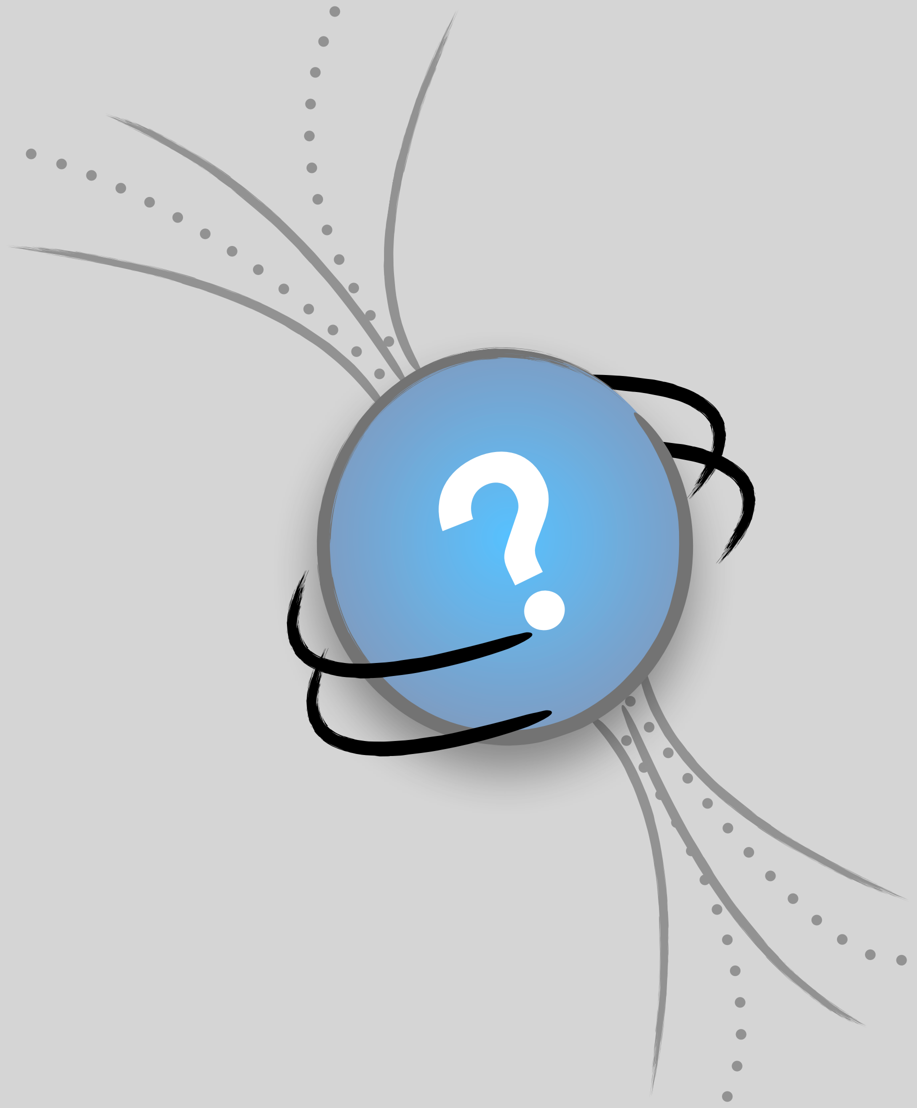
1290 UNIVERSIDADE D COIMBRA

CAMK PAN

fct Fundação para a Ciência e a Tecnologia



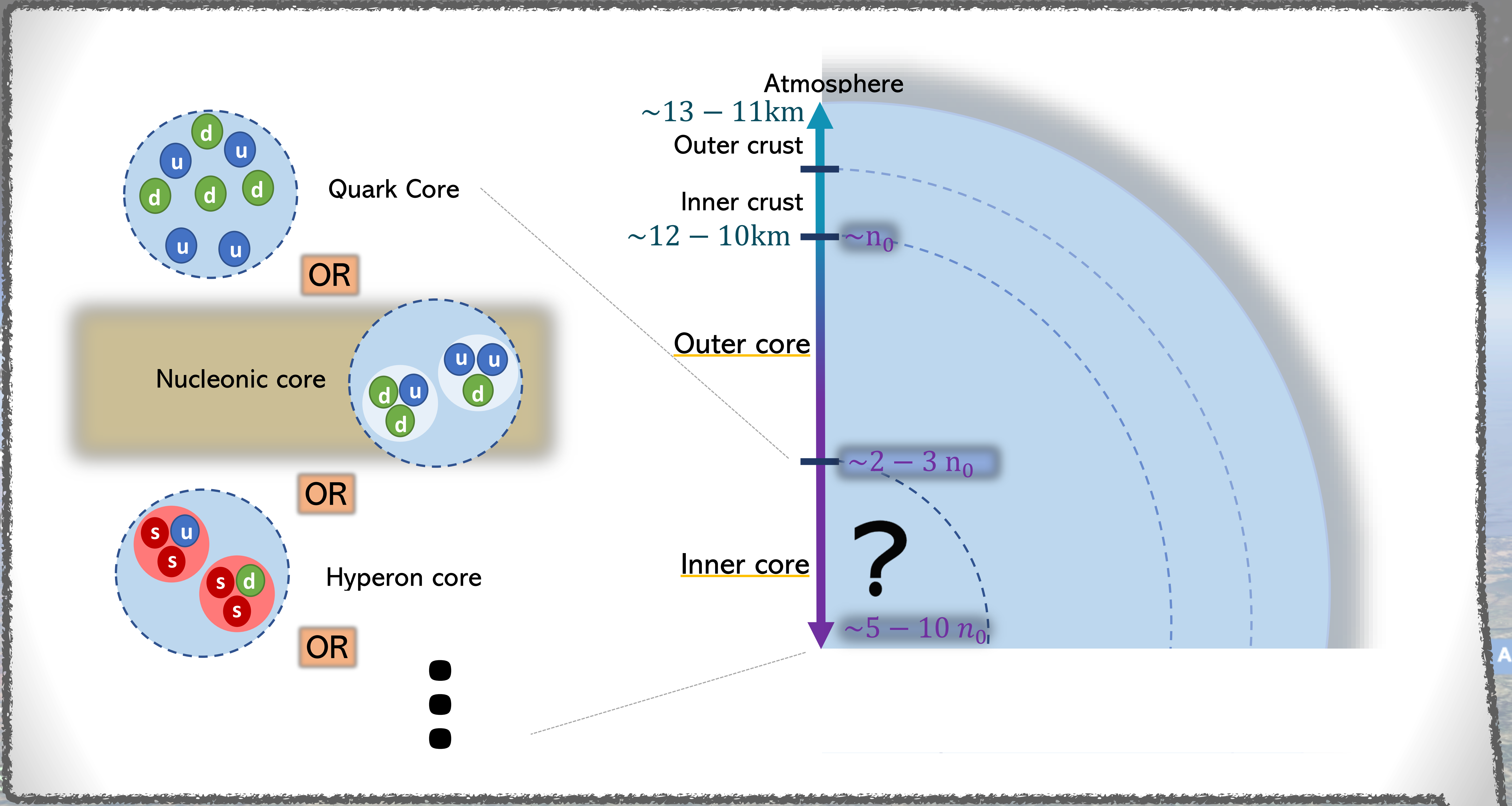
How effectively can Neural Posterior Estimation infer the Neutron Star Equation of State

Valéria Carvalho
Constança Providência, Michał Bejger, Márcio Ferreira

Scales 1° general Meeting

26 June 2026

What's inside a neutron star?



$$M = 1 \sim 2M_\odot$$

$$R \approx 10^{-5}\, R_\odot$$

$$T=0$$

The equation of state: A bridge between micro and macro

Equation of State(EoS)

$$p(\varepsilon)$$

Speed of Sound

$$c_s^2(n) = \frac{dp(n)}{d\varepsilon(n)}$$

Trace Anomaly

$$\Delta(n) = \frac{1}{3} - \frac{p(n)}{\varepsilon(n)}$$

Tolman-Oppenheimer-Volkoff equations

$$\frac{dP(r)}{dr} = -\frac{\varepsilon(r)m(r)}{r^2}\left(1+\frac{P(r)}{\varepsilon(r)}\right)\left(1+\frac{4\pi r^3 P(r)}{m(r)}\right)\left(1-\frac{2m(r)}{r}\right)^{-1}$$

$$\frac{dm(r)}{dr} = 4\pi r^2 \varepsilon(r)$$

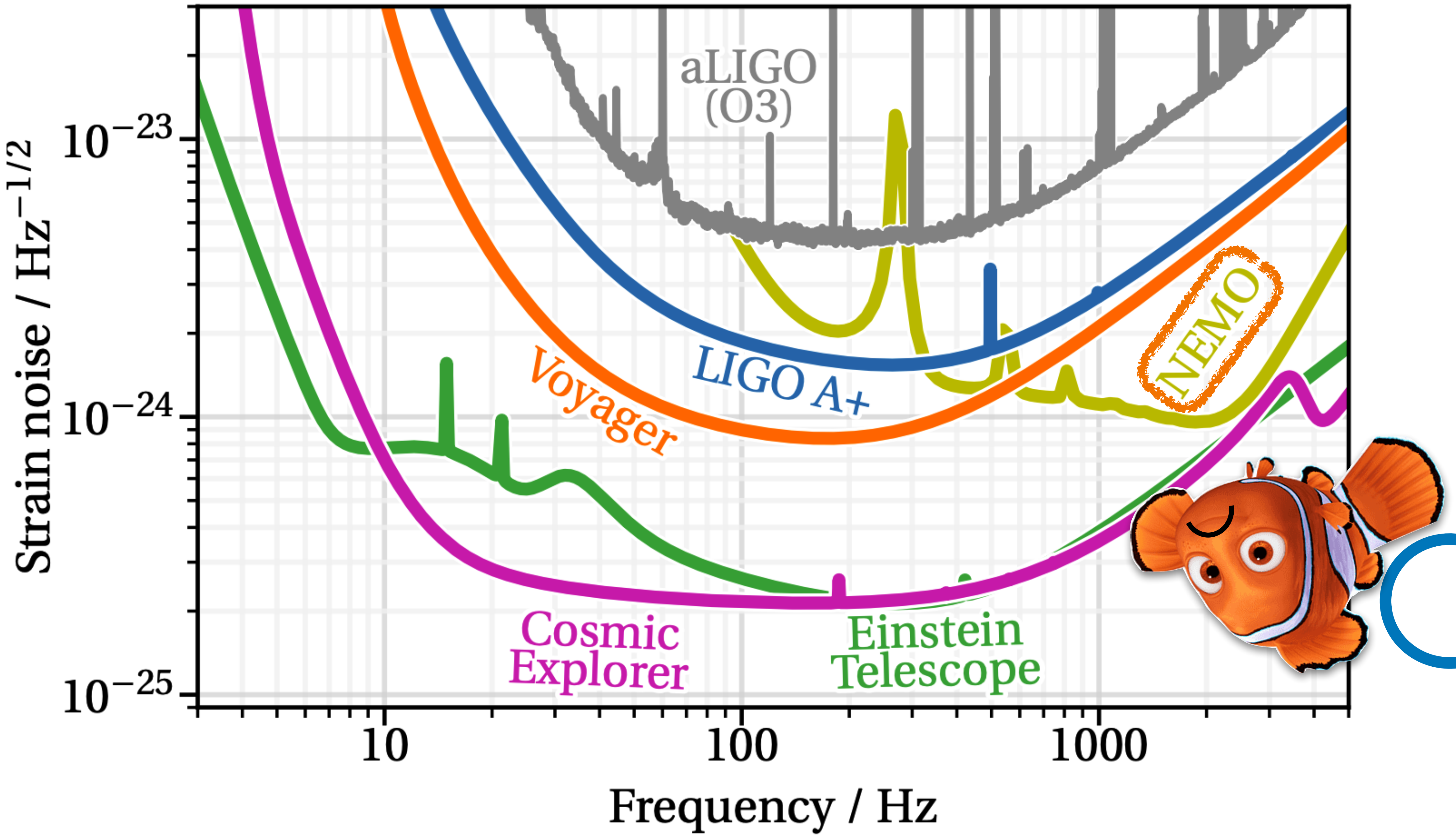
Tidal deformability

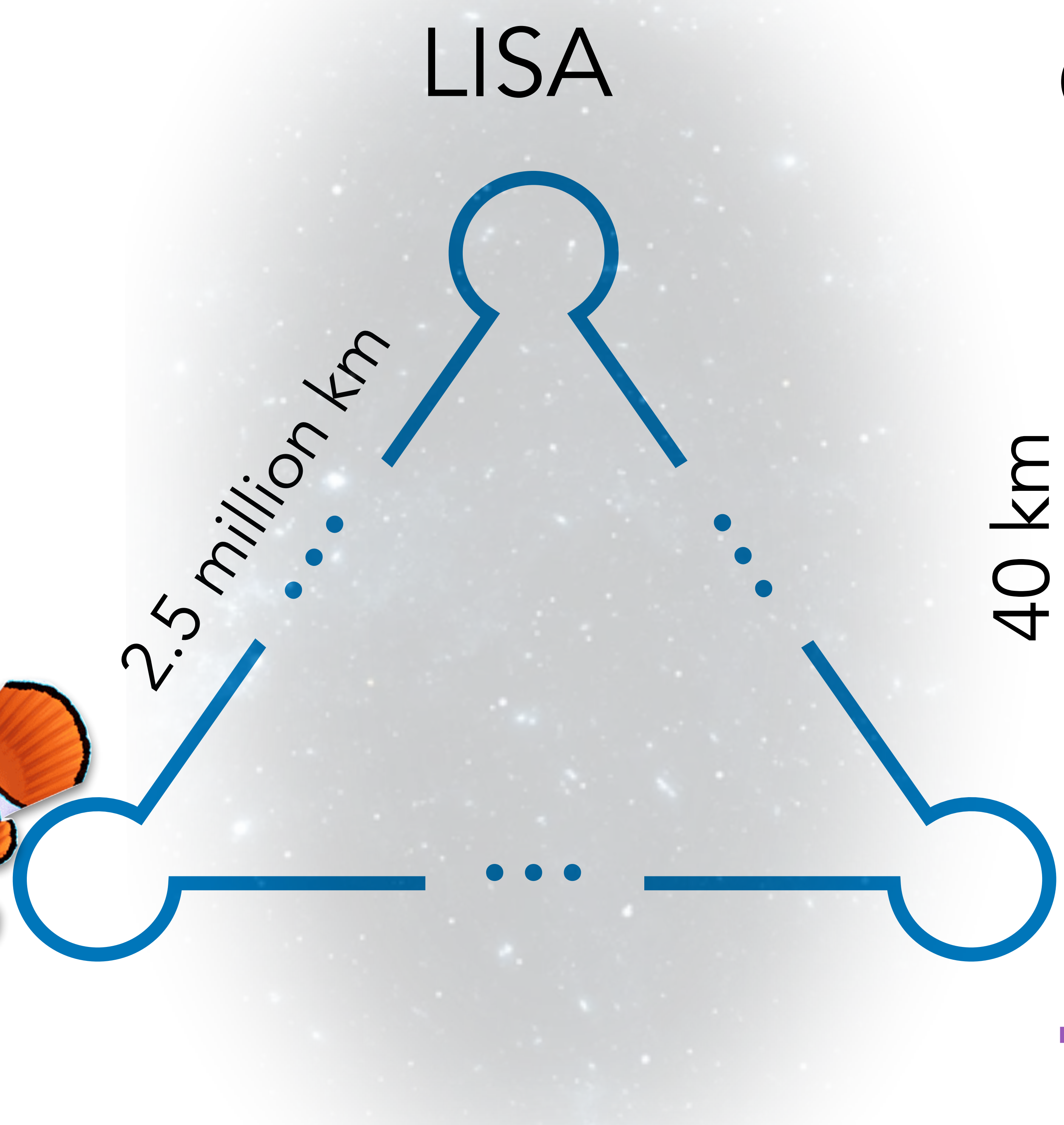
$$\Lambda = \frac{2}{3}k_2 C^{-5}, \qquad C = \frac{M}{R}$$

