Bayesian inference of hybrid stars with large quark cores

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Why are Neutron Stars cool?

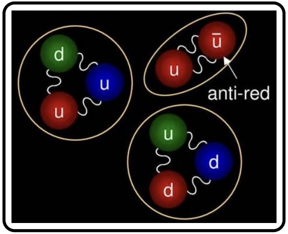
$M \approx 1.4 M_\odot$

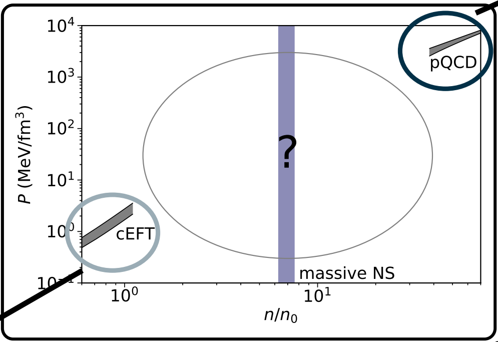
$R \approx 14$ km

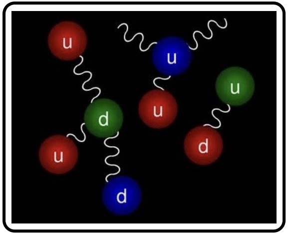
$R \approx 14$ km

Matter at extreme densities

Quark phase



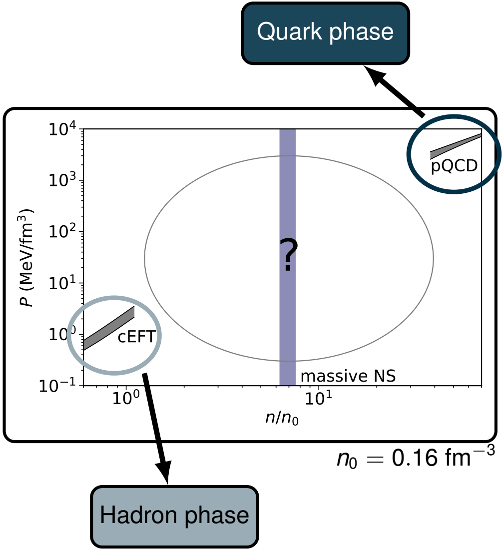




$n_0 = 0.16\ \text{fm}^{-3}$

Hadron phase

A question to be answered



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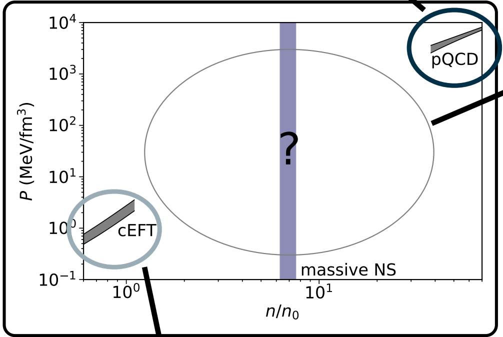
A question to be answered

Quark phase

There is a phase transition in this unknown region



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Hadron phase

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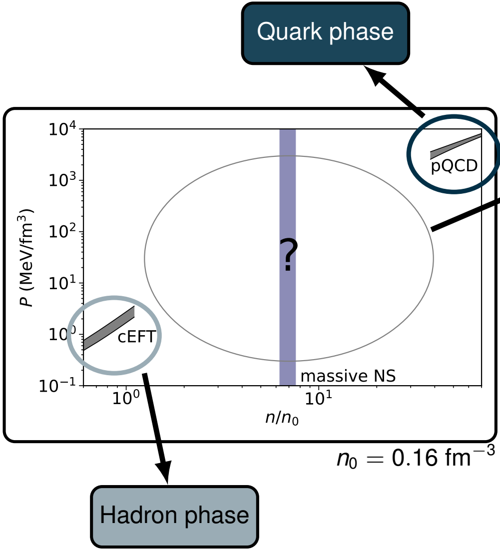

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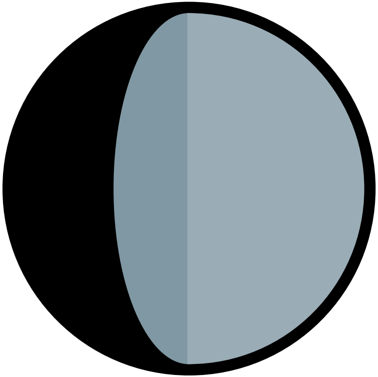
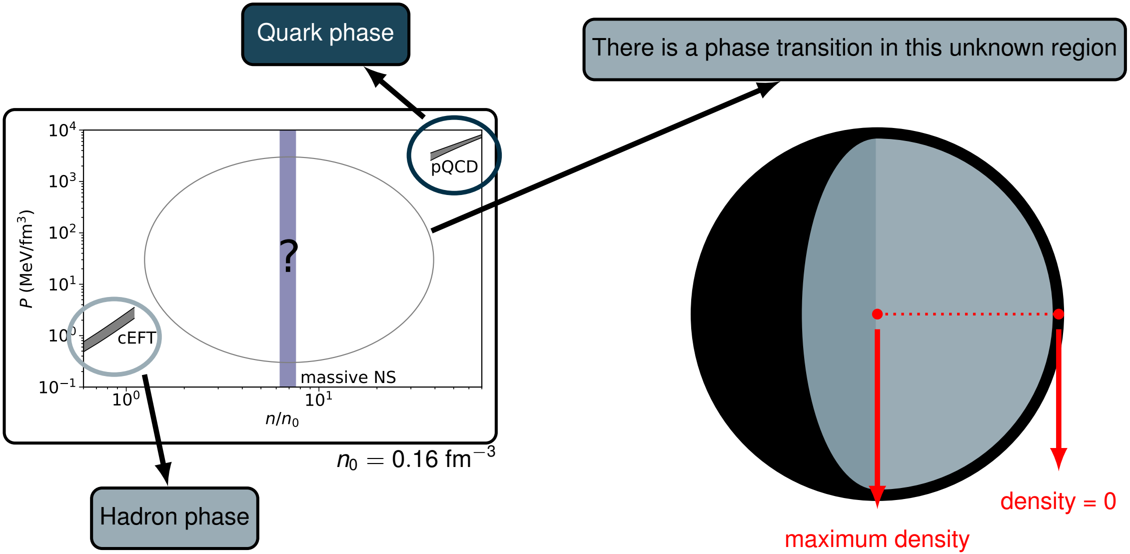


