Perturbative and colorful lectures on **Strong Interactions**

lecture 3/3

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1. A few words about QCD
   1. running of α_S
   2. asymptotic freedom and confinement
2. quarks and gluons from e^+e^- annihilation
   1. quark electric charges
   2. quark and gluon spins
   3. QCD gauge parameters
3. Structure of hadrons
   1. elastic scattering
   2. deep inelastic scattering
   3. proton structure functions
   4. DGLAP evolution equation
4. hadron - hadron interactions
   1. jet production
   2. Drell-Yan process

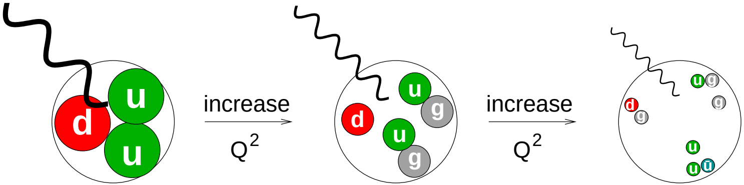
Compton wave length

To resolve small distances (Δr) we need small de Broglie wave length:

$$\lambda\!\!\!^{-} = \lambda/2\pi = \Delta r/2\pi = \frac{\hbar\, c}{p} \quad \Rightarrow \quad \Delta r = \frac{h\, c}{p} \tag{1}$$

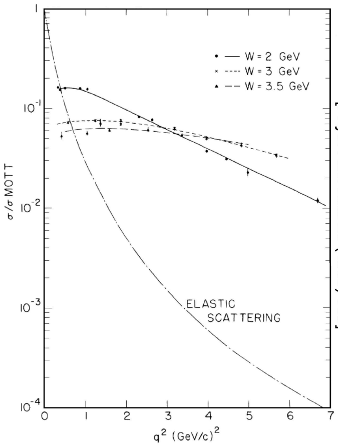
As $E^2 = c^2 p^2 + m^2 c^4$:

$$\lambda\!\!\!^{-} = \frac{\hbar\, c}{\sqrt{E^2 - m^2 c^4}} \overset{E^2 \ll -m^2 c^4}{\simeq} \frac{\hbar\, c}{Q}$$

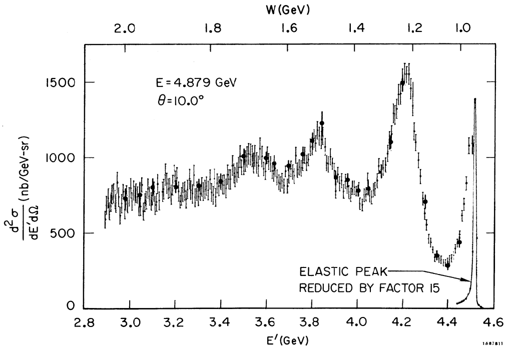


Deep Inelastic Scattering

1967 SLAC - e beam up to 21 GeV



[Phys. Rev. Lett. 23 (1969) 935]



$$W^2 = (q+p)^2 = X^2 = q^2 + 2p\cdot q + m_p^2$$

$$\boxed{x = \frac{Q^2}{2\,p\cdot q} \Rightarrow W^2 = m_p^2 + \frac{1-x}{x}Q^2}$$

- Elastic case: $\delta(E - E' + q^2/2m_p)$

$$W^2 = m_p^2 + \frac{1-x}{x}Q^2 = m_p^2 \qquad \rightarrow \qquad x = 1$$

- deep inelastic case: $x \ll 1$

- in between: resonances

$\Rightarrow$ in the deep inelastic (continuum) region need to parametrise the $\gamma^* - p$ interaction in the most general form

$$W_{\mu\nu} = -W_1 g_{\mu\nu} + \frac{W_2}{m_p^2} p_\mu p_\nu + \frac{W_3}{m_p^2}(p_\mu q_\nu + q_\mu p_\nu)$$
$$+ \frac{W_4}{m_p^2}(p_\mu q_\nu - q_\mu p_\nu) + \frac{W_5}{m_p^2} q_\mu q_\nu + \frac{W_6}{m_p^2} \varepsilon_{\mu\nu\rho\sigma} p^\rho q^\sigma$$

For a single photon exchange, imposing hadronic current conservation and due to the symmetry of $L^{\mu\nu}$, only 2 terms survive.

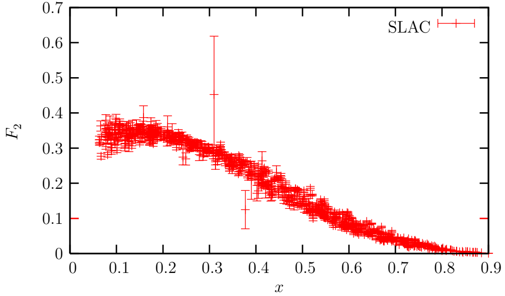
In Björken limit : $Q^2 \to \infty$, $s \to \infty$ and x fixed

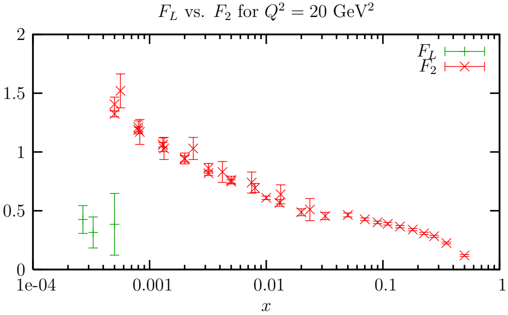
$$\boxed{\frac{d^2\sigma}{dx\,dQ^2} = \frac{4\,\pi\,\alpha^2}{x\,Q^4}\left[x\,y^2\,F_1(x,Q^2) + (1-y)\,F_2(x,Q^2)\right]}$$

with $y = \frac{p\cdot q}{p\cdot k}$

$$\frac{d^2\sigma}{dx\,dQ^2} = \frac{2\,\pi\,\alpha^2}{x\,Q^4}\left\{\left[1+(1-y)^2\right]F_2(x,Q^2) - y^2\,F_L(x,Q^2)\right\}$$

SLAC DIS data: scale invariance



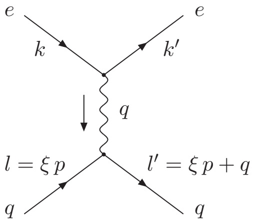


$$F_2(x, Q^2) \to F_2(x)$$

$F_L(x, Q^2)$ small but not zero

DIS in a naive quark model

quark: spin 1/2 point like particle of fractional electric charge e_q



$$\frac{d^2\hat{\sigma}_{eq\to eq}}{dx\, dQ^2}(\xi) = \frac{2\,\pi\, e_q^2\, \alpha^2}{Q^4}\left[1 + (1-y)^2\right]\delta(x-\xi)$$

$\Rightarrow$ simple physical interpretation of x : proton momentum fraction carried by the interacting quark

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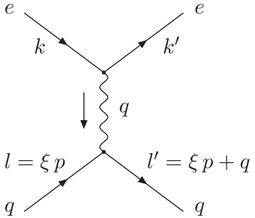

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interacting on the proton is the sum of interaction on all quarks flavours times the probability to find such a quark, integrated over their internal momentum fraction:

$$\begin{aligned}\frac{d^2\sigma_{ep\to eX}}{dx\, dQ^2} &= \int_0^1 d\xi \sum_q f_q(\xi)\frac{d^2\hat{\sigma}_{eq\to eq}}{dx\, dQ^2}(\xi, Q^2)\\ &= \frac{2\,\pi\,\alpha^2}{Q^4}\int_0^1 d\xi \sum_q e_q^2\, f_q(\xi)\left[1 + (1-y)^2\right]\delta(x-\xi)\end{aligned}$$

Comparing

$$\frac{d^2\sigma_{ep\to eX}}{dx\,dQ^2} = \frac{2\,\pi\,\alpha^2}{Q^4}\int_0^1 d\xi \sum_q e_q^2\, f_q(\xi)\,\left[y^2 + 2(1-y)\right]\;\delta(x-\xi)$$

to the DIS general expression:

$$\boxed{\frac{d^2\sigma}{dx\,dQ^2} = \frac{4\,\pi\,\alpha^2}{x\,Q^4}\left[x\,y^2\,F_1(x,Q^2) + (1-y)\,F_2(x,Q^2)\right]}$$

one gets a simple interpretation of the SF in the naive quark model:

$$F_1(x,Q^2) = \frac{1}{2}\int_0^1 d\xi \sum_q e_q^2\, f_q(\xi)\,\delta(x-\xi) = \frac{1}{2}\sum_q e_q^2\, f_q(x)$$

$$F_2(x,Q^2) = \int_0^1 d\xi \sum_q e_q^2\, x\, f_q(\xi)\,\delta(x-\xi) = \sum_q e_q^2\, x\, f_q(x)$$

⇒ simple interpretation:

$$F_1(x,Q^2) = \frac{1}{2}\sum_q e_q^2\, f_q(x) = \frac{1}{2}\sum_q e_q^2\,[q(x)+\bar{q}(x)] = F_1(x),$$
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⇒ scale invariance

$\Rightarrow$ simple interpretation:

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$\Rightarrow$ scale invariance

Callan-Gross relation:

$$F_2(x) = 2\,x\,F_1(x) \qquad \Rightarrow \qquad F_L(x) = F_2(x) - 2\,x\,F_1(x) = 0\,.$$

(due to helicity conservation: F_L is associated to spin flip)

very nice success - but not the full story

Using neutrons

Assumption ($SU(2)$ isospin): neutron is just proton with $u \Leftrightarrow d$:

proton = uud; neutron = ddu

Isospin: $u_n(x) = d_p(x)\,, \qquad d_n(x) = u_p(x)$

$$F_2^p = \frac{4}{9}u_p(x) + \frac{1}{9}d_p(x)$$
$$F_2^n = \frac{4}{9}u_n(x) + \frac{1}{9}d_n(x) = \frac{4}{9}d_p(x) + \frac{1}{9}u_p(x)$$

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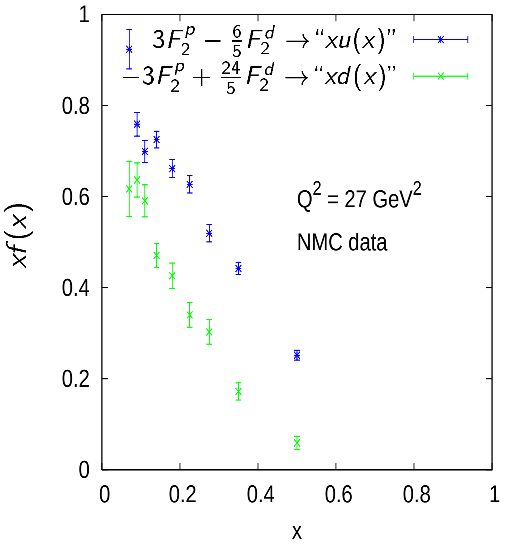
Linear combinations of F_2^p and F_2^n give separately $u_p(x)$ and $d_p(x)$.

