SCALES of the Vortex Avalanche in a Pulsar Glitch

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Vortex or flux-line avalanches, as distinct from large fluctuations in continuous dynamics (vortex / flux creep, or SOC models etc) require the **breaking of scale invariance.**

This talk presents **a simple model for avalanches**, based on intrinsic scales of pinning forces and of substrates sustaining pinning centers, for applications to pulsar glitches and to laboratory condensates.

Pulsar glitches: very fast (unresolved) rise, slow decay:

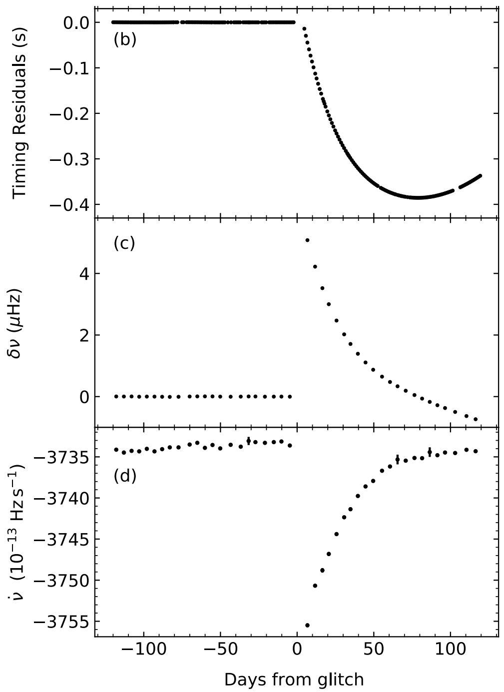

Figure 5. A glitch in the Crab pulsar (t_g = MJD 53067.1) with $\Delta\nu = 6.37\,\mu$Hz and $\Delta\dot{\nu} = -23.02 \times 10^{-13}$ Hz s^{-1}. The data was

(From review by Antonopoulou, Haskell, Espinoza, 2022)

Experiments and simulations with uniform distributions of pinned vortices show no difference between rise and decay timescales of events: eg classical experiments of Tsakadze & Tsakadze 1980,

& recent simulations & experiments –eg Novotny, Talir & Varga PRL 2026 Superfluid He^4

But also see Elena Poli's talk this morning, showing fast rise slower decay of glitches in simulation of vortex pinning to super solid.

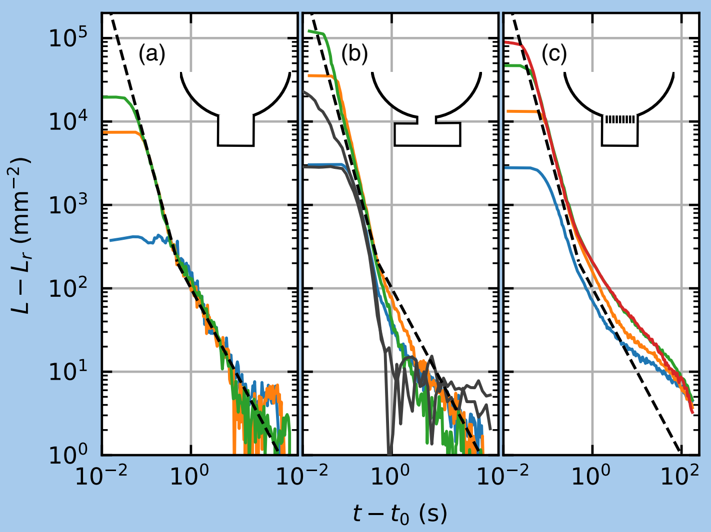

FIG. 2. Vortex line density decay from all three geometries for a range initial densities L_0 and similar probe velocity amplitudes $v_p \approx 12$ cm/s for all, except for the two gray curves in panel (b), for which $v_p \approx 4.2$ cm/s. The dashed straight lines indicate $50/t^2$ (for $t < 0.5$ s) and $100/t$ (for $t > 0.5$ s) and are identical for all three. The delay on the order of 10 ms is consistent with the inverse linewidth of the fourth sound resonance.