A question to be answered



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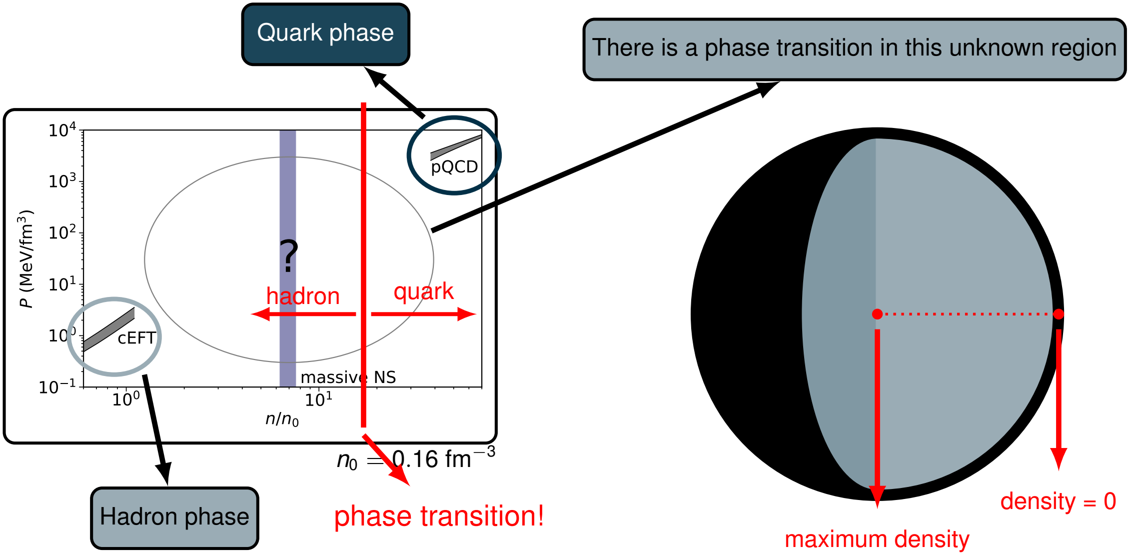

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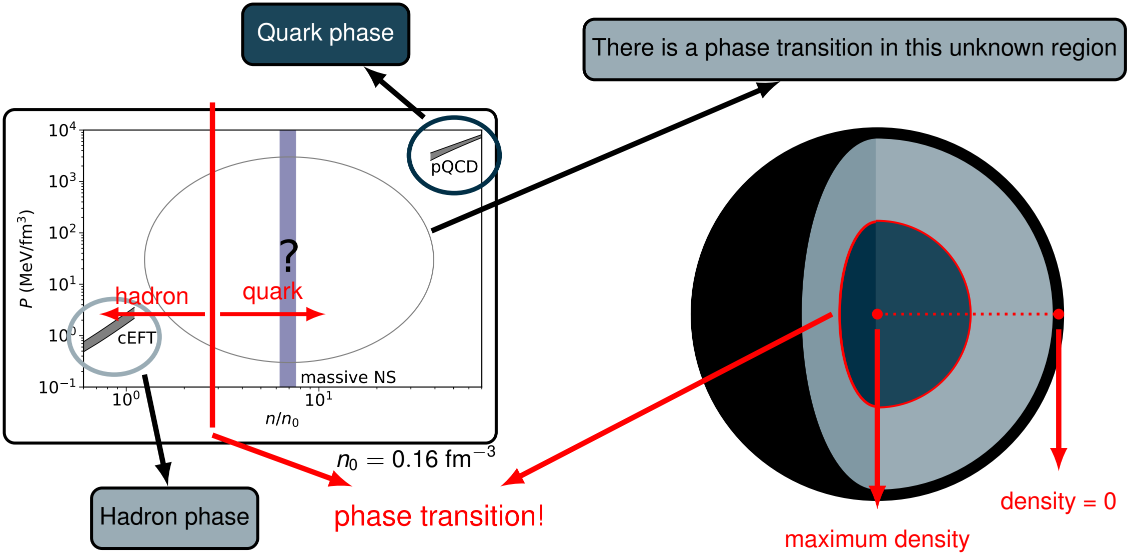
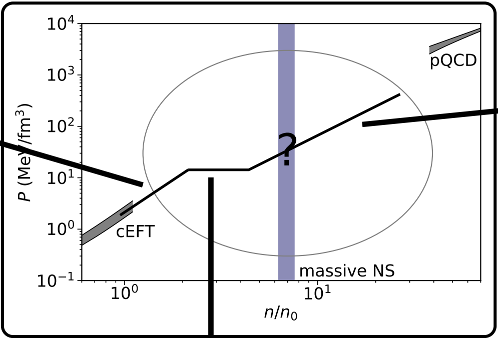


Objective:
Explore the possibility of
neutron stars with large quark cores

Exploring the unknown region

We must to assume models for the hadron and quark phases

Hadron equation



Quark equation

$n_0 = 0.16\ \mathrm{fm}^{-3}$

Phase transition

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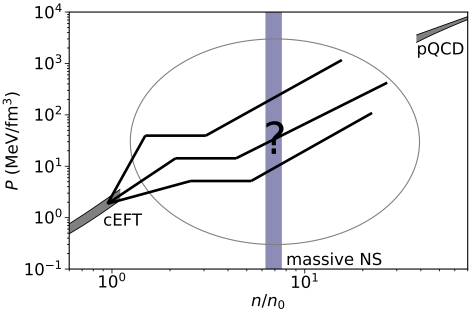
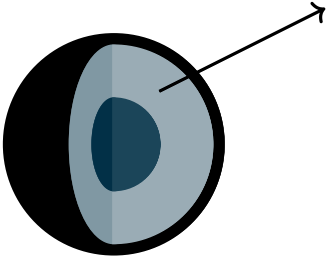


$n_0 = 0.16\ \text{fm}^{-3}$

By changing some parameters, we get more equations

Objective: Explore the possibility of neutron stars with large quark cores



Hadron phase

Relativistic Mean Field (RMF)[1]

