SCALES 1st General Meeting
Superfluid Condensates in Astrophysics & Laboratory Experiments
University of Coimbra · 22–26 June 2026

The dense-matter equation of state

from cold neutron stars to proto-neutron stars

STATUS · METHODS · SIGNATURES

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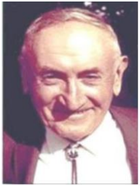
fct Fundação para a Ciência e a Tecnologia

Neutron stars in the newspapers in 1934!

Los Angeles Times 19 January 1934.

W Baade

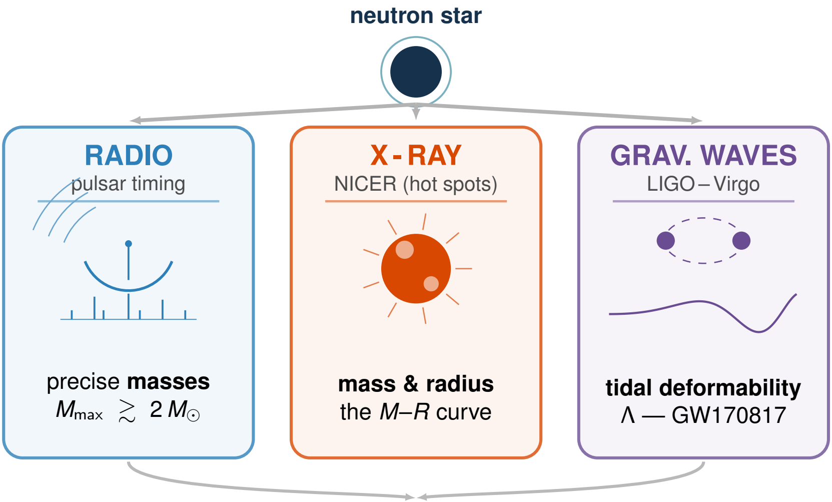


F Zwicky

1933: Astronomers **Walter Baade** and **Fritz Zwicky** proposed the existence of of neutron stars with a radius of approximately **10 km**, resulting from a supernova!

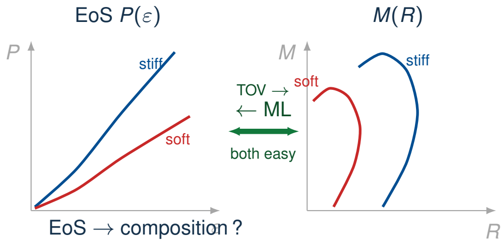
"Cosmic rays are caused by exploding stars which burn with a fire equal to 100 million suns and then shrivel from 1/2 million mile diameters to little spheres 14 miles thick, says Prof. Fritz Zwicky, Swiss Physicist."

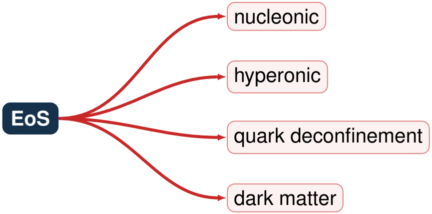
Neutron stars: one object, three messengers

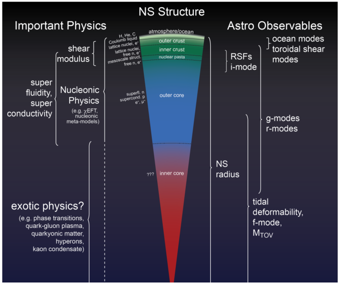


$\Rightarrow$ jointly constrain the dense-matter equation of state

EoS ⇄ M–R both ways easy — composition is hard







what's inside — and how the core may be probed

(oscillation modes: not yet observed)

Neill et al., arXiv:2606.09621 (2026); presented at this meeting

model-dependent inverse problem

***M*, *R* pin the EoS — *not* what's inside.**

The dense-matter EoS: two philosophies

the dense-matter EoS

AGNOSTIC

model-independent · no composition

- **speed of sound c_s^2**
  Tews et al., PRC ’18 · Annala et al., Nat. Phys. ’20
- **piecewise polytrope**
  Read et al., PRD ’09
- **Gaussian process**
  Landry & Essick, PRD ’19
- **GP / “bridge” prior**
  Gorda et al., ApJ ’26 · Kurkela & Malik, arXiv:2606.25186