J. S. Tsakadze and S. J. Tsakadze

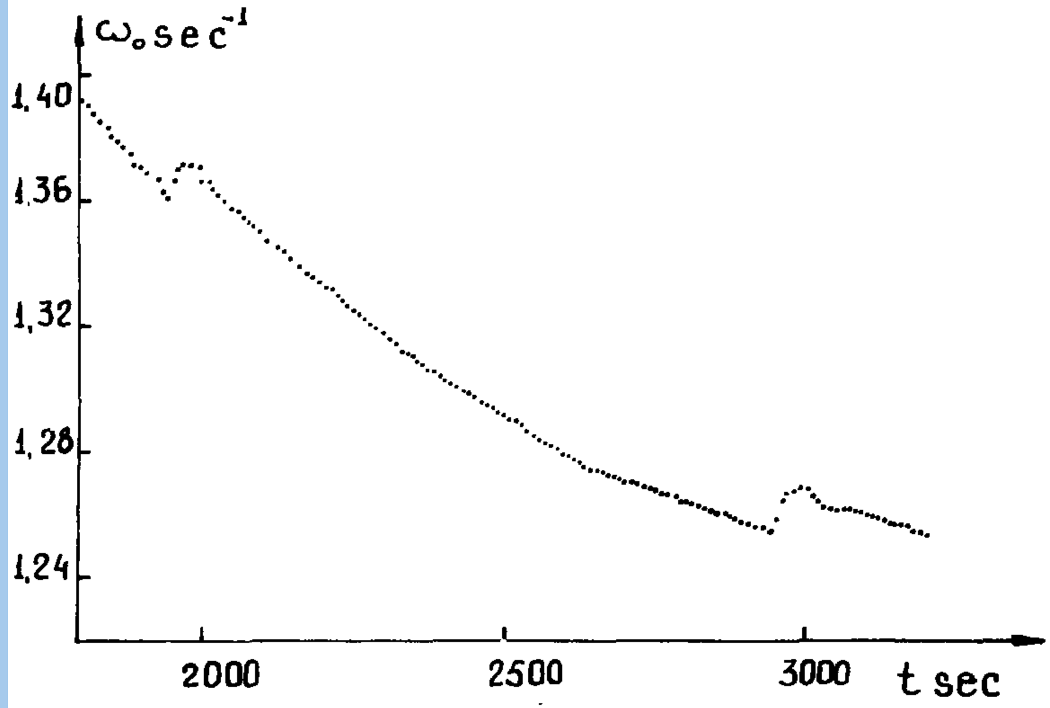

Fig. 25. Time dependence of the rotation velocity of a light vessel containing superfluid liquid. $T = 1.52$ K.

Distribution of glitch sizes is not scale invariant:

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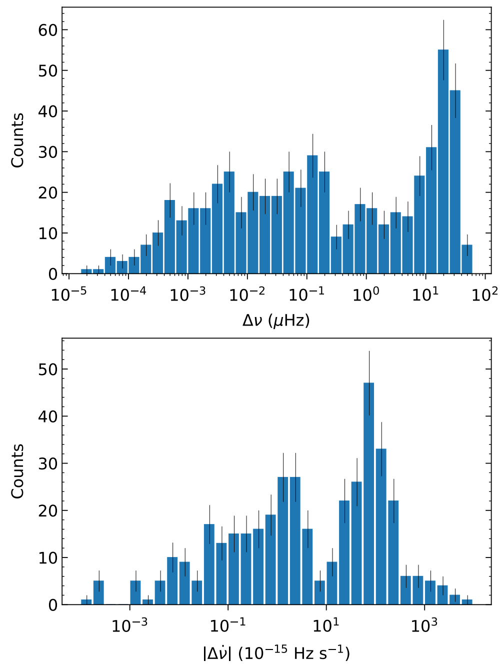


Figure 10. The distribution of glitch amplitudes $\Delta\nu$ (upper panel) and $|\Delta\dot{\nu}|$ (lower panel, data shown only for glitches where $\Delta\dot{\nu} < 0$) in logarithmic scales. Uncertainties are calculated as $\sqrt{N}$, where N is the number of elements in each bin. Glitch parameters were retrieved from the online Jodrell Bank Glitch Catalogue [27].

(From review by Antonopoulou, Haskell, Espinoza, 2022)

The concept of flux creep:

Anderson & Kim (1964): "Hard Superconductivity: Theory of the Motion of Abrikosov Flux Lines," *Reviews of Modern Physics*.

Kim, Hempstead, & Strnad (1963): "Flux Creep in Hard Superconductors," *Physical Review*.

