Warsaw University of Technology
Faculty of Physics

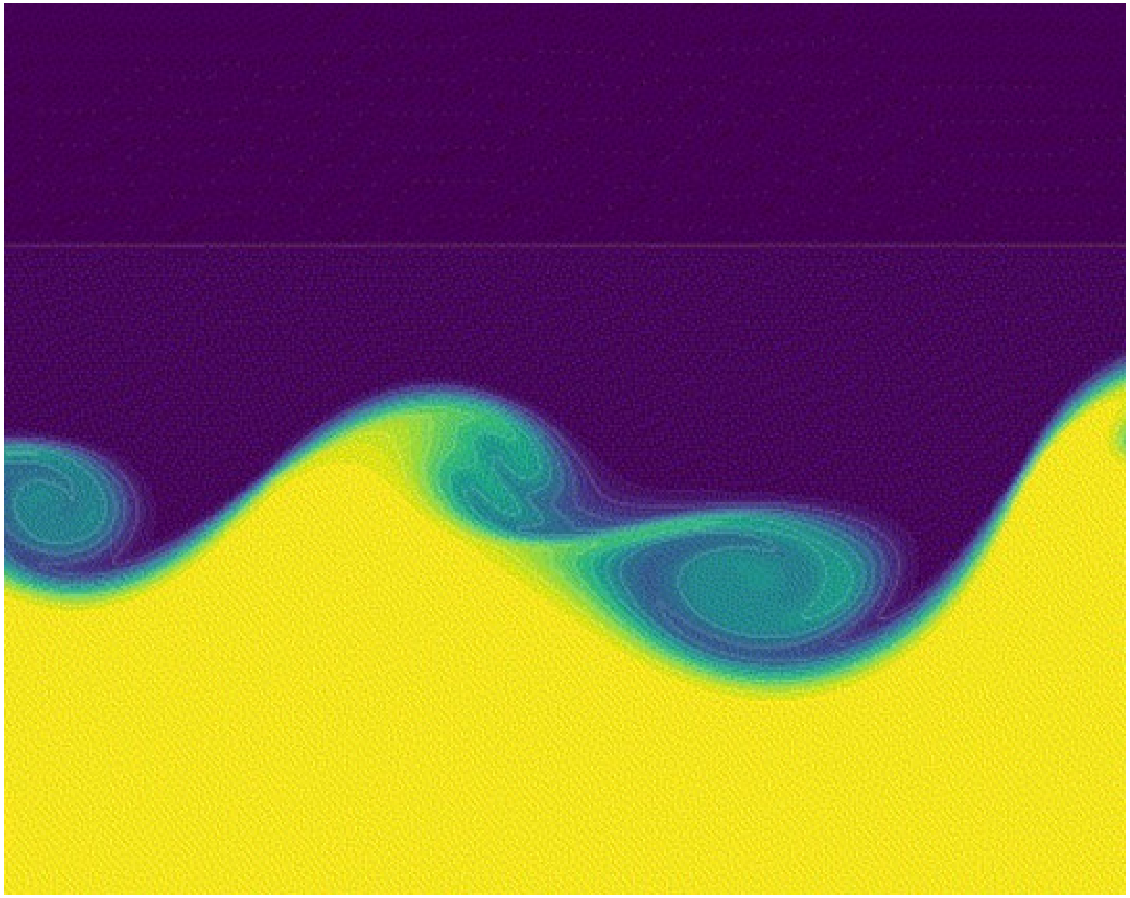
Kelvin-Helmholtz Instability in Fermionic Superfluids: Numerical Approach

SCALES 1. General Meeting
22.06.2026

Michał Śliwiński, MS

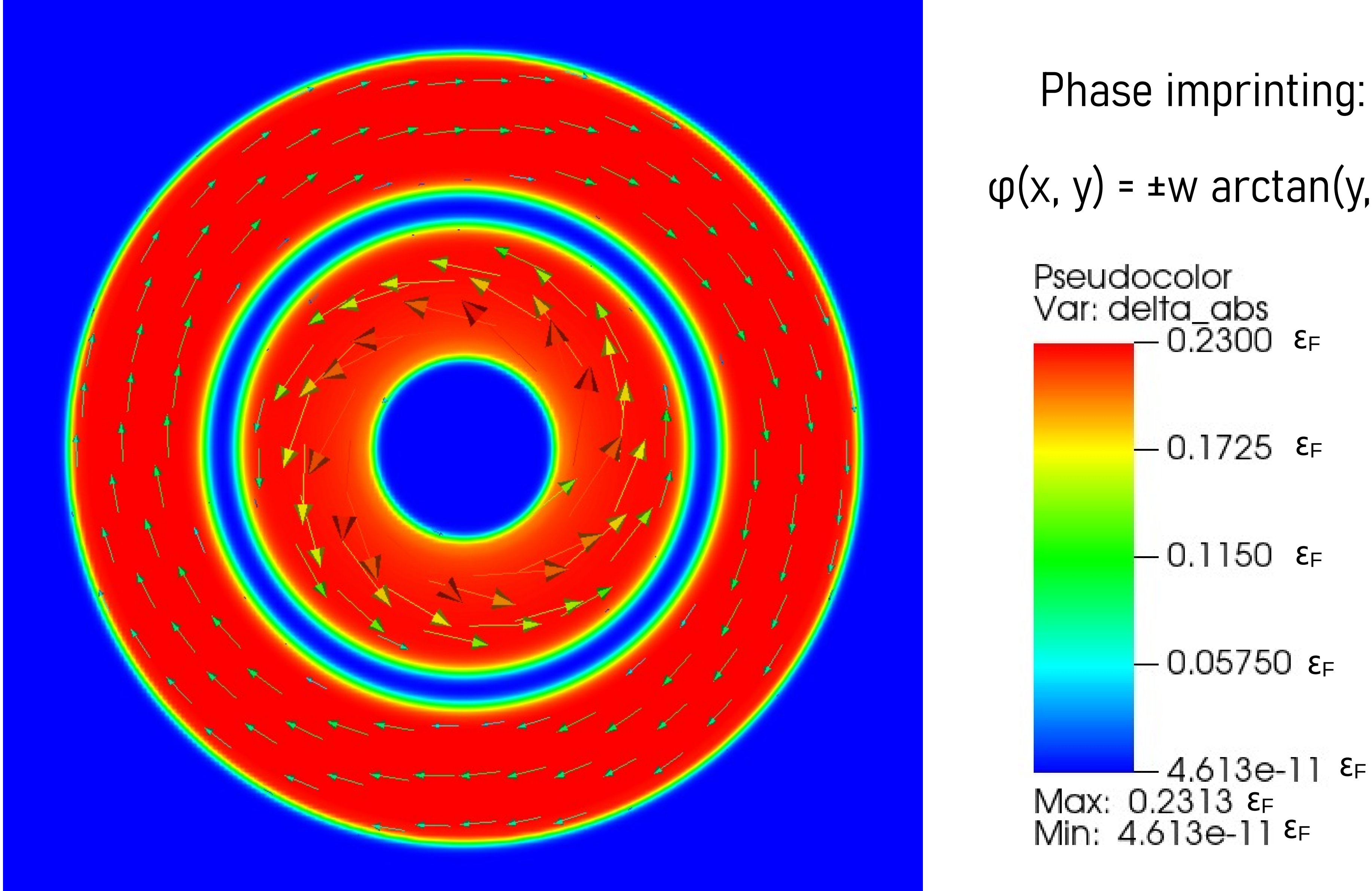
Classical KHI

Visualisation done with a script published in: [1] (Herho et al, 2025)



Phase imprinting:

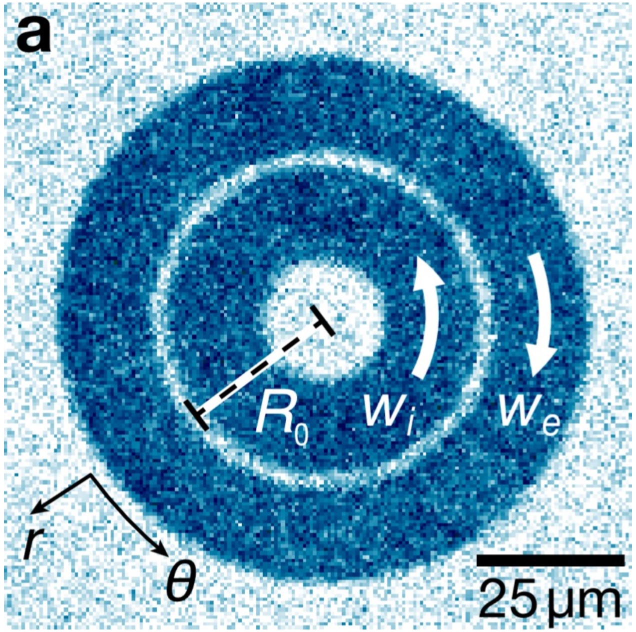
$$\varphi(x, y) = \pm w \arctan(y, x)$$

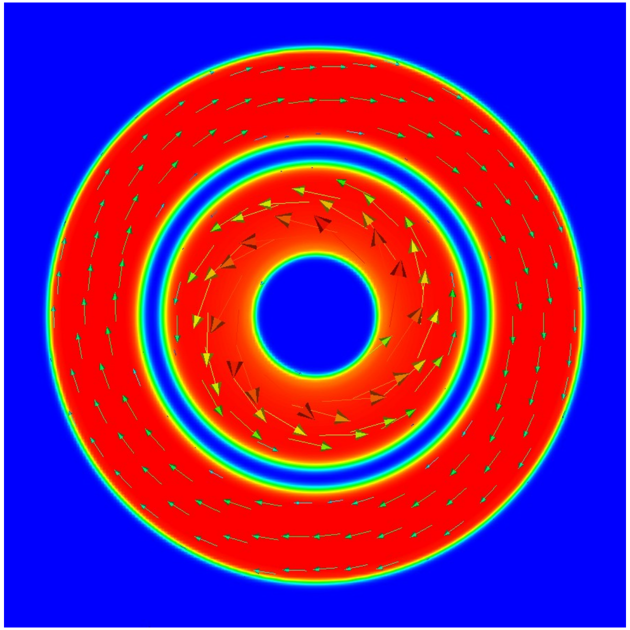


Systems investigated

We use W-SLDA Toolkit [4], that is being developed on Faculty of Physics.

[2] D. Hernández-Rajkov, et al: *Connecting shear flow and vortex array instabilities in annular atomic superfluids.* Nat. Phys., 20(6): 939–944, March 2024.





