FÍSICA E INSTRUMENTAÇÃO
DE ALTAS ENERGIAS COM O LHC-CERN@IFUSP
2026/05/26

$D_s^+ D_s^-$ Correlations: $X(3930)$ vs. "$X(3960)$"

Reporter:

Hao-Nan Liu (BUAA & USP)

Supervisors:

Prof. Li-Sheng Geng,

Prof. Alberto Martínez Torres (Since 2026-01)

Collaborators:

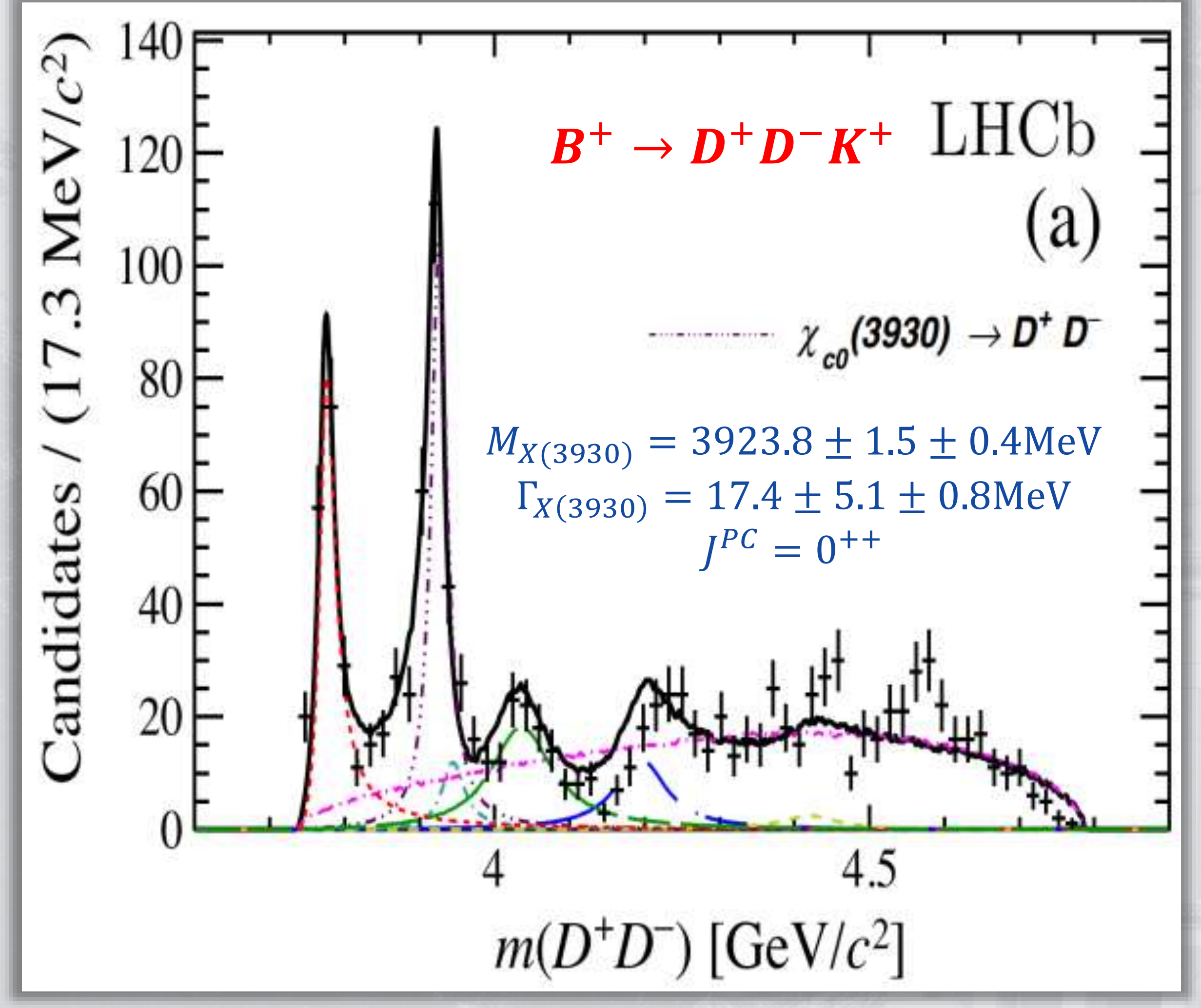
Zhi-Wei Liu

Luciano Abreu

Based on: arXiv:2511.19098 [hep-ph]

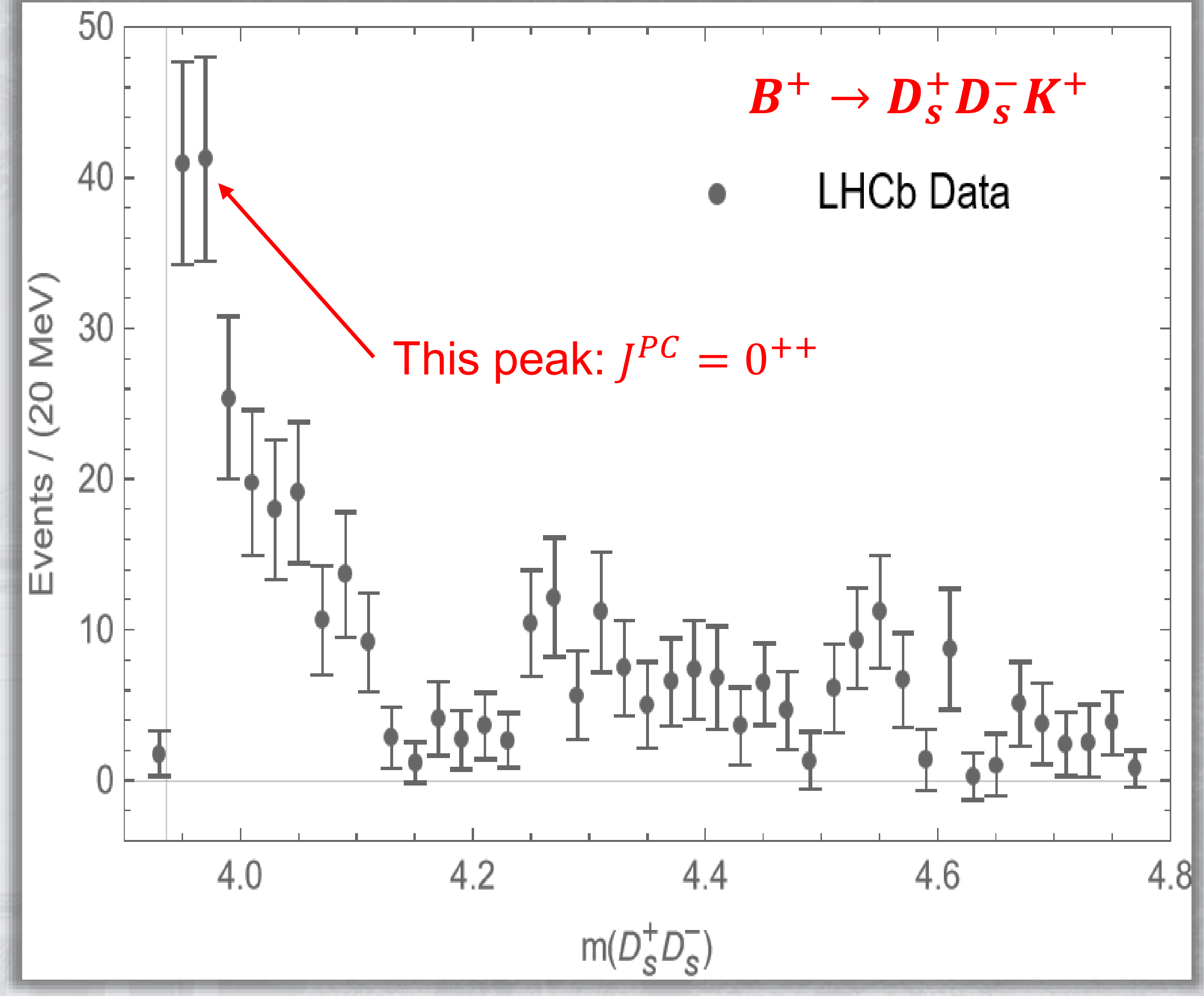
The invariant mass distribution of $D_{(s)}^+ D_{(s)}^-$

$X(3930)$



LHCb Collaboration,
Phys. Rev. D 102, 112003 (2020)

"$X(3960)$"



LHCb Collaboration,
Phys. Rev. Lett. 131, 071901 (2023)

The invariant mass spectrum of $D_s^+ D_s^-$

A qualitative figure showing the influence of a state with a width below the threshold.

