Perturbative and colorful lectures on **Strong Interactions**

lecture 1/3

Laurent Favart
IIHE - Université libre de Bruxelles

Sinop Gerze Deydaa Particle Physics Summer School

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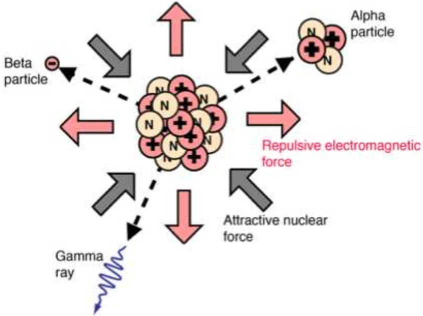
1. A few words about QCD
   1. running of α_S
   2. asymptotic freedom and confinement
2. quarks and gluons from e^+e^- annihilation
   1. quark electric charges
   2. quark and gluon spins
   3. QCD gauge parameters
3. Structure of hadrons
   1. elastic scattering
   2. deep inelastic scattering
   3. proton structure functions
   4. DGLAP evolution equation
4. hadron - hadron interactions
   1. jet production
   2. Drell-Yan process

The so rich nucleus



What makes the nucleons hold together in a nucleus ?

$$^{A}_{Z}M = Z \cdot m_p + (A - Z) \cdot m_n - E_L \quad \Rightarrow \quad E_L \simeq 8\ MeV$$

$\Rightarrow$ named **Strong interaction**

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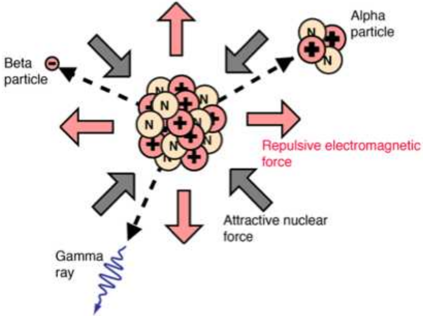

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In a nucleus there is a competition between EM, Strong and Weak interactions

- EM: repulsion between protons
- Strong: bounding between nucleons
- Weak and EM decays

A few preliminary remarks

	proton	neutron
mass	938.280 MeV	939.573 MeV
lifetime	stable	898 ± 16 sec
charge	$+1$	0
spin	1/2	1/2
magnetic moment	$2.793\mu_N$	$-1.913\mu_N$

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lifetimes and masses

- $\tau_n \simeq 15$ min - very long for wi
  $n \to p\, e^-\, \bar{\nu}_e$
- $\tau_p > 10^{33}$ y - stable (if free state)
- $m_n/m_p = 1.0014$!
  $\Rightarrow$ very similar internal bounding fields

Huge bounding energy: GeV !

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- if $m_n \searrow$: $He/H \nearrow$ - $\nexists$ stars like $\odot$

- if $m_n \nearrow$: all atoms unstable (except H)

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magnetic moment

Bohr magneton: $\mu_N = e\hbar/2m_N c$

we would naively expect $\mu_p = 1$ and $\mu_n = 0$

$\Rightarrow$ first sign of nucleon charged sub-structure !

Proton structure - and QCD

- in baryons, the 3 valence quarks are bound by gluon exhanges

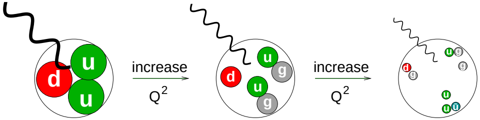
- the gluons carry a color and an anti-color (or a linear combination), e.g.

$$r\bar{g},\quad r\bar{b},\quad g\bar{r},\quad g\bar{b},\quad b\bar{r},\quad b\bar{g},\quad \sqrt{1/2}\,\{r\bar{r}-g\bar{g}\},\quad \sqrt{1/6}\,\{r\bar{r}+g\bar{g}-2b\bar{b}\}$$

but the color singlet state is excluded by hand to avoid free gluons

$$G_0=\sqrt{1/3}\,\{r\bar{r}+g\bar{g}+b\bar{b}\}$$

- the proton structure depends on the resolution scale - seeing more or less quantum fluctuations in $q\bar{q}$ (sea quarks)



- the QCD coupling constant, α_S, also depends on the resolution scale

- Part 1 -

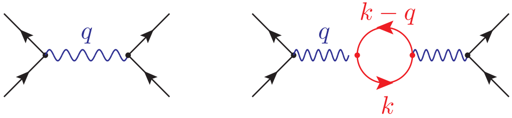
α_S running, asymptotic freedom and confinement