PHENOMENOLOGICAL

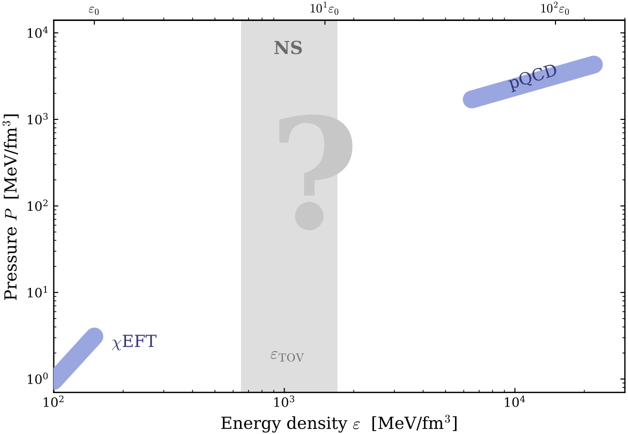
microscopic · gives composition

RMF — widely used

- **non-linear (const. couplings)**
  Boguta & Bodmer, NPA ’77 · Lalazissis et al., PRC ’97
- **density-dependent couplings**
  Typel & Wolter, NPA ’99 · Lalazissis et al., PRC ’05 · Malik et al., ApJ ’22

complementary: flexibility *vs.* physical composition

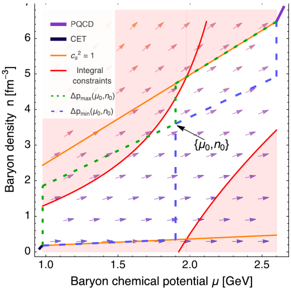
Agnostic EoS: bridging χEFT $\rightarrow$? $\rightarrow$ pQCD



the **agnostic** task: connect the two *ab-initio* anchors across the unknown core

methods differ in *how* they interpolate: c_s^2 · **polytrope** · **Gaussian process** · **bridge prior**

The Komoltsev–Kurkela construction (μ–n plane)



Komoltsev & Kurkela, PRL **128**, 202701 (2022), Fig. 2

Causality → slope. At $T = 0$,

$$c_s^2 = \frac{n}{\mu}\frac{d\mu}{dn} = \left(\frac{d\ln n}{d\ln\mu}\right)^{-1}$$

so $c_s^2 \le 1 \Leftrightarrow \dfrac{dn}{d\mu} \ge \dfrac{n}{\mu}$: the $c_s^2 = 1$ line is the *shallowest* allowed slope (steeper = softer; vertical = 1st-order transition).

Why this plane? At $T = 0$, $d\varepsilon = \mu\, dn$ and $\boxed{dp = n\, d\mu}$, so

$$\Delta p = \int_{\mu_{\rm CET}}^{\mu_{\rm pQCD}} n\, d\mu$$

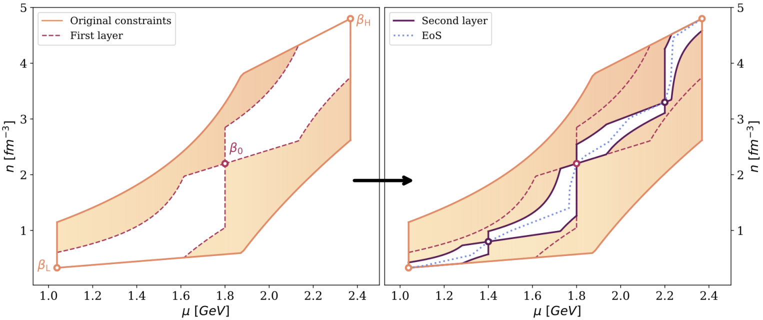
is the *area under* $n(\mu)$ — and it is *fixed* ($= p_{\rm pQCD} - p_{\rm CET}$).

