Backreaction from gauge fields produced during inflation

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Overview

Magnetic fields in the Universe

Magnetic fields exist in all astrophysical objects on all observable scales of the visible Universe:

- **Neutron stars**: $\sim 10^{12} - 10^{15}$ G
- **Stars**: $\sim 1 - 10^{3}$ G
- **Planets**: $\sim 10^{-3} - 10$ G
- **Galaxies**: $\sim 10^{-5} - 10^{-6}$ G
- **Galaxy clusters**: $\sim 10^{-6} - 10^{-7}$ G

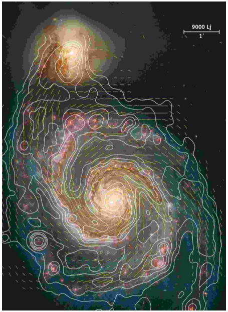


Figure: Optical and radio image of the Whirlpool galaxy M51 with MF configuration. Credit MPIfR Bonn.

Since 2010, there is an evidence of MF detection also on a cosmological scale — **in the cosmic voids**: $10^{-16}\,\text{G} \lesssim B_0 \lesssim 10^{-10}\,\text{G}$

Summary of constrains on MF

- Constraint from **below** is from the analysis of γ-radiation of blazars: $B \geq 10^{-16}$ G.

  [Tavecchio *et al.*, MNRAS **406**; Ando & Kusenko, ApJL **722**; Neronov & Vovk, Science **328** (2010)]

- Constrains from **above** follow from the analysis of the anisotropy spectrum of CMB and UHECR deviation: $B \leq 10^{-10}$ G.

  [Neronov et al., arXiv:2112.08202]
  [Jedamzik & Saveliev, PRL **123** (2019)]

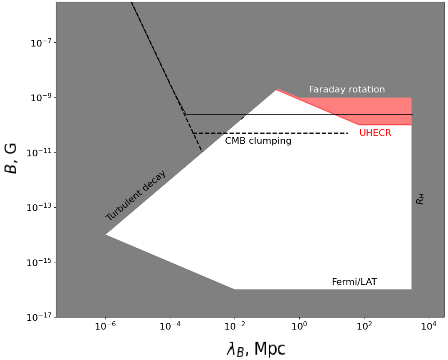


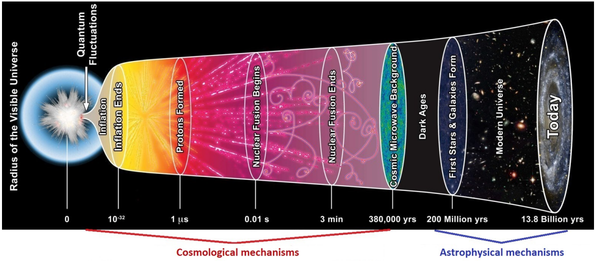
Figure: Summary of constrains on B and λ_B [Neronov et al., arXiv:2112.08202]

For the MF correlation length: $30\,\text{kpc} < \lambda_B < 300\,\text{Mpc}$ (90% C.L.)
[Alves Batista, Saveliev, ApJL **902**, L11 (2020)].

When and how could MFs arise in the Universe?

There are two different hypotheses for the generation of seed MF:

- **Astrophysical** — generation during structure formation: Biermann battery, adiabatic contraction, dynamos, ...
- **Cosmological** — generation in the early Universe: phase transitions, reheating, inflation, ...



The presence of magnetic fields in voids has already been confirmed!
This is very strong argument in favor of **cosmological origin** of these MF!

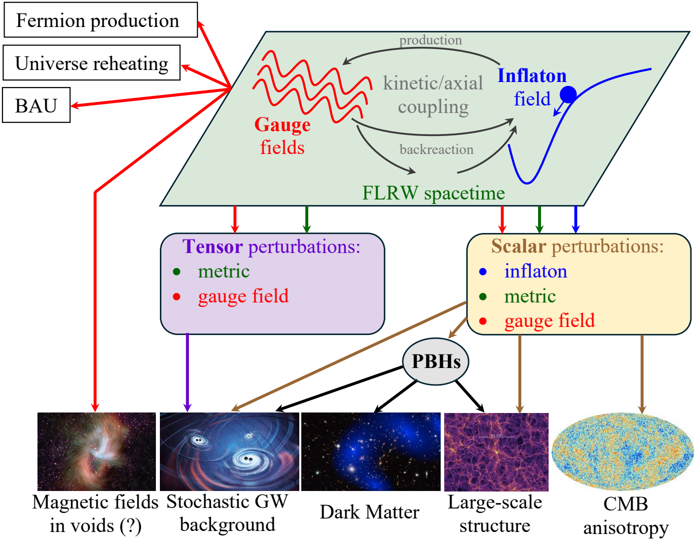
Cosmological mechanisms of MF generation

- Generation during phase transitions (electroweak, QCD). But the correlation length of such MF is of the order of the Hubble horizon at the time of phase transition, which is much smaller than Mpc today.
- Generation due to chiral anomaly. Requires initial chiral asymmetry in the fermion sector of the Standard Model.
- Generation during inflation and/or reheating. Requires conformal invariance violation. Probably the only way to generate MF with a correlation length of the order of kiloparsecs and more.

