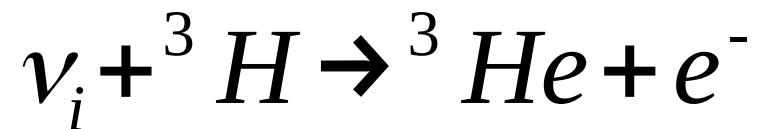


Question

Is there an experimental way of directly showing that the neutrino is a Dirac particle?

Dirac vs Majorana

Consider neutrino capture on Tritium



$$Amp \sim 1 - 2h \sqrt{E^2 - m^2}$$

Where h is the helicity of the incoming neutrino

Dirac vs Majorana

$$Amp \sim 1 - 2h \sqrt{E^2 - m^2}$$

If neutrinos are relativistic : $Amp \sim 1 - 2h E$

Right handed helical initial states ($h = \frac{1}{2}$) are suppressed

Dirac neutrinos : Antineutrinos won't interact.

Majorana neutrinos : The right-handed helical state, which is equivalent to the antineutrino, will be suppressed

$$\Gamma (Majorana) = \Gamma (Dirac)$$

Dirac vs Majorana

$$Amp \sim 1 - 2h \sqrt{E^2 - m^2}$$

If neutrinos are nonrelativistic : $Amp \sim 1$

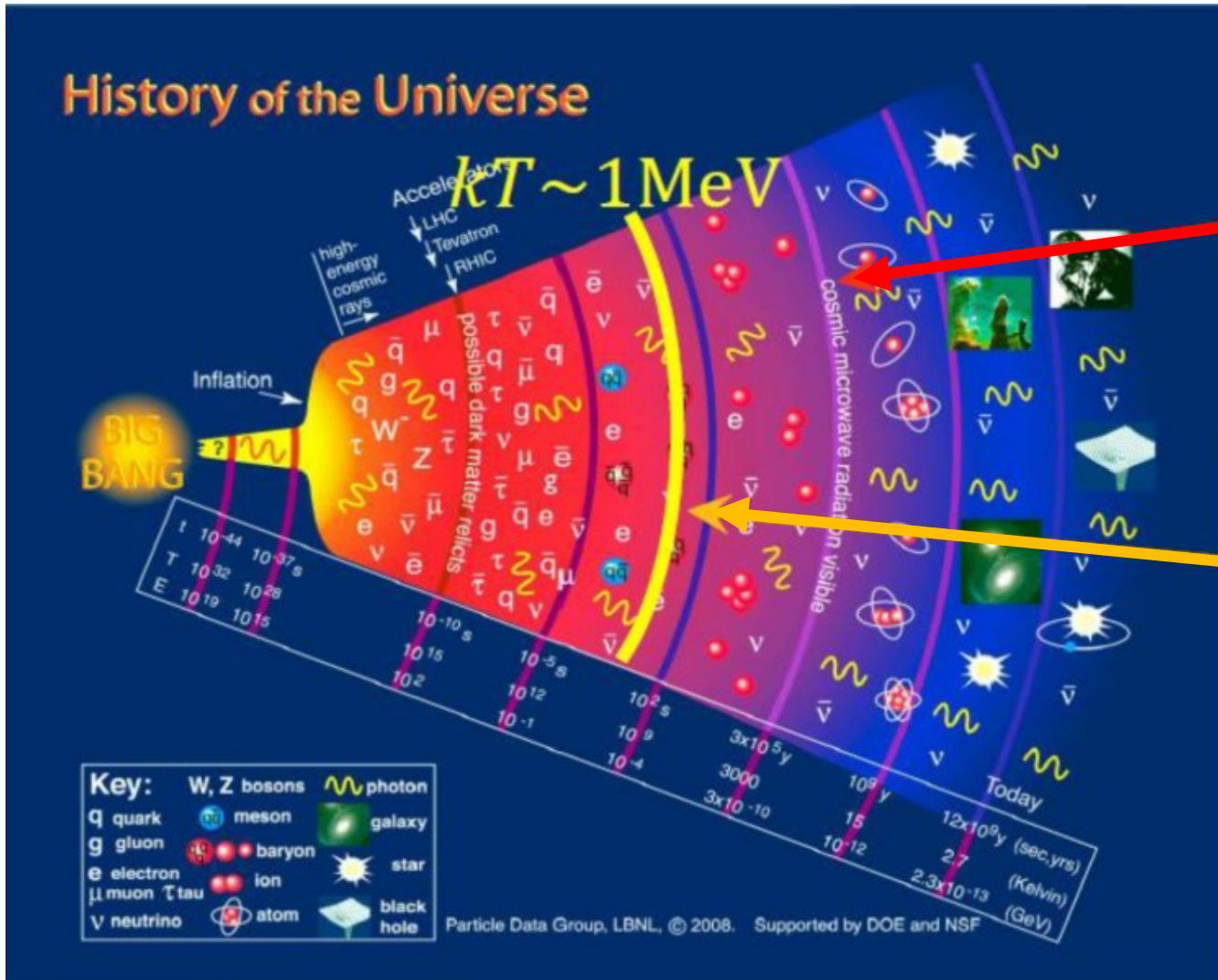
No helicity suppression

Dirac neutrinos : Antineutrinos will still not interact

Majorana neutrinos : No suppression; RH and LH helical states can interact.

$$\Gamma (Majorana) = 2 \Gamma (Dirac)$$

Relic Neutrinos



CMB

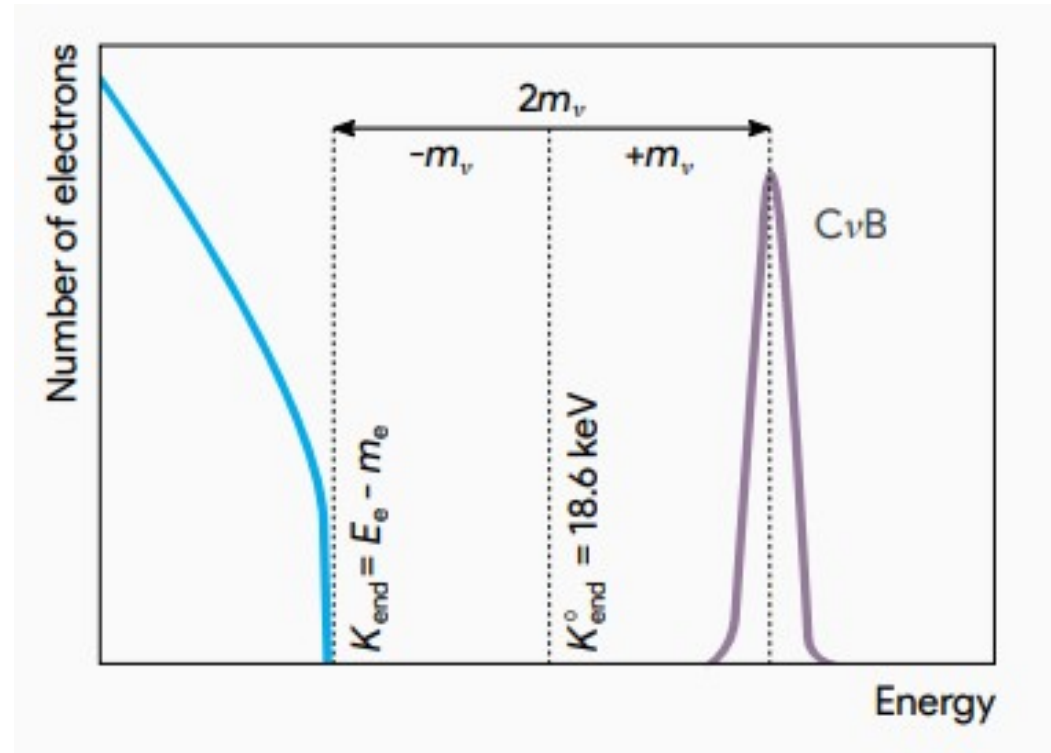
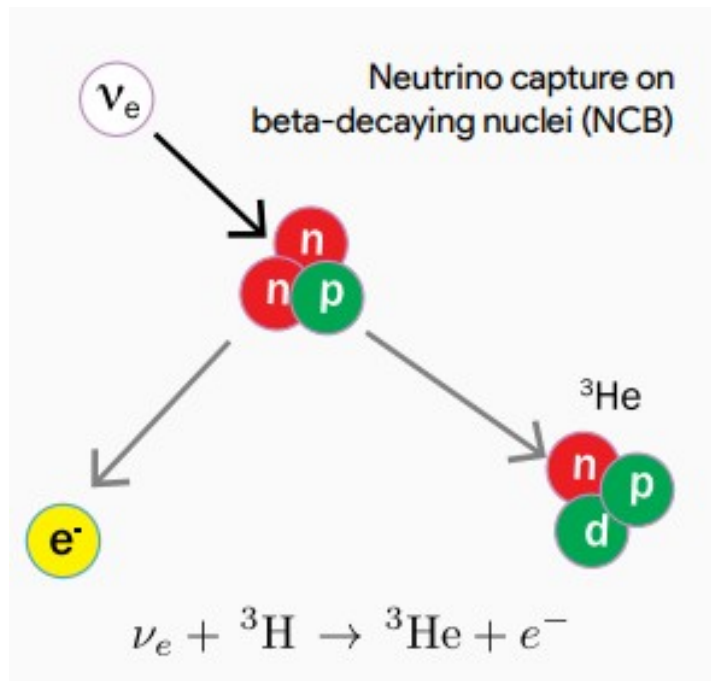
$C \nu B$

Cosmological
Neutrino
Background

Relic Neutrinos

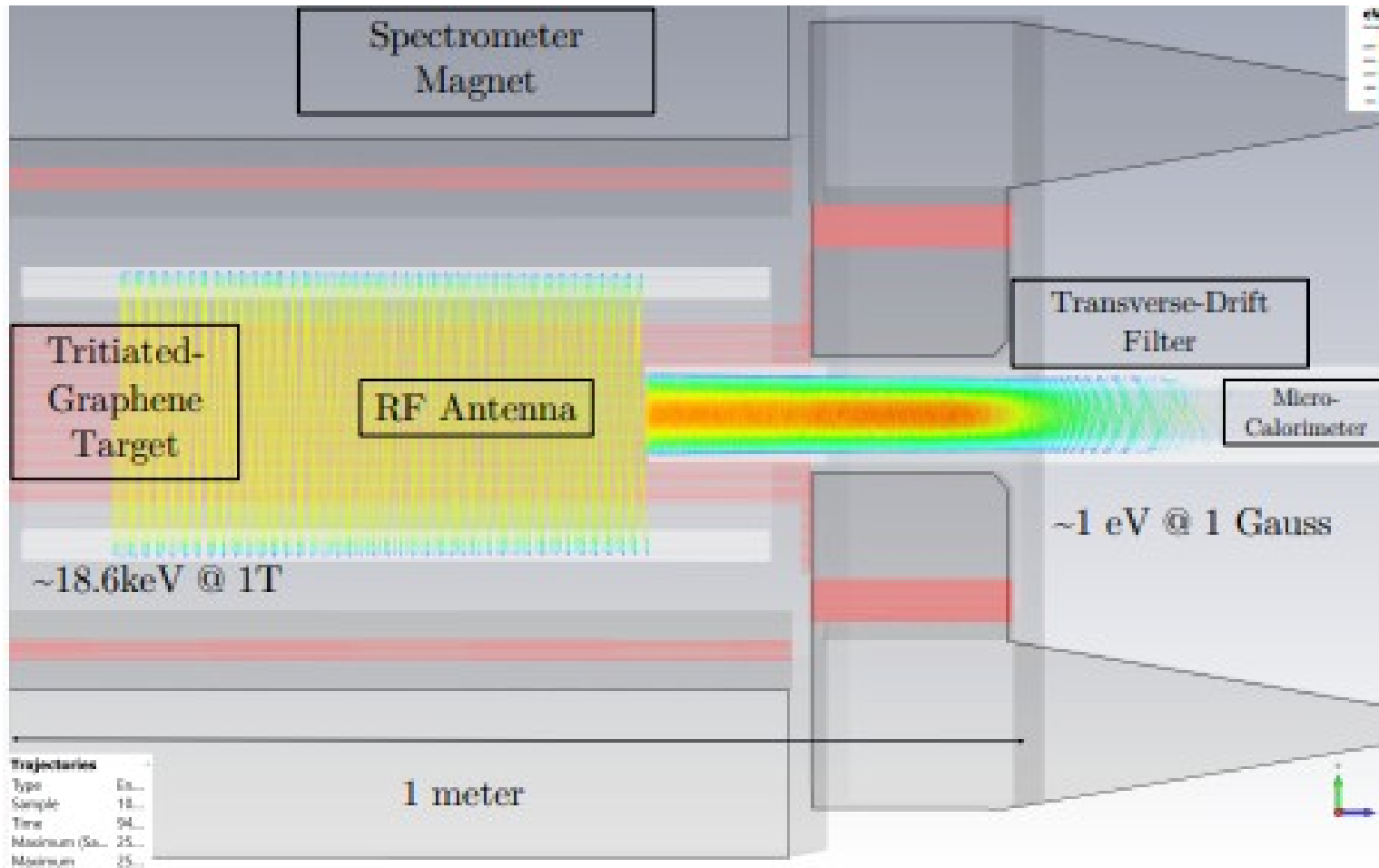
- ▶ Thought to have frozen out when the universe was around 1 second old
- ▶ Number density around $56 / \text{cm}^3$
- ▶ As the universe has expanded, these relic neutrinos have gotten colder : temperature is now around 2K
- ▶ Kinetic energy around $10^{-4} - 10^{-6} \text{ eV}$ – very non-relativistic (even with a neutrino mass around 10 meV)
- ▶ BUT : neutrino interaction cross sections are very small at these energies making detection extremely difficult

PTOLEMY



- ▶ Similar to beta decay experiments
- ▶ Look for a spike in event rate at an energy of $2Q_{\text{value}}$
- ▶ Need lots of tritium source and extremely good energy resolution.
- ▶ Can't copy KATRIN - need a solid tritium source

