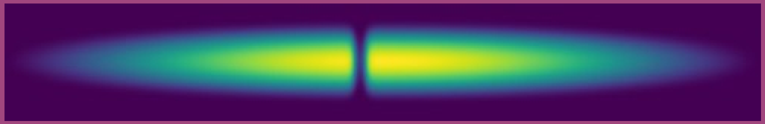
Josephson effects and atomtronic circuits in atomic superfluids

Klejdja Xhani

Politecnico of Torino



Talk outline

1) From superconductors to atomic Josephson junctions

2) Dynamical regimes in elongated 3D junctions

3) Critical current and dissipation across atomic Josephson junctions

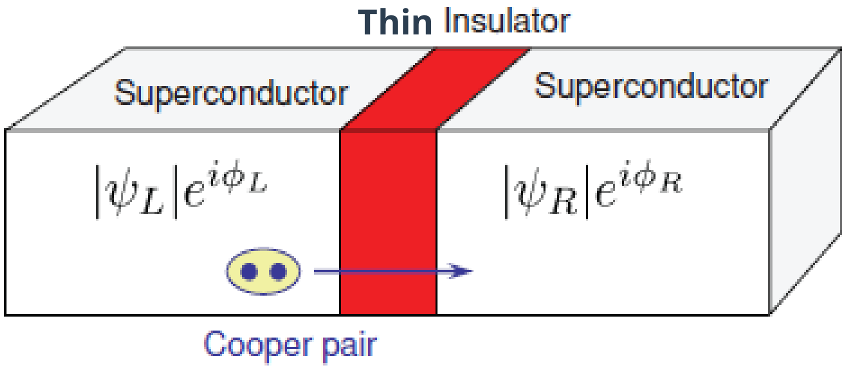
4) Thermal damping of Josephson currents

5) Stability and dissipation in ring circuits

Josephson effect in superconductors

B. D. Josephson, Phys. Letters 1, 251 (1962)

SIJ



J.Q. You and F. Nori Nature 474,7353 (2011)

Overlapping between the two wavefunctions→ Tunneling of Cooper pairs→ Supercurrent (no dissipation)

Josephson relations:

$$I(t)=I_c \sin\varphi(t) \qquad \varphi=\phi_L-\phi_R \qquad I_c=2eE_J/\hbar$$

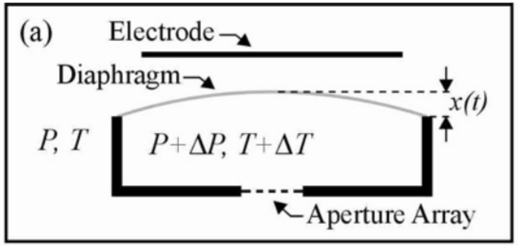
$$d\varphi/dt=-\Delta\mu/\hbar \qquad \Delta\mu=2eV$$

DC- Josephson effect:
V=0, I constant (direct current)

AC-Josephson effect:
V finite, I alternating $\nu=2eV/\hbar$

Josephson junctions across quantum fluids

Superfluid helium



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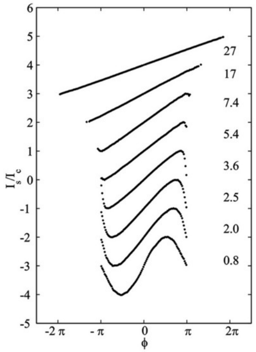

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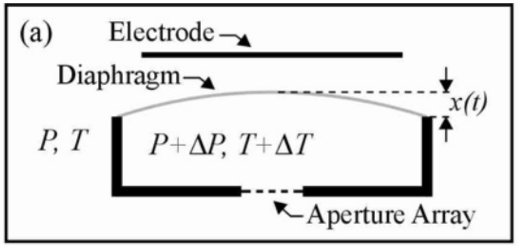

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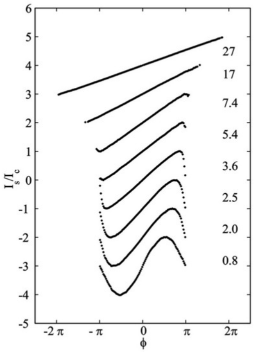

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Ultracold atoms

non linear interactions→ Macroscopic quantum self-trapping

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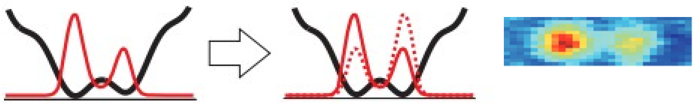

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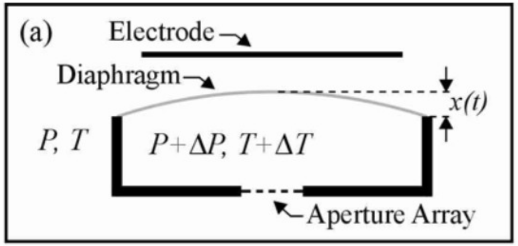

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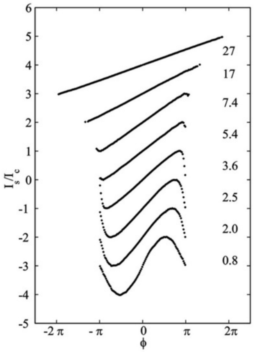

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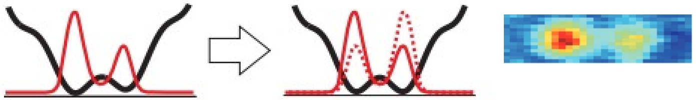

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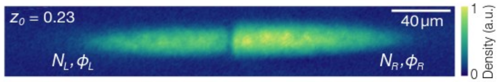

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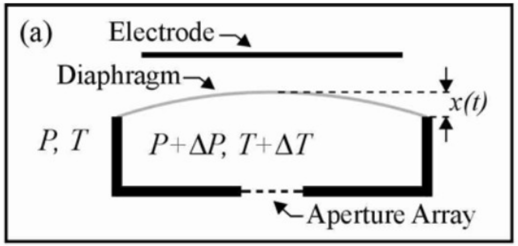

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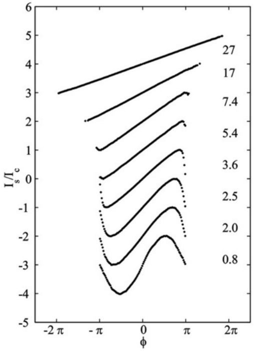

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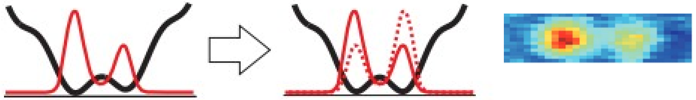

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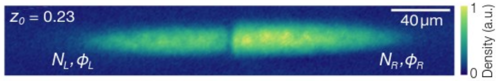

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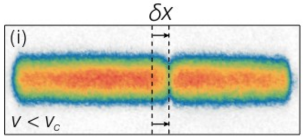

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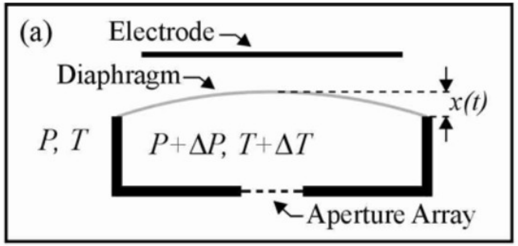

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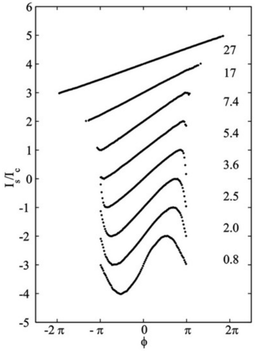

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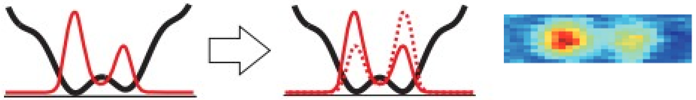

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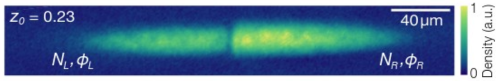

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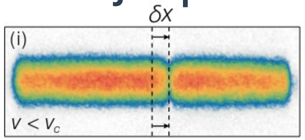

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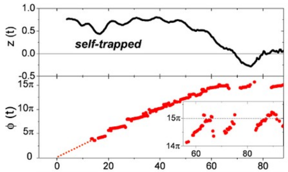
Exciton-polariton condensate

Josephson plasma

and self-trapping regime



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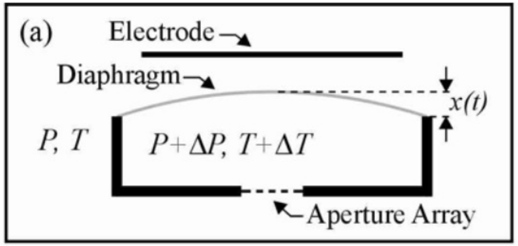

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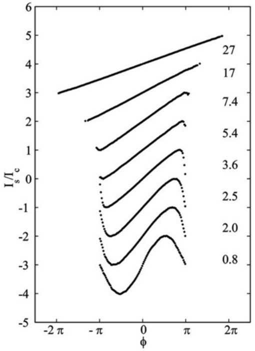

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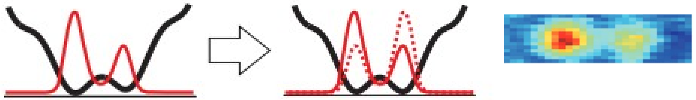

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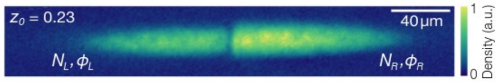

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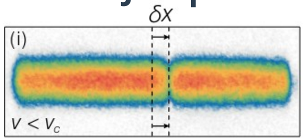

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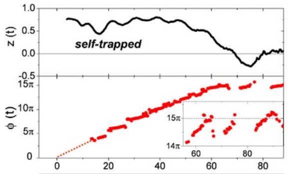

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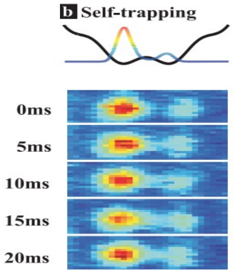
Josephson currents in neutron stars

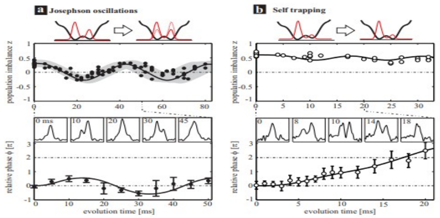
at the interface between S-wave and P-wave paired superfluids

A. Sedrakian's talk

Dynamical regimes in an atomic Josephson junctions

Josephson plasma → Self-trapping





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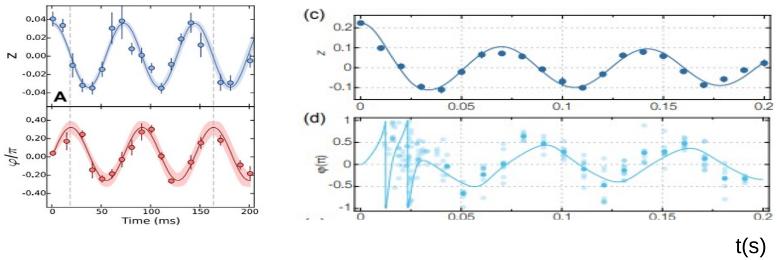
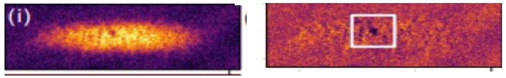
Josephson plasma → Dissipative regime





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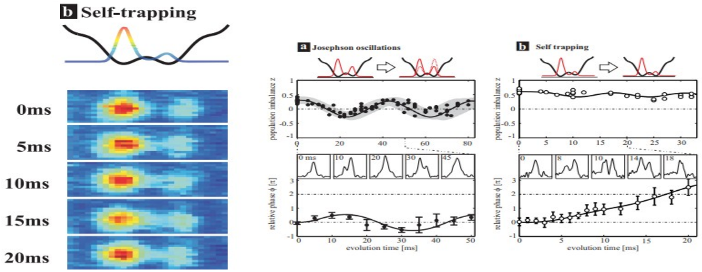
Dynamical regimes in atomic Josephson junctions

Josephson plasma → Self-trapping



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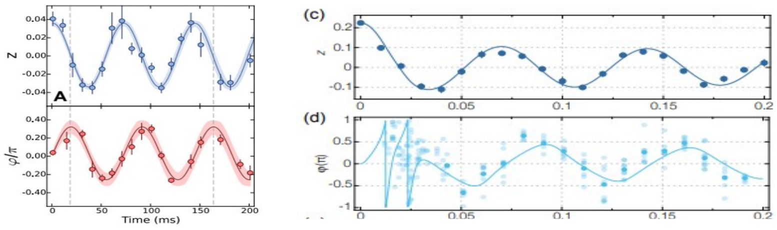

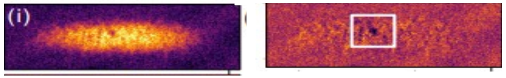

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C. Vianello ... arXiv:2...

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Eck... 4 03...
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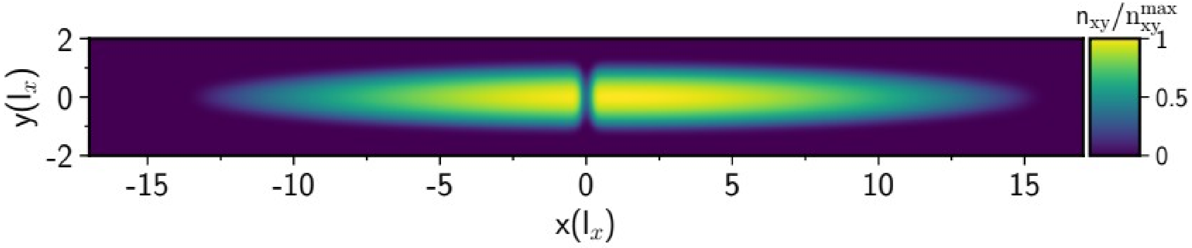
Atomic Josephson Junction: cigar-shaped 3D geometry

Numerical density profile



K. Xhani *et al, Phys. Rev. Lett. 124, 045301 (2020)*

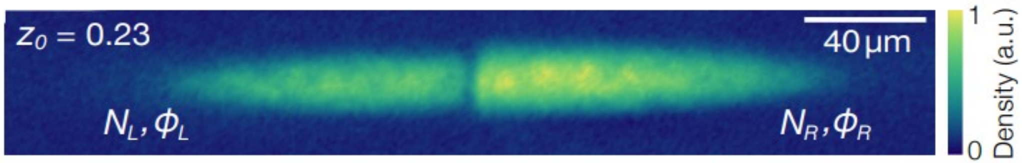
K Xhani *et al, New J. Phys. 22, 123006 (2020)*

K. Xhani *et al, Phys. Rev. Research 4, 033205 (2022)*

G. Wlazłowski, ***K. Xhani*** *et al, Phys. Rev. Lett. 130, 023003 (2023)*

Xhani, K.*; Proukakis, N.P. , Atoms 2025, 13, 68(2025)*

Motivated by ^{6}Li experiment:



LENS ^{6}Li experiment:

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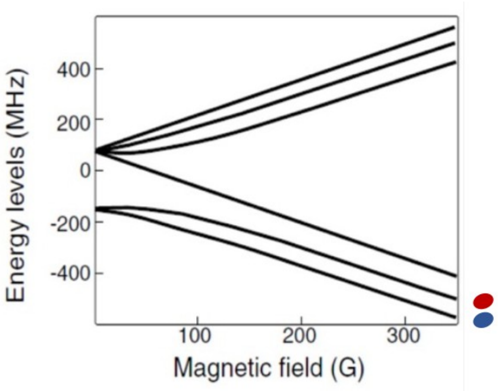
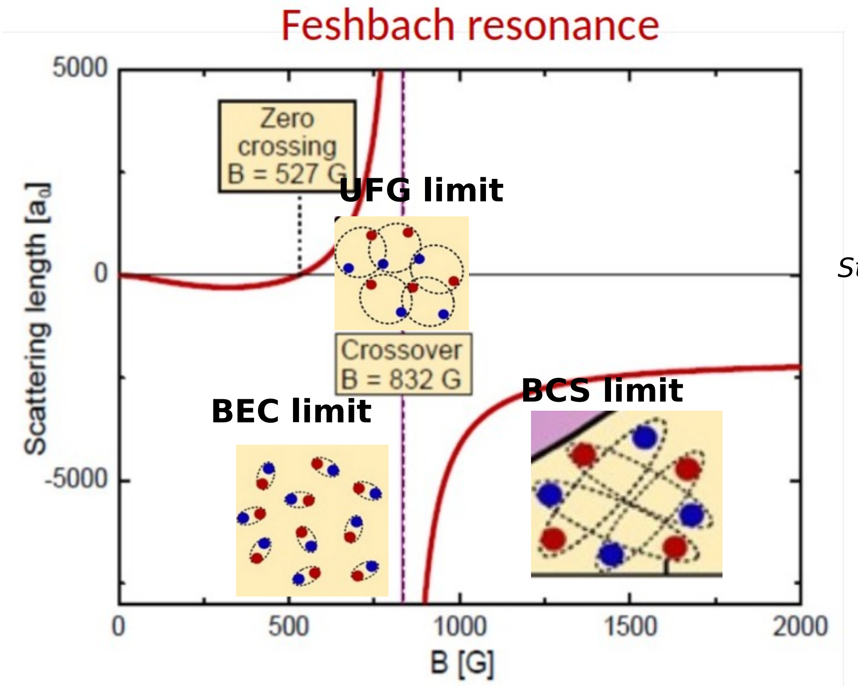
Atomic superfluids: from BEC to BCS





From molecular Bose-Einstein condensate limit (BEC limit)

↓

Strongly interacting unitary Fermi gases (UFG limit) $k_F \xi \approx 1$

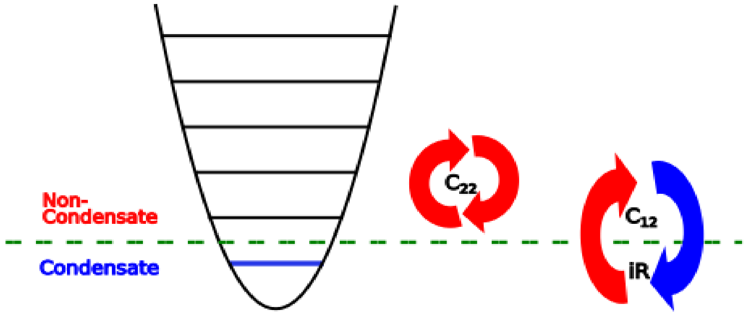
↓

Weakly attractive Bardeen–Cooper–Schrieffer (BCS limit) $k_F \xi > 1$

From bosonic to fermionic superfluids with a single experimental set-up

BEC limit	UFG limit	BCS limit
$1/(k_F a) > 0$	$1/(k_F a) \simeq 0$	$1/(k_F a) < 0$
GPE-ZNG	TDSLDA	

BEC limit: Finite temperature Zaremba-Nikuni-Griffin (ZNG) model



BEC density

$$n_c = |\psi|^2$$

Thermal density

$$n' = \int \frac{d^3p}{(2\pi\hbar)^3} f$$

MEAN FIELD COUPLING

$$U(r) = V(r) + 2g(n_c + n')$$

$$R(r,t) = (\hbar/n_C)\int dp C_{12}[f]$$

Dissipative GPE

$$i\hbar\frac{\partial\phi}{\partial t} = \left(-\frac{\hbar^2\nabla^2}{2m} + V_{TRAP} + g(n_C + 2n') - iR\right)\phi$$