Screening effect in QED

- in QFT the vacuum fluctuates in virtual particle-antiparticle pairs
- in presence of an electric charge, these pairs get polarised
- this leads to an effective charge which depends on the distance to the probe

At LO :

$$(-i)e_0\gamma^\mu \cdot (-i)\frac{g_{\mu\nu}}{q^2} \cdot (-i)e_0\gamma^\nu = ie_0^2\gamma^\mu \frac{g_{\mu\nu}}{q^2}\gamma^\nu$$



with one loop :

$$(-i)e_0\gamma^\mu(-i)\frac{g_{\mu\rho}}{q^2}(-1)\int \frac{d^4k}{(2\pi)^4} Tr\left[(-i)e_0\gamma^\rho \frac{i(\not{k}+m)}{k^2-m^2}(-i)e_0\gamma^\lambda \frac{i(\not{k}-\not{q}+m)}{(k-q)^2-m^2}\right](-i)\frac{g_{\lambda\nu}}{q^2}(-i)e_0\gamma^\nu$$

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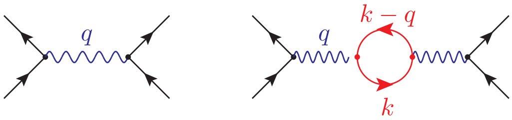

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fermion loop

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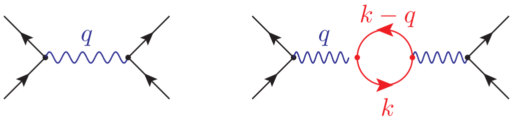

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trace because we sum over fermions spin states does the trace converge ?

$$\int \frac{d^4k}{(k^2 - m^2)^2} = \int \frac{k^3}{(k^2 - m^2)^2} \, dk \; d\Omega$$

divergent for $k \to \infty$

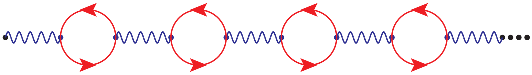
$$\int^{\infty} \frac{dk}{k} \to \int^{\mu_R} \frac{dk}{k} \qquad \to \text{we introduce a cutoff}$$

the integral give a log, the **effective** charge is given by:

$$e^2_{eff}(1 loop) = i e_0^2 \gamma^\mu \frac{g_{\mu\nu}}{q^2} \gamma^\nu \left[1 + \frac{e_0^2}{12\pi^2} \ln\left(\frac{m^2 - q^2}{\mu_R^2} \right) + \frac{e_0^2}{12\pi^2} F(q^2) \right]$$

where $F(q^2)$ is a finite function vanishing for $q^2 \to \infty$.

Adding more loops

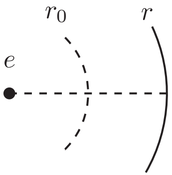


$$e^2_{eff} = e_0^2 \left[1 + \frac{e_0^2}{3\pi} \ln\left(\frac{m^2 - q^2}{\mu^2_{UV}}\right) + \left(\frac{e_0^2}{3\pi} \ln\left(\frac{m^2 - q^2}{\mu^2_{UV}}\right)\right)^2 + \cdots \right]$$

behaves arithmetic sequence, whose sum is (using $Q^2 = -q^2$):

$$e^2_{eff}(Q^2) = \frac{e_0^2}{1 - \frac{e_0^2}{3\pi} \ln \frac{Q^2}{\mu_R^2}}$$

Interpretation : using the distance $r^2 \sim 1/Q^2$ (sign in front of ln changes)



$$e^2(r) = \frac{e^2(r_0)}{1 + \frac{2e^2(r_0)}{3\pi} \ln \frac{r}{r_0}}$$

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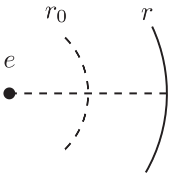

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if $r \gg r_0$ $\nearrow$ $\simeq \frac{e^2(r_0)}{e^2(r_0)\cdots} \rightarrow \alpha = 1/137$

independ. of r_0

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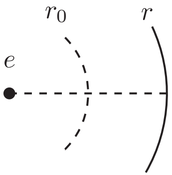

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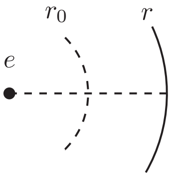