$R_0 k_F$ =~ 50.0

Gaussian barrier $\sigma_{Gauss} k_F$ = 2.36

winding number	w	±6,7,8	±6,7,8	±4
regime	$(ak_F)^{-1}$	-0.0 (UFG)	-0.3 (BCS)	-1.0 (dBCS)
temperature	T/T_C	0, 33, 66%	0, 45, 66%	0, 33%

W-SLDA

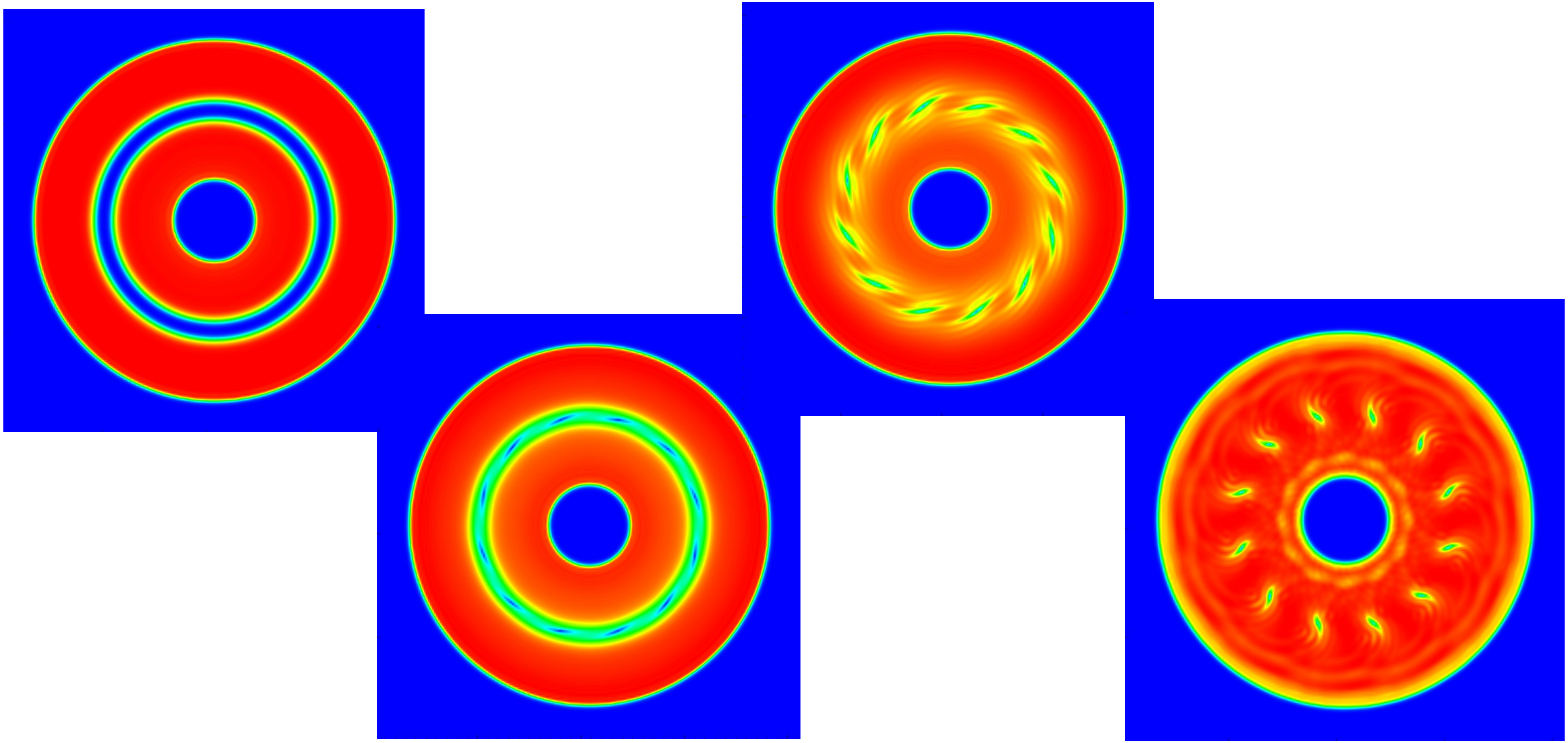
We use W-SLDA Toolkit [4], that is being developed on Faculty of Physics, WUT.

$$E_0 = \int \mathcal{E}[\rho_\sigma(\boldsymbol{r}), \tau_\sigma(\boldsymbol{r}), \boldsymbol{j}_\sigma, \nu(\boldsymbol{r})]\, d\boldsymbol{r}$$

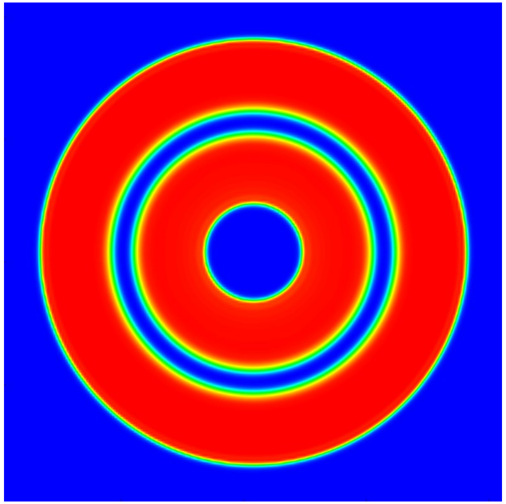
$$\begin{pmatrix} h_a(\boldsymbol{r}) - \mu_a & \Delta(\boldsymbol{r}) + \Delta_{\text{ext}}(\boldsymbol{r}) \\ \Delta^*(\boldsymbol{r}) + \Delta^*_{\text{ext}}(\boldsymbol{r}) & -h^*_b(\boldsymbol{r}) + \mu_b \end{pmatrix} \begin{pmatrix} u_{n,a}(\boldsymbol{r}) \\ v_{n,b}(\boldsymbol{r}) \end{pmatrix} = E_n \begin{pmatrix} u_{n,a}(\boldsymbol{r}) \\ v_{n,b}(\boldsymbol{r}) \end{pmatrix}$$

$$\mathrm{i}\frac{\partial}{\partial t} \begin{pmatrix} u_{n,a}(\boldsymbol{r},t) \\ v_{n,b}(\boldsymbol{r},t) \end{pmatrix} = \begin{pmatrix} h_a(\boldsymbol{r},t) - \mu_a & \Delta(\boldsymbol{r},t) + \Delta_{\text{ext}}(\boldsymbol{r},t) \\ \Delta^*(\boldsymbol{r},t) + \Delta^*_{\text{ext}}(\boldsymbol{r},t) & -h^*_b(\boldsymbol{r},t) + \mu_b \end{pmatrix} \begin{pmatrix} u_{n,a}(\boldsymbol{r},t) \\ v_{n,b}(\boldsymbol{r},t) \end{pmatrix}$$

Dynamical effects in the system



Dynamical effects in the system

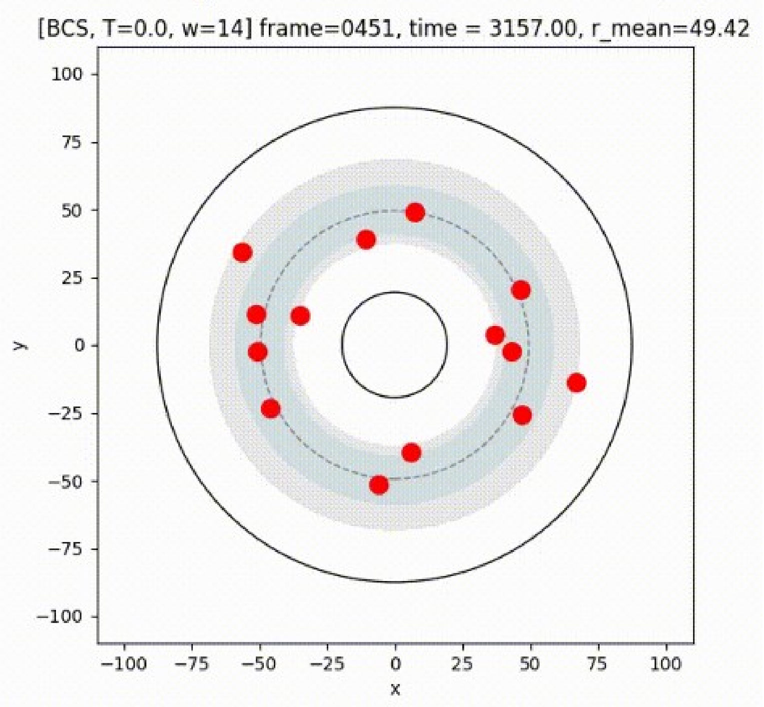


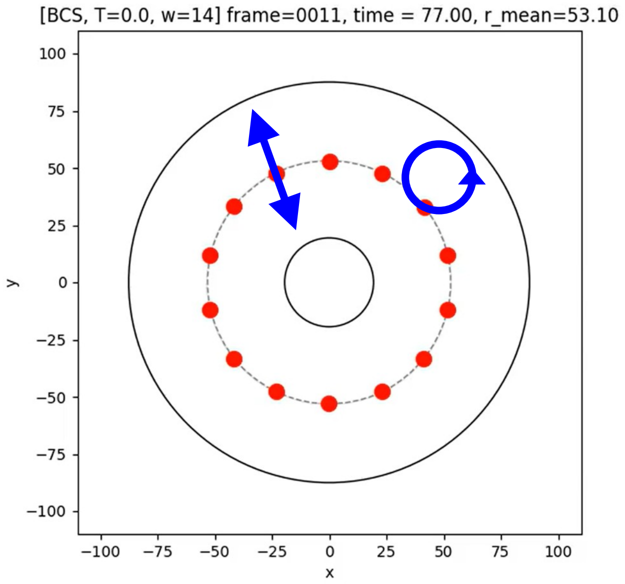
← Animation

Dynamical effects in the system

The vortex necklace tightens and relaxes before KHI.

We take R_0 for the very moment when: $STD(R_i) * k_F = 1.0$

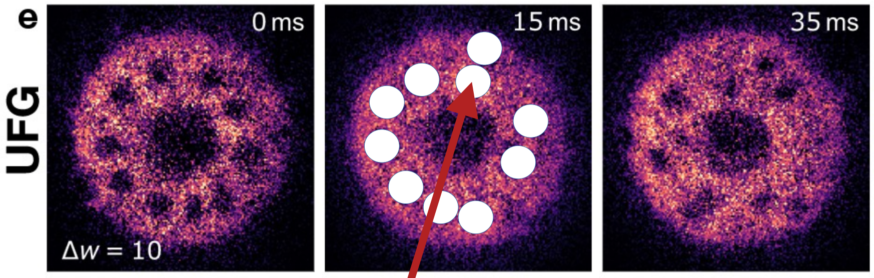


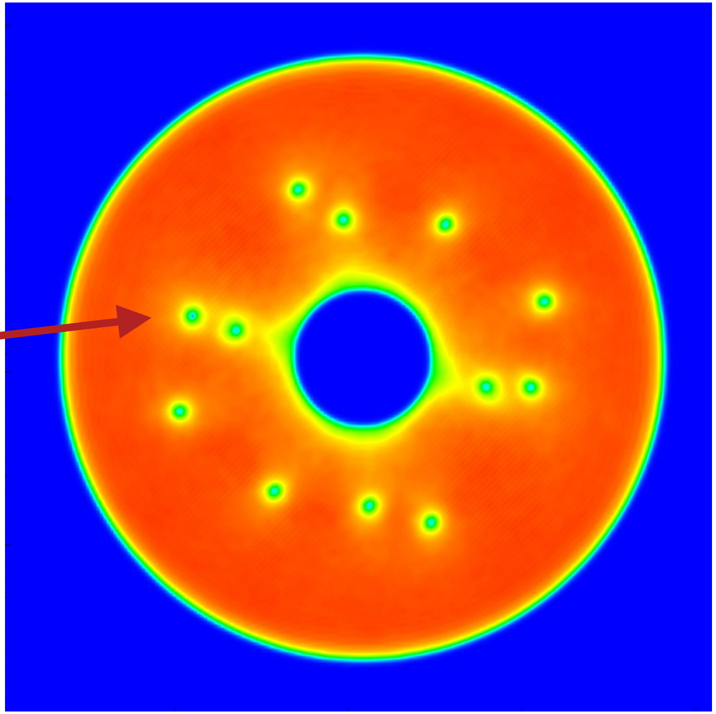


Radial squishing and relaxing

Precession-like movement

Dynamical effects in the system





Asymetrical pairing of vortices, but from different sources:

experiment: non-ideal phase imprinting
W-SLDA: elyptical shear layer between rings

SLDA result (T/T_c = 66%; UFG)

[2] D. Hernández-Rajkov, et al:
Connecting shear flow and vortex array instabilities
in annular atomic superfluids.
Nat. Phys., 20(6): 939–944, March 2024.