Pulsar Glitches as sudden unpinning: Anderson & Itoh 1975

Vortex creep in interaction with sudden unpinning:

Alpar, Anderson, Pines, Shaham, 1980 - 1984

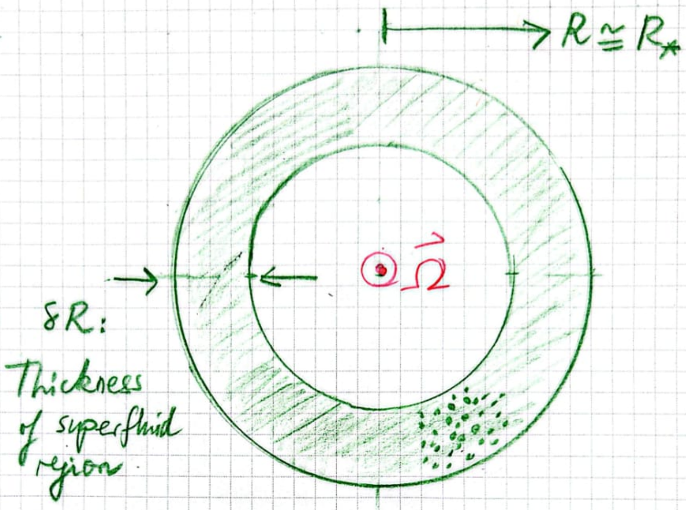
Vortex traps, vortex free regions, ideas for avalanche models:

Alpar, Cheng, Pines, Shaham late 1980s.

Vortex Creep and Avalanches

- **Vortex creep** is driven by the lag $\omega \equiv \Omega_s - \Omega_c$ between the rotation rate Ω_s. of the pinned superfluid and Ω_c , that of the neutron star crust/ container of the rotating superfluid.
- $\omega > 0$ because of pinning.
- When the distribution of the pinned vorticity is uniform, there are no scales; vortices creep radially outward from the rotation axis, spinning the pinned superfluid down and transferring angular momentum to the crust/container -an "internal torque".
- In the absence of any scales the distribution of event sizes $\delta\Omega_c \propto \delta\Omega_s \propto \delta N_v$ where δN_v is the number of vortices involved in the fluctuation, should be a power law.
- The timescale of both the rise and fall of fluctuations should be the same, $\tau_{rise} \sim \tau_{decay} \propto \delta\Omega_c / |\dot{\Omega}|$, where $|\dot{\Omega}|$ is the spin-down rate of the neutron star/ lab container.

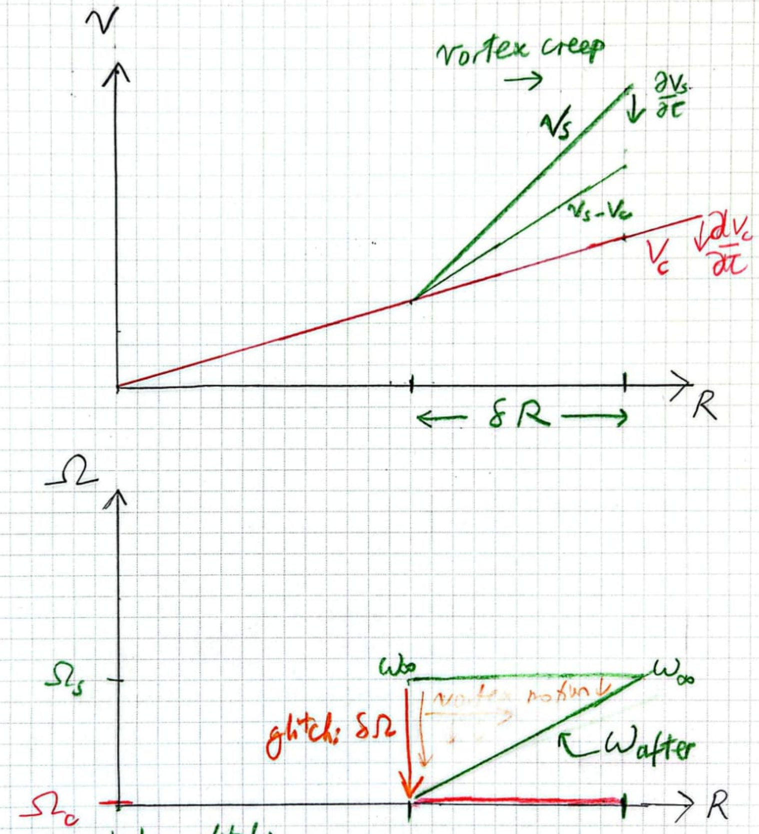
- Pinned vortices will be clustered into "traps" with size SCALES dictated by the strength of pinning forces and the critical strain angle of the solid (template) containing the pinning centers.
- Vortices will creep outward from the center of each trap.
- Around each vortex trap there is a vortex free region. Any vortices that unpin and spill into vortex free regions will move with the superfluid flow, very fast, in the azimuthal direction with respect to the rotation axis. These unpinned, free vortices will also move slightly radially outward, by a distance $\Delta R \equiv \zeta(2\pi R)$ per azimuthal turn $2\pi R$ due to the dissipative drag forces arising from electron and phonon scattering. The small scattering angle ζ, related to the mutual friction parameters, can be calculated for a given model of the drag forces.
- The superfluid velocity, with respect to the velocity of the pinned vortices, is limited by the critical velocity difference v_{cr} that can be maintained by the maximum available pinning force. This is almost reached at the boundaries of the trap.
- When a trap releases vortices, unpinned vortices will move rapidly in the surrounding vortex free regions, encountering and triggering other traps building into a relay race, an AVALANCHE.
- The fast motion of the avalanche has a very short TIMESCALE compared to creep.
- The number of vortices in traps and the number of traps joining the avalanche determine the glitch magnitude SCALES.