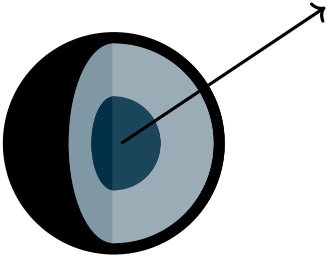
$$\mathcal{L}_N = \bar{\Psi}\left[\gamma^\mu\left(i\partial_\mu - g_\omega\omega_\mu - g_\rho \boldsymbol{t}\cdot\boldsymbol{\rho}_\mu\right) - (m - g_\sigma\phi)\right]\Psi,$$

$$\mathcal{L}_M = \frac{1}{2}\left[\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2\right] - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\boldsymbol{\varrho}_{\mu\nu}\cdot\boldsymbol{\varrho}^{\mu\nu} + \frac{1}{2}m_\varrho^2\boldsymbol{\varrho}_\mu\cdot\boldsymbol{\varrho}^\mu,$$

$$\mathcal{L}_{NL} = -\frac{1}{3}bmg_\sigma^3(\sigma)^3 - \frac{1}{4}cg_\sigma^4(\sigma)^4 + \frac{\xi}{4!}g_\omega^4(\omega_\mu\omega^\mu)^2 + \Lambda_\omega g_\varrho^2\boldsymbol{\varrho}_\mu\cdot\boldsymbol{\varrho}^\mu g_\omega^2\omega_\mu\omega^\mu,$$

[1] Tuhin Malik et al. "Spanning the full range of neutron star properties within a microscopic description". In: *Phys. Rev. D* 107.10 (2023), p. 103018. DOI: 10.1103/PhysRevD.107.103018. arXiv: 2301.08169 [nucl-th].

Objective: Explore the possibility of neutron stars with large quark cores



Quark phase

NJL (Nambu-Jona-Lasinio)	MFTQCD (Mean Field Theory of QCD)[2]
• t' Hooft term; • 5 different types of interactions.	• from the QCD Lagrangian • decomposition of gluon field in soft and hard momentum components.

[2] D. A. Fogaca and F. S. Navarra. "Gluon condensates in a cold quark–gluon plasma". In: *Phys. Lett. B* 700 (2011), pp. 236–242. DOI: 10.1016/j.physletb.2011.05.011. arXiv: 1012.5266 [hep-ph].

Nambu-Jona-Lasinio (NJL)

We use the following SU(3) NJL lagrangian:

$$\mathcal{L} = \bar{\psi}\left(i\partial\!\!\!/ - m + \mu\gamma^0\right)\psi + \frac{G}{2}\left[\left(\bar{\psi}\lambda_a\psi\right)^2 + \left(\bar{\psi}i\gamma^5\lambda_a\psi\right)^2\right] + \mathcal{L}_{\text{'t Hooft}} + \mathcal{L}_I$$

- standard NJL term
- 't Hooft term
- added interation terms

Nambu-Jona-Lasinio (NJL)

$$
\begin{aligned}
\mathcal{L}_{\mathrm{I}} = &-G_{\omega}\left[\left(\bar{\psi}\gamma^{\mu}\lambda_{0}\psi\right)^{2}+\left(\bar{\psi}\gamma^{\mu}\gamma_{5}\lambda_{0}\psi\right)^{2}\right] \\
&-G_{\rho}\sum_{a=1}^{8}\left[\left(\bar{\psi}\gamma^{\mu}\lambda^{a}\psi\right)^{2}+\left(\bar{\psi}\gamma^{\mu}\gamma_{5}\lambda^{a}\psi\right)^{2}\right] \\
&-G_{\omega\omega}\left[\left(\bar{\psi}\gamma^{\mu}\lambda_{0}\psi\right)^{2}+\left(\bar{\psi}\gamma^{\mu}\gamma_{5}\lambda_{0}\psi\right)^{2}\right]^{2} \\
&-G_{\sigma\omega}\sum_{a=0}^{8}\left[\left(\bar{\psi}\lambda_{a}\psi\right)^{2}+\left(\bar{\psi}i\gamma^{5}\lambda_{a}\psi\right)^{2}\right]\left[\left(\bar{\psi}\gamma^{\mu}\lambda_{0}\psi\right)^{2}+\left(\bar{\psi}\gamma^{\mu}\gamma_{5}\lambda_{0}\psi\right)^{2}\right] \\
&-G_{\rho\omega}\sum_{a=1}^{8}\left[\left(\bar{\psi}\gamma^{\mu}\lambda_{0}\psi\right)^{2}+\left(\bar{\psi}\gamma^{\mu}\gamma_{5}\lambda_{0}\psi\right)^{2}\right]\left[\left(\bar{\psi}\gamma^{\mu}\lambda_{a}\psi\right)^{2}+\left(\bar{\psi}\gamma^{\mu}\gamma_{5}\lambda_{a}\psi\right)^{2}\right]. \qquad (1)
\end{aligned}
$$

$$P \rightarrow P + B. \qquad (2)$$

Mean Field Theory of QCD (MFTQCD)

- Start with the QCD lagrangian;
- Assume we can decompose the gluon field in[3]
    - low momentum components (soft gluons);
    - high momentum components (hard gluons);

$$\tilde{G}^{a\mu}(k) = \tilde{A}^{a\mu}(k) + \tilde{\alpha}^{a\mu}(k) \qquad (3)$$

$$P = \frac{27}{2}\xi^2\rho_B^2 - B + P_F, \qquad (4)$$

$$\epsilon = \frac{27}{2}\xi^2\rho_B^2 + B + \epsilon_F, \qquad (5)$$

where

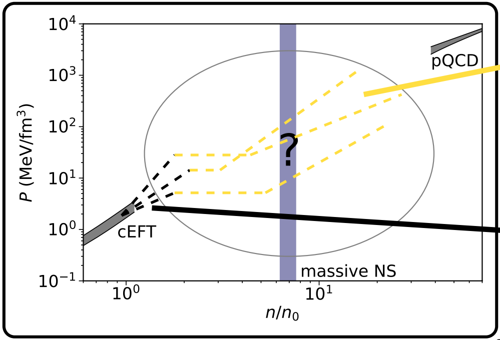
- $\xi = g/m_G$ and $m_G = \frac{9}{32}g^2\mu_0^2$;
- $B = \frac{9}{4(34)}g^2\phi_0^4$;
- P_F and ϵ_F are the pressure and energy density of a non-interacting Fermi gas of quarks and electrons.

[3] Fogaca and Navarra, "Gluon condensates in a cold quark–gluon plasma".