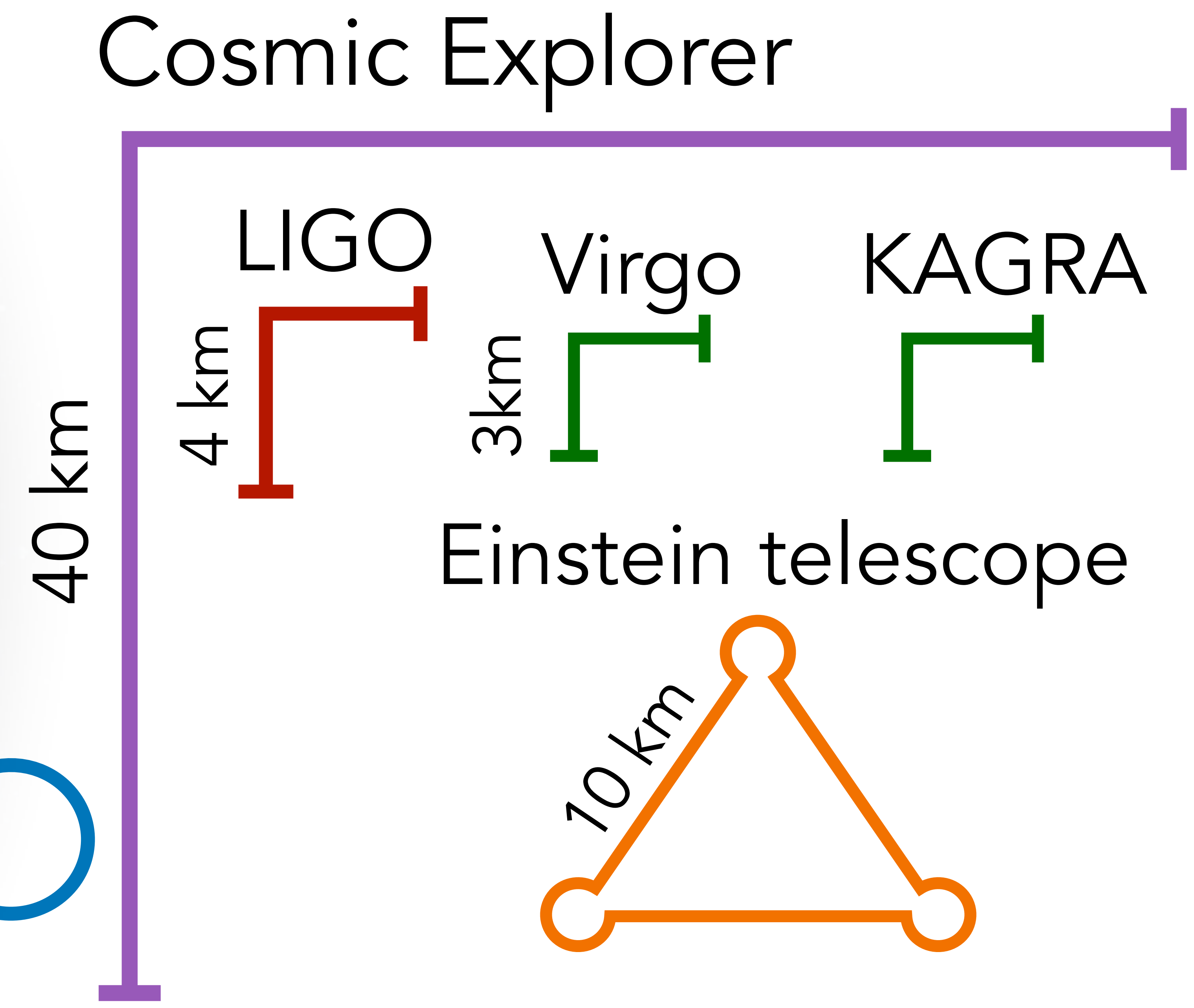
Current observations

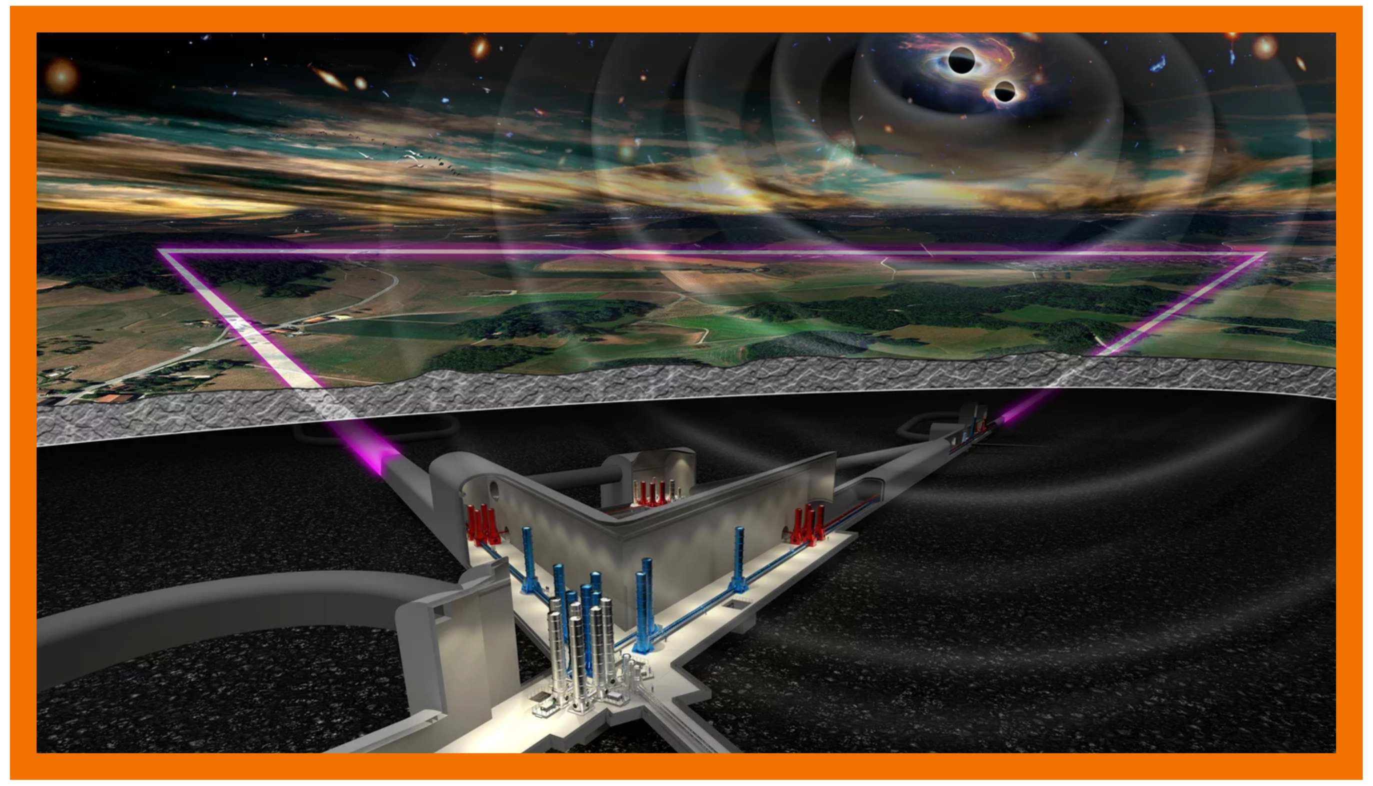
A golden age of data is coming- GW future detectors

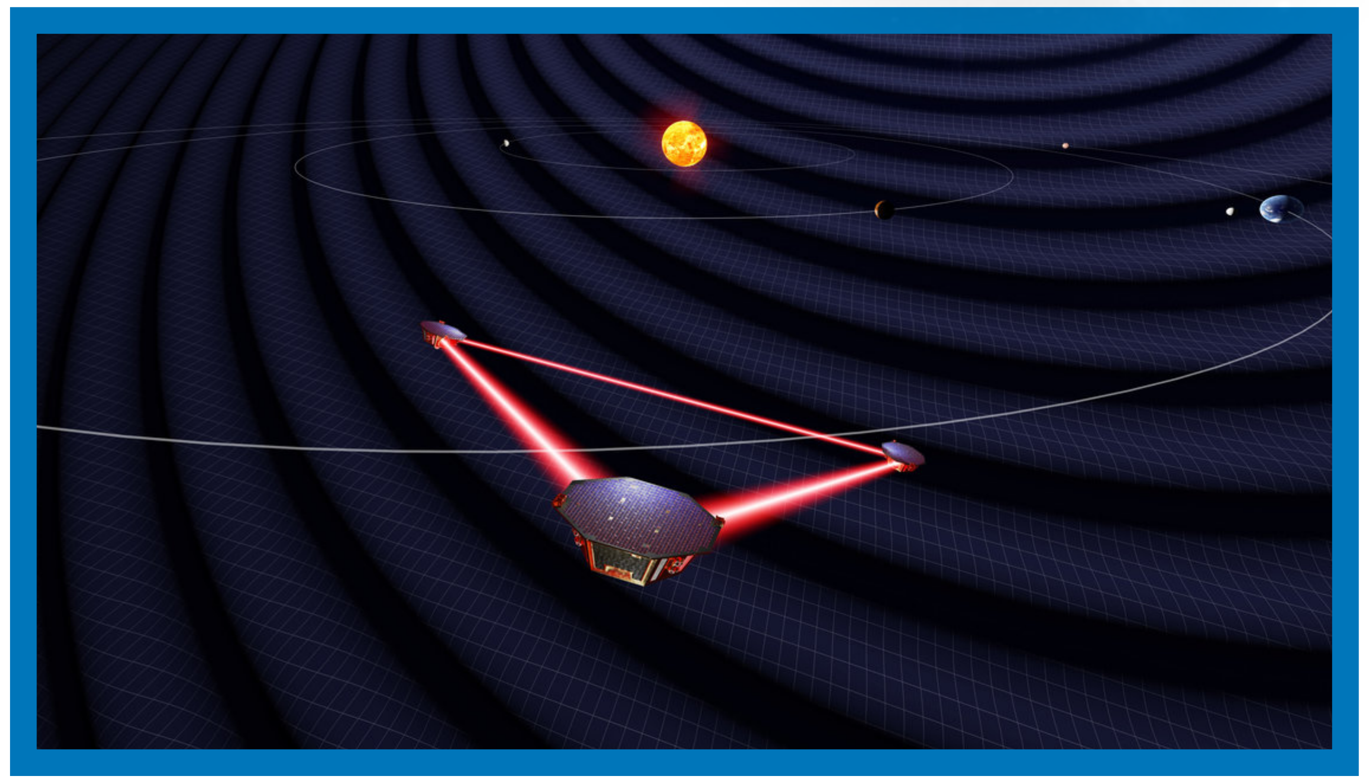
© Cosmic Explorer











The challenge: A sparse and noisy inverse problem

For current observations:

- Sparse coverage of observables,
- Uncertainties and degeneracies.

For future observations:

- Big amount of data,
- Smaller uncertainties, fewer degeneracies.

Likelihood-based or simulation-based?

Bayes' theorem $p(\theta \,|\, d) = \dfrac{p(d \,|\, \theta) p(\theta)}{p(d)}$

Likelihood-based Inference

Evaluate $p(d \,|\, \theta)$

- MCMC
- VI

Requires a likelihood

Slow inference

Simulation-based Inference

Sample $d \sim p(d \,|\, \theta)$

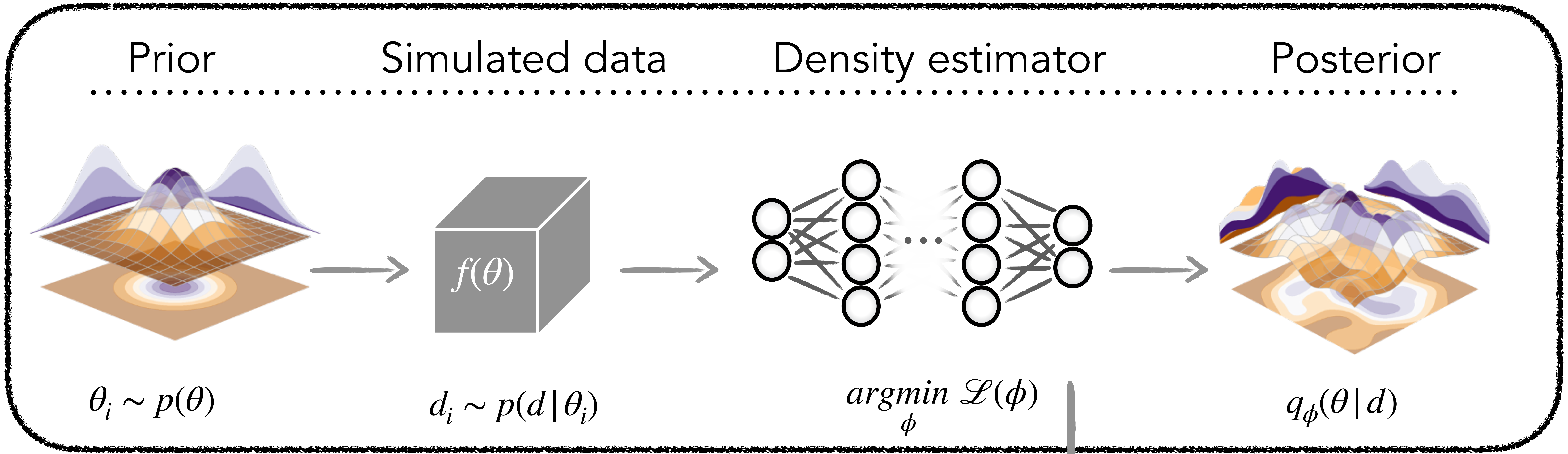
- Classical
  - ABC
- Neural
  - NPE
  - NLE

Doesn't require a likelihood

Slow training

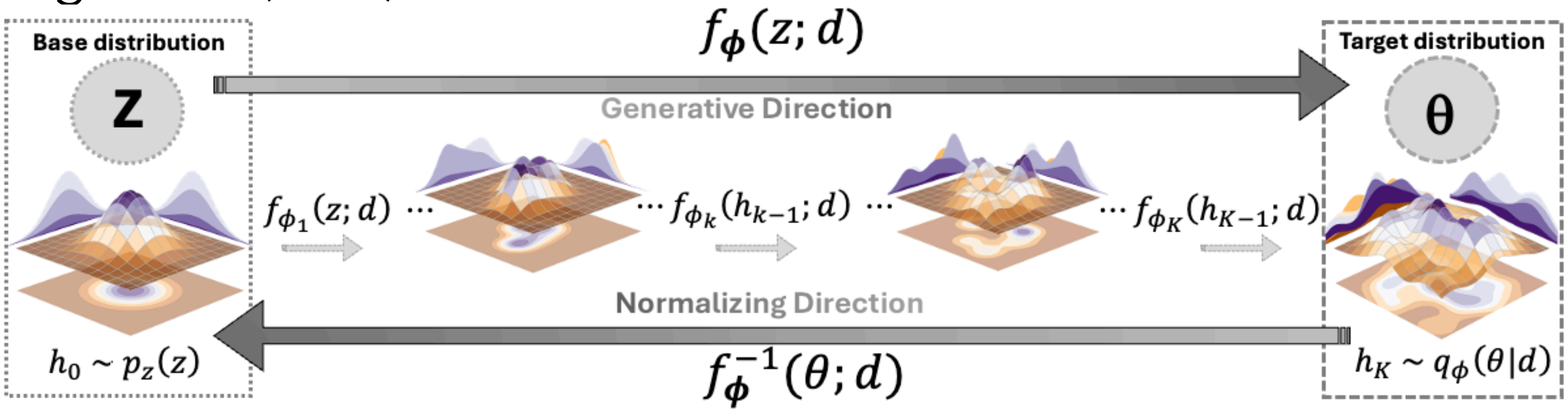
Fast Inference

Neural posterior estimation

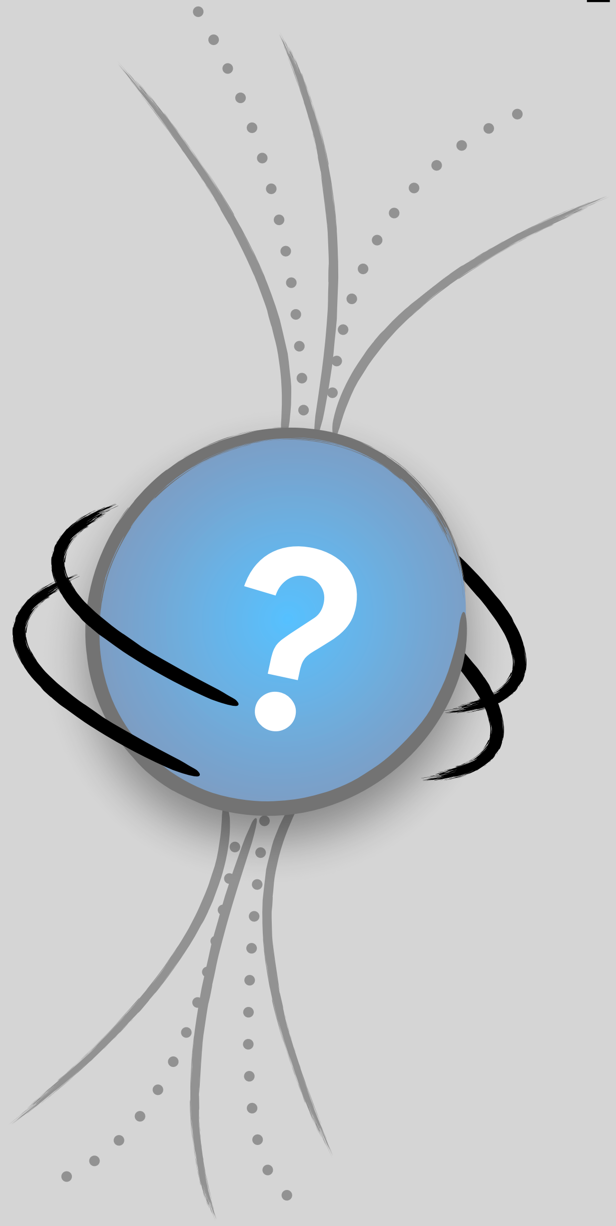


Conditional Normalizing Flows (CNF)