M. Bayar, A. Feijoo, and E. Oset, Phys. Rev. D 107, 034007 (2023)

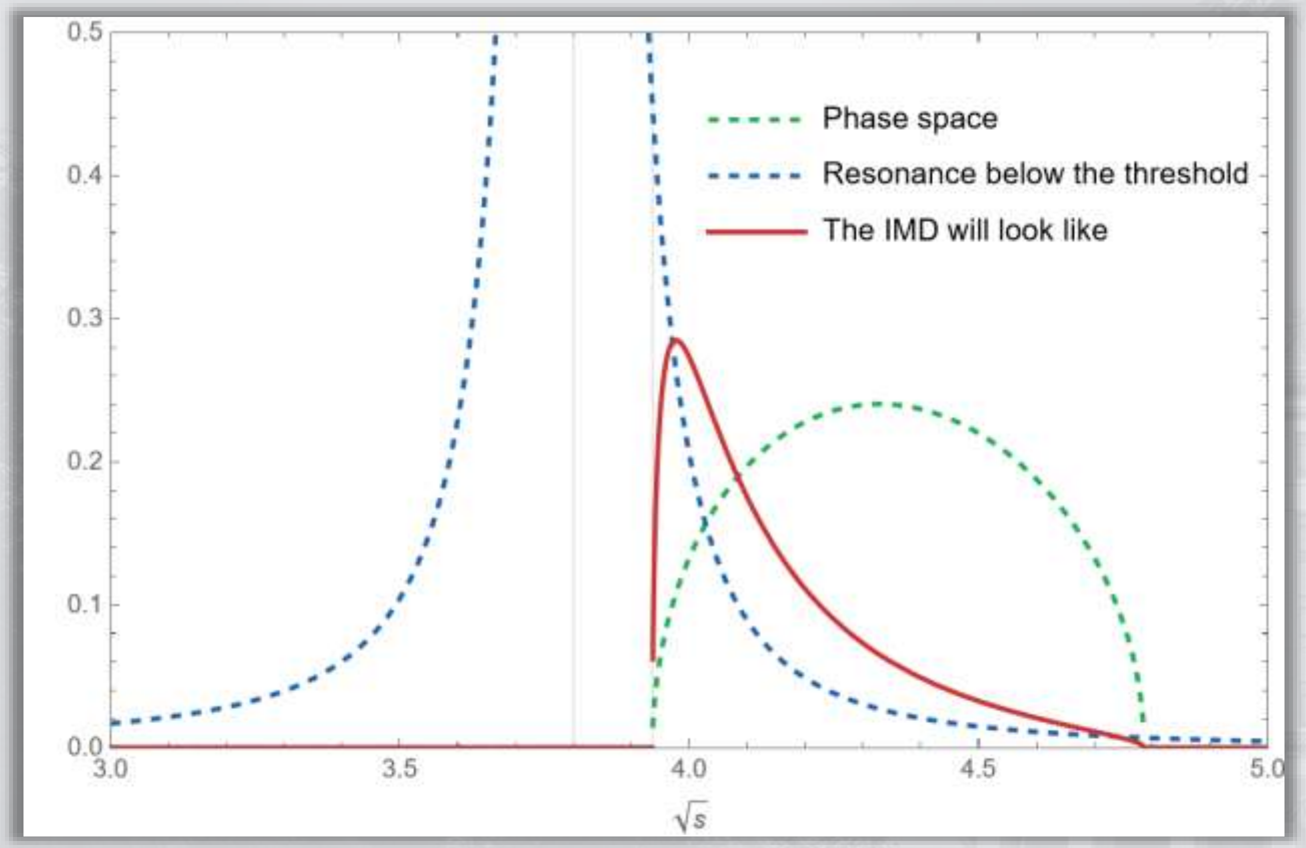


TABLE I. Masses and widths of the poles dynamically generated by the model, as well as, the corresponding modulus of the couplings $|g_i|$.

	M [MeV]	Γ [MeV]	$\lvert g_{\bar{D}D}\rvert$ [MeV]	$\lvert g_{\bar{D}_s D_s}\rvert$ [MeV]
Pole I	3699.04	...	14509.0	5707.2
Pole II ($X_0(3930)$)	3932.72	12.32	2889.5	10018.0

——A work on the $D\bar{D}$ and $D_s\bar{D}_s$ coupled channel system.

The invariant mass spectrum of $D_s^+ D_s^-$

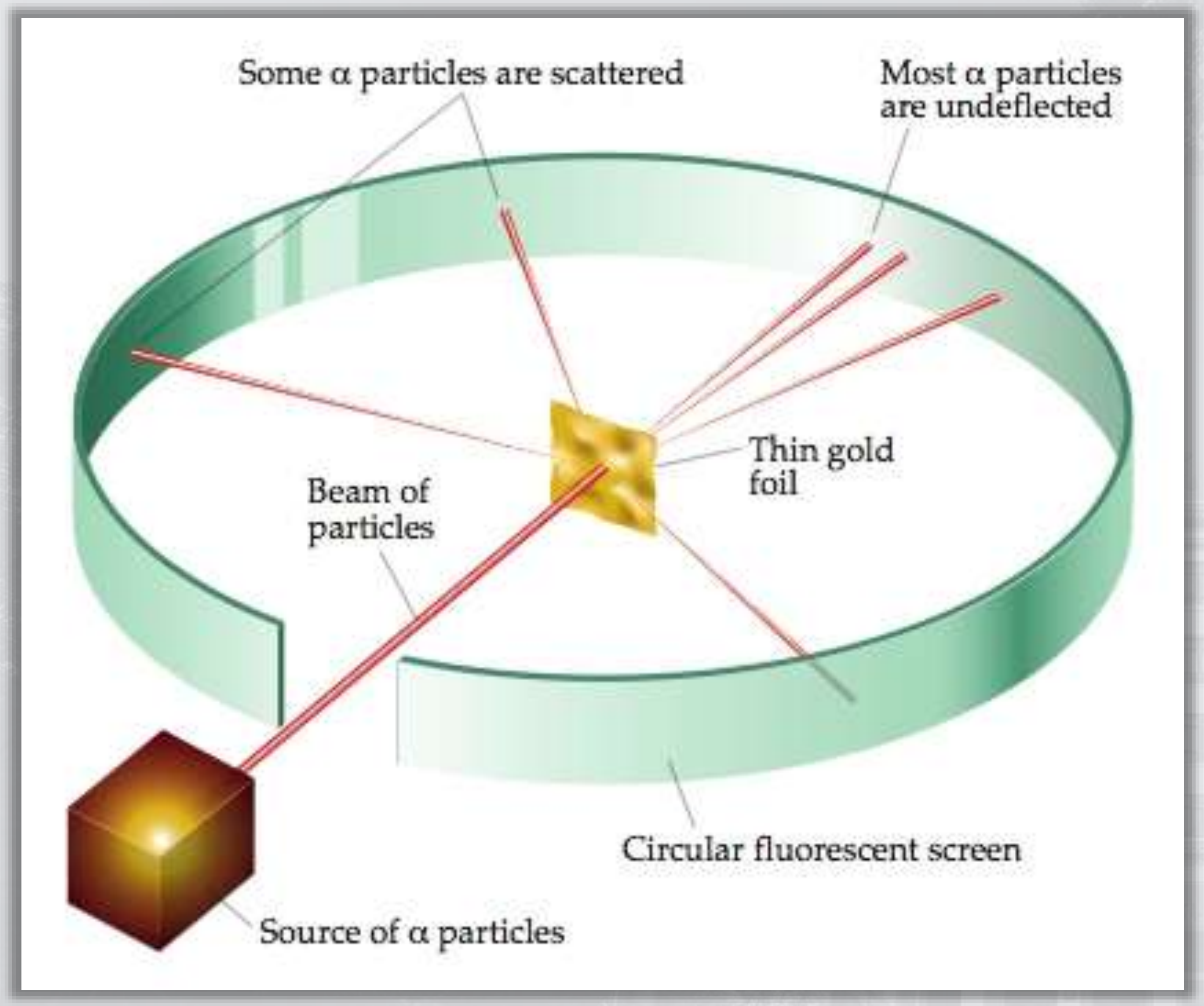
It seems difficult to determine from the IMD alone whether this state is above or below the threshold

If only there were other "observables" to help us distinguish.

CF? CF!

The Femtoscopic Correlation Function (CF)

1. How can we study interactions through experiments?



The scattering experiment:

We used to have the two subjects collide with each other
And
observe the scattering results.

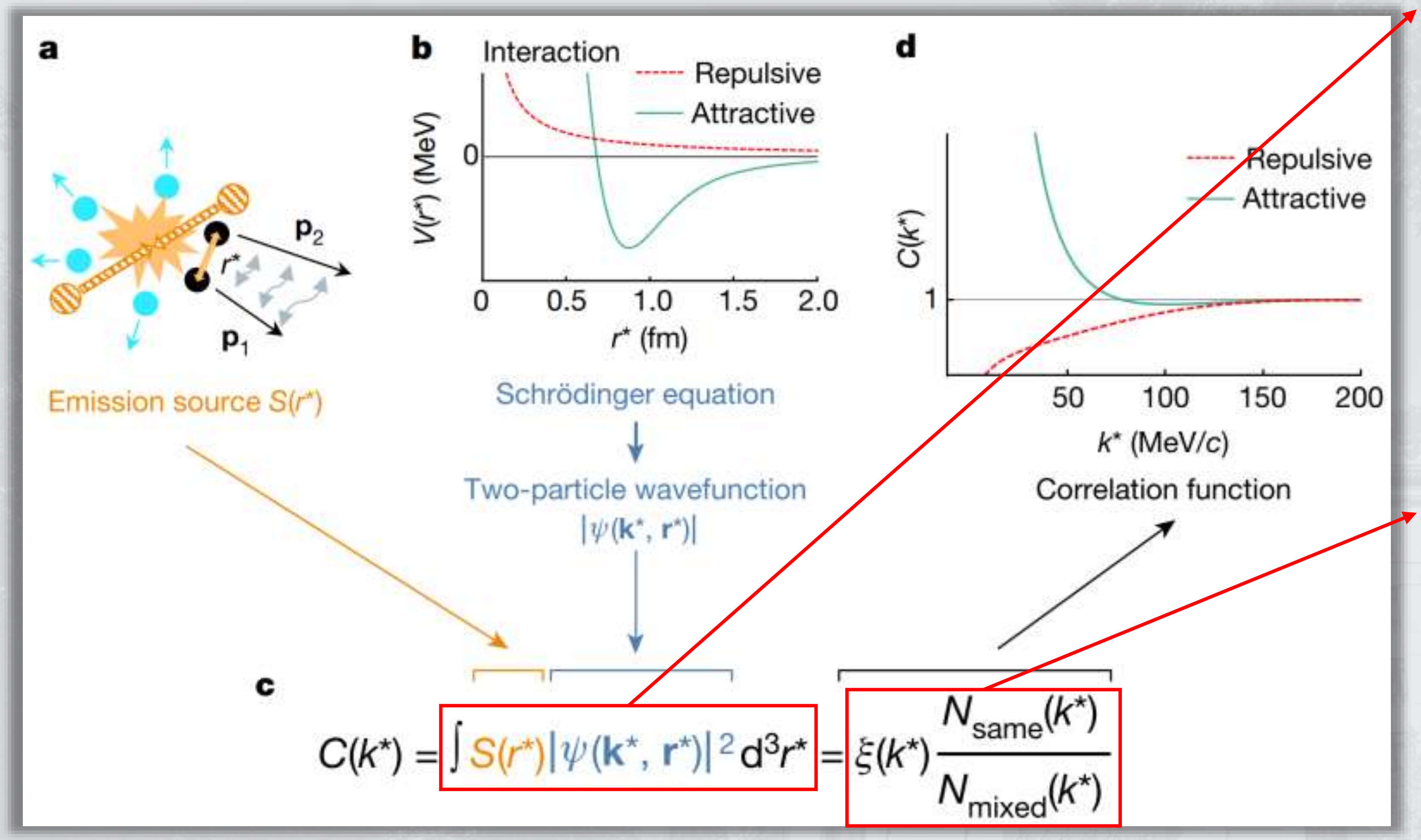
BUT,
What if they are unstable?