If MF were generated during or after inflation before the BBN, we get a new source of information about a very early Universe:

- Distribution of the light elements in the Universe.
- CMB spectrum
- Large-scale structure of the Universe.
- Primordial gravitational waves.
- **Magnetic fields in voids are a new source of information !!!**

Inflationary gauge-field production



Credit: O. Shmahalo, D. Harvey, R. Massey, H. Ebeling,J.-P. Kneib, Millenium Simulation Project, NASA, ESA, Planck Collaboration

[Turner'88]
[Ratra'91]
[Garretson'92]
[Anber'06]
[Martin'08]
[Durrer'13]

[Seery'08]
[Barnaby'11]
[Domcke'20]
[Durrer'24]

[Kobayashi'14]
[Hayashinaka'16]
[Domcke'18]
[Sobol'20]

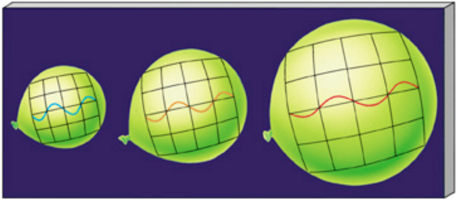
How can the EM field arise at the inflationary stage of the evolution of the Universe?

- In the conformal time $\eta(t) = \int^t \frac{dt}{a(t)}$ FLRW metric is **conformally flat**: $ds^2 = dt^2 - a^2(t)d\mathbf{x}^2 = a^2(\eta)\left(d\eta^2 - d\mathbf{x}^2\right) \Rightarrow g_{\mu\nu} = a^2(\eta)\eta_{\mu\nu}$
- The action for EM is **conformally invariant**:

$$S_{EM} = -\frac{1}{4}\int d^4x\,\sqrt{-g}\,g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta} = \left| \begin{array}{c} \sqrt{-g} = a^4, \\ g^{\mu\nu} = a^{-2}\eta^{\mu\nu} \end{array} \right| =$$

$$= -\frac{1}{4}\int d^4x\,\eta^{\mu\alpha}\eta^{\nu\beta}F_{\mu\nu}F_{\alpha\beta} \quad - \quad \text{the same as in Minkowski space.}$$

- Solutions of EOMs are EM waves, the frequency of which undergoes a redshift.
- **Generation of the EM field in the early Universe requires the breaking of conformal invariance of the action!!!**



Magnetogenesis from general kinetic and axial couplings

Consider an Abelian gauge field A_μ interacting with the inflaton field ϕ via kinetic and axial couplings

$$S_{\rm KA}[g_{\mu\nu},\phi,A_\mu]=\int d^4x\sqrt{-g}\Big[\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-V(\phi)-\frac{1}{4}I_1(\phi)F_{\mu\nu}F^{\mu\nu}-\frac{1}{4}I_2(\phi)F_{\mu\nu}\tilde{F}^{\mu\nu}\Big],$$

$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu,\quad \tilde{F}^{\mu\nu}=\frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$ – dual EMF tensor

- **The function $I_1(\phi)$ [$I_2(\phi)$] describes the kinetic [axial] coupling of the GF to the inflaton.**
- **The kinetic coupling function $I_1(\phi)$ must be (i) positive to ensure positive-definiteness of the energy density; (ii) $I_1(\phi)\geq 1$ to avoid the strong coupling problem.**
- **The axial coupling function $I_2(\phi)$ may be completely arbitrary since it does not enter the energy-momentum tensor.**

Equations of motion

- **Friedmann eq.:** $H^2 = \frac{1}{3M_{\rm P}^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi) + \frac{I_1(\phi)}{2}\langle \boldsymbol{E}^2 + \boldsymbol{B}^2\rangle\right]$
- **Klein–Gordon eq.:** $\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = \frac{1}{2}\frac{dI_1}{d\phi}\langle \boldsymbol{E}^2 - \boldsymbol{B}^2\rangle + \frac{dI_2}{d\phi}\langle \boldsymbol{E}\cdot\boldsymbol{B}\rangle$
- **Maxwell equations:** $\dot{\boldsymbol{E}} + 2H\boldsymbol{E} - \frac{1}{a}\mathrm{rot}\,\boldsymbol{B} = -\frac{\dot{I}_1}{I_1}\boldsymbol{E} - \frac{\dot{I}_2}{I_1}\boldsymbol{B},$

$$\dot{\boldsymbol{B}} + 2H\boldsymbol{B} + \frac{1}{a}\mathrm{rot}\,\boldsymbol{E} = 0, \quad \mathrm{div}\,\boldsymbol{E} = 0, \quad \mathrm{div}\,\boldsymbol{B} = 0.$$