PTOLEMY



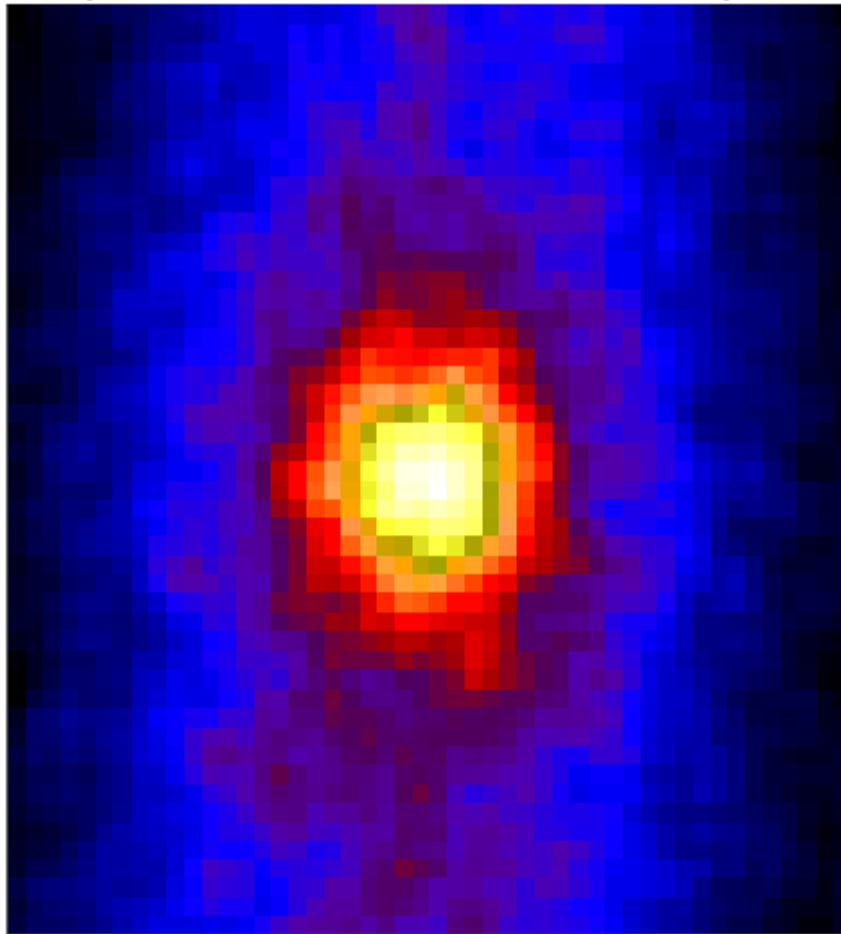
$$\Gamma(\textit{Dirac}) \sim 5 / \textit{year} \quad \Gamma(\textit{Majorana}) \sim 10 / \textit{year}$$

Lecture 3

The neutrino oscillation industry

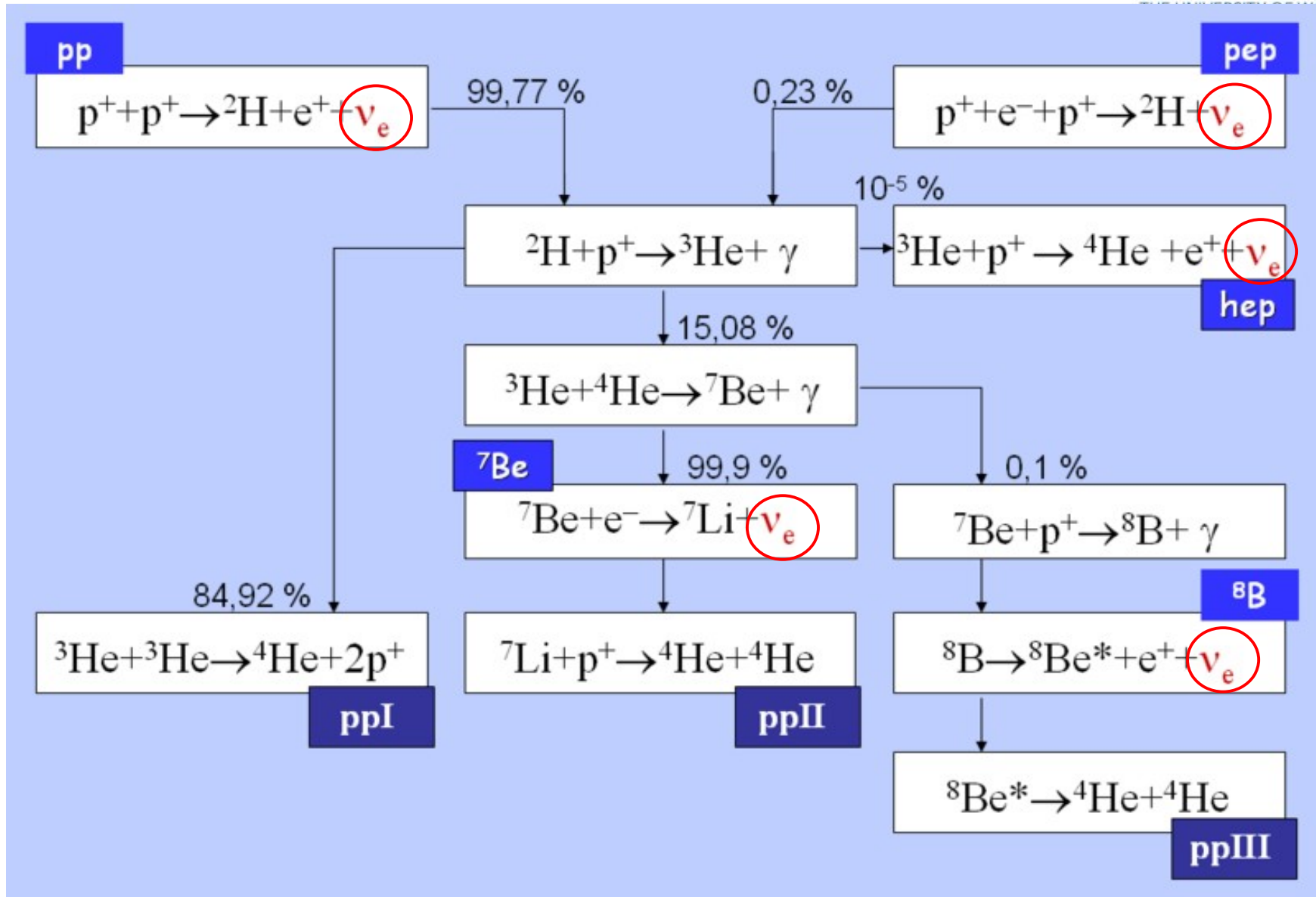
Solar Neutrinos

SuperK : Solar neutrino-gram

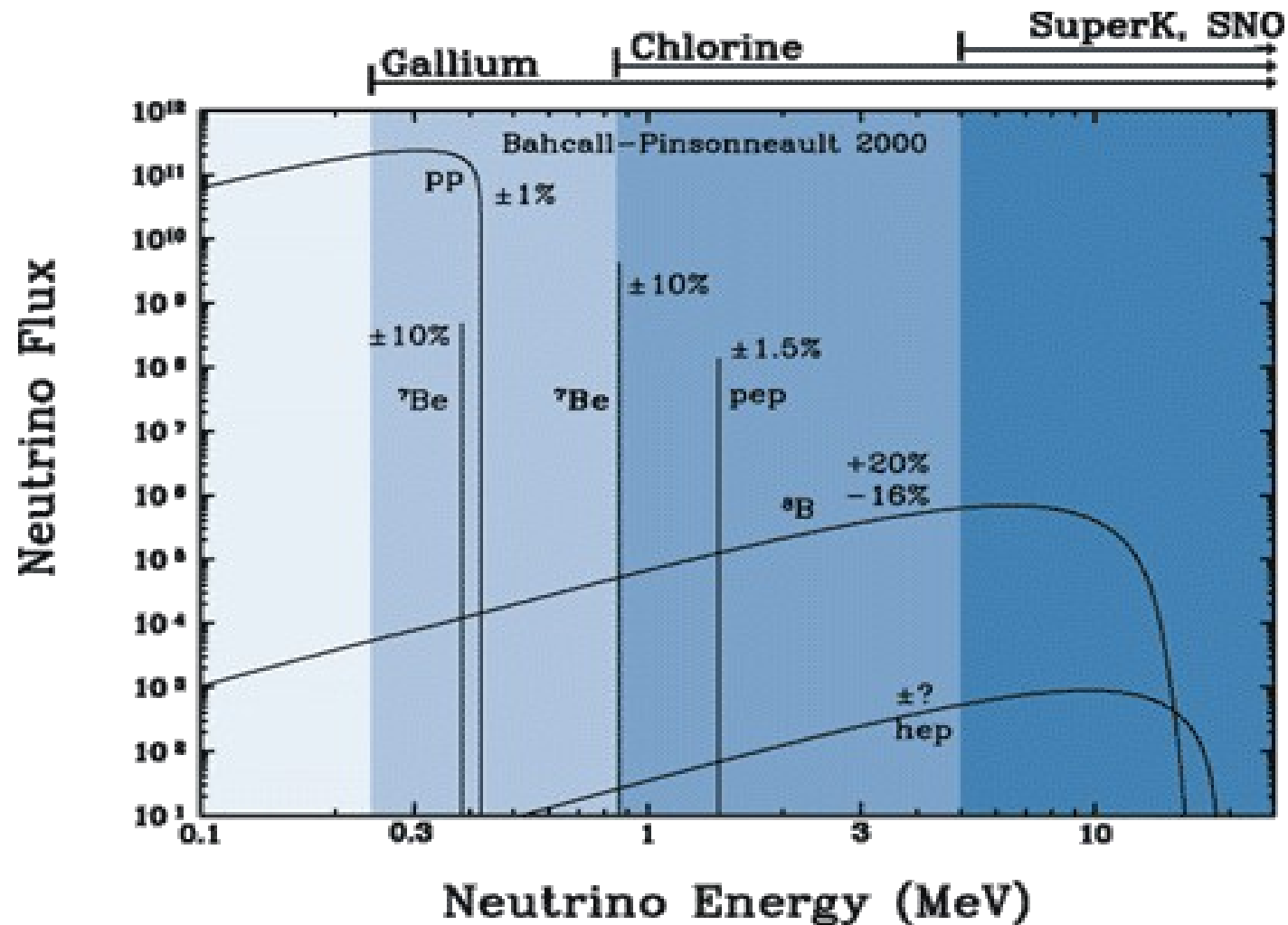


- Light from the solar core takes a million years to reach the surface
- Fusion processes generate electron neutrinos which take 2s to leave
- Solar neutrinos are a direct probe of the solar core
- Roughly 4.0×10^{10} solar ν_e per cm^2 per second on earth