Causality $c_s^2 \le 1$ caps the slope; stability $\Rightarrow n\uparrow$ with μ.

Through $\{\mu_0, n_0\}$ the max- & min-pressure paths must still reach pQCD with that area — else excluded (pink).

Comment. A bound rigorous only at $\sim 40\, n_0$ still squeezes the EoS at ~ 2–$5\, n_0$.

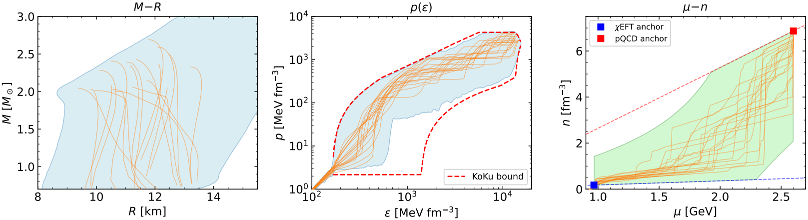
Fractal bridge prior



recursive causal bridging $\beta_{\rm L} \to \beta_0 \to \beta_{\rm H}$, layer by layer ($\sim$10) $\Rightarrow$ one EoS sample

Gorda et al., ApJ 2026 (bridge prior)

What does an EoS look like?



three views of one ensemble: *M–R* (what we observe) · $p(\varepsilon)$ (the EoS) · μ–*n* (construction plane)

each curve = one allowed EoS, anchored to χEFT & pQCD

fractal-bridge ensemble · Kurkela & Malik, arXiv:2606.25186

EoS: relativistic mean-field description

RMF Lagrangian for stellar matter

- Lagrangian density
  - Lorentz-covariant Lagrangian with baryon & meson fields
  - causal by construction

$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_{\mathrm{meson}} + \mathcal{L}_{\mathrm{NL}}$$

- Baryonic contribution:

$$\mathcal{L}_N = \bar{\Psi}\left[\gamma^\mu\left(i\partial_\mu - \Gamma_\omega A_\mu^{(\omega)} - \Gamma_\varrho\, \boldsymbol{t}\cdot\boldsymbol{A}_\mu^{(\varrho)}\right) - (m - \Gamma_\sigma\phi)\right]\Psi$$

- Meson contribution: $\mathcal{L}_{\mathrm{meson}} = \mathcal{L}_\sigma + \mathcal{L}_\omega + \mathcal{L}_\rho$

- (i) density-dependent couplings (DDH, DDB); (ii) non-linear meson terms (NL)

Density-dependent coupling schemes

- **DDH** (Typel & Wolter, Nucl. Phys. A 656, 331, 1999)
  - $\Gamma_i(n_B) = \Gamma_i(n_0)\, f_i(x),\;\; x = n_B/n_0,\;\; f_i(x) = a_i \dfrac{1 + b_i(x + d_i)^2}{1 + c_i(x + d_i)^2}$ $(i = \sigma, \omega)$
  - $\Gamma_\rho(n_B) = \Gamma_\rho(n_0)\, \exp[-a_\rho(x - 1)]$
- **GDFM** (Gögelein et al., PRC 77, 025802, 2008)
  - $\Gamma_i(n_B) = a_i + (b_i + d_i x^3)\, e^{-c_i x} \quad (i = \sigma, \omega, \rho)$
- **DDB** (Malik et al., ApJ 930, 17, 2022)
  - $\Gamma_M(n_B) = \Gamma_{M,0}\, h_M(x),\;\; x = n_B/n_0$
  - isoscalar: $h_M(x) = \exp[-(x^{\alpha_M} - 1)]$; isovector: $h_\rho(x) = \exp[-a_\rho(x - 1)]$

The nuclear matter parameters (NMP)