Friedmann and Klein–Gordon eqs. are classical and the gauge field is quantum. ⇒ One need to take the VEV of gauge-field quantities.
In Coulomb gauge, $A_\mu = (0, \boldsymbol{A})$ and $\mathrm{div}\,\boldsymbol{A} = 0$, the gauge field operators have the form

$$\hat{\boldsymbol{A}}(t,\boldsymbol{x}) = \int \frac{d^3\boldsymbol{k}}{(2\pi)^{3/2}\sqrt{I_1}} \sum_{\lambda=\pm}\left[\boldsymbol{\epsilon}^\lambda(\boldsymbol{k})\hat{a}_{\boldsymbol{k},\lambda}A_\lambda(t,k)e^{i\boldsymbol{k}\cdot\boldsymbol{x}} + \mathrm{h.c.}\right]$$

$$\hat{\boldsymbol{E}} = -\frac{1}{a}\dot{\hat{\boldsymbol{A}}}, \qquad \hat{\boldsymbol{B}} = \frac{1}{a^2}\mathrm{rot}\,\hat{\boldsymbol{A}}.$$

Time evolution of mode functions $A_\lambda(t,k)$

$$\frac{d^2A_\lambda(z,k)}{dz^2}+\left[1-2\lambda\frac{aH}{k}\xi(z)-\left(\frac{aH}{k}\right)^2 s(z)\right]A_\lambda(z,k)=0.$$

where $z=k\eta$, $\eta=\int^t dt'/a(t')$ **is conformal time and**

$$\xi(t)=\frac{\dot{I}_2}{2H\,I_1},\qquad s(t)=\frac{\dot{I}_1}{2HI_1}+\frac{\ddot{I}_1}{2H^2I_1}-\frac{\dot{I}_1^2}{4H^2I_1^2},$$

Approximation: de Sitter space $\eta\simeq-1/(aH)$. **Then,**

$$\boxed{\frac{d^2A_\lambda(z,k)}{dz^2}+\left[1+\lambda\frac{2\xi(z)}{z}-\frac{s(z)}{z^2}\right]A_\lambda(z,k)=0}$$

At early times, i.e., for $|z|\gg|2\xi|$ and $|z|\gg\sqrt{|s|}$ $\rightarrow$ oscillator-like equation $\rightarrow$ the mode function oscillates in conformal time and corresponds to vacuum fluctuations of GF $\rightarrow$ impose the boundary condition for the mode function in the form of Bunch-Davies vacuum solution: $A_\lambda(z,k)=\frac{1}{\sqrt{2k}}e^{-iz},\quad -z\gg1.$

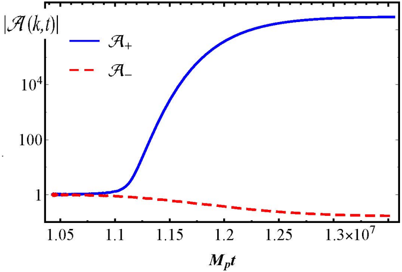
In "subhorizon" regime the effect of kinetic/axial couplings is negligible. *Subhorizon modes are* **not relevant** *for description of magnetogenesis.*

Standard mode-by-mode approach

Eq. for the mode function admits a solution in terms of Whittaker functions $W_{\varkappa,\mu}(2iz)$ Taking into account the Bunch-Davies boundary condition, we obtain the positive-frequency solution

$$\boxed{A_\lambda(z,k) = \frac{1}{\sqrt{2k}} e^{\frac{\pi\lambda\xi(t_\mathrm{h})}{2}} W_{\varkappa,\mu}(2iz),}, \quad \varkappa = -i\lambda\xi(t_\mathrm{h}), \quad \mu = \sqrt{\frac{1}{4} + s(t_\mathrm{h})}$$

If there is **no backreaction**,
Fourier modes of GF evolve independently



Only one of the two polarizations is amplified therefore, the generated MF could be **helical**!
$\mathcal{H} \sim \int k^3(|\mathcal{A}_+|^2 - |\mathcal{A}_-|^2)dk \neq 0.$

$$\rho_B = \sum_{\lambda=\pm1} \int_0^{k_\mathrm{h}} \frac{dk}{k} \frac{k^5}{4\pi^2 a^4} |\mathcal{A}_\lambda(t,k)|^2, \quad \rho_E = \sum_{\lambda=\pm1} \int_0^{k_\mathrm{h}} \frac{dk}{k} \frac{k^3}{4\pi^2 a^2} |\dot{\mathcal{A}}_\lambda(t,k)|^2.$$

Track of backreaction of the generated fields

- **The Friedmann and Klein–Gordon equations describe inflationary background.**

$$H^2 = \frac{1}{3M_{\mathrm{P}}^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi) + \frac{I_1(\phi)}{2}\langle \boldsymbol{E}^2 + \boldsymbol{B}^2\rangle\right]$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = \frac{1}{2}\frac{dI_1}{d\phi}\langle \boldsymbol{E}^2 - \boldsymbol{B}^2\rangle + \frac{dI_2}{d\phi}\langle \boldsymbol{E}\cdot\boldsymbol{B}\rangle$$

