Aalto University
School of Science

Vortex pinning in a spin-triplet superfluid

Riku Rantanen, Rida Alrahmanlokman Shahin, and Vladimir Eltsov

Aalto University (Helsinki, Finland)

ROTA group – Topological Quantum Fluids

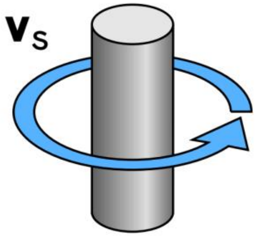
- Superfluid **^{3}He** and ^{4}He
- Unique **rotating cryostat** with temperatures as low as 140µK
- **Vortices**, quantum turbulence, magnon BECs, time crystals

Vladimir Eltsov

Outline

Introduction: Superfluid ^{3}He

Part 1: Vortices in superfluid ^{3}He



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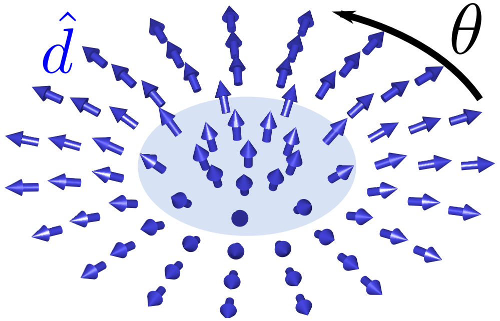
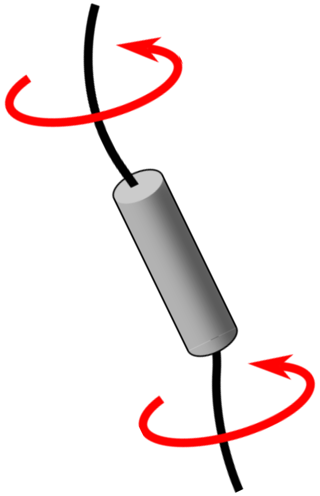
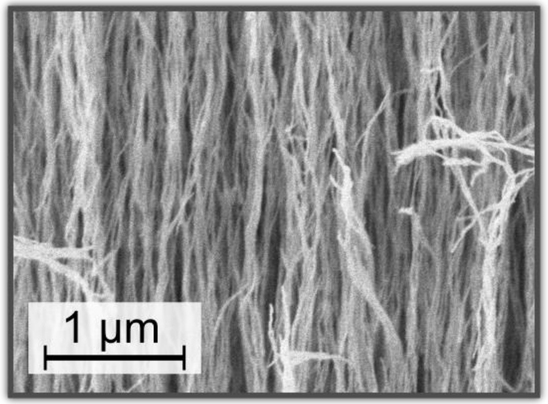


Part 2: Vortex pinning in ^{3}He

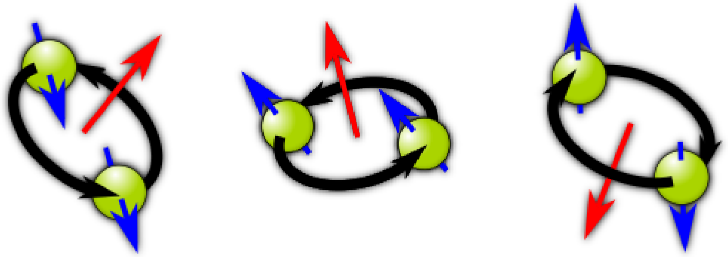




Superfluid ^{3}He

Fermionic superfluid

Spin-triplet **p-wave** Cooper pairs



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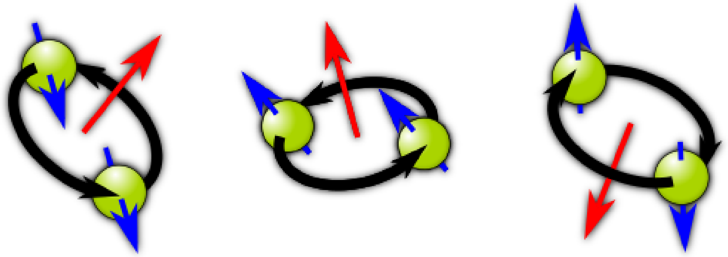

In addition to phase coherence, pairing breaks rotational symmetry SO(3) in **orbital** and **spin** space

$$S_z = -1, 0, 1$$
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$$A = \begin{bmatrix} A_{xx} & A_{xy} & A_{xz} \\ A_{yx} & A_{yy} & A_{yz} \\ A_{zx} & A_{zy} & A_{zz} \end{bmatrix}$$

Broken symmetry state described by a
3 x **3** complex order parameter matrix

A!

