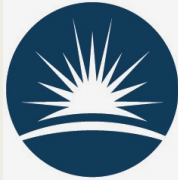


Categorical Symmetries,

Anomalies &

RG flows



Science & Technology
Facilities Council

Maths / Physics retreat

12/03/2024

All Souls College

Christian

Copetti

The big Problem: Strongly coupled QFTs

UV

CFT

$$+\mu \int \mathcal{O}$$

IR

???

RG flow

???

Can be



Gapless



Gapped

- CFT
- Goldstone modes
- Top. order (SSB)
- SPT

More can be said if \mathcal{C} preserves a **SYMMETRY**

\mathcal{C} along the flow

SYMMETRY := set of top. defects in the QFT

$| = \curvearrowright$

Gen. of "usual" notion

□ Def. can live in $\text{codim} > 1$

$$U_\alpha = \exp(i\alpha \int_\Sigma d\Sigma_\mu J^\mu)$$

$$\partial_\mu J^\mu = 0$$

□ Can be "non-invertible" i.e. not implemented

by unitary map: $\mathcal{H} \rightarrow \mathcal{H}$

In the IR \mathcal{C} acts on  $\mathcal{D}_{\mathcal{C}}$

Thus understanding the possible actions of \mathcal{C}
gives constraints on IR phases.

Some research avenues

○ "Holographic" descr.
of symmetry (SYMFT)

○ Modif. of basic concepts
(e.g. crossing symmetry)

○ Constr. on extended
defects (DCFT)

Physics

○ BSM physics
(Naturalness)

○ Anomalies for cat-symm.
(Fiber Functors)

○ Class. of TQFTs

Maths

○ Class. of symm.
structures (Higher Cat.)

Example $\mathcal{C} = \text{Ising}$ ($d=2$)

1 ; \mathcal{Z} ; \mathcal{N}

$\mathcal{Z} = \text{spin flip } \mathbb{Z}_2 \text{ symmetry}$

$\mathcal{N} = \text{KW self-duality symmetry}$

$$\mathcal{Z} \times \mathcal{Z} = 1 \quad ; \quad \mathcal{Z} \times \mathcal{N} = \mathcal{N} = \mathcal{N} \times \mathcal{Z}$$

$$\mathcal{N}^2 = 1 + \mathcal{Z}$$

Anomalous :

Admits no
gapped action
without SSB

Symm of unitary minimal models

M_3

M_4

|||

|||

Ising

3 Ising

"gapped"

$c = \frac{1}{2}$

$c = \frac{7}{10}$

✓

Anomaly : minimal rep of \mathcal{E} on \mathcal{H}_{IR}

is 3-dimensional

$|+\rangle$ $|-\rangle$ $|0\rangle$

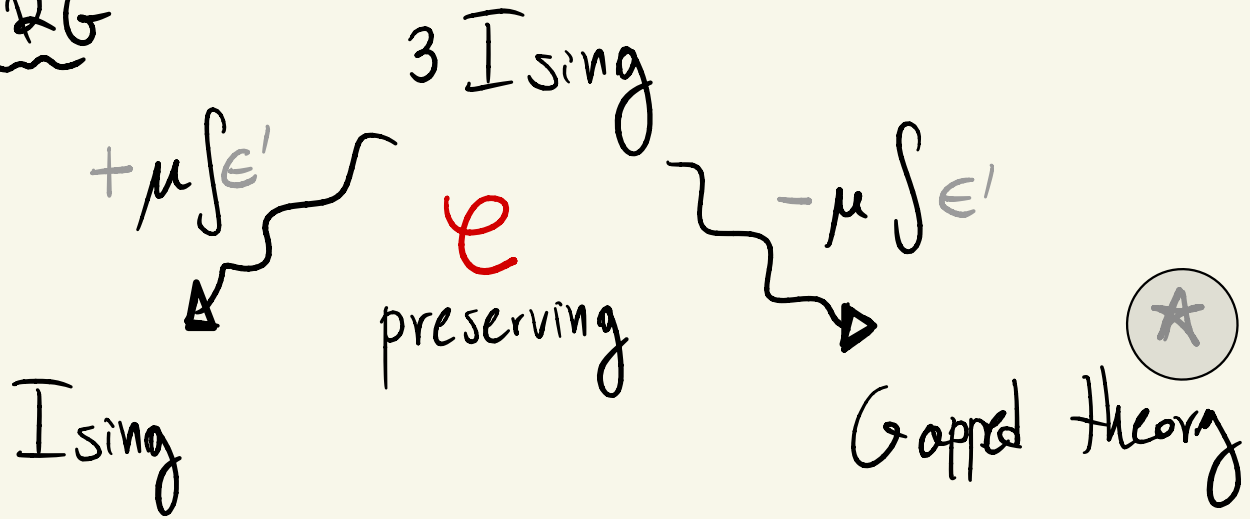
$$\mathcal{N}|+\rangle = |-\rangle$$

$$\mathcal{N}|+\rangle = \mathcal{N}|-\rangle = |0\rangle$$

$$\mathcal{N}|0\rangle = |0\rangle$$

$$\mathcal{N}|0\rangle = |+\rangle + |-\rangle$$

Int. RG



\star Must have at least 3 vacua

(Confirmed by lattice simulations)

\oplus

\ominus

0

$d=4$ generalizations

- Duality symm. $\left\{ \begin{array}{l} \mathcal{N}=4 \text{ SYM } \tau=i \quad (\& \text{ low SUSY}) \\ \text{QED-like theories (ABJ anom.)} \end{array} \right.$

$$\mathcal{N} \times \bar{\mathcal{N}} = \text{Cond}(\mathbb{Z}_N)$$

○ RG flows

- SUSY breaking of $\mathcal{N}=4$

$\mathcal{N}=1^*$
(gapped)

\rightsquigarrow
Konishi
(gapped) (FT?)

$20'$ - Modif. crossing in
4d?? (WIP)

○ Pheno impl.

- small v moves due
to monopoles

Thank
you!