- **The gauge field enters these equation.**
- **When it becomes strong enough the slow-roll dynamics is modified: the system enters the regime of strong backreaction.**

1. **All Fourier modes of the GF become coupled ⇒ One has to evolve all of them simultaneously** [Domcke'2020; Cheng'2016; Notari'2016]

   OR

2. **One can work in position space and describe the GF in terms of quantities which include all relevant Fourier modes ⇒ Gradient-expansion formalism**

G-E formalism [Gorbar, Schmitz, Sobol, SV, PRD **104** (2021); PRD **105** (2022)]

Introduce the bilinear scalar quantities which are the vacuum expectation values of different scalar products of $\boldsymbol{E}$, $\boldsymbol{B}$ and their spatial derivatives

$$\mathcal{E}^{(n)} = \frac{I_1(\phi)}{a^n} \langle \boldsymbol{E} \cdot \mathrm{rot}^n \boldsymbol{E} \rangle , \qquad \mathcal{B}^{(n)} = \frac{I_1(\phi)}{a^n} \langle \boldsymbol{B} \cdot \mathrm{rot}^n \boldsymbol{B} \rangle$$

$$\mathcal{G}^{(n)} = -\frac{I_1(\phi)}{2a^n} \langle \boldsymbol{E} \cdot \mathrm{rot}^n \boldsymbol{B} + \boldsymbol{B} \cdot \mathrm{rot}^n \boldsymbol{E} \rangle .$$

Direct application of Maxwell's equations to these GF correlators gives a system of coupled equations with vanishing right-hand sides

$$\dot{\mathcal{E}}^{(n)} + (n+4)H\mathcal{E}^{(n)} + \frac{\dot{I}_1}{I_1}\mathcal{E}^{(n)} - 2\frac{\dot{I}_2}{I_1}\mathcal{G}^{(n)} + 2\mathcal{G}^{(n+1)} = 0$$

$$\dot{\mathcal{G}}^{(n)} + (n+4)H\mathcal{G}^{(n)} - \frac{\dot{I}_2}{I_1}\mathcal{B}^{(n)} + \mathcal{B}^{(n+1)} - \mathcal{E}^{(n+1)} = 0$$

$$\dot{\mathcal{B}}^{(n)} + (n+4)H\,\mathcal{B}^{(n)} - \frac{\dot{I}_1}{I_1}\mathcal{B}^{(n)} - 2\mathcal{G}^{(n+1)} = 0$$

This would indeed be correct if the functions $\mathcal{E}^{(n)}$, $\mathcal{G}^{(n)}$, $\mathcal{B}^{(n)}$ would include the contributions of all Fourier modes of the gauge field.

Boundary terms

- However, we have to separate the physically relevant contribution of modes amplified due to kinetic/axial coupling from vacuum-like modes giving the UV divergent contribution.
- **We introduce the cutoff momentum $k_{\rm h}$ which separates these two types of modes.**

Any bilinear function $X^{(n)}$ has the following spectral representation

$$X = \int_0^{k_{\rm h}(t)} \frac{dk}{k}\frac{dX}{d\ln k} \quad \rightarrow \quad (\dot{X})_b = \left.\frac{dX}{d\ln\, k}\right|_{k=k_{\rm h}} \cdot \frac{d\ln k_{\rm h}}{dt}$$

There are two sources of time dependence:

1. **The spectral density depends of $\mathcal{A}_\lambda(k,t)$ and its derivatives**
2. **Upper limit $k_{\rm h} = aH\left(|\xi| + \sqrt{\xi^2 + |s|}\right)$ is also time dependent**

Number of physically relevant GF modes grows during inflation which is the reason for appearance of boundary terms on r.h.s. of GEF equations.

Exact expressions for the boundary terms

$$[\dot{\mathcal{E}}^{(n)}]_{\rm b} = \frac{d\ln k_{\rm h}(t)}{dt}\frac{1}{4\pi^2}\left(\frac{k_{\rm h}(t)}{a(t)}\right)^{n+4} \times \sum_{\lambda=\pm 1}\lambda^n E_\lambda(\xi(t),\, s(t),\, \sigma(t)), \quad (1)$$

$$[\dot{\mathcal{G}}^{(n)}]_{\rm b} = \frac{d\ln k_{\rm h}(t)}{dt}\frac{1}{4\pi^2}\left(\frac{k_{\rm h}(t)}{a(t)}\right)^{n+4} \sum_{\lambda=\pm 1}\lambda^n G_\lambda(\xi(t),\, s(t),\, \sigma(t)), \quad (2)$$

$$[\dot{\mathcal{B}}^{(n)}]_{\rm b} = \frac{d\ln k_{\rm h}(t)}{dt}\frac{1}{4\pi^2}\left(\frac{k_{\rm h}(t)}{a(t)}\right)^{n+4} \sum_{\lambda=\pm 1}\lambda^n B_\lambda(\xi(t),\, s(t)), \quad (3)$$