- ☑ Invertible
- ☑ Flexible
- ☑ Bijective



How effectively can Neural Posterior Estimation infer the Neutron Star Equation of State



Quantities we aim to predict : $p(EoS\,|\,O)$

Two model-agnostic EoS parameterizations :

Piecewise Polytropics (PT) PRD **111**,023035 (2025)

Gaussian Processes (GP) *Nat Commun* **14**, 8451 (2023)

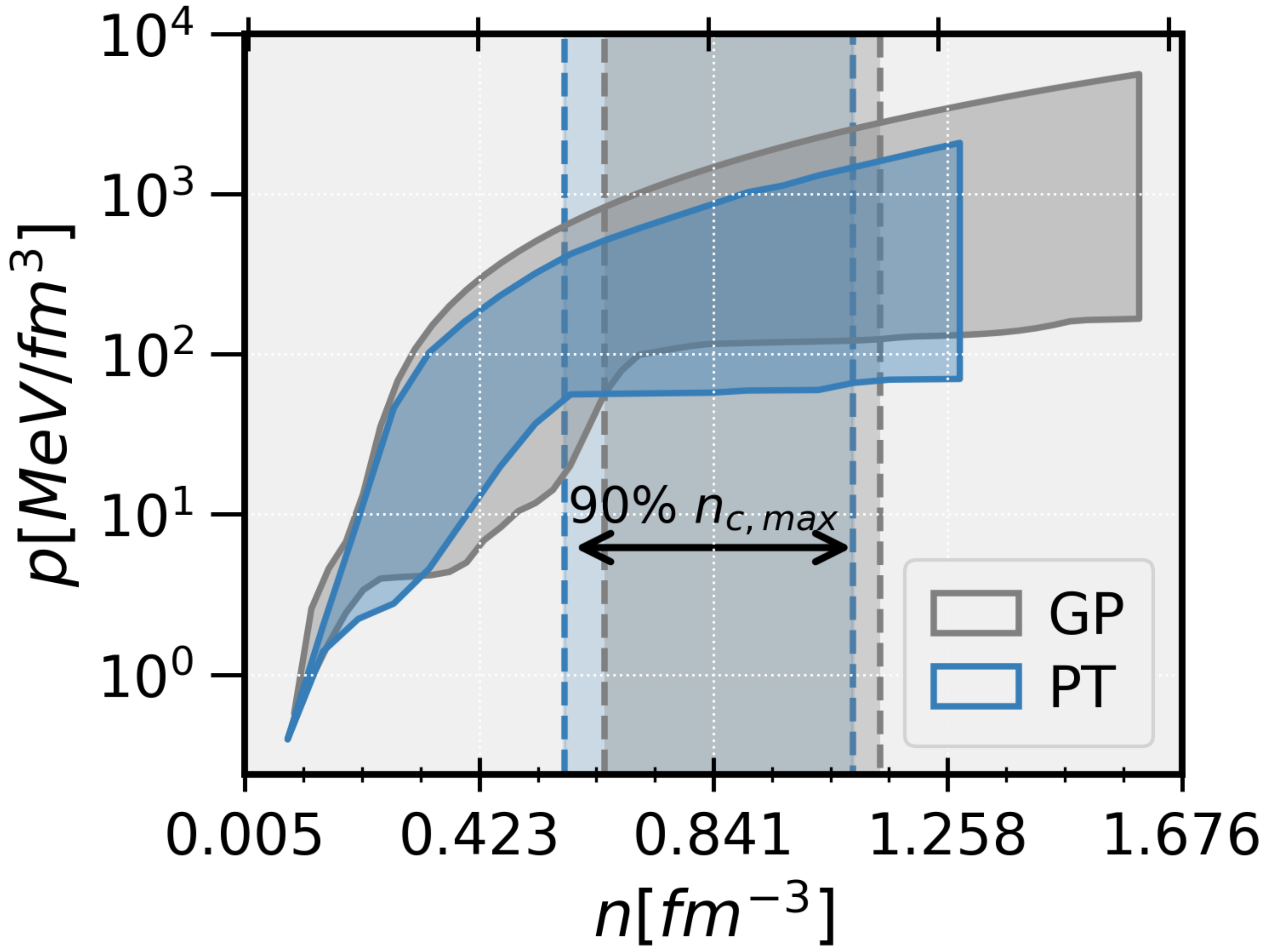
The predicted physical quantities :

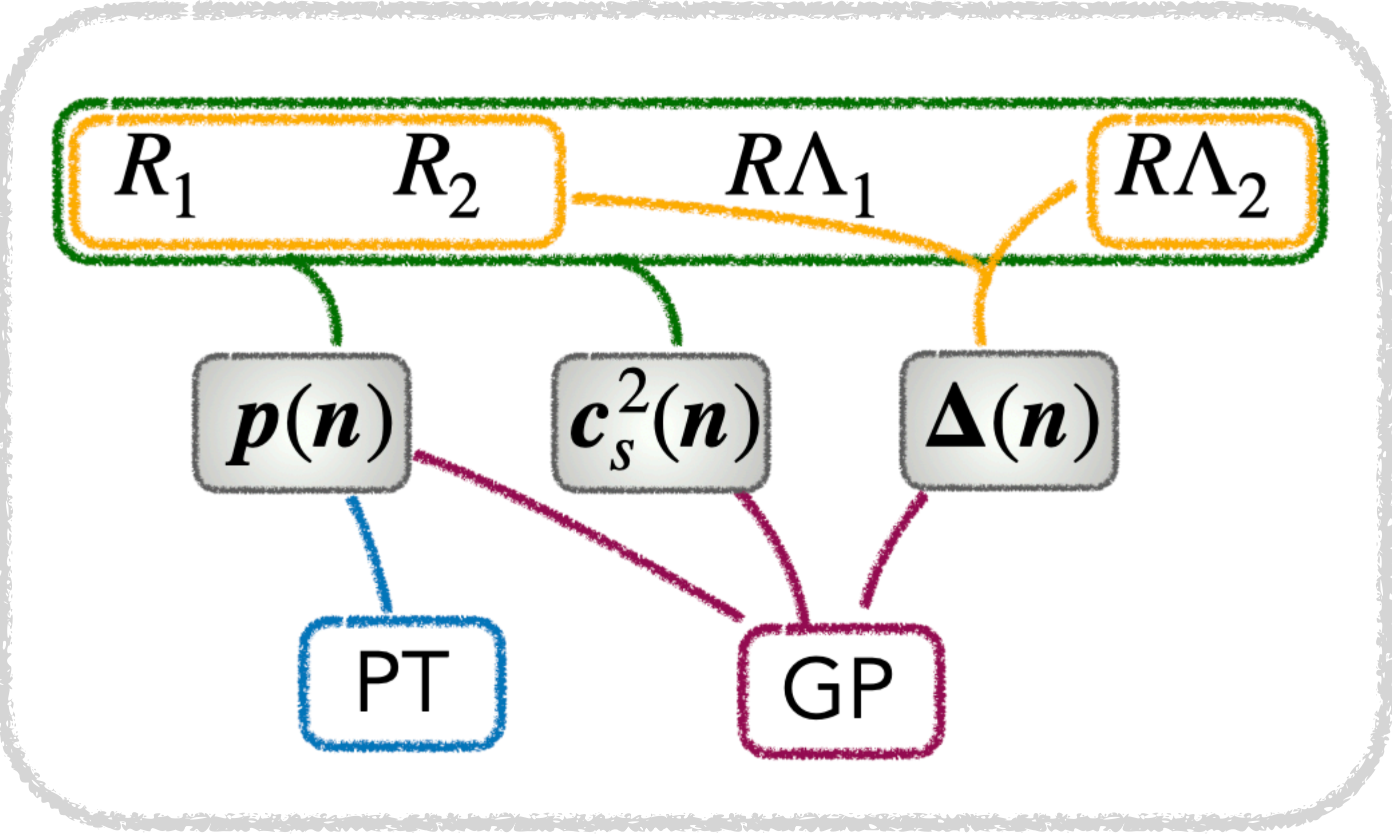
$$\begin{cases} \boldsymbol{p(n)} = [p(n_1), \cdots, p(n_{20})], \\ \boldsymbol{c_s^2(n)} = [c_s^2(n_1), \cdots, c_s^2(n_{20})], \\ \boldsymbol{\Delta(n)} = [\Delta(n_1), \cdots, \Delta(n_{20})], \end{cases}$$

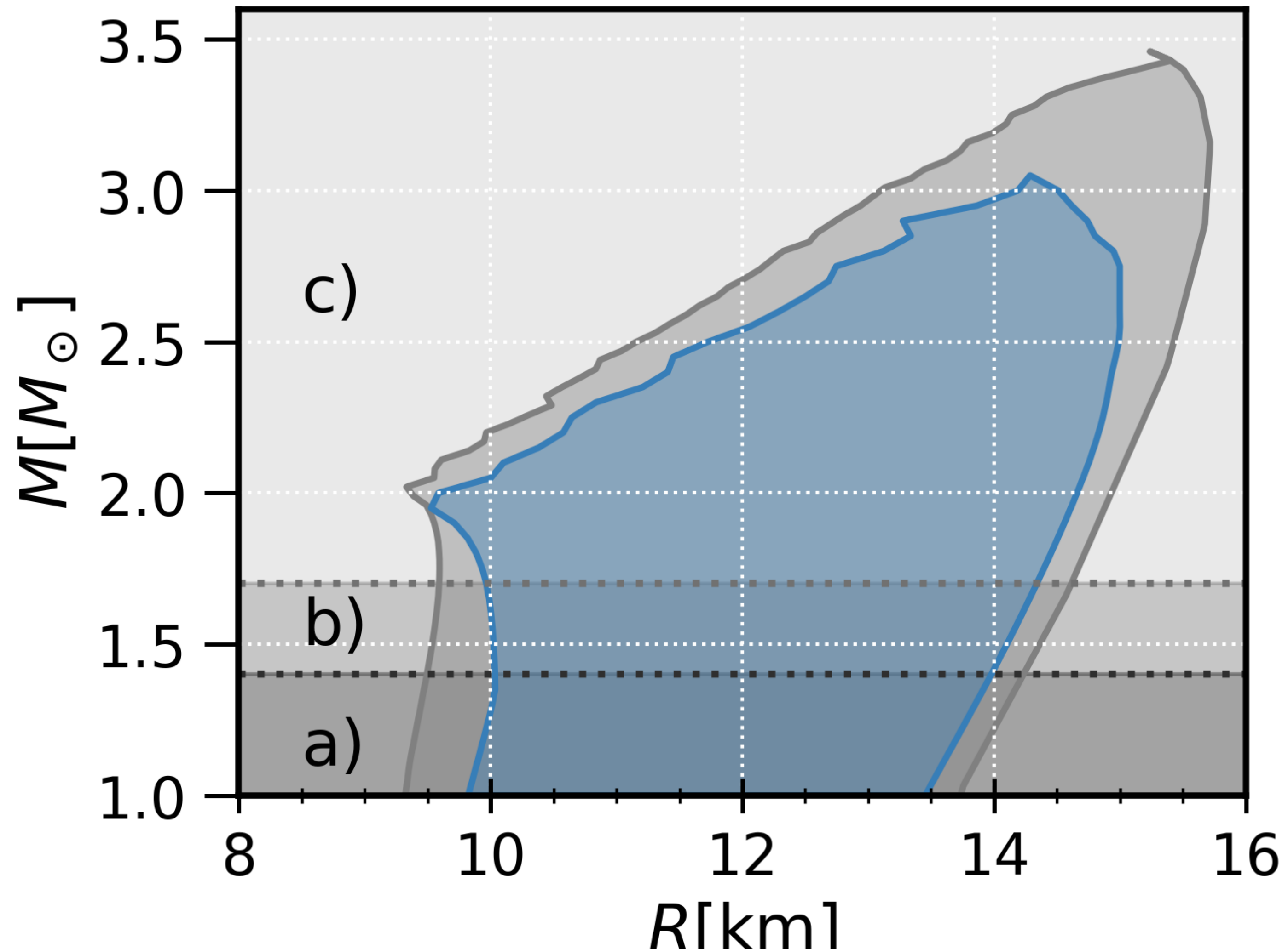
The conditioned quantities :

$$\begin{cases} R_x = [M_1, \cdots, M_{15}, R_1, \cdots, R_{15}], x \in [1,2] \\ R\Lambda_x = [M_1, \cdots, M_{15}, R_1, \cdots, R_{15}, M_1^*, \cdots, M_{15}^*, \Lambda_1, \cdots, \Lambda_{15}] . \end{cases}$$

$x = 1$ without noise, $x = 2$ with gaussian noise.

