Superfluid fraction in the inner crust of neutron stars

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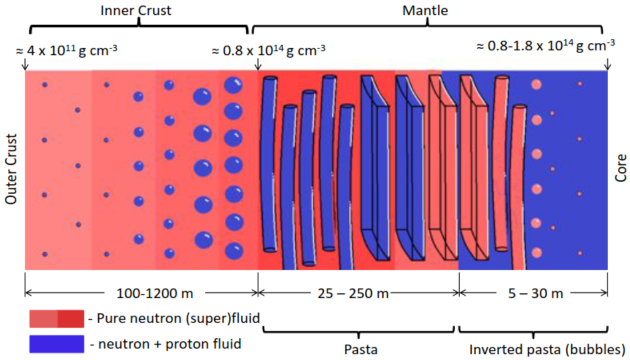
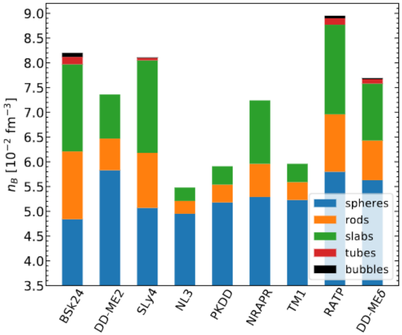
Outline

- **Introduction**: inner crust of neutron stars and its superfluid fraction
- **Formalism**: HFB with Bloch boundary conditions and Two-Fluid model
- **Results**: fully self-consistent HFB and linear response in BCS approximation

Inner crust of neutron stars

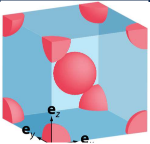




W.G. Newton et al,
Sym.En.,In.Crust,Gl.Mod. (2011)

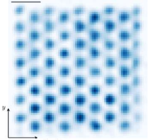
H. Dinh Thi et al,
A&A 654, A114 (2021)

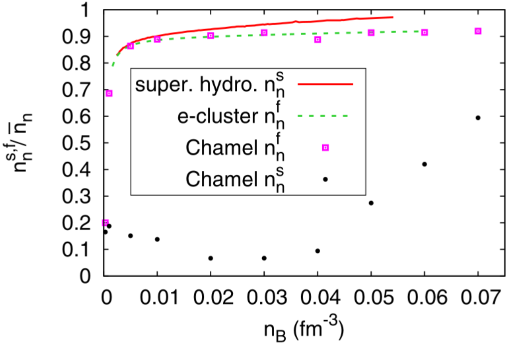
What about the superfluid fraction?



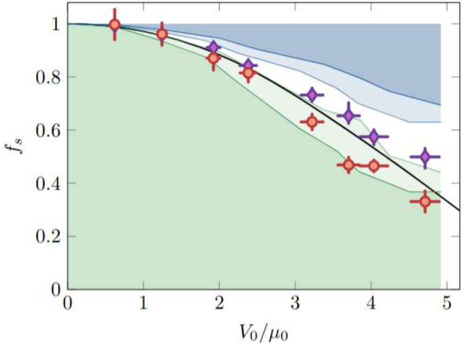
inner crust of neutron stars

weakly interacting BEC





N. Martin & M. Urban,
Phys. Rev. C 94, 065801 (2016)



F. Rabec et al,
Phys. Rev. Lett. 136, 133401 (2026)

Hartree-Fock-Bogoliubov

$$\begin{pmatrix} h-\mu & -\Delta \\ -\Delta^\dagger & -\bar{h}+\mu \end{pmatrix} \begin{pmatrix} U^*_\alpha \\ -V^*_\alpha \end{pmatrix} = E_\alpha \begin{pmatrix} U^*_\alpha \\ -V^*_\alpha \end{pmatrix}$$

the HFB equations in this form allow to compute the ground state properties of our system, moreover a flow is introduced performing a Galilean boost of the mean-field

$$h_{kk'} = \left(\frac{\hbar^2}{2m^*}\right)_{kk'} k \cdot k' + U_{kk'} - (k+k') \cdot J_{kk'} - \hbar\, k \cdot v\, \delta_{kk'} \tag{1}$$

$$U_{kk'} = -\sum_{pp'} V_{kpk'p'}\rho_{p'p} \rightarrow \text{Skyrme potential (SLy4 \& BSk24)} \tag{2}$$

$$\Delta_{kk'} = -\sum_{pp'} V_{kk'p'p}\kappa_{p'p} \rightarrow \text{separable interaction (fitted on } V_{\text{low-k}}) \tag{3}$$