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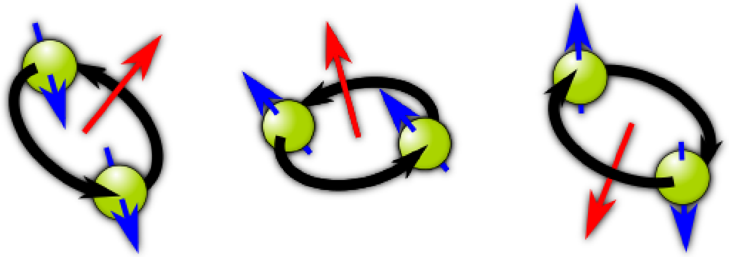

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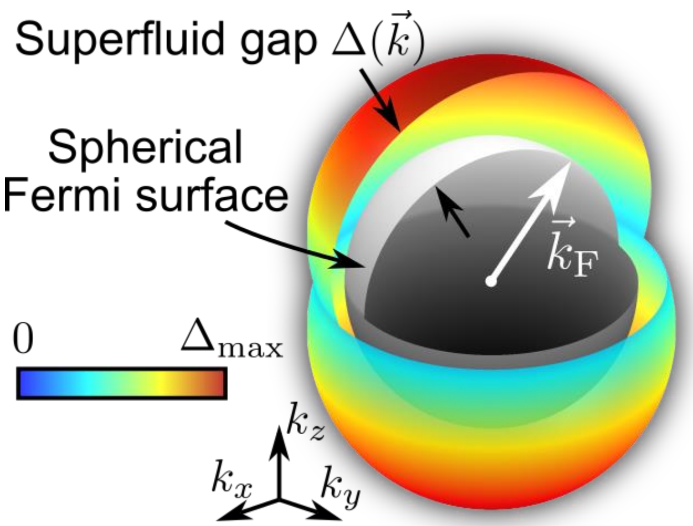

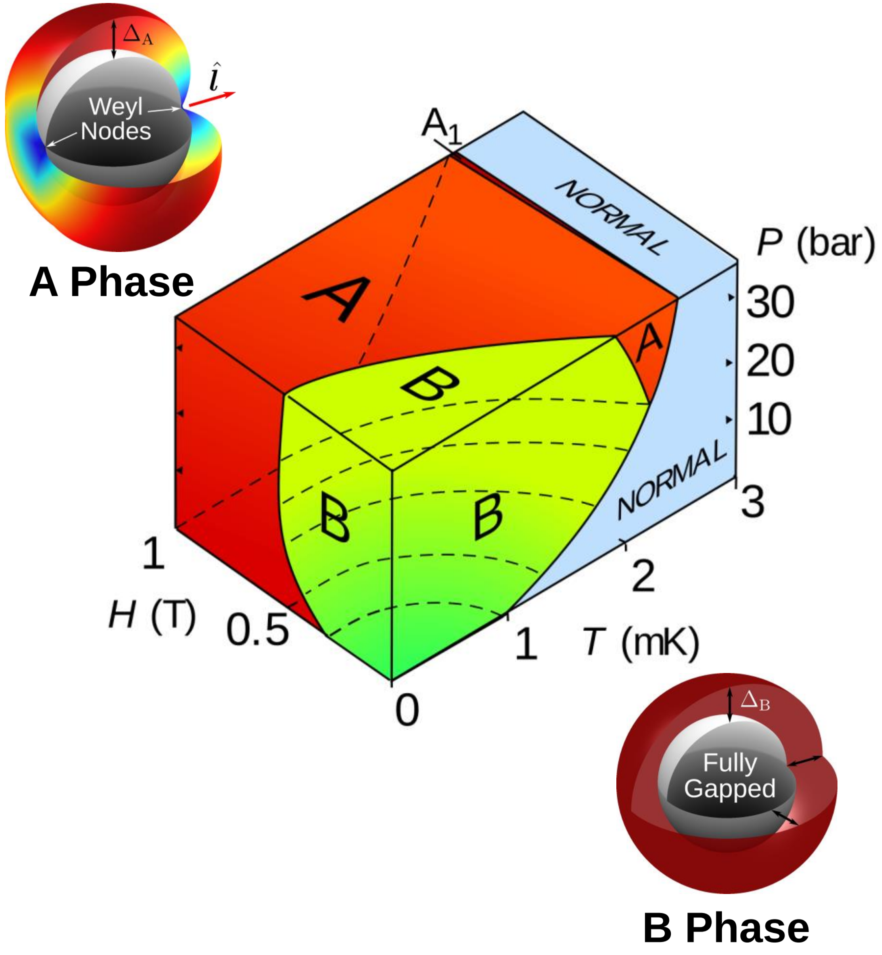

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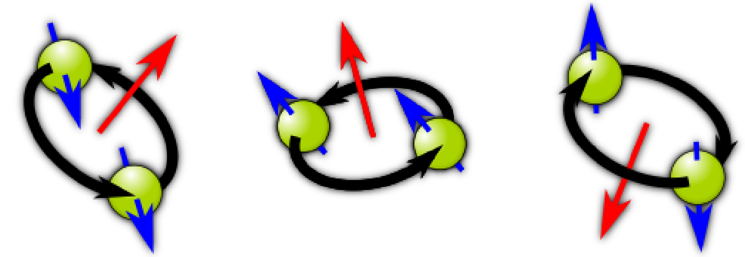