(Which means beams and targets are hard to get)
Direct scattering experiments are inpossible!

→ **The Femtoscopic Correlation Function!**

The Femtoscopic Correlation Function (CF)

2. What is the Correlation Function (CF) ?



ALICE Collaboration, Nature 588 (2020) 232

Theoretically:

A source-modulated relative momentum distribution

Source Function $S(\boldsymbol{r})$: The information from **"the Former Stage"**

Wave Function $\psi(\boldsymbol{k},\boldsymbol{r})$: The information of **interaction**

Experimentally:

An experimentally-corrected relative momentum distribution

$\xi(k)$: The **corrections** of the experimental effects

$N_{same}(k)$: the relative momentum distribution in **same events**

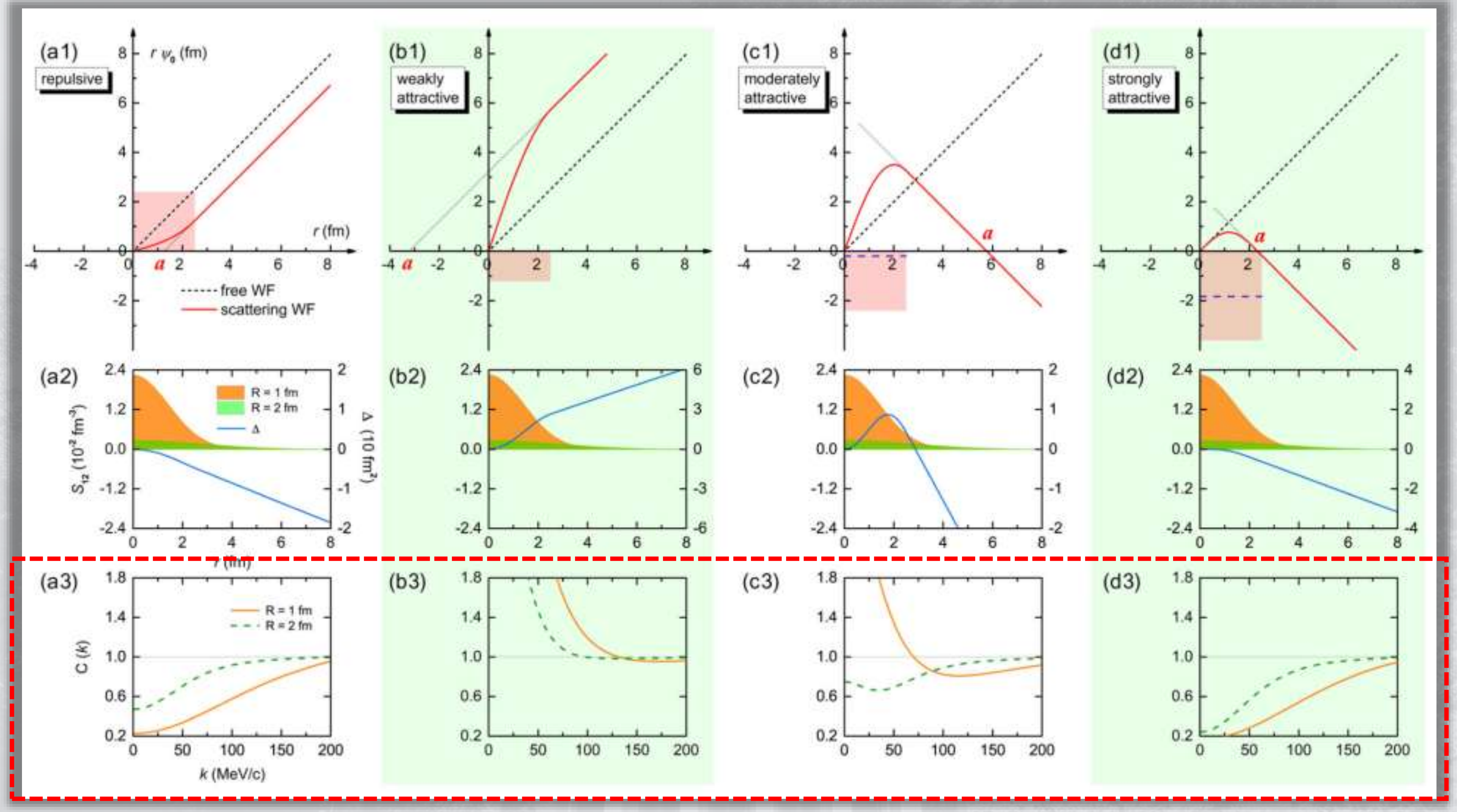
$N_{mixed}(k)$: the relative momentum distribution in **mixed events**

The Femtoscopic Correlation Function (CF)

北京航空航天大學
BEIHANG UNIVERSITY

Zhi-Wei Liu, Jun-Xu Lu, and Li-Sheng Geng,
Physical Review D, 2023, 107(7): 074019

3. CFs with bound states

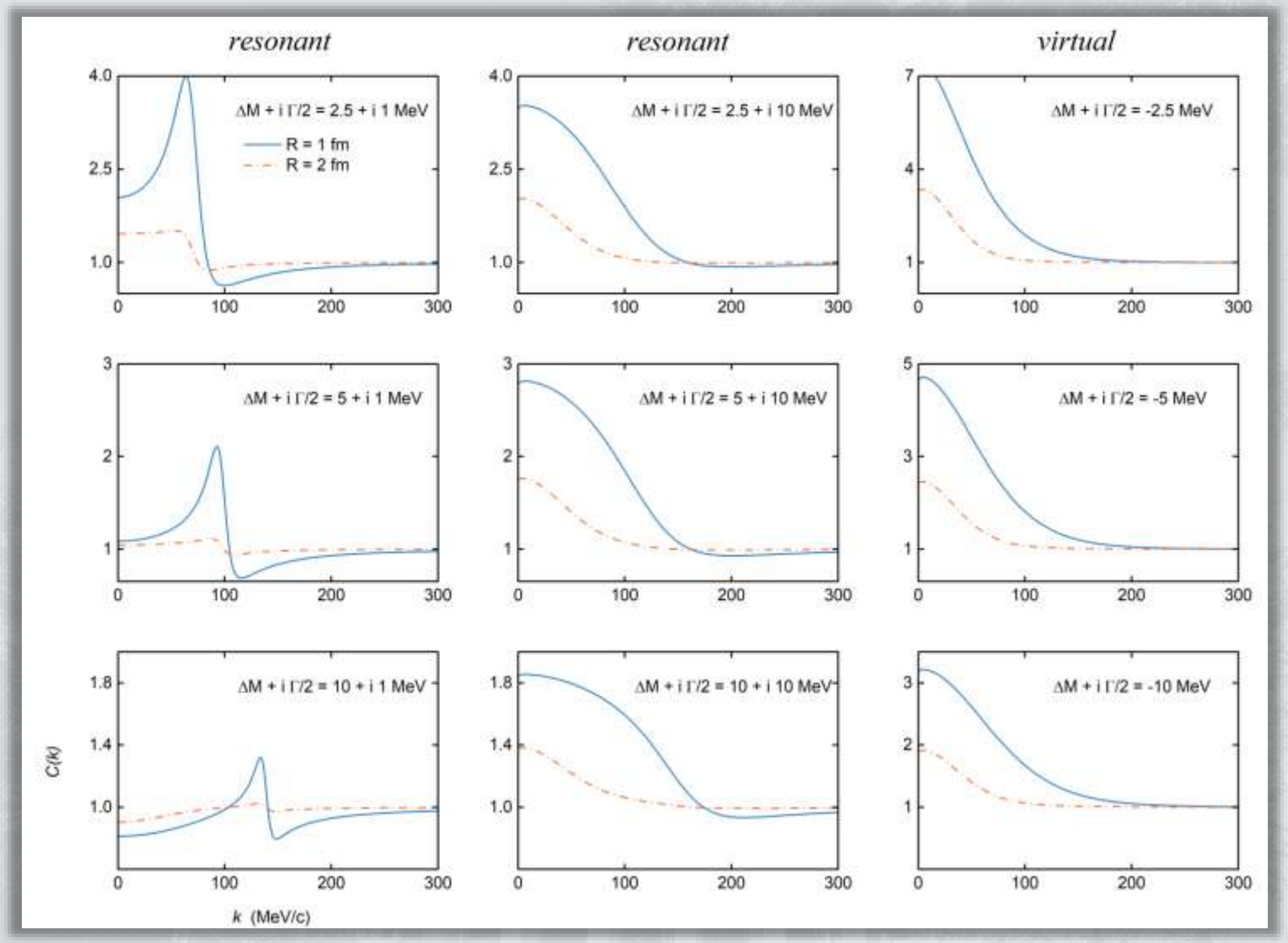


$$C(k) = 1 + \int S_{12}(\boldsymbol{r})[|\psi_0(k,r)|^2 - |j_0(kr)|^2]\, d\boldsymbol{r}$$