Quantum Boltzmann

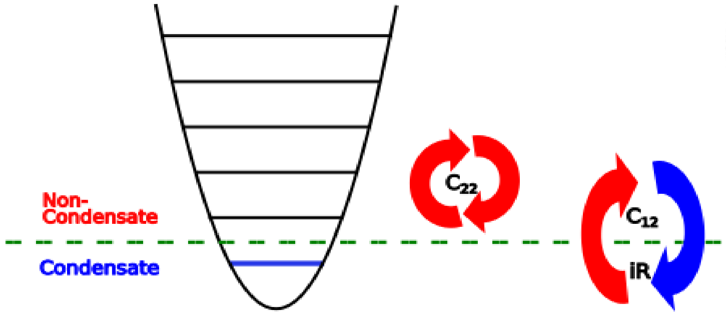
Thermal collisions

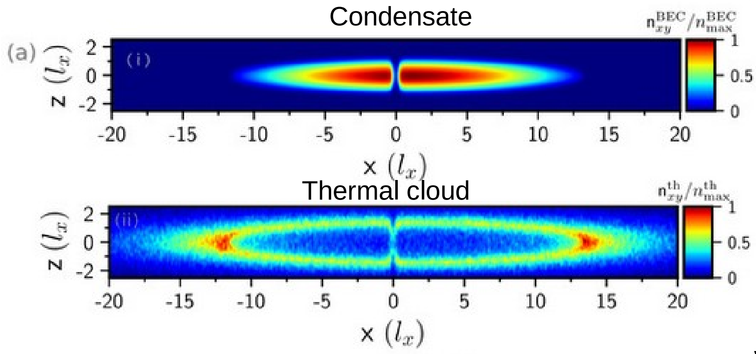
$$\frac{\partial f}{\partial t} + \frac{p}{m}\cdot\nabla f - \nabla U \cdot \nabla_p f = C_{12} + C_{22}$$

EXCHANGE OF ATOMS BETWEEN SUB-SYSTEMS

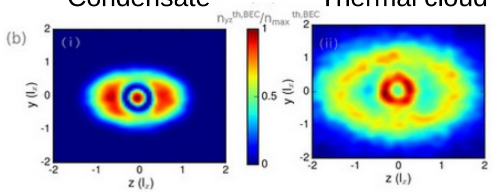
Zaremba, Nikuni & Griffin
J Low Temp Phys 116, 277 (1999)

BEC limit: Finite temperature Zaremba-Nikuni-Griffin (ZNG) model





Vortex rings



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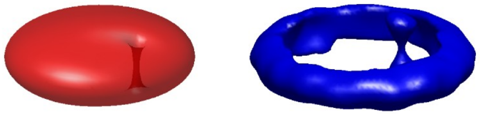
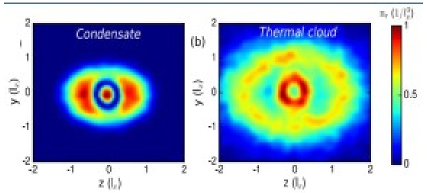
J Low Temp Phys 116, 277 (1999)

Finite temperature Zaremba-Nikuni-Griffin model

Solving self-consistently a kinetic finite temperature model known as ZNG model.
Coupled dynamical condensate and thermal cloud

Vortex dynamics

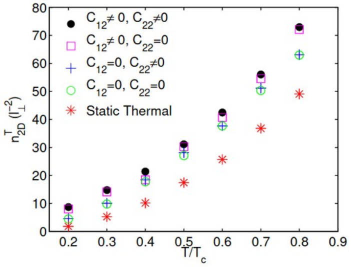




A. J. Allen et al., Phys. Rev. A 87, 013630 (2013)
B. Jackson et al., Phys. Rev. A 79, 053615 (2009)

K. Xhani et al, Phys. Rev. Research 4, 033205 (2022)

Xhani, K.; Proukakis, N.P. , Atoms 2025, 13, 68 (2025)



Increased thermal density at vortex cores with T→ increased vortex mass

Mutual frictions coefficients

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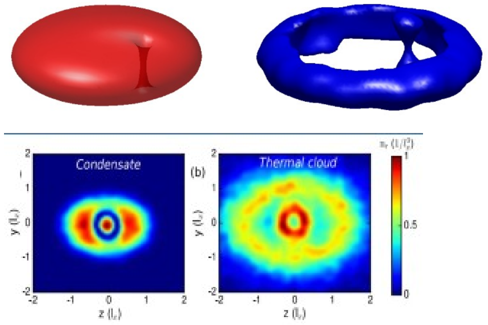

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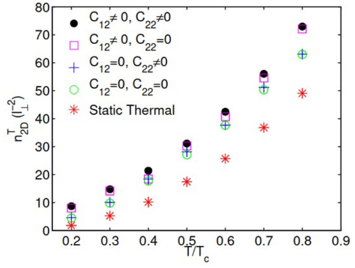

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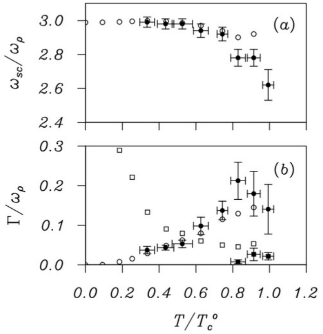
Mutual frictions coefficients

Collective modes

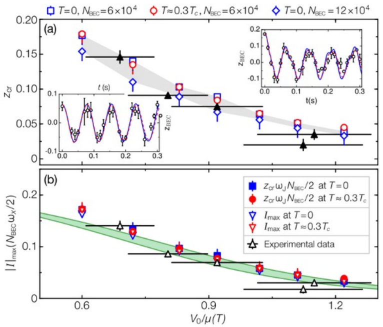


B. Jackson et al., 2002,
Phys. Rev. Lett. 88,180402

B. Jackson et al., 2001,
Phys. Rev. Lett. 87, 100404

K. Xhani et al, 2020,
Phys. Rev. Lett. 124, 045301

Josephson effect



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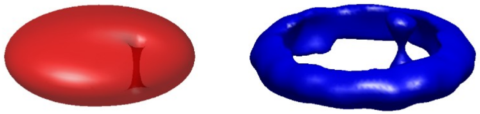

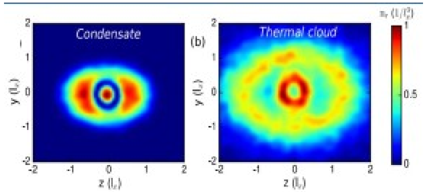

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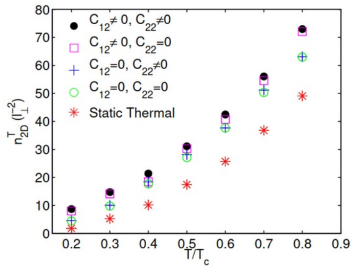

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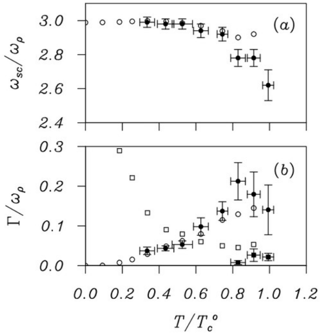

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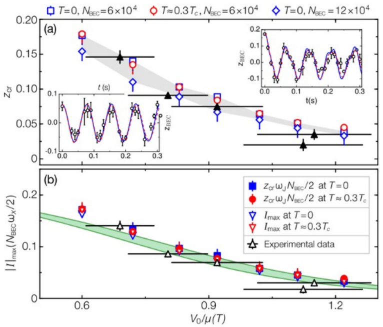
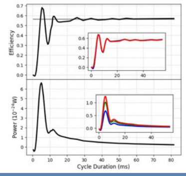
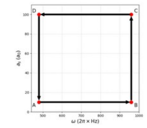


Poster by Henry Harper-Gardner

Performance and Dynamics of an Ultracold Atomic Superfluid as a Quantum Engine Working Fluid





BEC limit at T=0: Gross-Pitaevskii equation

At T=0, the ZNG model reduces to Gross-Pitaevskii equation

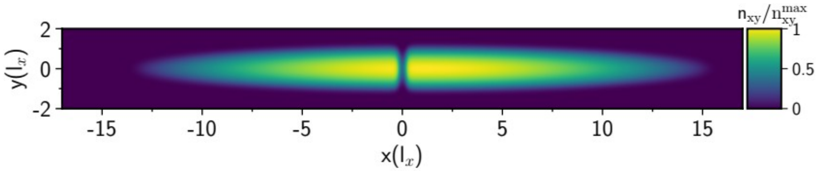
$$i\hbar \frac{\partial \psi(\mathbf{r},t)}{\partial t} = -\frac{\hbar^2}{2M}\nabla^2 \psi(\mathbf{r},t) + V_{ext}\,\psi(\mathbf{r},t) + g_M |\psi(\mathbf{r},t)|^2 \psi(\mathbf{r},t)$$

Trapping potential

barrier

$$V_{trap}(x,y,z) = (1/2)M(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2) + V_0 \cdot \exp(-2x^2/w^2)$$

$\omega_x = 2\pi\,\mathrm{x}\,15\,\mathrm{Hz}$ $(\omega_x, \omega_y, \omega_z) = (1:12:10)\,\omega_x$ $w = 4\xi$



$g_M = \frac{4\pi\hbar^2 a_M}{M}$ $N = 6\,x\,10^4$ $1/(k_F a) \simeq 4.6$

molecular BEC limit

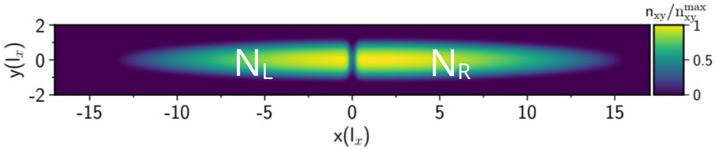
BEC limit at T=0: Gross-Pitaevskii equation

At T=0, the ZNG model reduces to Gross-Pitaevskii equation

$$i\hbar\frac{\partial\psi(\mathbf{r},t)}{\partial t}=-\frac{\hbar^2}{2M}\nabla^2\psi(\mathbf{r},t)+V_{ext}\,\psi(\mathbf{r},t)+g_M|\psi(\mathbf{r},t)|^2\,\psi(\mathbf{r},t)$$

Trapping potential

$$V_{trap}(x,y,z)=(1/2)M(\omega_x^2x^2+\omega_y^2y^2+\omega_z^2z^2)+V_0\cdot\exp(-2x^2/w^2)$$



Equilibrium state: $V_{ext}=V_{trap}-\epsilon x$

Dynamics: $V_{ext}=V_{trap}$

⇩

Population imbalance

$$z(t)=\frac{N_R(t)-N_L(t)}{N}$$

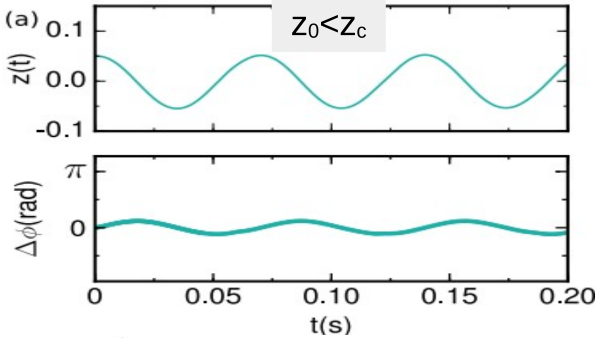
$$z_0=z(t=0)\neq 0$$

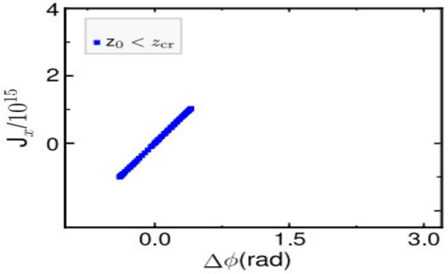
$$\Delta\phi_0=0$$

➡ Josephson current $I(t)=-\frac{N}{2}\frac{dz}{dt}$

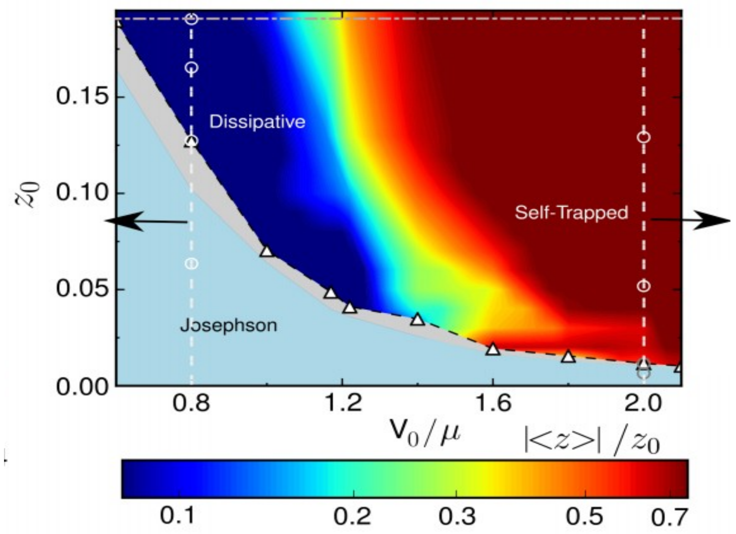
Phase diagram of a 3D bosonic Josephson junctions

Josephson plasma regime





Fixed $w = 4\,\xi$, variable barrier heights $[0.6, 2.1]\mu$



Undamped coherent z(t) and $\Delta\,\phi$ oscillations around zero values

Linear current-phase relation

K. Xhani et al, Phys. Rev. Lett. 124 045301 (2020)

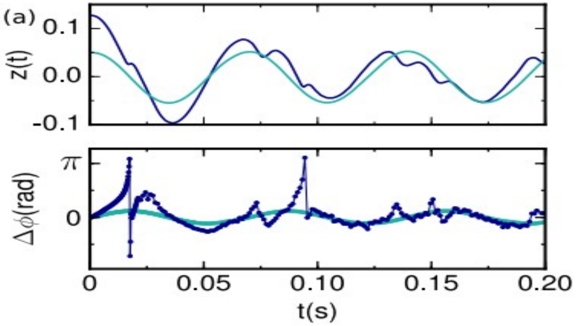
K. Xhani, PhD thesis, http://theses.ncl.ac.uk/jspui/handle/10443/5538

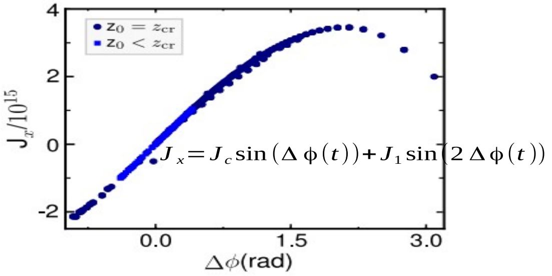
K Xhani et al., New J. Phys. 22, 123006 (2020)

Phase diagram of a 3D bosonic Josephson junctions

Dissipative regime (dark blue)

$\langle z(t)\rangle / z_0 \approx 0$





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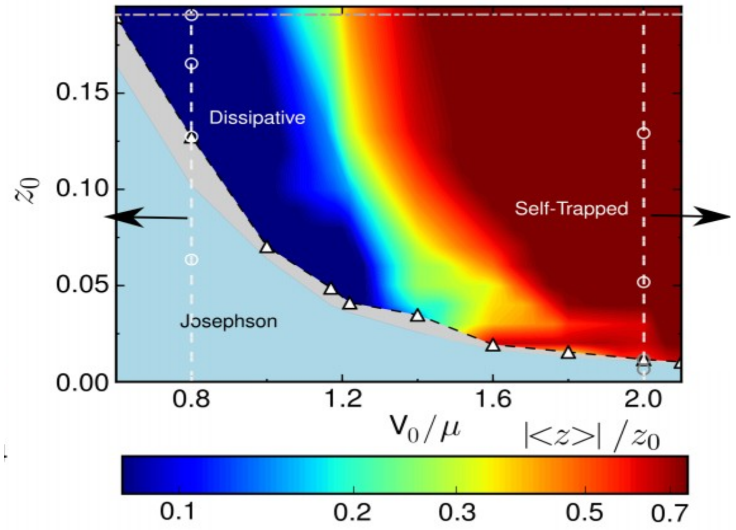


Damped Josephson oscillations

Phase-slippage

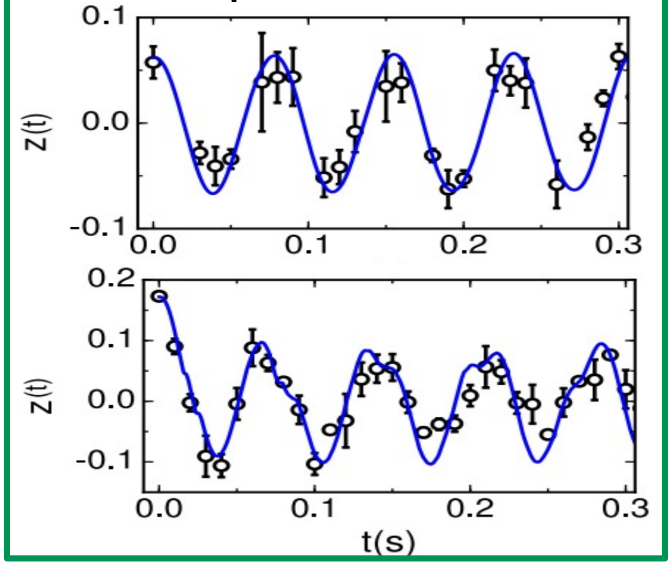
Sinusoidal current-phase relation

First and second order term in tunneling H

K. Xhani et al, Phys. Rev. Lett. 124 045301 (2020)

Phase diagram of a 3D bosonic Josephson junctions

Comparison with experimental data



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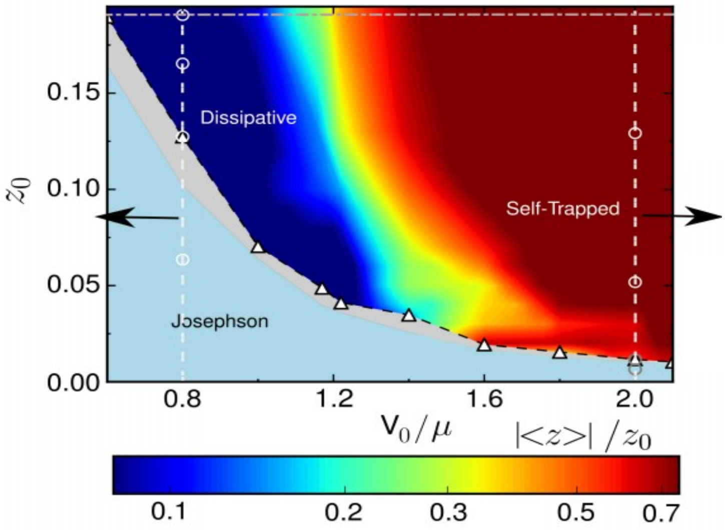

K. Xhani et al., Phys. Rev. Lett. 124 045301 (2020)

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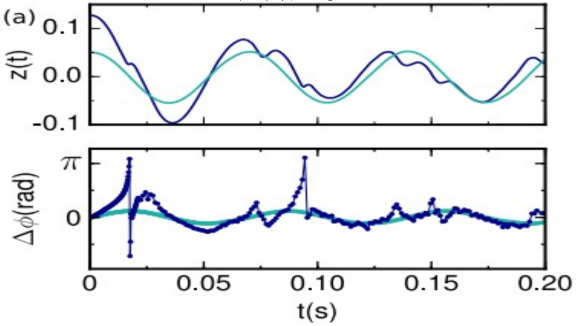
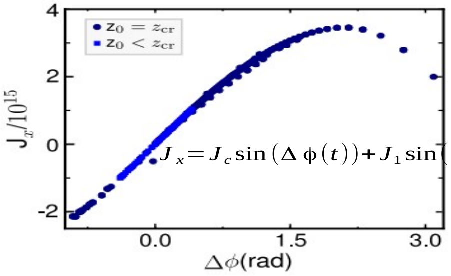
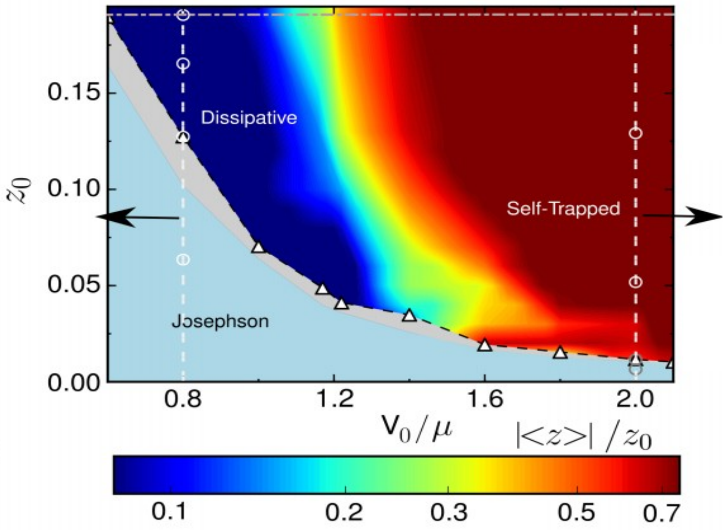