Aims of the research

Phase imprinting:

$\varphi(x, y) = \pm w \arctan(y, x)$

winding number

We use W-SLDA Toolkit [4], that is being developed on Faculty of Physics.
We try to:

1) Compare W-SLDA output the results reported in [2] with variable parameters:

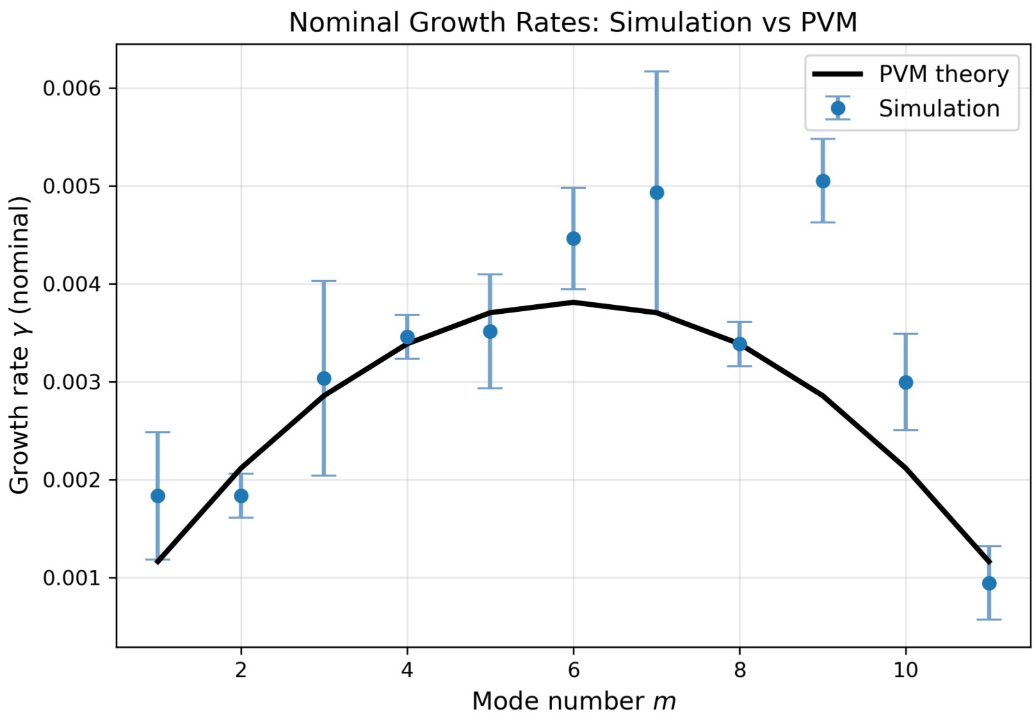
winding number	w	±6,7,8	±6,7,8	±4
regime	$(ak_F)^{-1}$	-0.0 (UFG)	-0.3 (BCS)	-1.0 (dBCS)
temperature	T/T_C	0, 33, 66%	0, 45, 66%	0, 33%

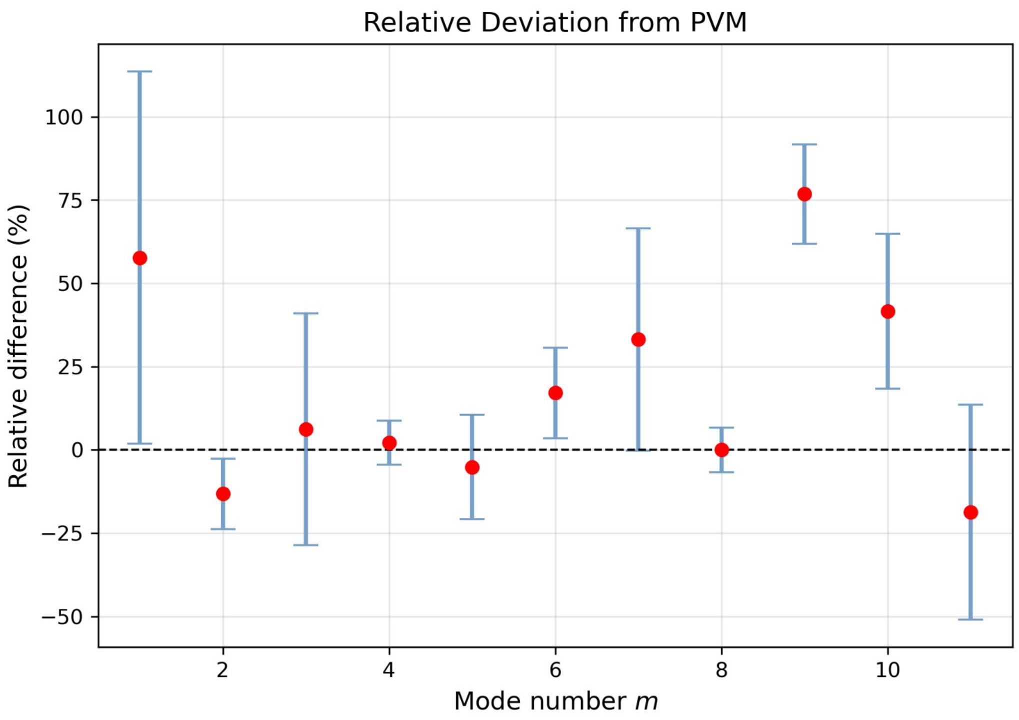
2) investigate the inner ring in the regime of $(ak_F)^{-1} = -1.0$ as antivortices tend to proliferate in the region where Δ is small

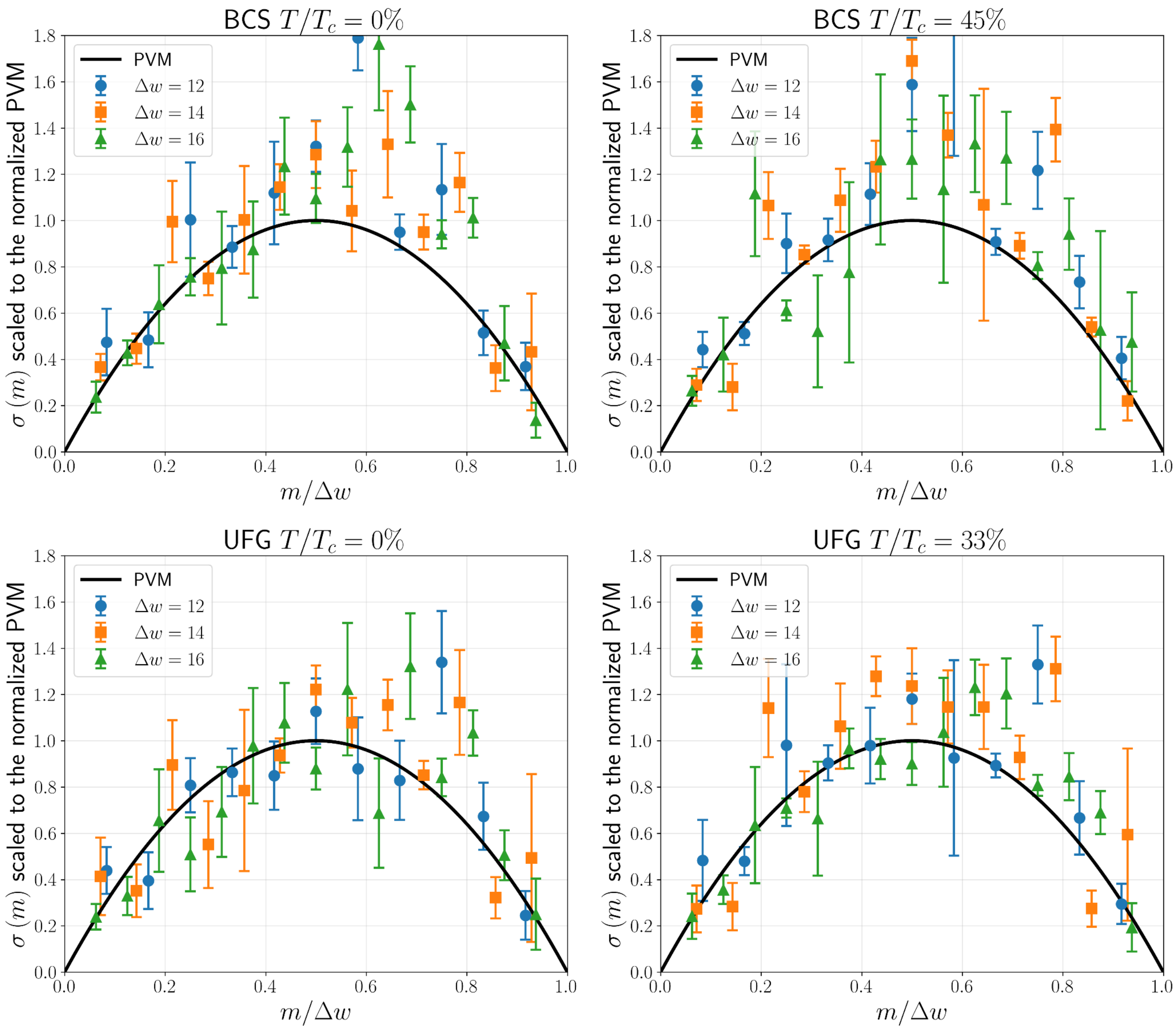
[2] D. Hernández-Rajkov, et al:
Connecting shear flow and vortex array instabilities in annular atomic superfluids.
Nat. Phys., 20(6): 939–944, March 2024.