The Femtoscopic Correlation Function (CF)

4. CFs with resonant and virtual states

Z.-W. Liu, J.-X. Lu, M.-Z. Liu, and L.-S. Geng, Sci. Bull. 70, 3515 (2025)



Framework of CFs (S-wave)

From the Koonin-Pratt formula:

$$C(k) = \int \mathrm{d}^3r\, S_{12}(\boldsymbol{r})|\Psi(\boldsymbol{k};\boldsymbol{r})|^2$$

The most common situation:

$$\Psi(\boldsymbol{k};\boldsymbol{r}) = e^{i\boldsymbol{k}\cdot\boldsymbol{r}} - j_0(kr) + \psi_0(\boldsymbol{k};\boldsymbol{r})$$

$$C(k) = 1 + \int 4\pi r^2 \mathrm{d}r\, S_{12}(r)[|\psi_0(k,r)|^2 - |j_0(kr)|^2]$$

For identical particles:

$$\Psi(\boldsymbol{k};\boldsymbol{r}) = \sqrt{2}\chi_s[\cos(\boldsymbol{k}\cdot\boldsymbol{r}) + \psi_0(\boldsymbol{k};\boldsymbol{r}) - j_0(kr)] + \sqrt{2}i\chi_t \sin(\boldsymbol{k}\cdot\boldsymbol{r})$$

$$C(k) = 1 - \frac{1}{2}e^{-4k^2R^2} + \frac{1}{2}\int 4\pi r^2 \mathrm{d}r\, S_{12}(r)[|\psi_0(k,r)|^2 - |j_0(kr)|^2]$$

$\boldsymbol{D_s^+ D_s^-}$ **!**

For pairs with Coulomb interaction:

$$\Psi(\boldsymbol{k};\boldsymbol{r}) = \Phi(\boldsymbol{k};\boldsymbol{r}) - F_0(kr) + \psi_0(\boldsymbol{k};\boldsymbol{r})$$

$$C(k) = \int \mathrm{d}^3\boldsymbol{r} S_{12}(\boldsymbol{r})|\boldsymbol{\Phi}(\mathbf{k};\mathbf{r})|^2 + \int 4\pi r^2 dr\, S_{12}(r)[|\psi_0(k,r)|^2$$

$$\Phi(\boldsymbol{k};\boldsymbol{r},z) = e^{-\frac{\pi\gamma}{2}}\Gamma(1+i\gamma)\mathrm{e}^{ikz}\ {}_1F_1(-i\gamma;1;ik(r-z))$$

j_ν & F_ν are spherical Bessel function & Coulomb function of order ν ,

$\Phi(\boldsymbol{k};\boldsymbol{r},z)$ is the complete Coulomb wave function

$\Gamma(z)$ is Euler gamma function,

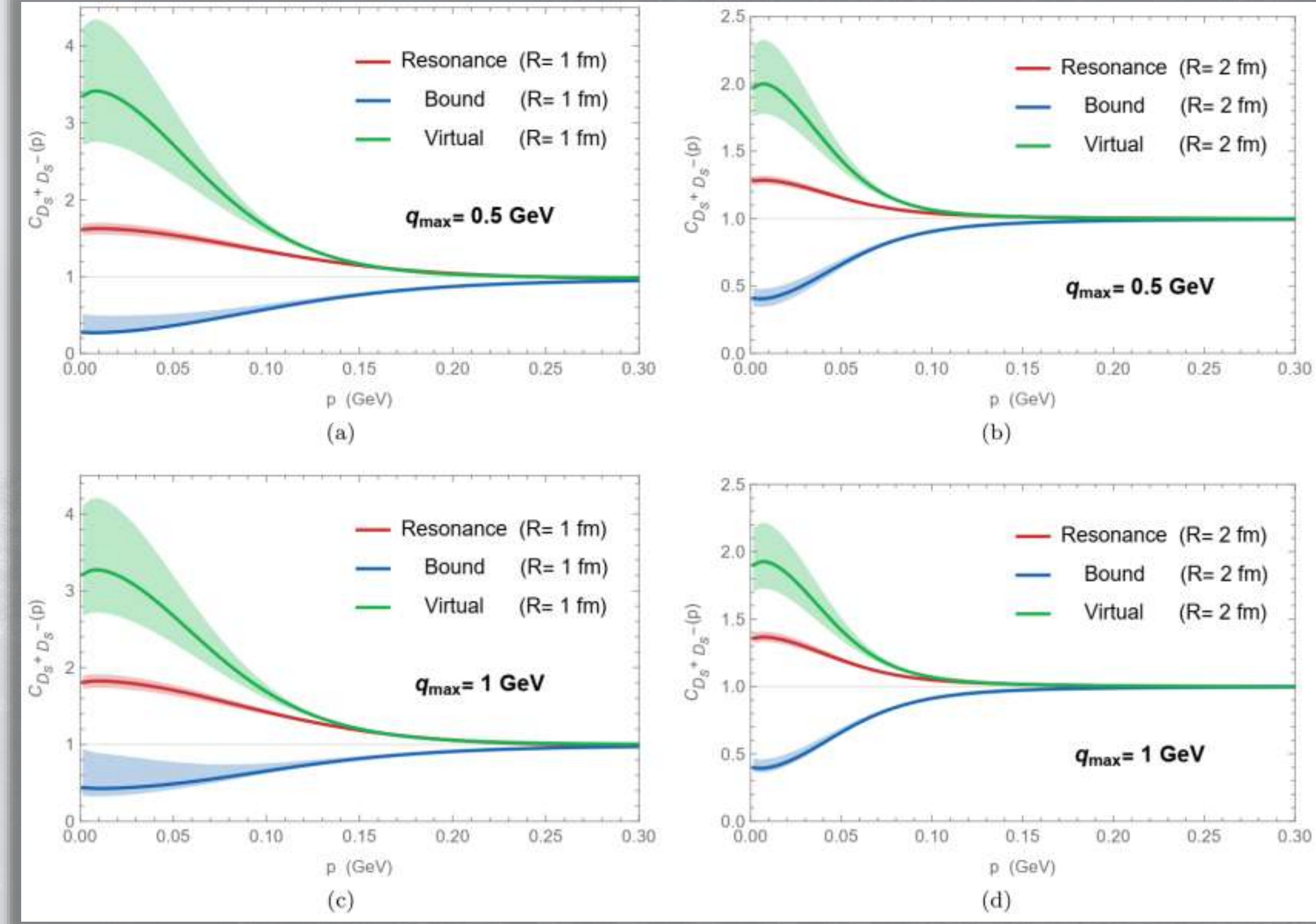
${}_1F_1(a;b,z)$ is the confluent hypergeometric function,

γ is the Sommerfeld parameter ($\gamma = Z_1Z_2\mu\alpha/k$)

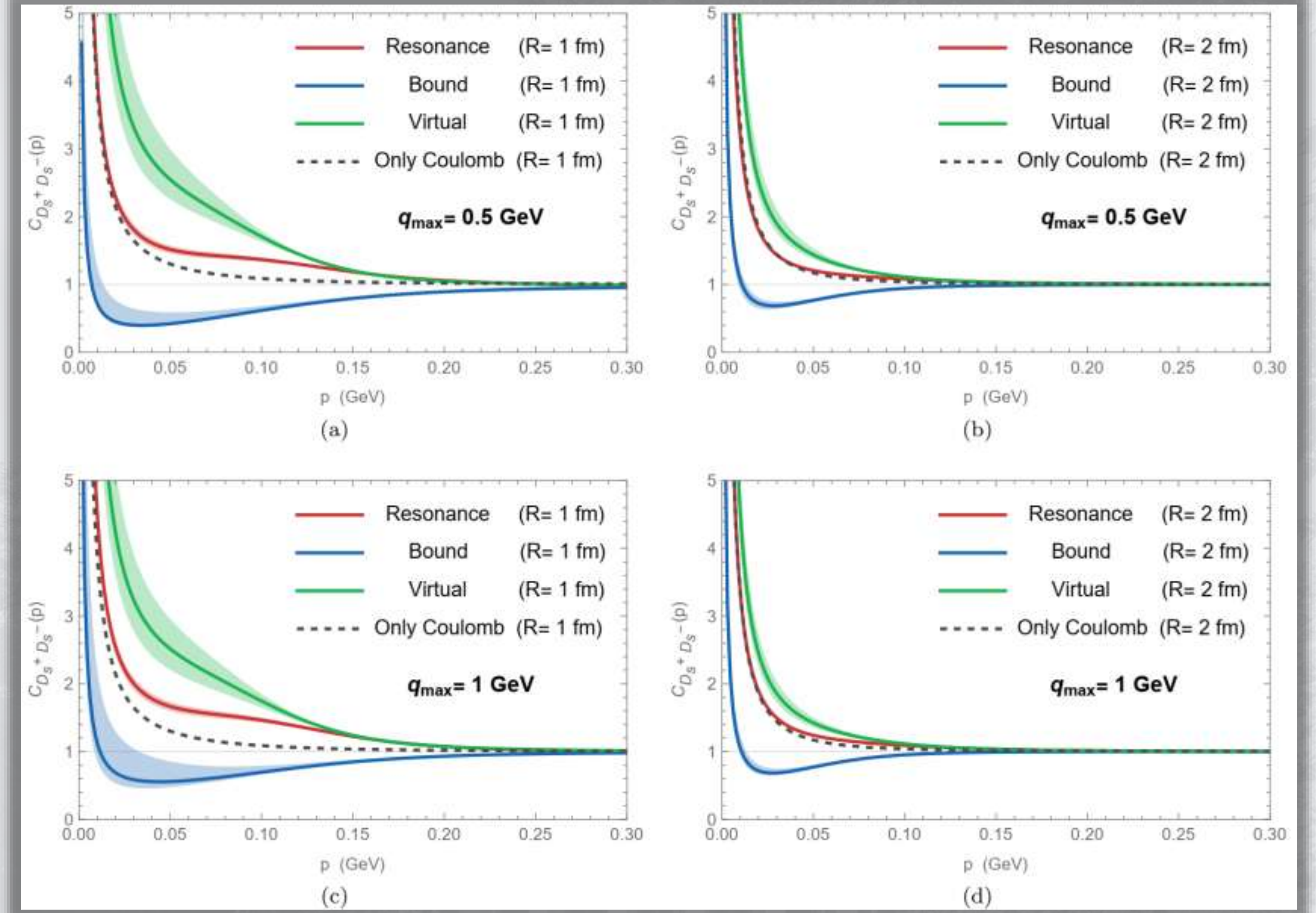
Here we adopt a Gaussian source function