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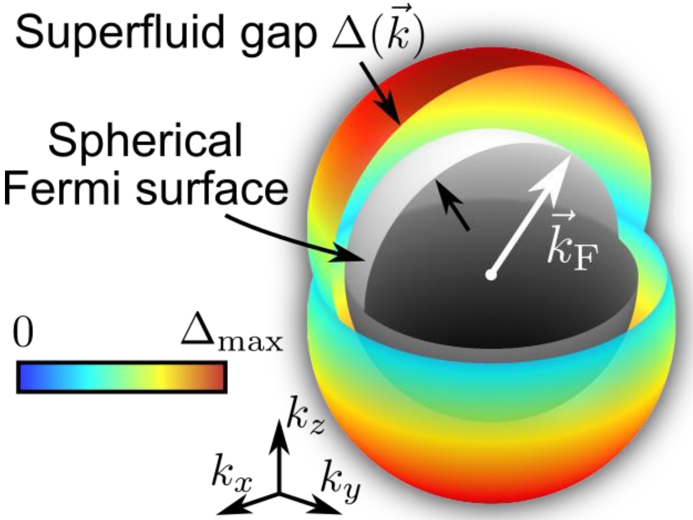
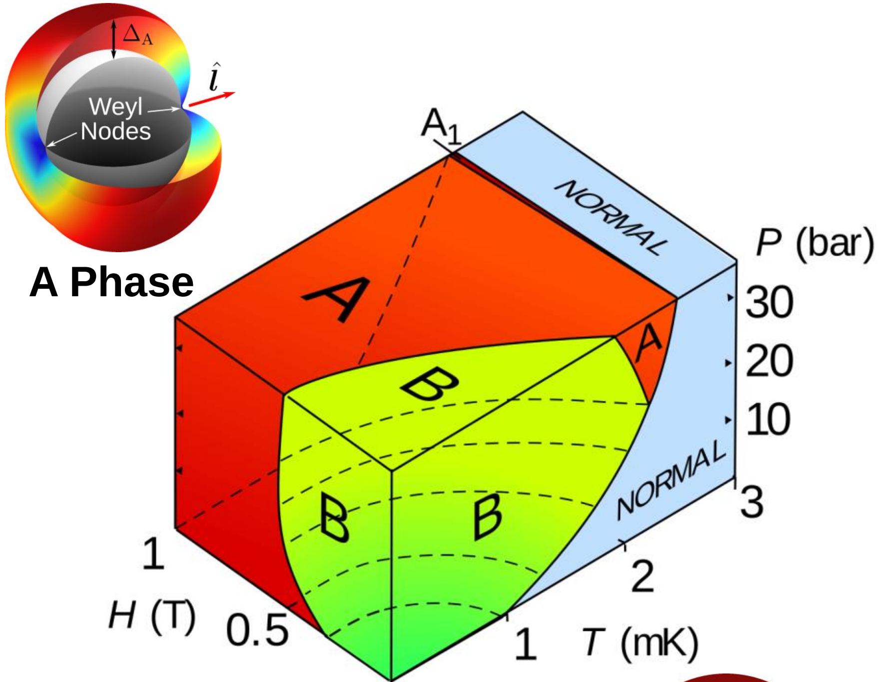
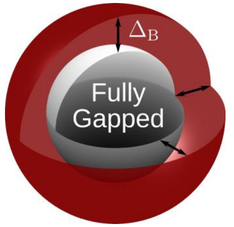
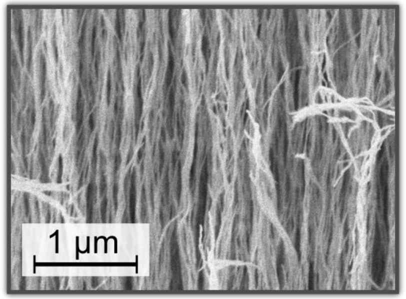