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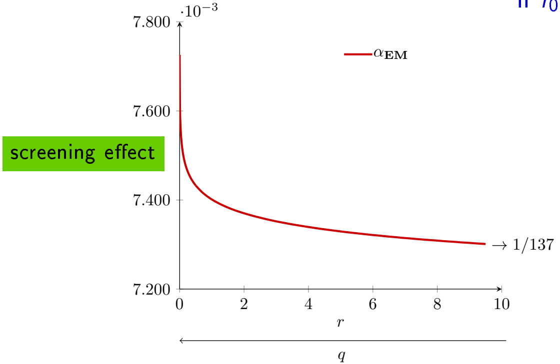
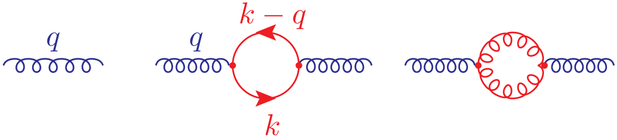


in QCD: situation partially similar, on additional gluon loop



- the fermion loop (N_f) is similar to the QED case
- the additional gluon loop comes with a $(+1)$
- sum over N_c colors

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu_R^2)}{1 - \frac{2N_f - 11N_c}{6\pi}\alpha_s(\mu_R^2)\ln\frac{Q^2}{\mu_R^2}}$$

- competition between N_f and N_c
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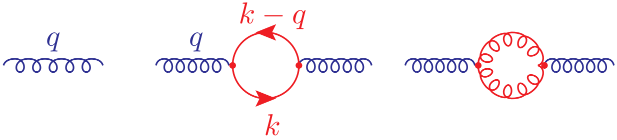

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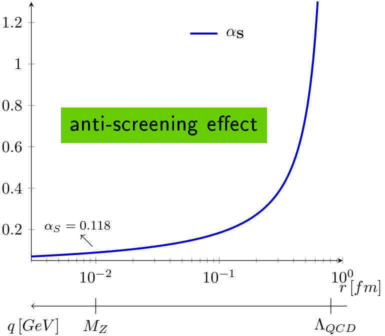

- QFT gives us the evolution but not the values
- in one direction or the other

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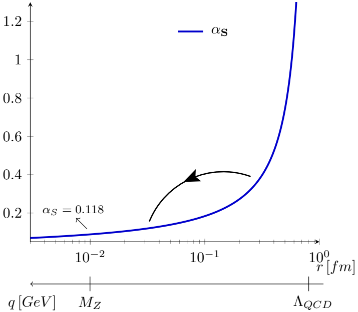
- Λ_{QCD} is defined as the divergent scale value:

$$\alpha_s(\Lambda_{QCD}^2) = \frac{\alpha_s(\mu_R^2)}{0}$$

- $\Lambda_{QCD} \simeq m_\pi \simeq 100$ MeV



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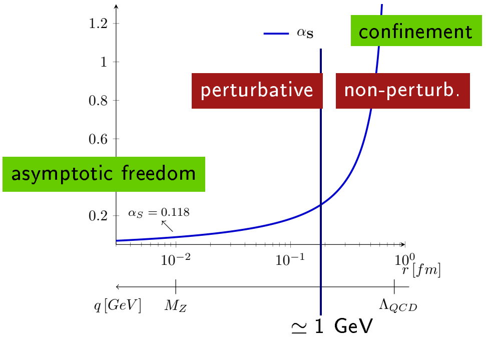
$$\alpha_s(\Lambda^2_{QCD}) = \frac{\alpha_s(\mu_R^2)}{0}$$

- $\Lambda_{QCD} \simeq m_\pi \simeq 100$ MeV
- to apply perturbative methods: $\alpha_S \ll 1$

i.e. $Q \gg \Lambda_{QCD}$

- if non-pert. (no Fey. diag, etc) but $\mathcal{L}_{QCD}$ is valid



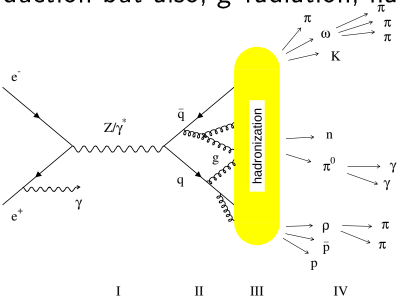
- Part 2 -

e^+e^- annihilation and QCD

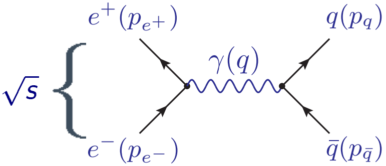
quark production from $e^- e^+$ annihilation

- the cleanest way to produce quarks is from $e^- e^+$ annihilations:
- no color charge in the initial state
- pure electroweak process (at LO)
- inital state momentum fully transfered to the $q\bar{q}$

$\to$ allows us to study q production but also, g radiation, hadronisation,...