Analysis

$$S(m,t) = \frac{1}{N_v} \sum_{j,l}^{N_v} e^{im(\theta_j(t) - \theta_l(t))}$$

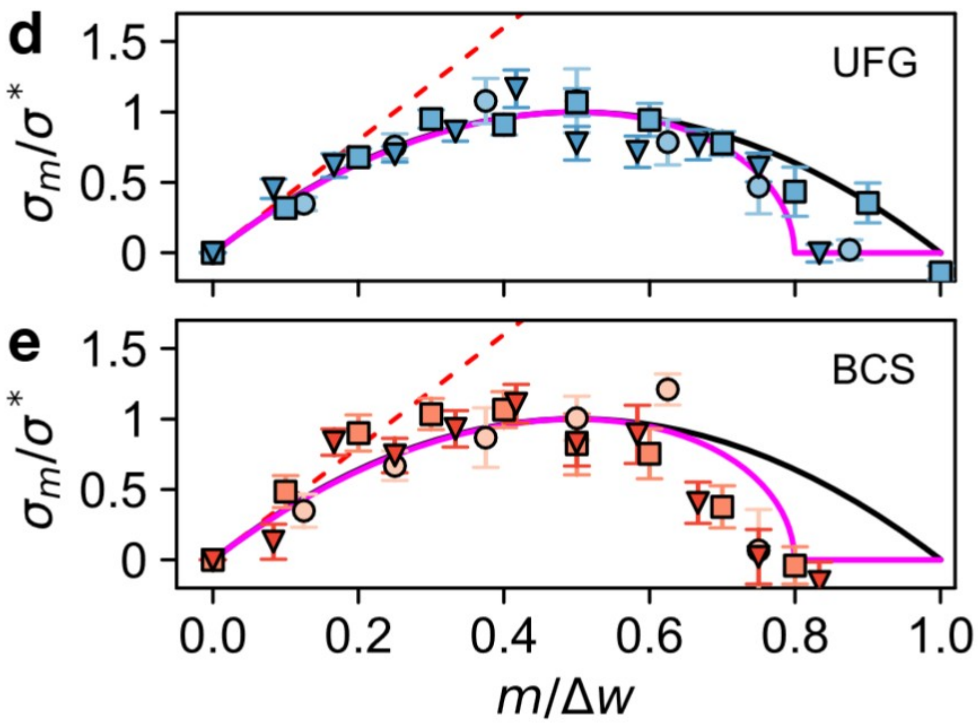


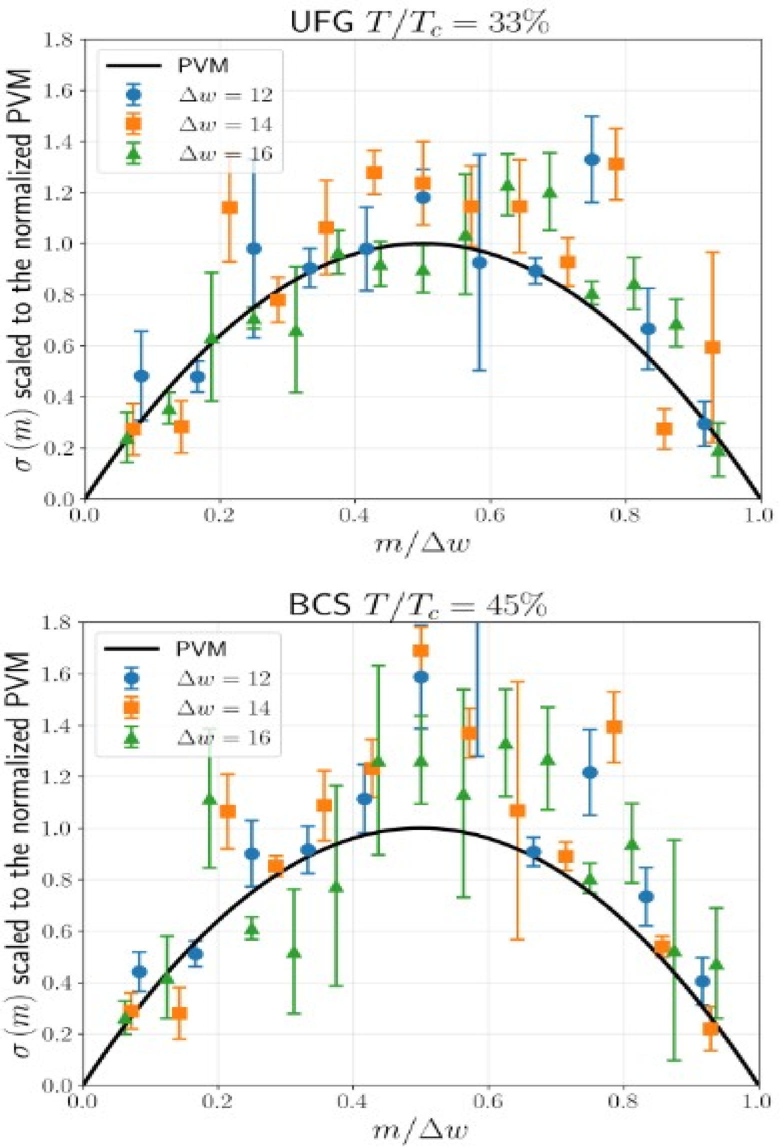



BCS $T/T_c = 0\%$
BCS $T/T_c = 45\%$
UFG $T/T_c = 0\%$
UFG $T/T_c = 33\%$
PVM
$\Delta w = 12$
$\Delta w = 14$
$\Delta w = 16$
$\sigma(m)$ scaled to the normalized PVM
$m/\Delta w$

Analysis

- winding number diff. = 8 (circle)
- winding number diff. = 10 (square)
- winding number diff. = 12 (triangle)





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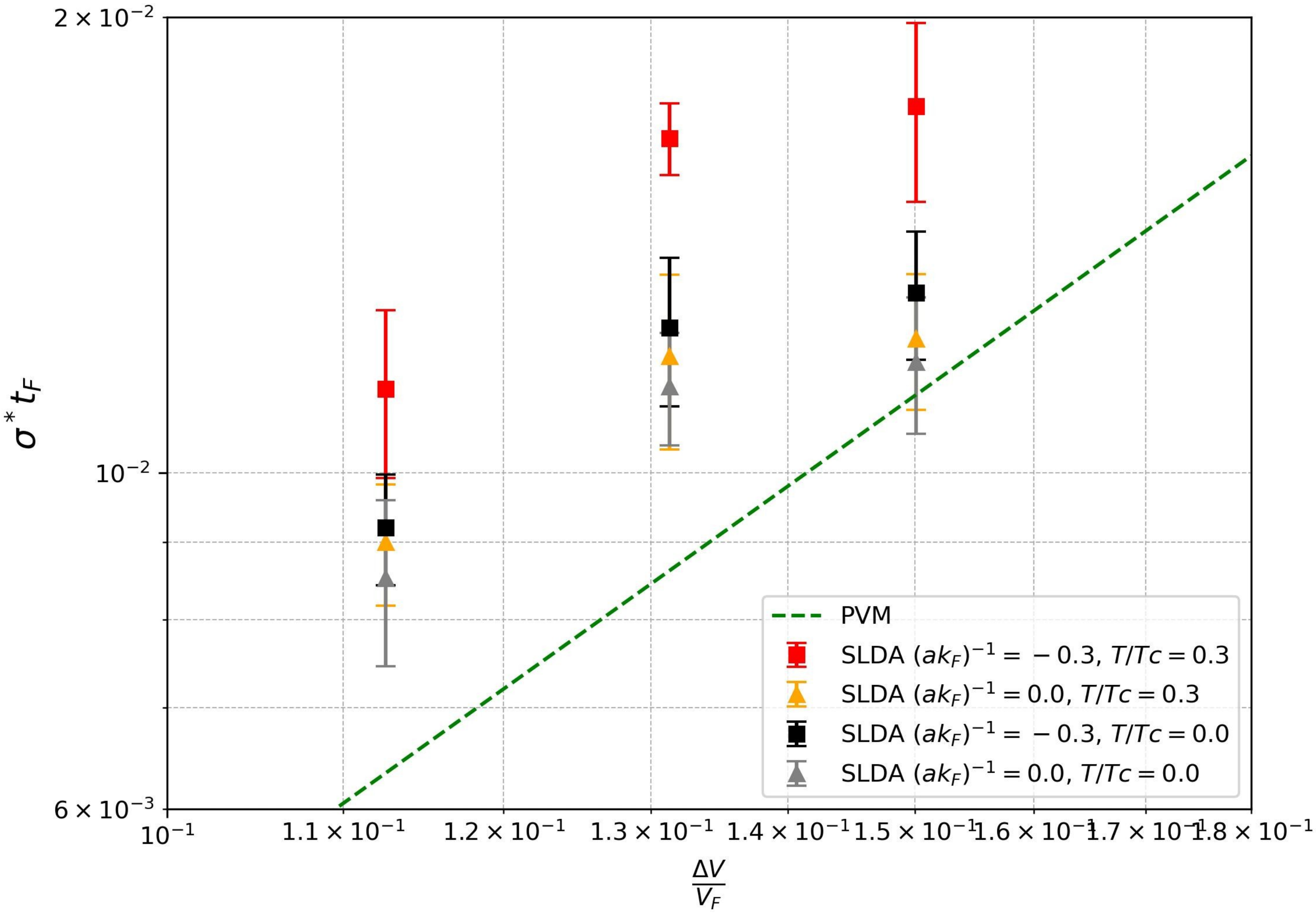
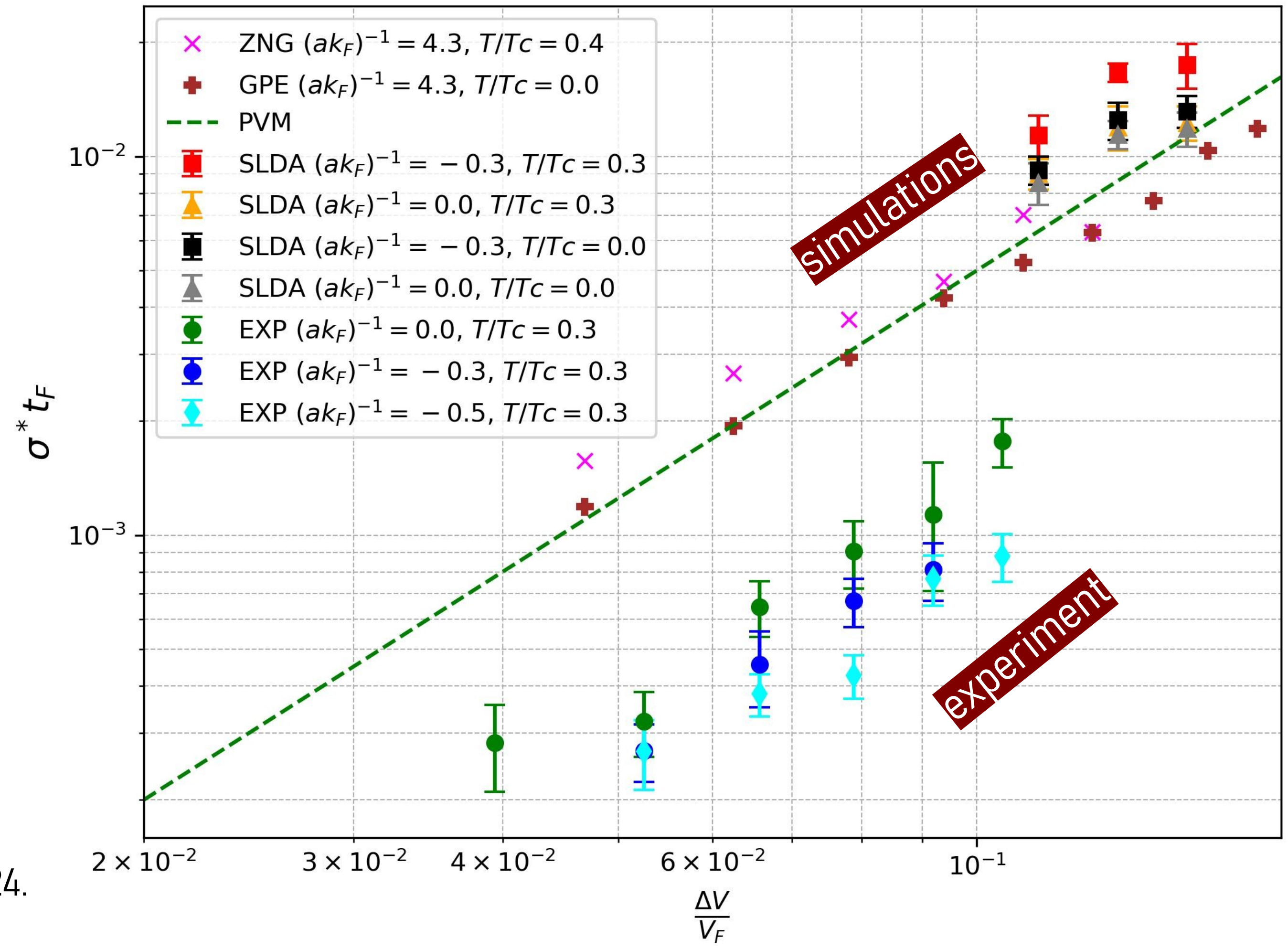


2×10^{-2}
10^{-2}
6×10^{-3}
$\sigma^* t_F$
10^{-1}
1.1×10^{-1}
1.2×10^{-1}
1.3×10^{-1}
1.4×10^{-1}
1.5×10^{-1}
1.6×10^{-1}
1.7×10^{-1}
1.8×10^{-1}
$\frac{\Delta V}{V_F}$
PVM
SLDA $(ak_F)^{-1} = -0.3$, $T/Tc = 0.3$
SLDA $(ak_F)^{-1} = 0.0$, $T/Tc = 0.3$
SLDA $(ak_F)^{-1} = -0.3$, $T/Tc = 0.0$
SLDA $(ak_F)^{-1} = 0.0$, $T/Tc = 0.0$

$$\tau_{\mathrm{KHI}} \sim 1/\sigma^*$$



[2] D. Hernández-Rajkov, et al:

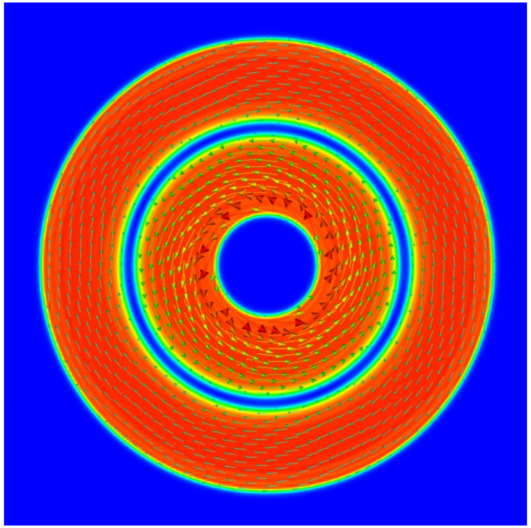
Connecting shear flow and vortex array instabilities in annular atomic superfluids.