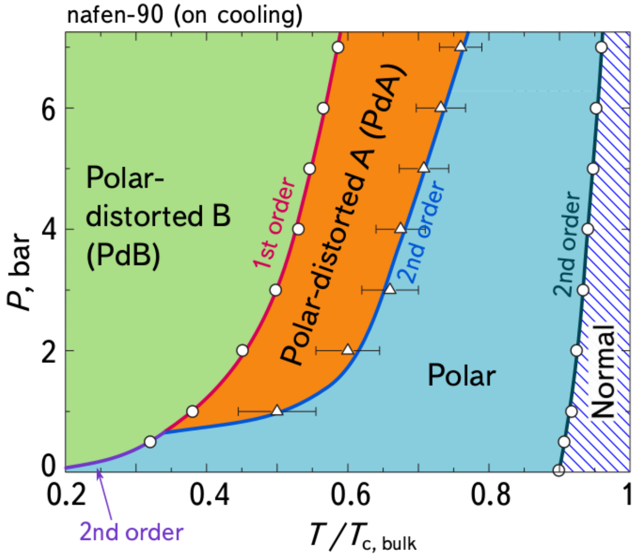


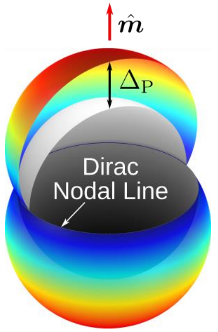
B Phase

New phases stabilized in **confinement:**



"nafen"





Polar phase

Part 1: Vortices in superfluid ^{3}He

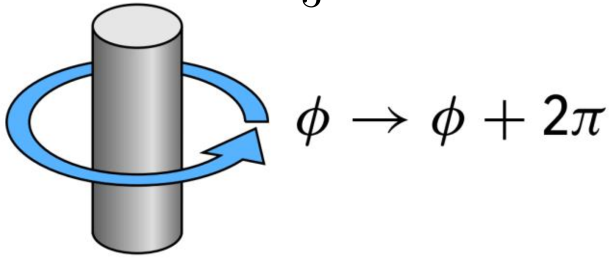
Quantized vortices

Conventional superfluids:
(^{4}He, s-wave superconductors, etc.)

Order parameter is a single scalar $\Psi = |\Psi| e^{i\phi}$

Superfluid velocity $v_{\rm s} = \frac{\hbar}{2m_3}\nabla\phi$

Irrotational
$\nabla \times v_{\rm s} = 0$



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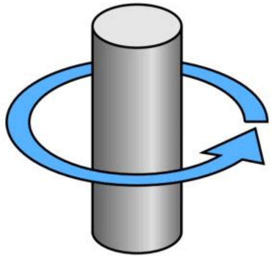

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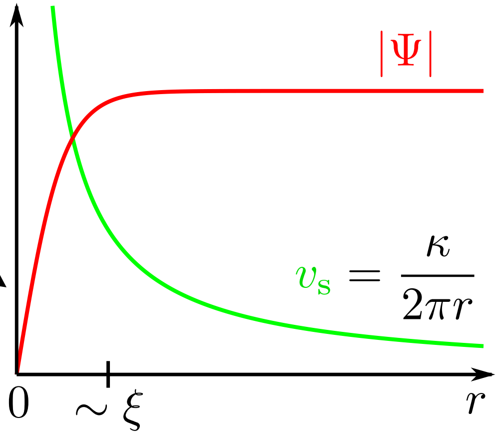
Superfluid suppression in the core costs energy!



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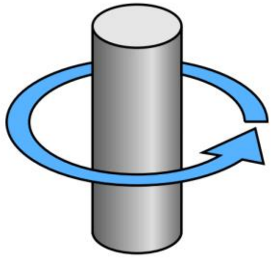

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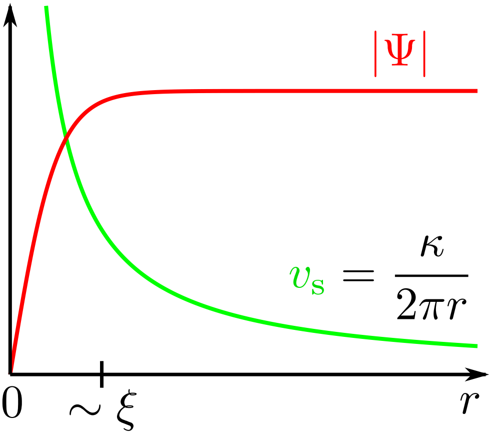

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Multicomponent order parameter:

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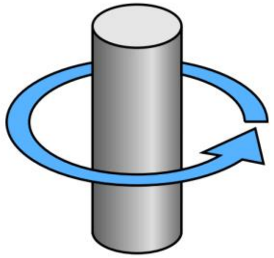

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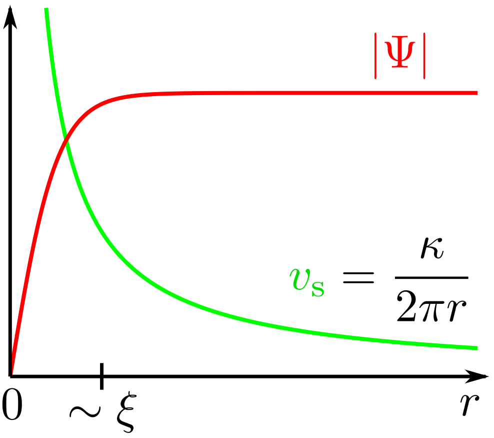