Ring: Pinned Inner Crust Superfluid

uniform density of quantised vortices

NO SCALES

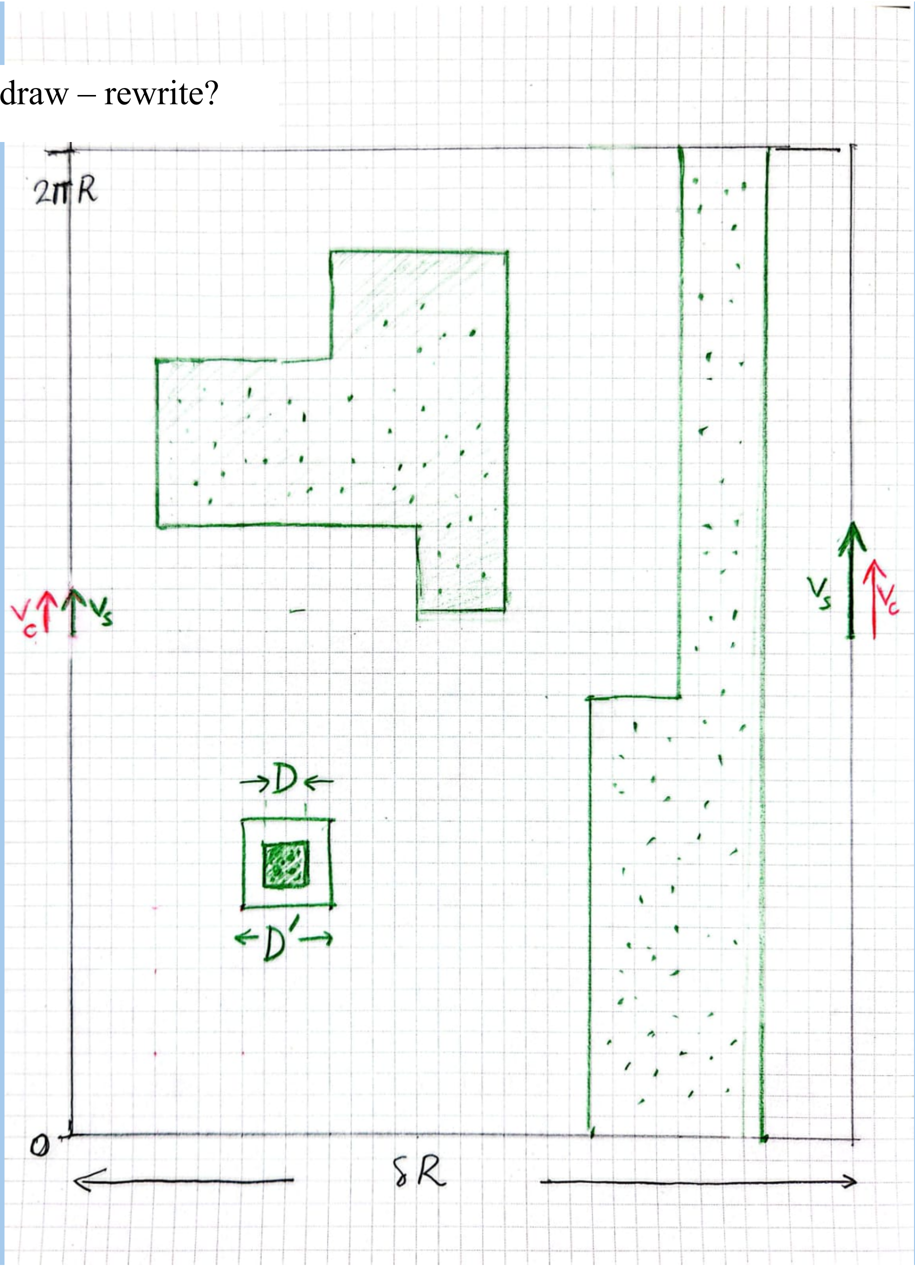


before glitch:
steady state $\omega_\infty = [\Omega_s - \Omega_c]_\infty \Rightarrow \dot{\Omega}_s = \dot{\Omega}_c$

glitch: sudden vortex motion

WHY SUDDEN?!

Keep – redraw – rewrite?



The pinned superfluid is a ring (cylindrical shell) of thickness δR and radius R , shown as a strip of length $2\pi R$. Vortex traps of size D, surrounded by vortex free regions out to size D' have been formed in a fraction η of the pinned superfluid, tiled with traps and surrounding vortex free regions (one example tile is shown).

The remaining (1 - η) of the pinned superfluid is larger, connected creep regions, dotted with uniform vortex density as shown in the figure.

- SCALES dictated by the strength of pinning forces and the critical strain angle of the solid (template) containing the pinning centers:

The size D of the traps scales with the critical strain angle

$$D \cong \delta R\, \theta_{cr}$$

Alpar&Pines: Electrons in the NS crust are relativistic; there is practically no screening; so the dimensionless critical strain angle $\theta_{cr} \cong \frac{Ze^2/a}{\hbar c k_F} \cong \alpha \frac{Z^{4/3}}{(3\pi^2)^{1/3}} \cong O(10^{-1})$;

here α is the fine structure constant, a is the inter-nucleus spacing and Z is the nuclear charge.

Molecular dynamics also gives $\theta_{cr} \cong O(10^{-1})$

(Horowitz-Kadau-Chugunov; Caplan, Smith, Bransgrove and Horowitz 2026)

- Around each vortex trap there is a vortex free region. Any vortices that unpin and spill into vortex free regions will move with the superfluid flow, very fast, in the azimuthal direction with respect to the rotation axis. These unpinned, free vortices will also move slightly radially outward due to the dissipative drag forces arising from electron and phonon scattering.
- When a trap goes critical, unpinned vortices will encounter and trigger other traps building into a relay race, an avalanche.