NJL-GW Set

NJL Set

r-NJL Set

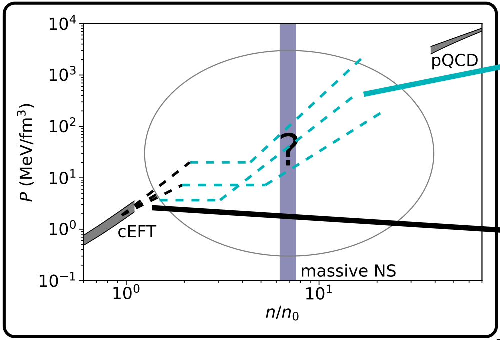


Quark model: NJL

Hadron model: RMF

$n_0 = 0.16$ fm^{-3}

MFTQCD Set

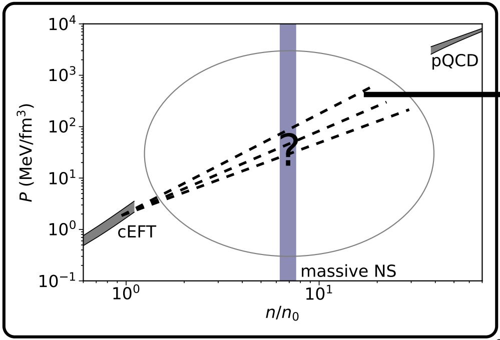


Quark model: MFTQCD

Hadron model: RMF

$n_0 = 0.16\ \text{fm}^{-3}$

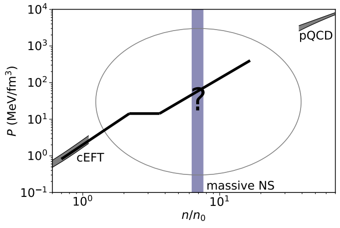
RMF Set



$n_0 = 0.16$ fm^{-3}

Hadron model: RMF

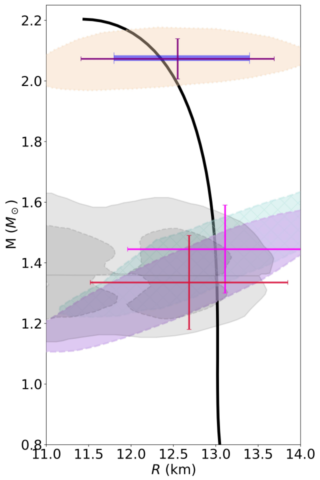
How we test the models



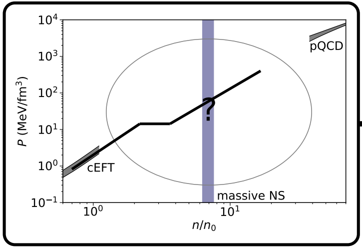
TOV equations

$$\frac{dP}{dr} = -\frac{[\epsilon(r)+P(r)]\left[M(r)+4\pi r^3 P(r)\right]}{r^2\left[1-\frac{2GM(r)}{r}\right]}$$

$$\frac{dM}{dr} = 4\pi r^2 \epsilon(r)$$



How we test the models



TOV equations

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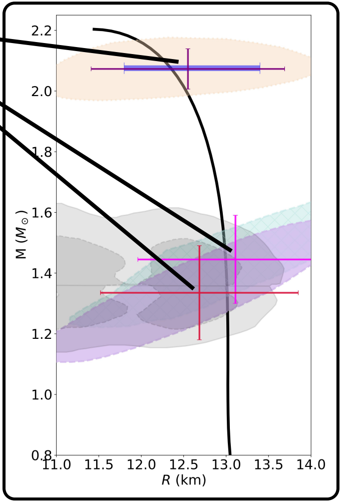
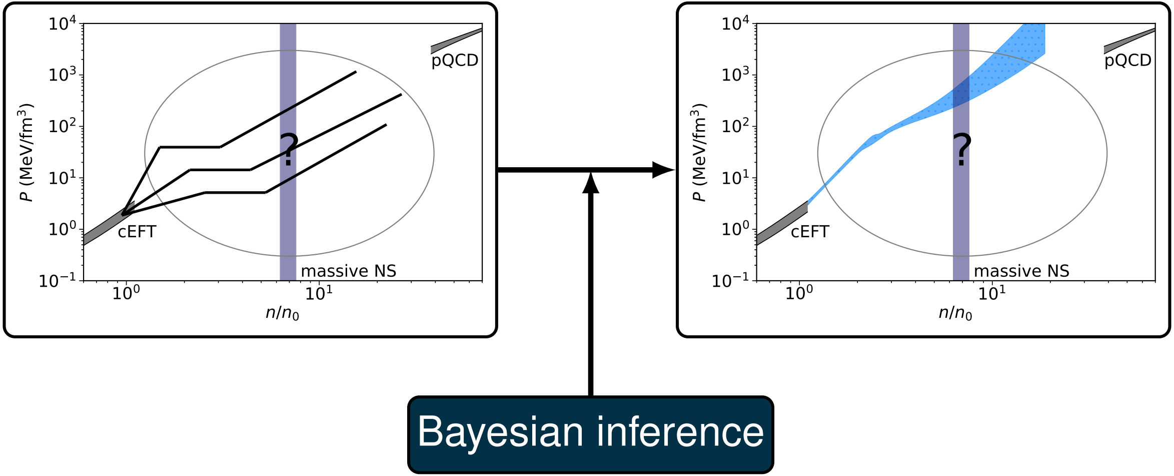
$$\frac{dM}{dr} = 4\pi r^2 \epsilon(r)$$

observational data



Building the hybrid sets of EoS



Bayesian Analysis: Posterior

$$\mathcal{P}(\Theta|D) \propto \mathcal{L}(D|\Theta)\,\pi(\Theta)$$

Posterior probability

Probability that the model Θ is true, given the data D.

Bayesian inference samples the parameters Θ to obtain the EoS with large posterior probabilities.