- at LO, for γ exchange only:

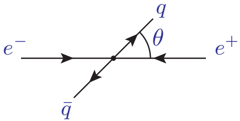


- negl. mass terms :

$$\overline{|\mathcal{M}|^2} = 8(4\pi)^2 \frac{\alpha^2}{q^4} N_c \, Q_i^2 \, \{(p_{e^+} \cdot p_q)(p_{e^-} \cdot p_{\bar{q}}) + (p_{e^+} \cdot p_{\bar{q}})(p_{e^-} \cdot p_q)\}$$

- only 1 degree of freedom
- in the c.m.s.:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}cos\theta} = \frac{\pi\alpha^2}{2s} N_c \, Q_i^2 \, (1 + \cos^2\theta)$$



acces to quark electric charges

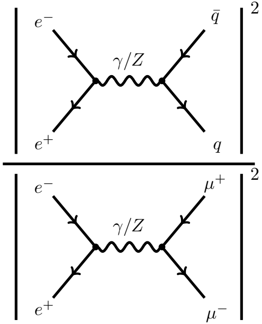
- as all hadrons come from quarks and that quarks always give hadrons :

$$\sigma(e^+e^- \to X(\text{any hadron})) = \sum_{i=1}^{N_f} \sigma(e^+e^- \to q\bar{q}) = \frac{4\pi\alpha^2}{3s} N_c \sum_{i=1}^{N_f} Q_i^2$$

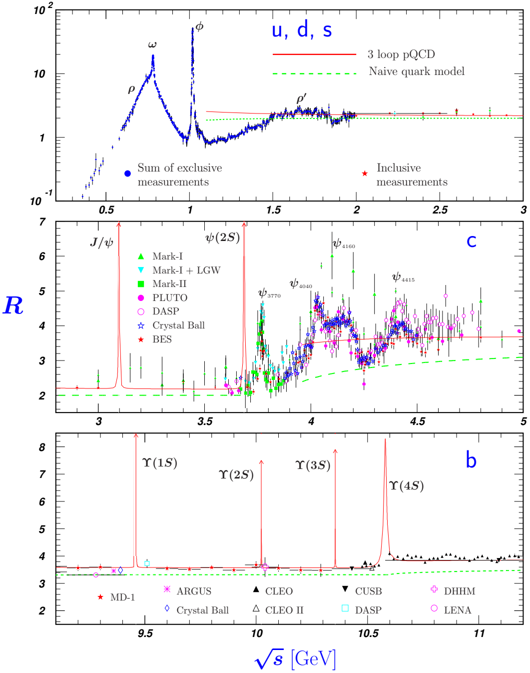
- we can study the observable:

$$R = \frac{\sigma(e^+e^- \to X)}{\sigma(e^+e^- \to \mu^+\mu^-)} = \frac{\frac{4\pi\alpha^2}{3s} N_c \sum_{i=1}^{N_f} Q_i^2}{\frac{4\pi\alpha^2}{3s}} = N_c \sum_{i=1}^{N_f} Q_i^2$$

$R \sim$



- is this very simple model realistic ?



$$R = \frac{\sigma(e^+e^- \to X)}{\sigma(e^+e^- \to \mu^+\mu^-)} = N_c \sum_{i=1}^{N_f} Q_i^2$$

$$= 3\left[\left(\frac{2}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 + \ldots\right]$$

$$= 2 \quad \text{for } 2m_s \ll \sqrt{s} \ll 2m_c$$

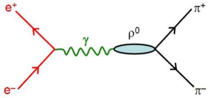
$$= 10/3 \quad \text{for } 2m_c \ll \sqrt{s} \ll 2m_b$$

$$= 11/3 \quad \text{for } 2m_b \ll \sqrt{s} \ll m_Z$$

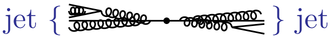
- amazing success if away from resonnances