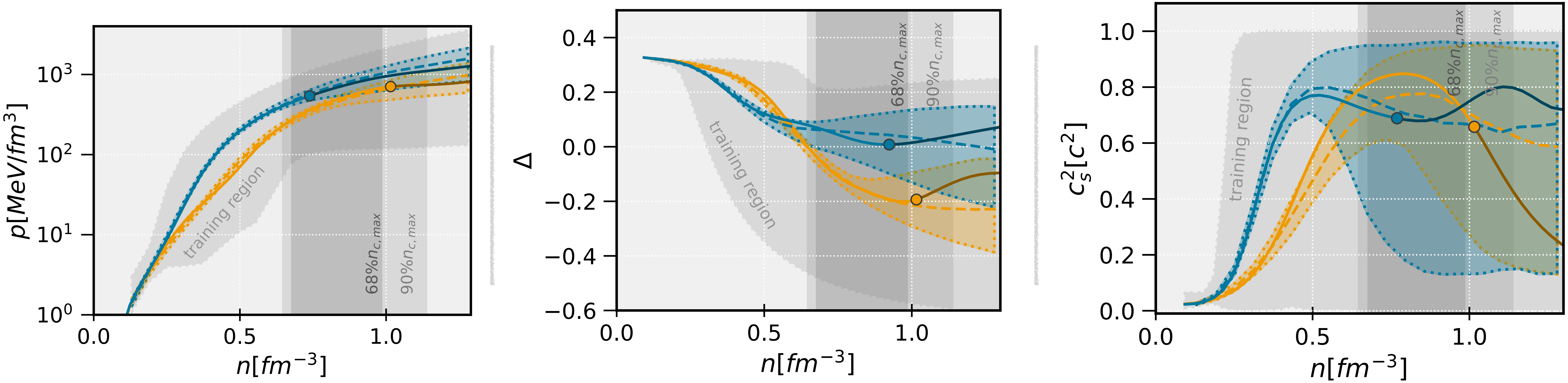






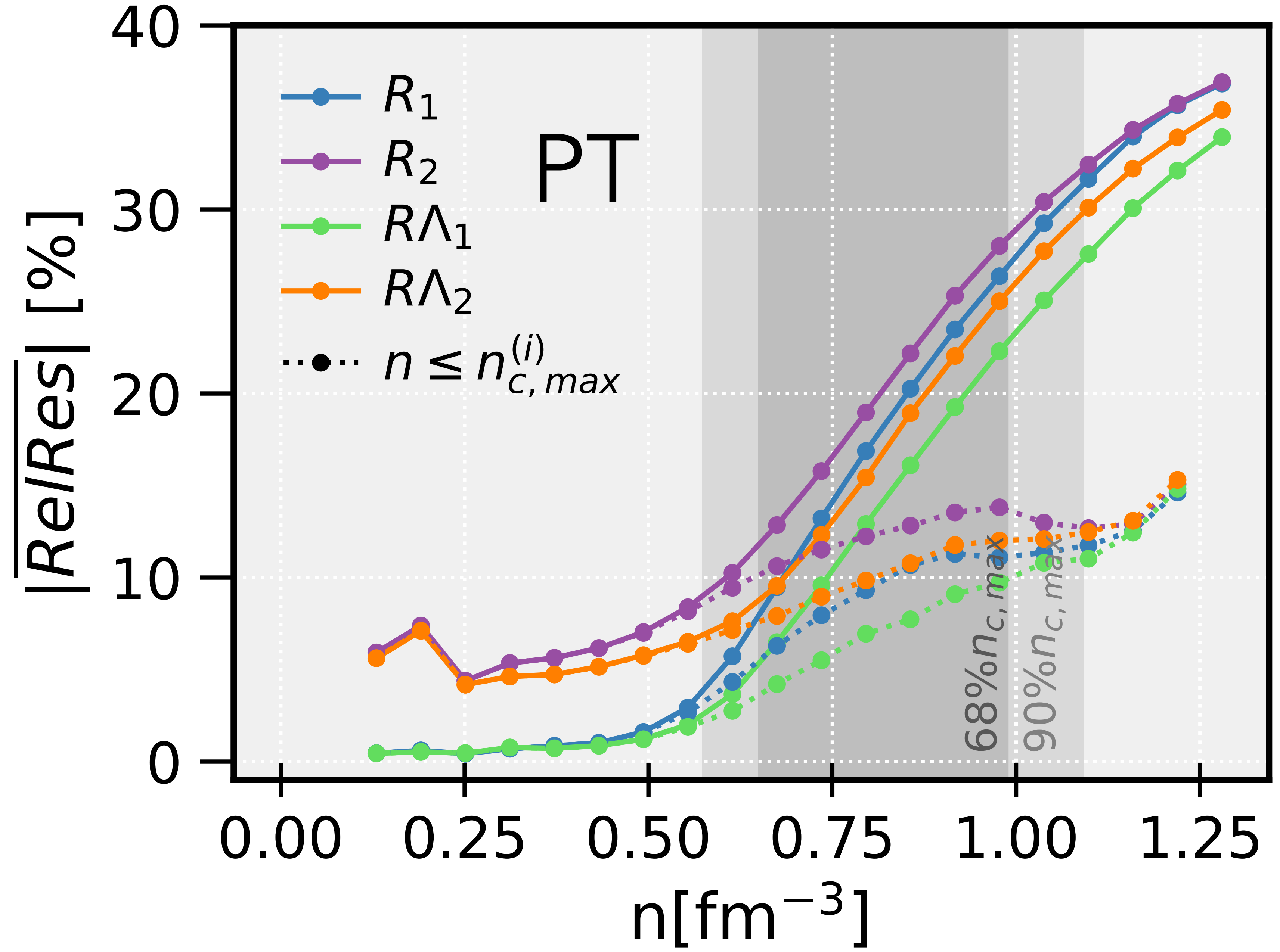
Key result: Accurate EoS reconstruction

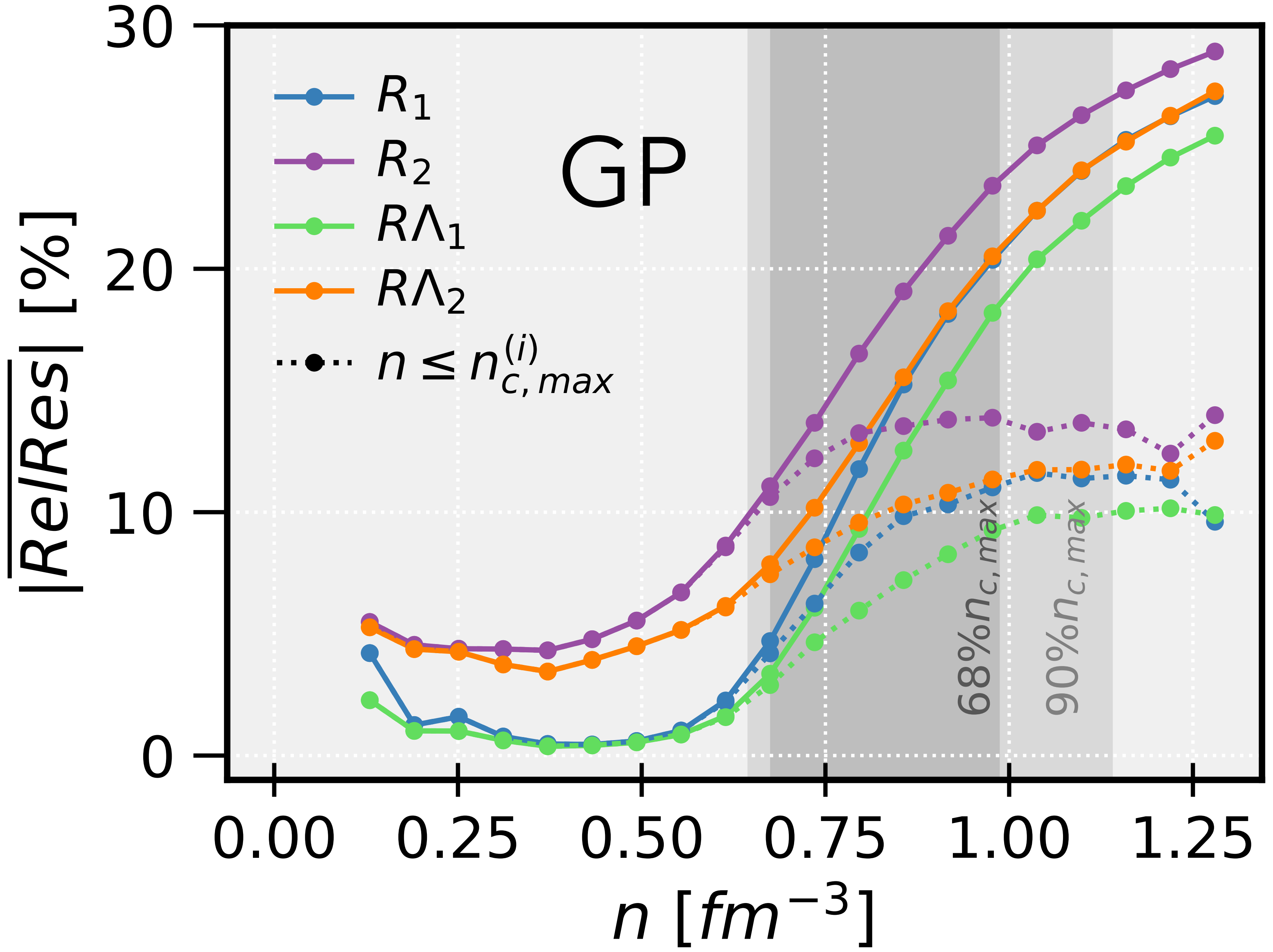
- Prediction for 2 samples of the test set $R\Lambda_2$ —, with a 90 % CI and median - - -,
- Increase in dispersion near maximum central density, represented by ●,
- The true EoS is always contained within the 90% credible interval.



Just for GP dataset

Relative error for pressure





$$\text{RelRes}^{(i)}(n) = \text{Med}_l \left[\frac{X_p^{(i,l)}(n) - X_T^{(i)}(n)}{X_T^{(i)}(n)} \right] \times 100$$

$$\begin{cases} |\overline{\text{RelRes}}|_{\leq n_{c,max}} = & \text{Med}_i \left| \text{RelRes}^{(i)}(n) \right| \quad \text{for} \quad n \leq n_{c,max}^{(i)} . \\ |\overline{\text{RelRes}}| = & \text{Med}_i \left| \text{RelRes}^{(i)}(n) \right| \quad . \end{cases}$$

l = Posterior sample, i = EoS at density n

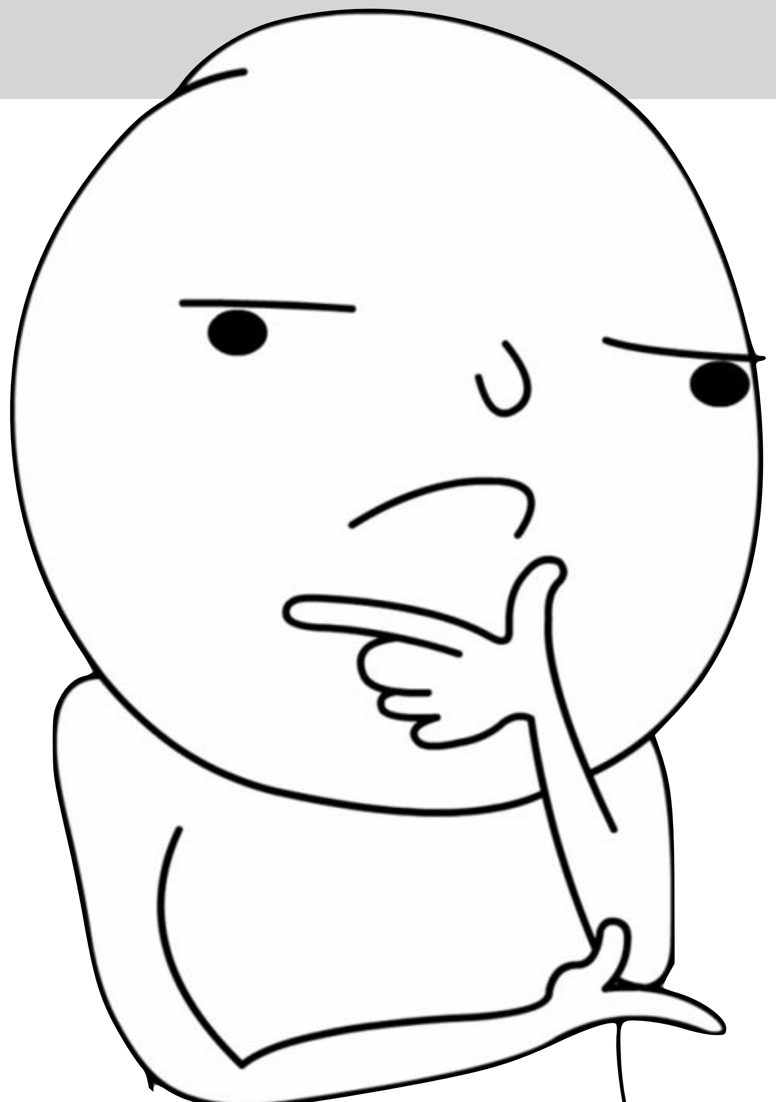
- Error increases with noise,
- Error decreases with tidal deformability,
- Error decreases when filtered for $n_{c,max}$.

Do we know the number of future observations?

No

Is each observation represented by a single point?

Also No

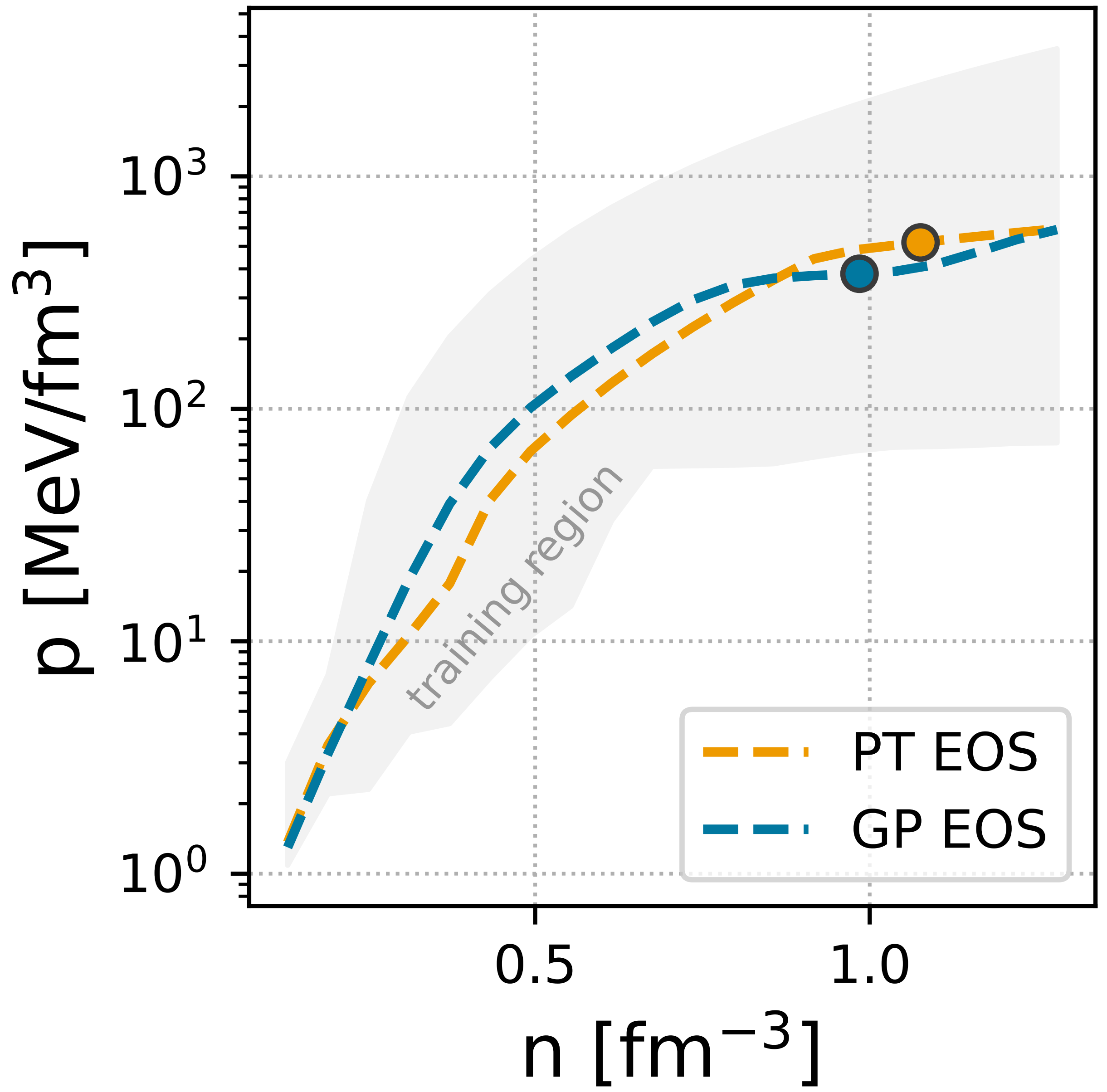


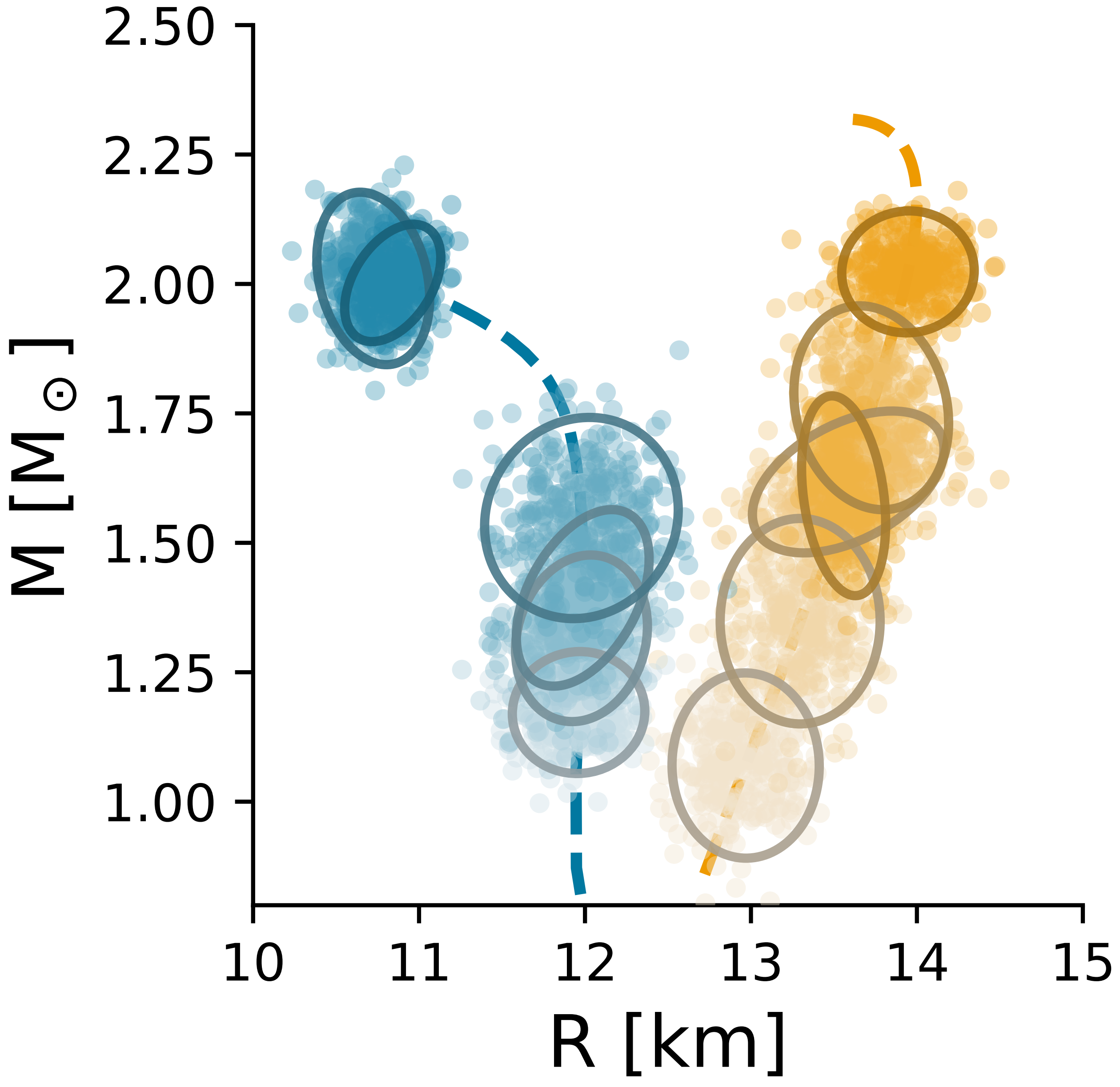
Generation of the mock dataset

- Number of observations?
- Mass distribution?
- Measurement uncertainties?