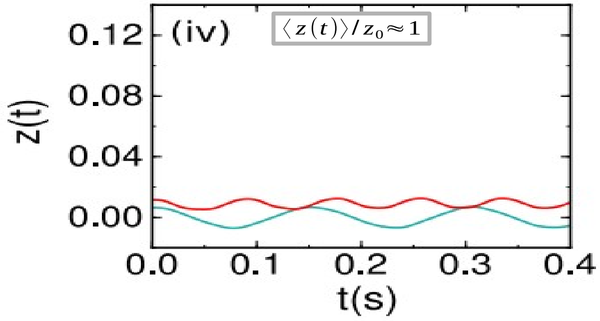


$J_x = J_c \sin(\Delta\phi(t)) + J_1 \sin(2\Delta\phi(t))$

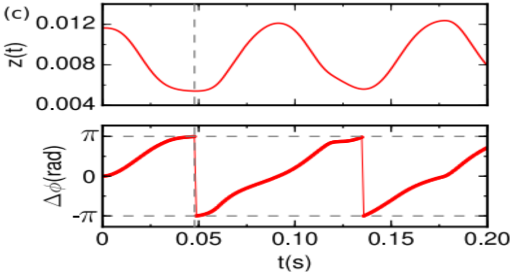
Fixed $w = 4\xi$, variable barrier heights $[0.6, 2.1]\mu$

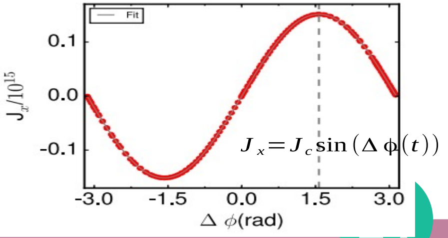


Self-trapping regime (red line)



$\langle z(t)\rangle / z_0 \approx 1$





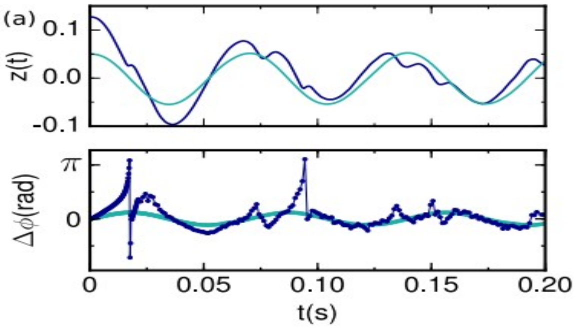
$J_x = J_c \sin(\Delta\phi(t))$

Self-trapped: Undamped coherent z(t) oscillations around non-zero values

$\Delta\phi$ 2π phase slippage or running phase regime

Sinusoidal current-phase relation with single-term

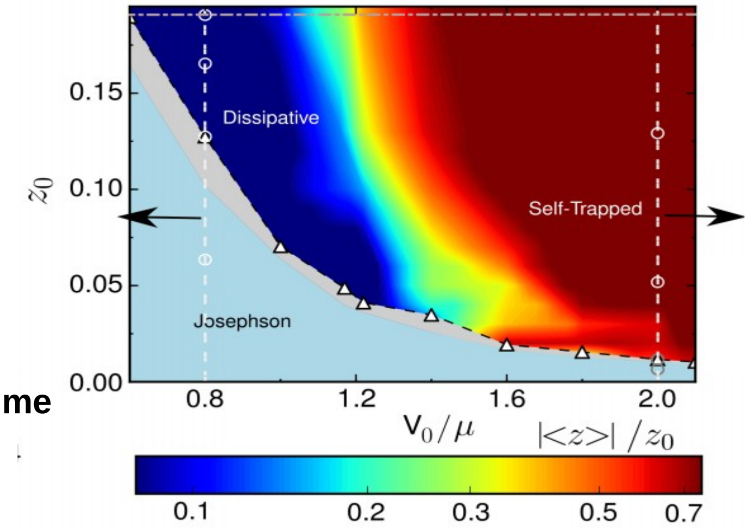
T=0: Phase diagram of a 3D bosonic Josephson junctions

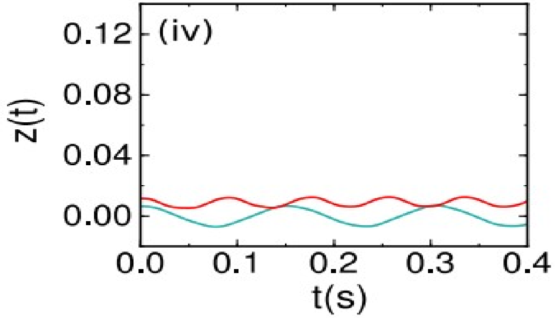


Low V_0/μ or small w/ξ

Josephson plasma to dissipative regime
upon increasing z_0 above z_c^d

Fixed $w = 4\xi$, variable barrier heights $[0.6, 2.1]\mu$





High V_0/μ or large w/ξ

Josephson plasma to Macroscopic Quantum Self-trapping regime
upon increasing z_0 above z_c^{ST}

Could all three dynamical regimes be observed within the same atomic Josephson junction?

Yes! Through barrier height/width and initial population imbalance's control

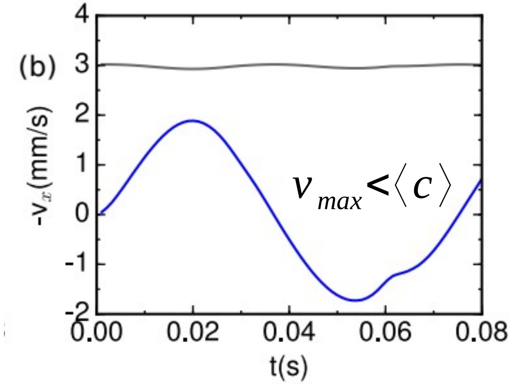
K Xhani et al, New J. Phys. 22, 123006 (2020)

T=0: Dissipation mechanisms of the Josephson oscillations

Vortex rings nucleation and dynamics

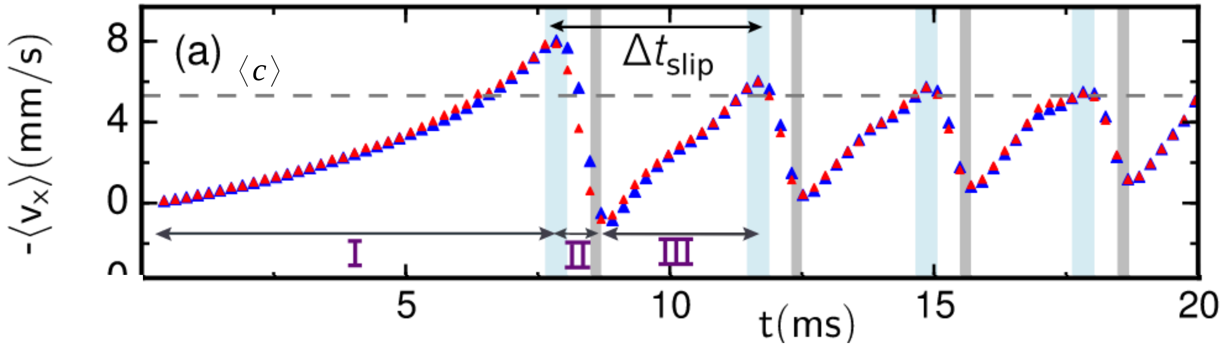
$V_0/\mu=0.8$

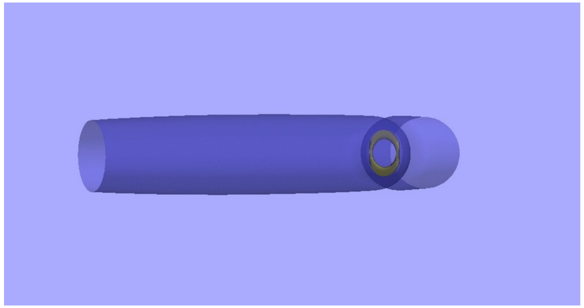
Josephson ‘plasma’ regime



$$\langle c\rangle = \sqrt{\mu/2M})$$

Dissipative regime $\langle v_{max}\rangle > \langle c\rangle$





Vortex rings generation

K. Xhani et al, Phys. Rev. Lett. 124, 045301 (2020)

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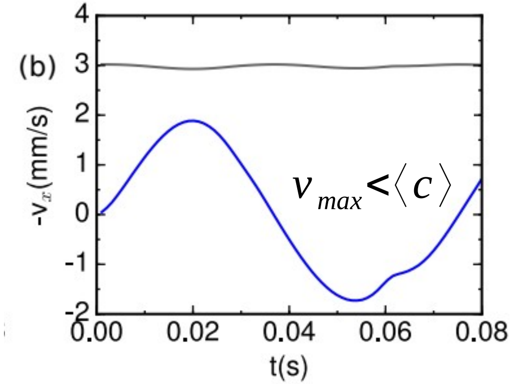
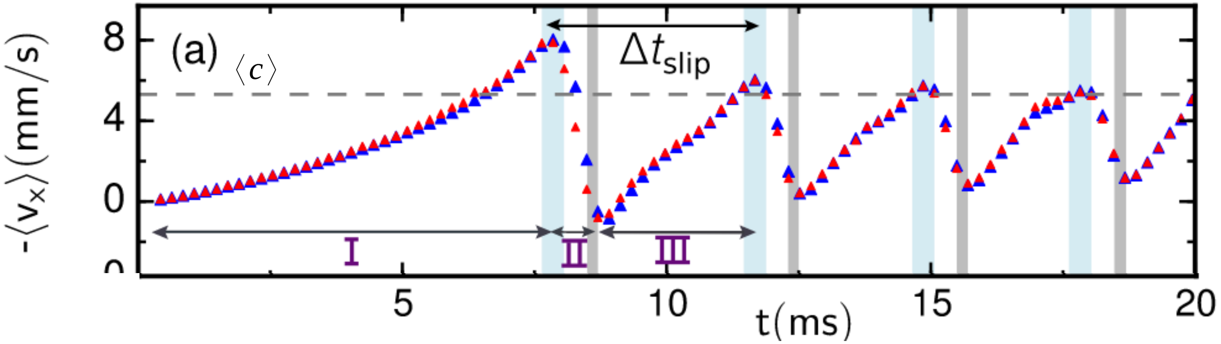


Dissipative regime $\langle v_{max} \rangle > \langle c \rangle$



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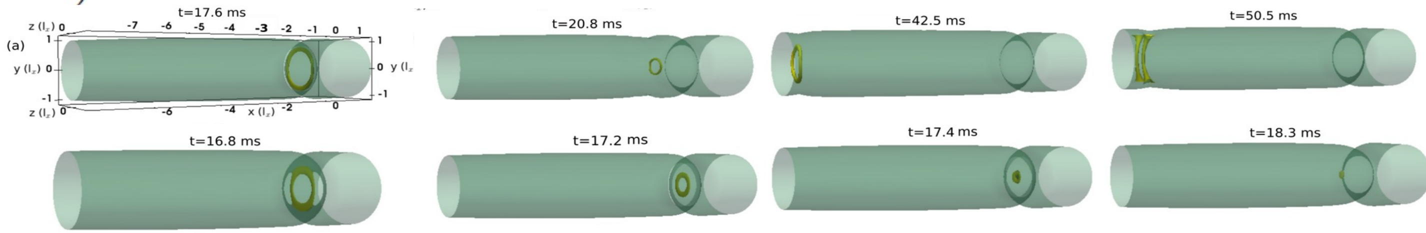

$V_0/\mu=0.6$

$V_0/\mu=0.8$

K Xhani et al., New J. Phys. 22, 123006 (2020)

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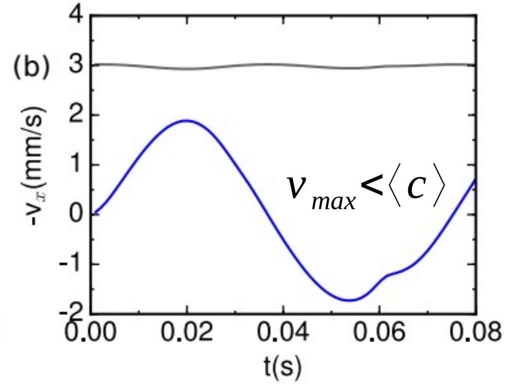

$$\langle c \rangle = \sqrt{\mu/2M})$$

Vortex rings generation

K. Xhani et al, Phys. Rev. Lett. 124, 045301 (2020)

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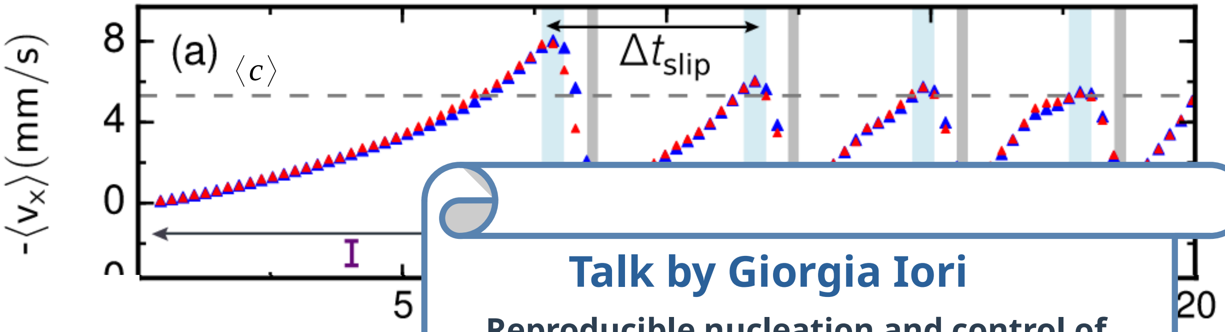
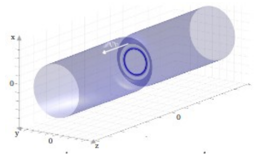
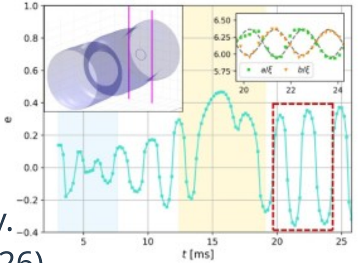


Talk by Giorgia Iori

Reproducible nucleation and control of stable quantum vortex rings in Bose-Einstein condensates

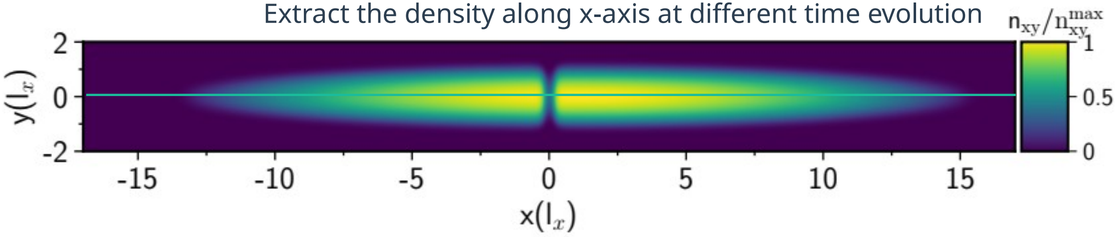


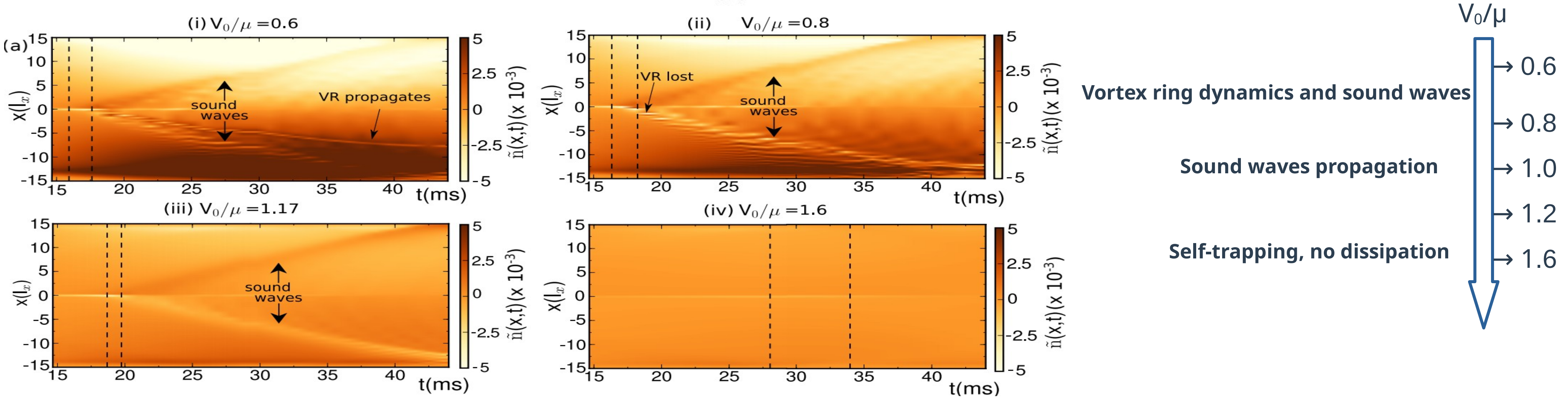


G. Iori et al., Phys. Rev. Research 8, L022043 (2026)

Dissipation mechanisms of a bosonic Josephson junction

Extract the density along x-axis at different time evolution





Vortex ring dynamics and sound waves

Sound waves propagation

Self-trapping, no dissipation

K Xhani et al., New J. Phys. 22, 123006 (2020)

Bosonic Josephson junction, role of finite temperature

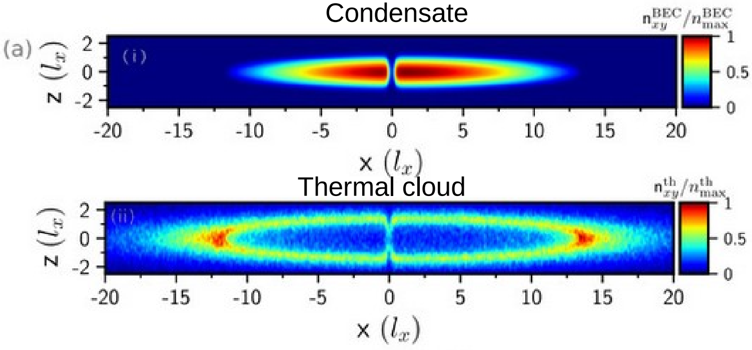
Collisionless limit

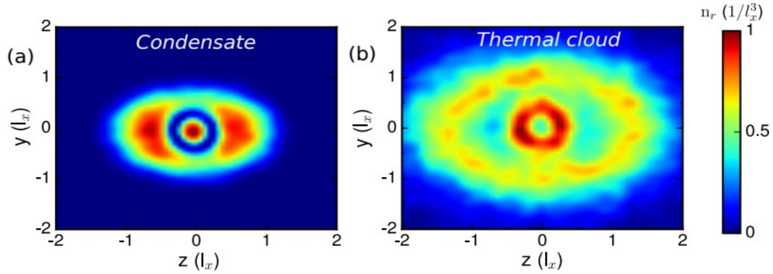
Dissipative GPE

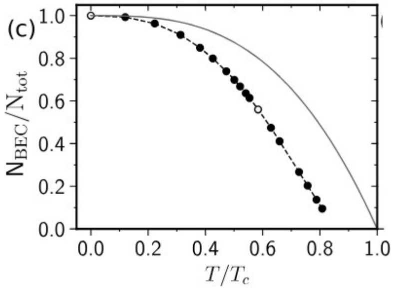
$$i\hbar\frac{\partial\phi}{\partial t} = \left(-\frac{\hbar^2\nabla^2}{2m} + V_{TRAP} + g\left(n_C + 2n'\right) - iR\right)\phi$$