where $k_{\rm h} = aH\left(|\xi| + \sqrt{\xi^2 + |s|}\right)$

$$E_\lambda(\xi,\, s,\, \sigma) = \frac{e^{\pi\lambda\xi}}{r^2}\Big|(ir - i\lambda\xi - \sigma)\, W_{-i\lambda\xi,\sqrt{s+\frac{1}{4}}}(-2ir) + W_{1-i\lambda\xi,\sqrt{s+\frac{1}{4}}}(-2ir$$

$$G_\lambda(\xi,\, s,\, \sigma) = \frac{e^{\pi\lambda\xi}}{r}\Big\{-\sigma\Big|W_{-i\lambda\xi,\sqrt{s+\frac{1}{4}}}(-2ir)\Big|^2 + \Re e\Big[W_{i\lambda\xi,\sqrt{s+\frac{1}{4}}}(2ir)\, W_{1-i\lambda\xi,}$$

$$B_\lambda(\xi,\, s) = e^{\pi\lambda\xi}\left|W_{-i\lambda\xi,\sqrt{s+\frac{1}{4}}}(-2ir)\right|^2, \quad (6)$$

here $r = r(\xi,\, s) = |\xi| + \sqrt{\xi^2 + |s|}$ and $\sigma(t) = \frac{i_1}{2HI_1}$

Full system of equations

$$H^2 = \frac{1}{3M_{\rm P}^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi) + \frac{1}{2}\left(\mathcal{E}^{(0)} + \mathcal{B}^{(0)}\right)\right],$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = \frac{1}{2I_1}\frac{dI_1}{d\phi}\left(\mathcal{E}^{(0)} - \mathcal{B}^{(0)}\right) - \frac{1}{I_1}\frac{dI_2}{d\phi}\mathcal{G}^{(0)}.$$

$$\dot{\mathcal{E}}^{(n)} + (n+4)H\mathcal{E}^{(n)} + \frac{\dot{I}_1}{I_1}\mathcal{E}^{(n)} - 2\frac{\dot{I}_2}{I_1}\mathcal{G}^{(n)} + 2\mathcal{G}^{(n+1)} = [\dot{\mathcal{E}}^{(n)}]_{\rm b},$$

$$\dot{\mathcal{G}}^{(n)} + (n+4)H\mathcal{G}^{(n)} - \frac{\dot{I}_2}{I_1}\mathcal{B}^{(n)} + \mathcal{B}^{(n+1)} - \mathcal{E}^{(n+1)} = [\dot{\mathcal{G}}^{(n)}]_{\rm b}.$$

$$\dot{\mathcal{B}}^{(n)} + (n+4)H\,\mathcal{B}^{(n)} - \frac{\dot{I}_1}{I_1}\mathcal{B}^{(n)} - 2\mathcal{G}^{(n+1)} = [\dot{\mathcal{B}}^{(n)}]_{\rm b}.$$

We trade an **infinite number of Fourier-modes** for an **infinite set** of scalar **functions** in the coordinate space – **what's the gain?**

The chain can be truncated!

Energy transfer

- **The time evolution of energy densities of inflaton $\rho_{\rm inf} = \dot{\phi}^2/2 + V(\phi)$ and the GF $\rho_{\rm GF} = 1/2(\mathcal{E}^{(0)} + \mathcal{B}^{(0)})$:**

$$\dot{\rho}_{\rm inf} + 3H(\rho_{\rm inf} + p_{\rm inf}) = -\frac{\dot{I}_1}{2I_1}(\mathcal{B}^{(0)} - \mathcal{E}^{(0)}) - \frac{\dot{I}_2}{I_1}\mathcal{G}^{(0)}, \tag{7}$$

$$\dot{\rho}_{\rm GF} + 3H(\rho_{\rm GF} + p_{\rm GF}) = \frac{\dot{I}_1}{2I_1}(\mathcal{B}^{(0)} - \mathcal{E}^{(0)}) + \frac{\dot{I}_2}{I_1}\mathcal{G}^{(0)} + [\dot{\rho}_{\rm GF}]_{\rm b}. \tag{8}$$

- **The sourse terms describe the energy transfer between the inflaton and gauge fields due to the kinetic and axial couplings**
- **There is an additional term on the right-hand side of Eq. (8) - the boundary term $[\dot{\rho}_{\rm GF}]_{\rm b} = \frac{1}{2}\Big([\dot{\mathcal{E}}^{(0)}]_{\rm b} + [\dot{\mathcal{B}}^{(0)}]_{\rm b}\Big)$. It acts as a source in the energy-conservation equation and describes the energy increase due to contributions of new modes crossing the horizon during inflation – "vacuum source term"**

Model with non-minimal coupling

[R.Durrer, O.Sobol, SV, PRD **106** (2022); PRD **108** (2023)]