From any RMF EoS $\rightarrow$ a handful of nuclear-matter parameters:

isoscalar: $E_0, \; K_0, \; J_0, \; Z_0$

isovector (symmetry): $E_{\rm sym}, \; L_{\rm sym}, \; K_{\rm sym}, \; J_{\rm sym}, \; \ldots$

via the Taylor (parabolic) expansion of $E/A(n, \delta)$ around saturation:

$$\frac{E_{\rm nuc}}{A}(n,\delta) = \frac{E_{\rm SNM}}{A}(n) + S(n)\,\delta^2,$$

$$\frac{E_{\rm SNM}}{A}(n) = E_0 + \frac{K_0}{2}\eta^2 + \frac{J_0}{3!}\eta^3 + \frac{Z_0}{4!}\eta^4,$$

$$S(n) = E_{\rm sym} + L_{\rm sym}\,\eta + \frac{K_{\rm sym}}{2}\eta^2 + \frac{J_{\rm sym}}{3!}\eta^3 + \frac{Z_{\rm sym}}{4!}\eta^4.$$

$$\delta = (n_n - n_p)/n, \qquad \eta = (n - n_0)/(3n_0)$$

Malik et al., PRC 98, 035804 (2018)

Bayesian inference of the EoS with RMF: a community effort

Phenomenological (covariant density-functional) priors $+$ nuclear & astro data $\rightarrow$ posterior EoS *with composition*. A growing body of work:

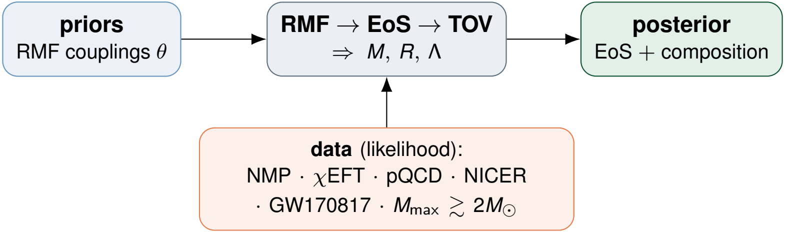
Traversi, Char & Pagliara (2020)
first Bayesian RMF EoS inference
ApJ 897, 165

Malik, Ferreira, Agrawal & Providência (2022)
density-dependent RMF (DDB)
ApJ 930, 17

Char, Mondal, Gulminelli & Oertel (2023)
generalized nucleonic relativistic DF
PRD 108, 103045

Li, Tian & Sedrakian (2025)
covariant DF, multimessenger data
PRC 111, 055804

Inferring the EoS: a Bayesian framework

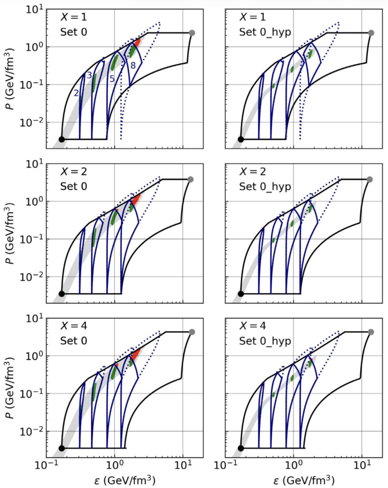


posterior on the EoS *and* its composition

engine: **CompactObject** (open source) — Huang, Malik et al., arXiv:2411.14615

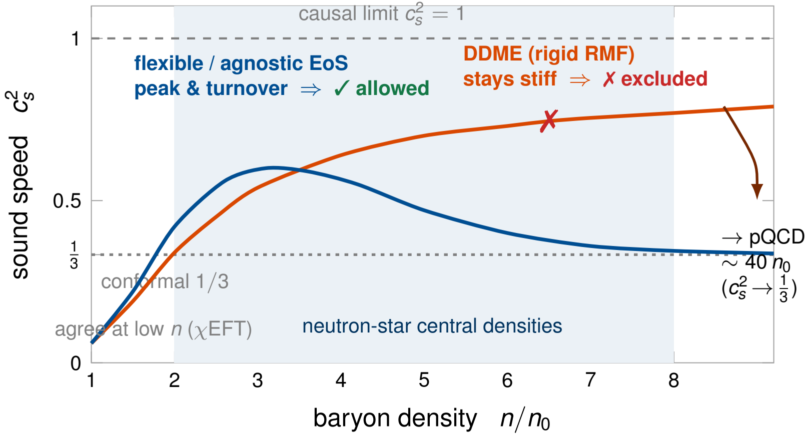
pQCD Constraints on Microscopic Models

Malik et al., Phys. Rev. D 107, 103018 (2023)