- The fast motion of the avalanche has a very short TIMESCALE compared to creep.
- The number of vortices in traps and the number of traps joining the avalanche determine the glitch magnitude SCALES.

- Vortices with area density n_0 such that $n_0\kappa = 2\Omega$, in a square of size *D'* are clustered, at higher density $n_0 + \delta n$, and higher superfluid circulation $(n_0 + \delta n)\kappa = 2\bar{\Omega}$ into each trap of size *D,* so

$$\frac{D'^2}{D^2} = \frac{\bar{\Omega}}{\Omega}.$$

- The circulation around the trap gives:

$\bar{\Omega} = 2v_{cr}/D$, while the critical speed is given by the Magnus Equation,

$\rho_s \kappa v_{cr} = f_{p,max}$, where ρ_s is the superfluid density and $f_{p,max}$ is the maximum pinning force per unit length of vortex line.

$$\frac{D'^2}{D^2} = \frac{2f_{p,max}}{\rho_s \kappa \ \Omega \ D} = \frac{2f_{p,max}}{\rho_s \kappa \ \Omega \ \delta R \ \theta_{cr}}$$, fixing $\frac{D'^2}{D^2}$ in terms of physical parameters.

- When N_{Ava} unpinned vortices move through the superfluid in an avalanche, the superfluid rotation rate suddenly decreases by $\delta\Omega = \dfrac{N_{Ava}\kappa}{2\pi r^2}$ in vortex free regions and by $\delta\Omega/2$ in vortex creep regions.
- The angular momentum transfer to the crust/container is observed as a glitch $\Delta\Omega$ in the crust/container rotation rate. Denoting by I_B and I_A the total moments of inertia of vortex free regions and vortex creep regions, respectively through which the avalanche moved, and by I_c the crust/container moment of inertia, one obtains:

$$I_B\delta\Omega + I_A\delta\Omega/2 = I_c\Delta\Omega$$

$$\frac{I_B}{I_A} \equiv \beta \ \rightarrow I_A\ \delta\Omega[\beta + 1/2] = I_c\Delta\Omega$$

The sudden decrease $\delta\Omega$ in the superfluid rotation rate stops vortex creep. The pinned superfluid stops spinning-down.
(Provided creep is in the nonlinear regime - in the linear regime creep is like linear drag forces; there's no deep offset.)