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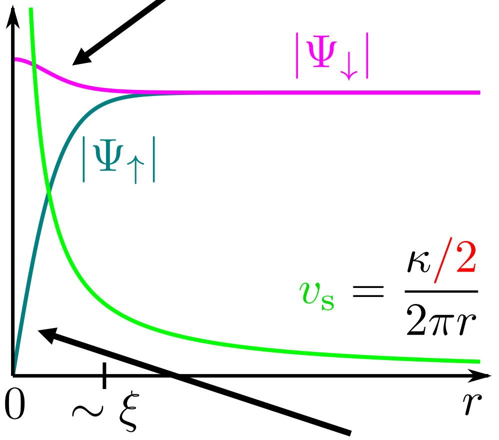
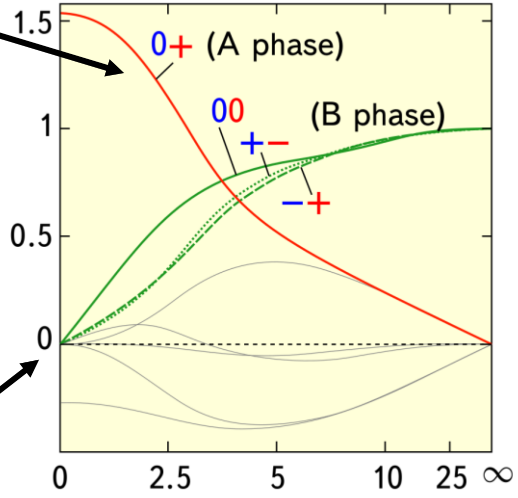
Cores are generally **not singular**

Suppression compensated by other components





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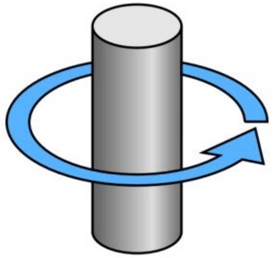

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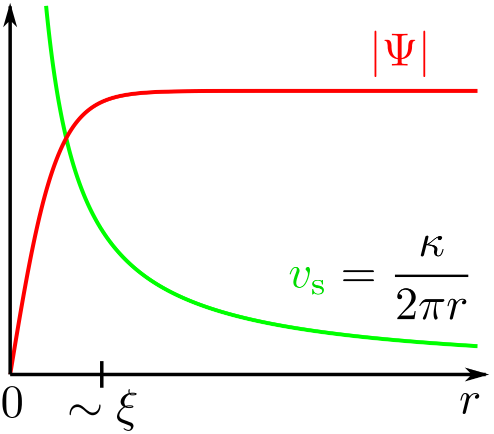

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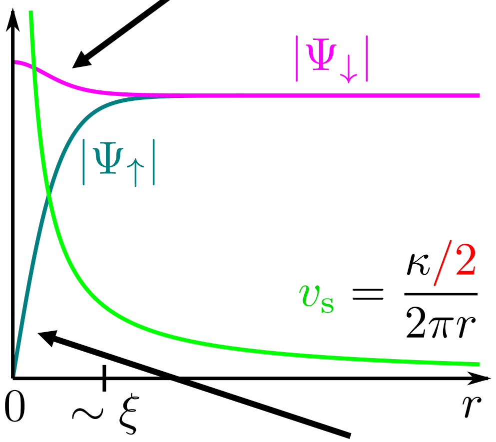

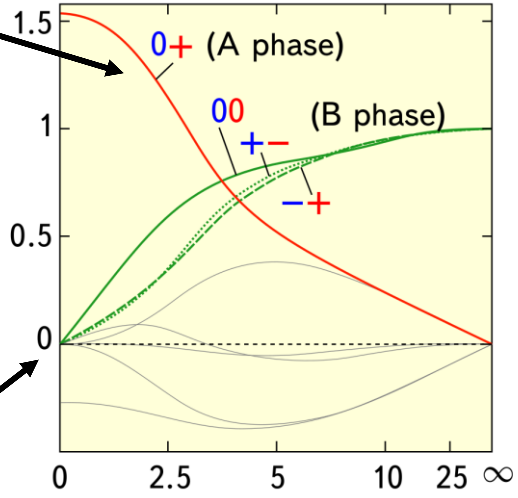

Components with winding are suppressed

Superflow **not necessarily irrotational** $\nabla \times v_{\mathrm{s}} \neq 0$

Vortices in the Polar phase

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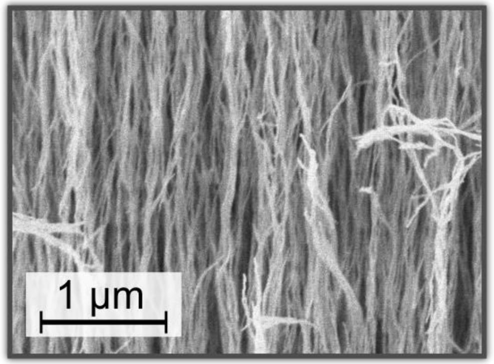