- P vs ε for **nucleonic** (Set 0) & **hyperonic** (Set 0_hyp) RMF EoS
- Blue bands: Komoltsev–Kurkela allowed regions at $2, 3, 5, 8\ n_s$
- **Green**: satisfy pQCD **Red**: excluded
- Rows: renormalization scale $X = 1, 2, 4$ — $X = 4$ **most restrictive**
- Hyperonic EoS **essentially all satisfy** pQCD
- Excluded models: **stiffer** high-density EoS (larger c_s^2, larger radii)

What does pQCD *actually* constrain?



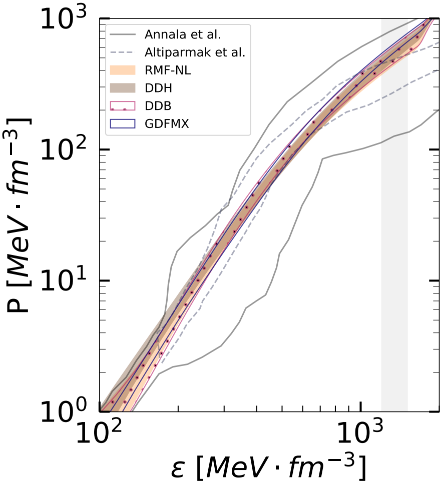
pQCD bounds **stiffness**, not M_{TOV} directly.

A rigid model can't turn over ⇒ excluded (its M_{TOV} capped); a flexible EoS reaches the *same* M_{TOV} via a c_s^2 peak ⇒ allowed.

Komoltsev & Kurkela, PRL 128, 202701 (2022) • Finch et al., PRD 112, 103023 (2025) • Mendes et al., arXiv:2605.18560 (2026)

schematic

Covariant DFs cross-compared: the EoS

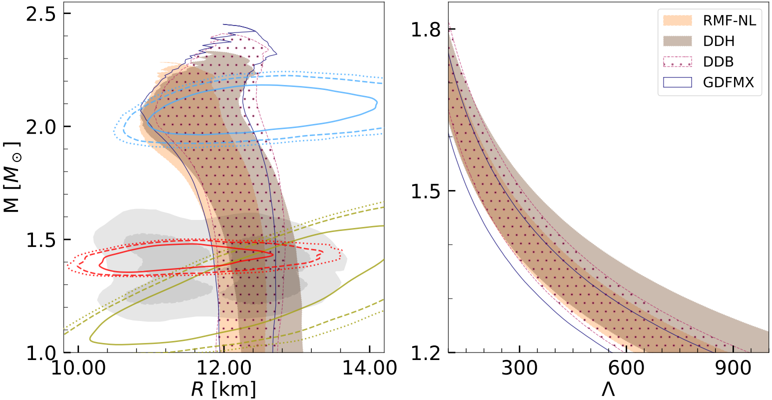


Four covariant DFs under the *same* nuclear + astro constraints:

- RMF-NL, DDH, DDB, GDFMX
- EoS posteriors **cluster tightly**
- sit **inside** the agnostic band (Annala; Altiparmak)

Cartaxo, Huang, Malik et al., ApJS 282, 33 (2026)

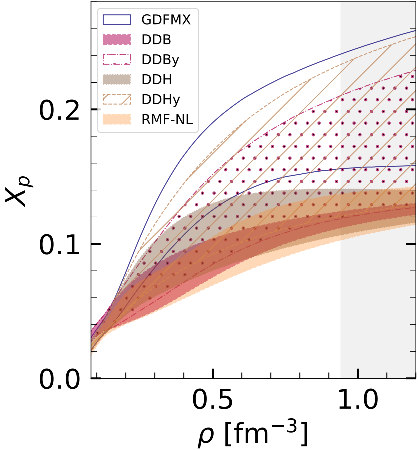
Covariant DFs cross-compared: M–R and tidal Λ



consistent M–R and M–Λ, matching NICER & GW170817

Cartaxo, Huang, Malik et al., ApJS 282, 33 (2026)