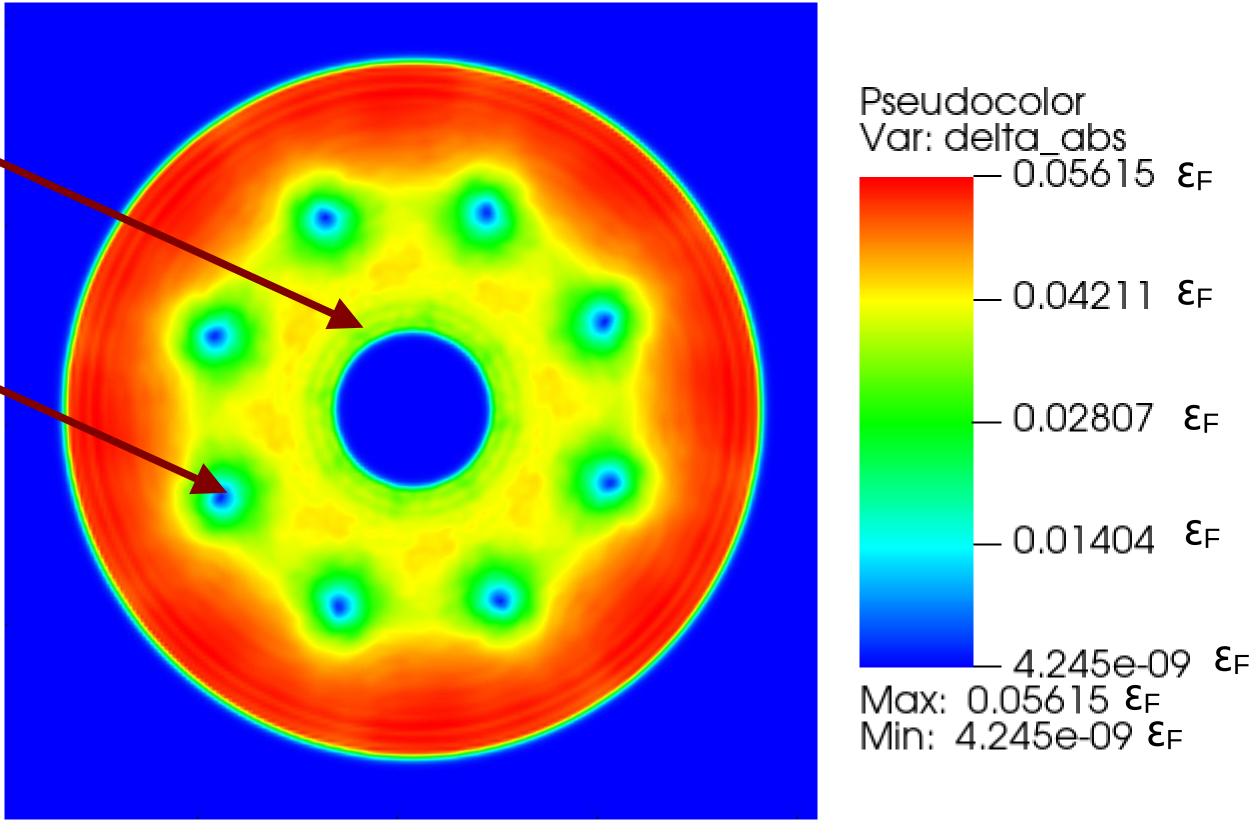
Nat. Phys., 20(6): 939–944, March 2024.

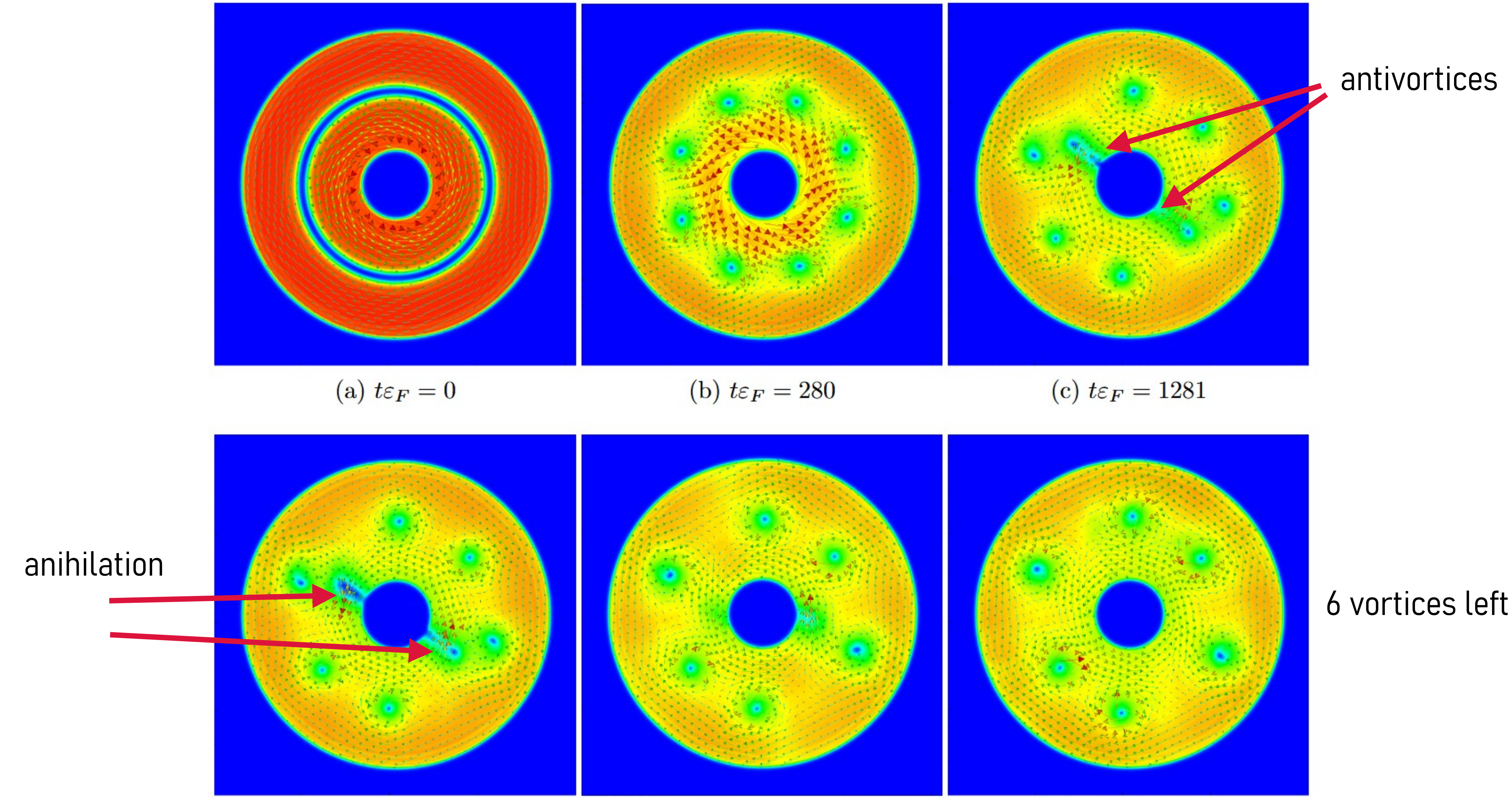
Analysis of deep BCS: $(ak_F)^{-1} = -1.0$

Pair breaking clearly visible

Vortices acumulate mass
(and grow in size)







(a) $t\varepsilon_F = 0$ (b) $t\varepsilon_F = 280$ (c) $t\varepsilon_F = 1281$

(d) $t\varepsilon_F = 1330$ (e) $t\varepsilon_F = 1505$ (f) $t\varepsilon_F = 1750$

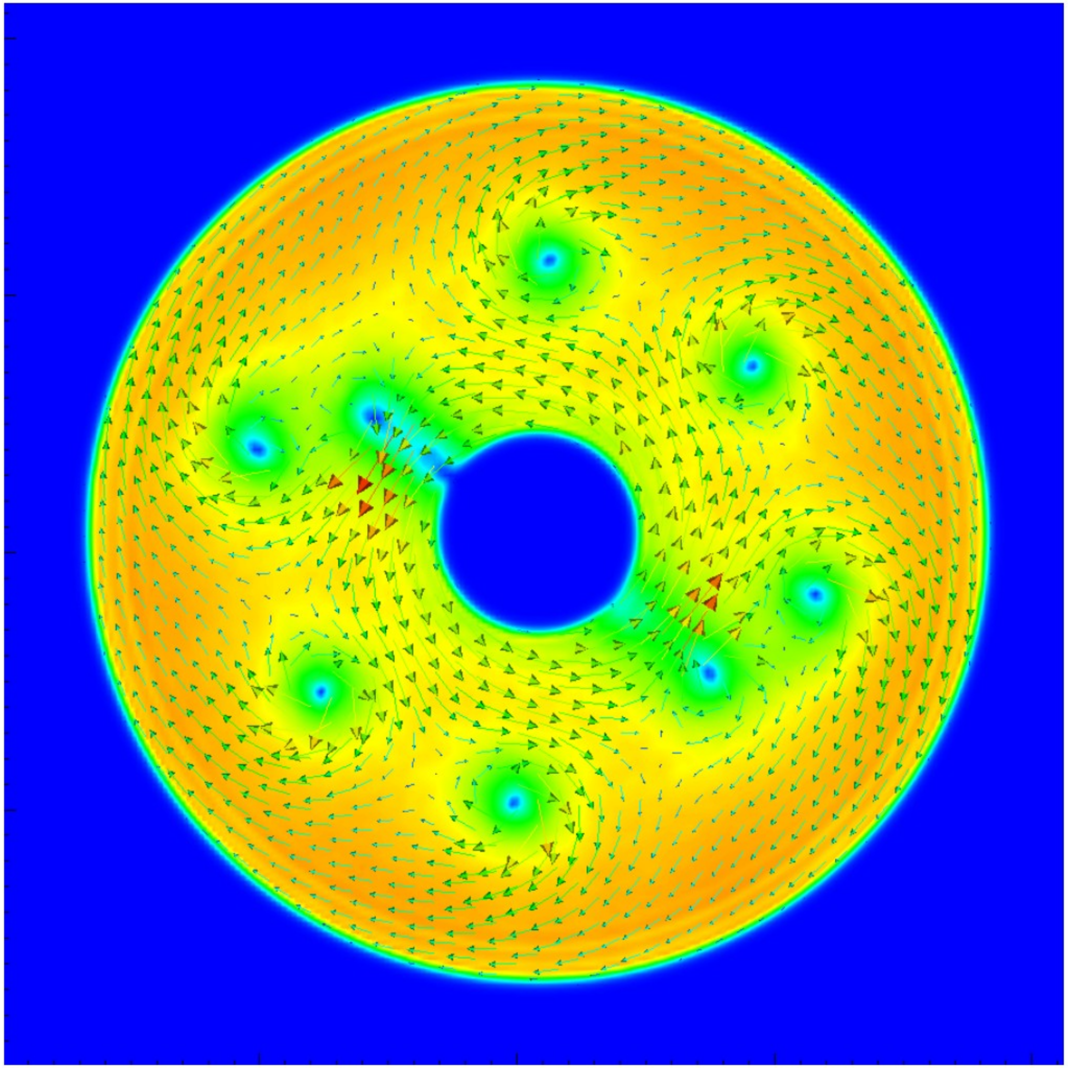
A new dynamical mechanism is observed. It governs the dispersion of vortices before KHI takes place.

$$\tau_V < \tau_{KHI}$$

Antivortices nucleate at the inner edge of the trapping potential.

Then they propagate outwards while drawing some of the vortices inwards.