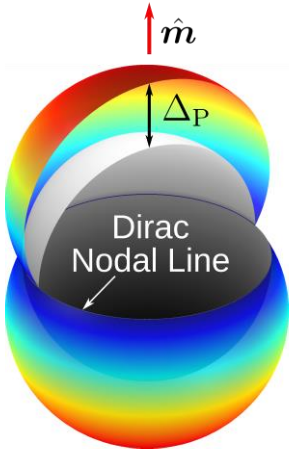

$$A_{\mu j} = \Delta_\mathrm{P} e^{i\phi} \hat{d}_\mu \hat{m}_j$$

Strands fix direction of $\hat{m} = \hat{z}$

$\hat{d}$ weakly oriented by **magnetic field**
and **spin**-**orbit** interaction $\hat{d} \perp \hat{H}, \hat{m}$

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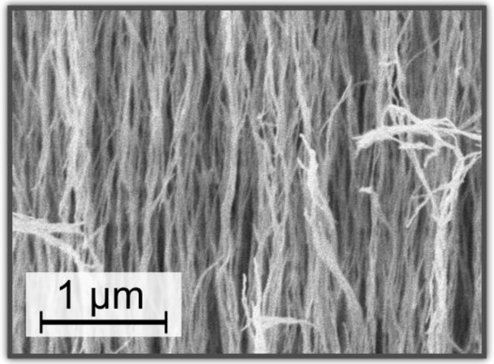

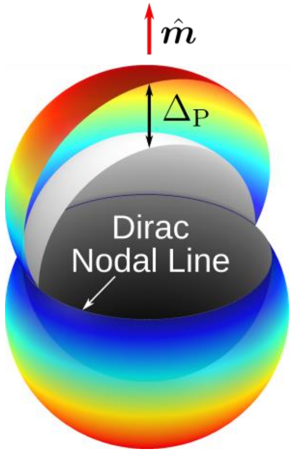

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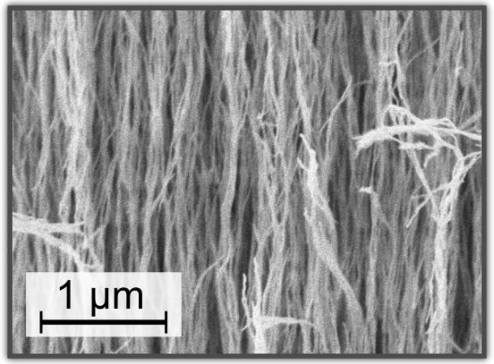

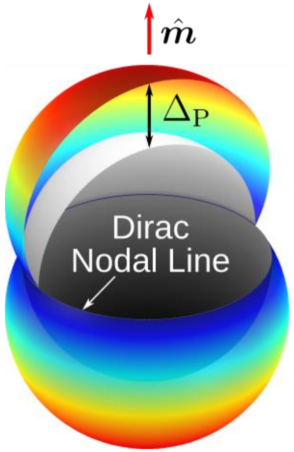

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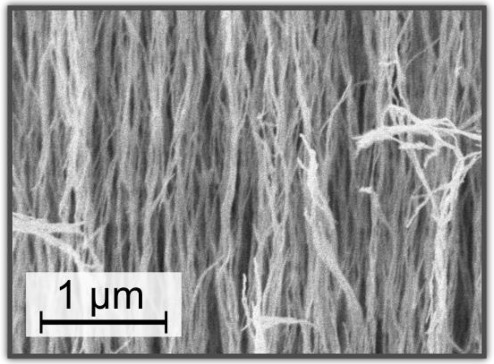

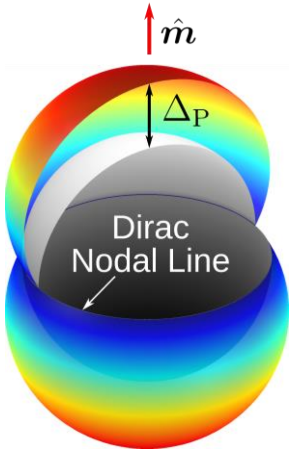

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Four possibilities:

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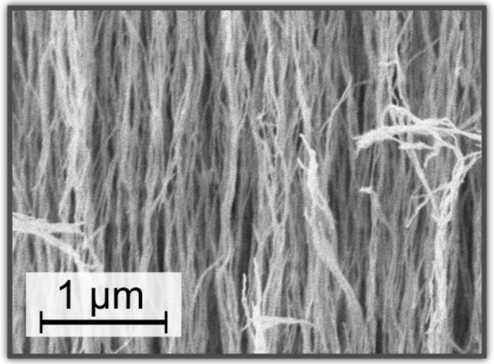
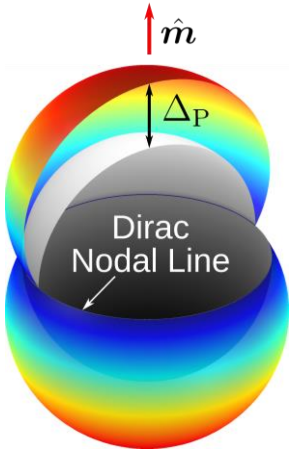
Vortices in the Polar phase