$$X_o = \left\{(M_{o,s}, R_{o,s})\right\}_{s=1}^{N_s}, X = \left\{X_o\right\}_{o=1}^{N_{\text{obs}}}.$$

$$N_{obs} = N_a + N_b + N_c \qquad N_a, N_b, N_c \geq 1$$





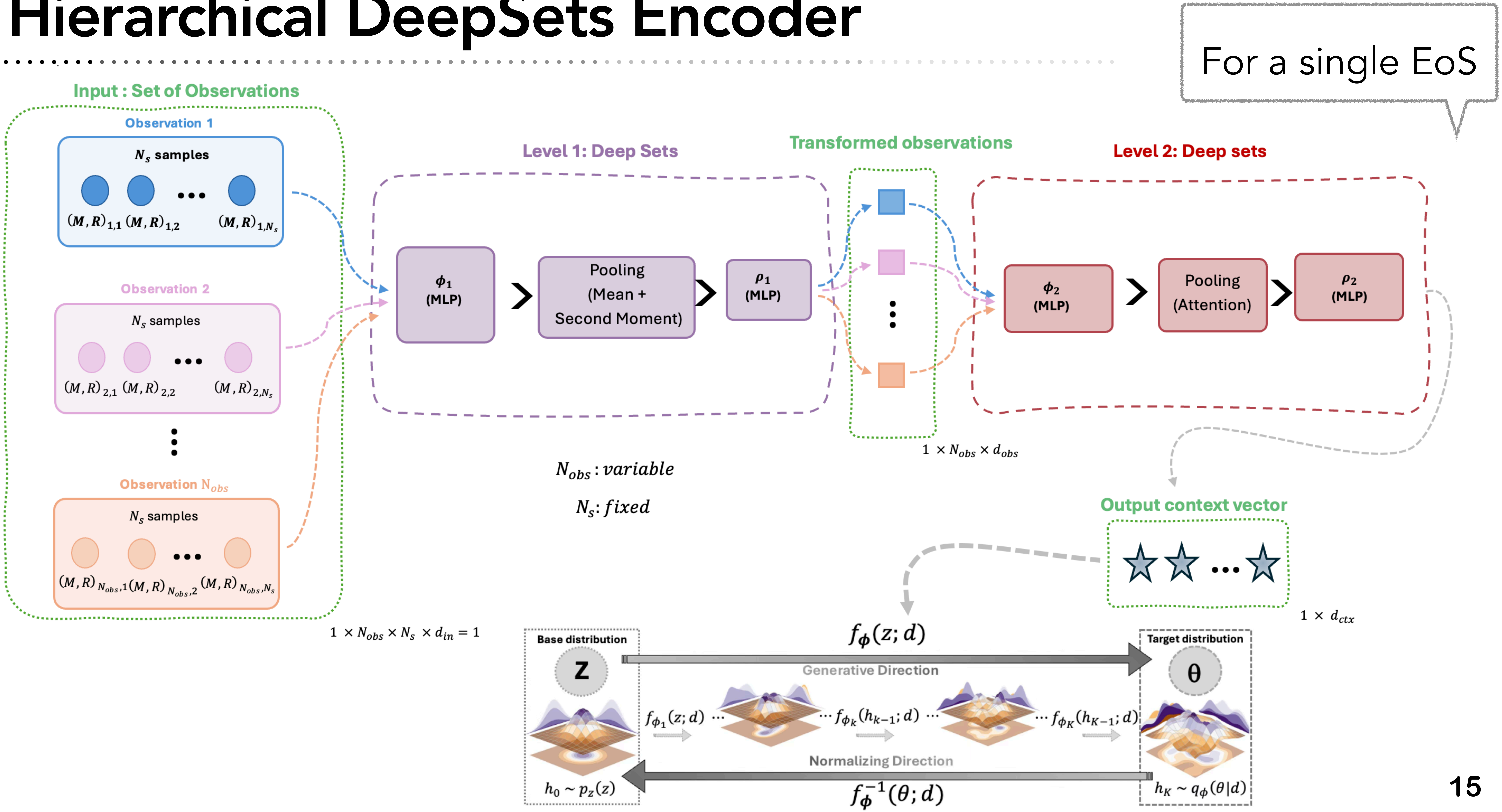
$$\begin{pmatrix} M_{o,s} \\ R_{o,s} \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} M_o \\ R_o \end{pmatrix}, \Sigma_o\right),$$

$$\Sigma_o = \begin{pmatrix} \sigma_{M,o}^2 & \rho_o\,\sigma_{M,o}\sigma_{R,o} \\ \rho_o\,\sigma_{M,o}\sigma_{R,o} & \sigma_{R,o}^2 \end{pmatrix}.$$

$$\sigma_M \sim \mathcal{U}(0.05, 0.10)\,M_\odot, \qquad \sigma_R \sim \mathcal{U}(0.10, 0.30)\,\text{km}$$

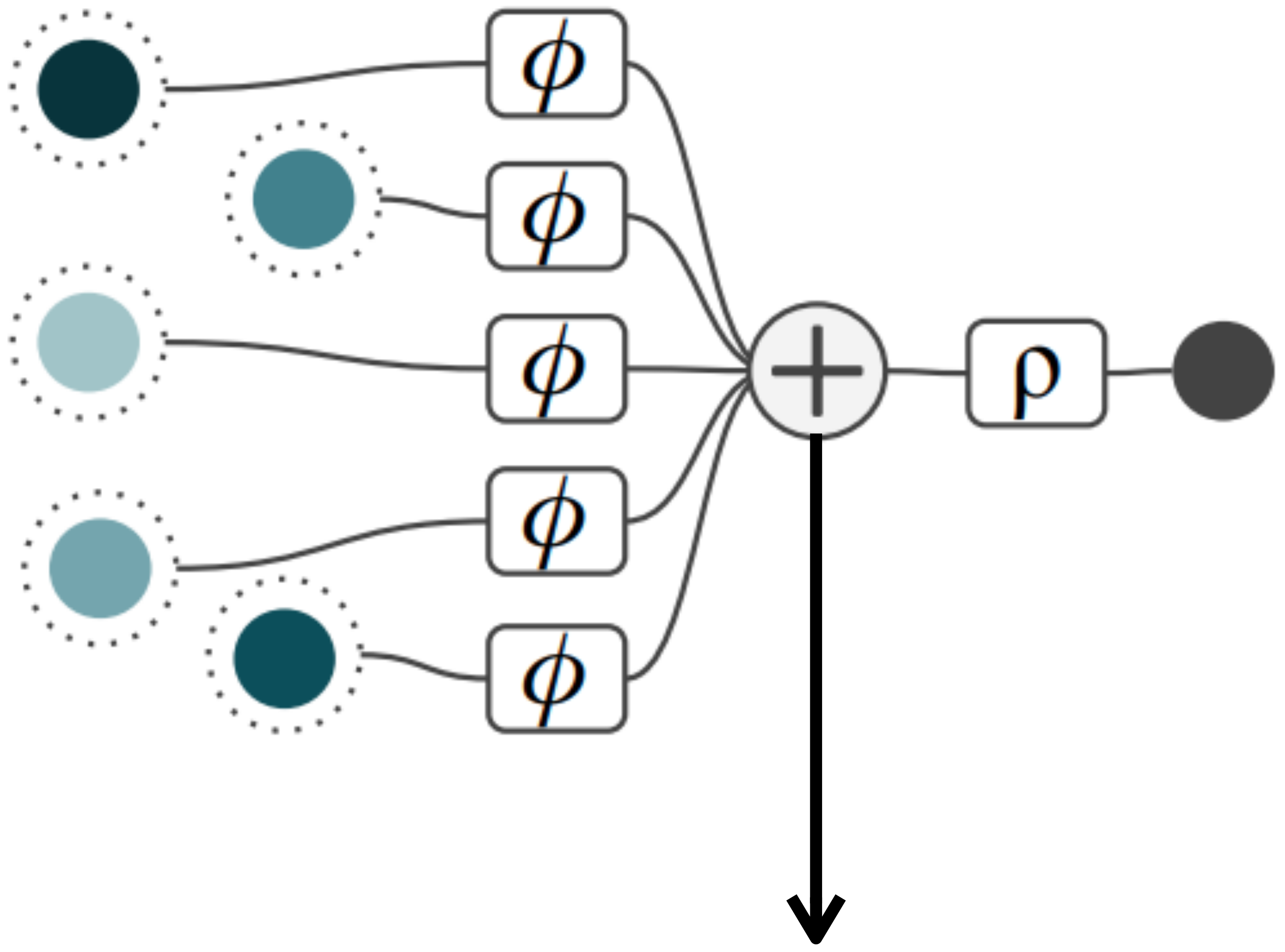
$$\rho_o = \mathcal{U}[-0.5, 0.5]$$

Hierarchical DeepSets Encoder



DeepSets

$$h_o = \rho_1\left(\text{Pool}\left(\{\phi_1(x_{o,s})\}_{s=1}^{N_s}\right)\right)$$



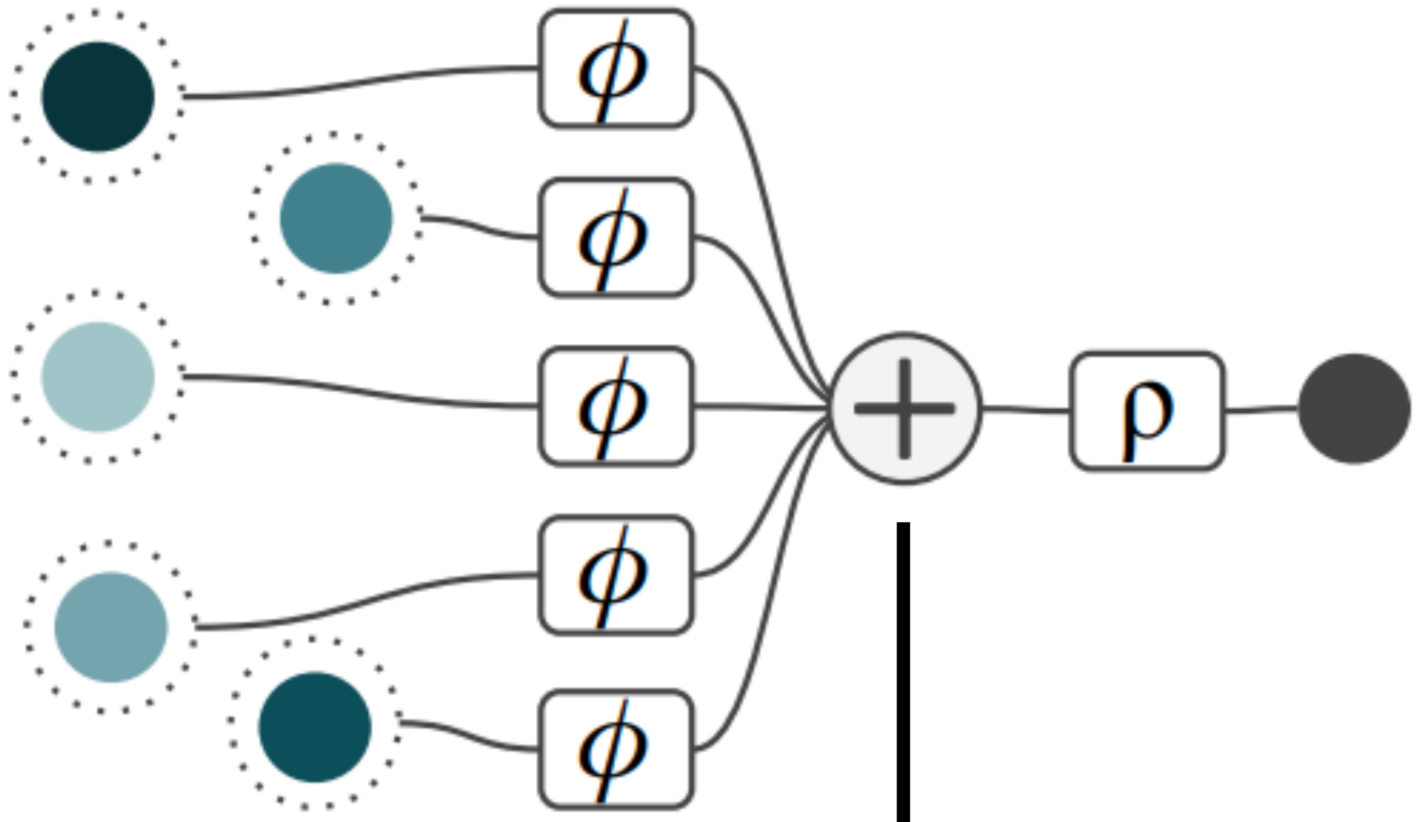
- Handles a variable number of observations
- Permutation invariant
- Produces a fixed-dimensional representation.

$$\text{Pool} = (\mu, \nu),$$

$$\mu = \frac{1}{N_s}\sum_{s=1}^{N_s} z_{o,s}, \qquad \nu = \frac{1}{N_s}\sum_{s=1}^{N_s} z_{o,s}^2, \, z_{o,s} = \phi_1(x_{o,s})\,.$$

DeepSets + Attention

$$h_o = \rho_1\left(\mathrm{Pool}\left(\{\phi_1(x_{o,s})\}_{s=1}^{N_s}\right)\right)$$



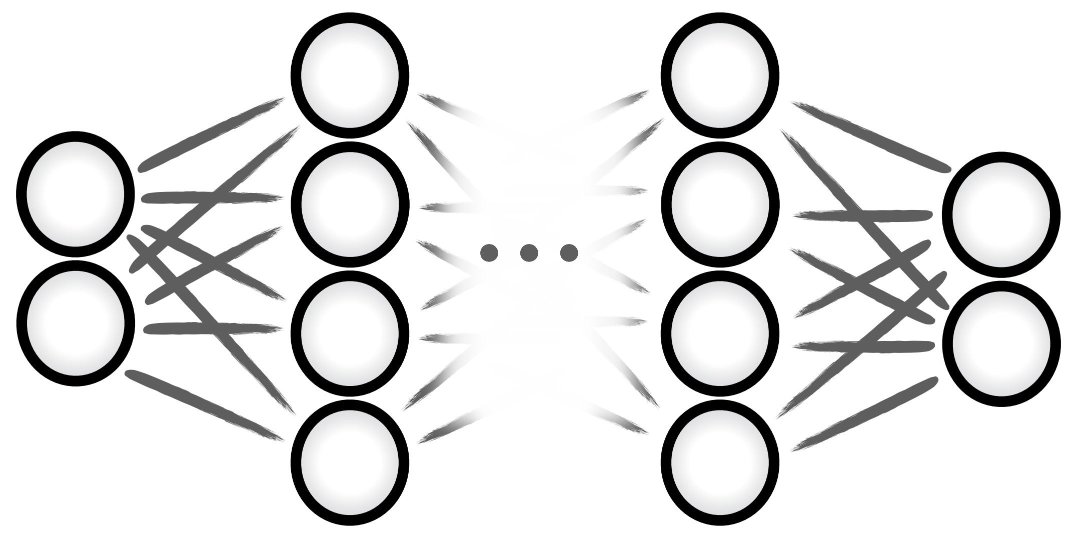
- Learnable weighted pooling
- Attention mechanism

Pool =

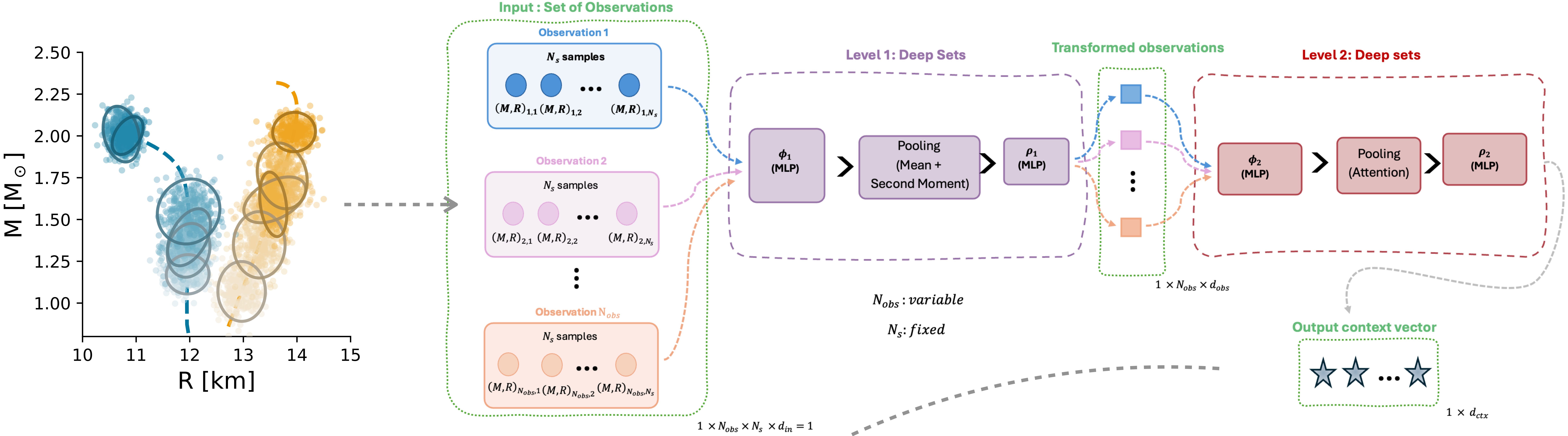
$$h = \rho_2\left(\sum_{o=1}^{N_{\mathrm{obs}}} \alpha_o \phi_2(h_o)\right), \qquad \alpha_o = \frac{\exp(g(\phi_2(h_o)))}{\sum_{o'} \exp(g(\phi_2(h_{o'})))}.$$