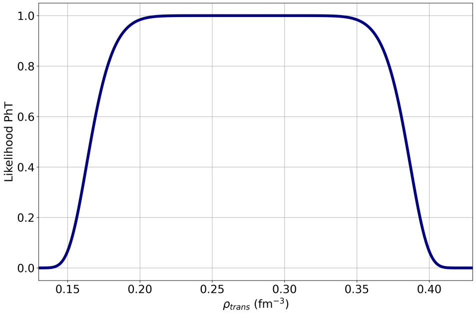
Bayesian Analysis: Likelihood

$$\mathcal{P}(\Theta|D) \propto \mathcal{L}(D|\Theta)\, \pi(\Theta)$$

Likelihood probability

Probability that the model Θ satisfies the data D.

exemple: phase transition density

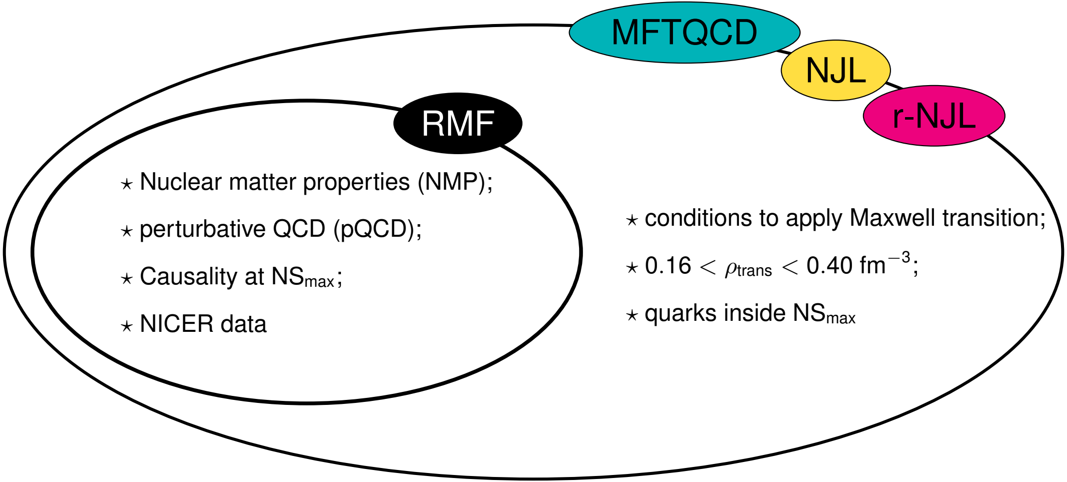


Bayesian Analysis: Likelihood

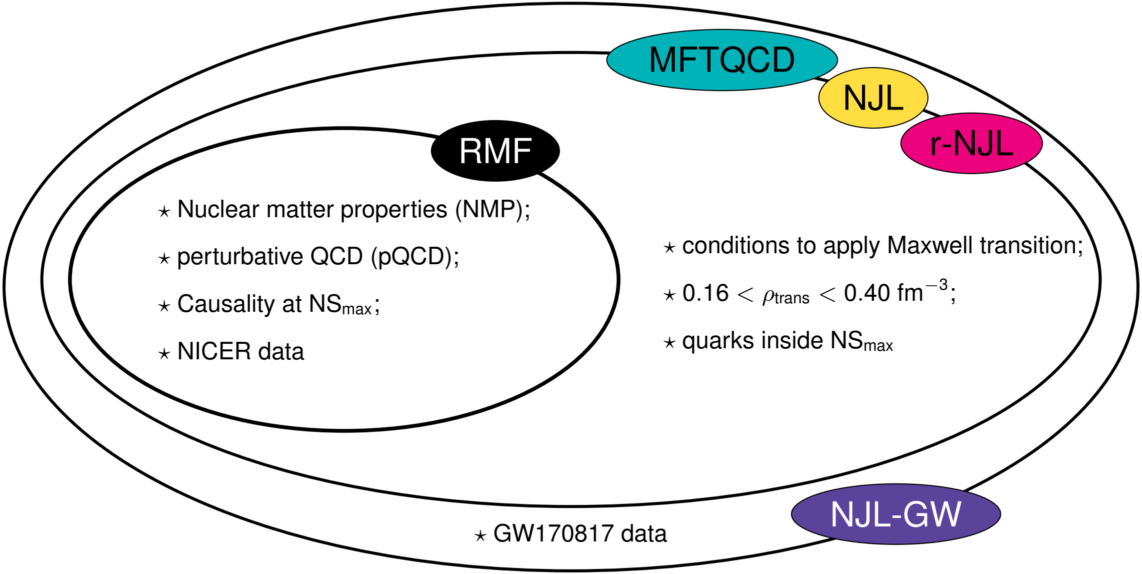
RMF

- ⋆ Nuclear matter properties (NMP);
- ⋆ perturbative QCD (pQCD);
- ⋆ Causality at NS_{max};
- ⋆ NICER data

Bayesian Analysis: Likelihood



Bayesian Analysis: Likelihood

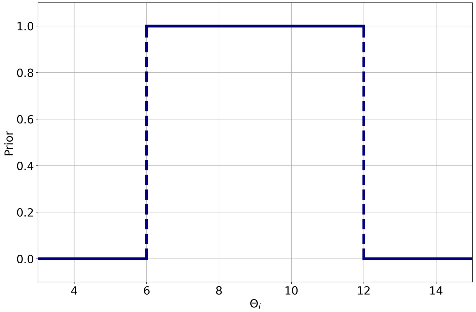


$$\mathcal{P}(\Theta|D) \propto \mathcal{L}(D|\Theta)\ \pi(\Theta)$$

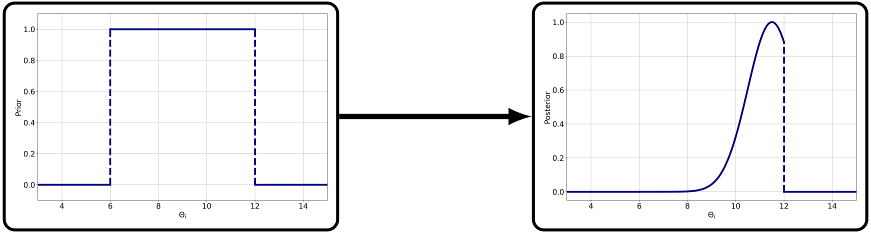
Prior probability

Probability that the model Θ is true before considering the data D.