- **The pulsar torque is now acting on less moment of inertia, leading to a glitch in the observed spin down rate:**
- $$\frac{\Delta\dot{\Omega}}{\dot{\Omega}} = \frac{I_A}{I}$$
- Vortex creep will restart gradually as the offset in the lag $\delta\omega = \delta(\Omega_s - \Omega_c) = \delta\Omega - |\dot{\Omega}|\, t$ heals due to the ongoing spin-down of the crust under the pulsar torque.
- Make the simplifying assumption that by the time t_g of the next glitch the offset in the lag is completely relaxed,

$$\delta\Omega = |\dot{\Omega}|\, t_g \quad ;$$

Putting all this together gives the ratio β of vortex free to vortex trap (creep) regions in terms of observed quantities:

$$\beta = \frac{\Delta\Omega}{|\Delta\dot{\Omega}|\, t_g} - 1/2$$

The value of β determined from glitch observations, together with the value of $(D/D')^2$ leads to an estimate of the tiled fraction η of the pinned superfluid:

$$\frac{I_B}{I_B} \equiv \beta = \frac{1-\eta(D/D')^2}{\eta(D/D')^2} \qquad \rightarrow \ \eta = \frac{1}{1+\beta}\frac{D'^2}{D^2} \ .$$

As a model for pulsar glitches, this simple avalanche model is completely solved in terms of observed parameters and physics of solid strength and pinning.

In experiments and simulations of pinned vortex or flux-line systems, η and $(D/D')^2$ are set as parts of the experiment or simulation design.

Bounds on the scattering angle ζ - I:

Unpinned vortices will encounter a number of traps on their paths given by

$N_{enc} = \frac{\eta\ 2\pi R\delta R}{l_{mfp}\ \Delta R}$ where the mean free path $l_{mfp} = \frac{D'^2}{D}$. Thus $N_{enc} = \frac{\eta}{\zeta}\frac{\Omega\ \delta R}{2\ v_{cr}}$. The number of vortices unpinned from each encountered trap is $N_{unpin} = \frac{N_{Ava}}{N_{enc}} = \frac{\zeta}{\eta}\frac{8\pi R\ v_{cr}}{\kappa}$. This must be

less than the number of vortices in a trap $N_{trap} = \frac{4D\ v_{cr}}{\kappa} = \frac{4\ \delta R\ \theta_{cr}\ v_{cr}}{\kappa}$,

leading to a bound $$\zeta < \frac{\eta\ \theta_{cr}}{2\pi}\frac{\delta R}{R} \ .$$

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Bounds on the scattering angle ζ - II:

The avalanche will take a time $\Delta t_{Ava} = \frac{\delta R}{v_r} = \frac{\delta R}{\zeta\, \Omega R} = \frac{\delta R\ P}{\zeta\ 2\pi R}$.

For one Vela pulsar glitch, there is an upper bound of 12 s on the glitch rise time (Palfreyman et al 2018), $\Delta t_{Ava} < 12\ s$, leading to the bound:

$$1.2 \times 10^{-3}\, \frac{\delta R}{R} < \zeta\, .$$

Notes in Conclusion

- **A simple 2-d avalanche model can be solved in terms basic physics of the crust solid and pinning forces and gives results consistent with observed glitch sizes in rotation rate and spin-down rate and with intervals between glitches.**

- **3-d effects like the spherical shape of the neutron star, vortex line bending and boundary layers are not included.**

- **2-d experiments and simulations can be done with thin systems.**

- **Experimental/simulation designs should have scales, eg by tiling the distribution of pinning centers.**

Other possibilities : vortex - flux sheets, hydrodynamic patterns, changes of geometry like pasta phases introducing fast - easy directions.

- (A simulation with anisotropic pinning distribution indicates a bimodal distribution of ‘glitch’ sizes: Anantharaman et al. 2026 to be submitted)

- **Experiments spinning the container down under a constant external torque are expected to produce occasional sudden spin-up glitches with slower relaxation time scales.**

- **Each glitch in the rotation rate of the container is expected to be accompanied by an also sudden step increase in the spin-down rate.**

- **This is not a percolation process as motion in the azimuthal direction is almost free, while motion in the radial direction is dissipative - but there is no percolation in 1-d.**