$$S_{12}(r) = \frac{1}{(4\pi)^{3/2}R^3}e^{-\frac{r^2}{4R^2}}$$

CFs without Coulomb interaction



CFs with Coulomb interaction



P A R T F O U R

Conclusion

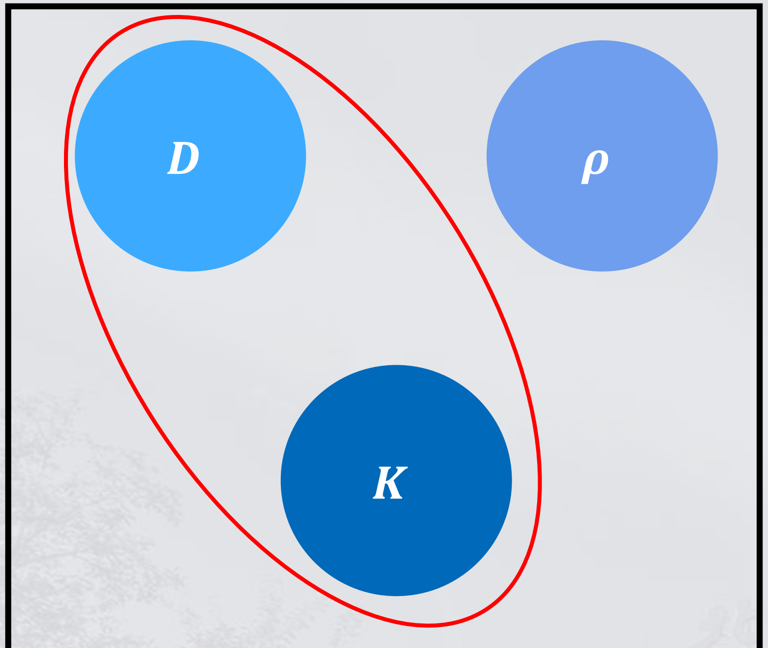
Conclusions and Outlook

- The invariant mass distribution alone can hardly distinguish whether a state is bound, virtual, or resonant.
- The Correlation Function can be used to distinguish the different scenarios of a state.
- Even in systems with Coulomb interaction, CFs can also work well to show the difference between scenarios.
- The resolution is expected to be better in smaller collision systems.

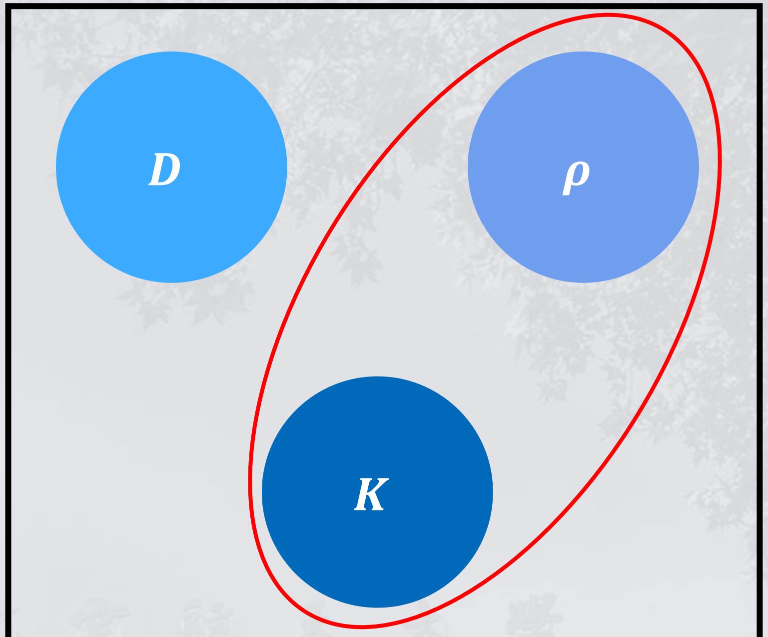
What if we consider simultaneously the CFs and scattering observable (e.g. Phase Shift)?

We are working on the $\kappa(700)$ resonance (πK interaction).

Extention to 3-body systems ($DK\rho$)



$D_{s0}(2317)$: $I(J^P) = 0(0^+)$



$K_1(1270)$: $I(J^P) = \frac{1}{2}(1^+)$

D: $I(J^P) = \frac{1}{2}(0^-)$

K: $I(J^P) = \frac{1}{2}(0^-)$

ρ: $I^G(J^{PC}) = 1^+(1^{--})$

To study the properties of :

1. $D_{s1}^*(2860)$: $I(J^P) = 0\ (1^-)$: Considered to be a $K_1(1270)D$ resonance.
2. The exotic state $c\bar{s}u\bar{d}$ ($I = 1$): Considered to be a $D_{s0}(2317)\rho$ resonance.

The reference studying the $X(2900)$ [$\bar{c}\bar{s}ud$]: *Malabarba, Khemchandani, Torres, and Oset PHYS. REV. D 107, 036016 (2023)*

Thanks for Your Attention

P A R T ? ? ?