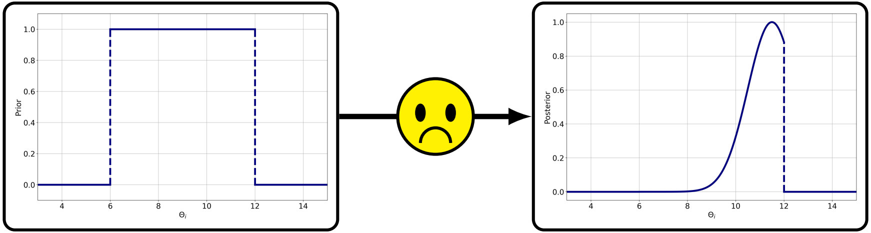
exemple: uniform distribution



Bayesian Analysis: Prior



Bayesian Analysis: Prior



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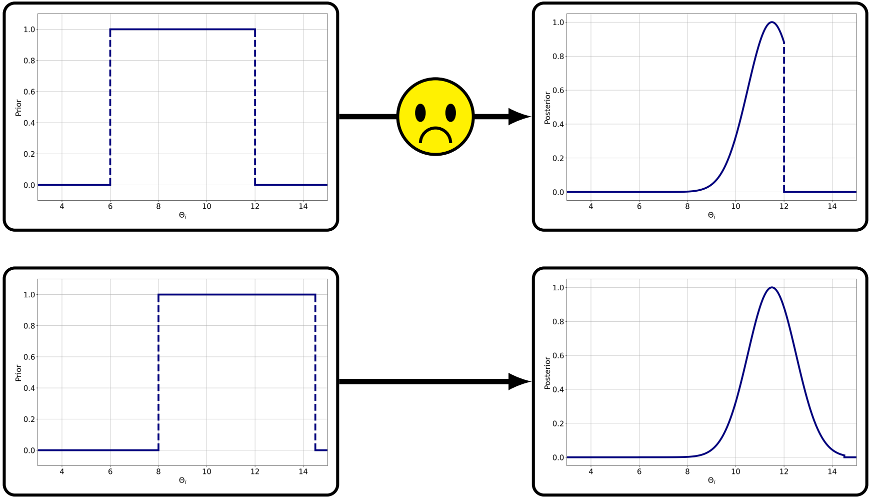
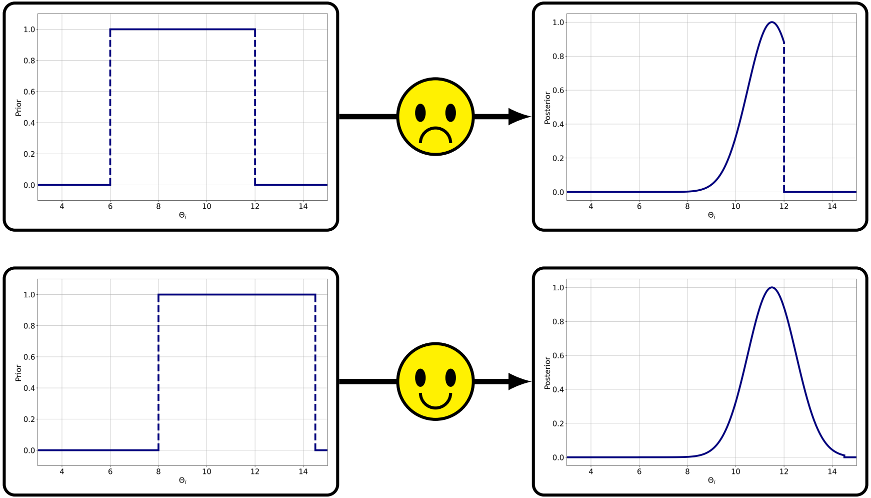


Bayesian Analysis: Prior



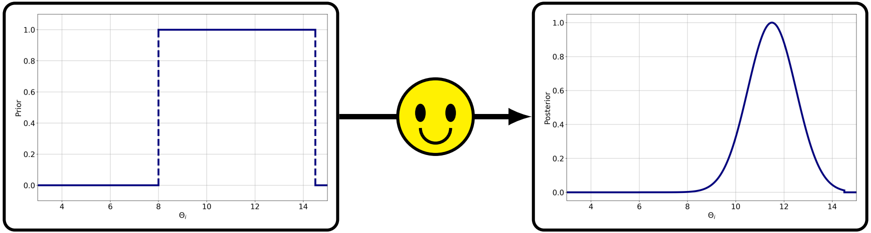
Bayesian Analysis: Prior

NJL

MFTQCD

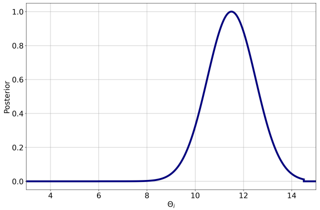
RMF

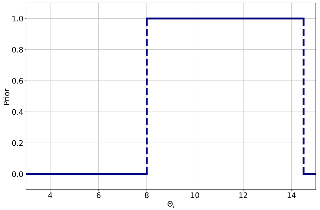
NJL-GW



Bayesian Analysis: Prior

Posterior distribution of RMF set



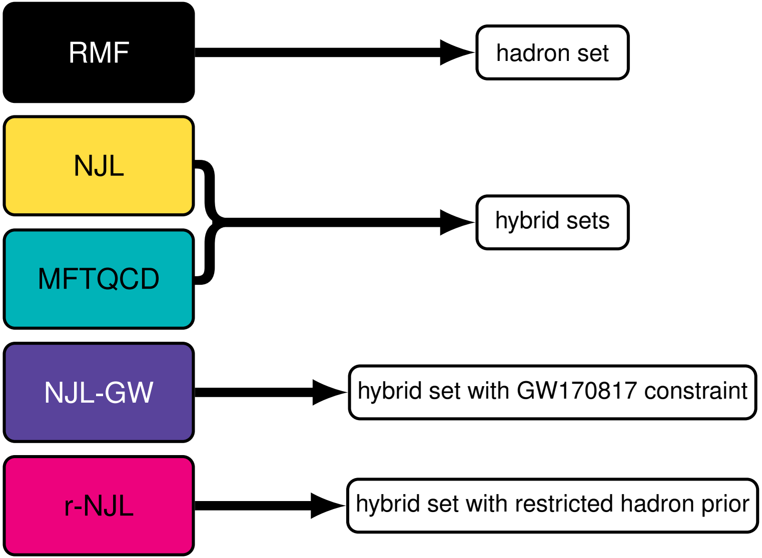


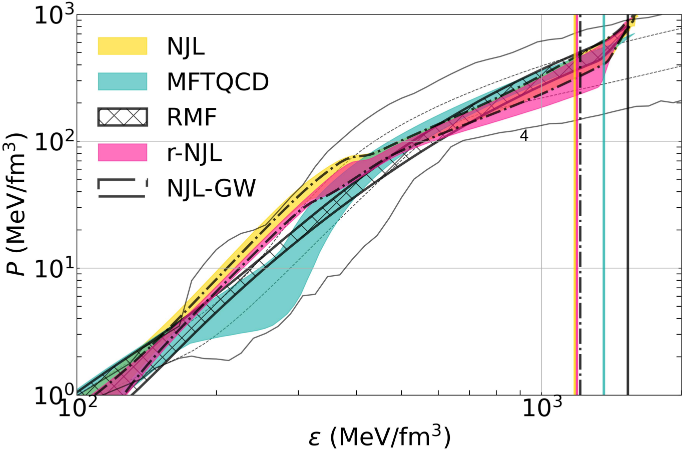
as the prior of

r-NJL

*for the hadron parameters

Summarizing...