Experimentally, get F_2^n from deuterons: $F_2^d = \dfrac{1}{2}(F_2^p + F_2^n)$

NMC proton & deuteron data



Combine F_2^p & F_2^d data, deduce $u(x)$, $d(x)$:

- Definitely more up than down (✓)

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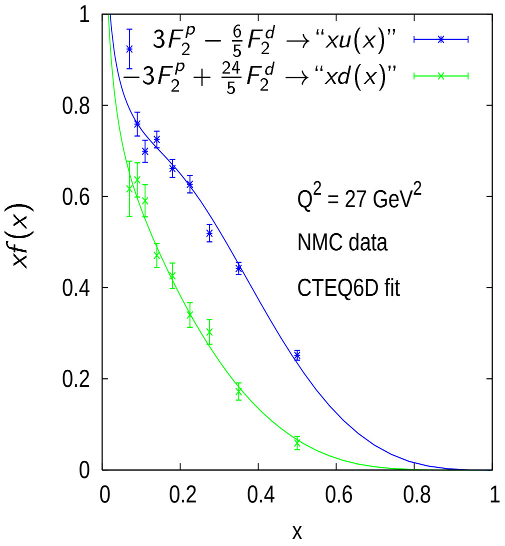

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<u>How much u and d?</u>

- Total $U = \int dx\, u(x)$
- $u(x) \sim d(x) \sim x^{-1.25}$

non-integrable divergence

So why do we say proton = uud?

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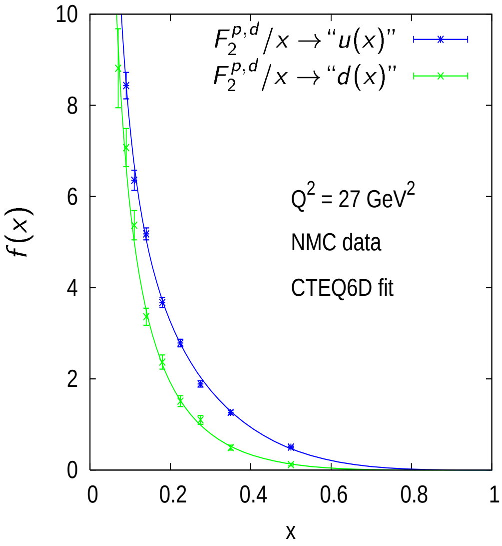

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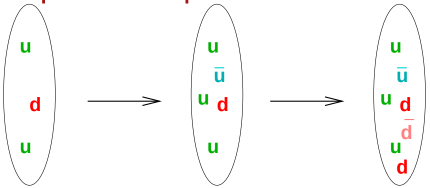
Anti-quarks in proton



How can there be infinite number of quarks in proton?

Proton wave function *fluctuates* — extra $u\bar{u}$, $d\bar{d}$ pairs (*sea quarks*) can appear:

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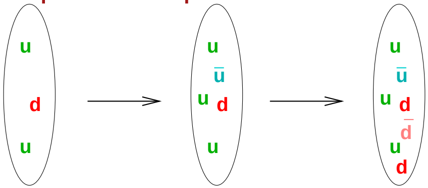

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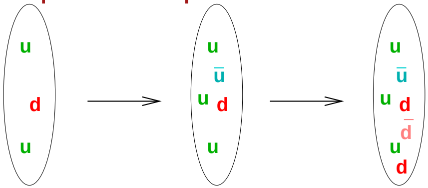
Proton wave function *fluctuates* — extra $u\bar{u}$, $d\bar{d}$ pairs (*sea quarks*) can appear:

Anti quarks also have distributions, $\bar{u}(x)$, $\bar{d}(x)$

$$F_2 = \frac{4}{9}(xu(x) + x\bar{u}(x)) + \frac{1}{9}(xd(x) + x\bar{d}(x))$$

NB: photon interaction $\sim$ square of charge

Anti-quarks in proton



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- Previous transparency: we were actually looking at $\sim u + \bar{u}$, $d + \bar{d}$
- Number of extra quark-antiquark pairs can be *infinite,* so

$$\int dx\,(u(x) + \bar{u}(x)) = \infty$$

as long as they carry little momentum (mostly at low x)

When we say proton has 2 up quarks & 1 down quark, we mean

$$\int dx\,(u(x) - \bar{u}(x)) = 2\,, \qquad \int dx\,(d(x) - \bar{d}(x)) = 1$$

$u - \bar{u} = u_V$ is known as a *valence* distribution.

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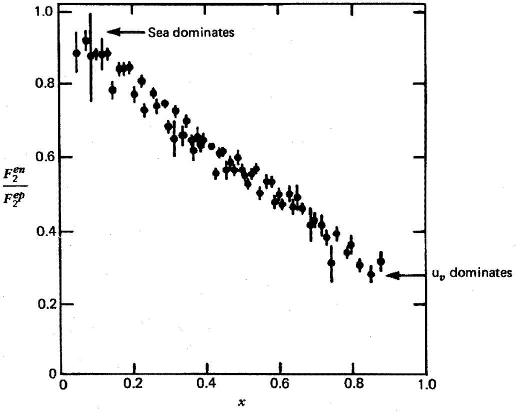
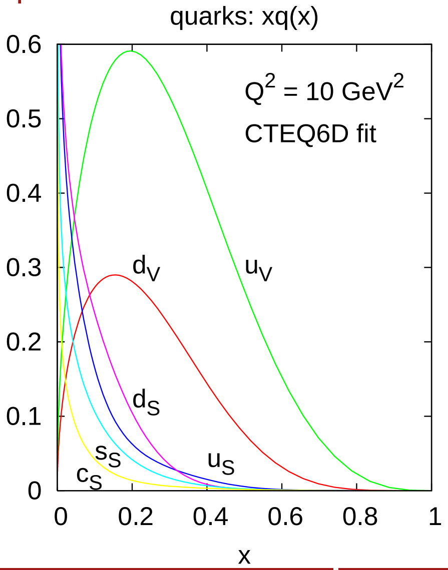


$$\lim_{x\to 1} \quad \frac{F_2^n}{F_2^p} \to \frac{u_V + 4d_V}{4u_V + d_V} \simeq \frac{1}{4}$$

$$\lim_{x\to 0} \quad \frac{F_2^n}{F_2^p} \to 1$$

All quarks



These & other methods $\rightarrow$ whole set of quarks & anti quarks

NB: also strange and charm quarks

- valence quarks ($u_V = u - \bar{u}$) are *hard*
  $x \rightarrow 1 : xq_V(x) \sim (1-x)^3$
  $x \rightarrow 0 : xq_V(x) \sim x^{0.5}$
- sea quarks ($u_S = 2\bar{u}, \ldots$) fairly *soft* (low-momentum)
  $x \rightarrow 1 : xq_S(x) \sim (1-x)^7$
  $x \rightarrow 0 : xq_S(x) \sim x^{-0.2}$

Momentum sum rule

Check momentum sum-rule (sum over all species carries all momentum):

$$\sum_i \int dx\, x\, q_i(x) = 1$$

q_i	momentum
d_V	0.111
u_V	0.267
d_S	0.066
u_S	0.053
s_S	0.033
c_S	0.016
total	0.546

Where is missing momentum?

Momentum sum rule

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$$\sum_i \int dx\, x\, q_i(x) = 1$$

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Where is missing momentum?

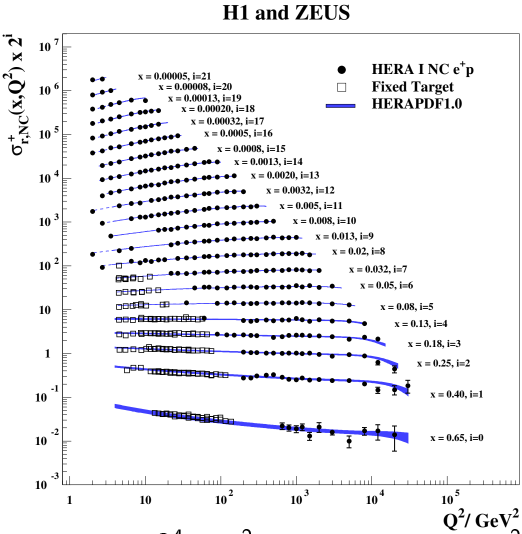
Only parton type we've neglected so far is the

gluon

Not directly probed by γ or $W^\pm$.

To discuss gluons we must go beyond 'naive' leading order picture, and bring in QCD effects...

DIS measurement



$$\sigma_r(x, Q^2) = \frac{1}{Y_+}\,\frac{x\,Q^4}{2\,\pi\,\alpha^2}\,\frac{d^2\sigma}{dx\,dQ^2} = F_2(x, Q^2) - \frac{y^2}{Y_+}\,F_L(x, Q^2).$$

Effect of Z exchange

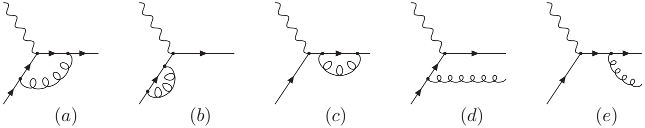
$$\frac{d^2\sigma(e^\pm p \to e^\pm X)}{dx\, dQ^2} = \frac{4\,\pi\,\alpha^2}{x\,Q^4}\Big[x\,y^2\,F_1^{NC}(x,Q^2) + (1-y)\,F_2^{NC}(x,Q^2) \mp y(1-y/2)F_3^{NC}(x,Q^2)\Big].$$

$$F_2^{NC}(x,Q^2) = 2xF_1^{NC}(x,Q^2) \sim \sum_q x\,[q(x)+\bar{q}(x)]$$

$$x\,F_3^{NC}(x,Q^2) \sim \sum_q x[q(x)-\bar{q}(x)]$$

$\Rightarrow$ allows to separate the valence and the sea

First order in α_S



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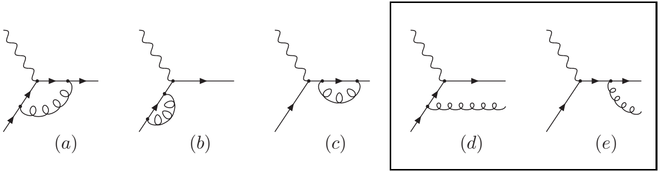

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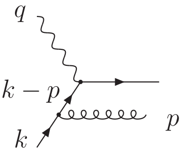

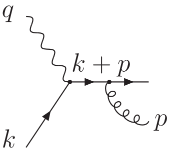

$$\overline{|\mathcal{M}|^2} = 32\pi^2(e_q^2\alpha\alpha_S)\frac{4}{3}\left[-\frac{t}{s}-\frac{s}{t}+\frac{2u\,Q^2}{st}\right]$$

$$\begin{aligned}
t &= (k-p)^2 = (q-k')^2 = -2pk(1-\cos\theta_{qg}) \\
s &= (k+q)^2 = (k'+p)^2 = \ 2pk'(1-\cos\theta_{q'g}) \\
u &= (q-p)^2 = (k-k')^2
\end{aligned}$$

First order in α_S

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double pole structure in t (left diag.) and in s (right diag.)