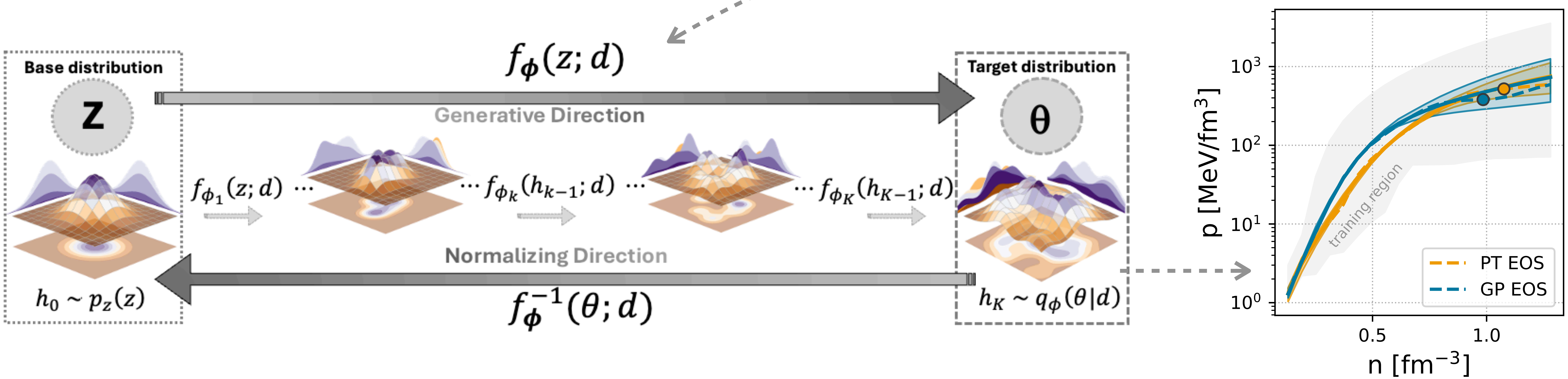
$$\sum_o \alpha_o = 1.$$

$g \rightarrow$ Score function

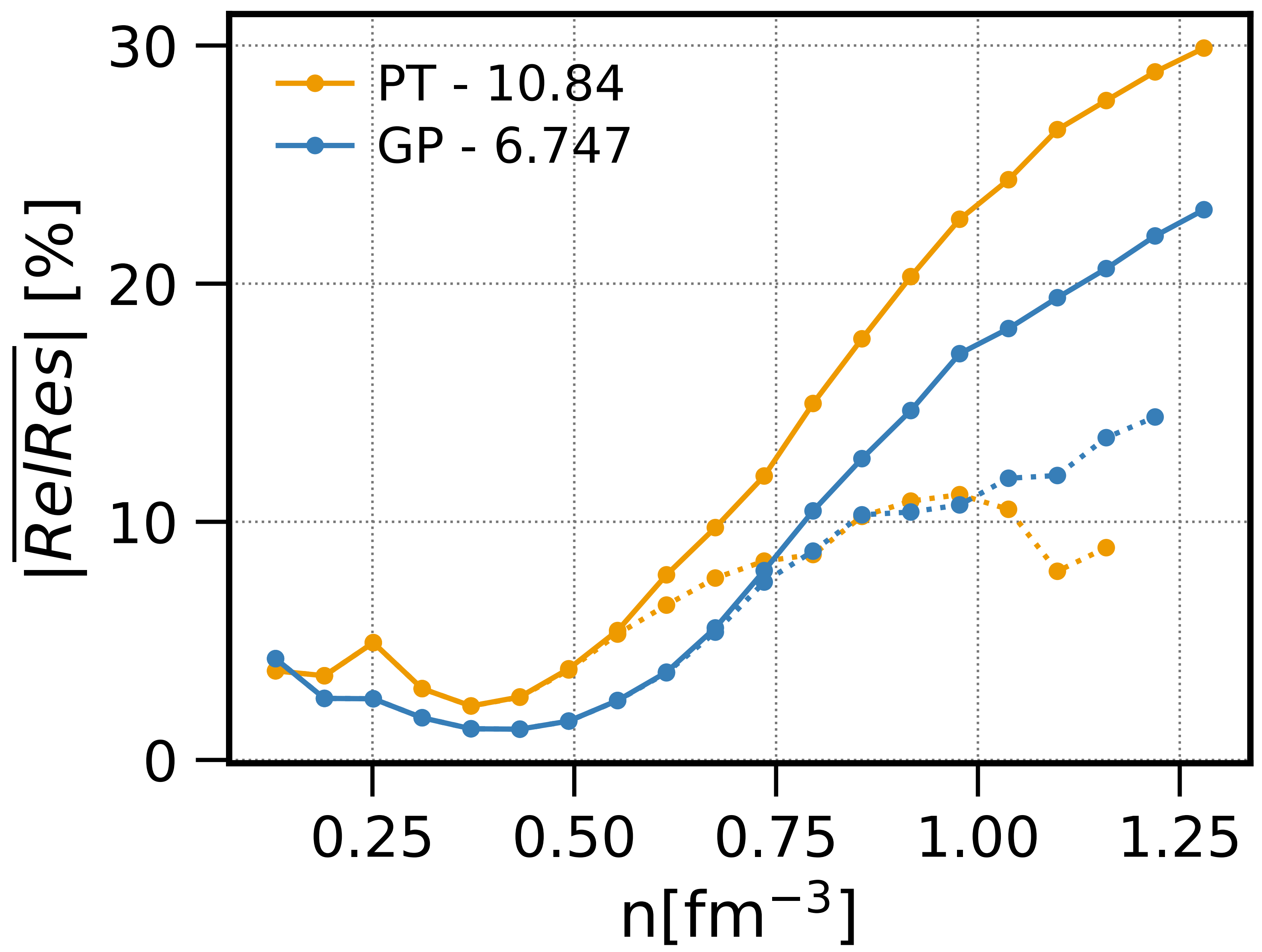


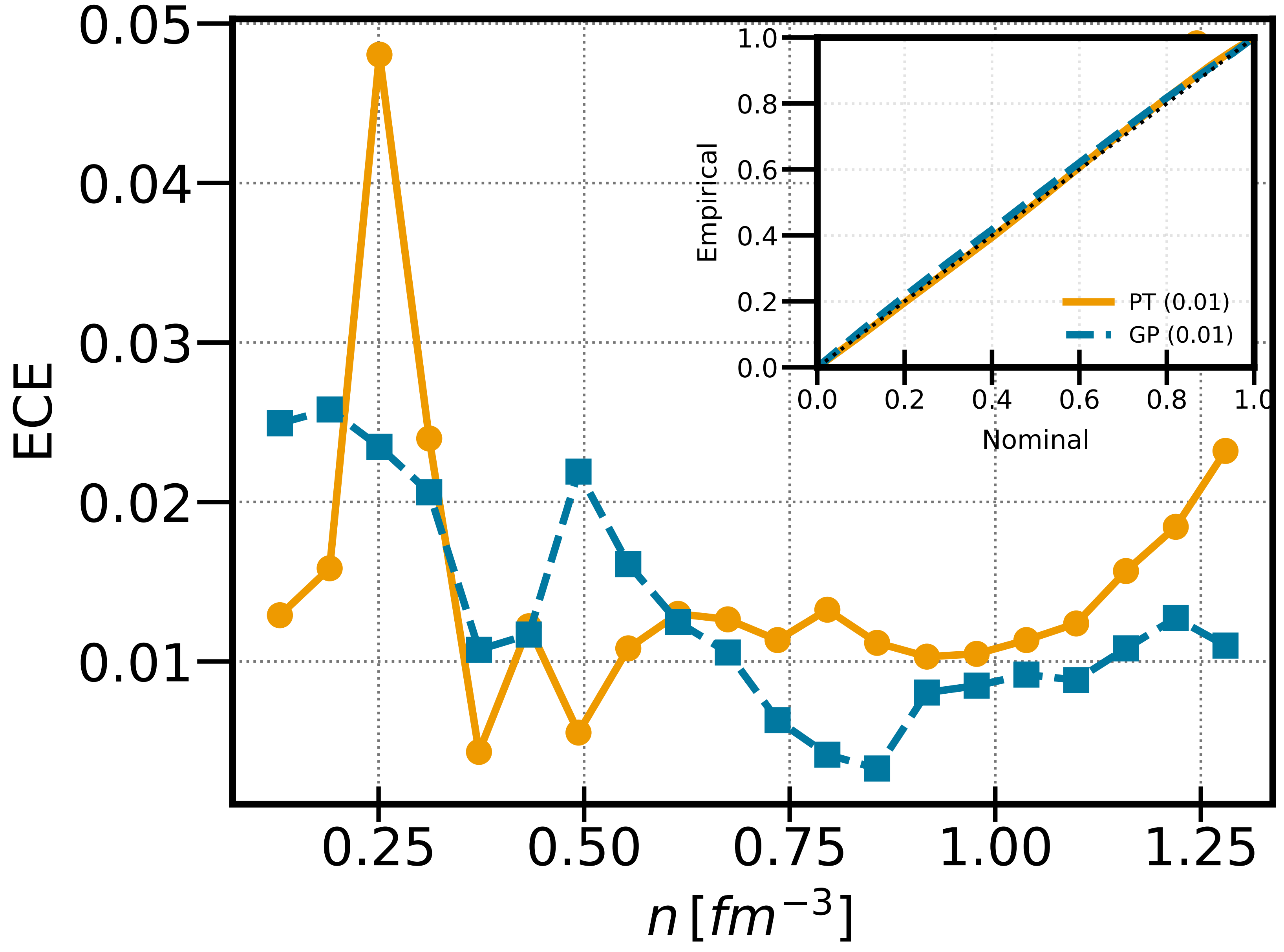
Hierarchical DeepSets Encoder





Preliminary results - Relative error

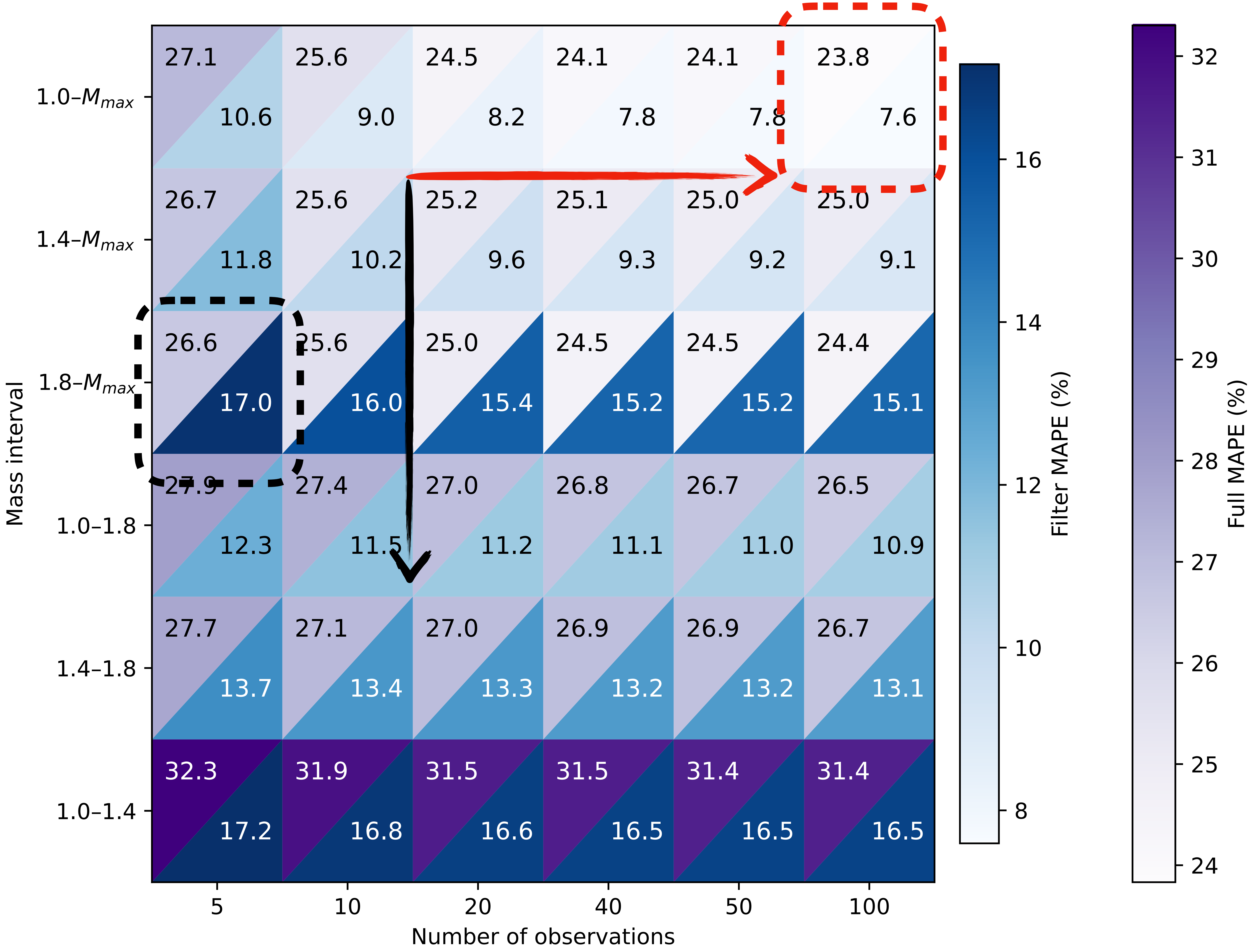




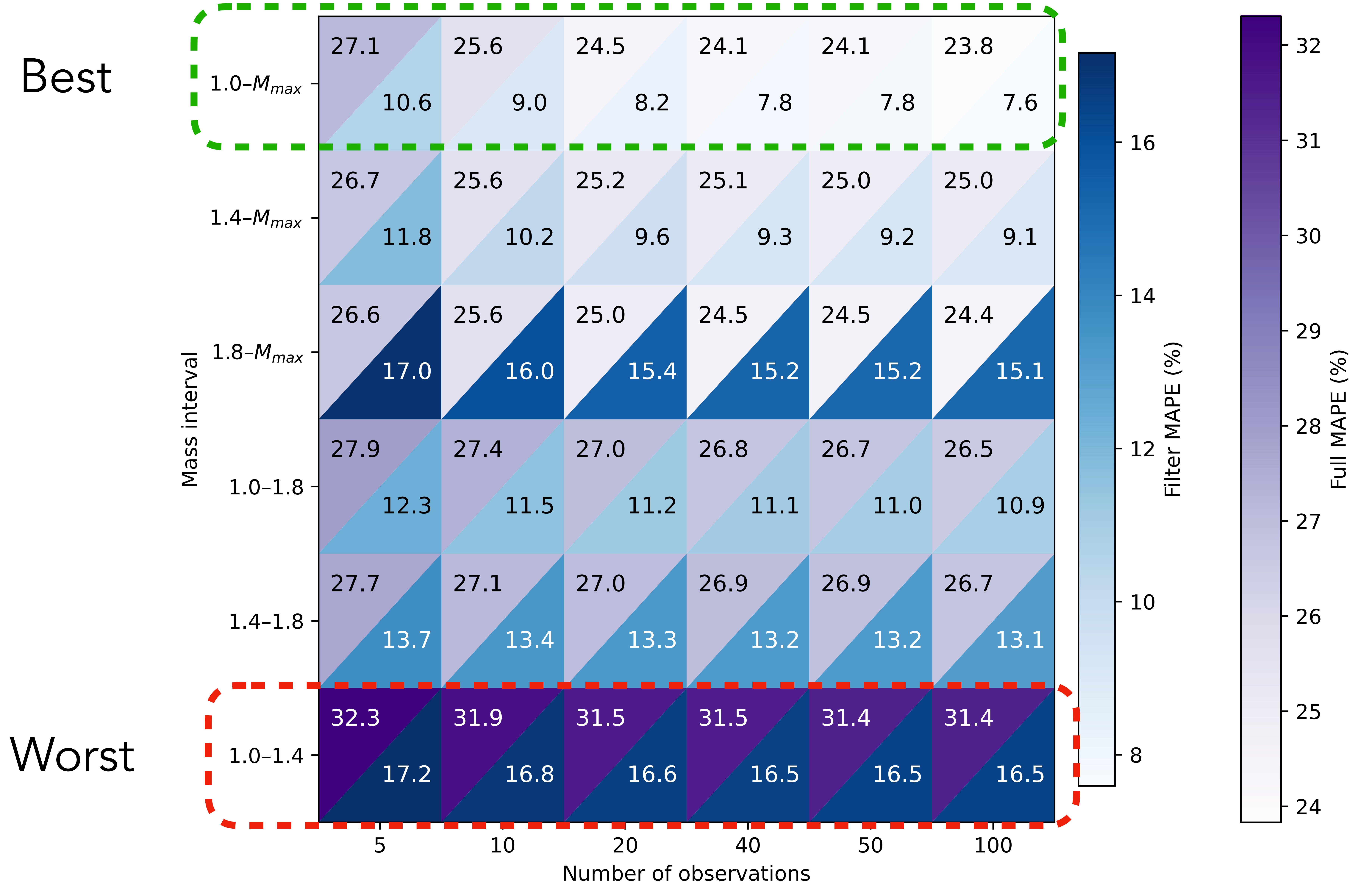
- Median reconstruction error below 12%
- Calibration error small = good coverage
- Reconstruction error reduced compared with the previous model