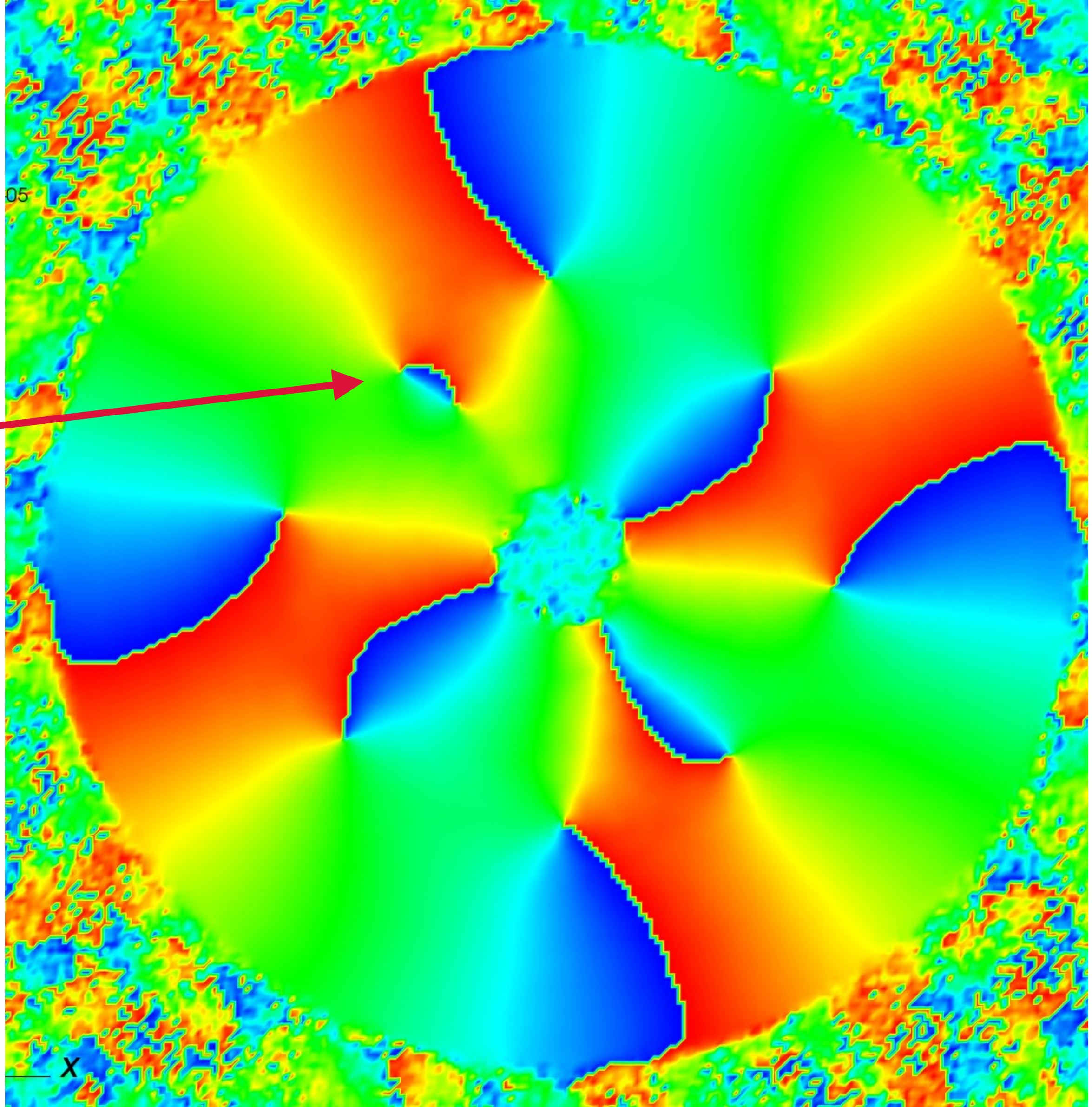
When an antivortex meets a vortex, the annihilate, releasing their enegy to the system.

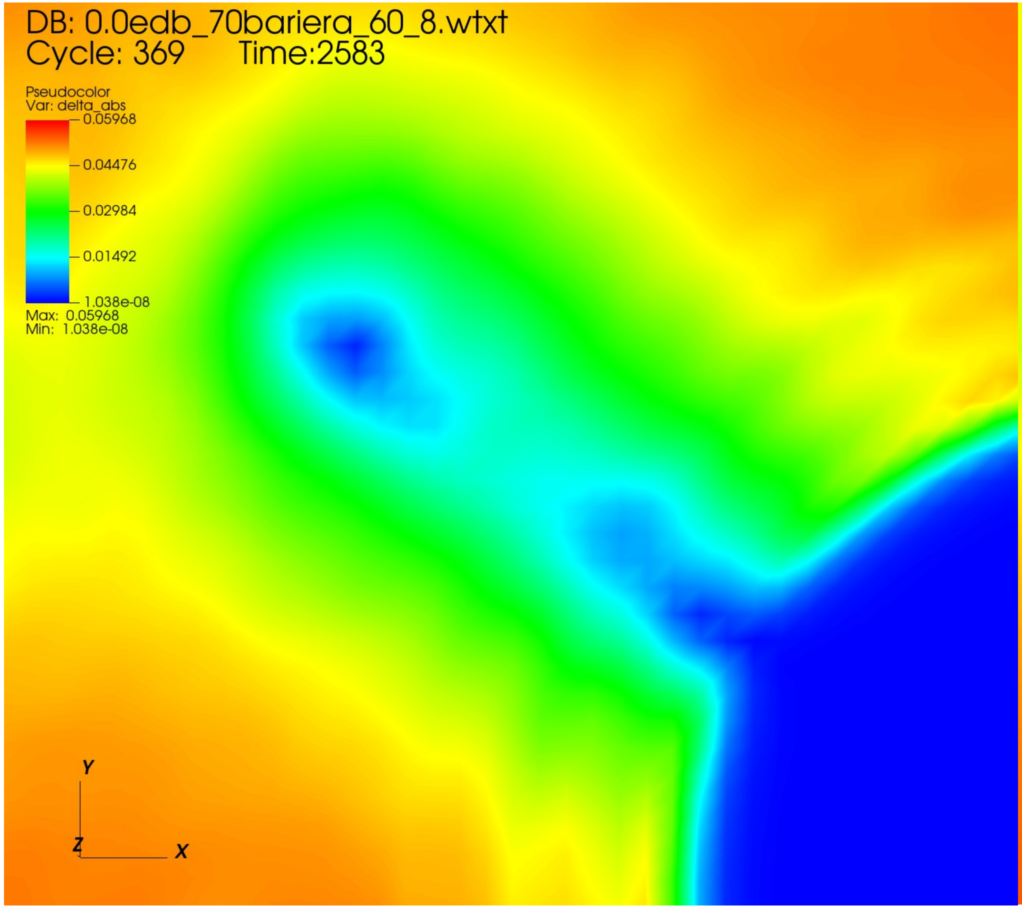


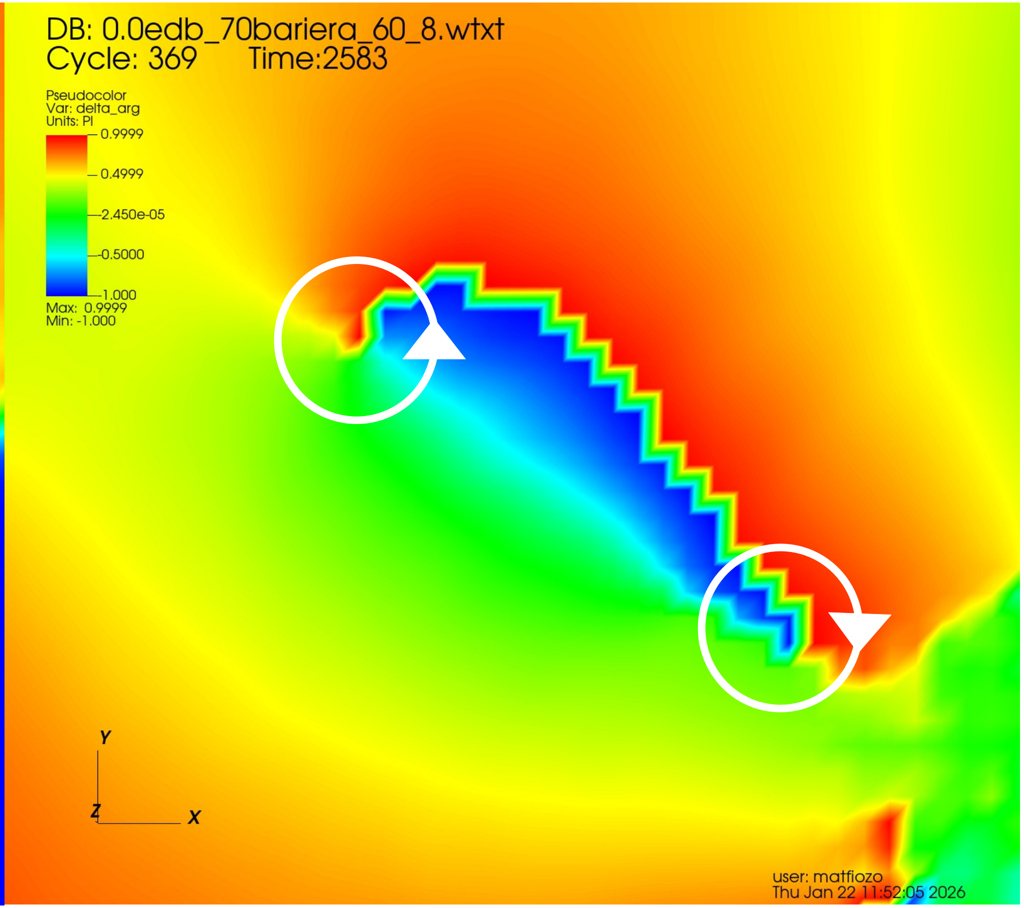
(c) $t\varepsilon_F = 1281$

a pair of

vortex-antivortex




DB: 0.0edb_70bariera_60_8.wtxt
Cycle: 369 Time:2583
Pseudocolor
Var: delta_abs
0.05968
0.04476
0.02984
0.01492
1.038e-08
Max: 0.05968
Min: 1.038e-08
Y
Z
X

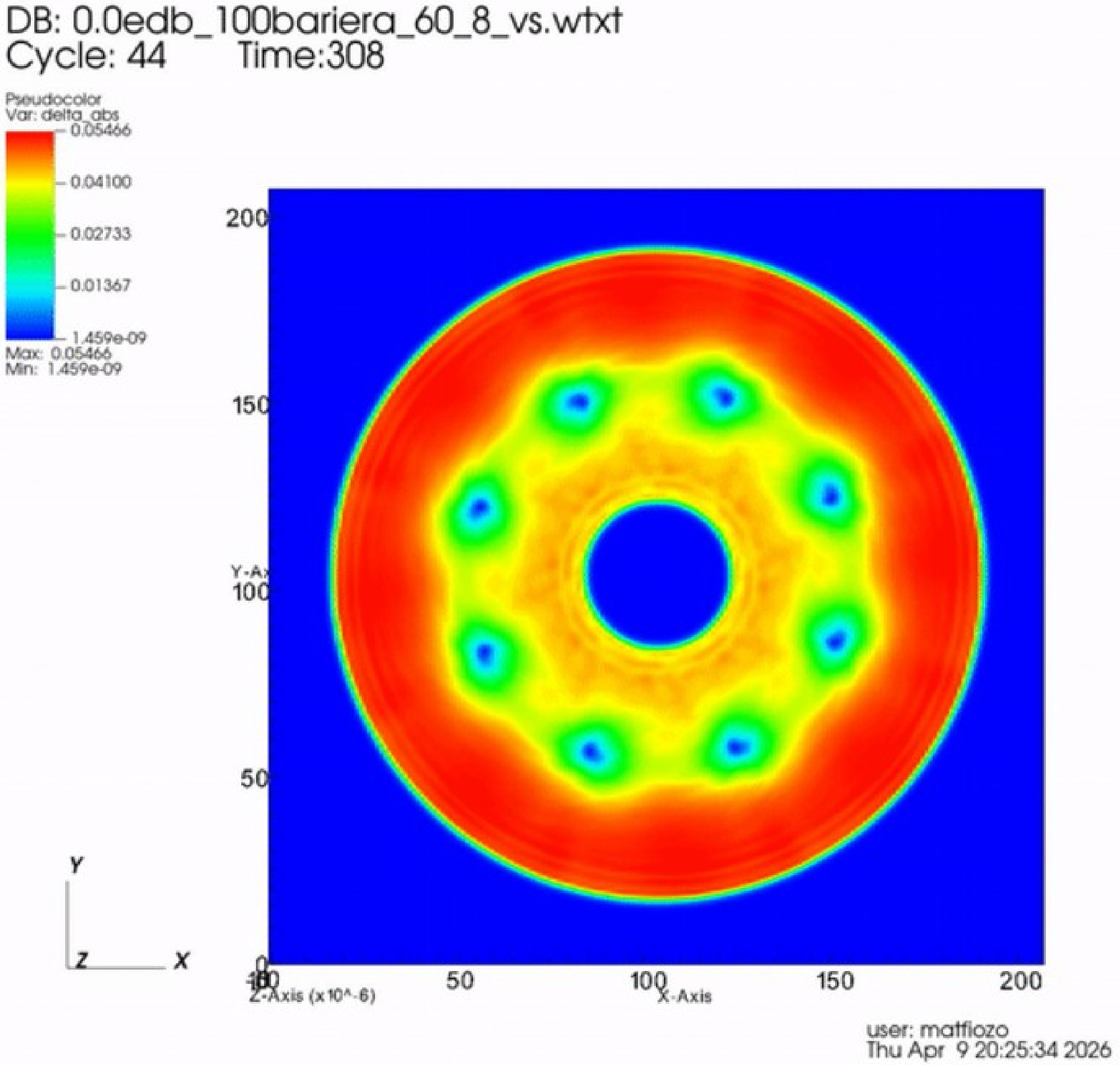

DB: 0.0edb_70bariera_60_8.wtxt
Cycle: 369 Time:2583
Pseudocolor
Var: delta_arg
Units: PI
0.9999
0.4999
2.450e-05
-0.5000
-1.000
Max: 0.9999
Min: -1.000
Y
Z
X
user: matfiozo
Thu Jan 22 11:52:05 2026

This process slows down the onsetting of KHI.