$t \to 0 : E_g \to 0$ or $g \parallel q$

$s \to 0 : E_g \to 0$ or $g \parallel q'$

First order in α_S: gluon radiation

At high energy ($-t \ll s$), using ($p_T = k' sin\theta$) :

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T^2}(z) = \frac{4\pi^2 \alpha e_q^2}{s} \frac{1}{p_T^2} \frac{\alpha_S}{2\pi} P_{qq}(z)$$

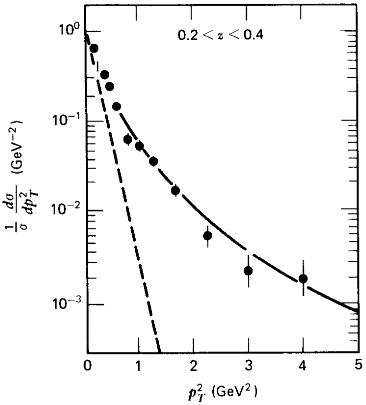
where the longitudinal momentum fraction of the incident quark is:

$$z \equiv \frac{Q^2}{2k \cdot q}$$

$P_{qq}(z)$ is the Splitting Function for a quark that keep a momentum fraction z after a gluon radiation.

$$P_{qq}(z) = \frac{4}{3} \frac{1+z^2}{1-z}$$

EMC $\nu N \to \mu X$ measurement [1980]



– – –– no gluon radiation included

——— gluon radiation included

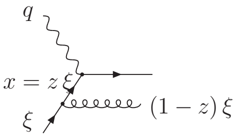
Measurement of the hadron with the largest P_T^2 w.r.t. the virtual photon.

Scaling Violation

These gluon radiation effects have to be included in the quark density.

After integration over p_T^2 between a cutoff (κ) and the maximum ($s/4 = Q^2(1-z)/(4z)$):

$$q(x, Q^2) = q_b(x) + \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} q_b(\xi) P_{qq}\left(\frac{x}{\xi}\right) \ln\left(\frac{Q^2}{\kappa^2}\right)$$



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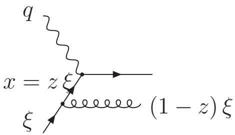

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$\boxed{\Rightarrow \text{Introduction of the Factorisation scale, } \mu_F\ !}$

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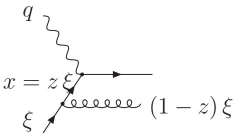
After integration over p_T^2 between a cutoff (κ) and the maximum ($s/4 = Q^2(1-z)/(4z)$):



$$\begin{aligned} q(x,Q^2) &= q_b(x) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\, q_b(\xi)\, P_{qq}\left(\frac{x}{\xi}\right)\, \ln\left(\frac{Q^2}{\kappa^2}\right) \\ &= q(x,\mu_F^2) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\, q_b(\xi)\, P_{qq}\left(\frac{x}{\xi}\right)\, \ln\left(\frac{Q^2}{\mu_F^2}\right) \\ &= q(x,\mu_F^2) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\, q(\xi,\mu_F^2)\, P_{qq}\left(\frac{x}{\xi}\right)\, \ln\left(\frac{Q^2}{\mu_F^2}\right) + \mathcal{O}(\alpha_s^2) \end{aligned}$$

$\Rightarrow$ Introduction of the Factorisation scale, μ_F !

$\Rightarrow$ The scale invariance is logarithmically broken !

DGLAP equation

$q(x, Q^2)$ being an observable, it cannot depend on μ_F :

$$\frac{\mathrm{d}}{\mathrm{d}\ln \mu_F^2}\, q(x, Q^2) = 0 \quad \Rightarrow \quad \frac{\mathrm{d}\, q(x, Q^2)}{\mathrm{d}\ln Q^2} = \frac{\alpha_s}{2\,\pi} \int_x^1 \frac{d\xi}{\xi}\, q(\xi, Q^2) P_{qq}\left(\frac{x}{\xi}\right)$$

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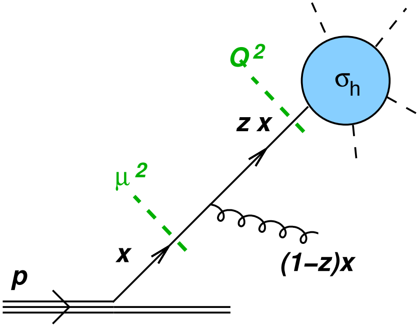

All processes

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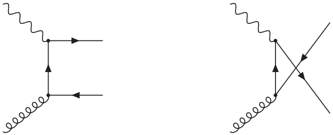
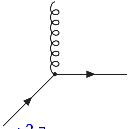
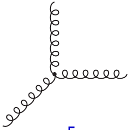


$$P_{qg}(z) = \frac{1}{2}\left[z^2 + (1-z)^2\right]$$



$$P_{gq}(z) = \frac{4}{3}\left[\frac{1+(1-z)^2}{z}\right]$$



$$P_{gg}(z) \;=\; 6\left[\frac{z}{(1-z)} + (1-z)(z+\frac{1}{z})\right]$$

- P_{qg}, P_{gg}: *symmetric $z \leftrightarrow 1-z$*
- P_{qq}, P_{gg}: *diverge for $z \to 1$ (soft gluon emission)*
- P_{gg}, P_{gq}: *diverge for $z \to 0$*

Implies PDFs grow for $x \to 0$

DGLAP equations: flavour structure

Proton contains both quarks and gluons — so DGLAP is a *matrix in flavour space:*

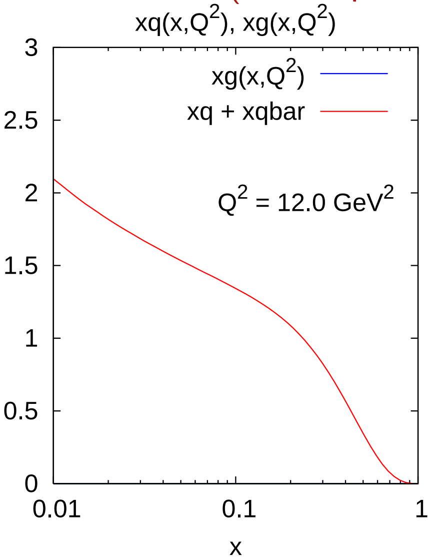
$$\frac{d}{d\ln Q^2}\begin{pmatrix} q_i(x,Q^2) \\ \bar{q}_i(x,Q^2) \\ g(x,Q^2) \end{pmatrix} = \frac{\alpha_s(Q^2)}{2\pi}\sum_{j=1}^{n_f}\int_x^1 \frac{d\xi}{\xi}\begin{pmatrix} P_{qq}\delta_{ij} & 0 & P_{qg} \\ 0 & P_{gq}\delta_{ij} & P_{qg} \\ P_{gq} & P_{gq} & P_{gg} \end{pmatrix}\begin{pmatrix} q_j(x,Q^2) \\ \bar{q}_j(x,Q^2) \\ g(x,Q^2) \end{pmatrix}$$

$$[P_{\bar{q}g} = P_{qg}]$$

$$q(x,Q^2) = q(x,\mu_F^2) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\, q(\xi,\mu_F^2)\, P_{qq}\left(\frac{x}{\xi}\right)\ln\left(\frac{Q^2}{\mu_F^2}\right) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\, g(\xi,\mu_F^2)\, P_{qg}\left(\frac{x}{\xi}\right)\ln\left(\frac{Q^2}{\mu_F^2}\right)$$

$\Rightarrow$ measuring the $d\sigma$ one can also access the gluon density !

Effect of DGLAP (initial quarks)



Take example evolution starting with just quarks:

$$\partial_{\ln Q^2} q = P_{qq} \otimes q$$
$$\partial_{\ln Q^2} g = P_{gq} \otimes q$$

- quark is depleted at large x
- gluon grows at small x

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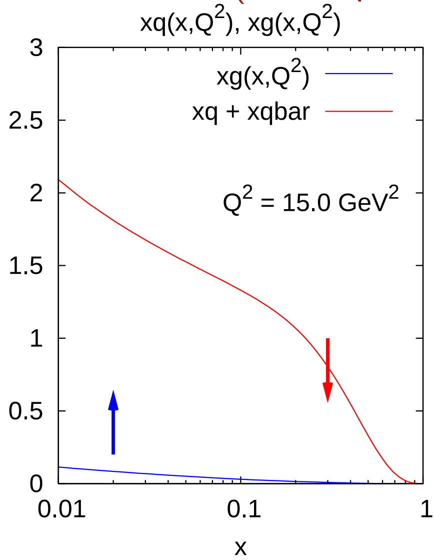

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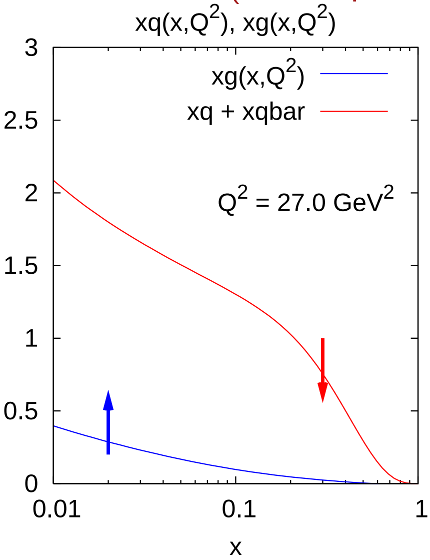

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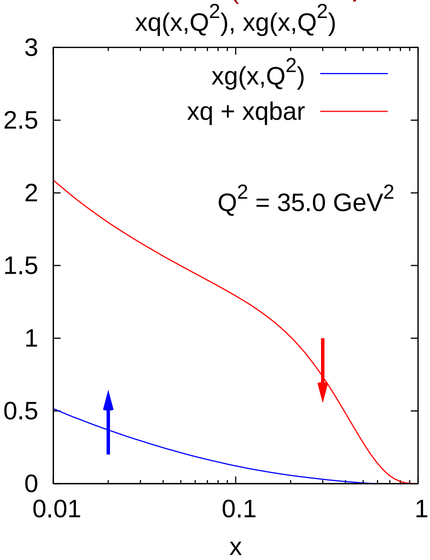

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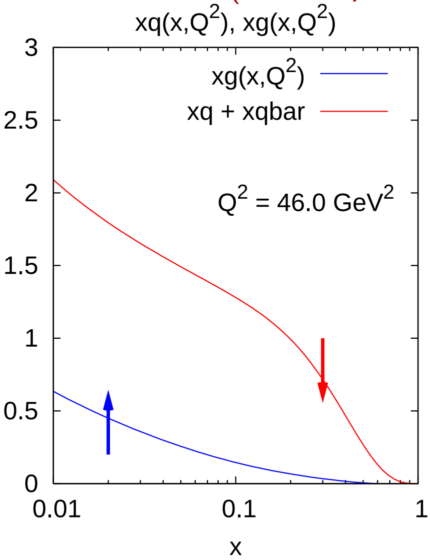

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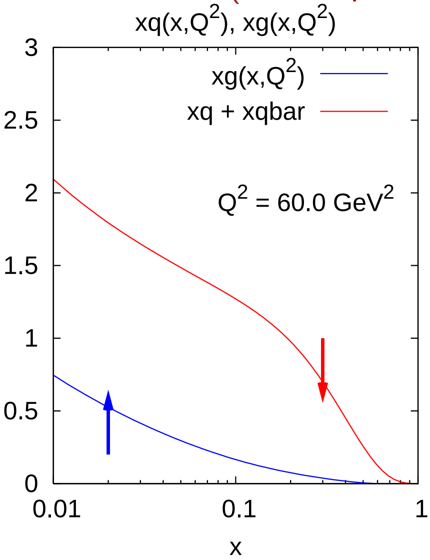

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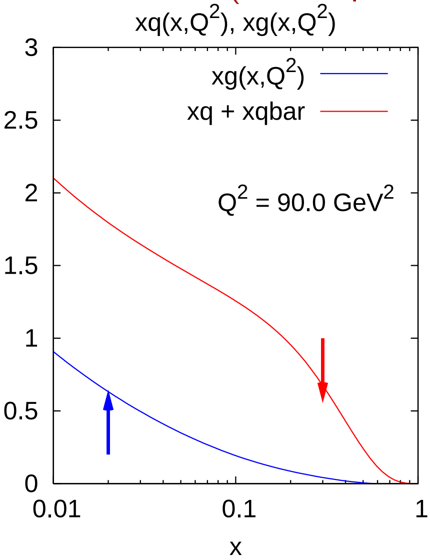