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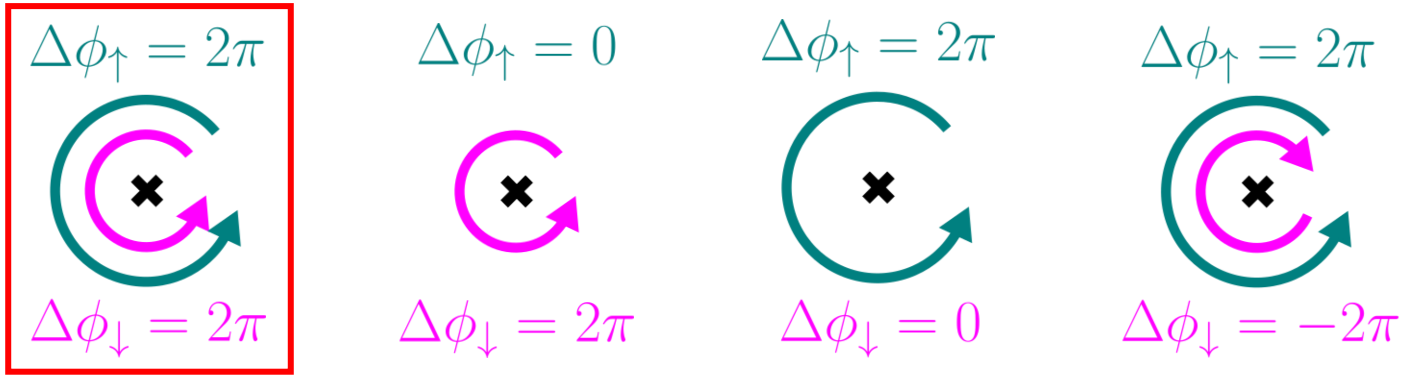
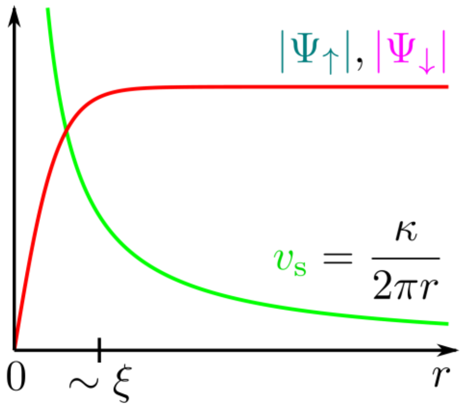
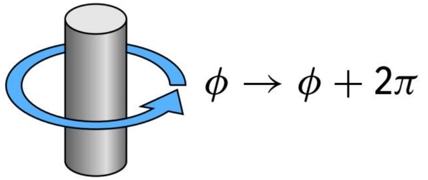


Single-quantum vortex



Singular, no superfluidity in the core

Equal winding in both components:
Phase winds by 2π
Uniform $\hat{d}$



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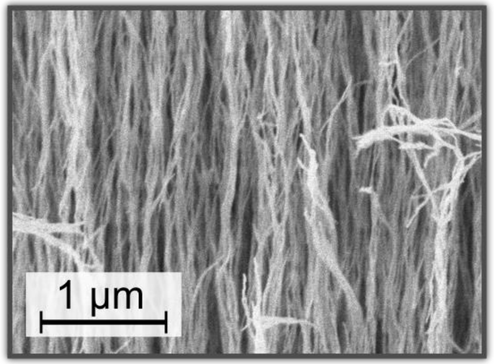