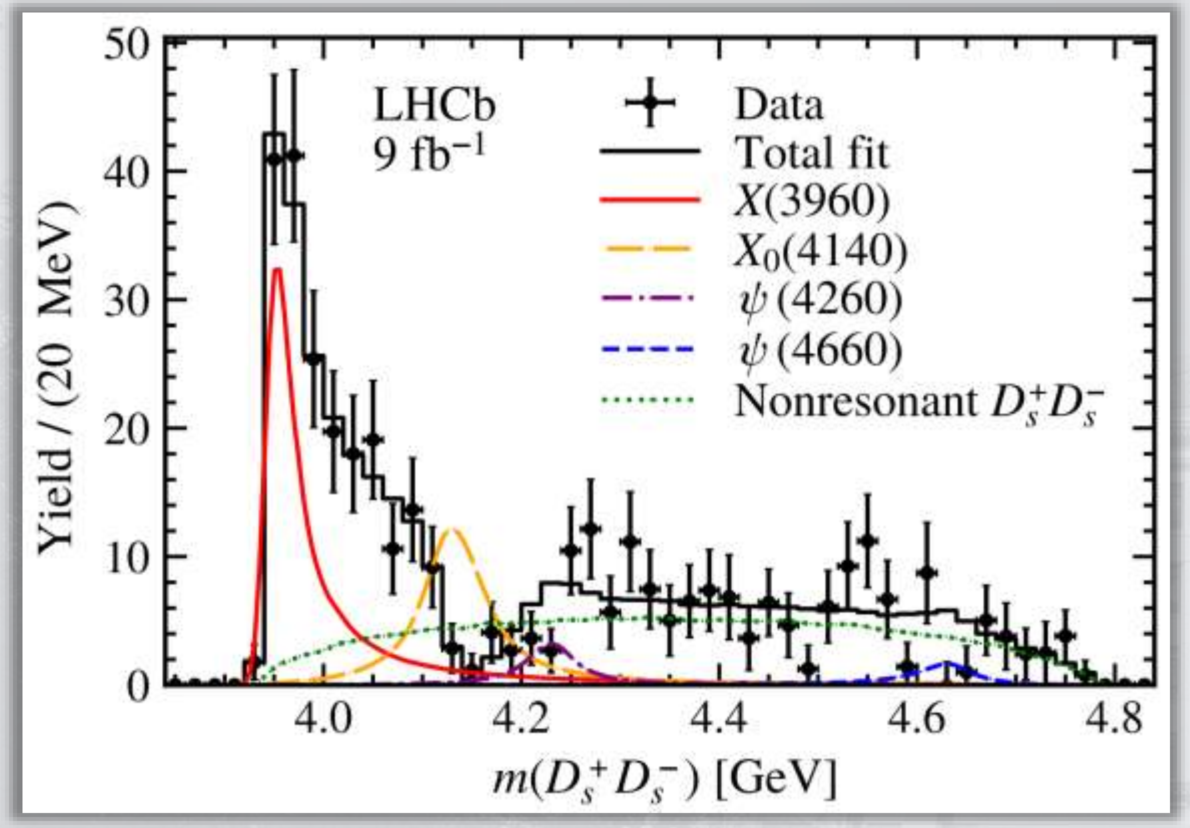
Backup Slides

The invariant mass spectrum of $D_s^+ D_s^-$

北京航空航天大學 BEIHANG UNIVERSITY

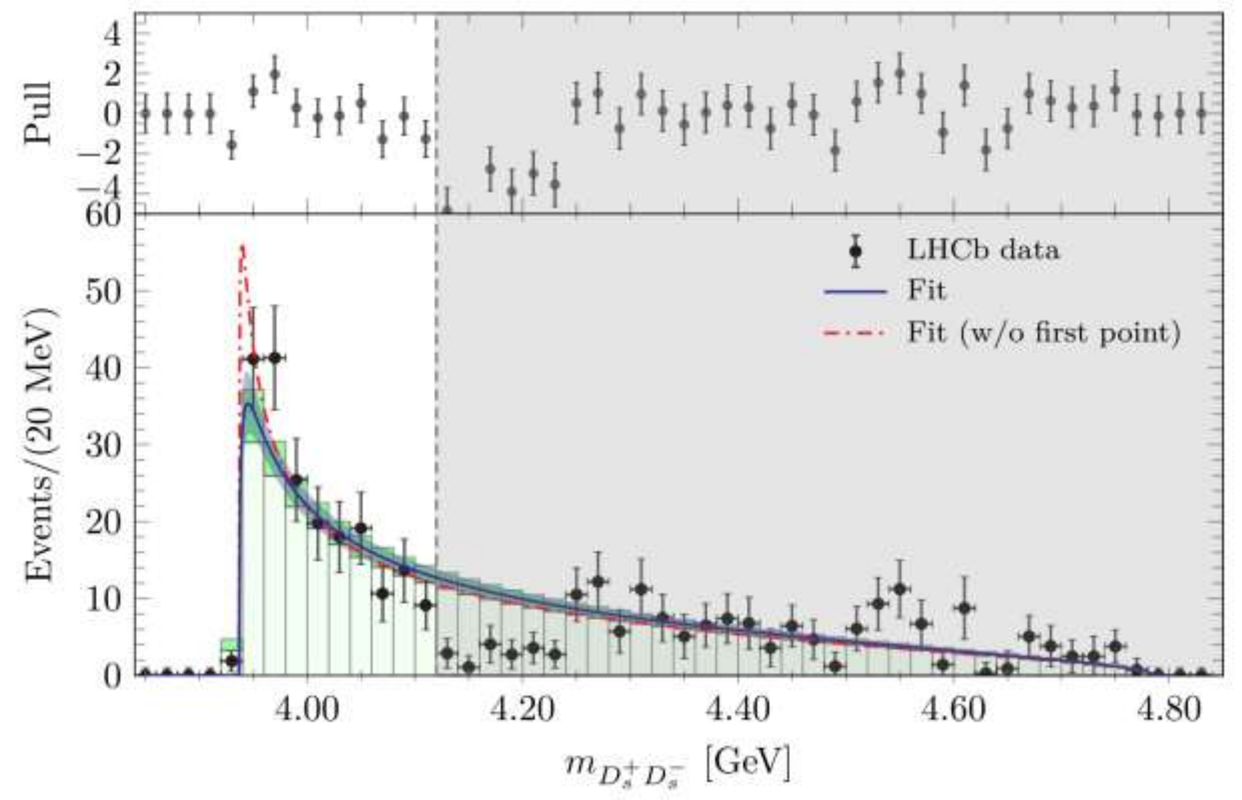
Different ways to understand the Invariant Mass Distribution (IMD) of $D_s^+ D_s^-$

Resonance



LHCb Collaboration,
Phys. Rev. Lett. 131, 071901 (2023)

Bound & Virtual

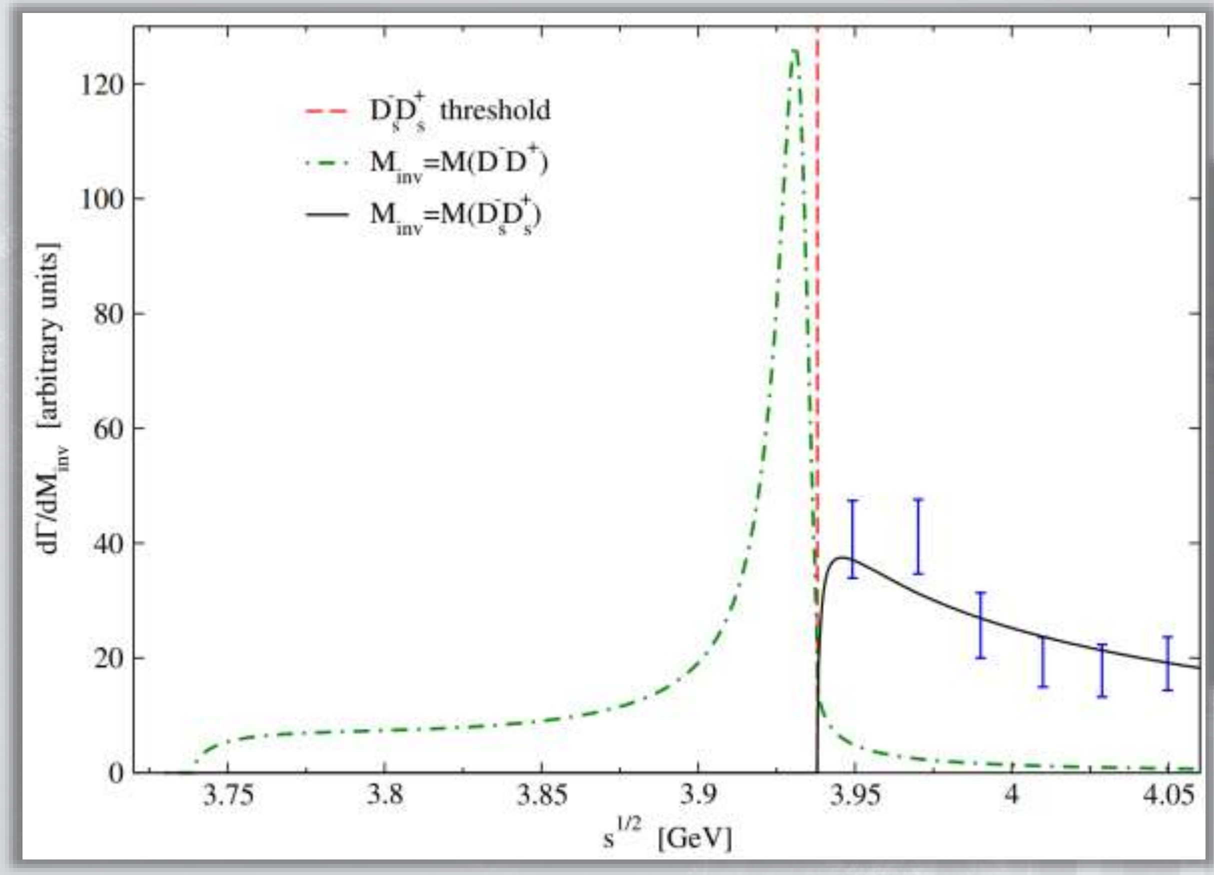


T. Ji, X.-K. Dong, M. Albaladejo, M.-L. Du, F.-K. Guo, and J. Nieves,
Phys. Rev. D 106, 094002 (2022),

The invariant mass spectrum of $D_s^+ D_s^-$

"The conclusion is then that there is not need to invoke a new $X(3960)$ state, and the experimental observation is due to the presence of the $X(3930)$."

——A work on the $D\bar{D}$ and $D_s\bar{D}_s$ coupled channel system.



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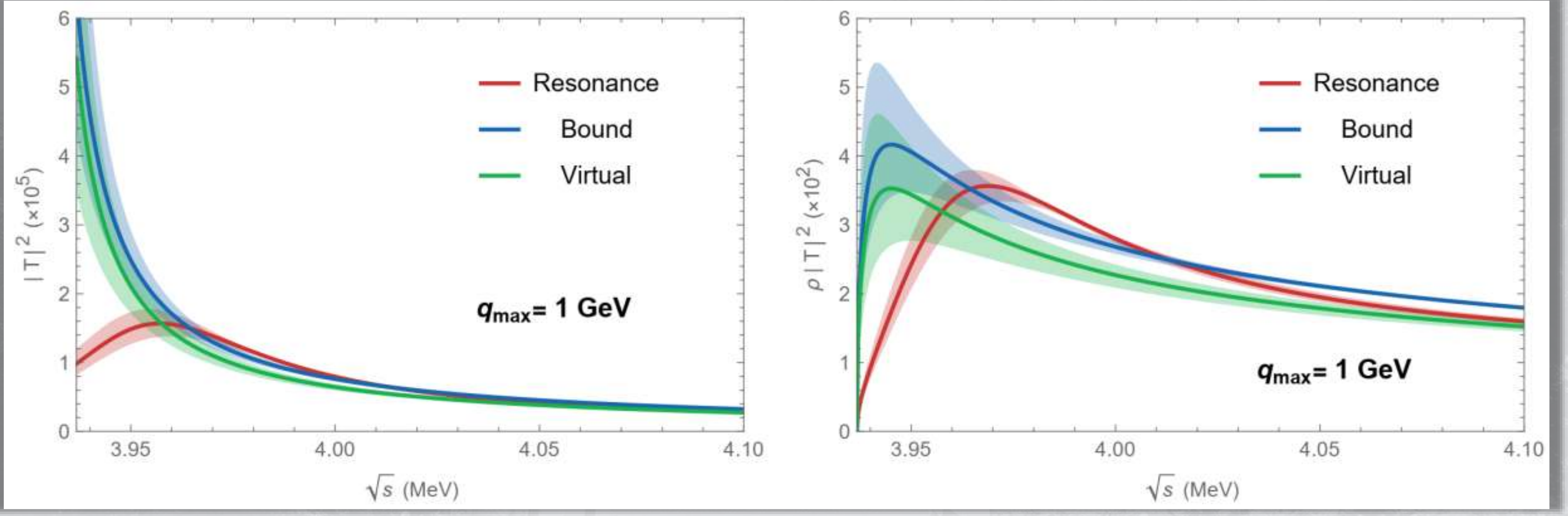
M. Bayar, A. Feijoo, and E. Oset, Phys. Rev. D 107, 034007 (2023)

$|T|^2$ and $\rho|T|^2$

$|T|^2$ $\rho|T|^2$



Coulomb interaction in CFs

We first divide the Coulomb interaction into short-range and long-range part

$$V^C_{total} = V^C \Theta(R_C - r) + V^C_{long} \Theta(r - R_C)$$

Then, the Coulomb potential in momentum space:

$$V^C(|\boldsymbol{p}' - \boldsymbol{p}|; R_C) = \int_0^{R_C} \mathrm{d}^3 r\, e^{i(\boldsymbol{p}'-\boldsymbol{p})\cdot \boldsymbol{r}} \frac{\varepsilon\alpha}{r}$$
$$= \frac{4\pi\varepsilon\alpha}{|\boldsymbol{p}' - \boldsymbol{p}|^2}[1 - \cos(|\boldsymbol{p}' - \boldsymbol{p}|R_C)]$$

The S-wave component:

$$V_0^C(p', p; R_C) = \frac{1}{2}\int_{-1}^{1} \mathrm{d}(\cos\theta_{\boldsymbol{p}\boldsymbol{p}'})\, V^C(|\boldsymbol{p}' - \boldsymbol{p}|; R_C)$$
$$= \frac{2\pi\varepsilon\alpha}{pp'}\left\{\mathrm{Ci}[|p' - p|R_C] - \mathrm{Ci}[|p' + p|R_C] + \ln\left(\frac{p' + p}{|p' - p|}\right)\right\}$$

Where the $\mathrm{Ci}[x] = \int_x^\infty \mathrm{d}t \frac{\cos t}{t}$ is the cosine integral function.