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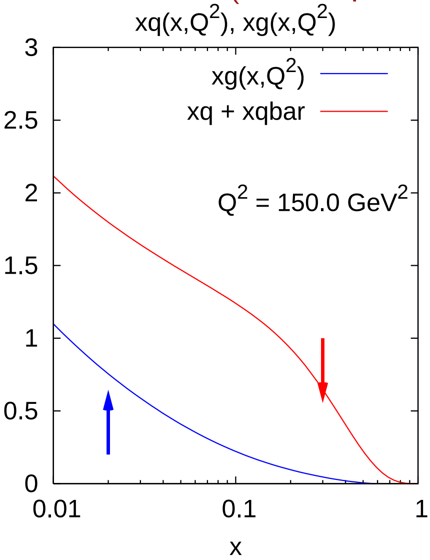

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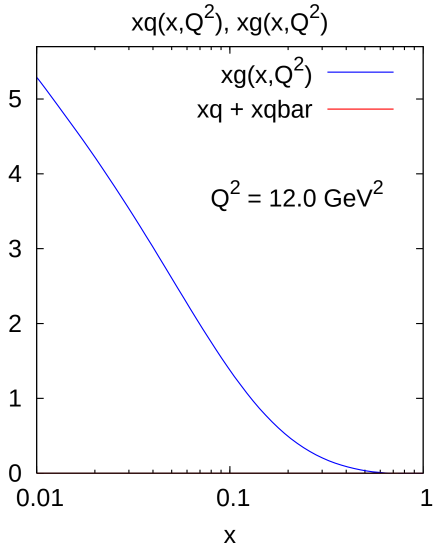

2nd example: start with just gluons.

$$\partial_{\ln Q^2} q = P_{qg} \otimes g$$
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- high-x gluon feeds growth of small x gluon & quark.

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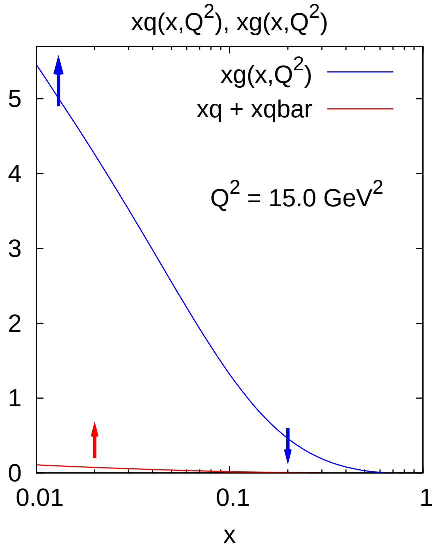

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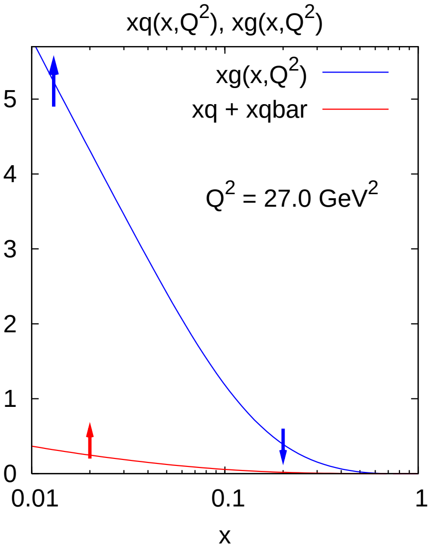

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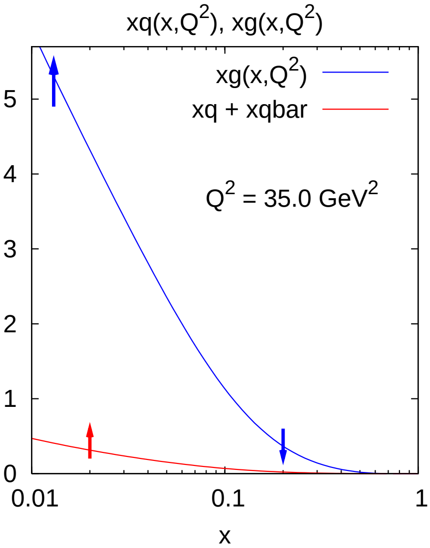

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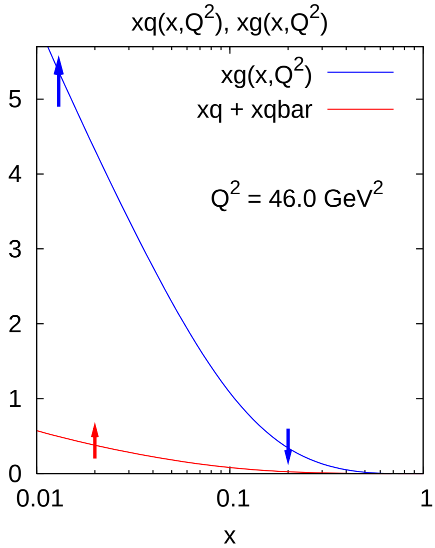

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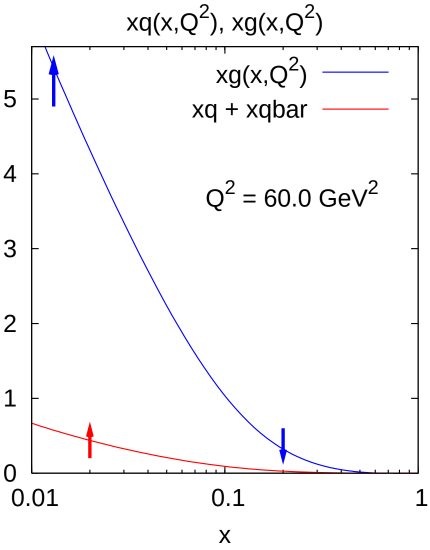

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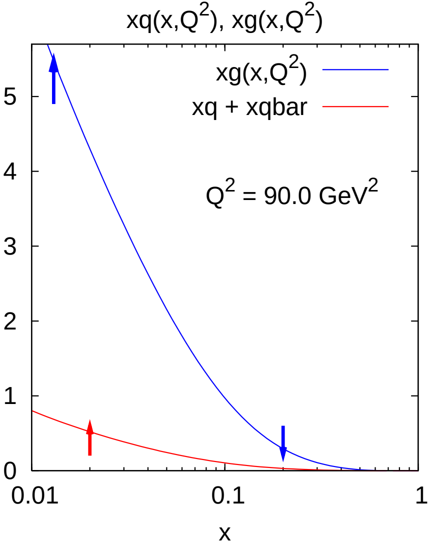

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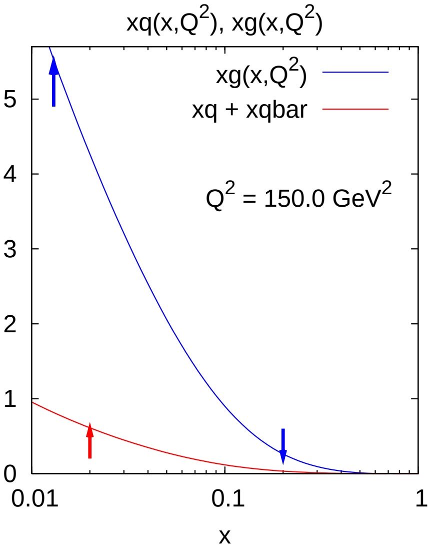

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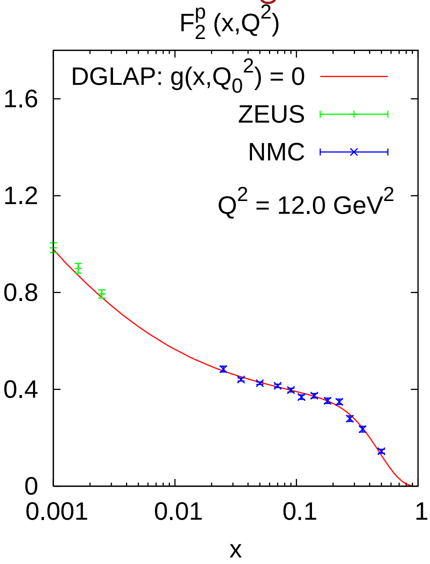
DGLAP evolution

- As Q^2 increases, partons lose longitudinal momentum; distributions all shift to lower x.
- stable point at $x = 0.18$ corresponding to the scaling observed at SLAC
- gluons can be seen because they drive the quark evolution.

Now consider data

DGLAP with initial gluon $= 0$



Fit quark distributions to $F_2(x, Q_0^2)$, at *initial scale* $Q_0^2 = 12\mathrm{GeV}^2$.

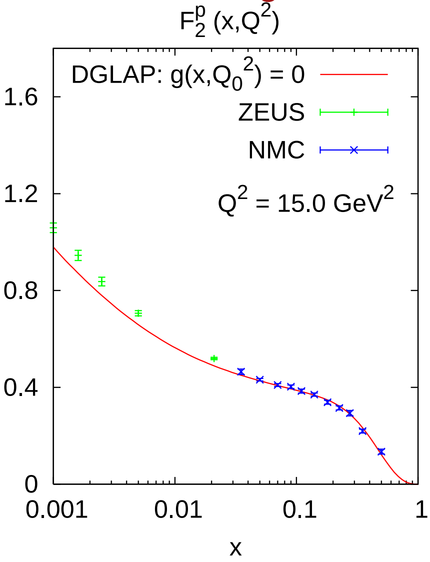
NB: Q_0 often chosen lower

Assume there is no gluon at Q_0^2:

$$g(x, Q_0^2) = 0$$

Use DGLAP equations to evolve to higher Q^2; compare with data.