Different mass intervals vs Number of observations

- The MAPE is much smaller for filter case,
- For 1000 EoSs in test set,



Different mass intervals vs Number of observations



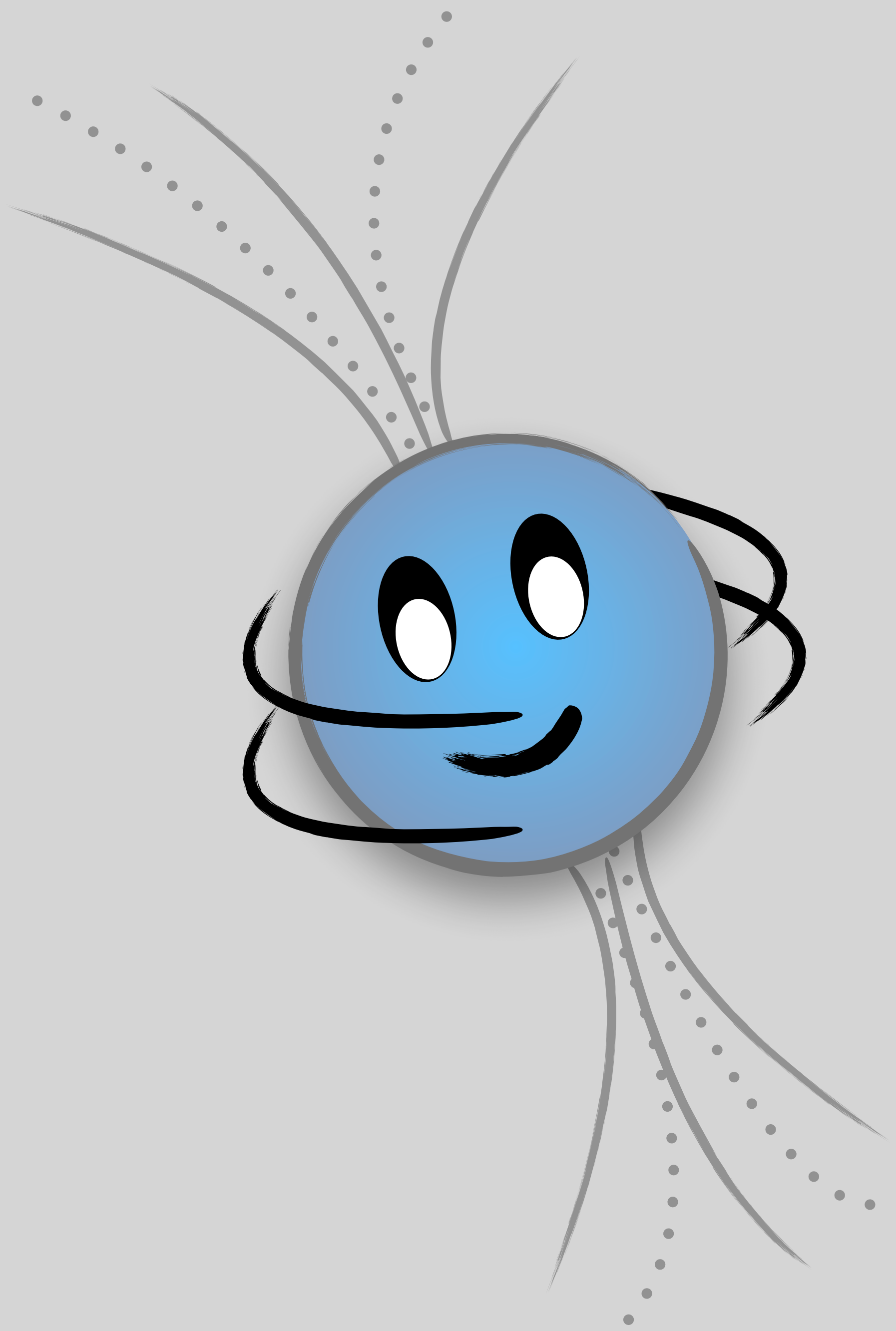
High-mass observations are the most informative.

Main take home message

- Hierarchical DeepSets enable inference from a variable number of observations.
- NPE accurately reconstructs the neutron star EoS.
- The posterior is well calibrated and provides reliable uncertainties.
- The new context-vector architecture improves reconstruction performance.
- ✦ The outlook: The framework is well suited for future multimessenger observations.

PRD **112,**083044

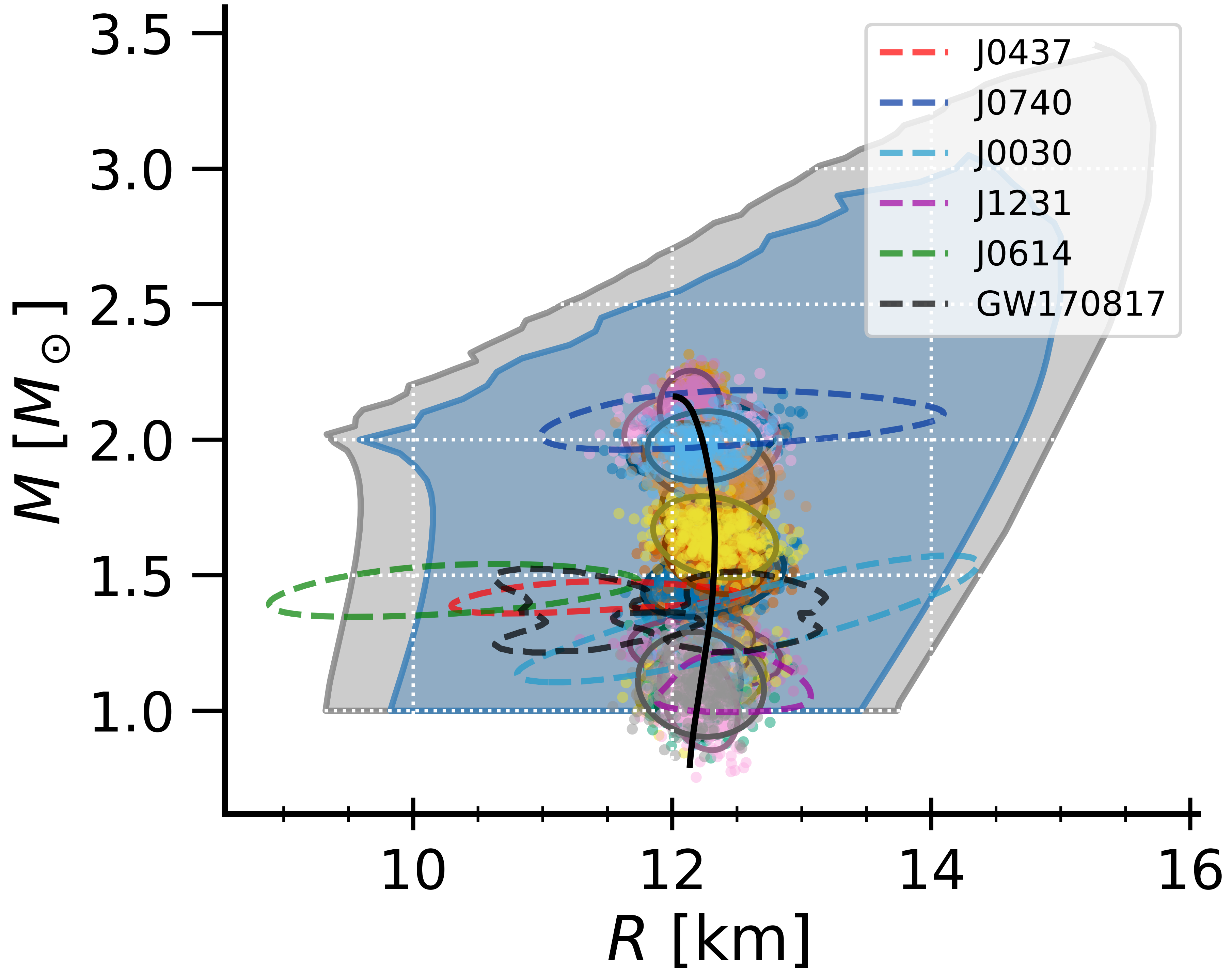
Thank you for your attention



More information available at:

val.mar.dinis@gmail.com

Backup



$$\hat{C}(c) = \frac{1}{N} \sum_{i=1}^{N} \mathbf{1}\left(\theta_i^{\text{true}} \in \text{CI}(c)\right),$$

$$\text{ECE} = \frac{1}{K} \sum_{k=1}^{K} \left| \hat{C}(c_k) - c_k \right| .$$

Dataset - Quantities we condition

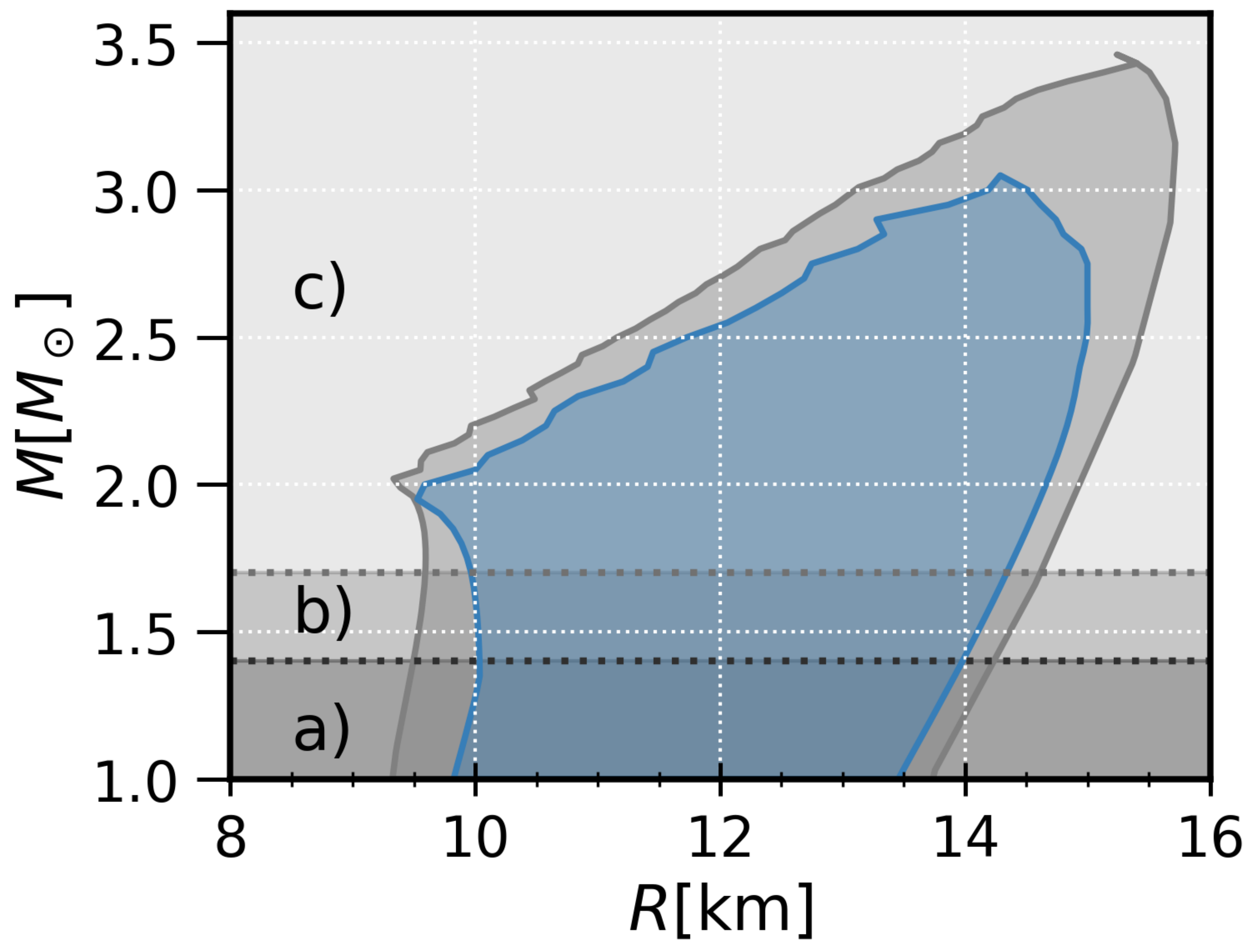
$$(M_o, R_o)\ o = 1, \cdots, N_{obs}\ N_{obs} = N_a + N_b + N_c$$