Then, the relativistic corrections:

$$V_0^{C,rel}(p, p'; \sqrt{s}) = \sqrt{2\omega_1(p)}\sqrt{2\omega_2(p)}\sqrt{\xi\,(p; s)}$$
$$\times V_0^C(p', p\,)$$
$$\times \sqrt{2\omega_1(p')}\sqrt{2\omega_2(p')}\sqrt{\xi\,(p'; s)}$$

The kinematic factor:

$$\xi(p; s) = 2\mu \frac{\sqrt{s} - \omega_1(p) - \omega_2(p)}{\frac{\lambda(s, m_1^2, m_2^2)}{4s} - p^2}$$

Where the $\mu = m_1 m_2/(m_1 + m_2)$ is the reduced mass, and $\lambda(s, m_1^2, m_2^2) = [s - (m_1 + m_2)^2][s - (m_1 - m_2)^2]$. The $\sqrt{\omega_i}$ term is related to the form of loop function.

Then, we have a complete S-wave V in the short-range:

$$V_0(p, p'; \sqrt{s}) = V_0^S + V_0^{C,rel}(p, p'; \sqrt{s})$$

P. Encarnacion, A. Feijoo, V. M. Sarti, and A. Ramos, Phys. Rev. D 111, 114013 (2025)

Coulomb interaction in CFs

Then, by solving the Bethe-Salpeter equation (Off-shell), We can obtain the T-matrix:

$$T(p,p';\sqrt{s}) = V_0(p,p';\sqrt{s}) + V_0(p,q;\sqrt{s})G_q(\sqrt{s})T(q,p';\sqrt{s})$$

But in our work, we make a **Born approximation** to the Coulomb interaction to simplify the calculation. And also obtain a T-matrix.

Then, the wave function can be obtained by the Lippmann-Schwinger equation:

$$\psi(k,r) = j_0(kr) + \int_0^{q_{max}} \frac{q^2 dq}{2\pi^2} \frac{1}{2\omega_1\, 2\omega_2} \frac{T(k,q;\sqrt{s})\, j_0(qr)}{\sqrt{s} - \omega_1 - \omega_2 + i\varepsilon}$$

This wave function contains the information of the strong interaction and the short-range Coulomb interaction, but does not have a proper asymptotic behavior.
Therefore, a correction is required.

The correction begins with forming two asymptotic wave functions:

$$\psi_{short}^{asy} = j_0 + h_0 \rho T$$

$$\psi_{long}^{asy} = F_0 + H_0 \rho \tilde{T}$$

Where the j_0 & h_0 are the spherical Bessel function & Hankel function; the F_0 & H_0 are the Coulomb function and Coulomb-Hankel function.

The $\tilde{T}$ can be obtain by the boundary condition:

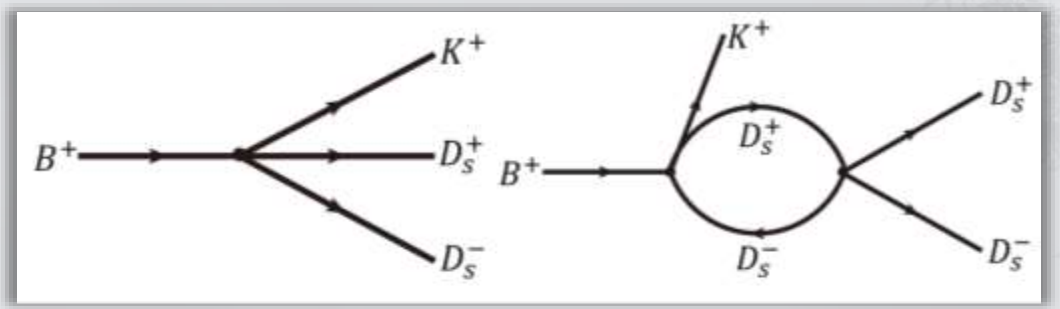
$$\left.\frac{\psi_{short}^{asy}{}'(k,r)}{\psi_{short}^{asy}(k,r)}\right|_{r=R_C} = \left.\frac{\psi_{long}^{asy}{}'(k,r)}{\psi_{long}^{asy}(k,r)}\right|_{r=R_C}$$

Then, the correction function is:

$$\zeta(k) = \psi_{long}^{asy}(k,R_C)/\psi_{short}^{asy}(k,R_C)$$

Finally, the wave function we use for CF writes: $\zeta(k)\psi(k,r)$.

Invariant mass distribution → potential

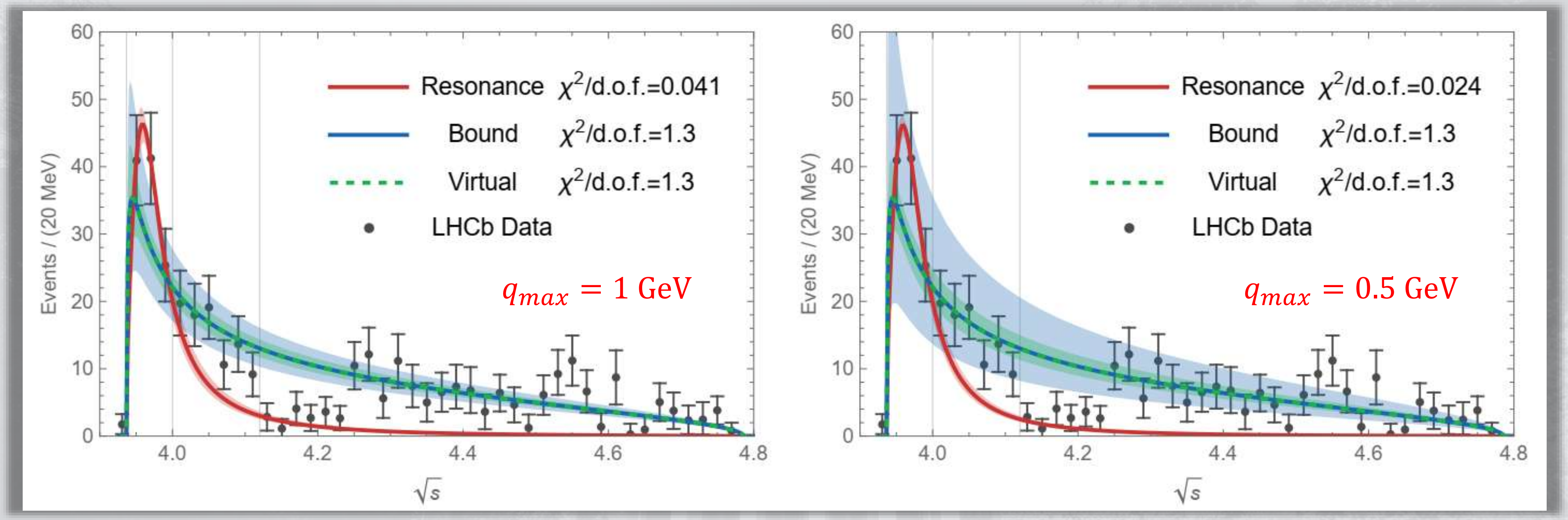


In our work: $V^S = a + bk^2$

(The parameter a and b are considered to be the LECs of EFT)

$$\frac{d\Gamma}{dM_{inv}} = \frac{1}{(2\pi)^3}\frac{1}{4m_B^2}p_{K^+}\tilde{p}_{D_s}|t|^2$$

$$t(M_{inv}) = c[1 + G(M_{inv})T^{(S)}(M_{inv})]$$



Parameters

$$V = a + b\,k^2$$

$$a_0 = \left.\frac{T^{(S)}}{8\pi\sqrt{s}}\right|_{s=s_{th}}$$

$$r_0 = \left.\frac{\partial^2}{\partial x^2}\left(-8\pi\sqrt{s}{T^{(S)}}^{-1} + ik\right)\right|_{s=s_{th}}$$