Solar neutrino - pp Cycle

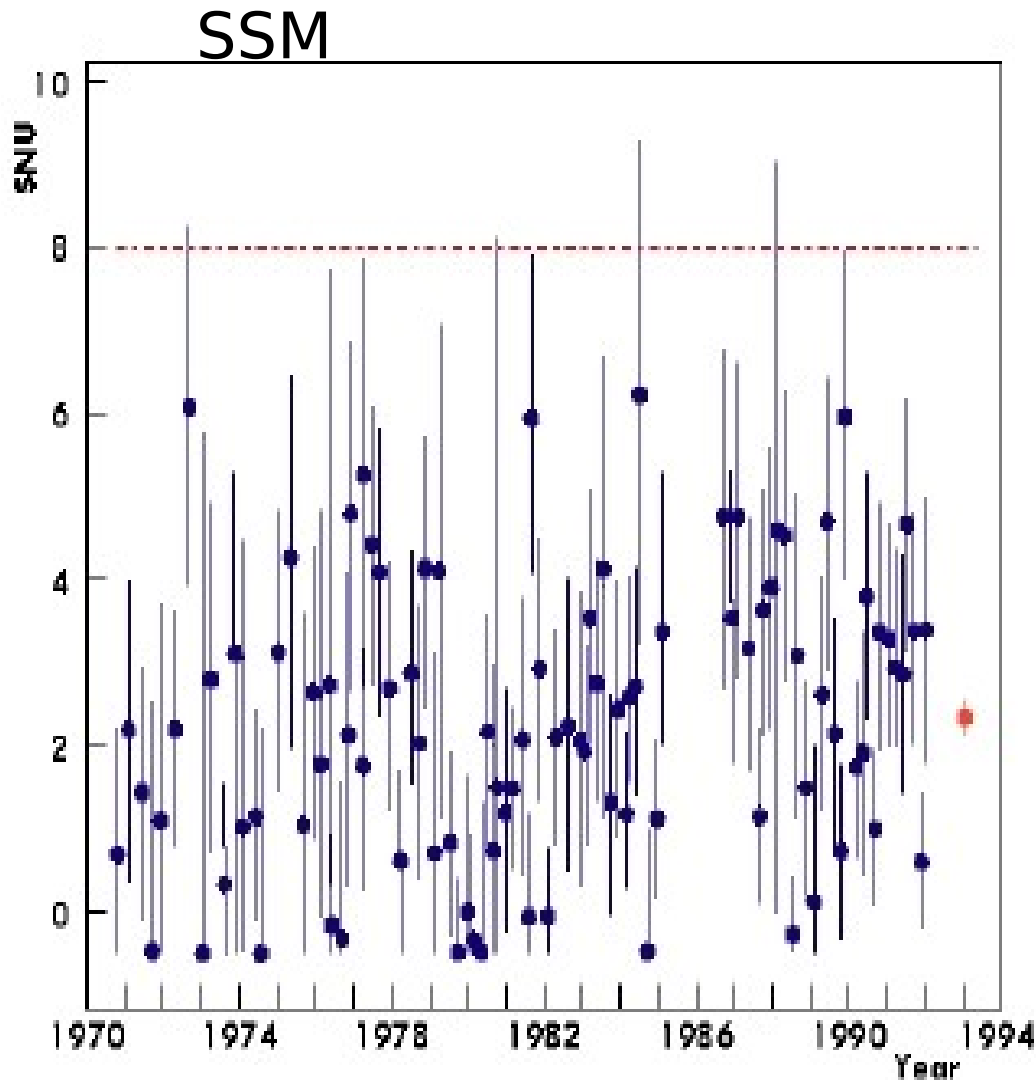


Solar Neutrino Flux



As predicted by Bahcall's Solar model

The Solar Neutrino Problem - Homestake



Homestake sensitive to ^8B and ^7Be *electron neutrinos*

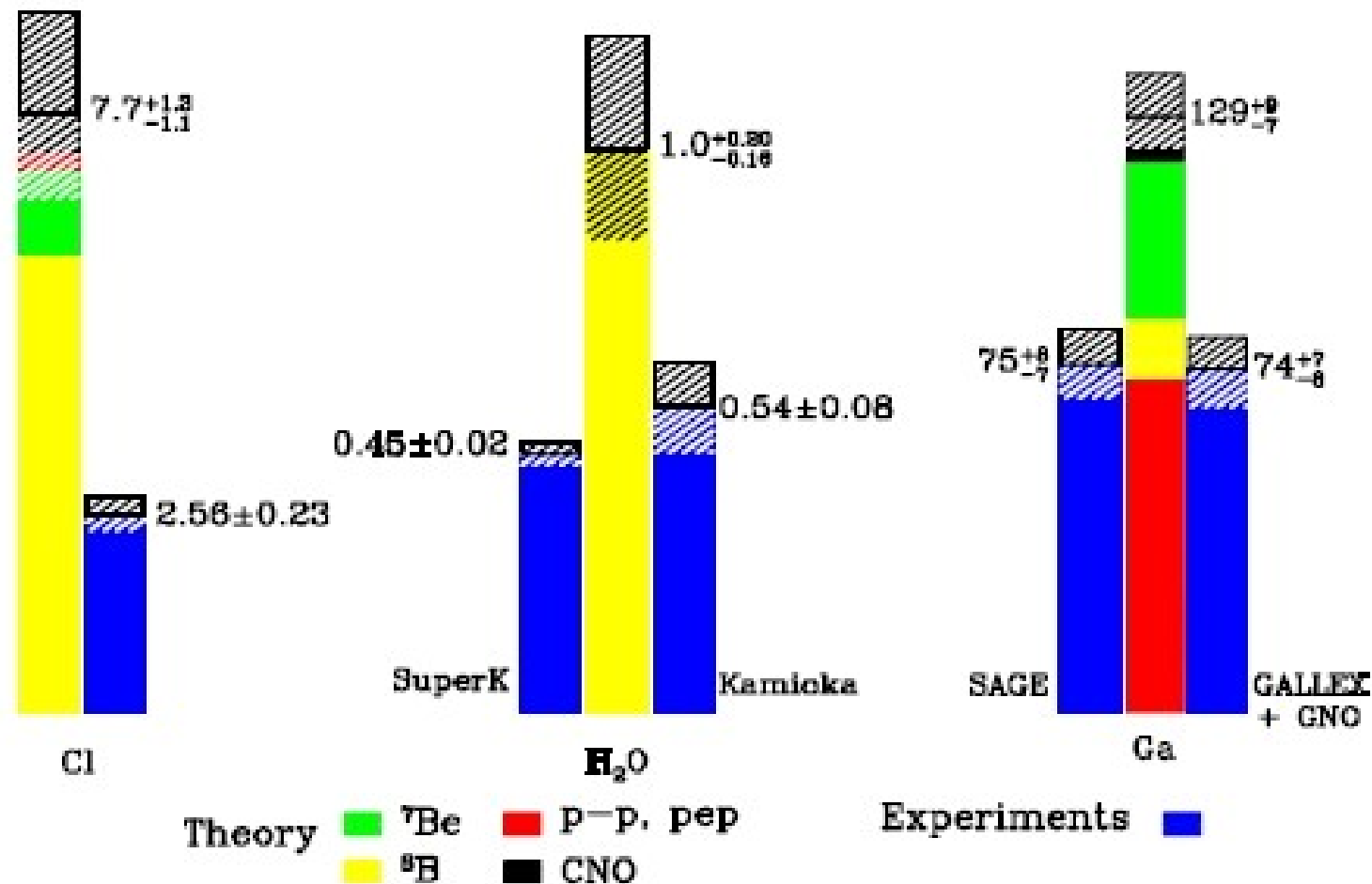
$E_\nu > 800 \text{ keV}$

Observe 1/3 of the expected number of solar neutrinos

1 SNU = 1 interaction per 10^{36} atoms per second

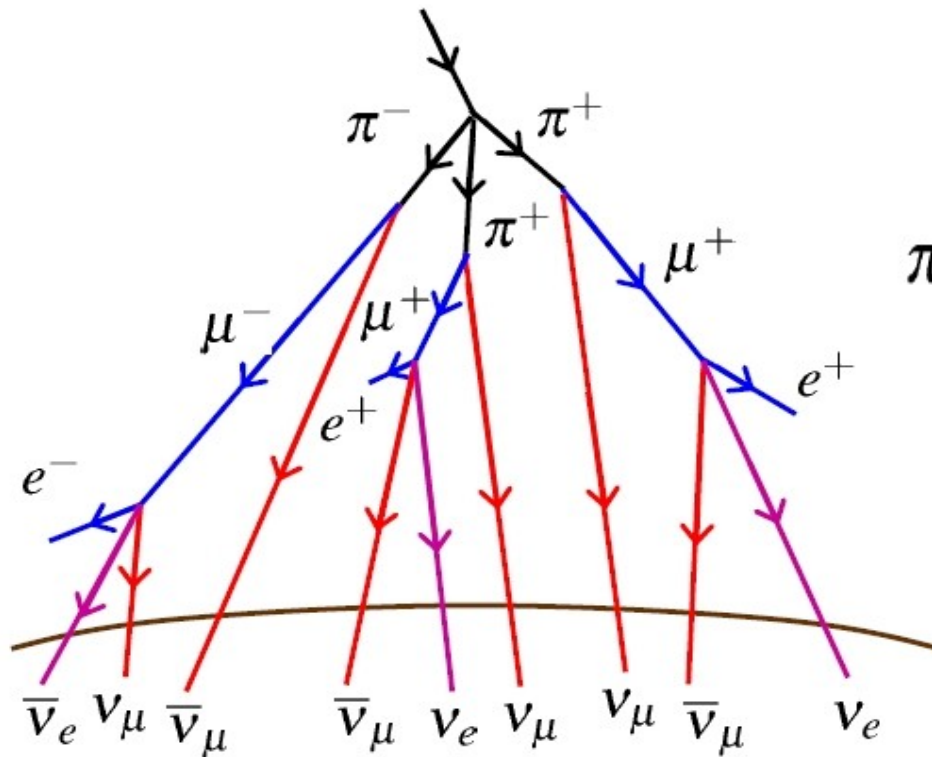
Experimental summary

Total Rates: Standard Model vs. Experiment Bahcall–Pinsonneault 2000

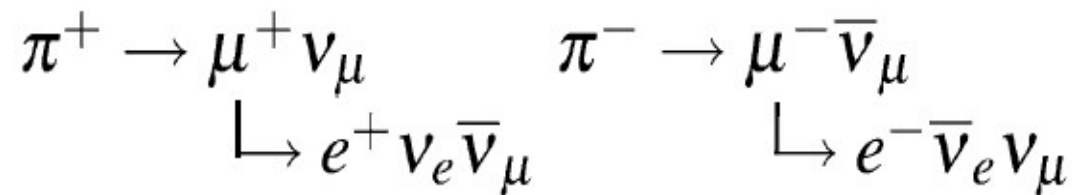


Atmospheric neutrinos

High energy cosmic rays interact in the upper atmosphere producing showers of mesons (mostly pions)

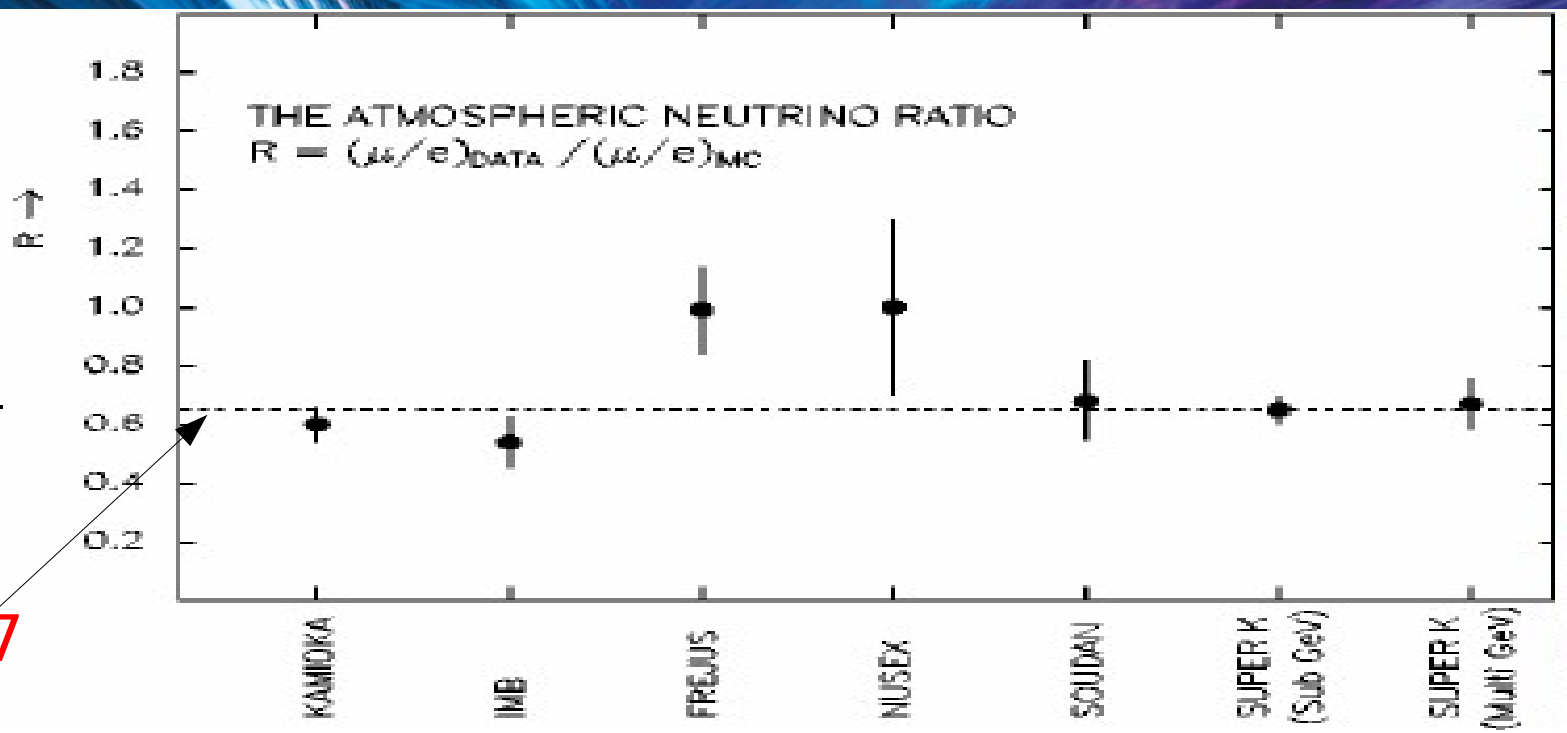


Neutrinos produced by



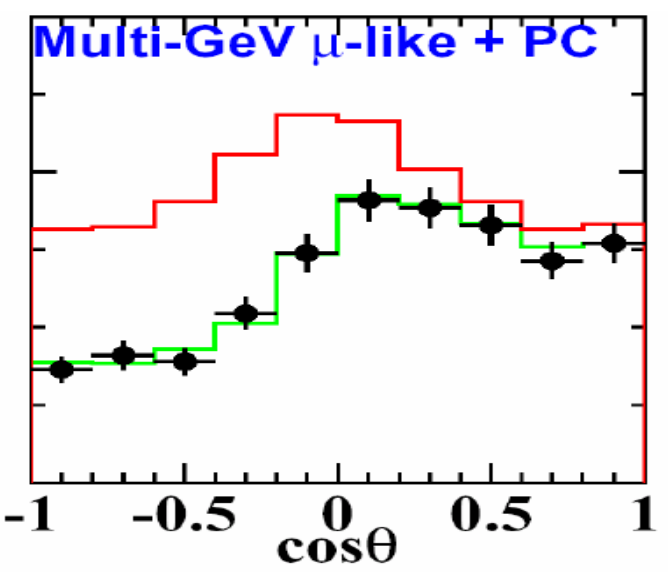
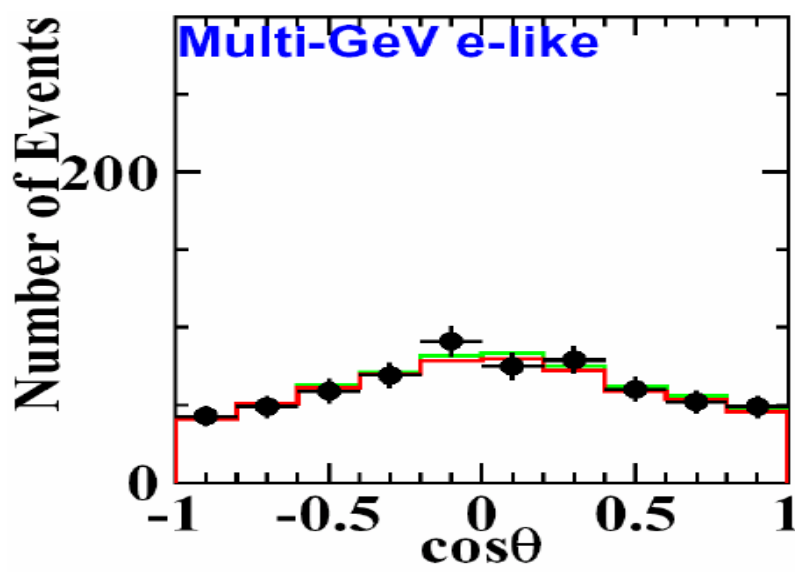
Expect
$$\frac{N(\nu_\mu + \bar{\nu}_\mu)}{N(\nu_e + \bar{\nu}_e)} \approx 2$$

At higher energies, the muons can reach the ground before decaying so ratio increases



$$R = \frac{(\mu/e)_{Data}}{(\mu/e)_{MC}}$$

R ~ 0.6 - 0.7



The Atmospheric Neutrino Anomaly

Neutrino Flavour Oscillations

Mixing

CKM
Mechanism

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L, \begin{pmatrix} c \\ s' \end{pmatrix}_L$$
$$d' = d \cos \theta_c + s \sin \theta_c$$
$$s' = -d \sin \theta_c + s \cos \theta_c$$

In the quark sector, the flavour eigenstates (those states which couple to the W/Z) are not identical to the mass eigenstates (those states which are eigenstates of the Hamiltonian)

Weak states \longrightarrow

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} 0.97 & 0.23 & 0.003 \\ 0.23 & 0.97 & 0.04 \\ 0.008 & 0.04 & 0.99 \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \longleftarrow \text{Mass states}$$

Mixing

CKM
Mechanism

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L, \begin{pmatrix} c \\ s' \end{pmatrix}_L \quad \begin{aligned} d' &= d \cos \theta_c + s \sin \theta_c \\ s' &= -d \sin \theta_c + s \cos \theta_c \end{aligned}$$

In the quark sector, the flavour eigenstates (those states which couple to the W/Z) are not identical to the mass eigenstates (those states which are eigenstates of the Hamiltonian)