Covariant DFs cross-compared: composition

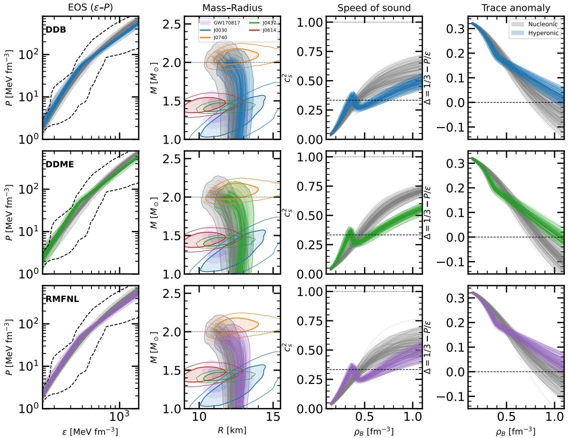


Same EoS, **different composition**:

- proton fraction $X_p(\rho)$ spreads widely across functionals
- sets the **direct-Urca** cooling threshold
- composition is the *model-dependent* part

Cartaxo, Huang, Malik et al., ApJS 282, 33 (2026)

Hyperons in RMF: a systematic Bayesian comparison

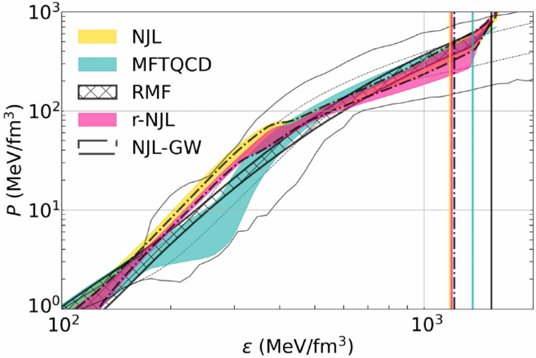


Three covariant-DF families — **DDB**, **DD-ME**, **RMF-NL** (rows); columns: EoS · M–R · c_s^2 · trace anomaly.

Nucleonic vs **hyperonic**: hyperons **soften** the core ⇒ smaller R, lower $M_{\max}$ — still meeting 2 $M_\odot$ & NICER.

Sanson, Malik & Providência, arXiv:2606.21617 (2026)

Nucleonic RMF can’t fill the agnostic space



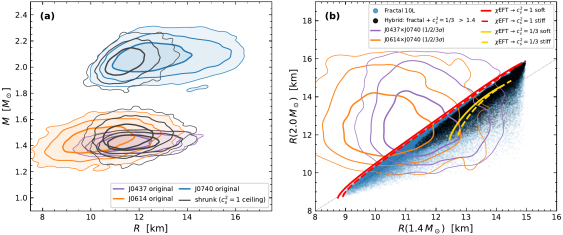
Nucleonic RMF (hatched) sits *inside* the agnostic band (grey); reaching the **soft (lower)** or **stiff (upper)** edges needs a **phase transition**.

→ details in **Milena’s talk** today.

Albino, Malik, Ferreira & Providência, PRD 113, 083019 (2026)

The causal ceiling shrinks the NICER radii

Impose the prior-independent ceiling on the **independent** NICER posteriors of J0437−4715, J0614−3329 & J0740+6620.

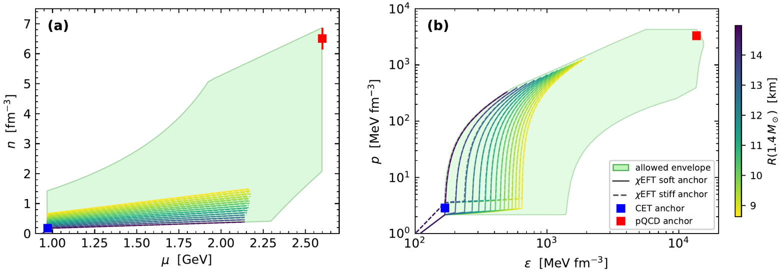


Causality **removes the large-radius tail** of J0740+6620 and retains only **7.5%** of the joint posterior — a transparent, prior-independent benchmark.

