_{\bar{3}3\mu} J_{\bar{3}\bar{3}}^\mu + \mathcal{L}_{\text{mass}} \right).$$

- The possible channels for  $I = 0$  are  $\pi\pi, K\bar{K}, \eta\eta, \eta\eta_c, D\bar{D}, D_s\bar{D}_s$ .
- We can have a  $J^P = 0^+$  state.

# PP sector: minimal $D\bar{D} - D_s\bar{D}_s$ coupled system

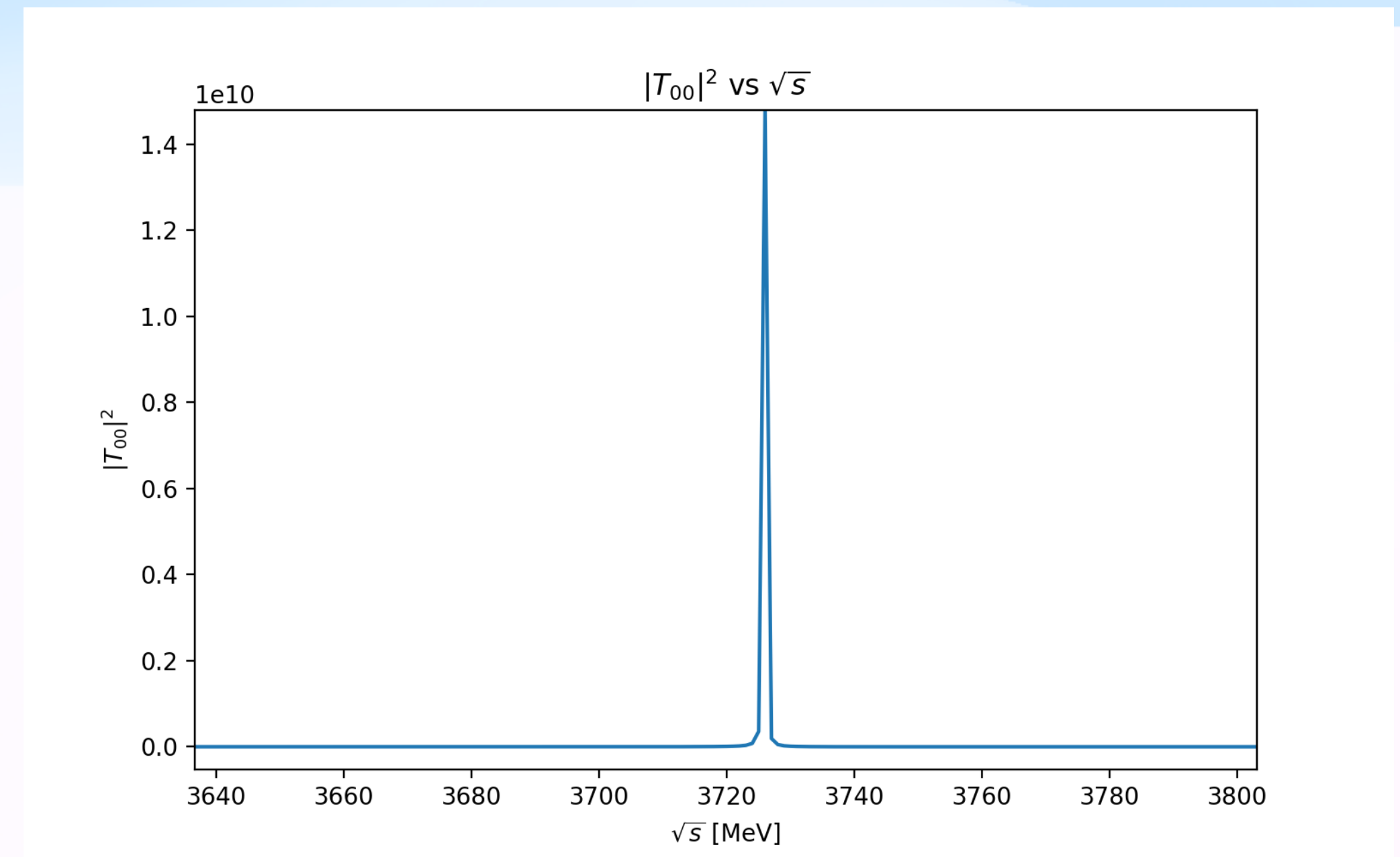
- Work in  $I = 0$ , s-wave ( $0^{++}$  sector as baseline)
- 2 channel basis:  $1 \equiv D\bar{D}$ ,  $2 \equiv D_s\bar{D}_s$
- After computing PP amplitudes, the kernel is organized as:

$$V(s) = \begin{pmatrix} A_1(s) & A_3(s) \\ A_3(s) & A_2(s) \end{pmatrix}$$

With

$$A_1(s) = \langle D\bar{D} | t | D\bar{D} \rangle, \quad A_2(s) = \langle D_s\bar{D}_s | t | D_s\bar{D}_s \rangle, \quad A_3(s) = \langle D\bar{D} | t | D_s\bar{D}_s \rangle$$

- With BSE, we get  $T(s)$



# VV sector

Molina, Oset. Phys.Rev.D 80 (2009) 114013

- We use the  $VV \rightarrow VV$  amplitudes given in Phys.Rev.D 80 (2009) 114013 for our Vector sector kernel  $V_{ij}(s)$ . They are obtained from the Lagrangian coming from the Hidden Gauge Symmetry formalism:

$$\mathcal{L} = -\frac{1}{4} \langle V_{\mu\nu} V^{\mu\nu} \rangle$$

- For the  $I = 0, \quad J = 0$  there are many possible channels  $D^* \bar{D}^*, D_s^* \bar{D}_s^*, K^* \bar{K}^*, \rho\rho, \omega\omega, \phi\phi, J/\psi J/\psi, \omega J/\psi, \psi J/\psi$ .
- We can, in principle, have  $J^P = 0^+, 2^+$  states, but we start working with  $0^+$ .

# VV sector: minimal $D^*\bar{D}^* - D_s^*\bar{D}_s^*$ coupled system

- Work with VV interactions already given in isospin basis and s-wave projected ( $I = 0, J = 0$ )