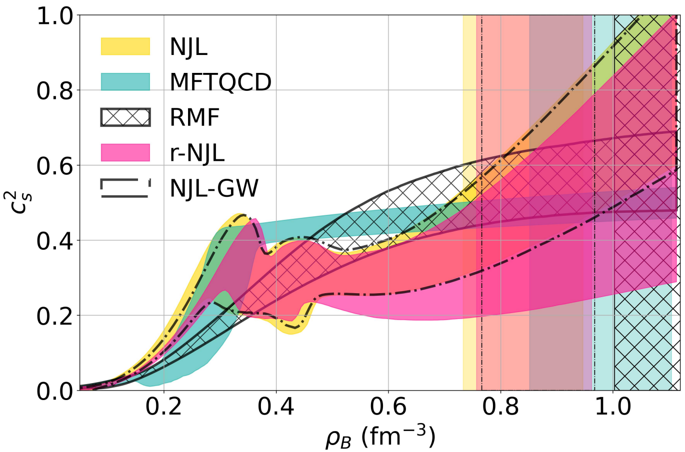
- Compatible with agnostic model[4];
- The three NJL sets have stiffer hadron phase;

ρ_{trans} (90% CI)

	median	min	max
NJL	0.353	0.304	0.388
MFTQCD	0.222	0.170	0.308
NJL-GW	0.362	0.314	0.391
r-NJL	0.369	0.333	0.395

[4] Sinan Altiparmak, Christian Ecker, and Luciano Rezzolla. "On the Sound Speed in Neutron Stars". In: *Astrophys. J. Lett.* 939.2 (2022), p. L34. DOI: 10.3847/2041-8213/ac9b2a. arXiv: 2203.14974 [astro-ph.HE].

Speech of Sound Squared

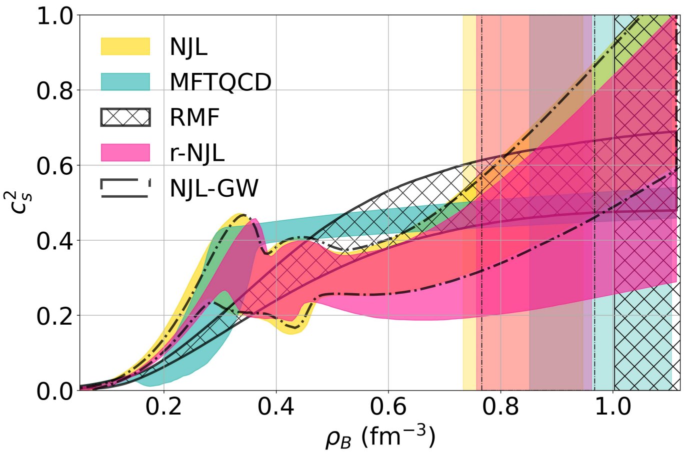


- The three NJL sets have two bumps: due to the phase transition and the onset of quark strange.

$c^2_{s,\max}$ (90% CI)

	median	min	max
NJL	0.529	0.329	0.776
MFTQCD	0.487	0.458	0.515
RMF	0.582	0.478	0.692
NJL-GW	0.564	0.340	0.795
r-NJL	0.388	0.200	0.744

Speed of Sound Squared

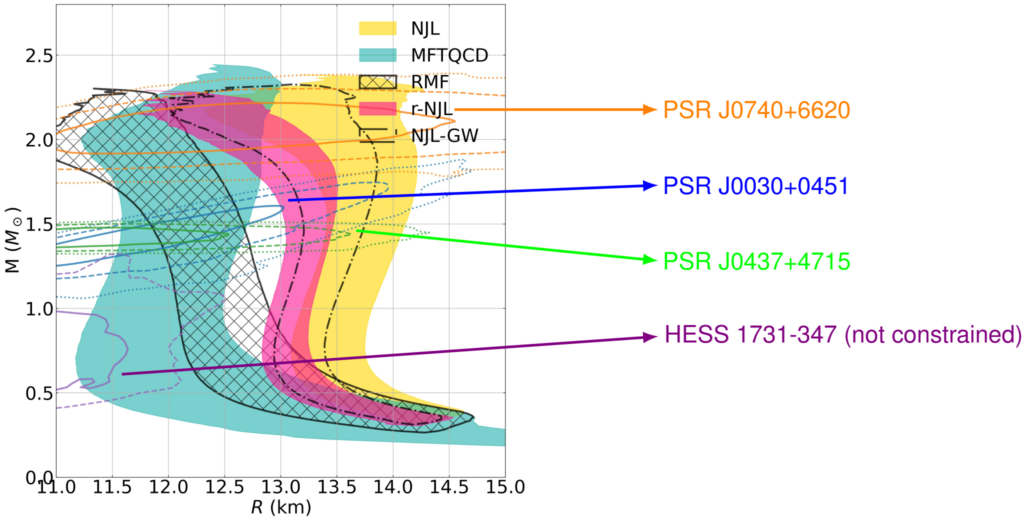


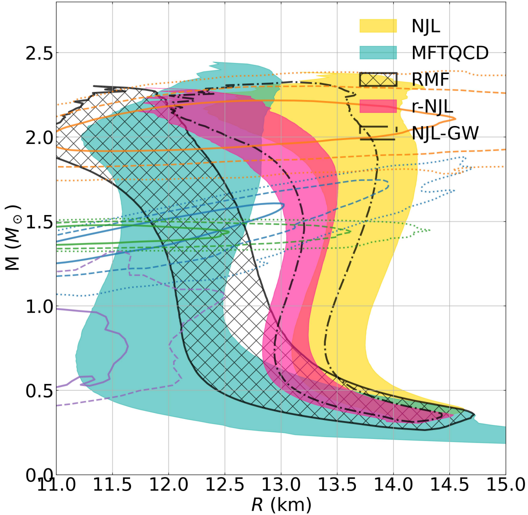
- The three NJL sets have two bumps: due to the phase transition and the onset of quark strange.