Kurkela & Malik, arXiv:2606.25186 (2026)

Origin of the ceiling: the stiffest causal EoS

For a given $R(1.4\,M_\odot)$, the largest $R(2.0\,M_\odot)$ comes from the **stiffest causal continuation** anchored to χEFT.



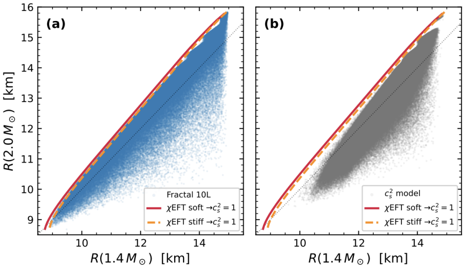
Inside the causal, thermodynamically-allowed **envelope** (green), a one-parameter family of **maximally-stiff** ($c_s^2 = 1$) EoS — pinned at the χ**EFT** anchor (low ε) and the **pQCD** anchor (high ε) — saturates the bound. Colour $= R(1.4\,M_\odot)$.

Kurkela & Malik, arXiv:2606.25186 (2026)

Causality alone bounds the radius difference

$\Delta R \equiv R(2.0\,M_\odot) - R(1.4\,M_\odot)$ probes high-density stiffening, softening & phase transitions.

$$R(2.0\,M_\odot) \;\leq\; 1.16\,R(1.4\,M_\odot) - 1.1\ \mathrm{km}$$



Anchored **only** to χEFT near saturation — a **closed-form, prior-independent** ceiling (saturated by an analytic one-parameter $c_s^2 = 1$ family); it caps *any* agnostic ensemble.

Kurkela & Malik, arXiv:2606.25186 (2026)

The mass–radius slope as a composition probe

Where new degrees of freedom switch on, the M–R curve **bends** — its local slope dR/dM carries **composition** information.

Slope as a learnable probe

The M–R slope encodes interior composition — recoverable from data.

Ferreira et al.,
PRD 112, 083058 (2025)

Hyperons leave a mark

Stellar properties flag the onset of **hyperons**.

Bauswein et al.,
PRR 8, 013253 (2026)
(incl. Chamel, Oertel, Tolos)

Bending angles $\rightarrow c_s^2$

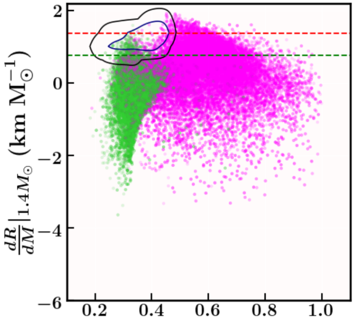
Two reference points $+$ the forward/backward bending of the M–R curve $\rightarrow c_s^2(\varepsilon)$, via **ML**.

Mukherjee & Mallick,
arXiv:2602.18191 (2026)

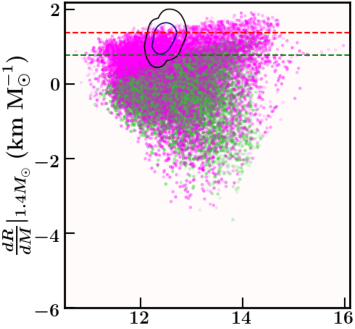
$\Rightarrow$ read composition *from observables* — our dR/dM discriminator next.

dR/dM: a new probe for NS composition

Kalita, Malik, Zhao, Kumar & Lattimer, arXiv:2510.23405



$dR/dM|_{1.4}$ vs central c_s^2 — **clean split**

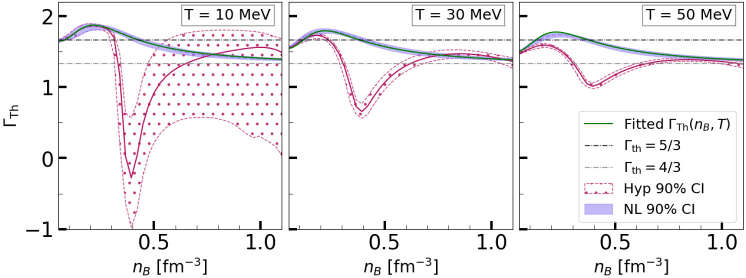


$dR/dM|_{1.4}$ vs $R_{1.4}$ — **they overlap**

hadronic (green) sit at **low** c_s^2; **quarkyonic** (magenta) reach **high** c_s^2.
dR/dM paired with central c_s^2 **separates** them — radius alone does not.