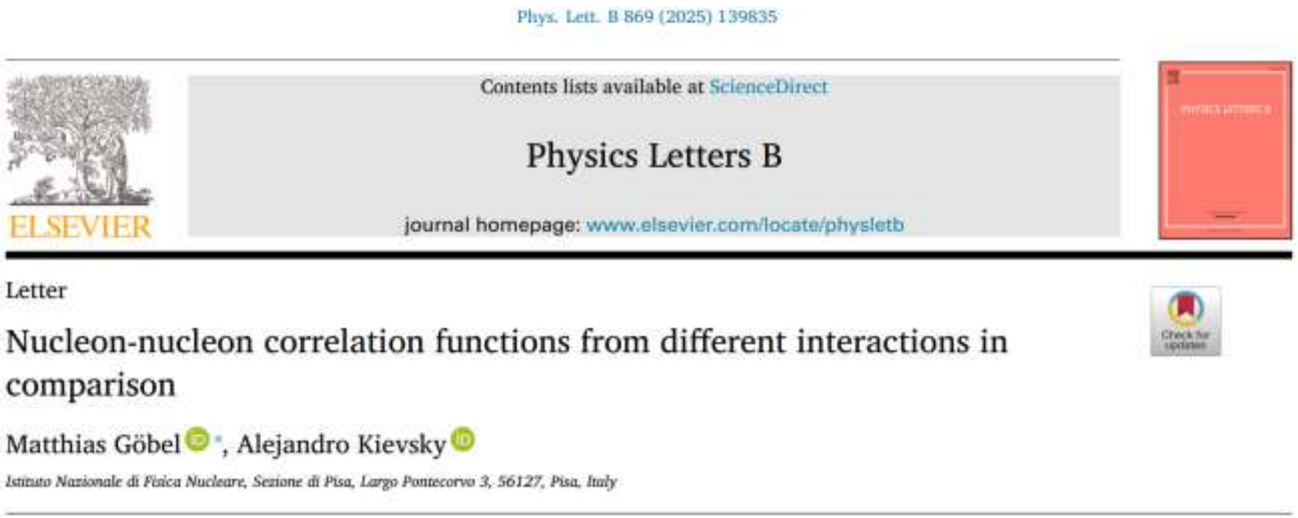
$$m_{X(3960)} = 3956 \pm 5 \pm 10 \text{ MeV}, \quad \Gamma_{X(3960)} = 43 \pm 13 \pm 8 \text{ MeV}$$

$$m_{X(3930)} = 3924 \pm 2 \text{ MeV}, \quad \Gamma_{X(3930)} = 17 \pm 5 \text{ MeV}$$

TABLE I. Relevant quantities obtained from the fits to the LHCb data in Ref. [1]: mass and width of the pole position, the LECs a, b, the scattering length a_0, and the effective range r_0. The $D_s^+ D_s^-$ threshold is 3936.7 MeV.

$q_{max} = 1.0$ GeV						
Scenario	M [MeV]	Γ [MeV]	a	b [10^{-6} MeV^{-2}]	a_0 [fm]	r_0 [fm]
Resonance	$3948.79^{+2.44}_{-3.33}$	$60.04^{+10.83}_{-8.72}$	-106.67 ± 3.53	-629.75 ± 73.62	$-0.63^{+0.06}_{-0.07}$	$-1.86^{+0.28}_{-0.30}$
Bound	$3928.27^{+3.55}_{-4.07}$	0	-202.61 ± 11.22	0.00	$1.60^{+0.48}_{-0.28}$	$0.28^{+0.001}_{-0.001}$
Virtual	$3928.28^{+3.31}_{-4.44}$	0	-132.57 ± 5.67	0.00	$-1.47^{+0.29}_{-0.43}$	$0.29^{+0.002}_{-0.002}$
$q_{max} = 0.5$ GeV						
Scenario	M [MeV]	Γ [MeV]	a	b [10^{-6} MeV^{-2}]	a_0 [fm]	r_0 [fm]
Resonance	$3949.43^{+2.04}_{-2.60}$	$63.75^{+8.30}_{-6.98}$	-136.79 ± 8.17	-1431.33 ± 145.39	$-0.48^{+0.05}_{-0.05}$	$-2.45^{+0.44}_{-0.51}$
Bound	$3928.27^{+3.85}_{-3.73}$	0	-517.19 ± 73.08	0.00	$1.60^{+0.53}_{-0.26}$	$0.51^{+0.002}_{-0.001}$
Virtual	$3928.28^{+3.24}_{-4.57}$	0	-220.19 ± 15.64	0.00	$-1.47^{+0.30}_{-0.42}$	$0.53^{+0.002}_{-0.002}$

Off-shell ambiguities


Phys. Lett. B 869 (2025) 139835

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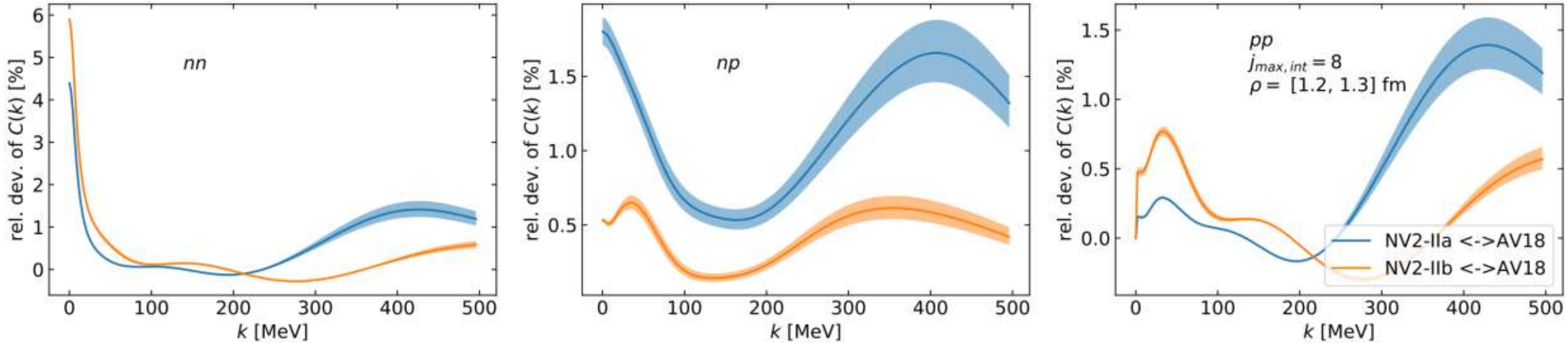
Letter

Nucleon-nucleon correlation functions from different interactions in comparison

Matthias Göbel *, Alejandro Kievsky

Istituto Nazionale di Fisica Nucleare, Sezione di Pisa, Largo Pontecorvo 3, 56127, Pisa, Italy

``The analysis of the differences between the correlations of the various interactions shows that for momenta between 0 and 500 MeV there are variations of up to 5.9 % for the nn system, of up to 1.8 % for the np system, and of 1.4 % for the pp system.''



Off-shell ambiguities

PHYSICAL REVIEW D **112**, 096006 (2025)

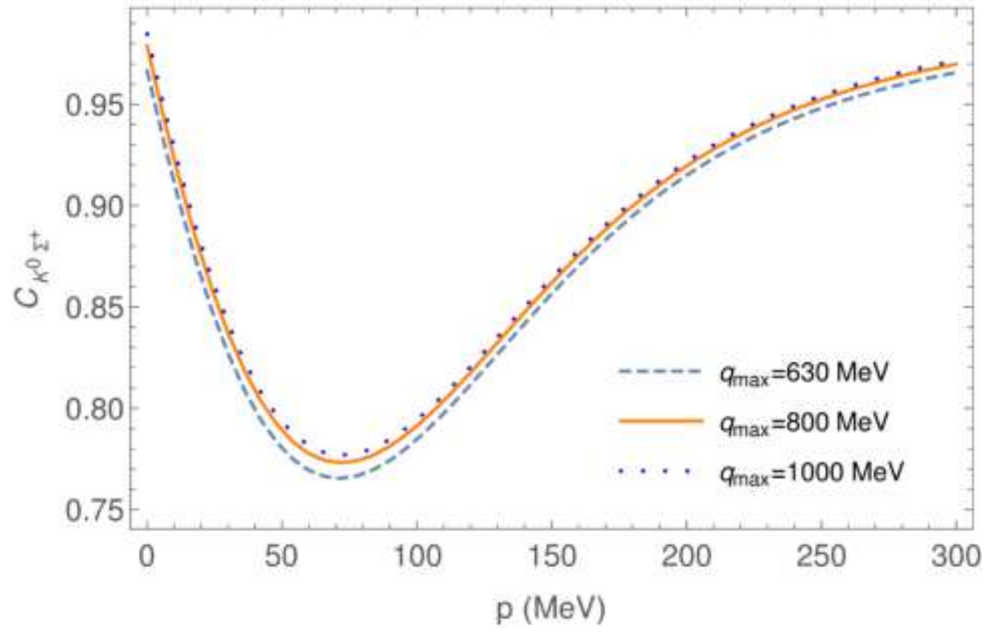
Determination of off-shell ambiguities in correlation functions: Strategies to minimize them

R. Molina* and E. Oset†

Departamento de Física Teórica and IFIC, Centro Mixto Universidad de Valencia-CSIC, Parc Científic UV, C/ Catedrático José Beltrán, 2, 46980 Paterna, Spain

(Received 28 July 2025; accepted 26 September 2025; published 6 November 2025)

``We find that using realistic interactions based on chiral dynamics the uncertainties are small, of the order of 2 - 3 %, but could be much bigger if other methods are used."



$$\mathcal{V}(\sqrt{s}; k, k') = V(\sqrt{s}) \cdot \theta(q_{\max} - k) \cdot \theta(q_{\max} - k'), \tag{1}$$

$$\mathcal{T}(\sqrt{s}; k, k') = T(\sqrt{s}) \cdot \theta(q_{\max} - k) \cdot \theta(q_{\max} - k'), \tag{2}$$

$$\psi(r, k) = j_0(kr) + T(\sqrt{s}) \cdot \theta(q_{\max} - k) \cdot \int_0^\infty \frac{d^3k'}{(2\pi)^3} \frac{E_1' + E_2'}{2E_1'E_2'} \frac{\theta(q_{\max} - k') \cdot j_0(k'r)}{s - (E_1' + E_2')^2 + i\varepsilon}, \tag{3}$$

PART ONE

Background

P A R T T W O

Framework & Method

P A R T T H R E E

Result

Contents

➢ Background Information

- The discovery of the $X(3930)$
- The invariant mass distribution of $D_s^+ D_s^-$
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➢ Framework

- The framework of the CF

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- The CF with/without Coulomb interaction

➢ Conclusions