- Action of the extended Starobinsky model

$$S=\int d^4x\sqrt{-g}\Big[-\frac{M_{\rm P}^2}{2}R+\frac{\xi_s}{1+\frac{\kappa_1}{M_{\rm P}^4}F_{\mu\nu}F^{\mu\nu}+\frac{\kappa_2}{M_{\rm P}^4}F_{\mu\nu}\tilde{F}^{\mu\nu}}R^2-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\Big]$$

- Switching to Einstein frame

$$\to\int d^4x\sqrt{-g}\Big[-\frac{M_{\rm P}^2}{2}R+\underbrace{\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi-V(\phi)}_{\text{inflaton}}\underbrace{-\frac{1}{4}I_1(\phi)F_{\mu\nu}F^{\mu\nu}}_{\substack{\text{kinetic coupling}\\ \text{GF to inflaton}}}\underbrace{-\frac{1}{4}I_2(\phi)F_{\mu\nu}\tilde{F}^{\mu\nu}}_{\substack{\text{axial coupling}\\ \text{of GF to inflaton}}}\Big]$$

with the Starobinsky potential $V(\phi)=\frac{M_{\rm P}^4}{4\xi_s}\left(1-e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm P}}}\right)^2$

and the kinetic and axial coupling functions

$$I_j=\delta_{j1}+\frac{\kappa_j}{\xi_s}\left[\exp\left(\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm P}}\right)-1\right]^2$$

Results: axial-dominated coupling

- **Significant gauge-field production occurs only if the axial coupling is much stronger than the kinetic one; i.e.,** $\kappa_2 \gg \kappa_1$
- **Since the coupling functions** $I_{1,2}$ **are decreasing in time, the generated field is also decreasing towards the end of inflation. Thus, the back- reaction is typically occurring some time before the end of inflation.**
- **Backreaction always occurs due to terms on the right-hand side of the equation** $\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = \frac{1}{2}\frac{dI_1}{d\phi}\langle \boldsymbol{E}^2 - \boldsymbol{B}^2\rangle + \frac{dI_2}{d\phi}\langle \boldsymbol{E}\cdot\boldsymbol{B}\rangle$

  Backreaction is getting relevant even in case when the GF energy density is much smaller than the one of the inflaton!
- **Qualitative features of BR regime:**
  1. **Inflation is extended**
  2. **Rolling of the inflaton is slowed down**
  3. **Oscillatory behavior of the generated fields**
- **Oscillations come from retardation between the changes in the inflaton field and the corresponding response in the GF**

Results: mixed axial and kinetic coupling

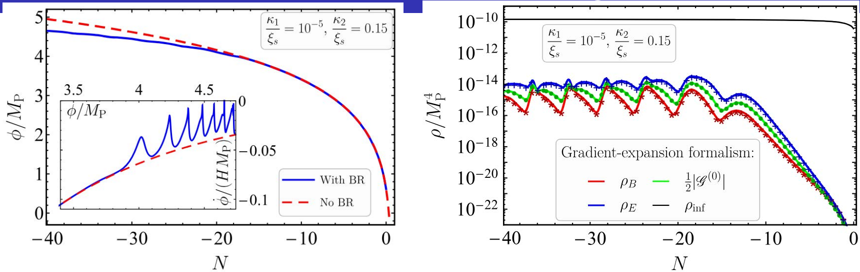


Figure: Evolution of the inflaton field (left) and energy densities (right) in the extended Starobinsky model with axion-dominated coupling $\kappa_2 = 0.15 \gg \kappa_1$. The strong backreaction occurs at more than 20 e-folds before the end of inflation.

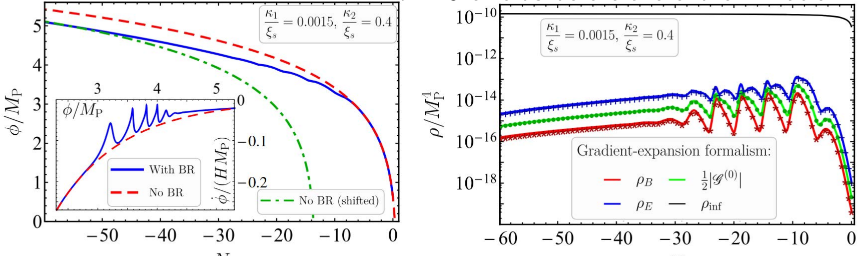
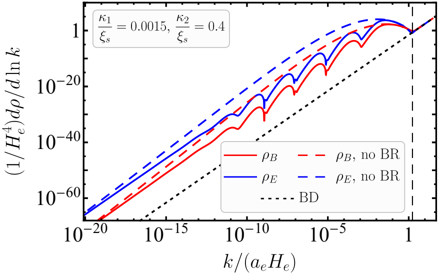


Figure: Same for $\kappa_2 = 0.4$, $\kappa_1 = 0.0015$. SBR occurs at $10 < N < 30$ e-folds