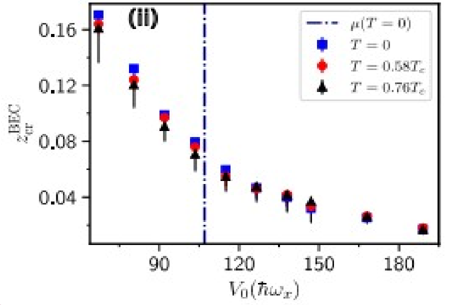
Quantum Boltzmann

$$\frac{\partial f}{\partial t} + \frac{p}{m}\cdot\nabla f - \nabla U\cdot\nabla_p f = C_{12} + C_{22}$$









Short time evolution

Critical imbalance not affected by thermal cloud as long as *we **fix condensate number and change N_{tot}** while changing T*

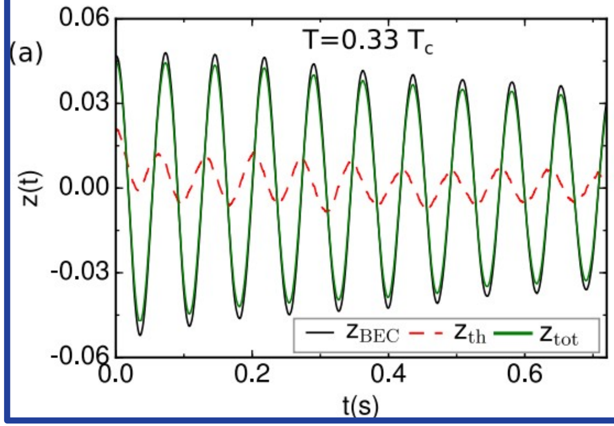
K Xhani and N P Proukakis, Phys. Rev. Research 4, 033205

Bosonic Josephson junction, role of finite temperature

Fixed N_{BEC} and V0 at different T

Damped Josephson plasma regime

Small T: $k_B T < V_0$



$$z_{\rm BEC}(t) = \frac{N_R^{BEC}(t) - N_L^{BEC}(t)}{N_R^{BEC}(t) + N_L^{BEC}(t)}$$

$$z_{\rm th} = \frac{N_R^{th}(t) - N_L^{th}(t)}{N_R^{th}(t) + N_L^{th}(t)}$$

$$z_{\rm tot} = \frac{N_R(t) - N_L(t)}{N_R + N_L} = \frac{N_R(t) - N_L(t)}{N_{tot}}$$

$$F(t) = a_J \cos(2\pi\nu_J t + \phi_J) \exp(-\gamma_J t) + a_i \cos(2\pi\nu_i t + \phi_i) \exp(-\gamma_i t)$$

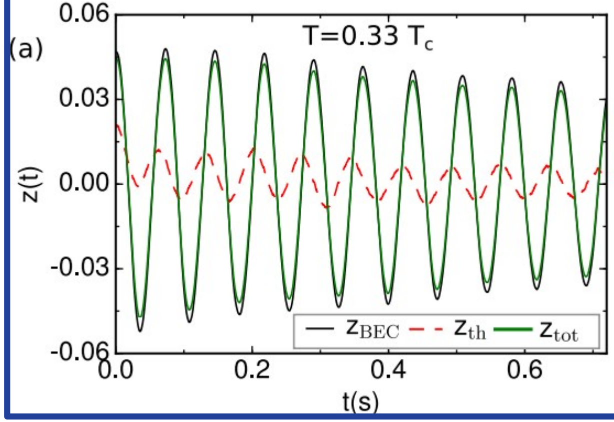
K Xhani and N P Proukakis, Phys. Rev. Research 4, 033205

Bosonic Josephson junction, role of finite temperature

Fixed N_{BEC} and V0 at different T

Damped Josephson plasma regime

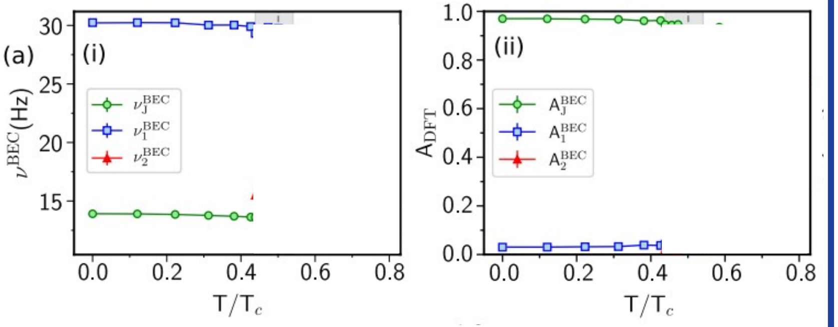
Small T: $k_B T < V_0$



Main component:
Josephson plasma frequency ν_J

Second component: $\nu_1 \approx 2\nu_J$

Dependence on $k_B T/V_0$



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$$z_{th} = \frac{N_R^{th}(t) - N_L^{th}(t)}{N_R^{th}(t) + N_L^{th}(t)}$$

$$z_{tot} = \frac{N_R(t) - N_L(t)}{N_R + N_L} = \frac{N_R(t) - N_L(t)}{N_{tot}}$$

$$F(t) = a_J \cos(2\pi\nu_J t + \phi_J)\exp(-\gamma_J t) + a_i \cos(2\pi\nu_i t + \phi_i)\exp(-\gamma_i t)$$

K Xhani and N P Proukakis, Phys. Rev. Research 4, 033205

Bosonic Josephson junction, role of finite temperature

Fixed N_{BEC} and V_0 at different T

Damped Josephson plasma regime

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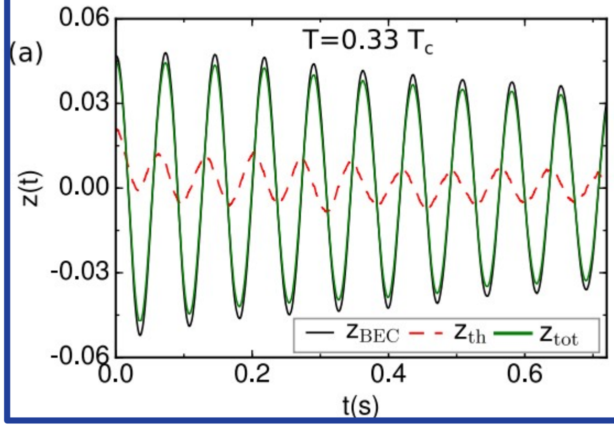
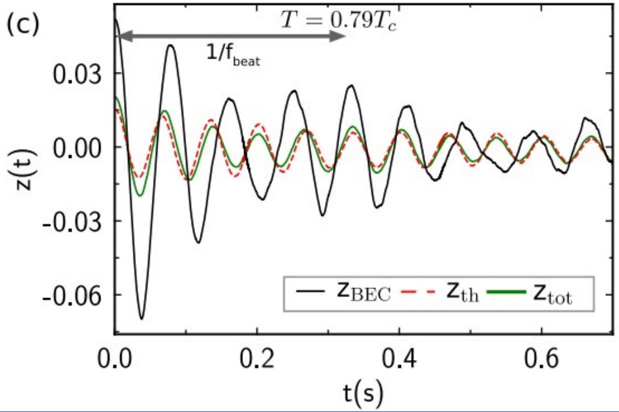
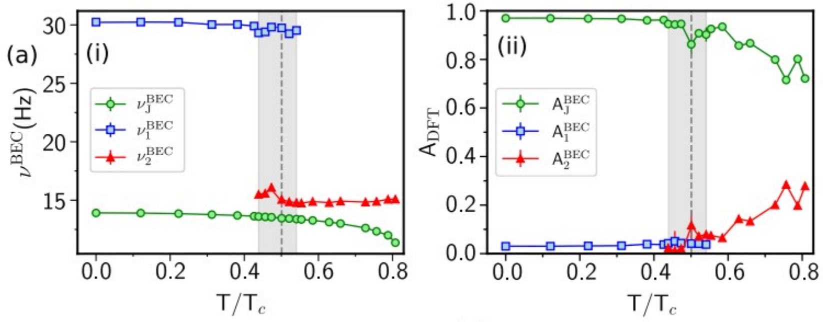


Large T: $k_B T>V_0$



Dependence on $k_B T/V_0$



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Large T: $k_B T>V_0$

Thermal cloud drives the condensate

Beating between condensate and thermal modes

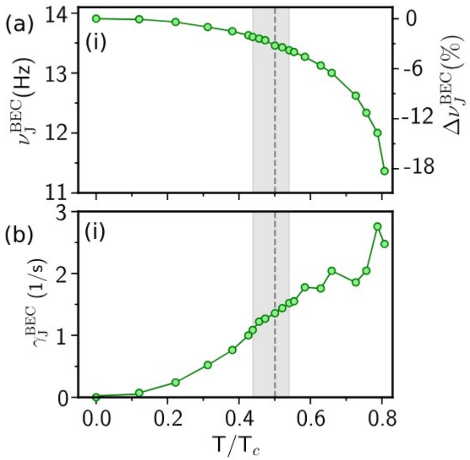
Main component: ν_J

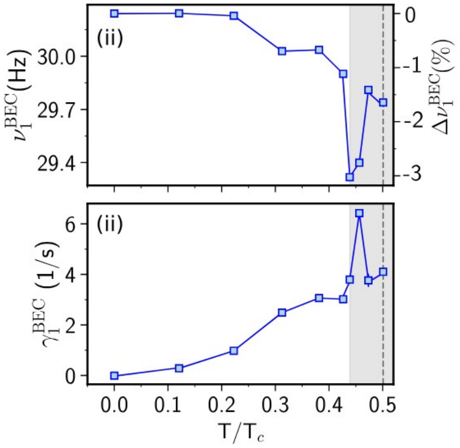
$\nu 1 \approx 2\nu_J$ component damped

New component: $\nu_2 = \nu_x \rightarrow$
induced by thermal cloud oscillations

Bosonic Josephson junction, role of finite temperature

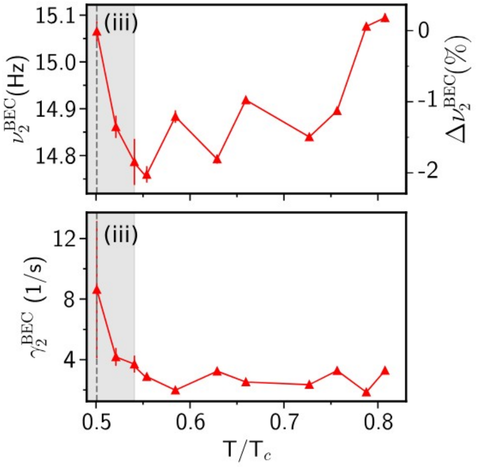
Damped Josephson plasma regime





ν_1 and ν_J values deacrease with T while their damping increase

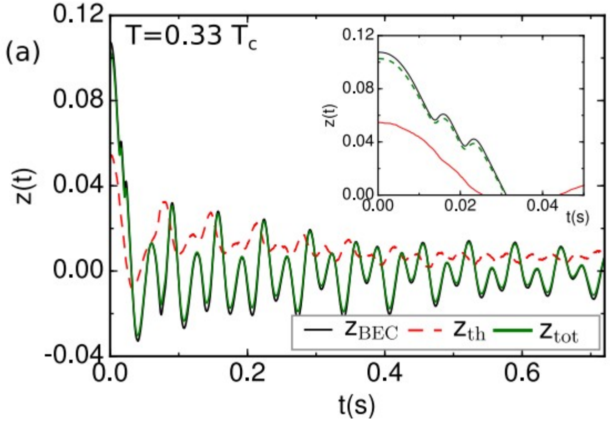
Opposite behaviour for ν_2

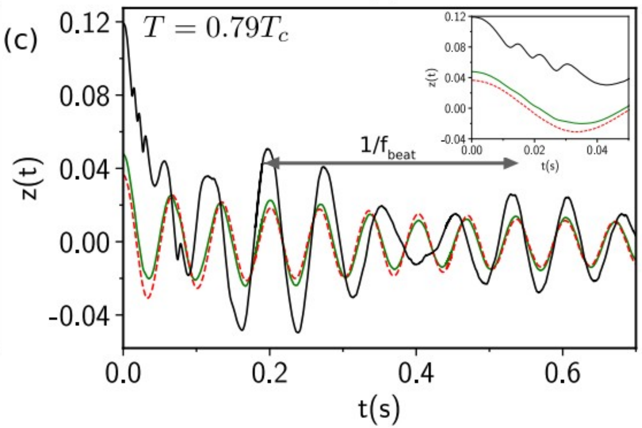


K Xhani and N P Proukakis, Phys. Rev. Research 4, 033205

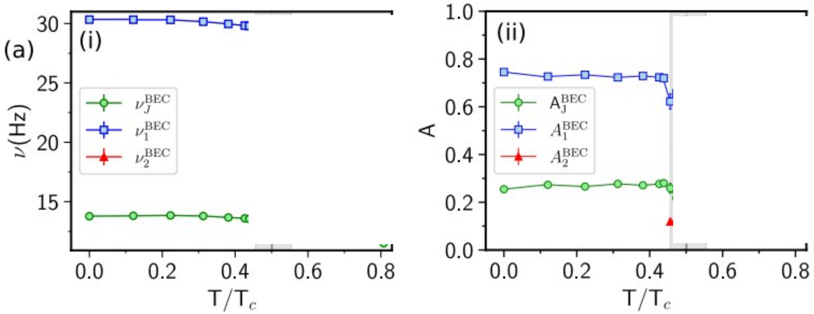
Bosonic Josephson junction, role of finite temperature

Vortex-induced dissipative regime





Dependence on k_BT/V_0



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Low T: $k_BT < V_0$

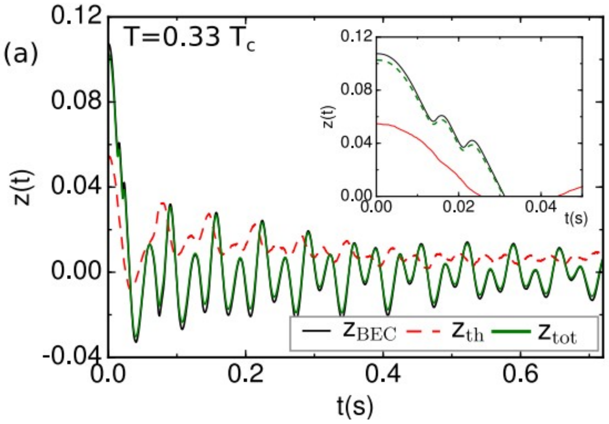
Main component : $\nu_1 \approx 2\nu_J$

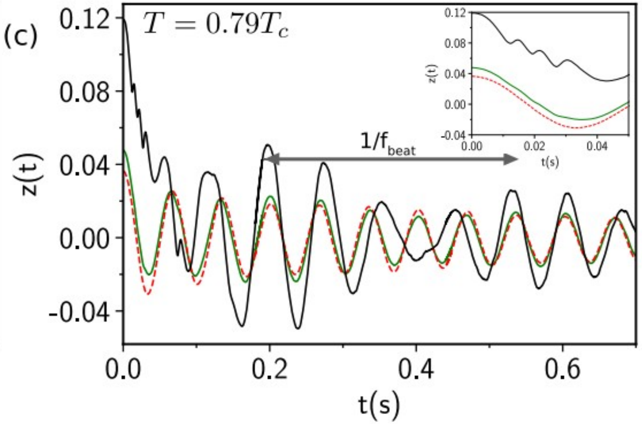
**Second component:
Josephson plasma frequency ν_J**

K Xhani and N P Proukakis, Phys. Rev. Research 4, 033205

Bosonic Josephson junction, role of finite temperature

Vortex-induced dissipative regime





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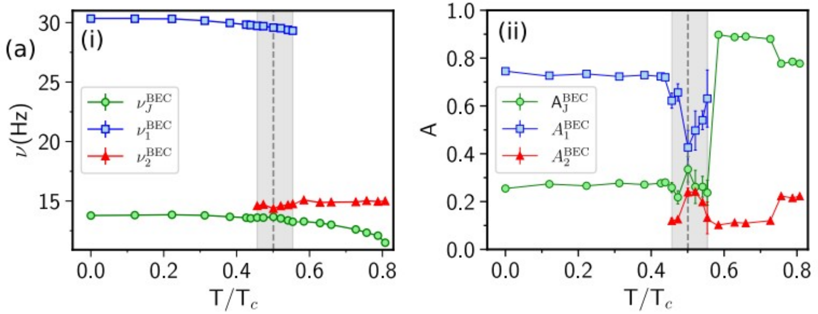

High T: $K_BT > V_0$

Main component : ν_J

Second component: ν_2

Beating between condensate and thermal cloud mode

$$z_{\rm BEC}(t) = \frac{N_R^{BEC}(t) - N_L^{BEC}(t)}{N_R^{BEC}(t) + N_L^{BEC}(t)}$$

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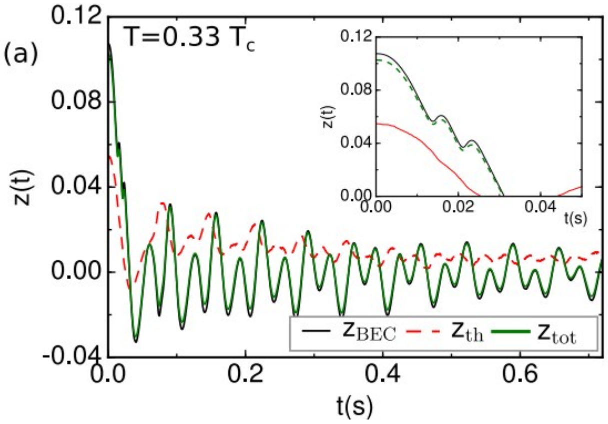
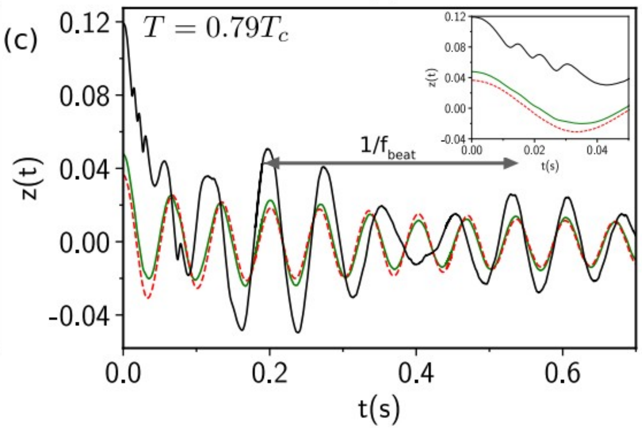
K Xhani and N P Proukakis, Phys. Rev. Research 4, 033205

Bosonic Josephson junction, role of finite temperature

Vortex-induced dissipative regime





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Dissipation mechanisms

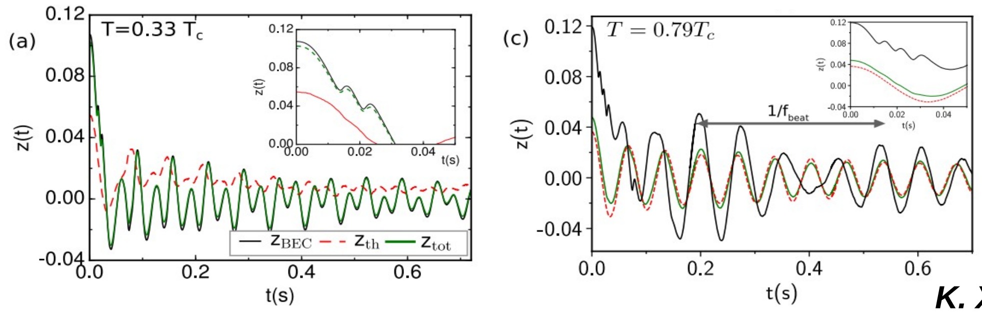
Short-time evolution:
Vortex-induced dissipation