$$N_a, N_b, N_c \geq 1$$

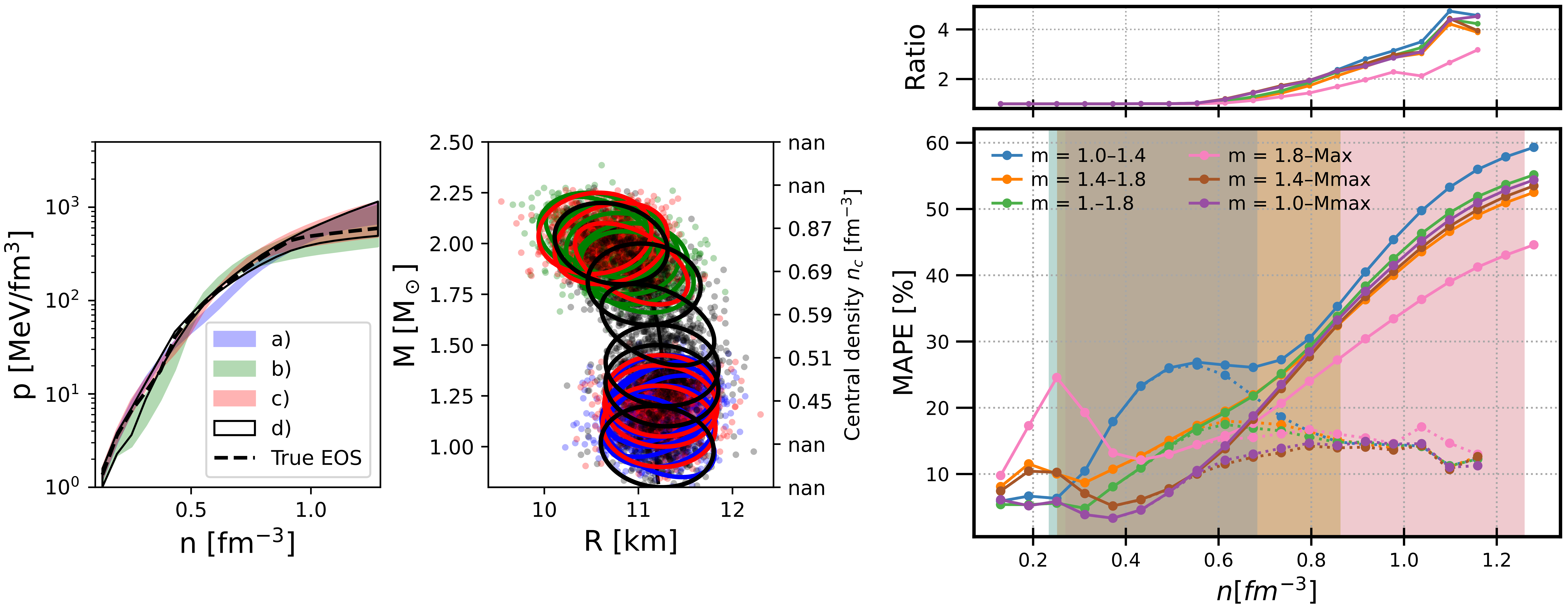
$$M_a \in [1.0, 1.4] M_\odot$$

$$M_b \in [1.4, 1.8] M_\odot$$

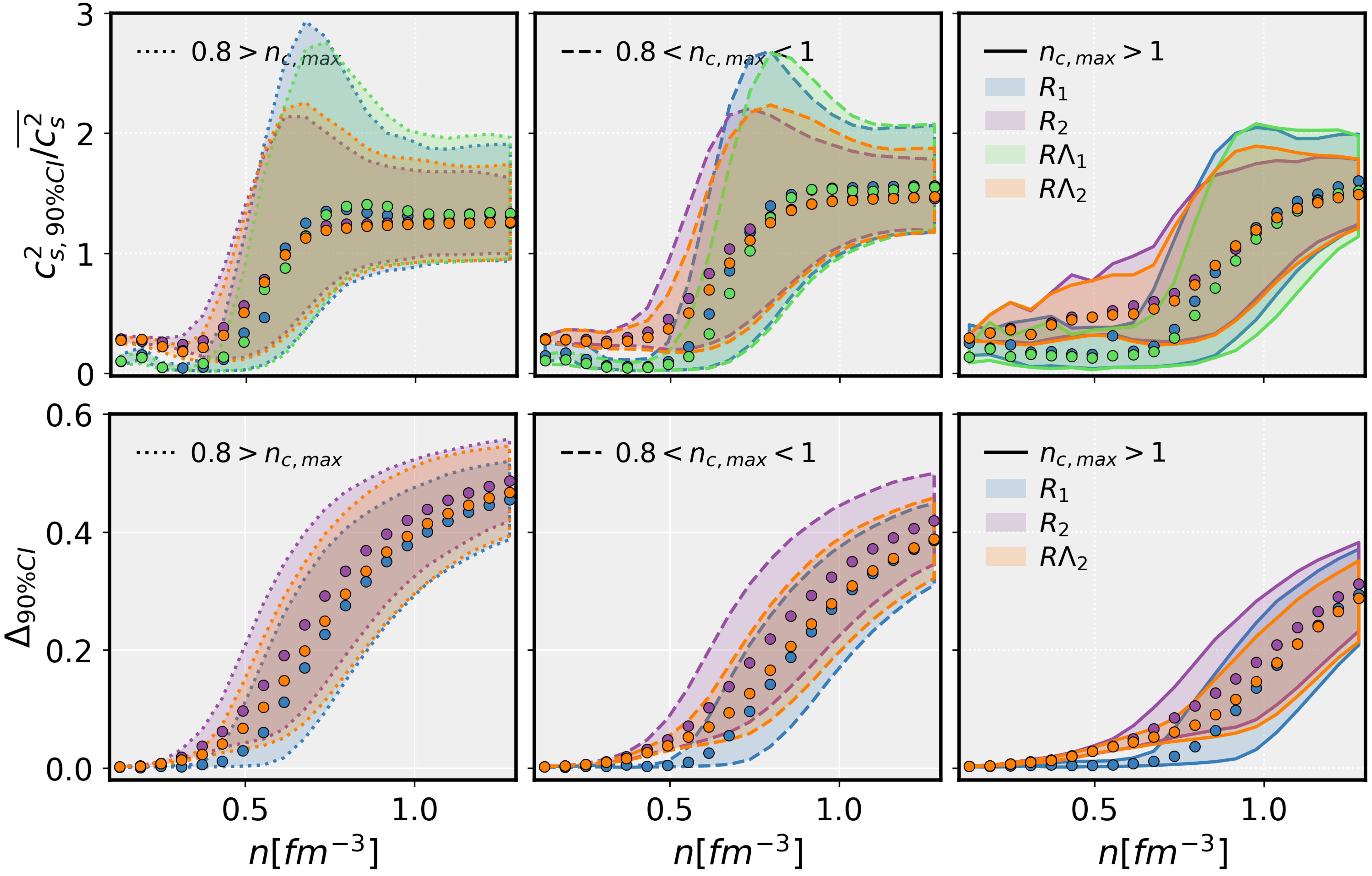
$$M_c \in [1.8, M_{max}(EoS_{ID})] M_\odot$$



Preliminary results



Effect of maximum central density

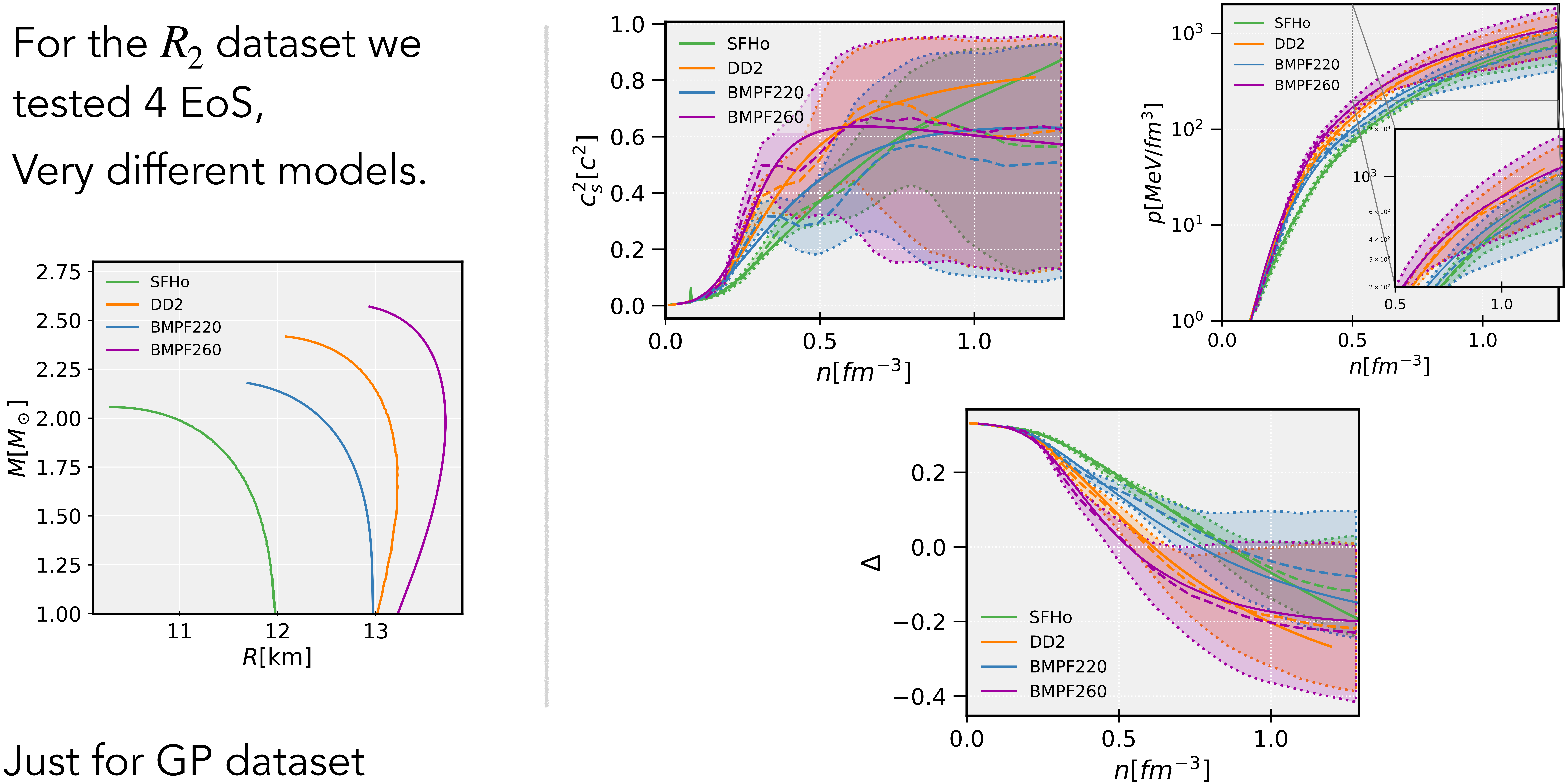


- Bands are the 90% CI,
- Dots are the mean.

- Predicted dispersion (CI) decrease with the increase of $n_{c,max}$.

Another dataset inference test

- For the R_2 dataset we tested 4 EoS,
- Very different models.



Just for GP dataset

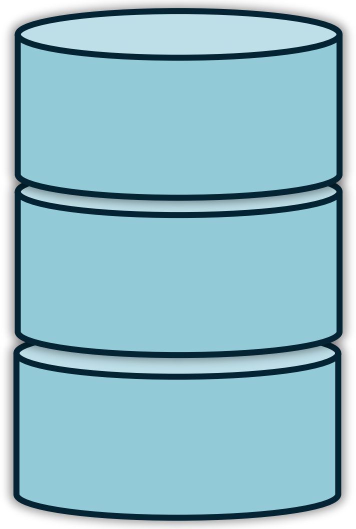
Deep Learning pipeline

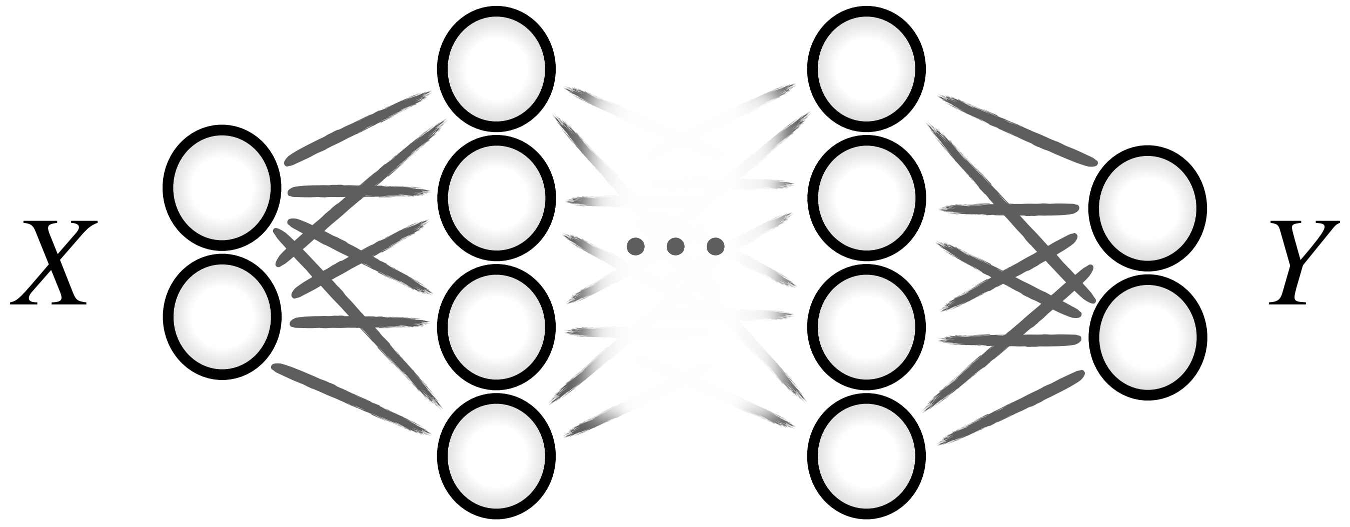
Deep Learning ⊂ Machine Learning ⊂ AI

Benefits :

- Handles complexity,
- Extremely Fast,
- Quantifies Uncertainty.

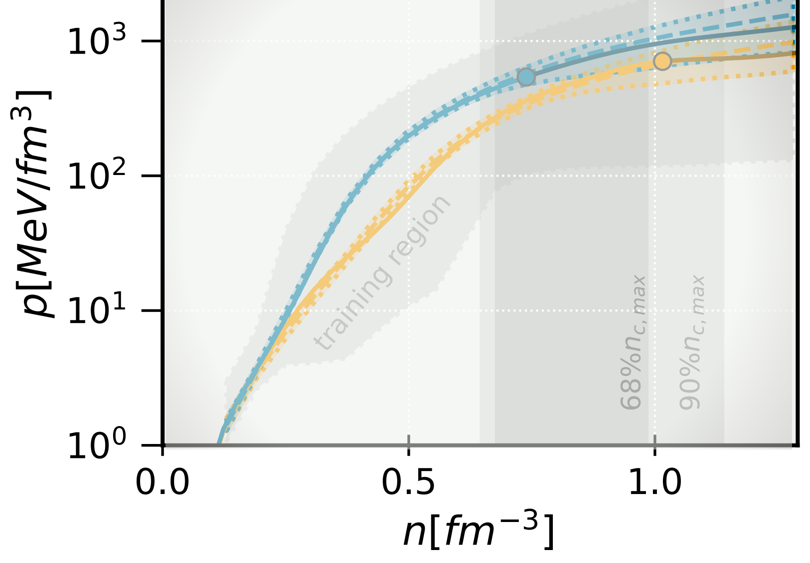
$\approx$ **70%** Prepare data — **28%** Build and train models — **2%** Deploy and predict



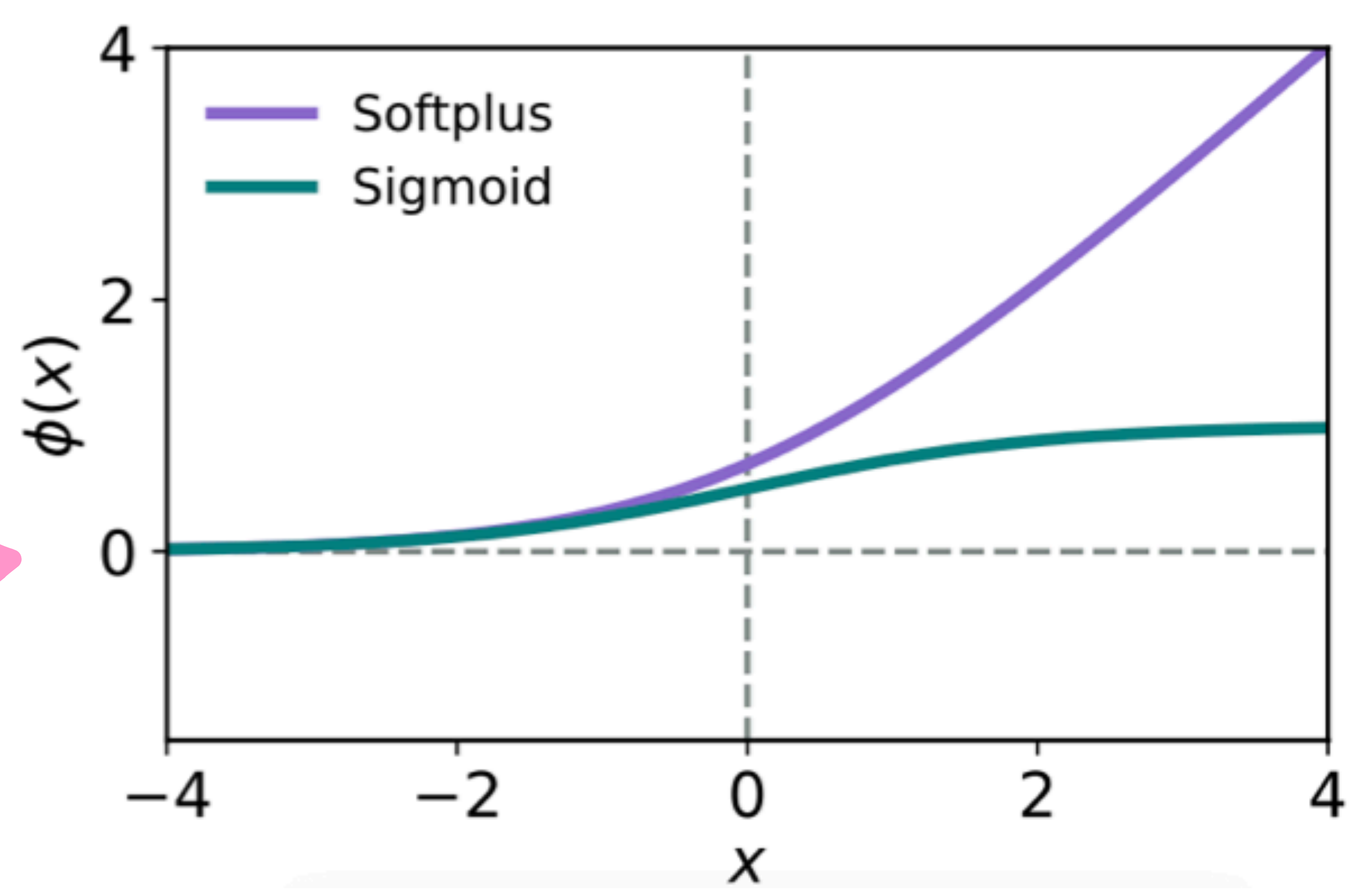


$f_\theta(x) : X \to Y$

Spoiler alert !



Deep Learning quick recap[1]



Note[1]: Notation on this slide only applies here