Hartree-Fock-Bogoliubov in a lattice

Periodicity can be imposed trough the Bloch's theorem,
in momentum space this introduces a decomposition in integer multiples of $2\pi/L$ and
a continuos momentum known as Bloch momentum

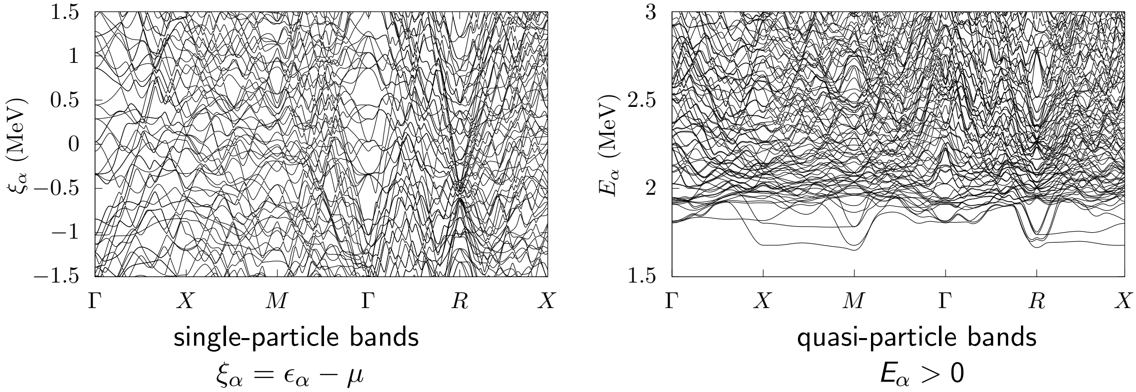
$$k = \frac{2\pi}{L} n + k_b \quad \text{with} \quad n \in \mathbb{Z} \;\; , \;\; k_b \in \left[-\frac{\pi}{L}, \frac{\pi}{L} \right)$$

as a consequence the HFB matrix has only integer indices, and it is diagonal in the Bloch and (if any) parallel momenta

$$\begin{pmatrix} h_{nn'}(k_b) - \mu & -\Delta_{nn'}(k_b) \\ -\Delta^*_{n'n}(k_b) & -h_{-n'-n}(k_b) + \mu \end{pmatrix} \begin{pmatrix} U^*_{n'\alpha}(k_b) \\ -V^*_{n'\alpha}(k_b) \end{pmatrix} = E_\alpha(k_b) \begin{pmatrix} U^*_{n\alpha}(k_b) \\ -V^*_{n\alpha}(k_b) \end{pmatrix}$$

Hartree-Fock-Bogoliubov in a lattice, a taste

Solving the HFB equations with Bloch boundary conditions means computing the band structure of the inner crust



$\Gamma - X - M - \Gamma - R - X$ is the high symmetrical path in the positive $k_b \in$ BZ

Two-Fluid model

Our setup gives us access to densities ρ, currents $\vec{j}$ and other relevant quantities.
Then with the so-called two-fluid model we can compute the superfluid fraction

$$\vec{j}_n = (\rho_n - \rho_S)\vec{v} + \rho_S \vec{v}_S \quad ; \quad \vec{j}_p = \rho_p \vec{v}$$

This relation has to be understood in an average sense ($\phi =$ phase of the gap: $\Delta = |\Delta| e^{i\phi}$)