ρ_{max} (90% CI)

	median	min	max
NJL	0.829	0.733	0.947
MFTQCD	0.976	0.852	1.104
RMF	1.103	1.003	1.212
NJL-GW	0.871	0.767	0.968
r-NJL	0.861	0.757	0.962

Mass-radius diagram



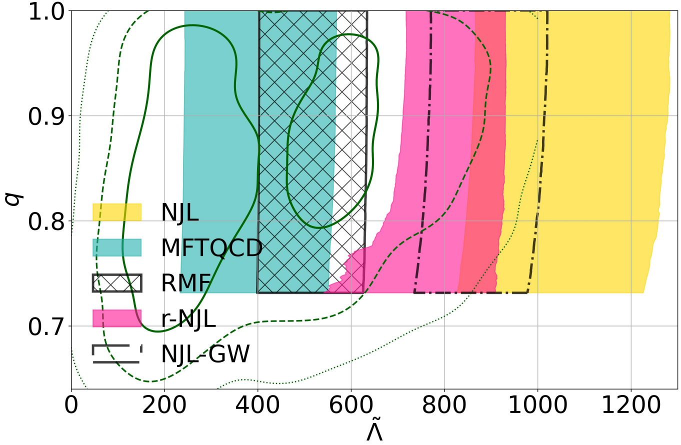


- The three NJL sets have larger radius;
- The MFTQCD set has smaller radius – compatible with HESS data.

M_{max} (90% CI)

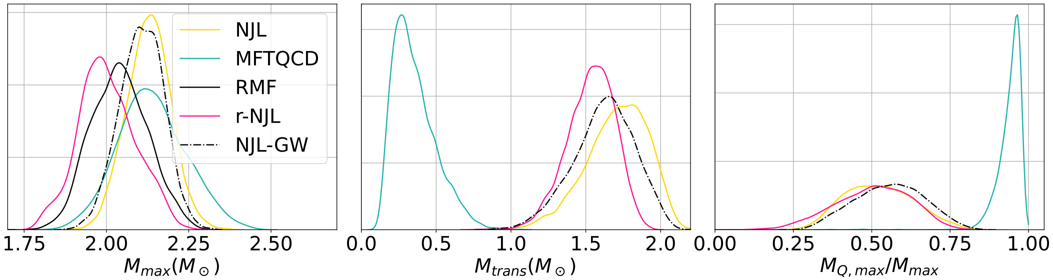
	median	min	max
NJL	2.130	2.018	2.236
MFTQCD	2.133	1.970	2.315
RMF	2.039	1.905	2.185
NJL-GW	2.108	1.993	2.212
r-NJL	1.996	1.863	2.154

Tidal Deformability



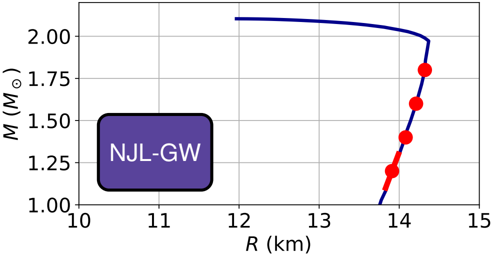
- binary mass ratio $q = M_2/M_1 < 1$;
- effective tidal deformability $\tilde{\Lambda}$;
- MFTQCD and RMF compatible with 68% CI;
- NJLs struggles to satisfy the GW170817 data.

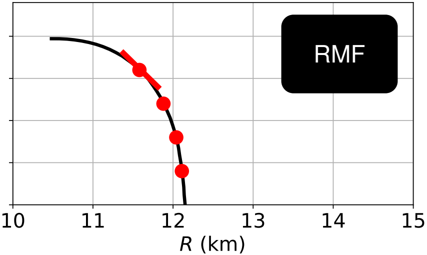
Distributions



- M_{trans}: minimum mass in which NSs have a quark core;
- NSs with $1.4M_\odot$: have (MFTQCD) or may have (NJLs) quark matter;
- NSs with $2M_\odot$: both models indicate the presence of quark matter.
- $M_{\text{Q,max}}/M_{\text{max}}$: fraction of quark matter at NS_{max};
- NJLs: $\sim$ 50% of the mass is from the quark core;
- MFTQCD: over 80% of the mass is from the quark core;

Slope of mass-radius diagram





M	$1.2M_\odot$		$1.4M_\odot$		$1.6M_\odot$		$1.8M_\odot$	
dM/dR	+	-	+	-	+	-	+	-
NJL	**6306**	208	**6008**	506	**4896**	1618	2509	**4005**
NJL-GW	**7268**	253	**6656**	865	**4699**	2822	1653	**5868**
r-NJL	**5180**	147	**4562**	765	2358	**2969**	206	**5121**
MFTQCD	**4392**	487	**4326**	553	**3459**	1420	1119	**3760**
RMF	822	**5215**	175	**5862**	70	**5967**	38	**5999**

NJL, NJL-GW, r-NJL, MFTQCD → positive

RMF → negative

Summarizing...

	NJL	NJL-GW	r-NJL	MFTQCD	RMF
agnostic models	yes	yes	yes	yes	yes
phase transition	$\sim 2\rho_0$	$\sim 2\rho_0$	$\sim 2\rho_0$	$\rho_0 < \rho_{\text{trans}} \lesssim 2\rho_0$	$\ldots$
NICER data	yes	yes	yes	yes	yes
GW170817 data	$\gtrsim 90\%$ CI	$\gtrsim 90\%$ CI	$\gtrsim 90\%$ CI	yes	yes
NSs with $1.4M_\odot$ are	undefined	undefined	undefined	hybrid NS	hadron NS
NSs with $2M_\odot$ are	hybrid NS	hybrid NS	hybrid NS	hybrid NS	hadron NS
fraction of quark core	$\sim 50\%$	$\sim 50\%$	$\sim 50\%$	$\gtrsim 80\%$	0%
slope dM/dR	positive	positive	positive	positive	negative

THANK YOU!

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