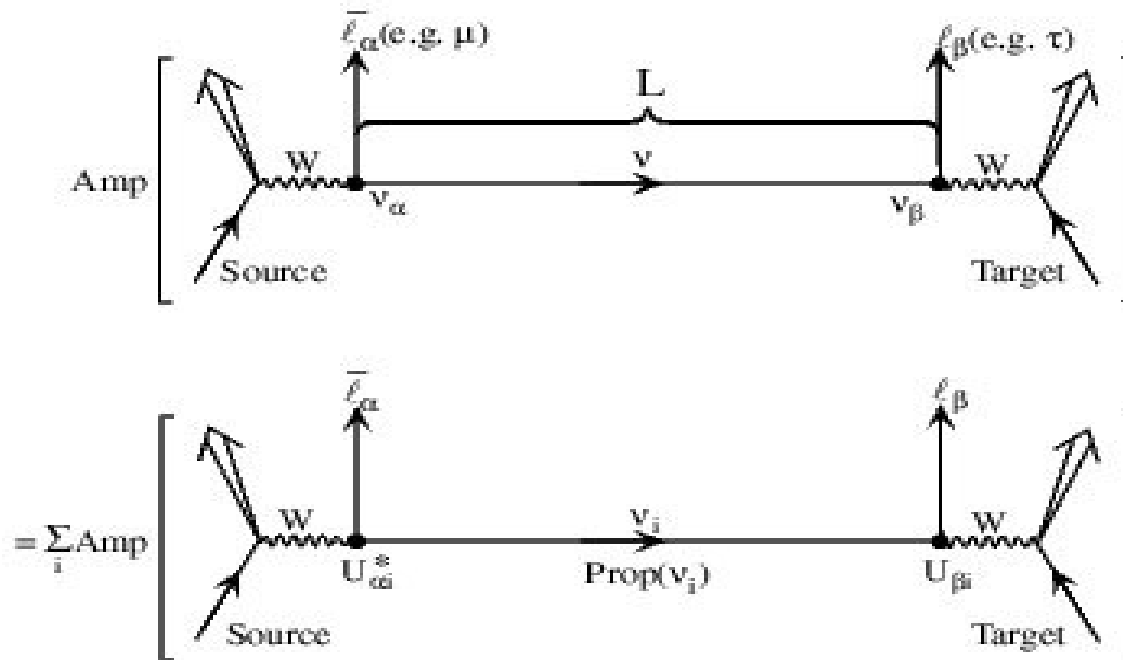
Weak states \longrightarrow

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

\longleftarrow **Mass states**

Unitary mixing matrix \longleftarrow

Neutrino Oscillations



$$\langle \nu_\alpha(0) | \nu_i(0) \rangle \langle \nu_i(0) | \nu_i(x_\mu) \rangle \langle \nu_\beta(x_\mu) | \nu_i(x_\mu) \rangle$$

$$|\langle \nu_\alpha(0) | \nu_\beta(x_\mu) \rangle|^2 \propto \left| \sum_j U_{\alpha i}^* e^{(i p_i^\mu x_\mu)} U_{\beta i} \right|^2$$

If we don't know which mass state was created then the the amplitude involves a coherent superposition of ν_i states

$$\text{Prob}(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{i>j} \Re(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin^2\left(\Delta m_{ij}^2 \frac{L}{4E}\right) + 2 \sum_{i>j} \Im(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin\left(\Delta m_{ij}^2 \frac{L}{2E}\right)$$

$$\Delta m_{ij}^2 = m_i^2 - m_j^2 \quad (\text{mass splitting})$$

L = distance from production to detection

E = Neutrino energy

- ▶ If $\Delta m_{ij}^2 = 0$ then neutrinos don't oscillate
- ▶ Absolute masses cannot be determined
- ▶ If there is no mixing (If $U_{ai} = 0$) neutrinos don't oscillate

$$\begin{aligned}
 \text{Prob}(\nu_\alpha \rightarrow \nu_\beta) = & \delta_{\alpha\beta} - 4 \sum_{i>j} \Re(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin^2\left(\Delta m_{ij}^2 \frac{L}{4E}\right) \\
 & + 2 \sum_{i>j} \Im(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin\left(\Delta m_{ij}^2 \frac{L}{2E}\right)
 \end{aligned}$$

$$\Delta m_{ij}^2 = m_i^2 - m_j^2 \quad (\text{mass splitting})$$

L = distance from production to detection

E = Neutrino energy

- ▶ One can detect flavour change in 2 ways : start with ν_α and look for ν_β (appearance) or start with ν_α and see if any disappears (disappearance)
- ▶ Flavour change oscillates with L/E. L and E are chosen by the experimenter to maximise sensitivity to a given Δm^2
- ▶ Flavour change doesn't alter total neutrino flux – it just redistributes it amongst different flavours (unitarity)

Two flavour oscillations

$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \Rightarrow U = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$$

$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{i>j} U_{\alpha i} U_{\beta i} U_{\alpha j} U_{\beta j} \sin^2 \left(\Delta m_{ij}^2 \frac{L}{4E} \right)$$

$P(\nu_\alpha \rightarrow \nu_\beta)$: Appearance Probability

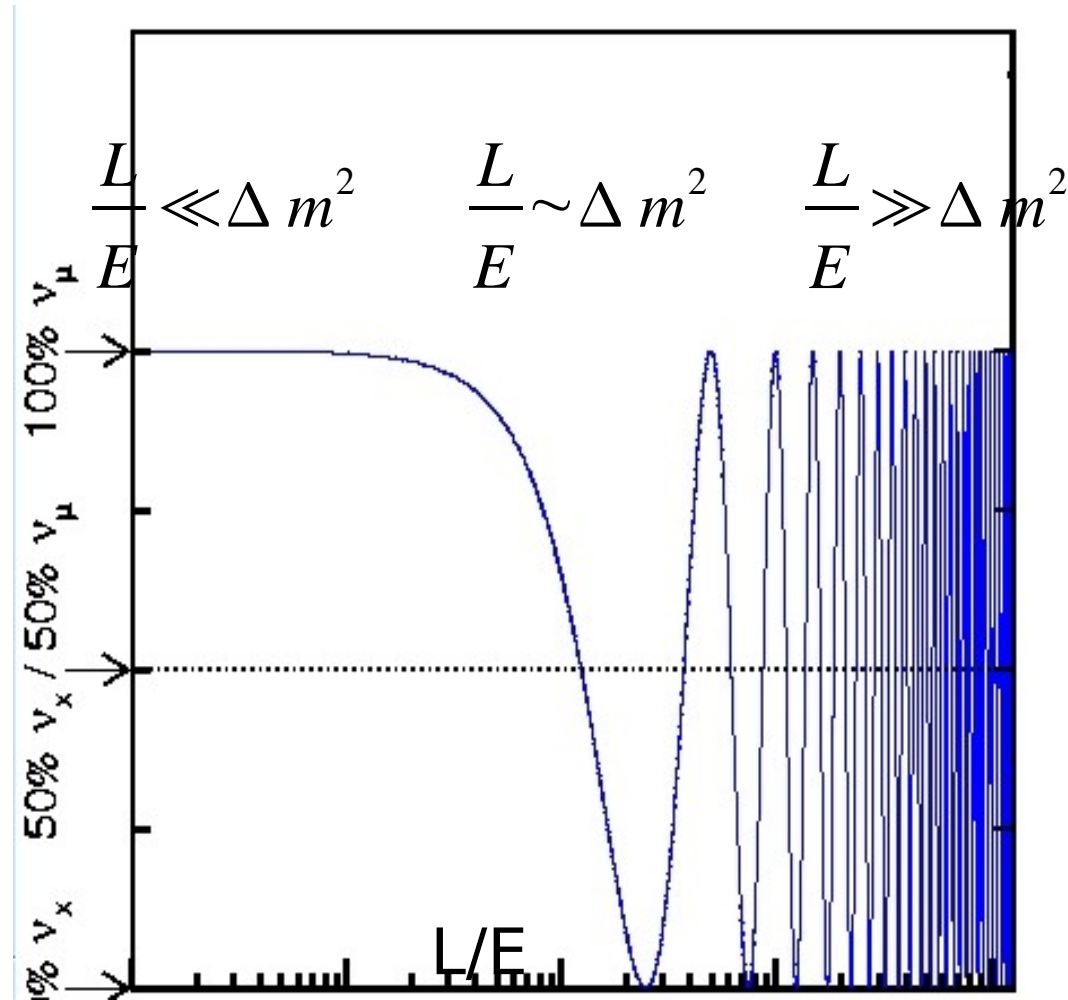
$P(\nu_\alpha \rightarrow \nu_\alpha)$: Survival Probability

$$P(\nu_\alpha \rightarrow \nu_\beta) = -4 (U_{\alpha 1} U_{\beta 1} U_{\alpha 2} U_{\beta 2}) \sin^2 \left(\Delta m_{ij}^2 \frac{L}{4E} \right)$$

$$= \sin^2(2\theta) \sin^2 \left(1.27 \Delta m^2 (\text{eV}^2) \frac{L(\text{km})}{E(\text{GeV})} \right)$$

(changing to useful units)

Survival Probability



$$P(\nu_\alpha(0) \rightarrow \nu_\alpha(x)) = 1 - \sin^2(2\theta) \sin^2\left(1.27 \Delta m^2 \frac{(L/\text{km})}{(E/\text{GeV})}\right)$$

Question : What would you observe if you were able to know which mass state propagated from source to detector?

Question : What would you observe if you were able to know what mass state propagated from source to detector?

$$Prob(v_\alpha \rightarrow v_\beta) \propto \left| \sum_i U_{\alpha i}^* e^{(i p_i^\mu x_{mu})} U_{\beta i} \right|^2$$

Question : What would you observe if you were able to know what mass state propagated from source to detector?

$$\begin{aligned}
 \text{Prob}(\nu_\alpha \rightarrow \nu_\beta) &\propto \sum_i |U_{\alpha i}^* e^{(i p_i^\mu x_\mu)} U_{\beta i}|^2 \\
 &\rightarrow \sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2
 \end{aligned}$$

- ▶ The propagation term is just a phase rotation so vanishes
- ▶ The probability is now a constant – there is flavour change if mixing can still happen – but now the oscillation has vanished, as the interference between mass states no longer exists.

Question : Why does the oscillation pattern vanish if we know the identities of the mass state ?

Three Flavour Oscillation

The three flavour case is more complicated, but no different

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \Leftrightarrow U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

U is the Pontecorvo-Maskawa-Nakayama-Sakata (PMNS) matrix

$$\begin{aligned} \text{Prob}(\nu_\alpha \rightarrow \nu_\beta) = & \delta_{\alpha\beta} - 4 \sum_{i>j} \Re(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin^2\left(\Delta m_{ij}^2 \frac{L}{4E}\right) \\ & + 2 \sum_{i>j} \Im(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin\left(\Delta m_{ij}^2 \frac{L}{2E}\right) \end{aligned}$$

Oscillation parameters

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}$$

2 independent Δm^2

$$\begin{aligned} \text{Prob}(v_\alpha \rightarrow v_\beta) = & \delta_{\alpha\beta} - 4 \sum_{i>j} \Re(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin^2\left(\Delta m_{ij}^2 \frac{L}{4E}\right) \\ & + 2 \sum_{i>j} \Im(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin\left(\Delta m_{ij}^2 \frac{L}{2E}\right) \end{aligned}$$

Oscillation parameters

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}$$

Three angles

$$\begin{aligned} \text{Prob}(v_\alpha \rightarrow v_\beta) = & \delta_{\alpha\beta} - 4 \sum_{i>j} \Re(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin^2(\Delta m_{ij}^2 \frac{L}{4E}) \\ & + 2 \sum_{i>j} \Im(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin(\Delta m_{ij}^2 \frac{L}{2E}) \end{aligned}$$

Oscillation parameters

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}$$

CP violating phase

$$\begin{aligned} \text{Prob}(\nu_{\alpha} \rightarrow \nu_{\beta}) = & \delta_{\alpha\beta} - 4 \sum_{i>j} \Re(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin^2(\Delta m_{ij}^2 \frac{L}{4E}) \\ & + 2 \sum_{i>j} \Im(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin(\Delta m_{ij}^2 \frac{L}{2E}) \end{aligned}$$

Oscillation parameters

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}$$

Extra Majorana phases

The extra Majorana matrix does not affect flavour oscillation processes....so is usually dropped. However it will affect the interpretation of neutrinoless double beta decay results

Explaining the solar data

Testing the oscillation hypothesis

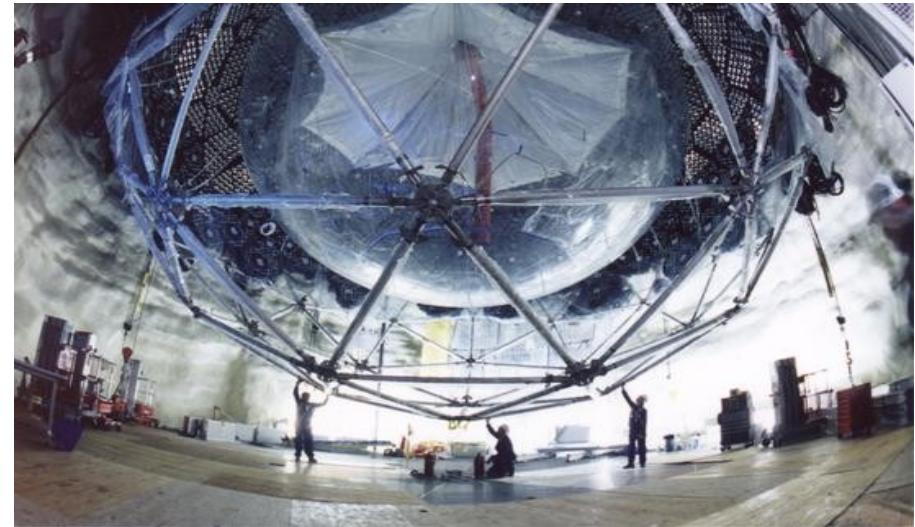
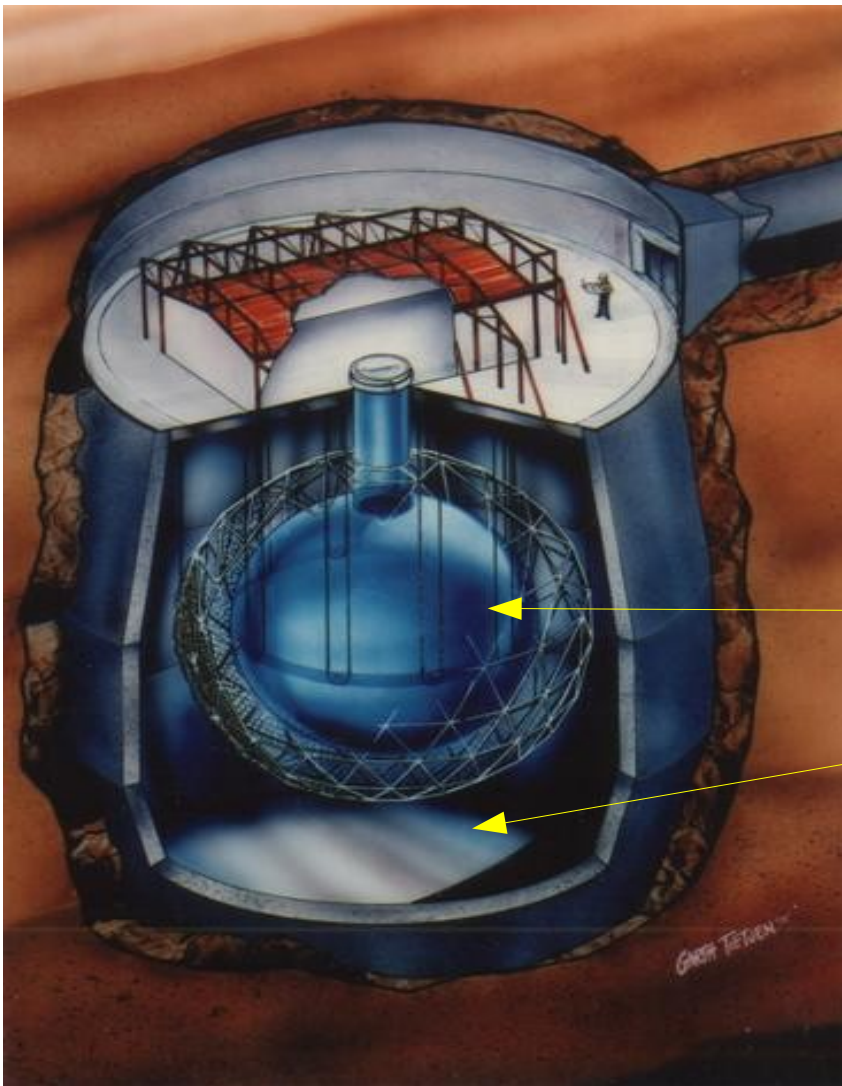
Solar neutrino problem

ν_e from sun would change to ν_μ or ν_τ . However these have too little energy to interact via the charged current, and all the detectors are only sensitive to charge current interactions.