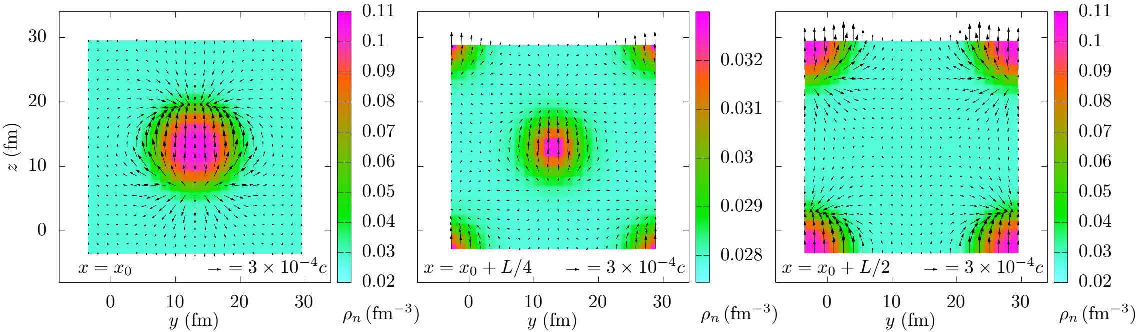
$$\vec{v}_S = \int_V \frac{d^3x}{V} \frac{\hbar}{2m} \vec{\nabla}\phi$$

Since our quantities are periodic the average superfluid velocity $\vec{v}_S$ is zero,
thus we are working in the reference frame in which the superfluid component is at rest

Crystal

body-centered cubic lattice

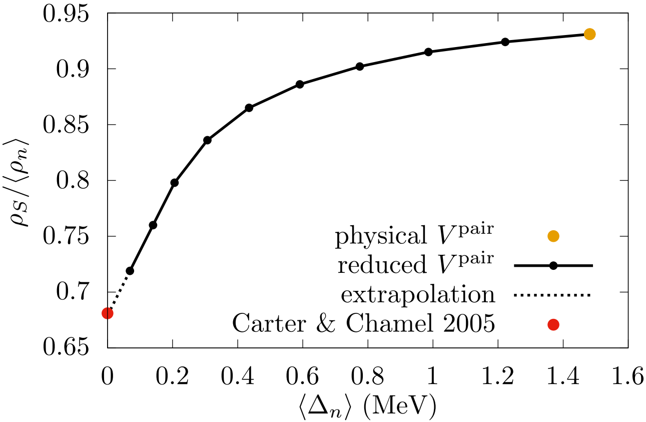
$\rho_b = 0.033\ \text{fm}^{-3}$; $L = 33$ fm ; $\rho_S/\rho_n = 92\%$



GA, T. Kaskitsi, M. Urban, arXiv:2512.18549

Bands effects VS pairing gap

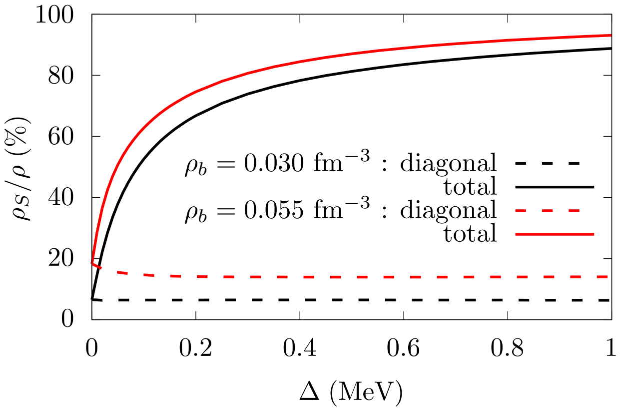
Normal band theory should be valid in the weak-coupling limit (pairing gap $\ll$ Fermi energy)



$\langle\rho_n\rangle = 0.059$ fm^{-3} ; $L = 27.17$ fm

GA & M. Urban, Phys. Rev. C 110, 065802 (2024)