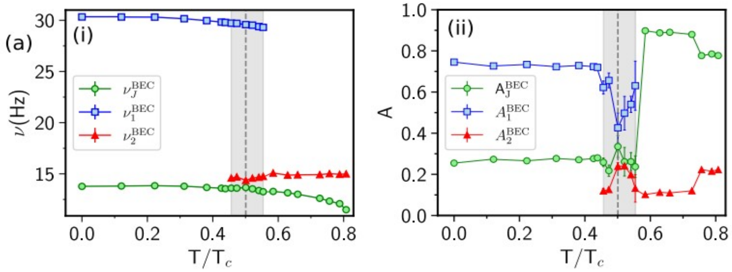
Long-time evolution:
Thermal cloud induced damping

K Xhani and N P Proukakis, Phys. Rev. Research 4, 033205

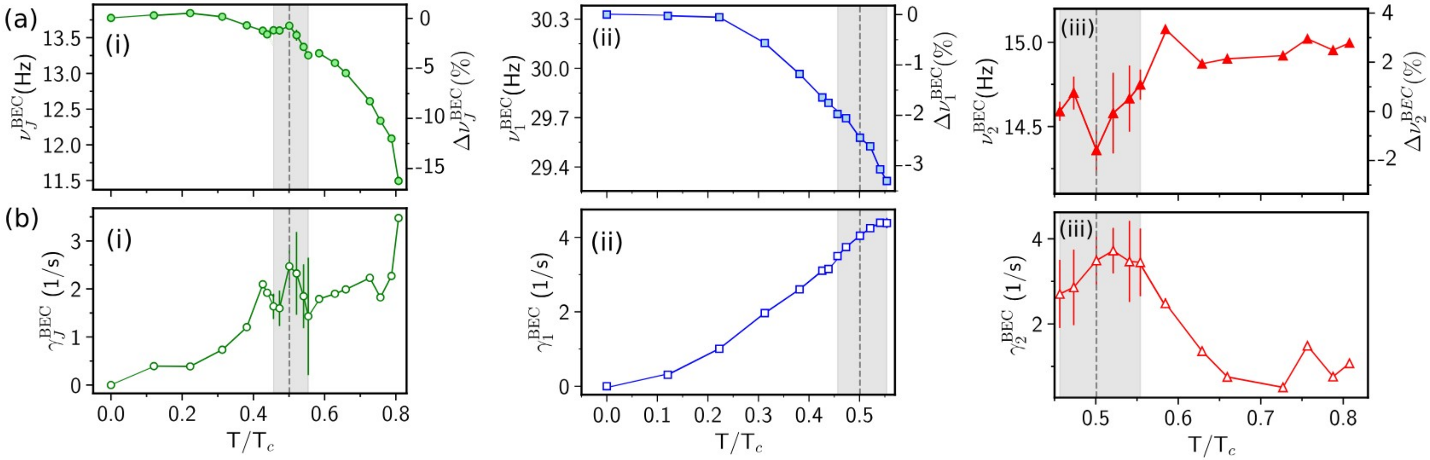
Bosonic Josephson junction: finite temperature

Vortex-induced dissipative regime





***K. Xhani** et al, Phys. Rev. Research 4, 033205 (2022)*

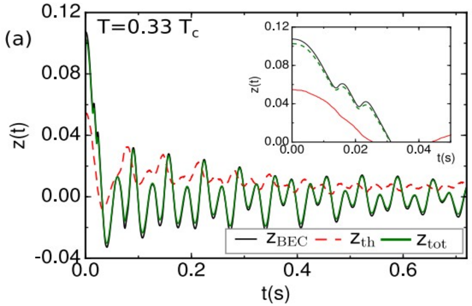


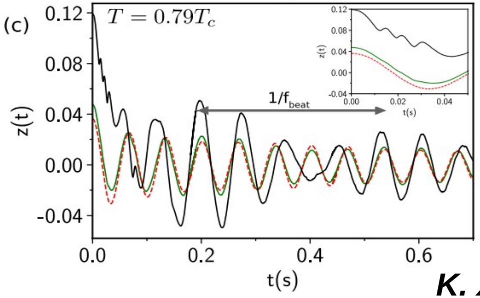
ν_1 and ν_J values deacrease with T while their damping increase

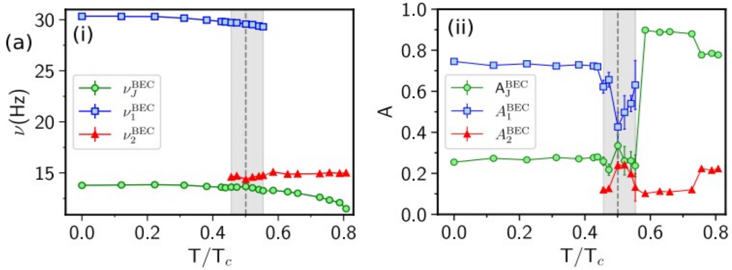
Opposite behaviour for ν_2

Bosonic Josephson junction: finite temperature

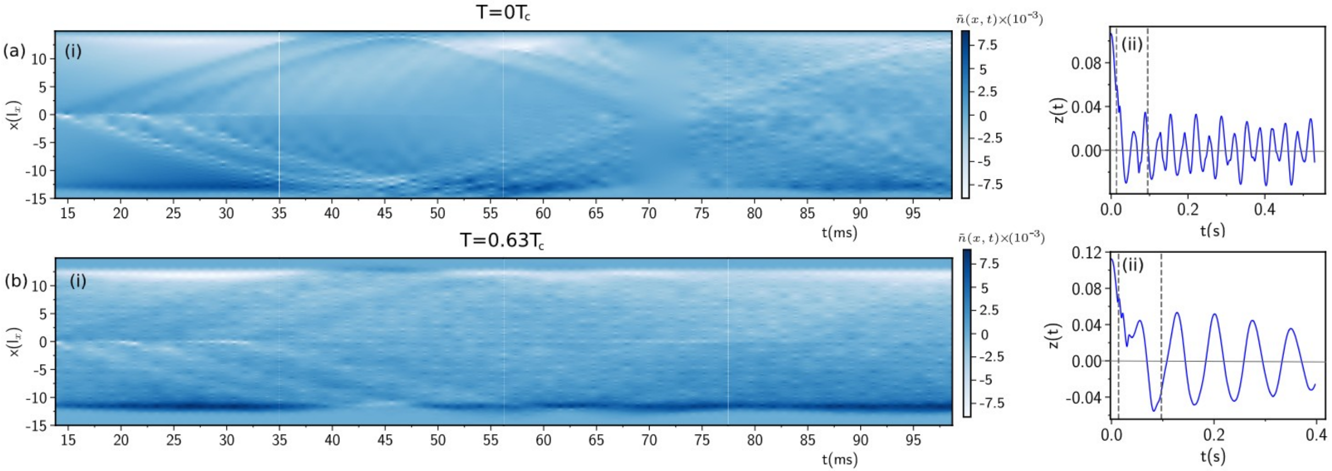
Vortex-induced dissipative regime







K. Xhani *et al, Phys. Rev. Research 4, 033205 (2022)*



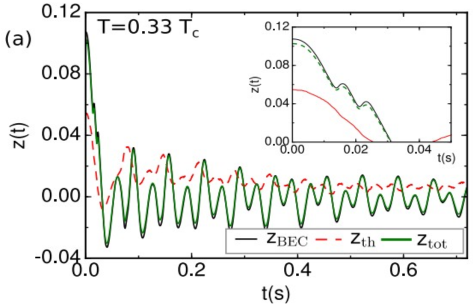
Damping of sound modes

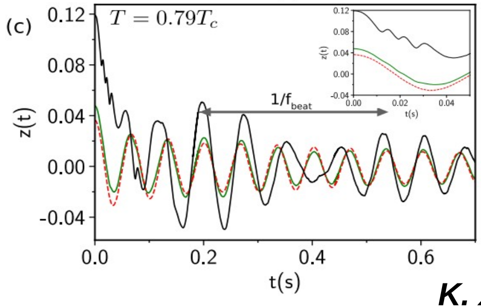
⇕ for $k_B T > V_0$

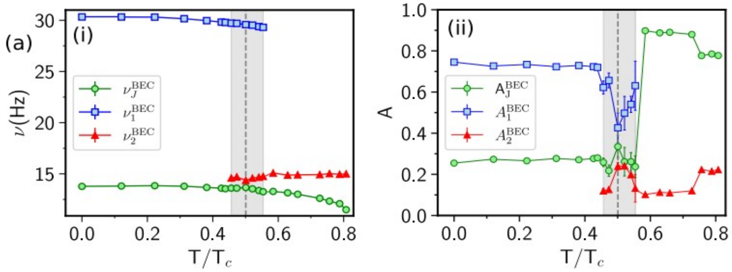
Damping of ν_1 component

Bosonic Josephson junction: finite temperature

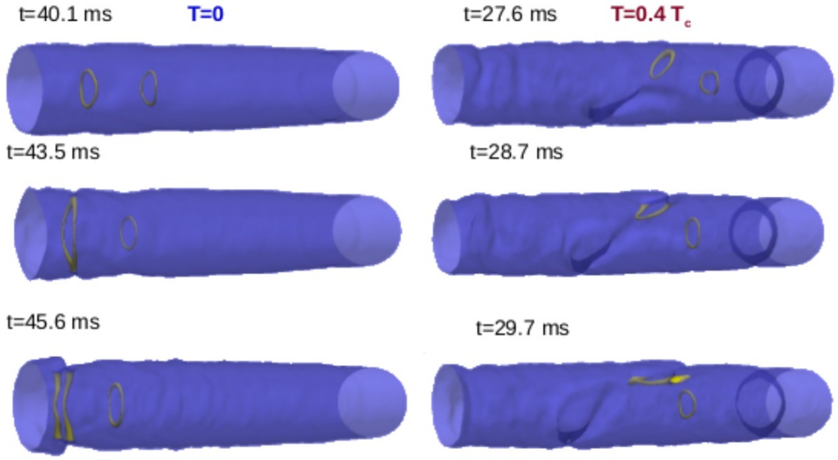
Vortex-induced dissipative regime







***K. Xhani** et al, Phys. Rev. Research 4, 033205 (2022)*

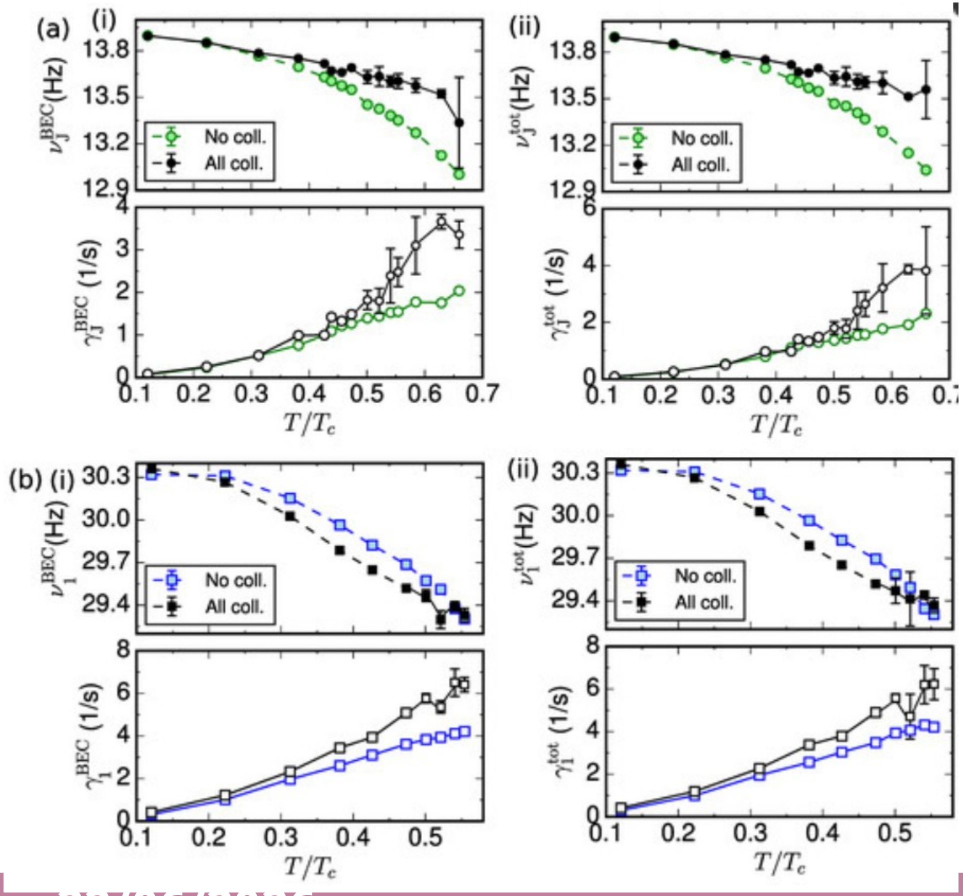


Thermal cloud drives vortex ring (VR) offcenter→ mutual friction

VR hits the boundary and breaks into single vortex line

***K. Xhani** et al, Phys. Rev. Lett. 124, 045301 (2020)*

Collisional limit: finite temperature



Josephson plasma

Vortex-induced dissipative regime

For T≤0.3 T_c collisions have no significant effect on the condensate dynamics

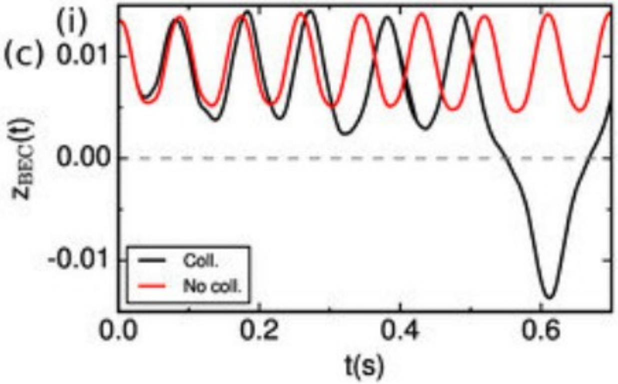
For higher T collisions increase the damping of condensate modes

Collisions slow down the decrease of ν_J with T

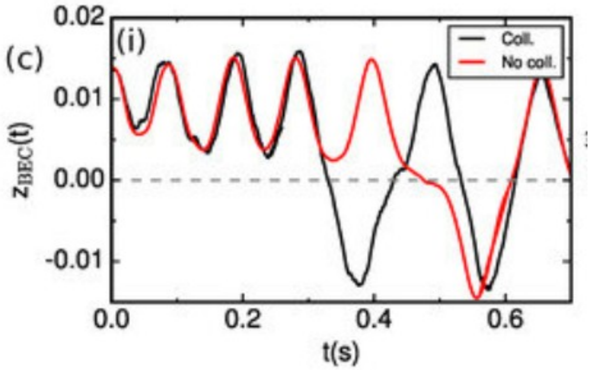
Xhani, K.; Proukakis, N.P. *Thermal–Condensate Collisional Effects on Atomic Josephson Junction Dynamics.* Atoms 2025, 13, 68.

Collisional limit and self-trapping: finite temperature

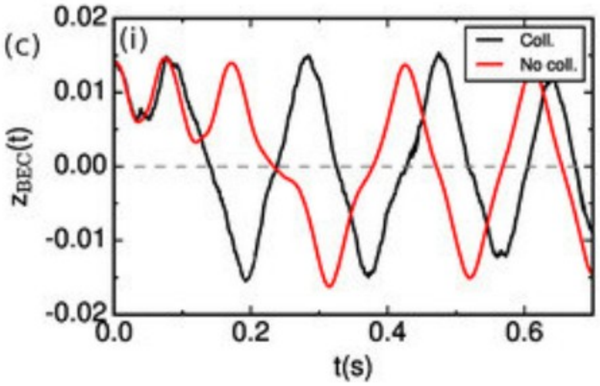
Self-trapping regime



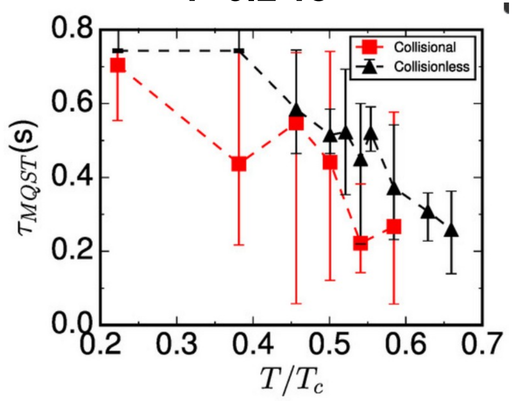
T=0.2 Tc



T=0.5 Tc



T=0.6 Tc



Collisions induce self-trapping decay even at low T

Agreement with experiments, no long-term self-trapping has been observed

Xhani, K.; Proukakis, N.P. Atoms 2025, 13, 68.

Fermionic Josephson junction: Theoretical model

Time-depedent Superfluid Local Density Approximation (SLDA) (a density functional theory)

$$i\frac{\partial}{\partial t}\begin{pmatrix} u_n(\mathbf{r},t) \\ v_n(\mathbf{r},t) \end{pmatrix} = \begin{pmatrix} h(\mathbf{r},t) & \Delta(\mathbf{r},t) \\ \Delta^*(\mathbf{r},t) & -h^*(\mathbf{r},t) \end{pmatrix} \begin{pmatrix} u_n(\mathbf{r},t) \\ v_n(\mathbf{r},t) \end{pmatrix}$$

$$h(\mathbf{r},t) = -\frac{\nabla^2}{2} + U(\mathbf{r},t) + V_{\text{ext}}(\mathbf{r},t) - \mu$$

$$U = \frac{\delta \mathcal{E}}{\delta n} \text{ and } \Delta = -\frac{\delta \mathcal{E}}{\delta \nu^*}$$

Anomalous density: $\nu = \sum_{E_n > 0} v_n^* u_n$

BCS, pairing potential: $\Delta(\mathbf{r},t) = -g\nu(\mathbf{r},t)$

$U_{BCS} \approx 0$

UFG: Pairing potential $\Delta^{(\text{UFG})} = -\frac{\gamma}{n^{1/3}}\nu$

$$U^{(\text{UFG})} = \frac{\beta(3\pi^2 n)^{2/3}}{2} - \frac{|\Delta|^2}{3\gamma n^{2/3}}$$

β and γ such that $\xi_0 \approx 0.4$ and the energy gap $\Delta/\epsilon_F \approx 0.5$

In agreement with Monte Carlo calculations and experimental data

Additional coupling between
the pairing and density modes

A. Bulgac et al.,Phys. Rev. A 76, 040502 (2007).

1million coupled equations to be solved
→ High performance supercomputer

Fermionic Josephson junction: Theoretical model

Time-depedent Superfluid Local Density Approximation (SLDA) (a density functional theory)

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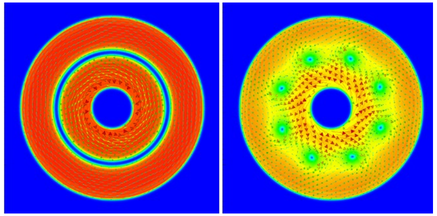
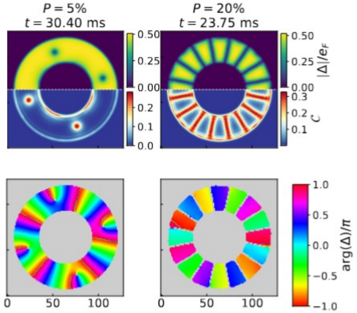
Talk by Michał Śliwiński

Kelvin-Helmholtz Instability in Fermionic Superfluids: Numerical Approach



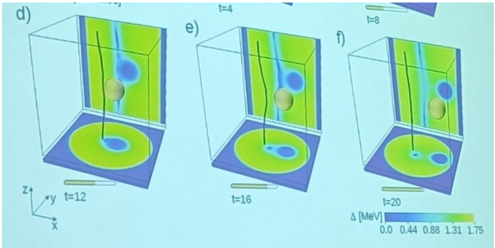
Talk by Bugra Tuzemen

Structure and Dynamics of Pairing Field in Inhomogeneous Fermi Superfluids



Talk by Daniel Pecak

Vortex-nucleus interaction in the inner crust of neutron star



Fermionic Josephson junction: Numerical simulations

1) Find the static solution in a tilted potential

$$i\frac{\partial}{\partial t} \rightarrow E_n$$

2) Solve the dynamics evolving $5\text{x}10^5$ quasiparticles states

Double-well potential-εx

in a Double-well potential

3D simulations but homogenous along 3rd direction: The quasi-particle wave function $\varphi_n(\mathbf{r},t) \equiv \varphi_n(x,y,t)e^{ik_z z}$

$$V_{\text{ho}}(x,y) = m\omega_x^2(x^2 + \lambda^2 y^2)/2 \qquad \lambda = \omega_y/\omega_x = 148/15 \qquad V_{\text{b}}(\mathbf{r}) = V_0 e^{-2x^2/w^2}$$