Non- ν_e component would effectively disappear, reducing the apparent ν_e flux.

Proof : Neutral current event rate shouldn't change.

Sudbury Neutrino Observatory



1000 tonnes of D_2O

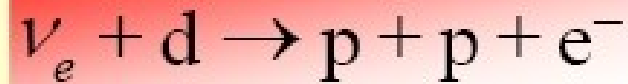
6500 tons of H_2O

Viewed by 10,000 PMTS

In a salt mine 2km underground
in Sudbury, Canada

SNO

CC

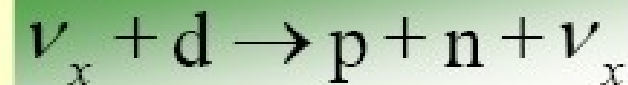


- $Q = 1.445 \text{ MeV}$
- good measurement of ν_e energy spectrum
- some directional info $\propto (1 - 1/3 \cos\theta)$
- ν_e only

Produces Cherenkov
Light Cone in D_2O

ν_e

NC

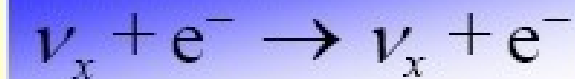


- $Q = 2.22 \text{ MeV}$
- measures total 8B ν flux from the Sun
- equal cross section for all ν types

n captures on deuteron
 $^2H(n, \gamma)^3H$

Observe $6.25 \text{ MeV } \gamma$
 $\nu_e + \nu_\mu + \nu_\tau$

ES

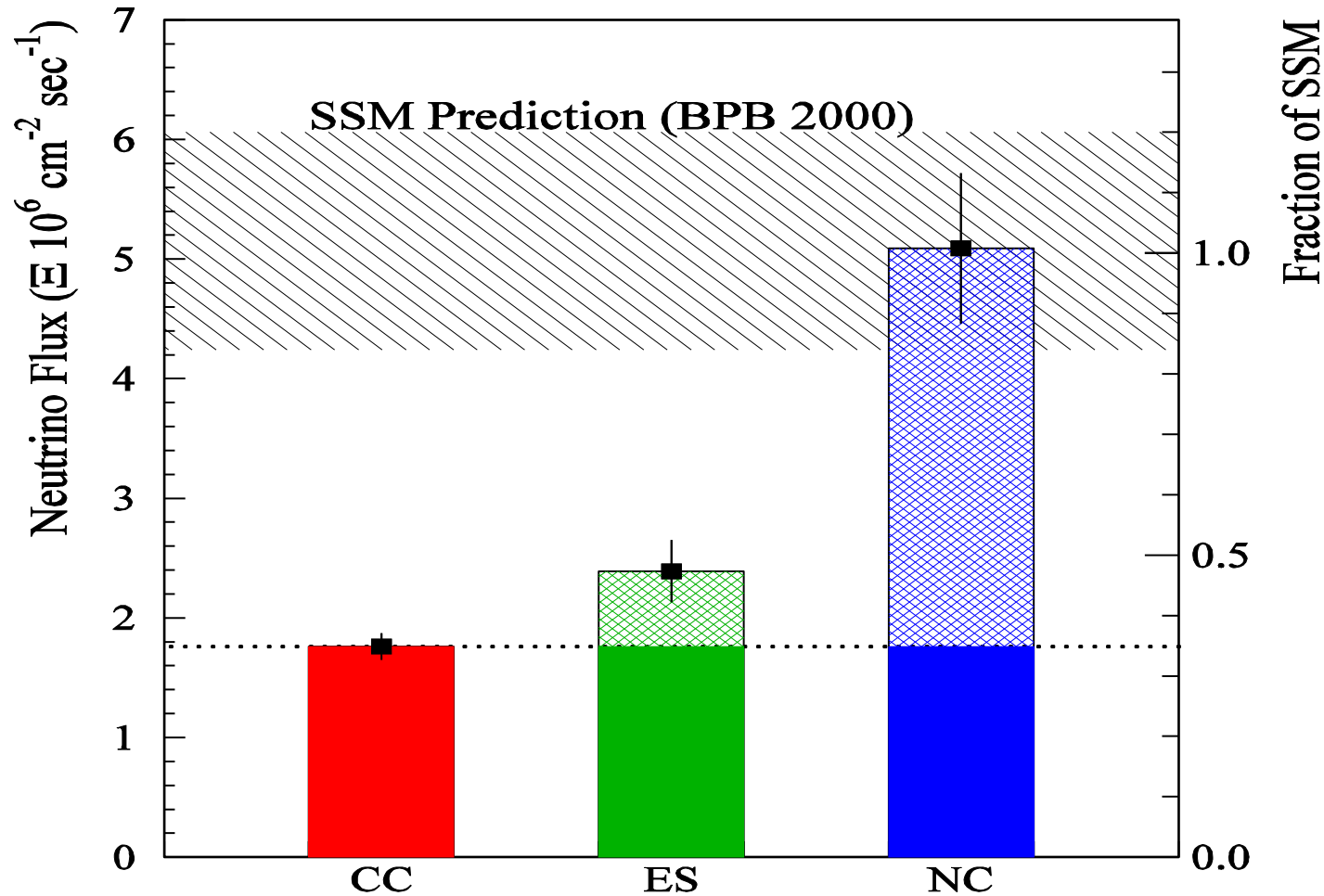


- low statistics
- mainly sensitive to ν_e , some ν_μ and ν_τ
- strong directional sensitivity

Produces Cherenkov
Light Cone in D_2O

$\nu_e + 0.15 * (\nu_\mu + \nu_\tau)$

SNO Results



5.3 σ appearance of $\nu_{\mu\tau}$ in a ν_e beam
Roughly 70% of ν_e oscillates away

Naively...

First instinct is to assume that neutrinos leave the sun as ν_e and oscillate on their way to the earth. Assuming this

$$L \sim 10^8 \text{ km}, E_\nu < 10 \text{ MeV} \rightarrow \Delta m^2 \sim 3 \times 10^{-10} \text{ eV}^2$$

Naively...

First instinct is to assume that neutrinos leave the sun as ν_e and oscillate on their way to the earth. Assuming this

$$L \sim 10^8 \text{ km}, E_\nu < 10 \text{ MeV} \rightarrow \Delta m^2 \sim 7 \times 10^{-5} \text{ eV}^2$$

Naively...

First instinct is to assume that neutrinos leave the sun as ν_e and oscillate on their way to the earth. Assuming this

$$L \sim 10^8 \text{ km}, E_\nu < 10 \text{ MeV} \rightarrow \Delta m^2 \sim 7 \times 10^{-5} \text{ eV}^2$$

Oscillations come from phase difference between mass states. In a vacuum the phase diff comes from free particle Hamiltonian. In a material there are interaction potentials as well

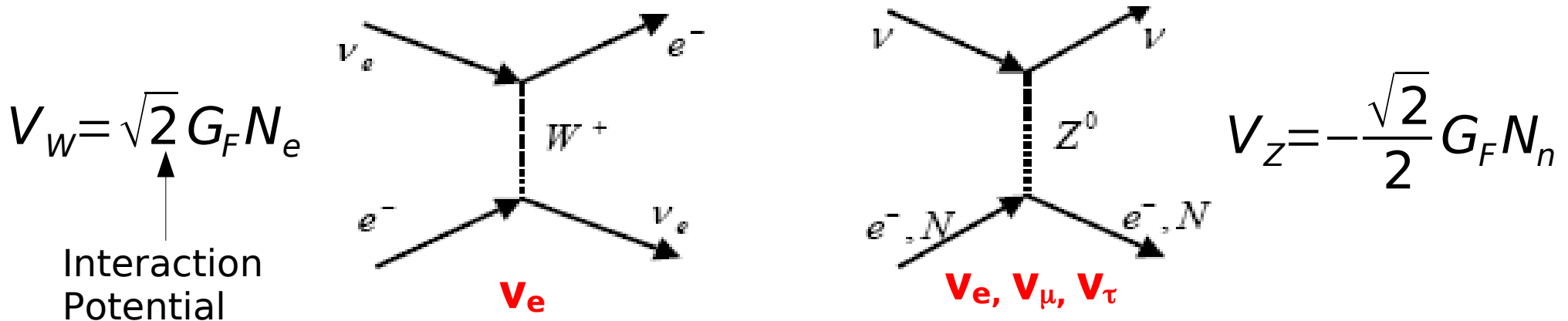
$$-i\hbar \frac{\partial \psi}{\partial t} = E \psi = \frac{-\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} \rightarrow -i\hbar \frac{\partial \psi}{\partial t} = (E + V) \psi = \frac{-\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2}$$

$$E_{vac}^2 - p^2 = m_{vac}^2 \rightarrow (E_{vac} + V)^2 - p^2 = m_{mat}^2 \rightarrow m_{mat} \approx \sqrt{m_{vac}^2 + 2 E_{vac} V}$$

c.f. effective mass of an electron in a semiconductor or light in glass

Oscillations in Matter

Electrons exist in standard matter – μ/τ do not. Electron neutrinos travelling in matter can experience an extra charged current interaction that other flavours cannot.



$$P(\nu_e \rightarrow \nu_e) = 1 - \sin^2(2\theta_M) \sin^2\left(\frac{\Delta m_M^2 L}{4E}\right)$$

Oscillation probability modified by matter effects

$$\Delta m_M^2 = \Delta m_V^2 \sqrt{\sin^2(2\theta_V) + (\cos 2\theta_V - \zeta)^2}$$

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta_V}{\sin^2 2\theta_V + (\cos 2\theta_V - \zeta)^2}$$

$$\zeta = \frac{2\sqrt{2} G_F N_e E}{\Delta m_V^2}$$

Implications

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta_V}{\sin^2 2\theta_V + (\cos 2\theta_V - \zeta)^2} \quad \zeta = \frac{2\sqrt{2}G_F N_e E}{\Delta m_{Vac}^2}$$

- If $\Delta m_{Vac}^2 = 0$ or matter is very dense, $\zeta = \infty$ and $\theta_M = 0$
- Similarly, if $\theta_{vac} = 0$, then $\theta_M = 0 \Rightarrow$ need mixing in vacuum
- If there is no matter, then $\zeta = 0$ and we have vacuum mixing
- At a particular electron density, dependent on Δm^2 ,

$$\zeta = \frac{2\sqrt{2}G_F N_e E}{\Delta m^2} = \cos 2\theta \Rightarrow \sin^2 2\theta_M = 1$$

Even if the vacuum mixing angle is tiny, there is a density for which the matter mixing angle is maximal

Mass hierarchy

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - \xi)^2} \quad \xi = \frac{2\sqrt{2}G_F N_e E}{\Delta m_V^2}$$

If mass of $\nu_1 <$ mass of ν_2 , $\Delta m_V^2 = m_1^2 - m_2^2 < 0$

$$\xi = -\frac{2\sqrt{2}G_F N_e E}{|\Delta m^2|} \rightarrow \sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta + |\xi|)^2}$$

Positive definite - no resonance

If mass of $\nu_1 >$ mass of ν_2 , $\Delta m^2 = m_1^2 - m_2^2 > 0$

$$\xi = \frac{2\sqrt{2}G_F N_e E}{|\Delta m^2|} \rightarrow \sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - |\xi|)^2}$$

Mass hierarchy

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - \xi)^2} \quad \xi = \pm \frac{2\sqrt{2}G_F N_e E}{|\Delta m_V^2|}$$