Geometric contribution in BCS approximation

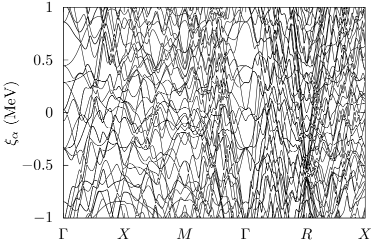


$$\rho_S^{ij} = \int_{\mathrm{BZ}} \frac{d^3k}{(2\pi)^3} \left[m \sum_{\alpha} \frac{\Delta^2}{E_{\alpha\boldsymbol{k}}^3} \frac{\partial \xi_{\alpha\boldsymbol{k}}}{\partial k^i} \frac{\partial \xi_{\alpha\boldsymbol{k}}}{\partial k^j} + \frac{2}{m} \sum_{\alpha\neq\beta} \frac{\Delta^2}{E_{\alpha\boldsymbol{k}} E_{\beta\boldsymbol{k}} (E_{\alpha\boldsymbol{k}} + E_{\beta\boldsymbol{k}})} p^i_{\alpha\beta\boldsymbol{k}} p^j_{\beta\alpha\boldsymbol{k}} \right]$$

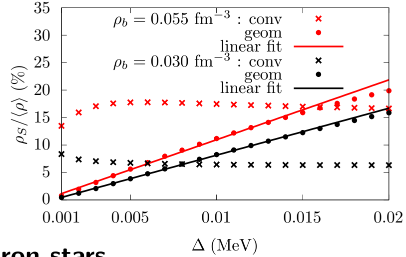
GA & M. Urban, Phys. Rev. Lett. 135, 132701 (2025)

Geometric contribution in neutron stars and condensed matter

band structure

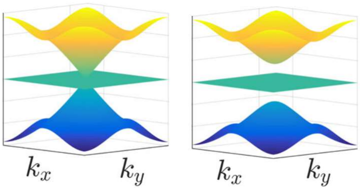


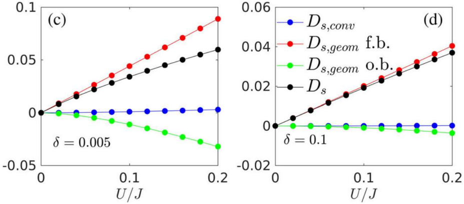
superfluid fraction(weight)



GA, Particles 2026

inner crust of neutron stars





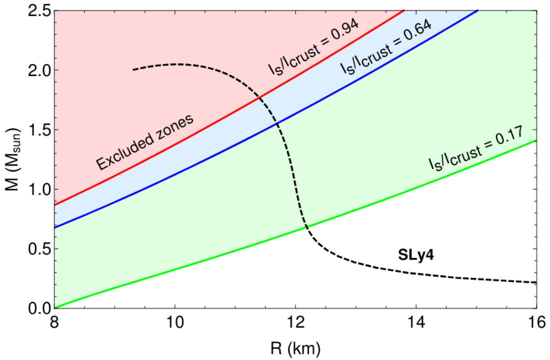
A. Julku et al, PRL 2016

Hubbard model on the Lieb lattice

Constraint from glitch activity

The superfluid density ρ_S is related to the superfluid moment of inertia of the crust I_S, then the ratio I_S/I can be used as a constraint thanks to its relation with the glitch activity $\mathcal{G}$

$$\frac{I_S}{I_{\text{crust}}} = \frac{1}{P_{\text{core}}} \int_{P_{\text{drip}}}^{P_{\text{core}}} \frac{\rho_S}{\rho_b} dP \quad , \quad \frac{I_S}{I} = \frac{I_S}{I_{\text{crust}}} \times \frac{I_{\text{crust}}}{I} > \mathcal{G}$$



N. Martin & M. Urban, Phys. Rev. C 94, 065801 (2016)

Conclusions

The self-consistent HFB calculations performed here accounted
for both band structure and superfluidity

- 1D: $\rho_S/\langle\rho_n\rangle \simeq 97\%$ No surprise.
- 2D: $\rho_S/\langle\rho_n\rangle \simeq 95\%$ Normal band theory underestimates the superfluid fraction !
- 3D: $\rho_S/\langle\rho_n\rangle \simeq 92\%$ Entrainment is very small because of the geometric contribution !!

The superfluid reservoir in the inner crust of neutron stars is big enough to explain
pulsar glitches with the crust only !!!

The geometric contribution could be non-zero also in the shallowest layers of the inner crust,
and it is expected to appear in all the superfluid response functions

Thanks for your attention!