Spectra of the produced gauge fields



In spectra, one can distinguish three regions with different properties

1. For modes crossing the horizon at more than 30 e-foldings before the end of inflation (when the inflaton field decreases monotonically) the spectral curves are monotonic and approach the unperturbed spectra in the limit $k \to 0$
2. **For modes which cross the horizon when strong backreaction occurs, the spectrum shows an oscillatory pattern which corresponds to the inflaton oscillations in the backreaction regime. On average, the spectrum is also blue-tilted with $n_B \approx 4$.**
3. The modes which cross the horizon during the second slow-roll phase (during the last 10 e-foldings of inflation) have a red-tilted spectrum

Impact of backreaction on the spectra

Backreaction has strong impact on the spectra of generated MF

- In the absence of backreaction, $\boxed{n_B = 4 - \frac{32\pi}{9}\frac{\kappa_2}{\xi_s}}$, so by choosing a sufficiently large value of parameter κ_2, it is possible to achieve a scale-invariant or red-tilted magnetic power spectrum and, thus, *obtain a large correlation scale for the produced magnetic field*
- **Backreaction drastically changes this behavior. For magnetic field modes which cross the horizon in the backreaction regime, the spectrum also reveals oscillatory behavior and the average spectral index appears to be close to $n_B = 4$. Since backreaction turns off a few e-foldings before the end of inflation, one can still get a part of the spectrum which is red-tilted, although for a limited range of modes spanning over 2–3 orders of magnitude. Therefore, the resulting coherence length of the produced gauge fields may be at most 2–3 orders of magnitude larger than the horizon size at the end of inflation**

Conclusion

Theories with decreasing coupling functions, although they seem to produce a larger coherence length of generated MF, *unavoidably run into a backreaction regime* which leads to the following:

1. It limits the resulting magnitude of the produced magnetic field because in the backreaction regime, the gauge-field energy density is smaller than that of the inflaton by several orders of magnitude, while once backreaction switches off the field can only decrease
2. *Backreaction does not allow for a significant increase in magnetic correlation length as it turns red-tilted spectrum into blue-tilted.*
3. Also, it leads to oscillatory features in the spectra of the produced gauge fields which can be potentially observable.

Outlook: The backreaction can be lifted in the presence of **the Schwinger pair production** (unavoidable in the case of Standard Model gauge fields). It would be interesting to study the resulting MFs in this case and their spectral properties.

Thank you for your attention!

Magnetogenesis in extended Starobinsky inflationmodel

- The GF production was studied in extended Starobinsky model

$$S[g_{\mu\nu}, A_\mu] = \int d^4x\sqrt{-g} \times \Big[-\frac{M_{\rm P}^2}{2}R + \frac{\xi_s}{1+\frac{\kappa_1}{M_{\rm P}^4}F_{\mu\nu}F^{\mu\nu}+\frac{\kappa_2}{M_{\rm P}^4}F_{\mu\nu}\tilde{F}^{\mu\nu}}R^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\Big], \quad (9)$$

including the GF nonminimally coupled to gravity. This model:
i) acquires a scalar degree of freedom which plays the role of the inflaton and has an asymptotically flat potential as favored by CMB observations;
(ii) its Lagrangian is quadratic in the gauge fields and has the form of kinetic and axial couplings to the inflaton;
(iii) for positive values of the coupling parameter κ_1 it avoids the strong coupling problem during inflation;
(iv) the absence of higher order terms in the gauge fields allows us to study the gauge field nonperturbatively and take into account its backreaction on the evolution of the inflaton

Action in the Einstein frame

1. Bring action (9) to the Einstein-Hilbert form, perform a two-step procedure **1)get rid of the R^2 term by performing a Legendre transform and introducing an additional scalar degree of freedom; 2) perform a conformal transformation of the metric.**
2. **Introduce the new auxiliary field** $\Psi = 1 - \frac{\xi_s}{M_{\rm P}^2 \Delta(F_{\mu\nu})} R$ **and express the spacetime curvature as** $R = \frac{M_{\rm P}^2}{\xi_s} \Delta(F_{\mu\nu})(1-\Psi)$.
3. **the Legendre transform of the function** f **reads** $F(\Psi,\, F_{\mu\nu}) = \Psi R - f(R,\, F_{\mu\nu}) = -\frac{M_{\rm P}^2}{2\xi_s} \Delta(F_{\mu\nu})(1-\Psi)^2$.
4. **Representing** f **as an inverse Legendre transform** $f(R,\, F_{\mu\nu}) = \Psi R - F(\Psi,\, F_{\mu\nu})$, **we obtain the action in the form** $S = \int d^4x \sqrt{-g}\left[-\frac{M_{\rm P}^2}{2} \Psi R - \frac{M_{\rm P}^4}{4\xi_s} \Delta(F_{\mu\nu})(1-\Psi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right]$
5. **In order to remove the extra multiplier** Ψ **in front of** R**, we perform the Weyl transformation** $g_{\mu\nu} = \Psi^{-1} \bar{g}_{\mu\nu}$**, under which the Ricci curvature scalar transforms as** $R = \Psi\left[\bar{R} - \frac{3}{2\Psi^2} \bar{g}^{\mu\nu} \partial_\mu \Psi \partial_\nu \Psi + 3 \bar{\nabla}_\mu \bar{\nabla}^\mu \ln \Psi \right]$.