DGLAP with initial gluon = 0



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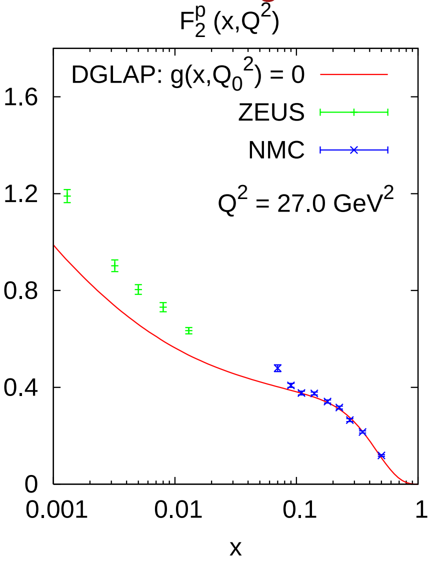

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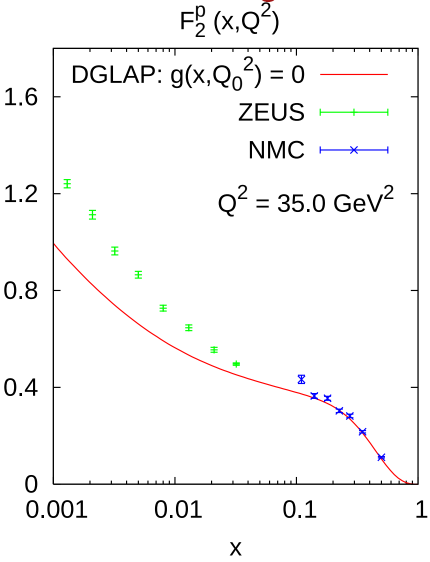

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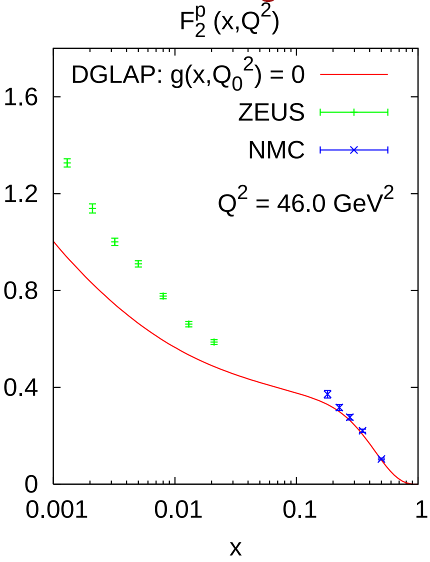

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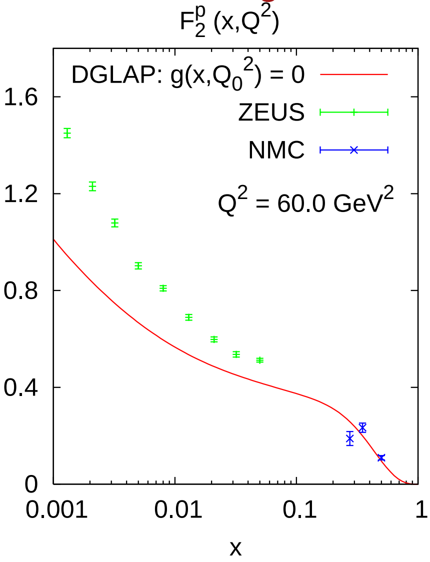

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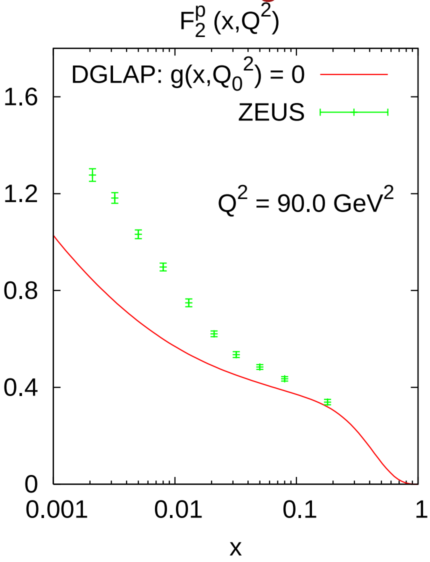

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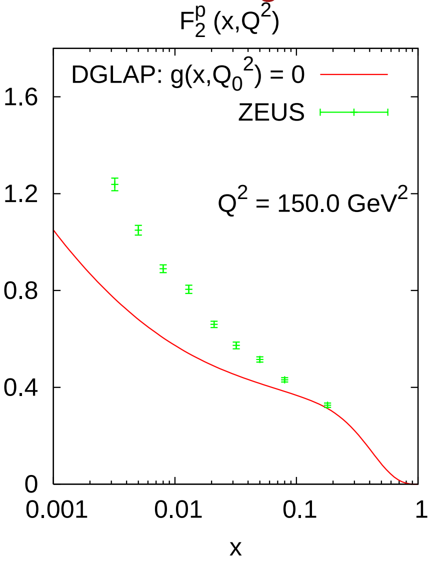

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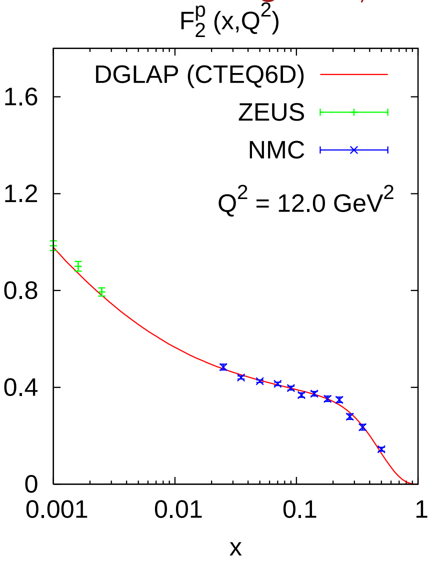
Use DGLAP equations to evolve to higher Q^2; compare with data.

Complete failure!

DGLAP with initial gluon $\neq 0$



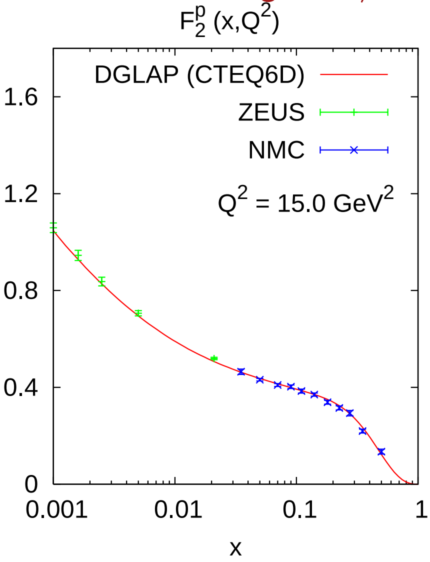
If gluon $\neq 0$, splitting $g \to q\bar{q}$ generates *extra quarks at large Q^2.*

➥ faster rise of F_2

Find a gluon distribution that leads to correct evolution in Q^2.

Done for us by CTEQ, MRST, ...
PDF fitting collaborations.

DGLAP with initial gluon $\neq 0$



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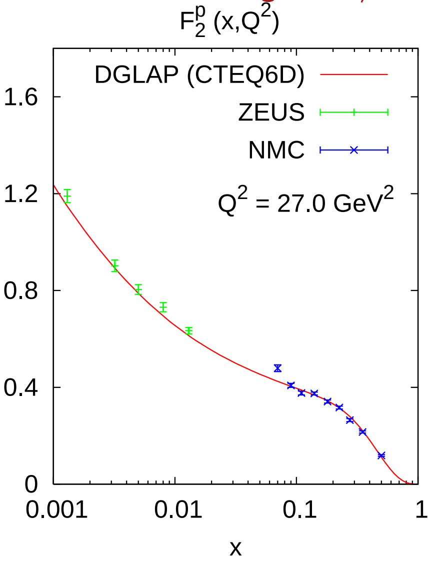

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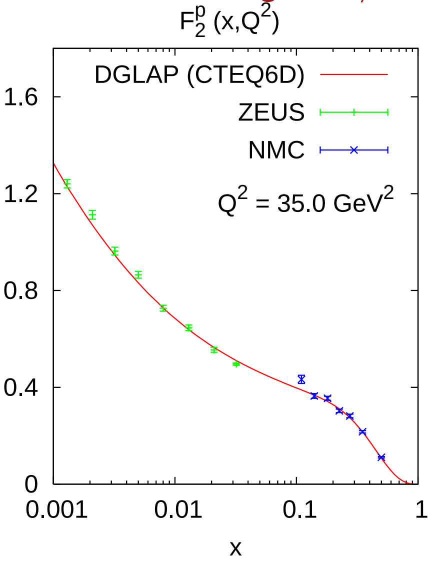

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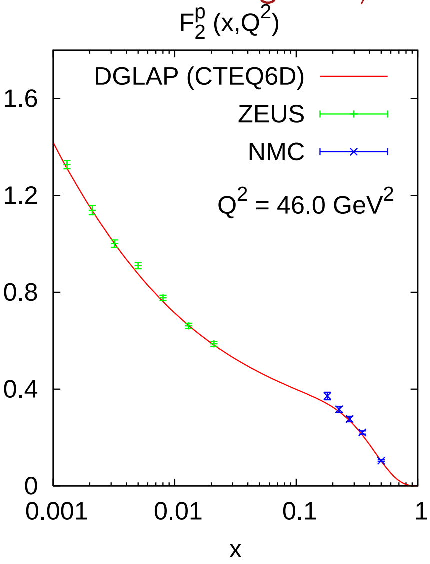

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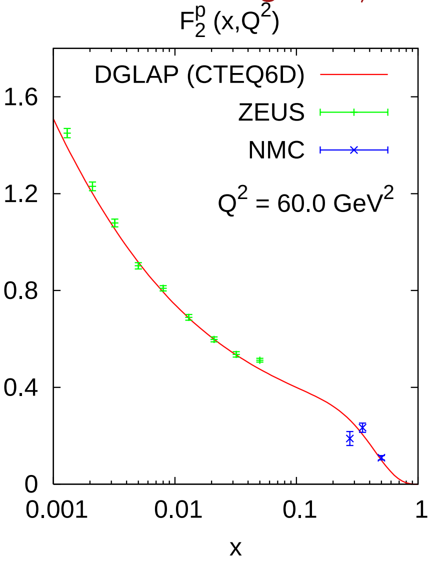

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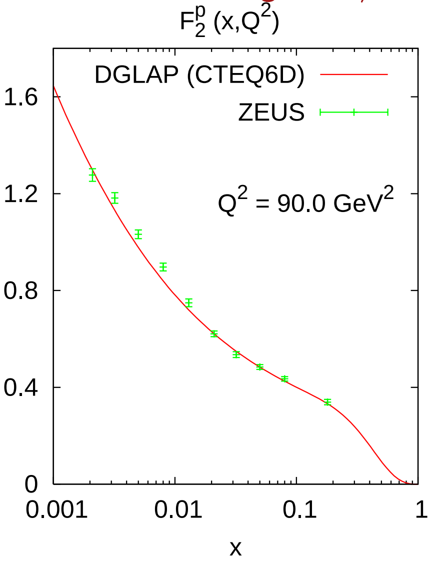

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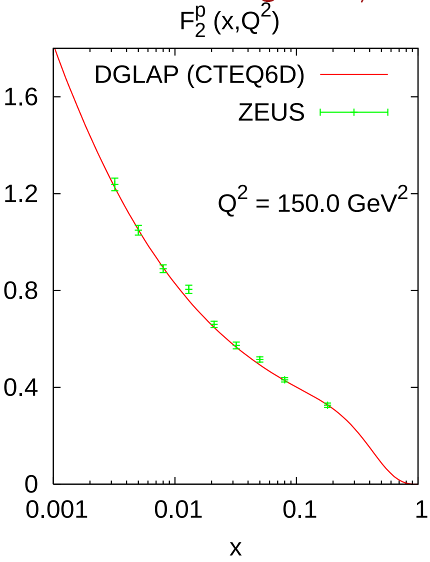

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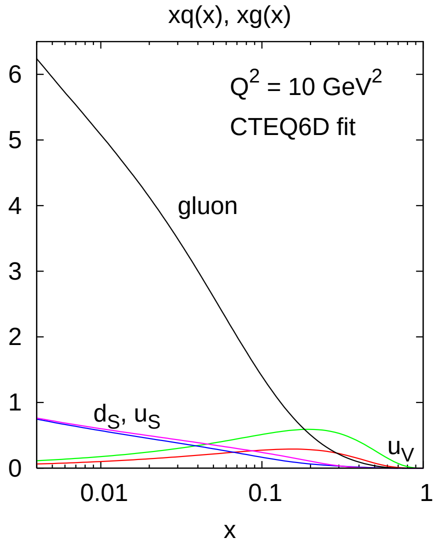
Find a gluon distribution that leads to correct evolution in Q^2.