Fixed barrier width wk_F=5.2

Variable barrier height

$$k_F = \sqrt{2\varepsilon_F} = (3\pi^2 n)^{1/3}$$

Grid points

$$N_x \times N_y \times N_z = 768 \times 96 \times 24$$

Investigate two-interaction regimes:

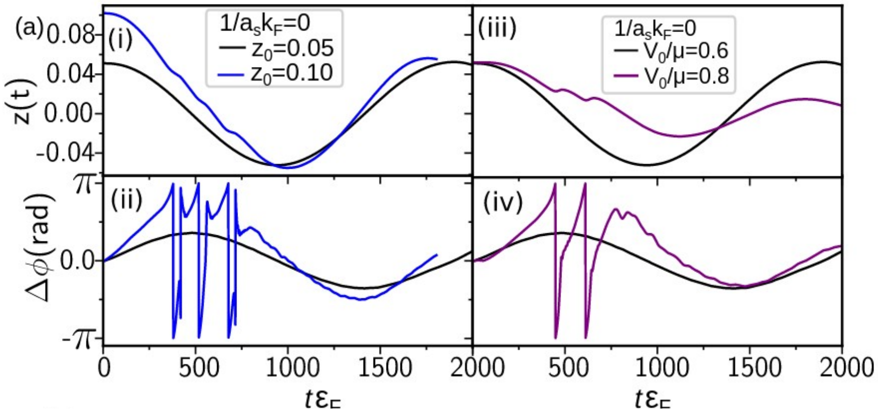
Strongly interacting or UFG limit ($1/k_F a=0$) and

Weakly attractive or BCS ($1/k_F a=-1$) regime

Fermionic Josephson junction: Dynamical regimes

G. Wlazłowski, **K. Xhani**, M. Tylutki, N. P. Proukakis, and P. Magierski, Phys. Rev. Lett. 130, 023003 (2023)

UFG limit



Transition from coherent Josephson plasma to phase-slippage induced dissipation

Fermionic Josephson junction: Dynamical regimes

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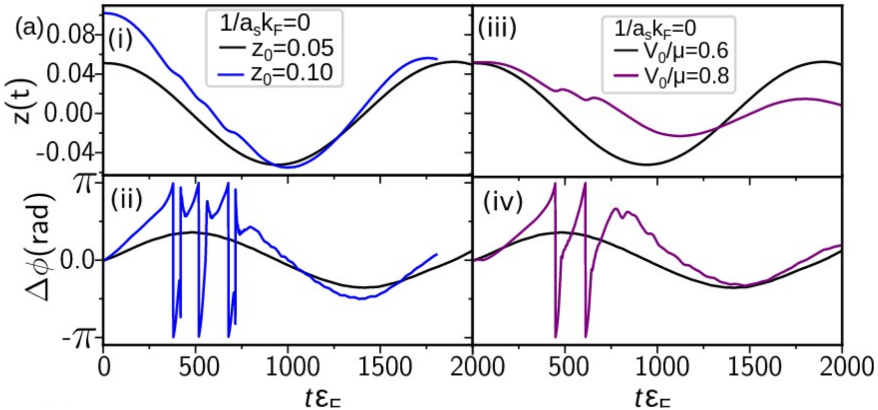
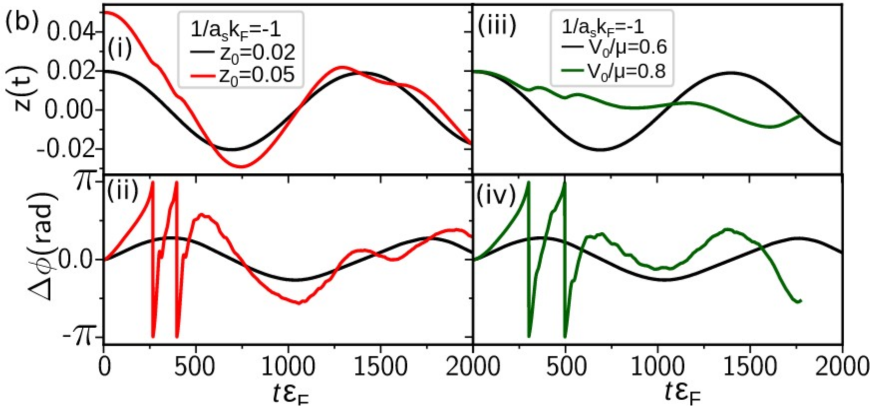


Transition from coherent Josephson plasma to phase-slippage induced dissipation

BCS limit



Transition from coherent Josephson plasma to phase-slippage induced dissipation

Similar z(t) or current temporal profiles

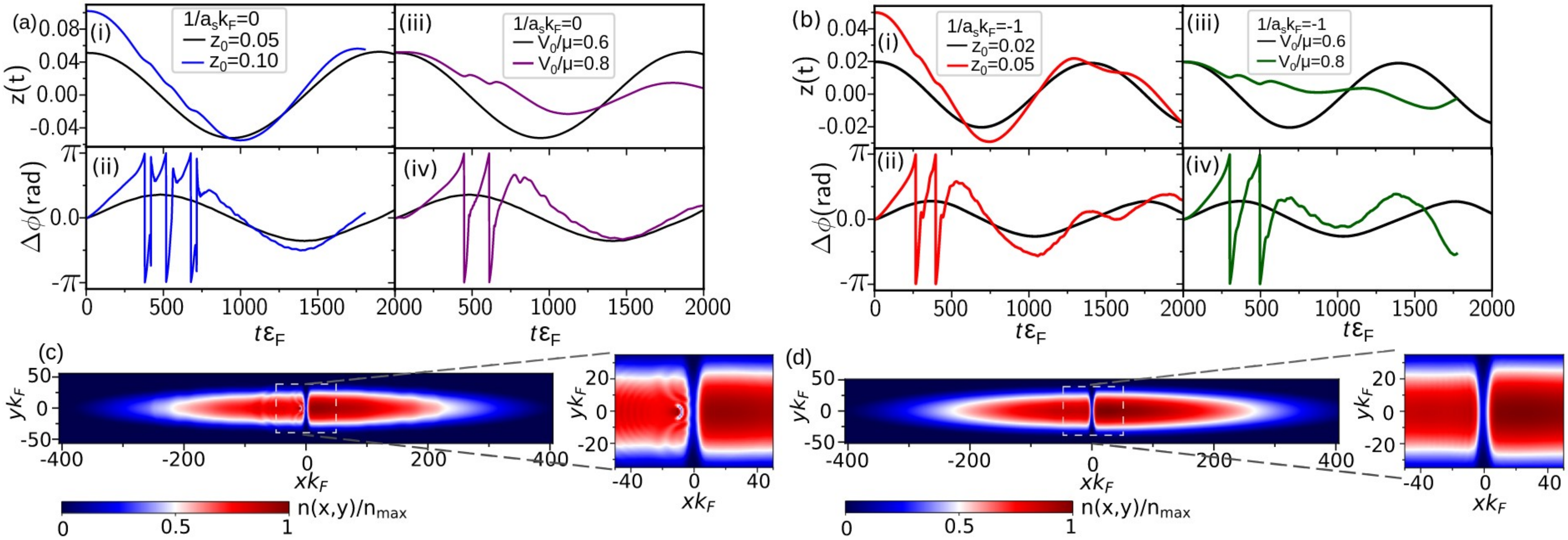
Lower values of the critical imbalance and of the critical current in BCS regime for the same barrier height

Consistent with the results in L. Salasnich et al., Laser Phys. 19, 636 (2009),
M. Zaccanti and W. Zwerger, Phys. Rev. A 100, 063601 (2019)

Fermionic Josephson junction: Dynamical regimes

G. Wlazłowski, **K. Xhani**, M. Tylutki, N. P. Proukakis, and P. Magierski, Phys. Rev. Lett. 130, 023003 (2023)

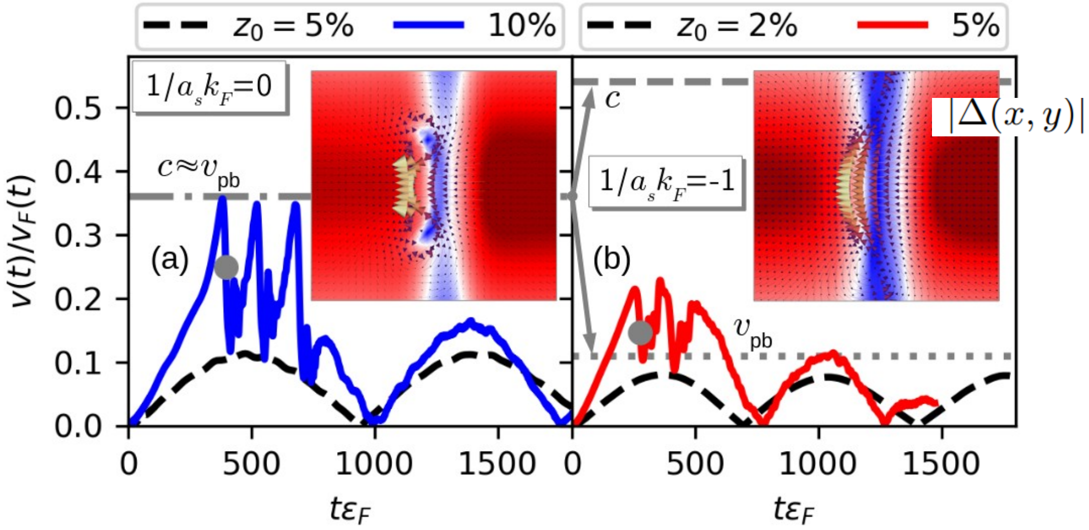
UFG limit **BCS limit**



Vortices propagation and sound waves **Only low-amplitude sound waves**

Critical velocity

fixed $wk_F = 5.2$ and fixed $V_0/\mu = 0.6$



UFG limit: v_{max} exceeds c

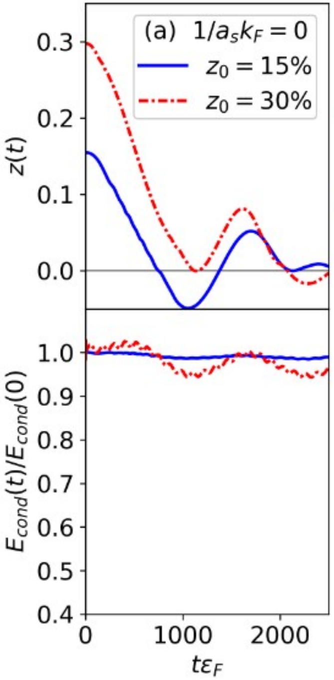
BCS limit: v_{max} exceeds v_{pb}

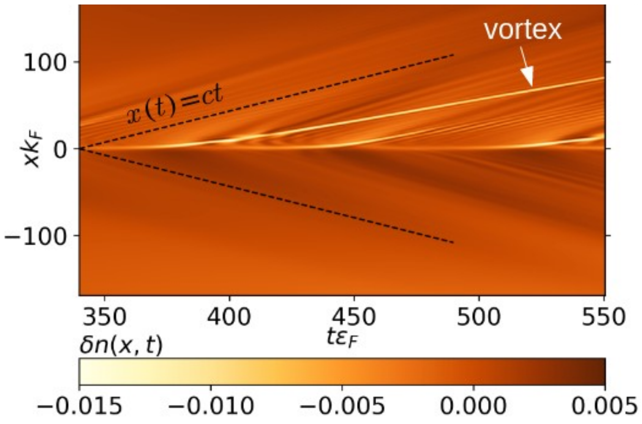
UFG limit vortex dipoles generation and propagation

$$\boldsymbol{j} = 2\sum_{E_n>0} \mathrm{Im}\left[v_n \boldsymbol{\nabla} v_n^*\right] \qquad v(t) \equiv |\boldsymbol{v}(0,t)|$$

$$\boldsymbol{v}(\boldsymbol{r},t) = \boldsymbol{j}(\boldsymbol{r},t)/n(\boldsymbol{r},t) \qquad v_F(t) = [3\pi^2 n(0,t)]^{1/3}$$

Dissipation mechanisms of a fermionic Josephson junction





Condensation energy

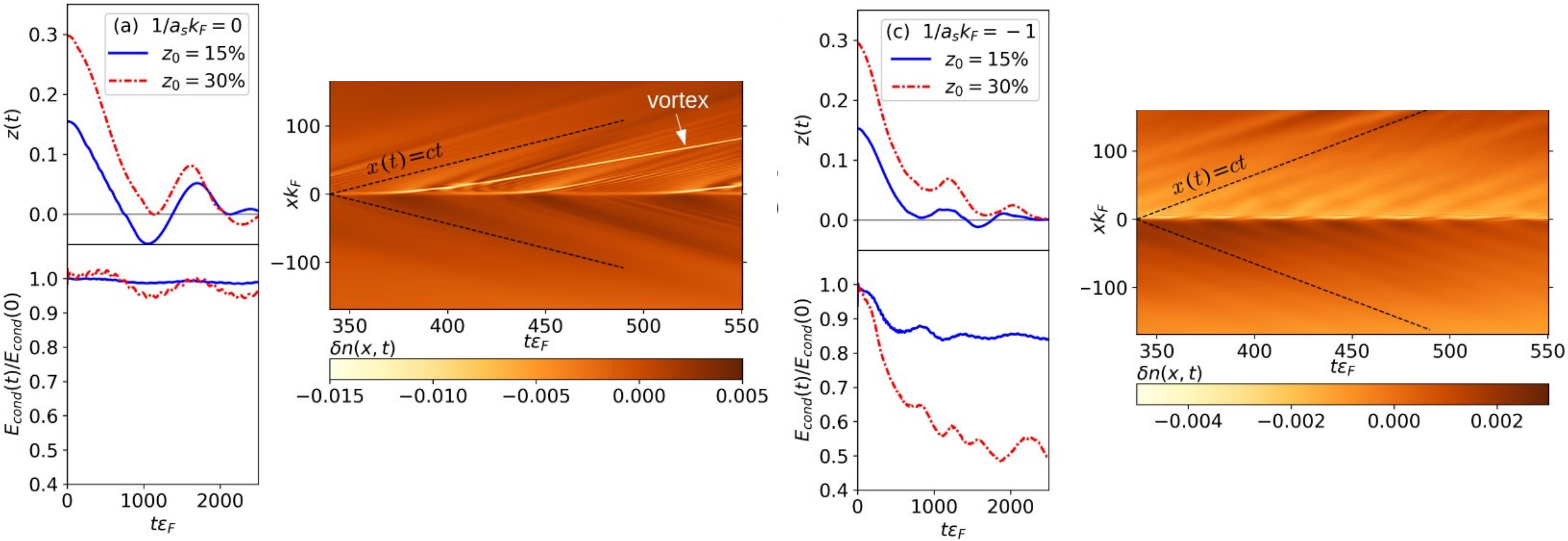
$$E_{\text{cond}} = \int \frac{3}{8} \frac{|\Delta(\mathbf{r})|^2}{\varepsilon_F(\mathbf{r})} n(\mathbf{r}) d\mathbf{r}$$

UFG limit dissipative mechanism: vortices and/or sound waves

BCS limit: mainly pair-breaking

Pair-breaking larger for larger z0 and from UFG to BCS limit

Dissipation mechanisms of a fermionic Josephson junction



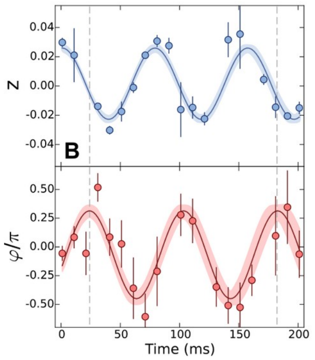
Condensation energy

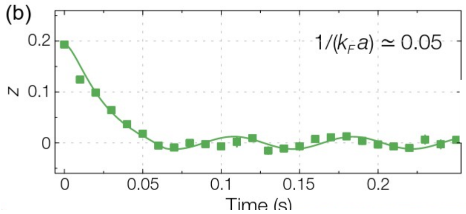
$$E_{\rm cond} = \int \frac{3}{8} \frac{|\Delta(\mathbf{r})|^2}{\varepsilon_F(\mathbf{r})} n(\mathbf{r}) d\mathbf{r}$$

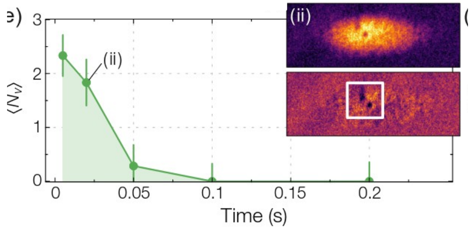
UFG limit dissipative mechanism: vortices and/or sound waves
BCS limit: mainly pair-breaking
Pair-breaking larger for larger z0 and from UFG to BCS limit

UFG limit: Comparison with experimental data

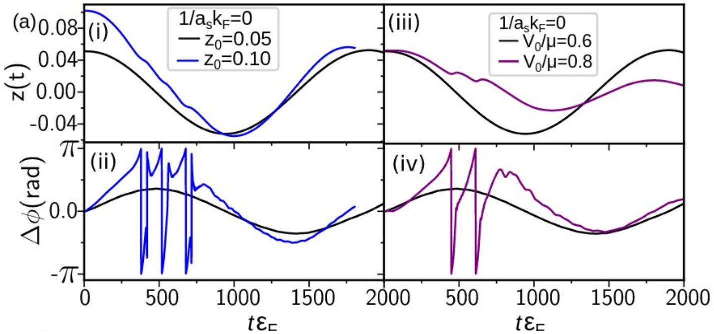
Experimental Dynamical regime: Josephson plasma and dissipative regime.
Dissipation induced by phase-slippage. Vortices propagate.

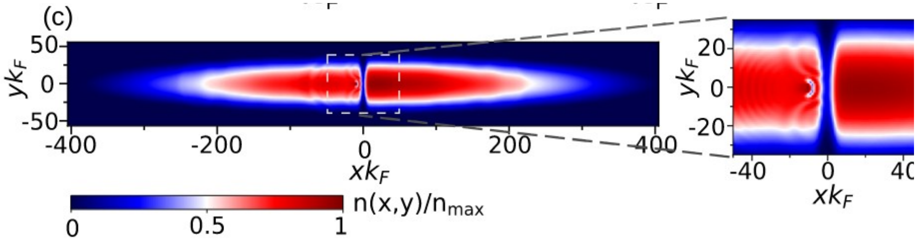






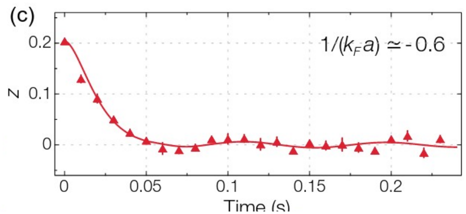
Theoretical Dynamical regime: Josephson plasma and dissipative regime.
Dissipation induced by phase-slippage. Vortices could propagate.