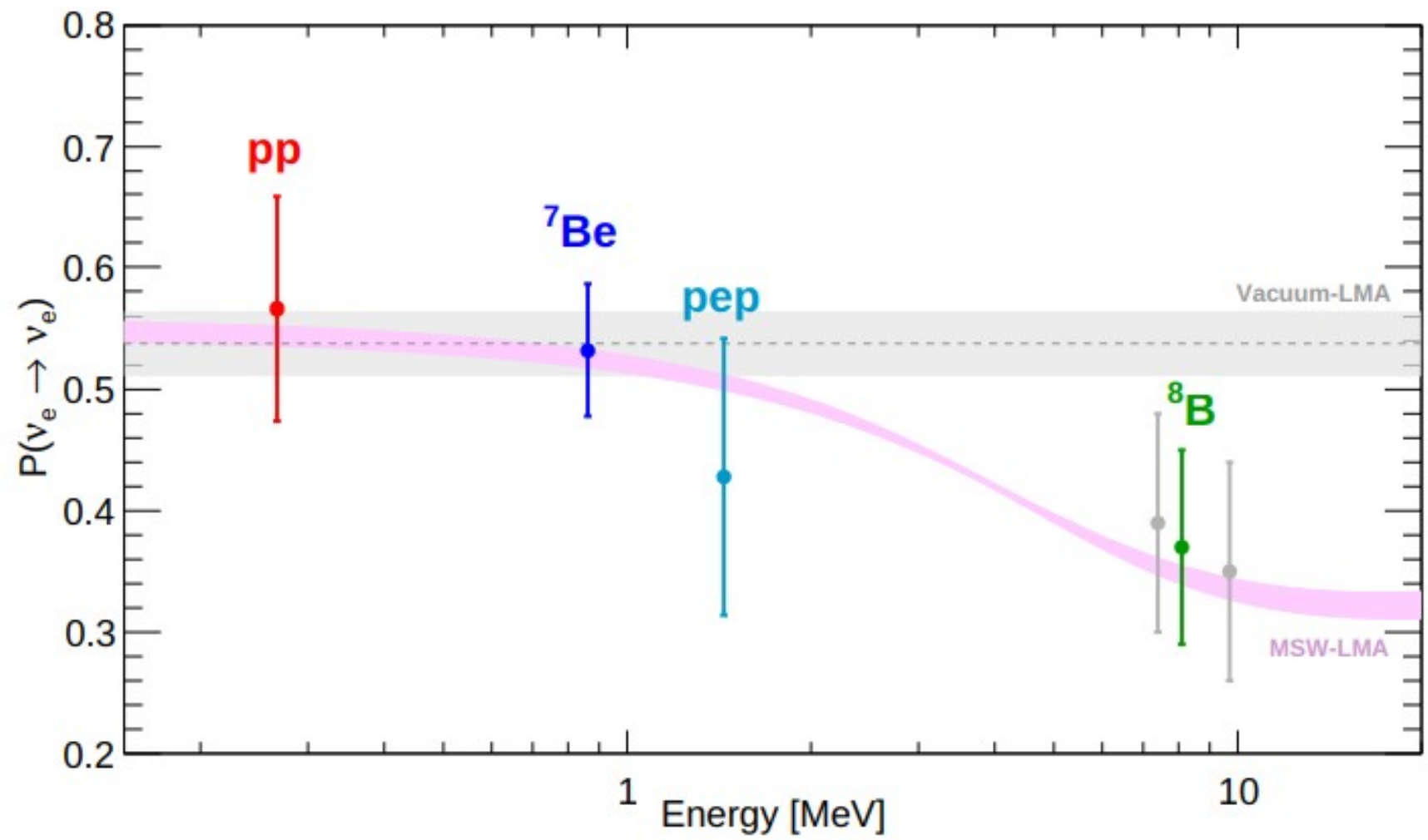
The effect of matter on neutrino oscillations can be used to measure the mass hierarchy.

Not all neutrinos oscillate via matter effects in their way out of the sun. Only those with energy greater than

$$E_{min} = \Delta m_V^2 \frac{\cos 2\theta_V}{2\sqrt{2}G_F N_e (core)}$$

can do this. In the sun this is around E_{min} of 1 MeV. pp neutrinos have energies < 0.5 MeV, so are subject to vacuum oscillations.

ν_e survival probability



Mixing matrix

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}$$

Solar sector

$$\theta_{e\mu} = 32.5^\circ \pm 2.4^\circ$$

$$\Delta m_{12}^2 = +7.9 \times 10^{-5} eV^2$$

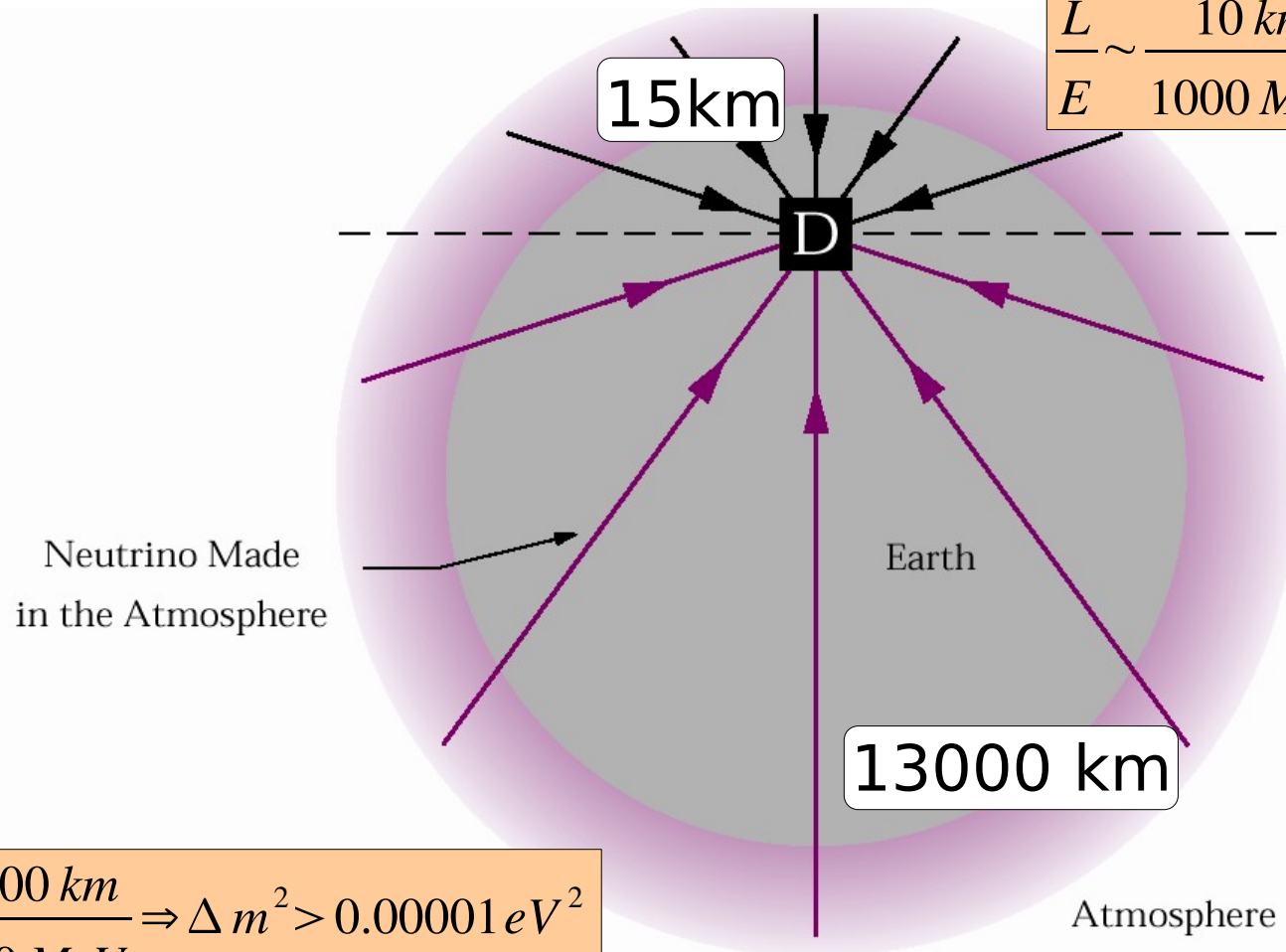
Solar oscillations occur *in* the sun via matter effects

Explaining the atmospheric data

Cosmic Labs

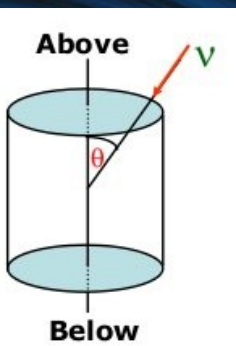
$$\cos \theta_{\text{zenith}} = 1.0$$

$$\frac{L}{E} \sim \frac{10 \text{ km}}{1000 \text{ MeV}} \Rightarrow \Delta m^2 > 0.01 \text{ eV}^2$$

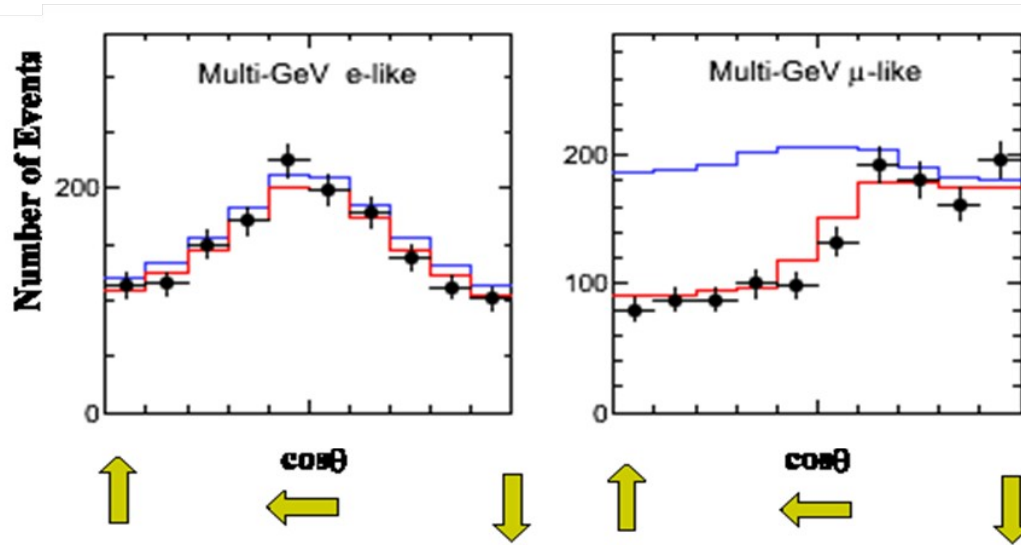


$$\frac{L}{E} \sim \frac{10000 \text{ km}}{1000 \text{ MeV}} \Rightarrow \Delta m^2 > 0.00001 \text{ eV}^2$$

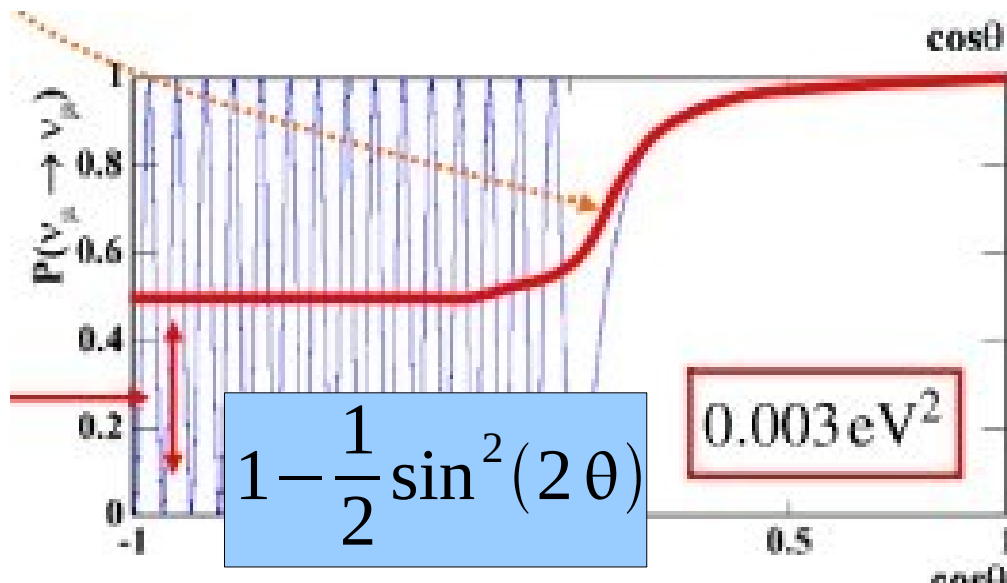
$$\cos \theta_{\text{zenith}} = -1.0$$



Atmospheric results



- Prediction for ν_e rate agrees with data.
- ν_μ disappear at large baseline consistent with $\nu_\mu \rightarrow \nu_\tau$
- Don't detect ν_τ as
 - below τ mass threshold
 - SuperK is awful at τ detection



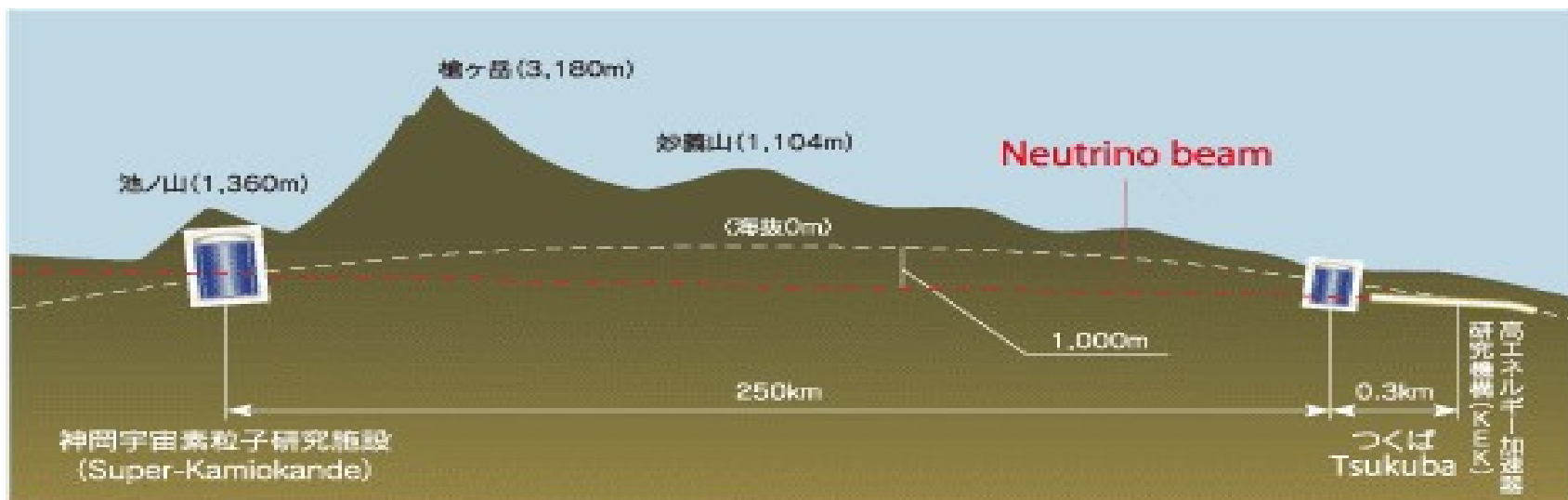
$$|\Delta m_{atmos}^2| \approx 0.0025 eV^2$$