Done for us by CTEQ, MRST, ...
PDF fitting collaborations.

Success!

Gluon distribution



Gluon distribution is **HUGE!**

Can we really trust it?

- Consistency: momentum sum-rule is now *satisfied*.

  NB: gluon mostly at small x

- Agrees with vast range of data

- such a set of q and g densities is called a PDF set for Parton Distribution Functions.

Higher-order calculations

$P_{ab}^{(1)}$: Curci, Furmanski & Petronzio '80

NLO:

$$P_{ab} = \frac{\alpha_s}{2\pi} P_{ab}^{(0)} + \left(\frac{\alpha_s}{2\pi}\right)^2 P_{ab}^{(1)}$$

$$P_{\rm ps}^{(1)}(x) = 4\, C_F n_f \left(\frac{20}{9}\frac{1}{x} - 2 + 6x - 4\mathrm{H}_0 + x^2\left[\frac{8}{3}\mathrm{H}_0 - \frac{56}{9}\right] + (1+x)\left[5\mathrm{H}_0 - 2\mathrm{H}_{0,0}\right]\right)$$

$$\begin{aligned}
P_{\rm qg}^{(1)}(x) = {}& 4\, C_A n_f \left(\frac{20}{9}\frac{1}{x} - 2 + 25x - 2p_{\rm qg}(-x)\mathrm{H}_{-1,0} - 2p_{\rm qg}(x)\mathrm{H}_{1,1} + x^2\left[\frac{44}{3}\mathrm{H}_0 - \frac{218}{9}\right] \right.\\
&\left. +4(1-x)\left[\mathrm{H}_{0,0} - 2\mathrm{H}_0 + x\mathrm{H}_1\right] - 4\zeta_2 x - 6\mathrm{H}_{0,0} + 9\mathrm{H}_0\right) + 4\, C_F n_f \left(2p_{\rm qg}(x)\left[\mathrm{H}_{1,0} + \mathrm{H}_{1,1} + \mathrm{H}_2 \right.\right.\\
&\left.\left. -\zeta_2\right] + 4x^2\left[\mathrm{H}_0 + \mathrm{H}_{0,0} + \frac{5}{2}\right] + 2(1-x)\left[\mathrm{H}_0 + \mathrm{H}_{0,0} - 2x\mathrm{H}_1 + \frac{29}{4}\right] - \frac{15}{2} - \mathrm{H}_{0,0} - \frac{1}{2}\mathrm{H}_0\right)
\end{aligned}$$

$$\begin{aligned}
P_{\rm gq}^{(1)}(x) = {}& 4\, C_A C_F \left(\frac{1}{x} + 2p_{\rm gq}(x)\left[\mathrm{H}_{1,0} + \mathrm{H}_{1,1} + \mathrm{H}_2 - \frac{11}{6}\mathrm{H}_1\right] - x^2\left[\frac{8}{3}\mathrm{H}_0 - \frac{44}{9}\right] + 4\zeta_2 - 2 \right.\\
&\left. -7\mathrm{H}_0 + 2\mathrm{H}_{0,0} - 2\mathrm{H}_1 x + (1+x)\left[2\mathrm{H}_{0,0} - 5\mathrm{H}_0 + \frac{37}{9}\right] - 2p_{\rm gq}(-x)\mathrm{H}_{-1,0}\right) - 4\, C_F n_f \left(\frac{2}{3}x \right.\\
&\left. -p_{\rm gq}(x)\left[\frac{2}{3}\mathrm{H}_1 - \frac{10}{9}\right]\right) + 4\, C_F^2 \left(p_{\rm gq}(x)\left[3\mathrm{H}_1 - 2\mathrm{H}_{1,1}\right] + (1+x)\left[\mathrm{H}_{0,0} - \frac{7}{2} + \frac{7}{2}\mathrm{H}_0\right] - 3\mathrm{H}_{0,0} \right.\\
&\left. +1 - \frac{3}{2}\mathrm{H}_0 + 2\mathrm{H}_1 x\right)
\end{aligned}$$

$$\begin{aligned}
P_{\rm gg}^{(1)}(x) = {}& 4\, C_A n_f \left(1 - x - \frac{10}{9}p_{\rm gg}(x) - \frac{13}{9}\left(\frac{1}{x} - x^2\right) - \frac{2}{3}(1+x)\mathrm{H}_0 - \frac{2}{3}\delta(1-x)\right) + 4\, C_A^2 \left(27 \right.\\
& +(1+x)\left[\frac{11}{3}\mathrm{H}_0 + 8\mathrm{H}_{0,0} - \frac{27}{2}\right] + 2p_{\rm gg}(-x)\left[\mathrm{H}_{0,0} - 2\mathrm{H}_{-1,0} - \zeta_2\right] - \frac{67}{9}\left(\frac{1}{x} - x^2\right) - 12\mathrm{H}_0\\
&\left. -\frac{44}{3}x^2\mathrm{H}_0 + 2p_{\rm gg}(x)\left[\frac{67}{18} - \zeta_2 + \mathrm{H}_{0,0} + 2\mathrm{H}_{1,0} + 2\mathrm{H}_2\right] + \delta(1-x)\left[\frac{8}{3} + 3\zeta_3\right]\right) + 4\, C_F n_f \left(2\mathrm{H}_0 \right.\\
&\left. +\frac{2}{3}\frac{1}{x} + \frac{10}{3}x^2 - 12 + (1+x)\left[4 - 5\mathrm{H}_0 - 2\mathrm{H}_{0,0}\right] - \frac{1}{2}\delta(1-x)\right) .
\end{aligned}$$

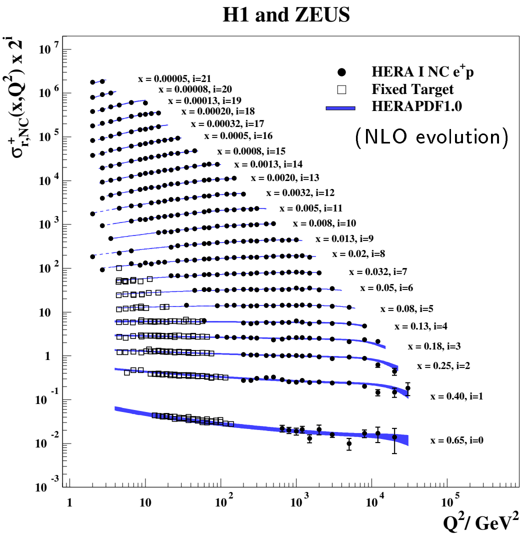
NNLO splitting functions

$P_{ab}^{(2)}$: Moch, Vermaseren & Vogt '04

NNLO:

$$P_{ab} = \frac{\alpha_s}{2\pi} P_{ab}^{(0)} + \left(\frac{\alpha_s}{2\pi}\right)^2 P_{ab}^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^3 P_{ab}^{(2)}$$

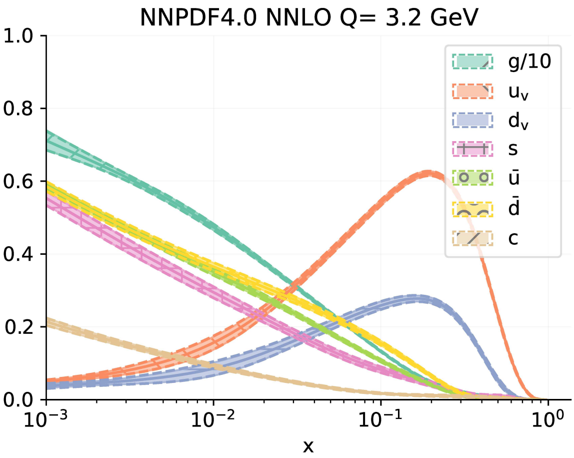
Compare to data



tremendous success

PDF uncertainties - state of the art

NNPDF4.0 [2022]



Major activity is translation of experimental errors (and theory uncertainties) into *uncertainty bands on extracted PDFs.*

PDFs with uncertainties allow one to estimate *degree of reliability* of future predictions

PDF uncertainties - state of the art

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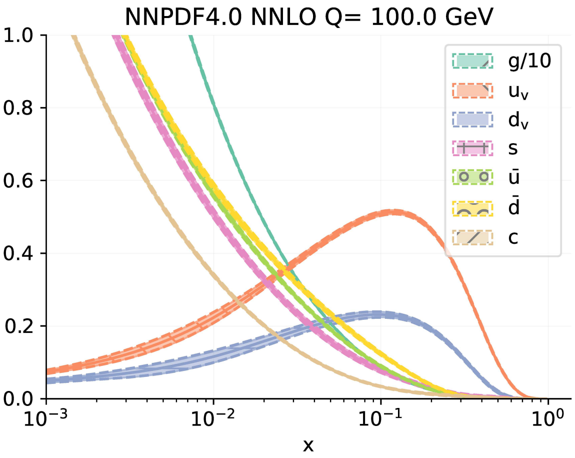


Major activity is translation of experimental errors (and theory uncertainties) into *uncertainty bands on extracted PDFs.*

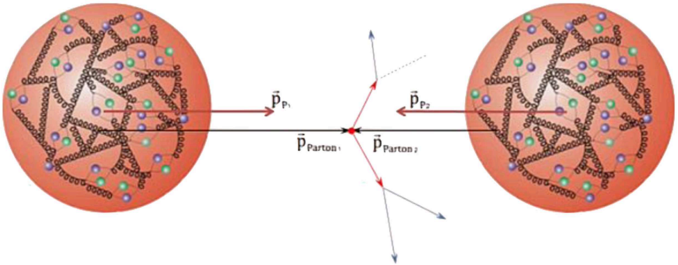
PDFs with uncertainties allow one to estimate *degree of reliability* of future predictions

Conclusion of this section and open questions

- In a few decades our understanding of the proton structure has drastically changed
- from a static model with 3 valence quarks to a dynamic object with a high gluon density ($\rightarrow$ what is a colour field ?)
- DGLAP: phenomenal succes of pQCD on a large phase space (4x4 orders of magnetude)
- Do we have to go beyond DGLAP (BFKL dynamics, saturation,...) ? attractive explanations but not fully proven yet.
- many other open questions today:
  1) transverse spacial quark and gluon distributions ?
  2) correlations between partons ?
  3) how is the proton spin mad by the partons ? (orbital angulair momentum of q and g)
  4) ...