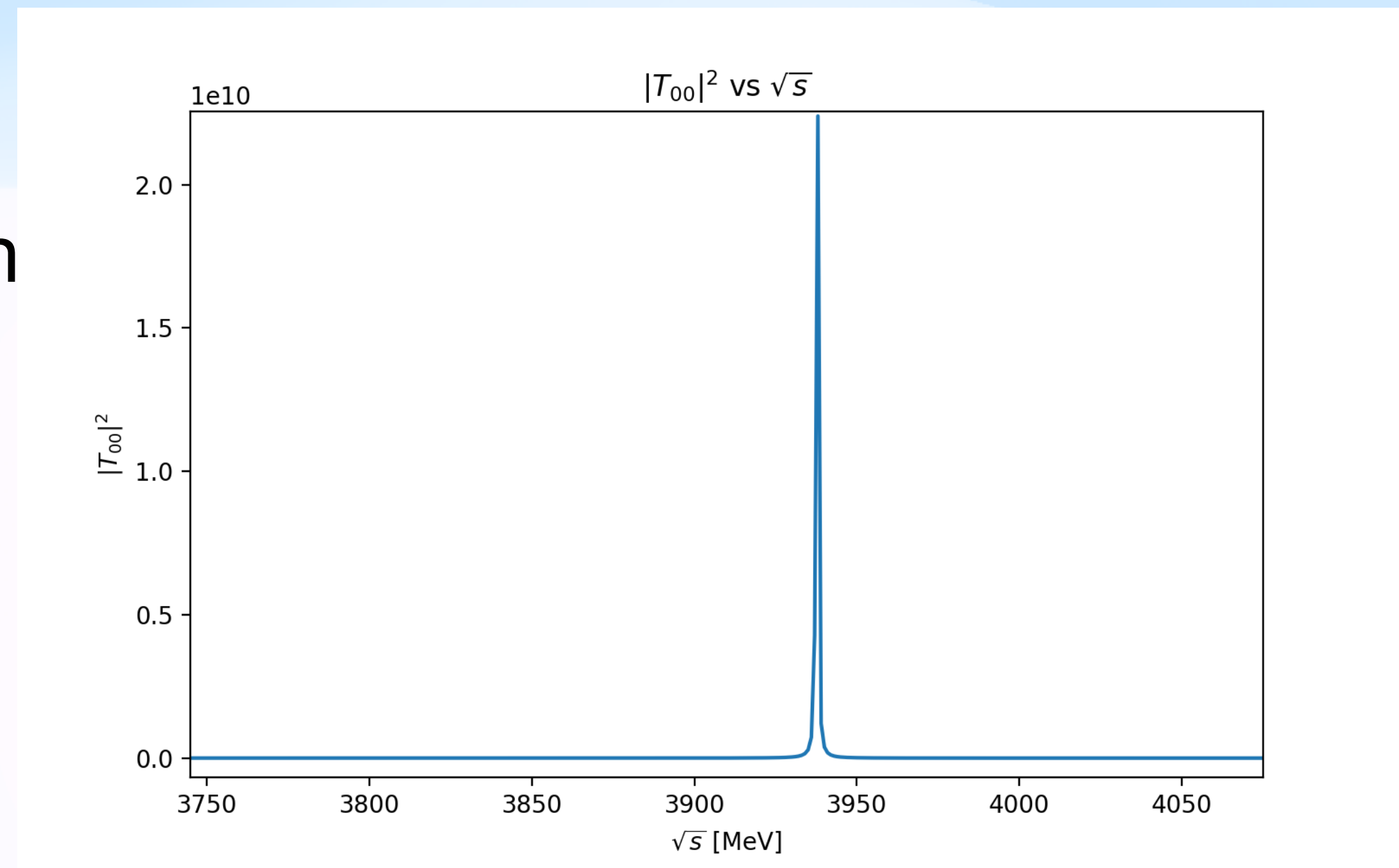
- 2 channel basis:  $1 \equiv D^*\bar{D}^*$ ,  $2 \equiv D_s^*\bar{D}_s^*$   

$$V(s) = \begin{pmatrix} A_1(s) & A_3(s) \\ A_3(s) & A_2(s) \end{pmatrix}$$

With Similarly, after computing VV amplitudes, th

$$A_1(s) = \langle D^*\bar{D}^* | t | D^*\bar{D}^* \rangle, \quad A_2(s) = \langle D_s^*\bar{D}_s^* | t | D_s^*\bar{D}_s^* \rangle, \quad A_3(s) = \langle D^*\bar{D}^* | t | D_s^*\bar{D}_s^* \rangle$$

- With BSE, we get  $T(s)$  (now with  $G(s)$  for two vectors)



# Mixed block $D\bar{D} \rightarrow D^*\bar{D}^*$

Martinez, et al. Phys.Rev.D. 90, 114023

- We use the Lagrangians given in Phys.Rev.D 90 (2014) 114023 to compute our amplitudes. We then project the amplitudes onto s-wave and spin 0 components for kernels  $V_{ij}(s)$ . The Lagrangians are:

$$\mathcal{L}_{PPV} = -ig_{PPV} \langle V^\mu [P, \partial_\mu P] \rangle,$$

$$\mathcal{L}_{VVP} = \frac{g_{VVP}}{\sqrt{2}} \epsilon^{\mu\nu\alpha\beta} \langle \partial_\mu V_\nu \partial_\alpha V_\beta P \rangle$$