$$\sin^2(2\theta_{atmos}) \approx 1.0$$

Accelerator Cross-check

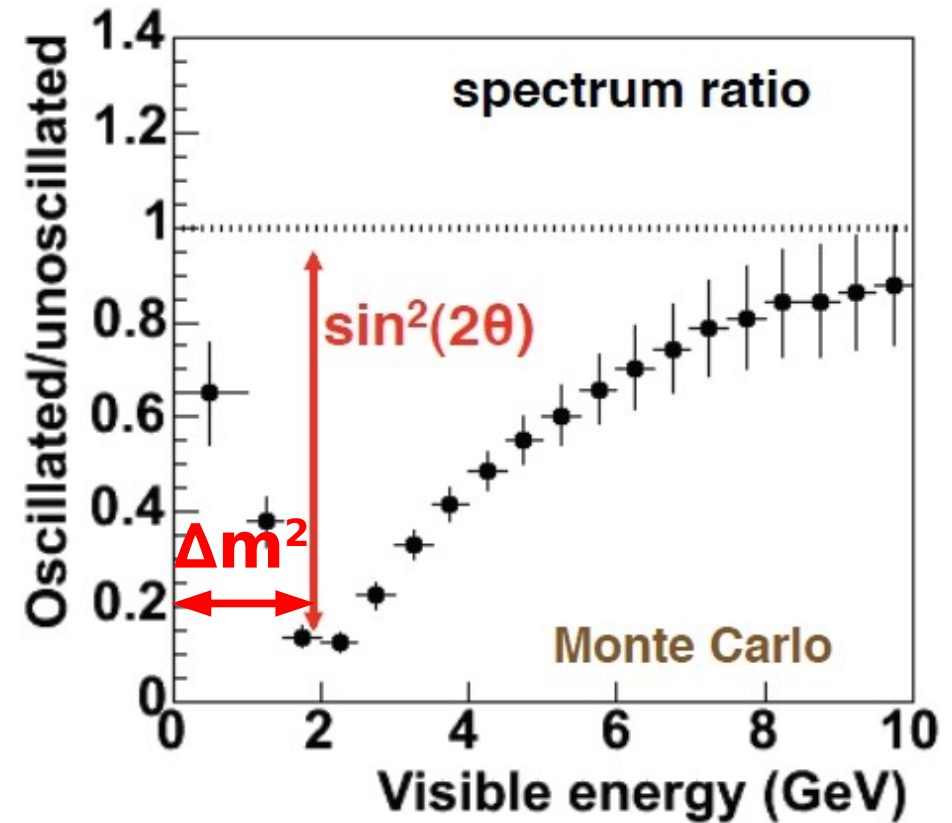
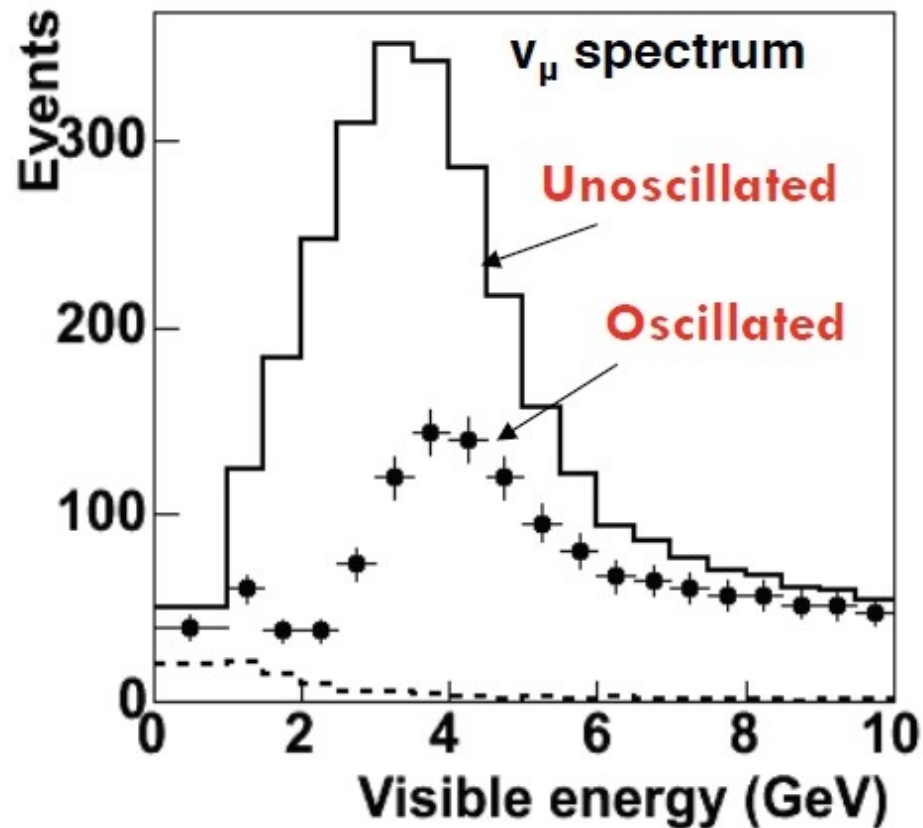
$$\Delta m_{atmos}^2 \approx 3 \times 10^{-3} eV^2 \rightarrow L/E \approx 400 km GeV^{-1}$$

$$L = 250 km \rightarrow E_\nu \approx 0.6 GeV$$



Beam events tagged using GPS at both near and far detector sites

Disappearance Experiments



$$P(\nu_\alpha \rightarrow \nu_\alpha) \rightarrow \frac{\Phi_\nu(@FD)}{\Phi_\nu(@ND)}$$

Φ_ν : Neutrino Flux

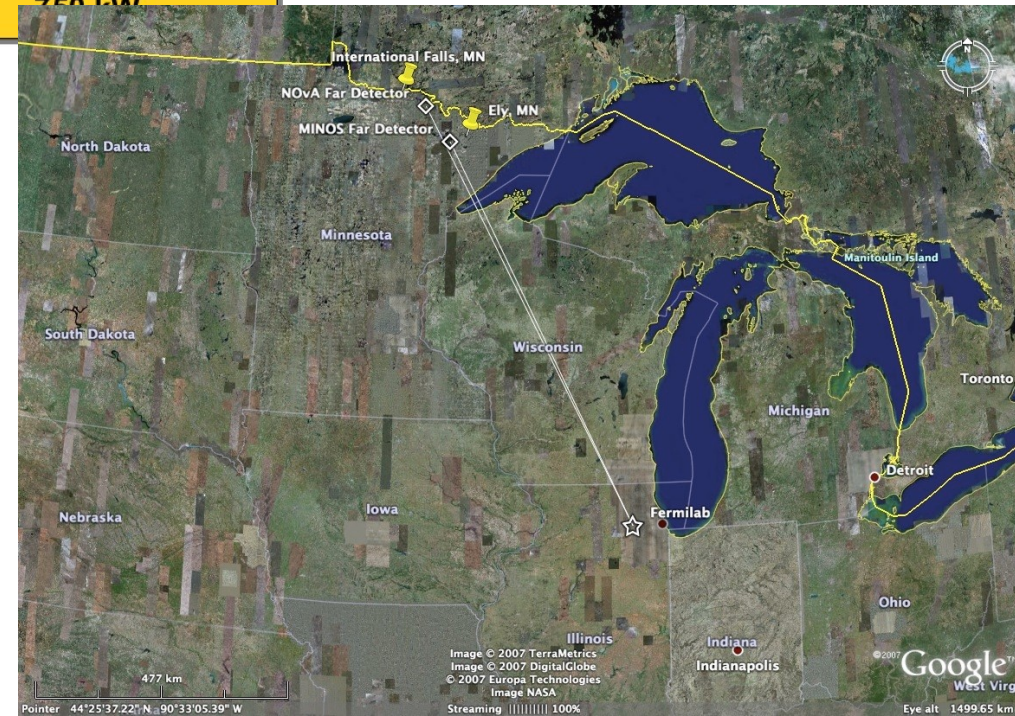
Use Near Detector to measure $\Phi_\nu(@ND)$

T2K and NOVA

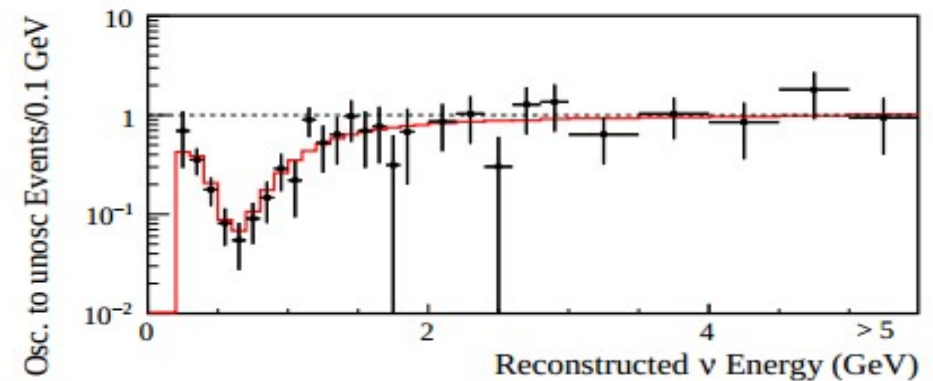
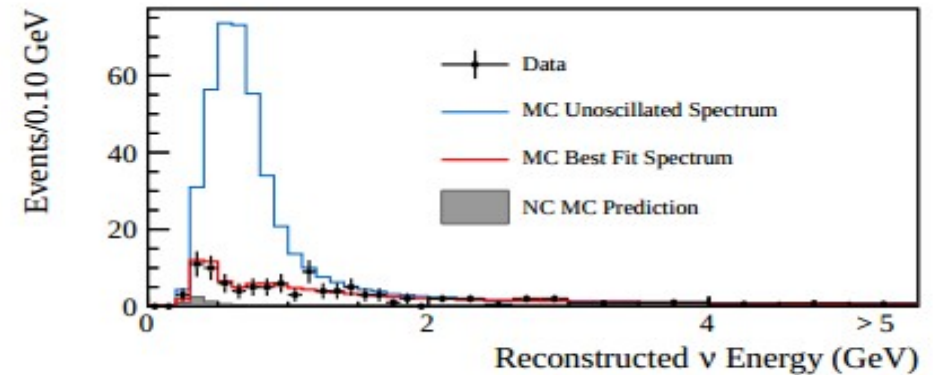
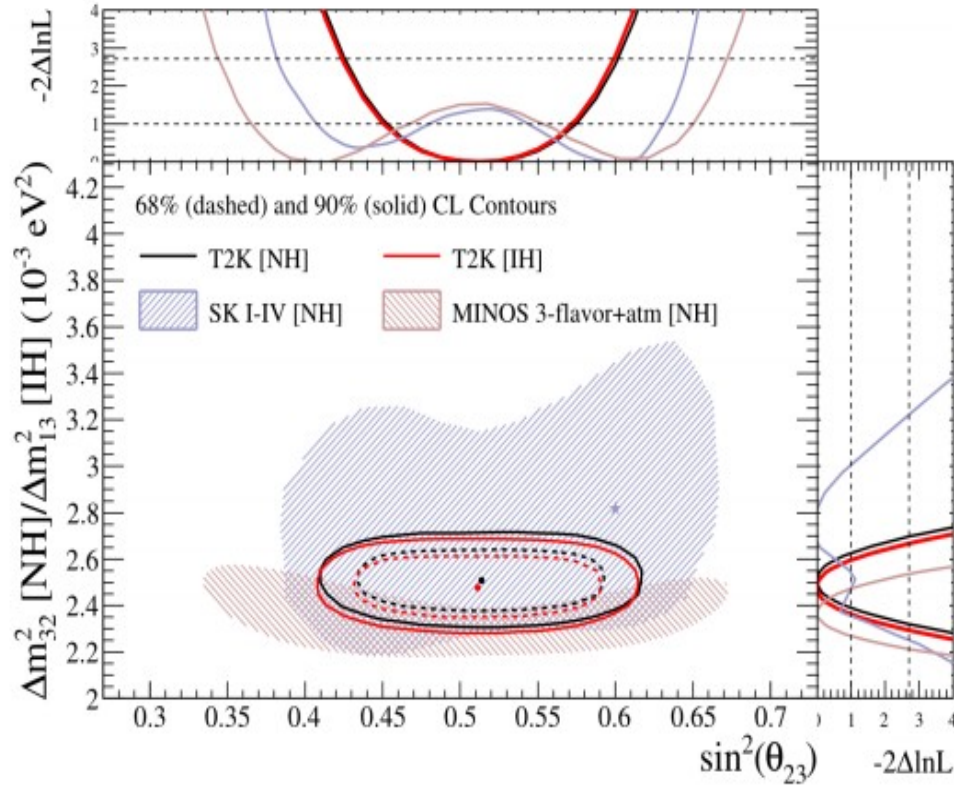


- ▶ JPARC to Kamioka
- ▶ $L = 295$ km
- ▶ $E_\nu \sim 0.6$ GeV
- ▶ Far Det : 22.6 kton water Cerenkov detector

- ▶ Fermilab to Ash River, MN
- ▶ $L = 810$ km
- ▶ $E_\nu \sim 2.0$ GeV
- ▶ Far Det : 14 kton of liquid scintillator (in bars)



T2K Disappearance



$$\frac{\# \text{ events observed}}{\# \text{ events expected}} = P(\nu_{\mu} \rightarrow \nu_{\mu}) = 1 - \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

$$|\Delta m_{23}^2| = (2.51 \pm 0.1) \times 10^{-3} \text{ eV}^2$$

$$\sin^2(\theta_{23}) = 0.514_{-0.056}^{+0.055} \rightarrow \theta_{23} = 45.8 \pm 3.2$$

(best fit)

Mixing matrix

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}$$

Solar sector : $\nu_{\mu} \rightarrow \nu_e$

$$\theta_{e\mu} = 33.7^\circ \pm 1.1^\circ$$

$$\Delta m_{12}^2 = +(7.54 \pm 0.24) \times 10^{-5} eV^2$$

Atmospheric sector

$\nu_{\mu} \rightarrow \nu_{\tau}$

$$\theta_{\mu\tau} = 42^\circ \pm 3.0^\circ$$

$$\Delta m_{23}^2 = |(2.43 \pm 0.06) \times 10^{-3}| eV^2$$

How do we measure θ_{13} ?