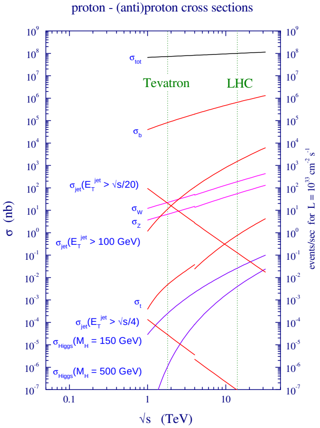
- Part 4 -

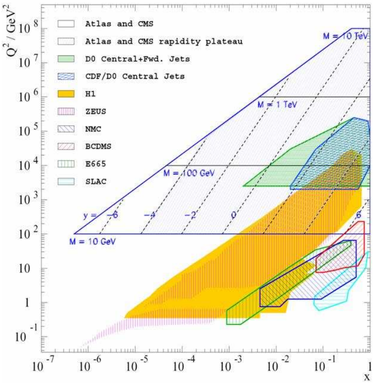
hadron - hadron interactions



High energy $p-p$ (or $p-\bar{p}$) interactions

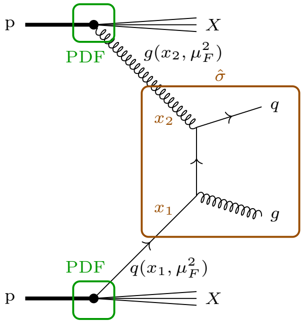
- most of the interactions are non perturbative - even at very high $\sqrt{s}$
- but the available energy makes high momentum transfer possible $\to$ perturbative interactions (scale $= Q^2$)



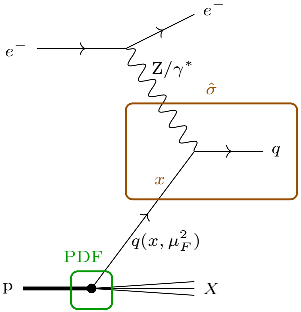


Cross section estimate: the master formula

hadron interactions



DIS



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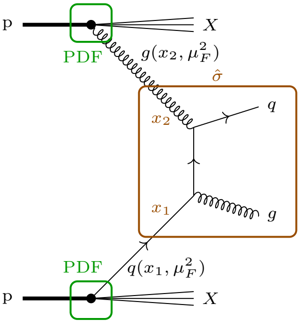
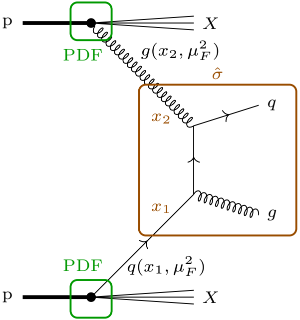


$$\sigma_{pp\to X(Q)} = \sum_{a,b=q,\bar{q},g} \int_0^1 dx_1\, dx_2\, f_{a/h}(x_1, \mu_F)\; f_{b/h'}(x_2, \mu_F)$$
$$\times\; \hat{\sigma}_{ab\to X(Q)}(x_1, x_2, Q, \mu_F, \alpha_s(\mu_R))$$
$$\times\; \theta(x_1 x_2\, s - Q^2)$$

Cross section estimate: the master formula

hadron interactions



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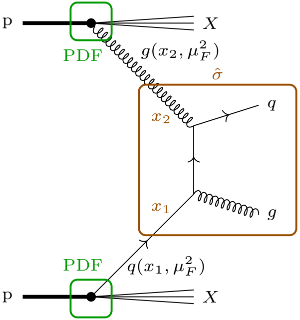
$$\times\; \hat{\sigma}_{ab\to X(Q)}(x_1, x_2, Q, \mu_F, \alpha_s(\mu_R))$$
$$\times\; \theta(x_1 x_2\, s - Q^2)$$

$$\begin{aligned} Q^2 &= M_{qg}^2 = (p_q + p_g)^2 \\ &= 2\, E_q\, E_g\, (1 - \cos\theta_{qg}) \\ &= 4\, E_q\, E_g = 4\, x_1\, p_1\, x_2\, p_2 = x_1\, x_2\, s \end{aligned}$$

Cross section estimate: the master formula

hadron interactions



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$$\times\, \hat{\sigma}_{ab\to X(Q)}(x_1, x_2, Q, \mu_F, \alpha_s(\mu_R))$$
$$\times\, \theta(x_1 x_2\, s - Q^2) + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{Q^2}\right)$$

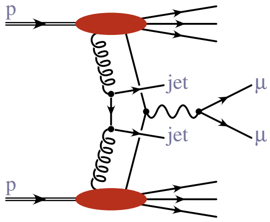
$\mathcal{O}\left(\frac{\Lambda^2_{QCD}}{Q^2}\right)$ vanishing term for $Q^2 \to \infty$

- that can break the factorisation
- called *higher twist*

Multiple Parton Interactions

high energy $\rightarrow$ high parton densities (at low x)
$\rightarrow$ probability of multiple partons scattering increases

Direct consequence of the composite nature of hadrons.
$\rightarrow$ Out of the frame of the QCD factorisation.
$\rightarrow$ No clear separation with single parton + splitting
$\rightarrow$ non-trivial changes of colour topology
$\rightarrow$ in case of two hard interactions: Double Parton Interaction (DPS)



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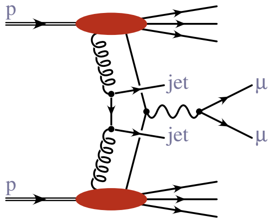
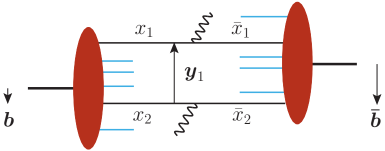


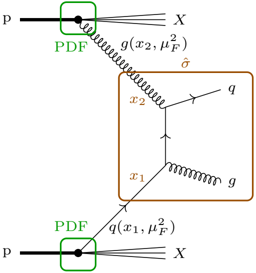
The correlation between the two partons depends on their relative distance:



$\Rightarrow$ possibility to learn about parton location in the proton (like GPDs ?).

Jet production

- The dijet (or total multijet) is the perturbative process with the highest cross section
- first test at a collider with higher energy to test QCD and the presence of new physics



$$\frac{d^3\sigma}{dy_c\, dy_d\, dp_T^2} = \frac{1}{16\pi s^2 x_1 x_2} \sum_{a,b,c,d=q,\bar{q},g} f_{a/h}(x_1, \mu_F)\; f_{b/h'}(x_2, \mu_F)$$

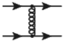
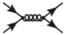
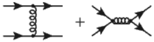
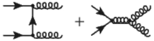
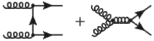
$$\times\, \overline{|\mathcal{M}(ab \to cd)|^2}\, \frac{1}{1+\delta_{cd}}$$

δ_{cd}: statistical factor for identical final state

- x_1 and x_2 can be accessed via:

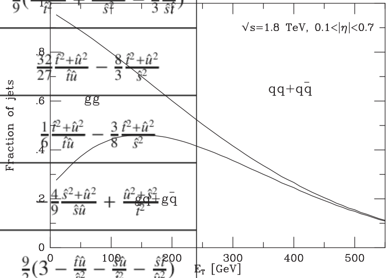
$$\tau = \frac{\hat{s}}{s} = \frac{M_{cd}^2}{s} = x_1 x_2 \qquad y_{cd} = \frac{y_c + y_d}{2} = \frac{1}{2} \ln \frac{x_1}{x_2}$$

Dijet cross section calculation (LO)

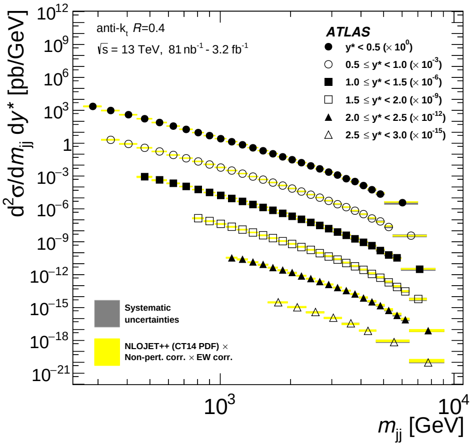
processus	diagrammes	$\overline{\|\mathcal{M}\|^2}/g^4$
$q\,q' \to q\,q'$		$\frac{4}{9}\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}$
$q\,q \to q\,q$		$\frac{4}{9}\left(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}+\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}\right)-\frac{8}{27}\frac{\hat{s}^2}{\hat{u}\hat{t}}$
$q\bar{q} \to q'\bar{q}'$		$\frac{4}{9}\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}$
$q\bar{q} \to q\bar{q}$		$\frac{4}{9}\left(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}+\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}-\frac{2}{3}\frac{\hat{u}^2}{\hat{s}\hat{t}}\right)$
$q\bar{q} \to gg$		$\frac{32}{27}\frac{\hat{t}^2+\hat{u}^2}{\hat{t}\hat{u}}-\frac{8}{3}\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}$
$gg \to q\bar{q}$		$\frac{1}{6}\frac{\hat{t}^2+\hat{u}^2}{\hat{t}\hat{u}}-\frac{3}{8}\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}$
$gq \to gq$		$-\frac{4}{9}\frac{\hat{s}^2+\hat{u}^2}{\hat{s}\hat{u}}+\frac{\hat{u}^2+\hat{s}^2}{\hat{t}^2}$
$gg \to gg$		$\frac{9}{2}\left(3-\frac{\hat{t}\hat{u}}{\hat{s}^2}-\frac{\hat{s}\hat{u}}{\hat{t}^2}-\frac{\hat{s}\hat{t}}{\hat{u}^2}\right)$

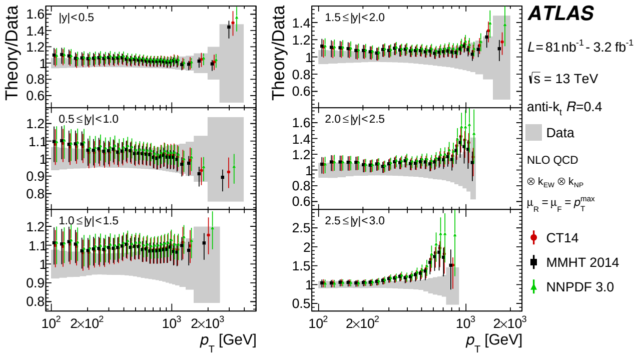
with

$$\hat{s} = (p_a + p_b)^2$$
$$\hat{t} = (p_a - p_c)^2$$
$$\hat{u} = (p_b - p_c)^2$$



Dijet cross section measurement

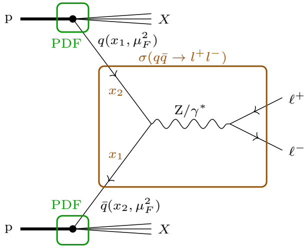




- this measurement constraints further gluon densities at large x

Drell-Yan process

- annihilation of a q with a $\bar{q}$ in hadronic interactions giving a charged lepton pair
- the only process in hadron interaction for which we are approaching percent level precision both experimentally and theoretically



at LO, on can derive easily $\sigma_{q\bar{q}\to l^- l^+}$ from:

$$\frac{d\sigma_{e^-e^+\to q\bar{q}}}{d\Omega} = \frac{\alpha^2}{4s} e_q^2 \left(1+\cos^2\theta\right) = \frac{\alpha^2}{4s} e_q^2 \frac{t^2+u^2}{s^2}$$

where a sum on final state colors was done, here we should take the average:

$$\frac{d\sigma_{q\bar{q}\to l^- l^+}}{d\Omega} = \frac{1}{3}\frac{\alpha^2}{4s_{q\bar{q}}} e_q^2 \frac{t^2+u^2}{s_{q\bar{q}}^2}$$

$$\sigma(q\bar{q} \to l^- l^+) = \frac{4\pi\alpha^2}{9s_{q\bar{q}}} e_q^2$$

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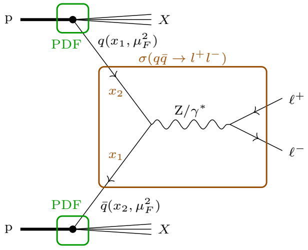


$$\frac{\mathrm{d}\sigma_{pp\to l^- l^+ X}}{\mathrm{d}Q^2} = \sum_{q,\bar{q}} \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \left[q(x_1)\bar{q}(x_2) + (q \leftrightarrow \bar{q})\right] \times \sigma(q\bar{q} \to e^- e^+)\, \delta(Q^2 - s_{q\bar{q}})$$