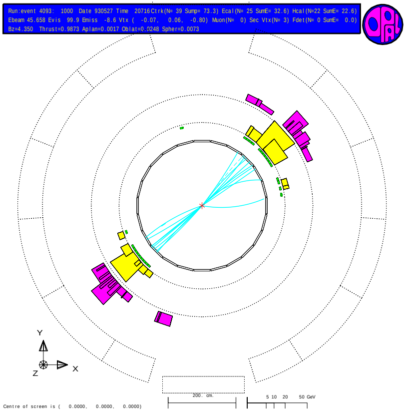
quark charges are as expected

- if close to a resonnance, the important npQCD corrections only in the numerator

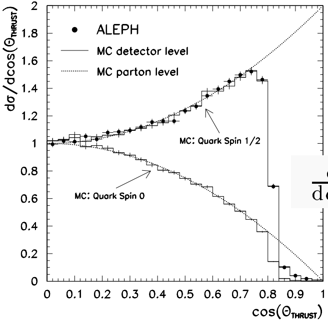


jets - during the had. process, hadrons are forming around the initial quark direction





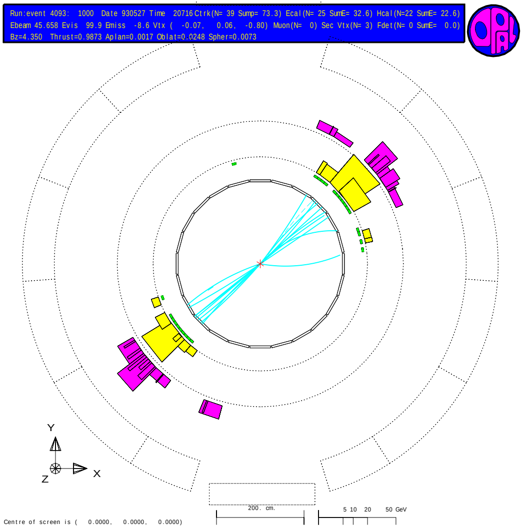
- two particles below to different jets only if $p_i \cdot p_j \gg \Lambda^2_{QCD}$
- angular measurement:

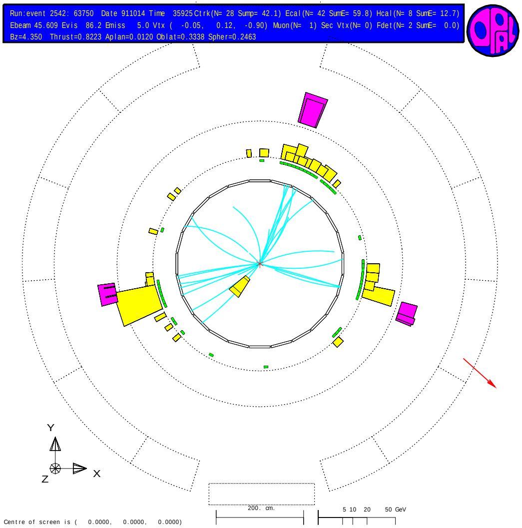


$$\frac{d\sigma}{d\cos\theta} \sim (1 + \cos^2\theta)$$

the quark are spin 1/2 !

- excellent agreement between jet and quark directions


Run:event 4093: 1000 Date 930527 Time 20716 Ctrk(N= 39 Sump= 73.3) Ecal(N= 25 SumE= 32.6) Hcal(N=22 SumE= 22.6)
Ebeam 45.658 Evis 99.9 Emiss -8.6 Vtx (-0.07, 0.06, -0.80) Muon(N= 0) Sec Vtx(N= 3) Fdet(N= 0 SumE= 0.0)
Bz=4.350 Thrust=0.9873 Aplan=0.0017 Oblat=0.0248 Spher=0.0073
Y
X
Z
200. cm.
5 10 20 50 GeV
Centre of screen is (0.0000, 0.0000, 0.0000)


Run:event 2542: 63750 Date 911014 Time 35925 Ctrk(N= 28 Sump= 42.1) Ecal(N= 42 SumE= 59.8) Hcal(N= 8 SumE= 12.7)
Ebeam 45.609 Evis 86.2 Emiss 5.0 Vtx (-0.05, 0.12, -0.90) Muon(N= 1) Sec Vtx(N= 0) Fdet(N= 2 SumE= 0.0)
Bz=4.350 Thrust=0.8223 Aplan=0.0120 Oblat=0.3338 Spher=0.2463
Y
X
Z
200. cm.
5 10 20 50 GeV
Centre of screen is (0.0000, 0.0000, 0.0000)

That's all for today