According to the PVM, the fewer vortices in the system, the smaller σ^* is, and then

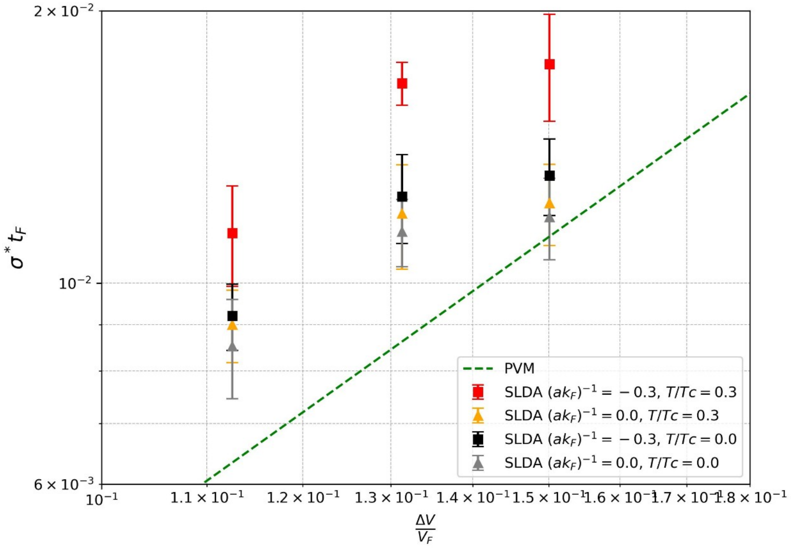
$$\tau_{\mathrm{KHI}} \sim 1/\sigma^*$$

the characteristic starting time of KHI becomes longer.



Conclusions

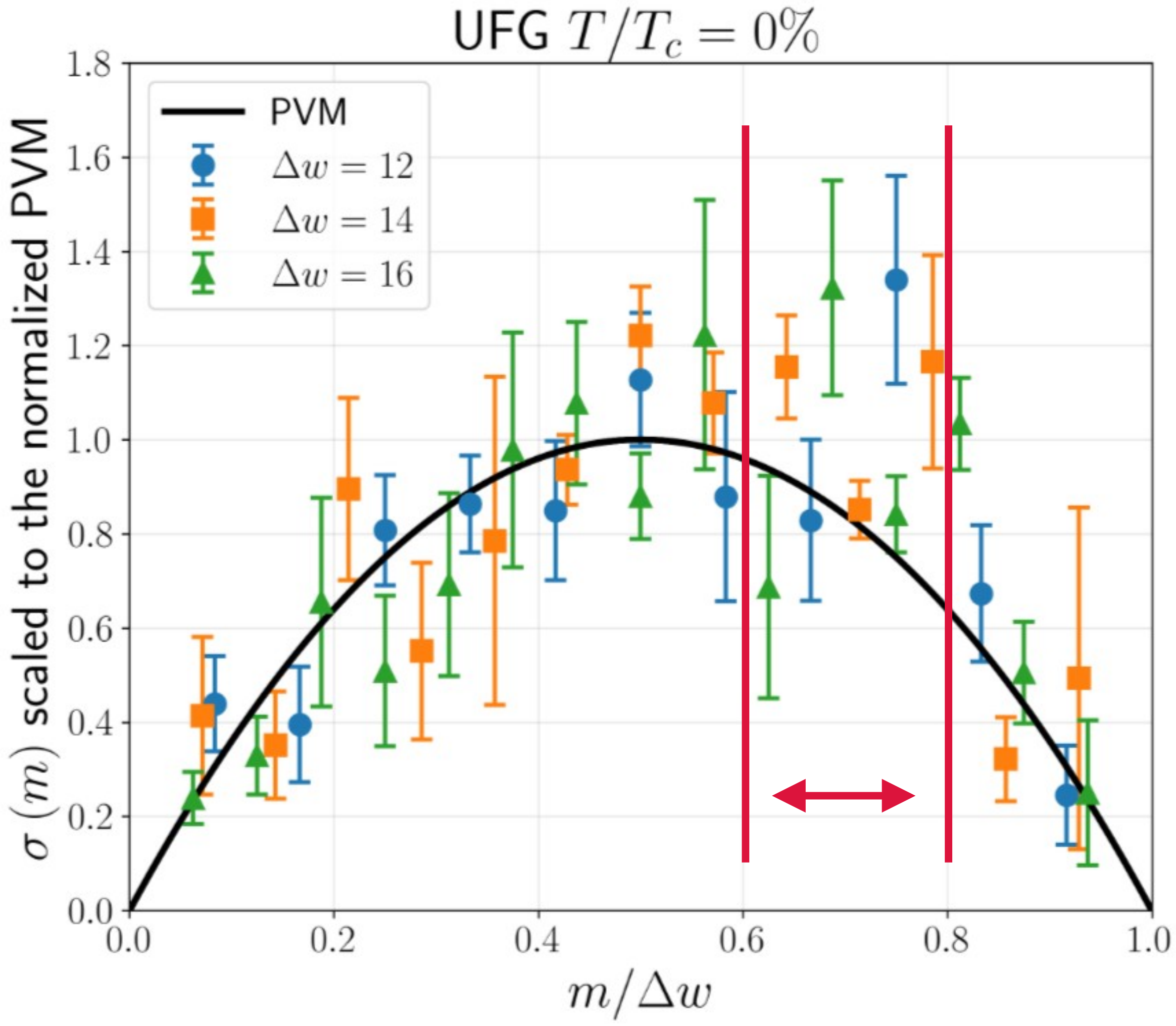
1. We do not confirm the exact sensitivity of the growth rate with respect to the interaction regime (BCS vs UFG) as the LENS group. Even for finite temperatures.



Conclusions

2. We systematically observe a deviation from the PVM for modes in the range

   $m/\Delta w \in (0.6, 0.8)$.

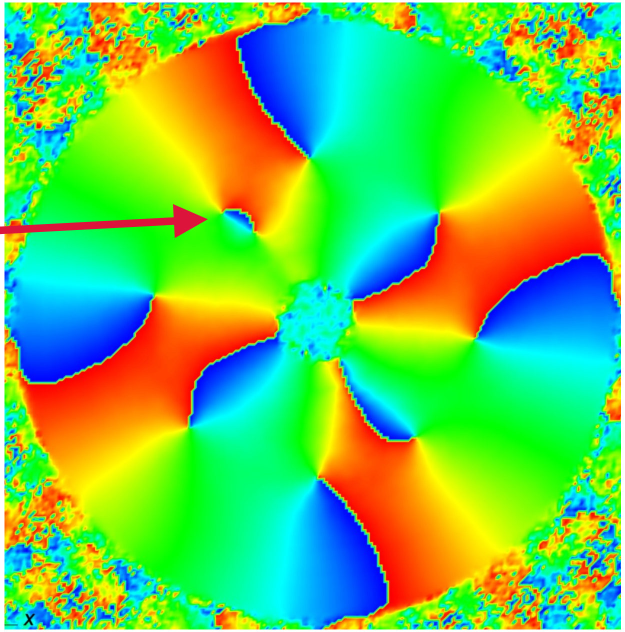


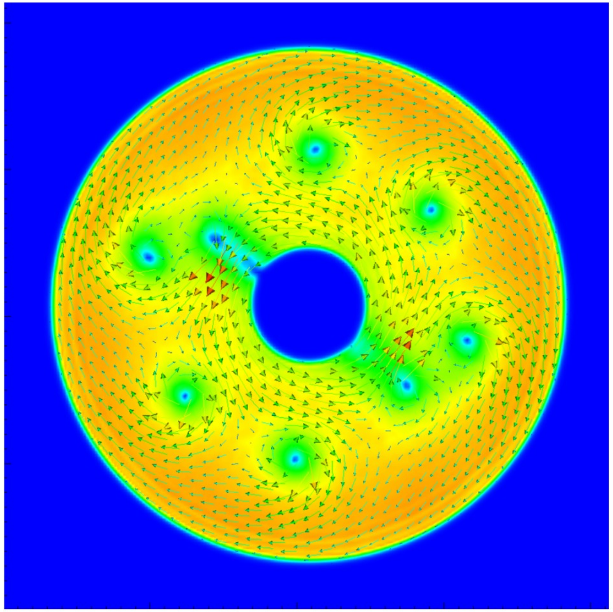
Conclusions

3. In the deep BCS regime, we find that the dynamics qualitatively change.

   A new mechanism is activated: the proliferation of vortices on the inner edge of the ring.

a pair





(c) $t\varepsilon_F = 1281$

Acknowledgements

The whole research was conducted in our working group:

Michał Śliwiński (1)

Klejdja Xhani (2) see: Josephson effects and atomtronic circuits in atomic superfluids
at: 16:40

Gabriel Wlazłowski (1,3)

(1) – Faculty of Physics, Warsaw University of Technology, Poland

(2) – Departament of Applied Science and Technology, Politecnico di Torino, Italy

(3) – Departament of Physics, University of Washington, Seattle, USA

Bibliography

[1] S. H. S. Herho, et al:
A Python Library for Idealized Two-Dimensional Incompressible Kelvin-Helmholtz Instability
Applied and Computational Mechanics 19 125-156, 2025

[2] D. Hernández-Rajkov, et al:
Connecting shear flow and vortex array instabilities in annular atomic superfluids.
Nat. Phys., 20(6): 939–944, March 2024

[3] Diego Hernandez-Rajkov:
Kelvin–Helmholtz Instability in Annular Fermi Superfluids. PhD thesis.
University of Florence, Florence, Italy, 2024.

[4] Gabriel Wlazłowski, Piotr Magierski, Michael McNeil Forbes, and Aurel Bulgac.
W-SLDA Toolkit: A simulation platform for ultracold Fermi gases.
2026.