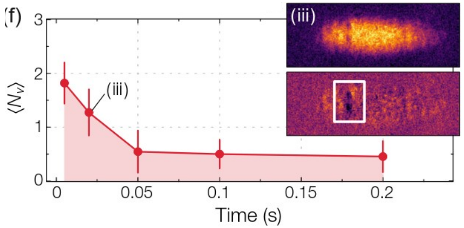




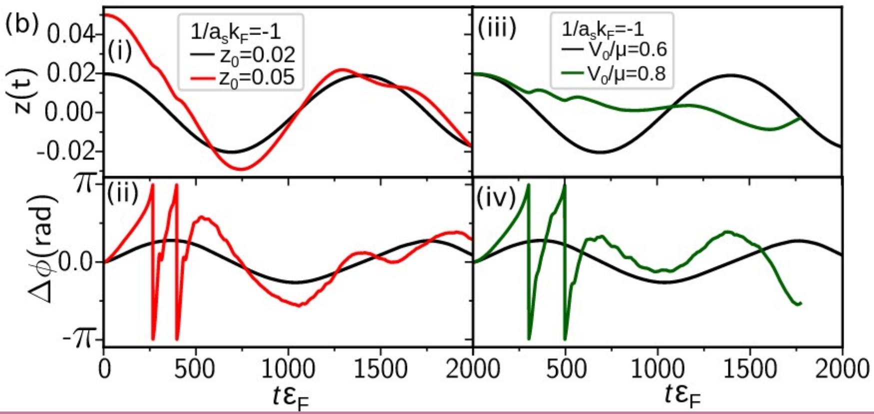
BCS limit: Comparison with experimental data

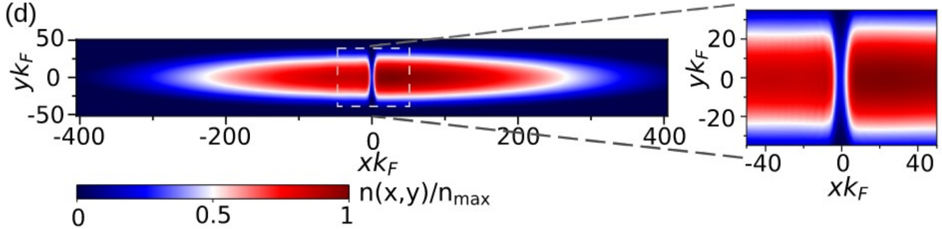
Experimental Dynamical regime: Josephson plasma and dissipative regime.
Dissipation induced by phase-slippage. Vortices propagate.





Theoretical Dynamical regime: Josephson plasma and dissipative regime.
Dissipation induced by pair-breaking. Phase-slippage present but Vortices do not propagate.

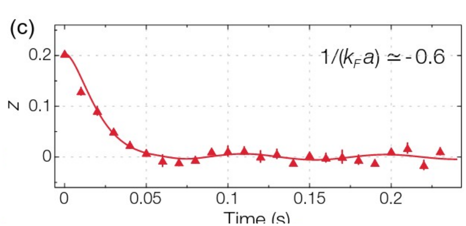


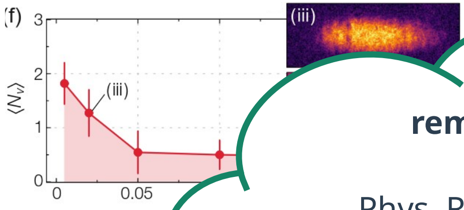


Deeper in BCS regime

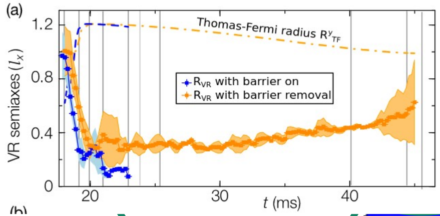
BCS limit: Comparison with experimental data

Experimental Dynamical regime: Josephson plasma and dissipative regime.
Dissipation induced by phase-slippage. Vortices propagate.

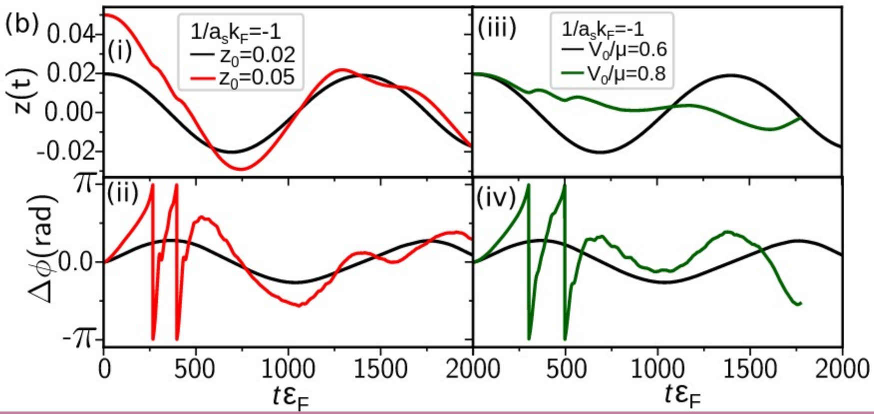


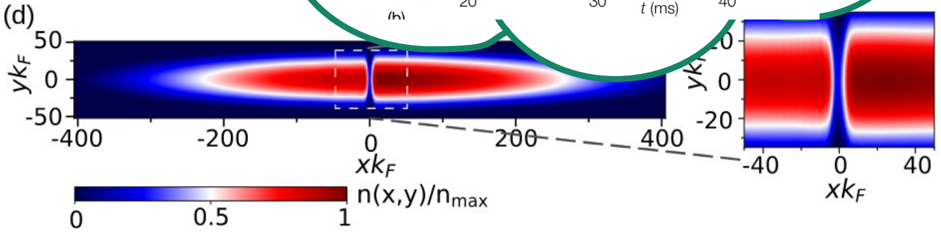


Experiment: barrier removal to detect vortices
K. Xhani et al,
Phys. Rev. Lett. 124, 045301 (2020)



Theoretical Dynamical regime: Josephson plasma a
Dissipation induced by pair-breaking. Phase-slippage present



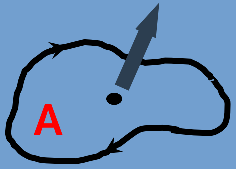


Deeper in BCS regime

Ultracold gases in ring traps

Madelung transformation: Superfluid wavefunction $\psi(\boldsymbol{r}) = \sqrt{\rho(\boldsymbol{r})} e^{i\phi(\boldsymbol{r})}$ $\quad \boldsymbol{v} = \frac{\hbar}{M} \nabla \phi$

If a hole is present



Multiple-connected geometry

Multiple-connected geometry

$$\Gamma = \oint_A \boldsymbol{v} \cdot d\boldsymbol{l} = \frac{\hbar}{M} \oint_A \nabla \boldsymbol{\phi} \cdot d\boldsymbol{l}$$

As Ψ is single-valued →

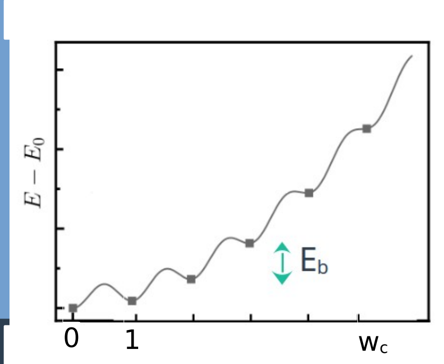
Γ is quantized

$$\Gamma = \frac{\hbar}{M} 2\pi w$$

As for a vortex or persistent current in a ring

winding number
w→integer number

Persistent current states, the minima of the washboard potential.



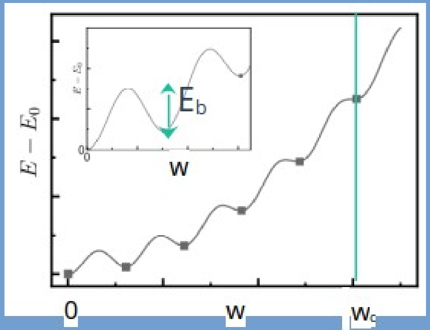
Dissipationless current for long time→ persistent current

Ultracold gases in ring traps

Circulation Γ is quantized

$$\frac{m}{\hbar}\oint_\Gamma d\mathbf{r}\cdot\mathbf{v}(\mathbf{r}) = 2\pi w,$$

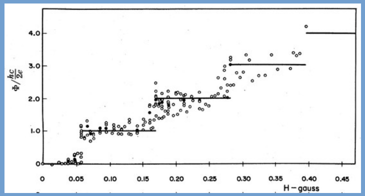
Persistent current states,
the minima of the washboard potential.



The energy barrier E_b goes to zero for $w \to w_c$

The presence of a defect decreases E_b and thus w_c

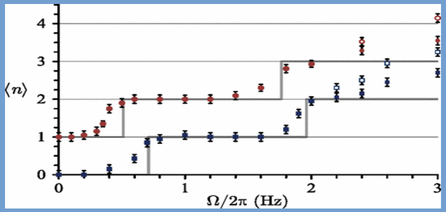
Superconducting rings pierced by magnetic fields



B. S. Deaver et al., PRL 7, 43 (1961)

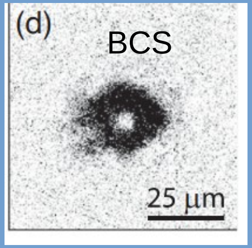
Landrò, E. et al, Eur. Phys. J. B 98, 7 (2025).

Stirring a beam at certain Ω



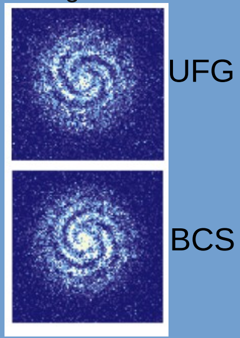
K. Wright et al,
PRL, 110, 025302(2013)

Stirring technique

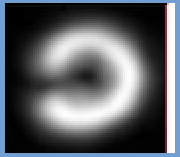


Y. Cai at al.,
PRL. 128, 150401 (2022)

Optical phase imprinting via DMD



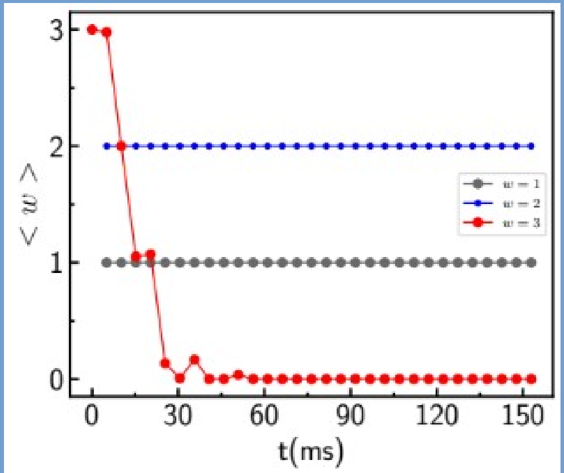
G. Del Pace, K.Xhani et al.,
Phys. Rev. X 12, 041037 (2022)



Voronova, N., et al.
Exciton-polariton ring Josephson junction.
Nat Commun 16, 466 (2025).

Single Josephson junction

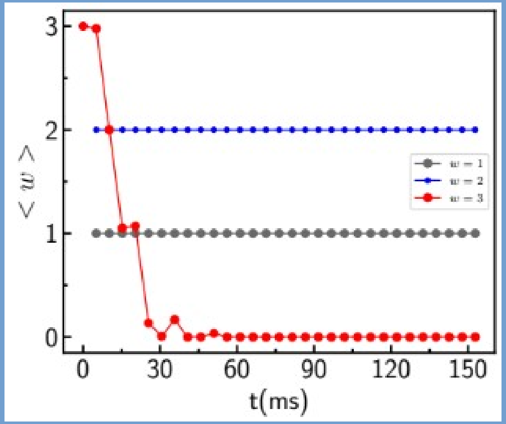
$V_0/\mu=1.4$



For $w_0 > w_c$, the current or w(t) decays
vortices emission

Single Josephson junction

$V_0/\mu = 1.4$

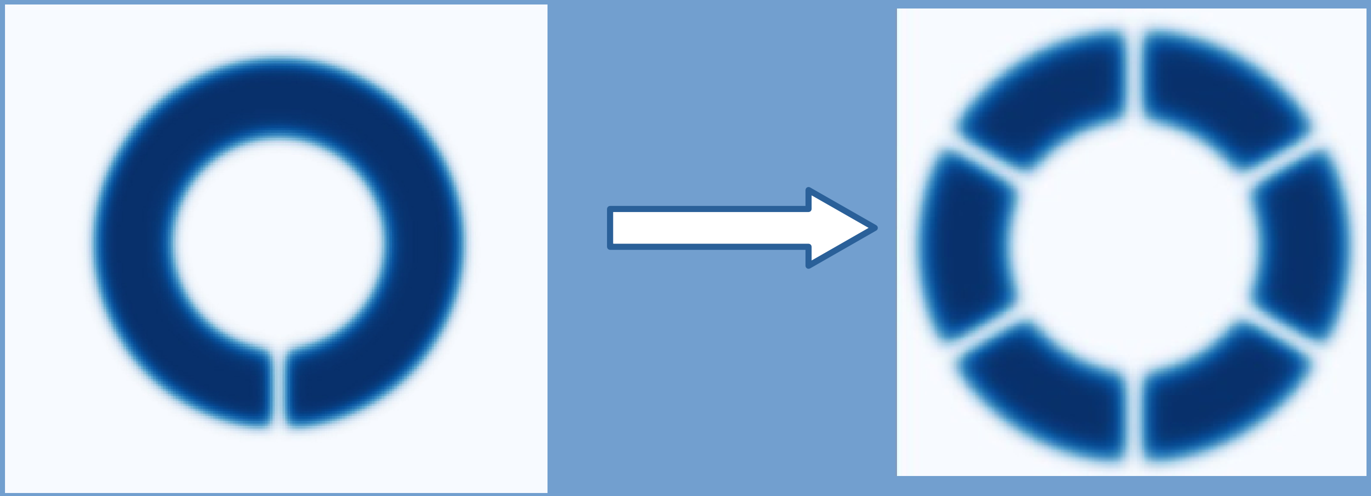


For $w_0 > w_c$, the current or w(t) decays vortices emission

time

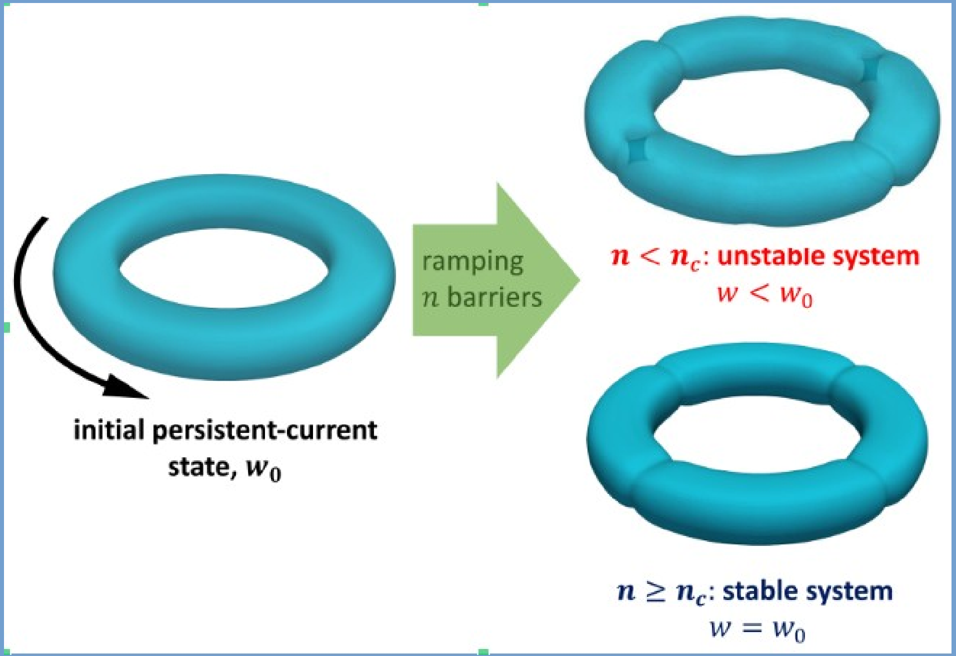
From single to many Josephson junctions

What happens if we add more barriers?



Josephson junction arrays

From single to many Josephson junctions

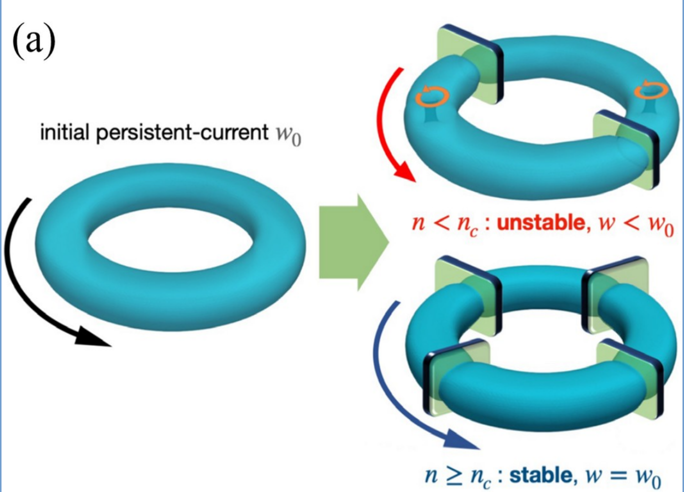


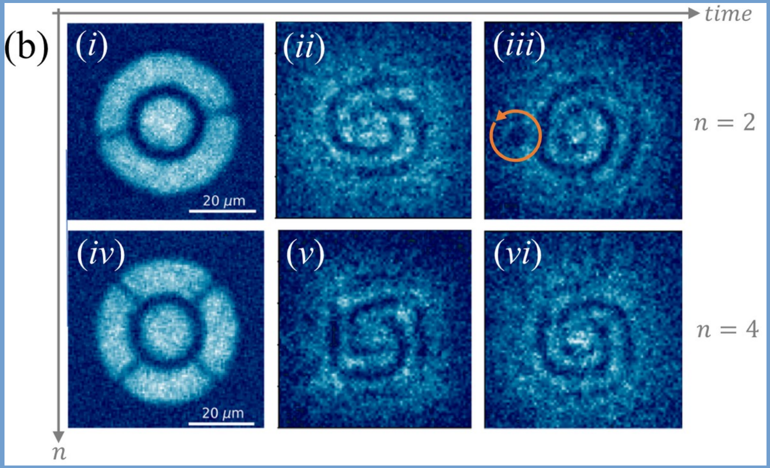
For fixed w_0, the current goes from **unstable to stable** by **increasing** the number of **barriers n** larger than a critical value n_c

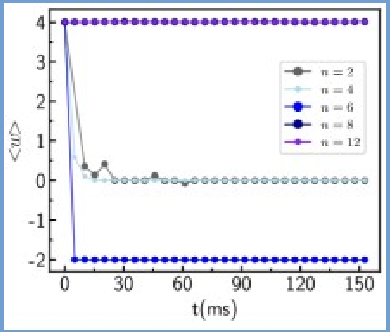
L. Pezzè*, K. Xhani*, C. Daix*, et al., Nature Commun 15, 4831 (2024).

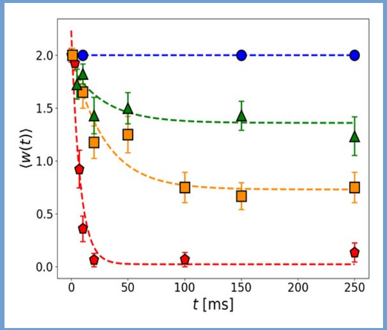
Stabilizing persistent currents in an atomtronic Josephson junction necklace (BEC limit)

We study the stability diagram of the superfluid by tuning both the initial circulation and the number of tunneling barriers (V0/μ>1).







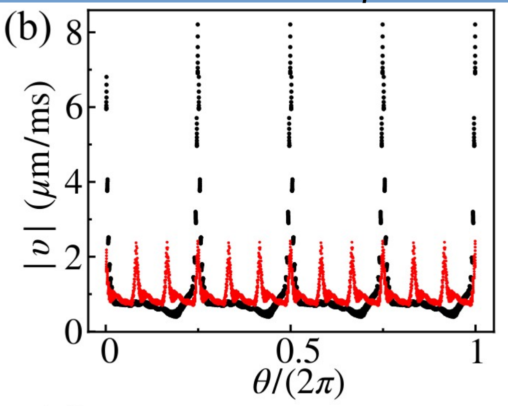


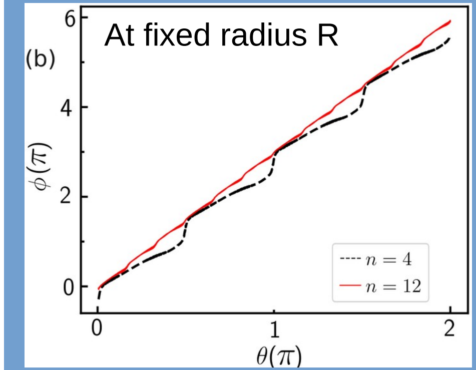
Both experiment and theory for fixed circulation the supercurrent becomes stable for a number of barriers larger than a critical value n_c.