Proto-neutron stars: the finite-T equation of state

The **thermal index** $\Gamma_{\mathrm{Th}} = 1 + p_{\mathrm{Th}}/\varepsilon_{\mathrm{Th}}$ sets how the pressure responds to heat.



Green: a closed-form *fitted* $\Gamma_{\mathrm{Th}}(n_B, T)$ for **nucleonic** matter — adds finite-T to any cold EoS.

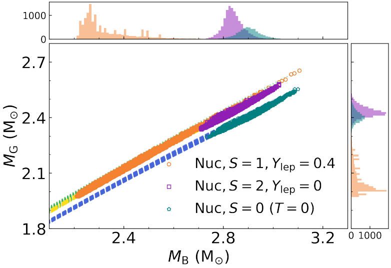
Hyperons (pink) **dip** sharply at the onset density — extra softening as new degrees of freedom appear.

Kumar, Malik, Mishra & Providência, PRD 112, 063042 (2025)

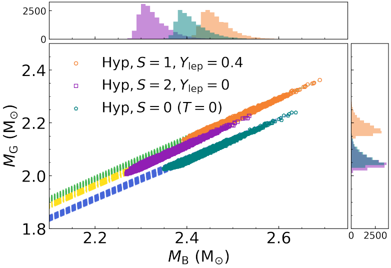
Proto-neutron stars: evolution at fixed baryonic mass

A newborn NS is **hot** & **neutrino-trapped**, then cools and deleptonizes at **conserved** baryonic mass M_B:

$$S = 1,\ Y_{\rm lep} = 0.4 \ \rightarrow\ S = 2,\ Y_{\rm lep} = 0 \ \rightarrow\ S = 0\,(T = 0)$$



M_G vs M_B — **nucleonic**



M_G vs M_B — **hyperonic** (softer, lower $M_{\rm max}$)

At fixed M_B: **hyperons** set in only *after* deleptonization, lowering the cold $M_{\rm max}$ — a PNS above the $T = 0$ limit is **metastable** and collapses to a black hole.

Kumar, Malik, Mishra & Providência, PRD 112, 063042 (2025)

Conclusions

- **Anchored, open in the middle.** χEFT (low n) and pQCD (high n) pin the ends; the NS core stays the frontier — *agnostic* (flexible) and *RMF / covariant-DF* (composition) approaches are converging.
- **Inference returns composition.** Bayesian RMF inference with open tools (**CompactObject**) gives the EoS *and* its composition — which agnostic priors cannot.
- **Model-independent limits sharpen it.** Causality bounds ΔR; pQCD bounds stiffness; NICER $+$ the causal ceiling shrink the allowed band.
- **New probes & signatures.** dR/dM, M–R "bending", hyperons, phase transitions — reading composition from observables.

Toward superfluidity: the EoS and composition we infer — density, proton fraction, m^* — are key inputs to the **pairing gaps** (hence T_c), to neutrino **cooling** (direct Urca), and to the **glitch** (crustal moment-of-inertia) reservoir; the finite-T EoS extends this to **proto-NS** & mergers.

Thank You!

Questions & Discussion

Contact & Resources

Email: `tm@uc.pt`

GitHub:
`https://github.com/tuhinbits`

EOS Database Package:
`https://github.com/tuhinbits/CompactObject-CEDF-EOS-Database-GUI`

Open-Source Code:
CompactObject

Acknowledgments

Computing Resources:
EuroHPC
EHPC-REG-2025R01-021

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