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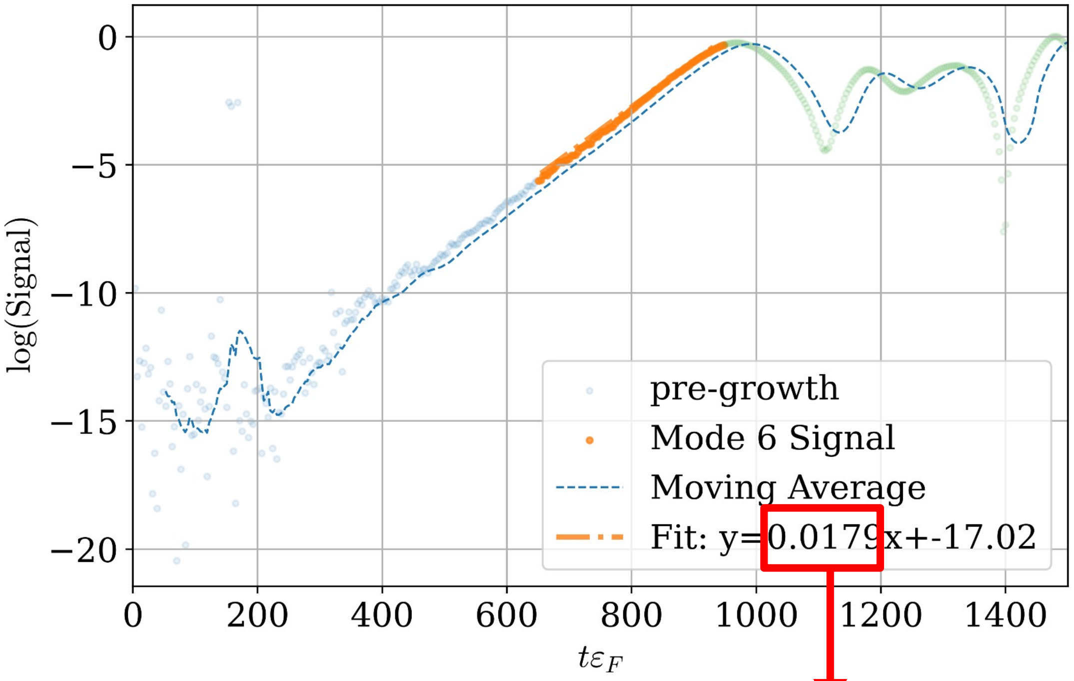
Analysis

We employ the structure factor analysis as described in [3].

$$S(m,t) = \frac{1}{N_v}\sum_{j,l}^{N_v} e^{im(\theta_j(t)-\theta_l(t))}$$



instability growth rate for mode 6: $\sigma_6 = 0.0179$

[3] Diego Hernandez-Rajkov: *Kelvin-Helmholtz Instability in Annular Fermi Superfluids. PhD thesis.* University of Florence, Florence, Italy, 2024.

SLDA equations of motion

$$\mathcal{E}(\mathbf{r}) = \alpha\frac{\tau_c(\mathbf{r})}{2} + \beta\frac{3(3\pi^2)^{2/3}n^{5/3}(\mathbf{r})}{10} + g_{eff}(\mathbf{r})|\nu_c(\mathbf{r})|^2 + V_{ext}(\mathbf{r})n(\mathbf{r}), \tag{5}$$

$$\frac{1}{g_{eff}(\mathbf{r})} = \frac{n^{1/3}(\mathbf{r})}{\gamma} + \Lambda_c(\mathbf{r}), \tag{6}$$

$$\tau_c(\mathbf{r}) = 2\sum_{E_k<E_c}|\nabla v_k(\mathbf{r})|^2, \quad \nu_c(\mathbf{r}) = \sum_{E_k<E_c} v_k^*(\mathbf{r})u_k(\mathbf{r}), \tag{7}$$

$$\begin{cases} [h(\mathbf{r}) - \mu]u_k(\mathbf{r}) + \Delta(\mathbf{r})v_k(\mathbf{r}) = E_k u_k(\mathbf{r}), \\ \Delta^*(\mathbf{r})u_k(\mathbf{r}) - [h(\mathbf{r}) - \mu]v_k(\mathbf{r}) = E_k v_k(\mathbf{r}), \end{cases} \tag{8}$$

$$h(\mathbf{r}) = -\frac{\alpha\nabla^2}{2} + U(\mathbf{r}), \tag{9}$$

$$U(\mathbf{r}) = \frac{\beta(3\pi^2 n(\mathbf{r}))^{2/3}}{2} - \frac{|\Delta(\mathbf{r})|^2}{3\gamma n^{2/3}(\mathbf{r})} + V_{ext}(\mathbf{r}), \tag{10}$$

$$\Delta(\mathbf{r}) := -g_{eff}(\mathbf{r})\nu_c(\mathbf{r}), \tag{11}$$

$$\Lambda_c(\mathbf{r}) = -\frac{k_c(\mathbf{r})}{2\pi^2\alpha}\left\{1 - \frac{k_0(\mathbf{r})}{2k_c(\mathbf{r})}\ln\frac{k_c(\mathbf{r}) + k_0(\mathbf{r})}{k_c(\mathbf{r}) - k_0(\mathbf{r})}\right\}, \tag{12}$$

$$E_c + \mu = \frac{\alpha k_c^2(\mathbf{r})}{2} + U(\mathbf{r}), \quad \mu = \frac{\alpha k_0^2(\mathbf{r})}{2} + U(\mathbf{r}). \tag{13}$$

SLDA equations of motion

Normal Density:

$$\rho_\sigma(\boldsymbol{r}) = \sum_{|E_n|<E_c} |v_{n,\sigma}(\boldsymbol{r})|^2 f_\beta(-E_n),$$

Kinetic Density:

$$\tau_\sigma(\boldsymbol{r}) = \sum_{|E_n|<E_c} |\boldsymbol{\nabla} v_{n,\sigma}(\boldsymbol{r})|^2 f_\beta(-E_n),$$

Current Density:

$$\boldsymbol{j}_\sigma(\boldsymbol{r}) = \sum_{|E_n|<E_c} \mathrm{Im}[v_{n,\sigma}(\boldsymbol{r})\boldsymbol{\nabla} v^*_{n,\sigma}(\boldsymbol{r})] f_\beta(-E_n),$$

Anomalous Density:

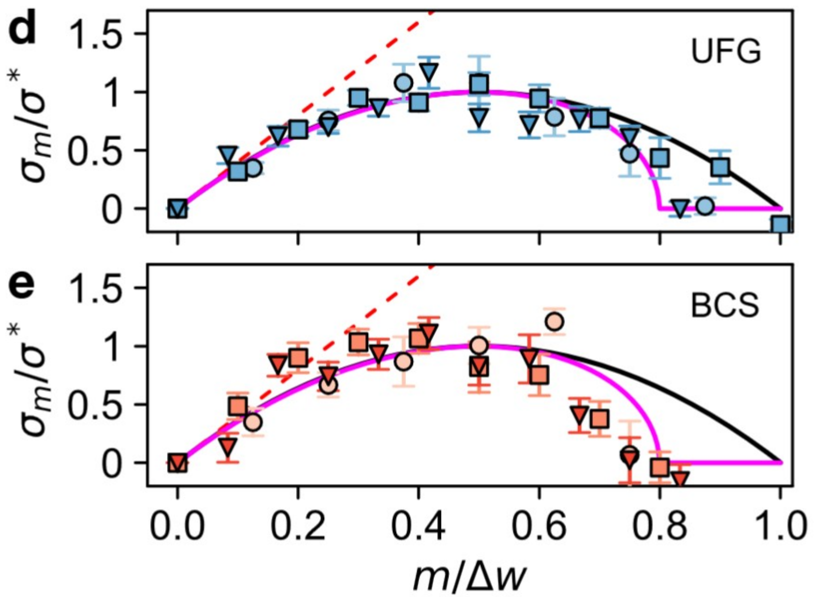
$$\nu(\boldsymbol{r}) = \frac{1}{2}\sum_{|E_n|<E_c} \left[u_{n,a}(\boldsymbol{r})v^*_{n,b}(\boldsymbol{r}) - u_{n,b}(\boldsymbol{r})v^*_{n,a}(\boldsymbol{r})\right] f_\beta(-E_n).$$

PVM

$$\sigma_{\mathrm{PVM}}(k, \Delta v) = \frac{\Gamma k}{2d_v}\left(1 - \frac{k d_v}{2\pi}\right)$$

$$m = kR_0$$

Rayleigh Model



Rayleigh model

In classical fluid mechanics, the problem of the stability of a finite-width shear layer was first analyzed by Rayleigh[17], who derived an interface-dependent growth rate as:

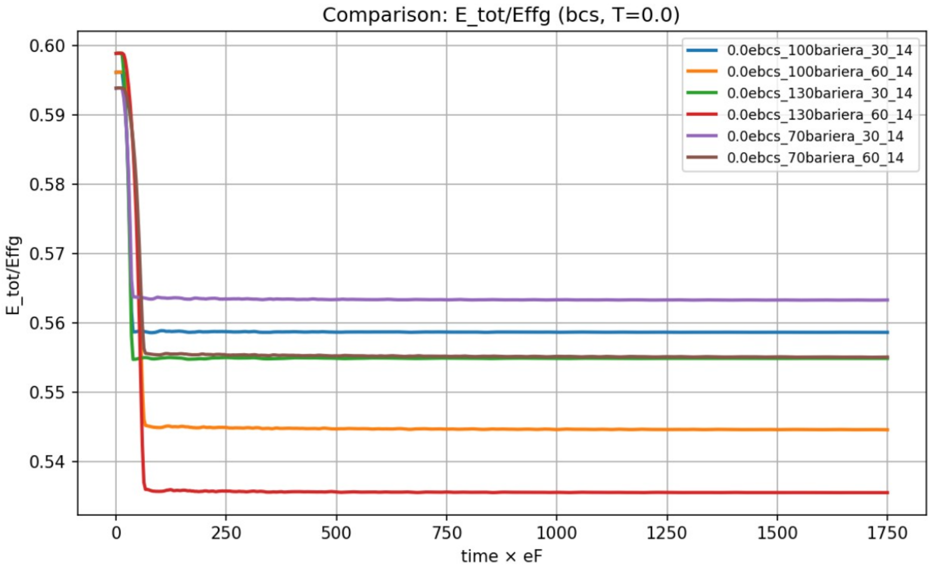
$$\sigma_R(k, \Delta v) = \mathrm{Im}\frac{\Delta v}{4\delta}\sqrt{(2k\delta - 1)^2 - e^{-4k\delta}}. \qquad (3)$$

Here, δ is the interface width and depends on the fluid's specifics and the flow shear velocity. According to Eq. (3), the instability only occurs for $k\delta \leq 0.64$, while the system is stable against perturbations with higher wavenumbers[19]. Similar to the PVM, Eq. (3) recovers Kelvin's rate for $k\delta \ll 1$.

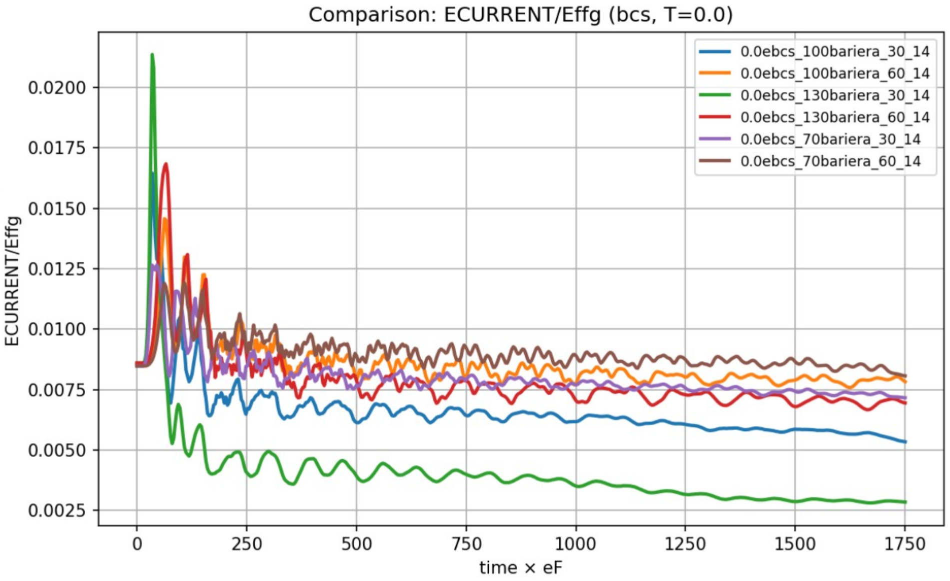
[17] L. Rayleigh, On the stability, or instability, of certain fluid motions, Proc. London Math. Soc. **s1-11**, 57 (1879).

[19] F. Charru, *Hydrodynamic Instabilities* (Cambridge University Press, 2011).

source: [2]



(a) BCS, T=0.0



(a) BCS, T=0.0