Action in the Einstein frame

This brings to the Einstein frame where the action takes the next form

$$S = \int d^4x\sqrt{-g}\Big[-\frac{M_{\rm P}^2}{2}R + \underbrace{\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - V(\phi)}_{\text{inflaton}} - \underbrace{\frac{1}{4}I_1(\phi)F_{\mu\nu}F^{\mu\nu}}_{\substack{\text{kinetic coupling}\\ \text{GF to inflaton}}} - \underbrace{\frac{1}{4}I_2(\phi)F_{\mu\nu}\tilde{F}^{\mu\nu}}_{\substack{\text{axial coupling}\\ \text{of GF to inflaton}}}\Big]$$

where the first term is the Einstein-Hilbert action for gravity. other terms describe the inflaton and gauge fields with kinetic and axial couplings. The potential of the inflaton field $V(\phi)$ coincides with corresponding expression for the Starobinsky model $V(\phi) = \frac{M_{\rm P}^4}{4\xi_s}\left(1 - e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm P}}}\right)^2$ **while the kinetic and axial coupling functions have the form**

$$I_j = \delta_{j1} + \frac{\kappa_j}{\xi_s}\left[\exp\left(\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm P}}\right) - 1\right]^2. \qquad (10)$$

Note that the dependence of these coupling functions on the inflaton field is not postulated or constructed by hand, but is deduced from action (9) by rewriting it in the Einstein frame

Why is the gradient expansion formalism needed?

1. **GF exists during inflation in the form of vacuum fluctuations**
2. **The kinetic and axial couplings to the inflaton field break the conformal invariance of the GF action and enable the amplification of its Fourier modes when they cross horizon, in a similar manner to the generation of primordial perturbations**
3. **Modes with wavelengths largely exceeding the radius of observable region behave as classical mean fields; however, their quantum origin implies that they are stochastic quantities; i.e., they are chaotically oriented in different regions of the Universe**
4. **This means that vector quantities as E or B average to zero and are not suitable for the description of the generated fields**
5. **With the aim to keep track of backreaction of the generated GF on background evolution one needs a tool which can take into account these nonlinear effects fully self-consistently. One such tool – the gradient expansion formalism (GEF).**

Exact expressions for the boundary terms

$$[\dot{\mathcal{E}}^{(n)}]_{\rm b} = \frac{d\ln k_{\rm h}(t)}{dt}\frac{1}{4\pi^2}\left(\frac{k_{\rm h}(t)}{a(t)}\right)^{n+4} \times \sum_{\lambda=\pm 1}\lambda^n E_\lambda(\xi(t),\, s(t),\, \sigma(t)), \quad (11)$$

$$[\dot{\mathcal{G}}^{(n)}]_{\rm b} = \frac{d\ln k_{\rm h}(t)}{dt}\frac{1}{4\pi^2}\left(\frac{k_{\rm h}(t)}{a(t)}\right)^{n+4} \sum_{\lambda=\pm 1}\lambda^n G_\lambda(\xi(t),\, s(t),\, \sigma(t)), \quad (12)$$

$$[\dot{\mathcal{B}}^{(n)}]_{\rm b} = \frac{d\ln k_{\rm h}(t)}{dt}\frac{1}{4\pi^2}\left(\frac{k_{\rm h}(t)}{a(t)}\right)^{n+4} \sum_{\lambda=\pm 1}\lambda^n B_\lambda(\xi(t),\, s(t)), \quad (13)$$

where $k_{\rm h} = aH\left(|\xi| + \sqrt{\xi^2 + |s|}\right)$

$$E_\lambda(\xi,\, s,\, \sigma) = \frac{e^{\pi\lambda\xi}}{r^2}\Big|(ir - i\lambda\xi - \sigma)\, W_{-i\lambda\xi,\sqrt{s+\frac{1}{4}}}(-2ir) + W_{1-i\lambda\xi,\sqrt{s+\frac{1}{4}}}(-2ir$$

$$G_\lambda(\xi,\, s,\, \sigma) = \frac{e^{\pi\lambda\xi}}{r}\Big\{-\sigma\Big|W_{-i\lambda\xi,\sqrt{s+\frac{1}{4}}}(-2ir)\Big|^2 + \Re e\Big[W_{i\lambda\xi,\sqrt{s+\frac{1}{4}}}(2ir)\, W_{1-i\lambda\xi,}$$

$$B_\lambda(\xi,\, s) = e^{\pi\lambda\xi}\left|W_{-i\lambda\xi,\sqrt{s+\frac{1}{4}}}(-2ir)\right|^2, \quad (16)$$

here $r = r(\xi,\, s) = |\xi| + \sqrt{\xi^2 + |s|}$ and $\sigma(t) = \frac{i_1}{2HI_1}$