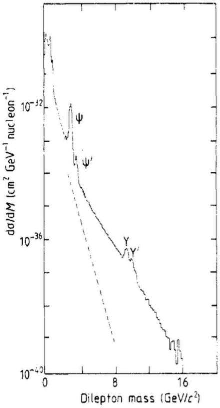
using $\tau = \frac{m_{ll}^2}{s} = \frac{Q^2}{s} = \frac{s_{q\bar{q}}}{s} = x_1 x_2$ and using the δ to get rid of the integral over x_2:

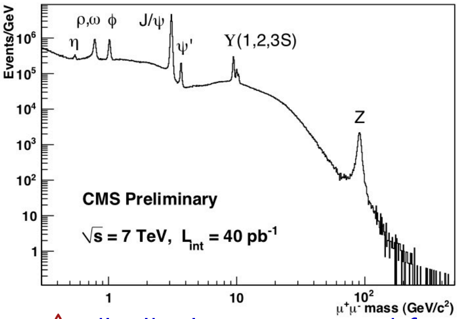
$$\frac{\mathrm{d}\sigma_{pp\to l^- l^+ X}}{\mathrm{d}\tau} = \frac{4\pi\alpha^2}{9s}\frac{1}{\tau} \sum_{q,\bar{q}} e_q^2 \int_\tau^1 \frac{\mathrm{d}x_1}{x_1} \left[q(x_1)\bar{q}(\tau/x_1) + (q \leftrightarrow \bar{q})\right]$$

$\Rightarrow$ scaling (as in DIS)

DY: Mass distribution

- here we derived only the one γ exchange
- but resonances have to be taken into account
- in particular the Z boson at the LHC/TeVatron

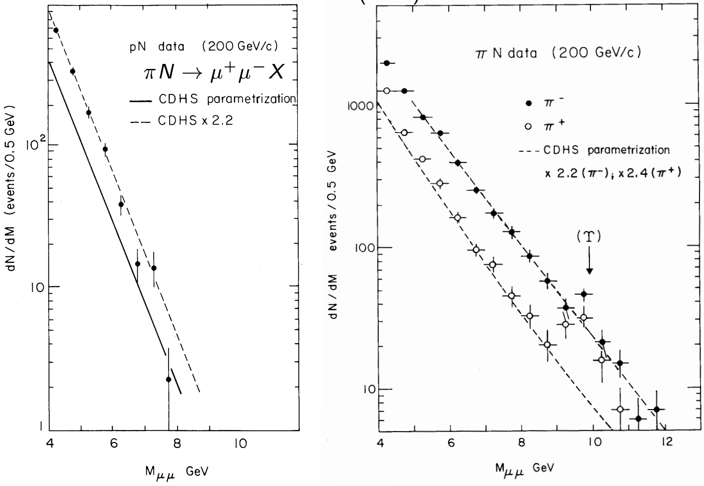




⚠ distribution not corrected for acceptance and efficiency effects !

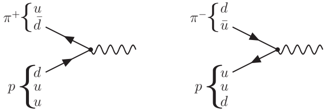
First DY Cross section measurements

NA3 - PLB (1979)



- (out of the resonance regions) scaling observed ! confirms the model i.e. also the QCD factorisation

- $\frac{\sigma(\pi^- N \to \mu^+\mu^-)}{\sigma(\pi^+ N \to \mu^+\mu^-)} \simeq 2$ as expected



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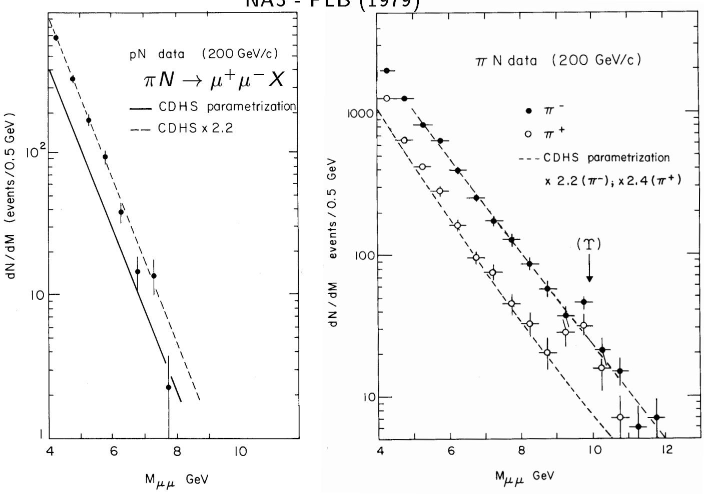

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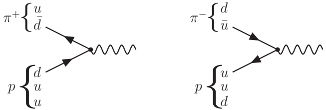


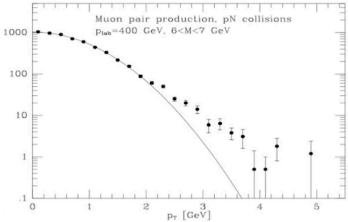
- but the normalisation is wrong by a factor 2.2 !

great success, but...

DY: transverse momentum

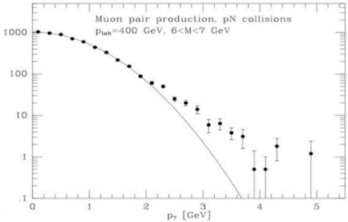
- in top of the normalisation problem, the transverse momentum of the lepton pair is not described

- at LO, everything is longitudinal (x of PDF are p longitudinal momentum fraction)



DY: transverse momentum

- in top of the normalisation problem, the transverse momentum of the lepton pair is not described



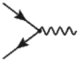
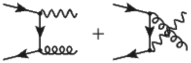
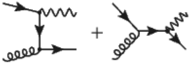
- at LO, everything is longitudinal (x of PDF are p longitudinal momentum fraction)

- low P_T part explained by the Fermi motion inside the proton:

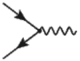
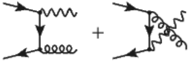
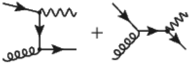
$\Delta p \geq \hbar/2\Delta x \simeq 113$ MeV for each transverse direction and each proton (of $\Delta x = 0.87$ fm)
$\Rightarrow$ typically 500 MeV (fit to date gives 760 MeV)

- missing NLO !

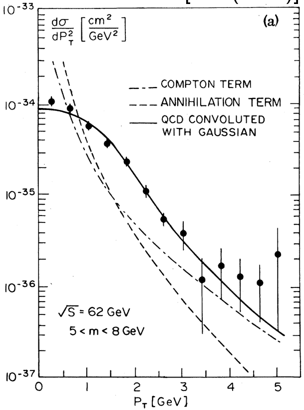
DY at NLO

LO	NLO	
	processus d'annihilation	processus QCD
$q+\bar{q}\to\gamma^*$	$q+\bar{q}\to g+\gamma^*$	$q+g\to q+\gamma^*$
		
1	$16\pi^2\alpha_S\alpha\frac{8}{9}\left[\frac{\hat{u}^2}{\hat{t}^2}+\frac{\hat{t}^2}{\hat{u}^2}+\frac{2M^2\hat{s}}{\hat{u}\hat{t}}\right]$	$16\pi^2\alpha_S\alpha\frac{1}{3}\left[-\frac{\hat{t}^2}{\hat{s}^2}-\frac{\hat{s}^2}{\hat{t}^2}-\frac{2M^2\hat{u}}{\hat{s}\hat{t}}\right]$

DY at NLO

LO	NLO	
	processus d'annihilation	processus QCD
$q+\bar{q}\rightarrow\gamma^*$	$q+\bar{q}\rightarrow g+\gamma^*$	$q+g\rightarrow q+\gamma^*$
		
1	$16\pi^2\alpha_S\alpha\frac{8}{9}\left[\frac{\hat{u}^2}{\hat{t}^2}+\frac{\hat{t}^2}{\hat{u}^2}+\frac{2M^2\hat{s}}{\hat{u}\hat{t}}\right]$	$16\pi^2\alpha_S\alpha\frac{1}{3}\left[-\frac{\hat{t}^2}{\hat{s}^2}-\frac{\hat{s}^2}{\hat{t}^2}-\frac{2M^2\hat{u}}{\hat{s}\hat{t}}\right]$

[ISR (1982)]



$\Rightarrow$ nicely describes the large P_T distribution

$\Rightarrow$ nicely describes the normalisation

- large NLO effect because a new type of diagram comes in, furthermore with (huge) gluon densities

complete success

DY constrains on PDF

- assuming hypercharge symmetry such as $u = u_p = d_n$ and et $d = d_p = u_n$,
- assuming there are only 2 flavours :

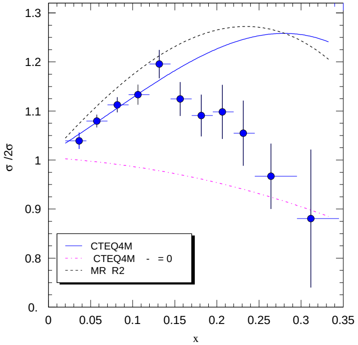
$$\sigma^{pp} \sim \frac{4}{9}u(x_1)\bar{u}(x_2) + \frac{1}{9}d(x_1)\bar{d}(x_2)$$

$$\sigma^{pn} \sim \frac{4}{9}u(x_1)\bar{d}(x_2) + \frac{1}{9}d(x_1)\bar{u}(x_2)$$

the ratio, using p and deuterium target:

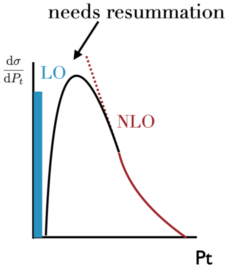
$$\frac{\sigma^{pd}}{2\sigma^{pp}} = \frac{(1+\frac{1}{4}\frac{d(x_1)}{u(x_1)})}{(1+\frac{1}{4}\frac{d(x_1)}{u(x_1)}\frac{\bar{d}(x_2)}{\bar{u}(x_2)})}(1+\frac{\bar{d}(x_2)}{\bar{u}(x_2)}) \simeq 1+\frac{\bar{d}(x_2)}{\bar{u}(x_2)}$$

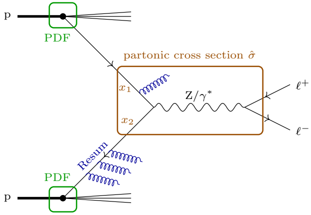
[E866 (2001)]



$\Rightarrow$ the sea distributions of $\bar{u}$ and $\bar{d}$ are different ! Surprise !

DY: state of the art

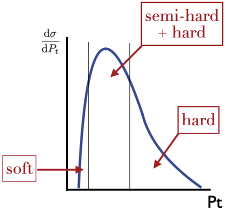




- the NLO contribution diverges at low P_t

- a realistic description of the P_t spectrum requires a resummation of many gluon radiation

- can be done in different ways:
  - analytic calculation
  - Monte Carlo Parton Showers (PS)
  - PDF $\rightarrow$ Transverse Momentum Distributions (TMDs)



Conclusion and Open questions

- The TeVatron & LHC opened a new range in energy
- allowed study processes at high scales and multijet production
- huge progresses have been achieved on all aspects
- a precise prediction (at % level) remains a challenge for many observables
- they are needed to measure the Higgs production (very close to Drell-Yan) and decay as precisely as possible
- and to put constrains on new physics