$\nu_\mu \rightarrow \nu_e$ oscillations with atmospheric L/E

$$P(\nu_\mu \rightarrow \nu_e) = \sin^2 2\theta_{13} \sin^2 \theta_{23} \sin^2 \left(1.27 \Delta m_{23}^2 \frac{L}{E} \right)$$

ν_e appearance in a ν_μ beam – ideal for *accelerator experiments*

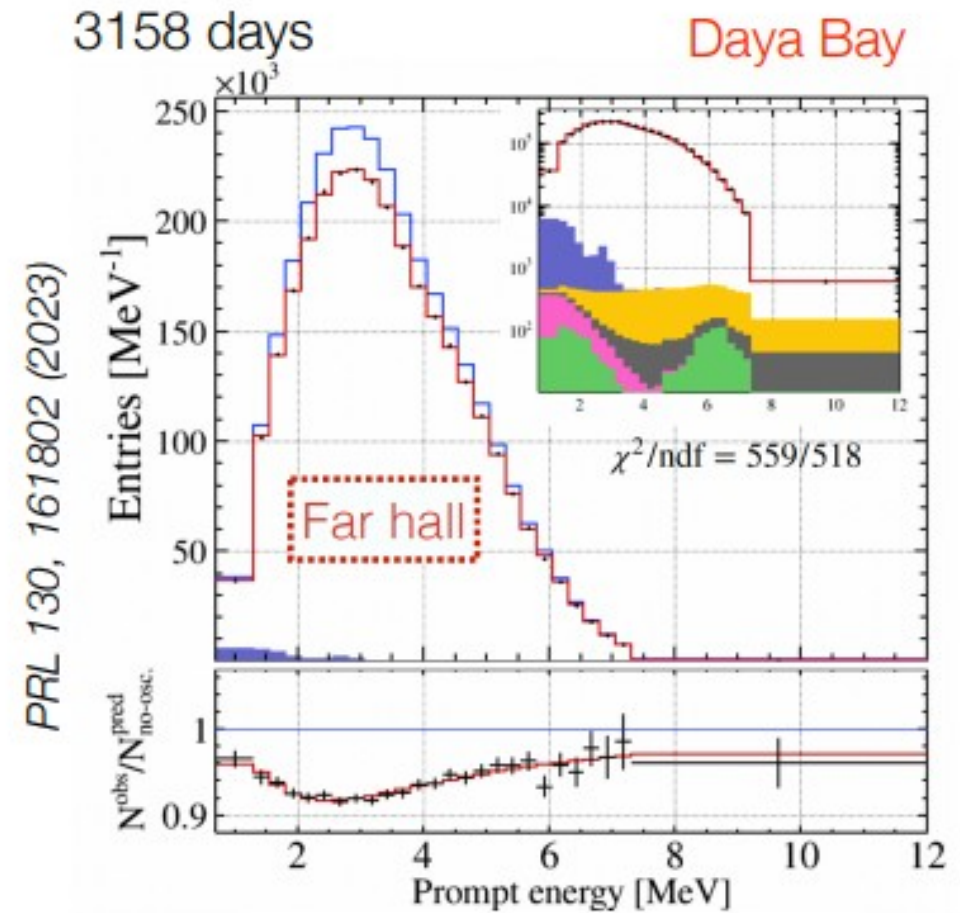
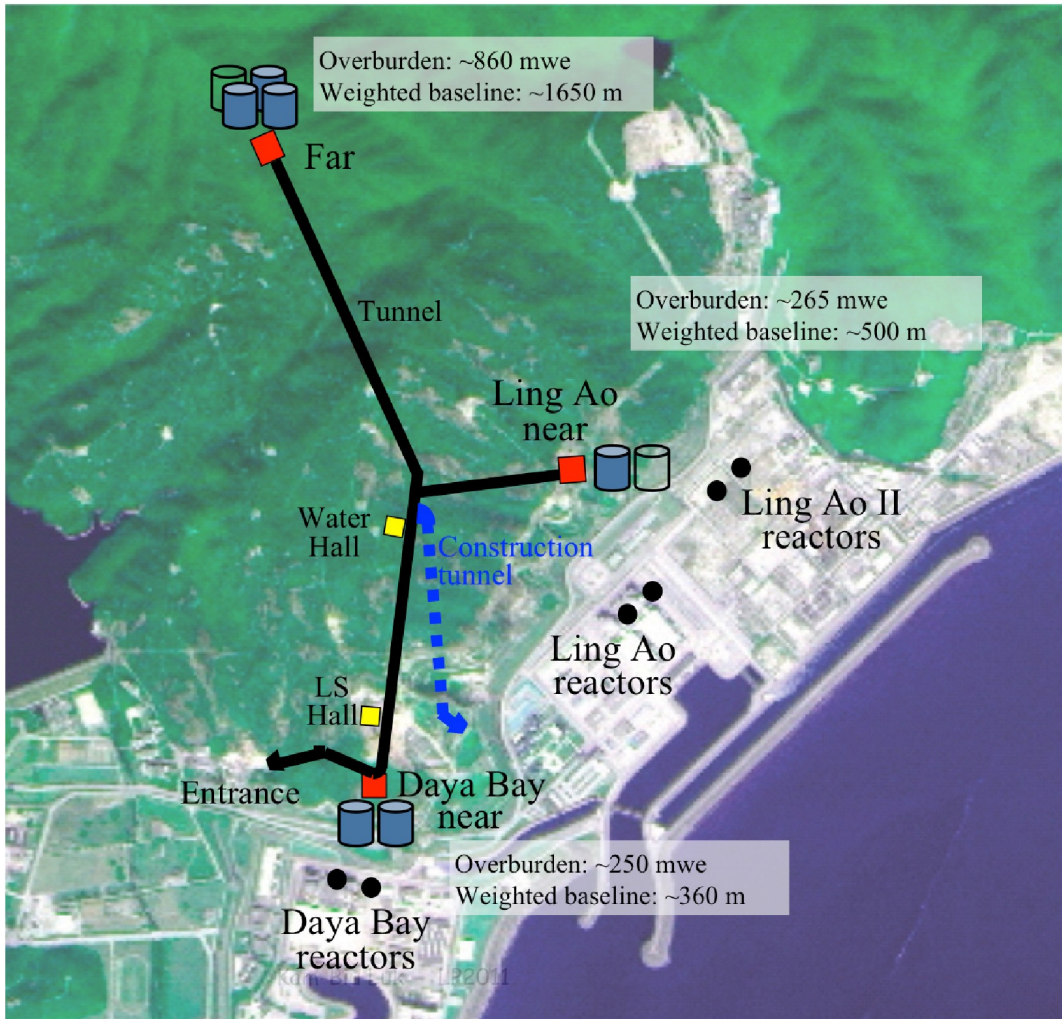
$\bar{\nu}_e \rightarrow \bar{\nu}_x$ disappearance oscillations with atmospheric L/E

$$p(\bar{\nu}_e \rightarrow \bar{\nu}_x) = 1 - \sin^2(2\theta_{13}) \sin^2 \left(1.27 \Delta m_{23}^2 \frac{L}{E} \right)$$

$\bar{\nu}_e$ disappearance – ideal for *reactor experiments*

Probability only a function of θ_{13}

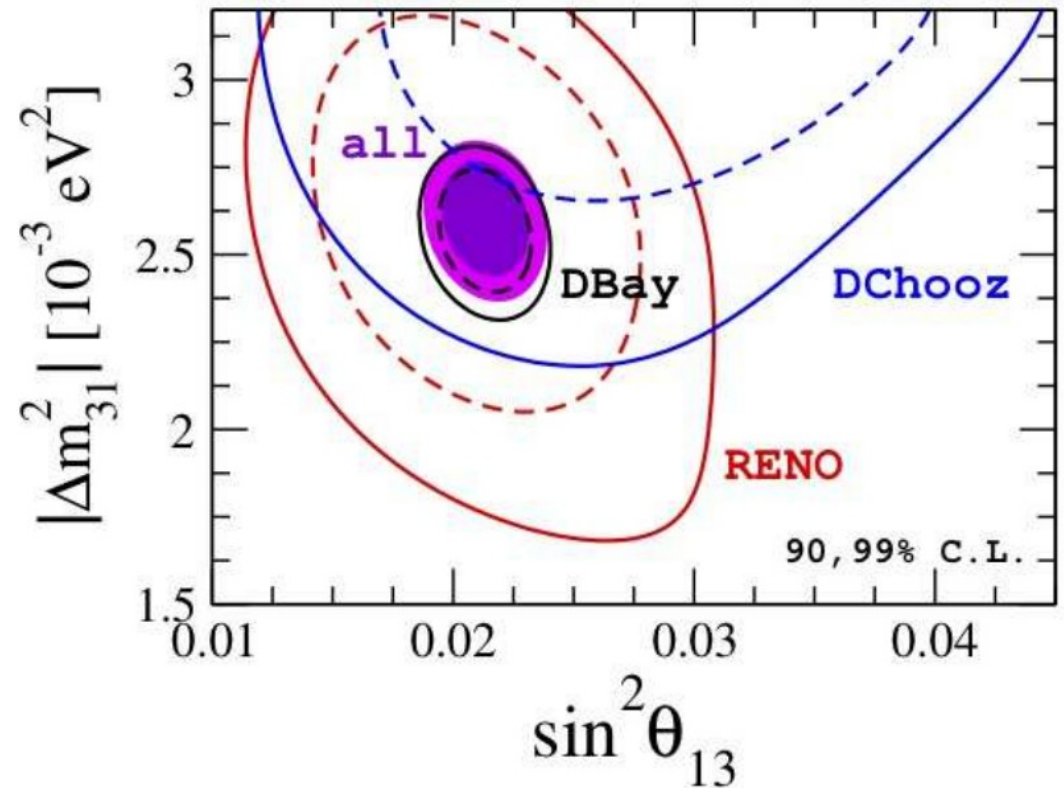
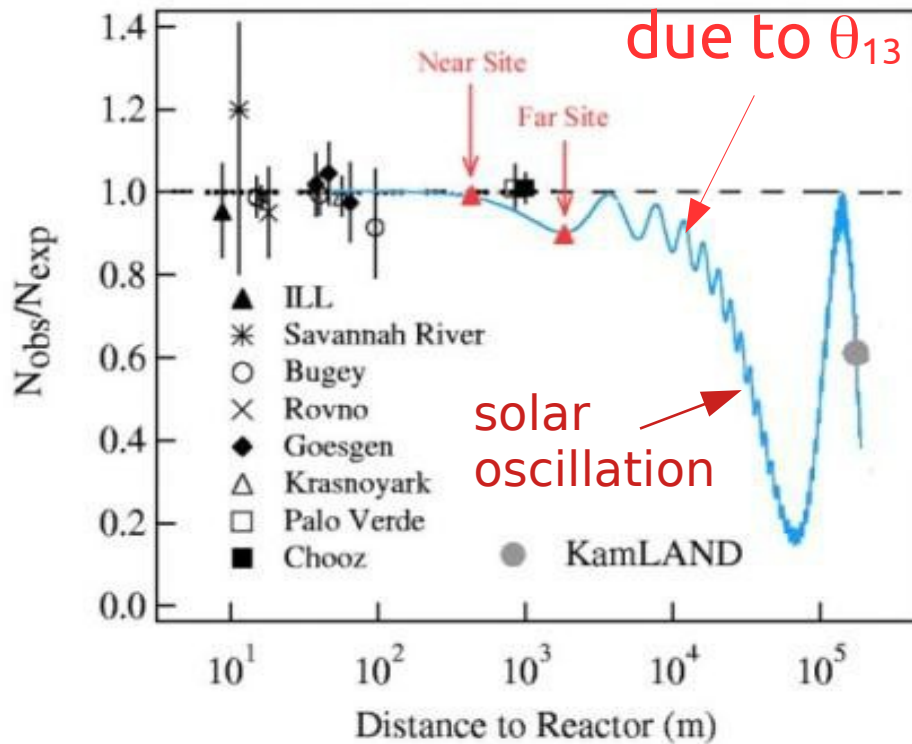
Example : Daya Bay



θ_{13} from reactors

$\bar{\nu}_e$ disappearance

subleading
oscillations
due to θ_{13}



$$\theta_{13} = (8.5 \pm 0.1)^\circ \text{ (NO(IO))}$$

3-Neutrino Mixing

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}$$

Solar sector

$$\nu_e \rightarrow \nu_\mu$$

$$\theta_{12} = 34.5^\circ \pm 1.1^\circ$$

$$\Delta m_{12}^2 = +7.56 \times 10^{-5} eV^2$$

13 Sector

$$\nu_\mu \rightarrow \nu_e$$

$$\theta_{13} = 8.44^\circ \pm 0.16^\circ$$

$$\Delta m_{23}^2 = |2.52 \times 10^{-3}| eV^2$$

Atmospheric sector

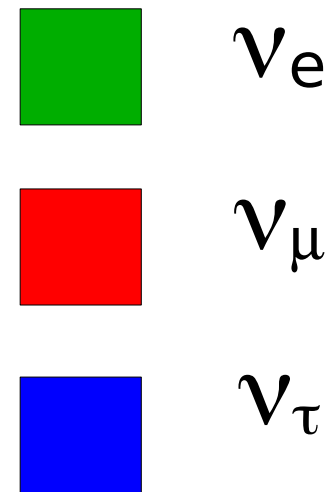
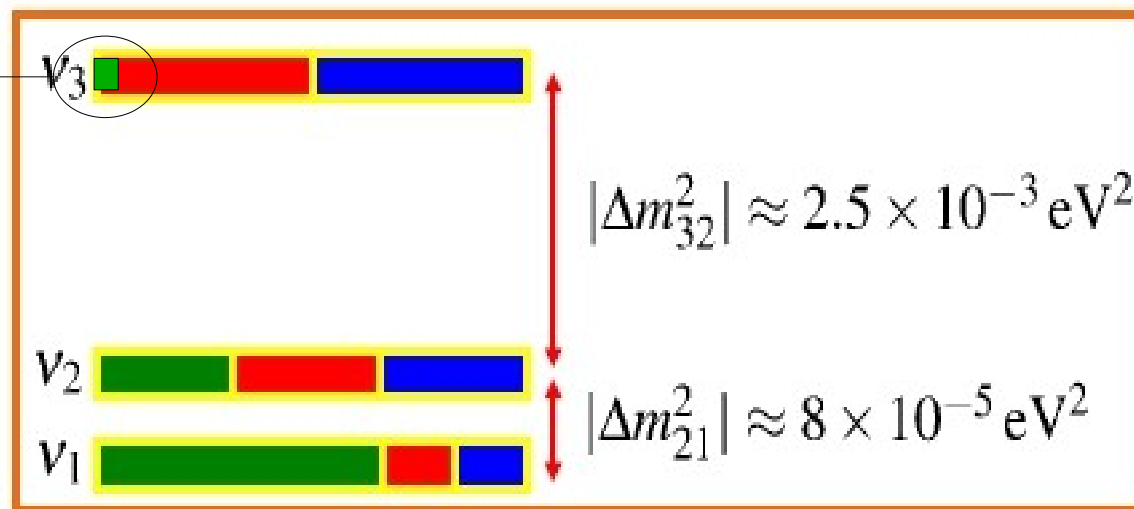
$$\nu_\mu \rightarrow \nu_\tau$$

$$\theta_{23} = 41.0(50.5)^\circ \pm 1.1^\circ$$

$$\Delta m_{23}^2 = |2.52 \times 10^{-3}| eV^2$$

Summary of Current Knowledge

θ_{13} : how much ν_e is in ν_3



$$U_{MNSP} \approx \begin{pmatrix} 0.82 & 0.54 & 0.14 \\ 0.35 & 0.56 & 0.68 \\ 0.35 & 0.55 & 0.69 \end{pmatrix}$$

Uncertainties on the order of 10%

Very very different from the quark CKM matrix