L. Pezzè*, K. Xhani*, C. Daix*, et al., Nature Commun 15, 4831 (2024).

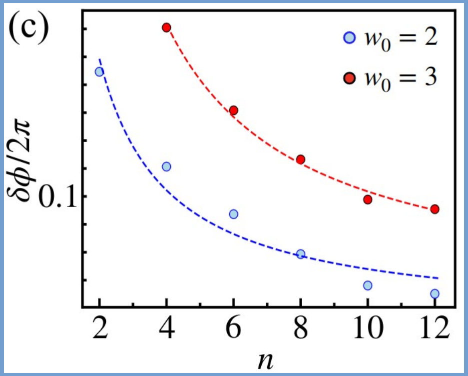
Critical current in a Josephson junction necklace: BEC limit

3D time-dependent GPE simulations

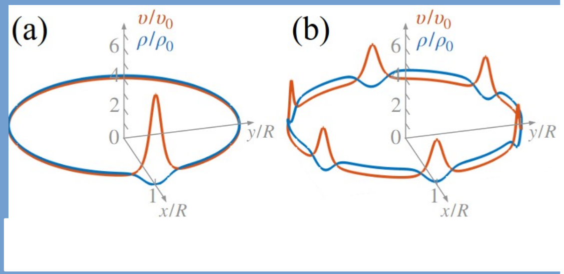


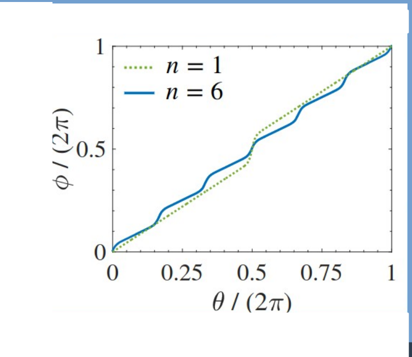


At fixed w0



1D stationary GPE simulations
(rotating also the barriers at Ω)





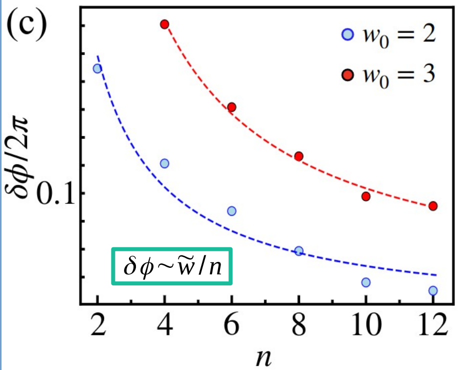
The Josephson phase δΦ decreases while increasing n

The maximum velocity descrease with junctions number n

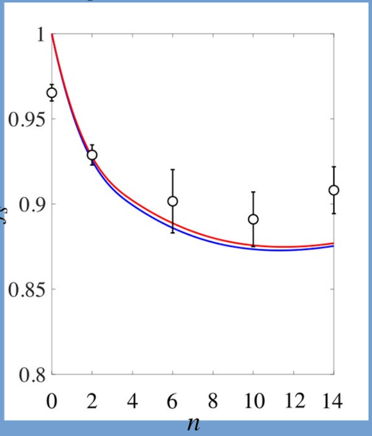
⇩

Stabilizing the supercurrent upon increasing n

Critical current in a Josephson junction necklace: BEC limit



Superfluid fraction



1D analytical model

$$\frac{m}{\hbar}\oint_{\Gamma} d\mathbf{r}\cdot\mathbf{v}(\mathbf{r}) = 2\pi w.$$

$$v_{\text{bulk}} + \frac{1}{2\pi}\int_0^{2\pi} d\theta\, v_{n-\text{peaks}}(\theta) = \frac{\hbar w}{mR}$$

The Josephson phase δΦ decreases while increasing n

$$\delta\phi = \frac{2\pi\tilde{w}}{n}\left(1-\frac{f(\tilde{w},n)}{2\pi\rho_{\text{bulk}}(\tilde{w},n)}\right)$$

$\tilde{w} = w - \Omega/\Omega_R$

$f(\tilde{w},n) \equiv (2\pi)^2\left[\int d\theta/\rho(\theta)\right]^{-1} \le f_s$

$\delta\phi \sim \tilde{w}/n$

$f(w=0)=f_s$ with f_s the superfluid fraction

Leggett's criteria

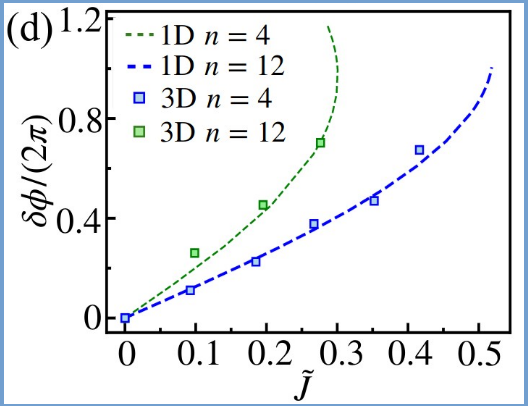
$$\iint \frac{dz\,dr\,r}{\frac{1}{d^2}\int_{\text{cell}}\frac{d\theta}{|\psi(\mathbf{r})|^2}} \le f_s \le \left(\frac{1}{d^2}\int_{\text{cell}}\frac{d\theta}{\iint dz\,dr\,r|\psi(\mathbf{r})|^2}\right)^{-1}.$$

$f(w, n) \le f_s$; $f_c = f(w=w_c, n)$

$d = 2\pi/n$

A. J. Leggett, "Can a solid be "superfluid"?," Phys. Rev. Lett., vol. 25, pp. 1543–1546, Nov 1970.
A. J. Leggett, "On the superfluid fraction of an arbitrary many-body system at t=0," Journal of Statistical Physics, vol. 93, pp. 927–941, Nov. 1998.

Critical current in a Josephson junction necklace: BEC limit



$$\delta\phi = \sin^{-1}(\tilde{J}/\tilde{J}_c)$$

Critical current ⟺ **$\delta\Phi_c = \pi/2$**

$$\frac{m}{\hbar}\oint_\Gamma d\boldsymbol{r}\cdot\boldsymbol{v}(\boldsymbol{r}) = 2\pi w.$$

$$v_{\text{bulk}} + \frac{1}{2\pi}\int_0^{2\pi} d\theta\, v_{n-\text{peaks}}(\theta) = \frac{\hbar w}{mR}$$

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$$\tilde{w} = w - \Omega/\Omega_R$$

$$\delta\phi \sim \tilde{w}/n$$

$\delta\Phi_c = \pi/2$

$$w_c \sim \tilde{n}$$

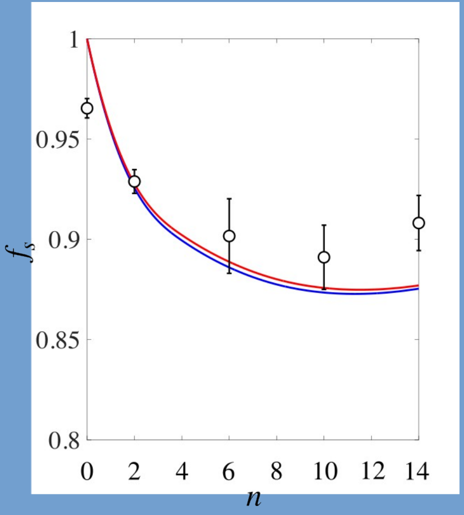
Critical current increases with n

$$J_c = J(\tilde{w} = \tilde{w}_c) \sim n f(\tilde{w} = \tilde{w}_c)$$

Critical current in a Josephson junction necklace: BEC limit

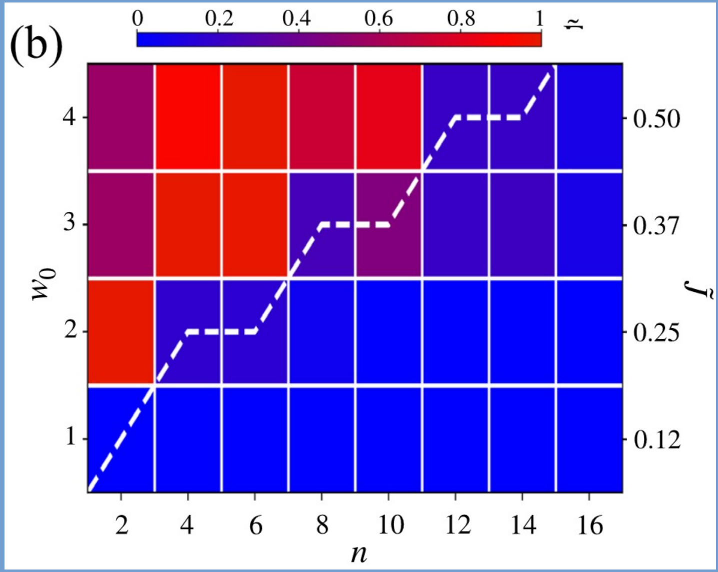
L. Pezzè*, K. Xhani*, C. Daix*, et al., Nature Commun 15, 4831 (2024).

Superfluid fraction slightly decreases with n



Both experiment and theory: Critical current increases with n

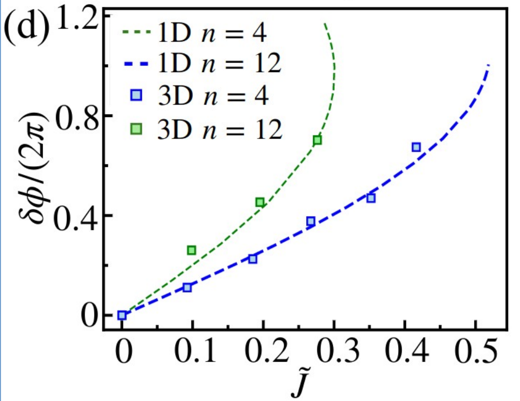
$$J_c = J(\tilde{w} = \tilde{w}_c) \sim n f(\tilde{w} = \tilde{w}_c)$$

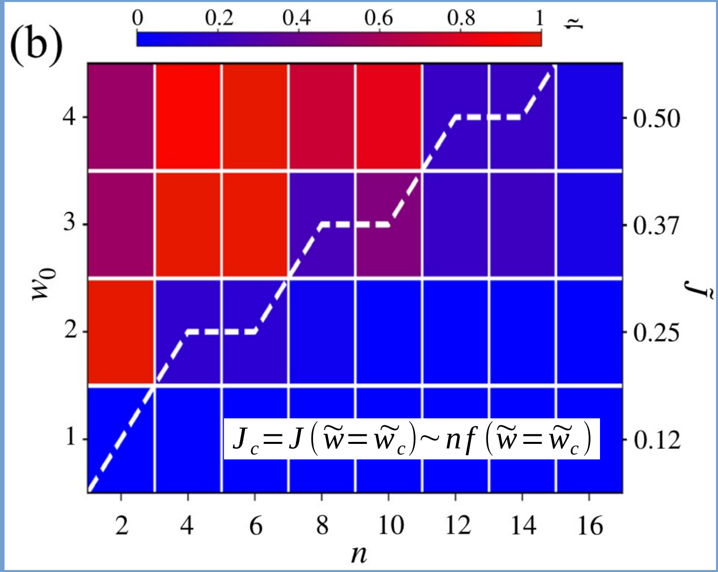


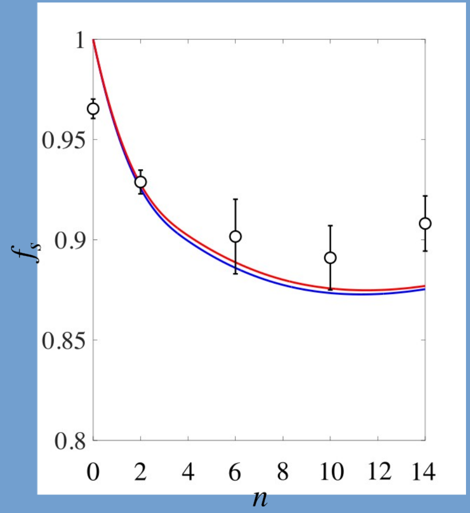
Stabilizing persistent currents in an atomtronic Josephson necklace

Sinusoidal current-phase relation

L. Pezzè*, K. Xhani*, C. Daix*, et al., Nature Commun 15, 4831 (2024).







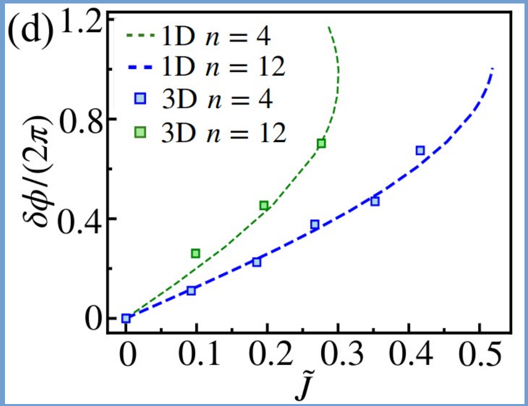
Although the superfluid fraction decreases with n, the critical current increases with the number of junctions

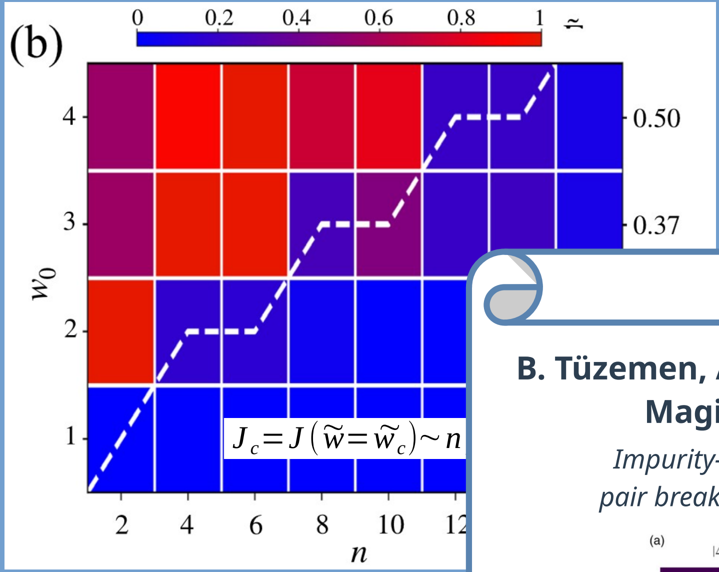
How would these results change for strongly interacting unitarity Fermi gas? **Work in progress**

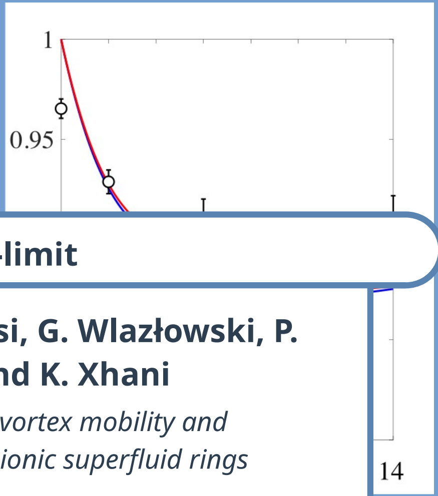
Stabilizing persistent currents in an atomtronic Josephson necklace

L. Pezzè*, K. Xhani*, C. Daix*, et al., Nature Commun 15, 4831 (2024).

Sinusoidal current-phase relation







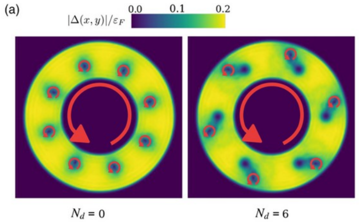
Although the superfluid fraction decreases with n, the critical curren

How would these results change for strongly interacting unitarity Fer

BCS-limit

B. Tüzemen, A. Barresi, G. Wlazłowski, P. Magierski, and K. Xhani

Impurity-controlled vortex mobility and pair breaking in fermionic superfluid rings



Phys. Rev. Research 8, 023204 (2026)

PI of the grant Fondo Italiano per la Scienza (FIS3)

(ERC-like funding by Italian Ministry of University and Research)

SANDI: Superflow Stability: Exploring Dynamics of Inhomogeneous Superfluids

The project study inhomogeneous superfluids dynamics; how thermal excitations, interfaces instability, spin imbalance and interactions shape superfluid’s and quantum vortex dynamics. Relevant for quantum technologies and **quantum simulations of neutron stars, superconductors and superfluid helium.**

Start date:
1st of May 2026

5 years
theoretical project

Call openings for Phd and postdocs positions

PI of the grant Fondo Italiano per la Scienza (FIS3)
(ERC-like funding)

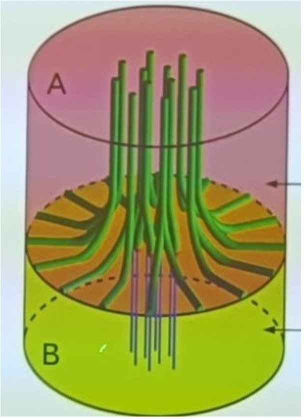
SANDI: Superflow Stability: Exploring Dynamics of Inhomogeneous Superfluids

The project study inhomogeneous superfluids dynamics; how thermal excitations, **interfaces instability**, spin imbalance and interactions shape superfluid’s and quantum vortex dynamics. Relevant for quantum technologies and **quantum simulations of neutron stars, superconductors and superfluid helium.**

Inspired by:

Studies of interface instability and vortex transport between A-B phases in superfluid ^{3}He

From V. Eltsov’s Scales talk

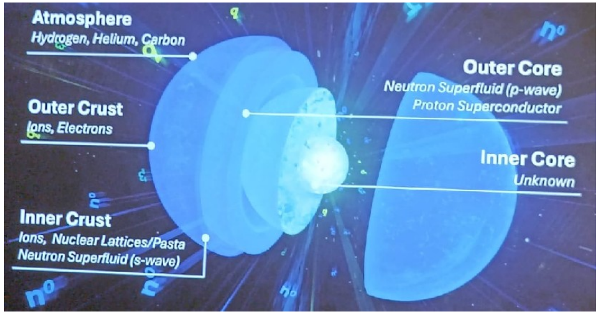


R. Blaauwgeers et al., 2002, https://doi.org/10.1103/PhysRevLett.89.155301

A P Finne, et al., 2006, 10.1088/0034-4885/69/12/R03

Neutron stars: Instability at the crust-core ($^1S_0$-$^3P_2$-superfluid) boundary

From G. Liu’s Scales talk



ANDERSON, P., ITOH, N., 1975, https://doi.org/10.1038/256025a0

A. Mastrano et al., 2005, https://doi.org/10.1111/j.1365-2966.2005.09219.x

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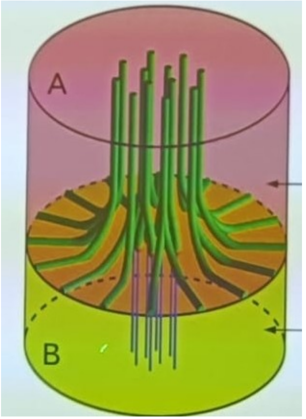
SANDI: Superflow Stability: Exploring Dynamics of Inhomogeneous Superfluids

The project study inhomogeneous superfluids dynamics; how thermal excitations, interfaces instability, **spin imbalance** and interactions shape superfluid's and quantum vortex dynamics. Relevant for quantum technologies and **quantum simulations of neutron stars, superconductors and superfluid helium.**

Studies of interface instability and
vortex transport between A-B phases in superfluid ^{3}He

From V. Eltsov's Scales talk



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Inspired by:

Spin-imbalanced fermionic superfluids:
FFLO states in superconductors

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Neutrons stars such as magnetars

High magnetic field induces spin polarization

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Dynamical phase diagram of bosonic Josephson junctions

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lens European Laboratory for Non-Linear Spectroscopy

Experiment by G. Roati's group

SISSA

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Dissipation in a Finite Temperature Atomic Josephson Junction

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Dissipative mechanisms in fermionic Josephson junctions

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Atomtronic Josephson necklace

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