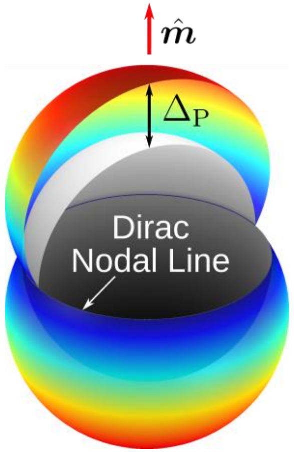

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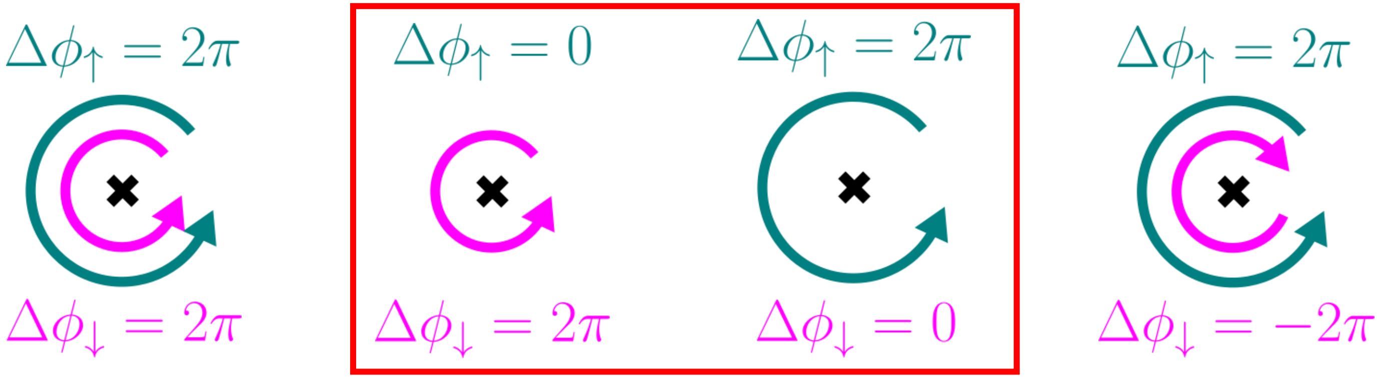
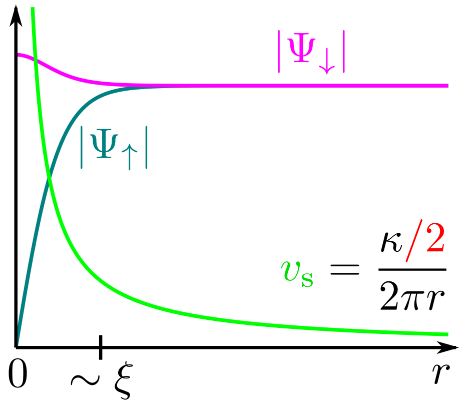
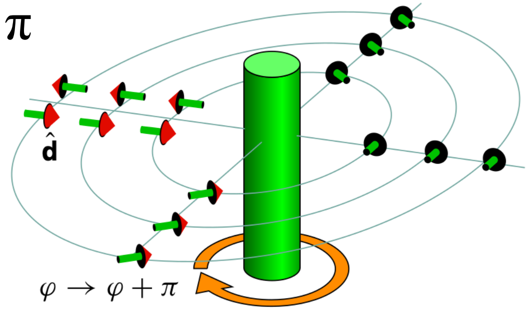


Half-quantum vortex

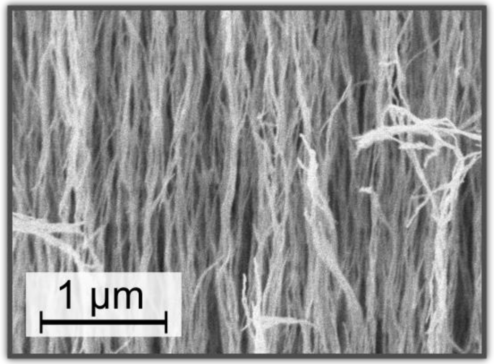


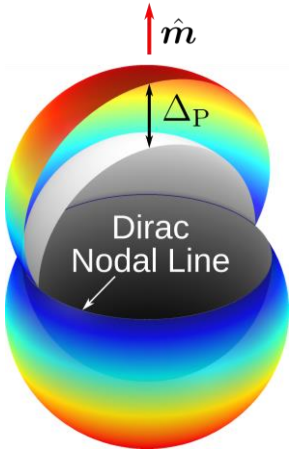
Non-singular

Winding in only one component:
Phase winds by π
$\hat{d}$ rotates by π



Vortices in the Polar phase





$$A_{\mu j} = \Delta_{\mathrm{P}} e^{i\phi} \hat{d}_\mu \hat{m}_j$$

Strands fix direction of $\hat{m} = \hat{z}$

$\hat{d}$ weakly oriented by **magnetic field** and **spin-orbit** interaction $\hat{d} \perp \hat{H}, \hat{m}$

Two degrees of freedom:
Phase ϕ and angle θ of $\hat{d}$ in the plane perpendicular to $\hat{H}$

Split into **spin up** and **spin down** superfluids:

$$\Psi_\uparrow = |\Psi_\uparrow| e^{i\phi_\uparrow} \qquad \Psi_\downarrow = |\Psi_\downarrow| e^{i\phi_\downarrow}$$

Phase: $\phi = \dfrac{\phi_\uparrow + \phi_\downarrow}{2}$ Angle of $\hat{d}$: $\theta = \dfrac{\phi_\uparrow - \phi_\downarrow}{2}$

Four possibilities:

$\Delta\phi_\uparrow = 2\pi$ $\Delta\phi_\uparrow = 0$ $\Delta\phi_\uparrow = 2\pi$ $\Delta\phi_\uparrow = 2\pi$

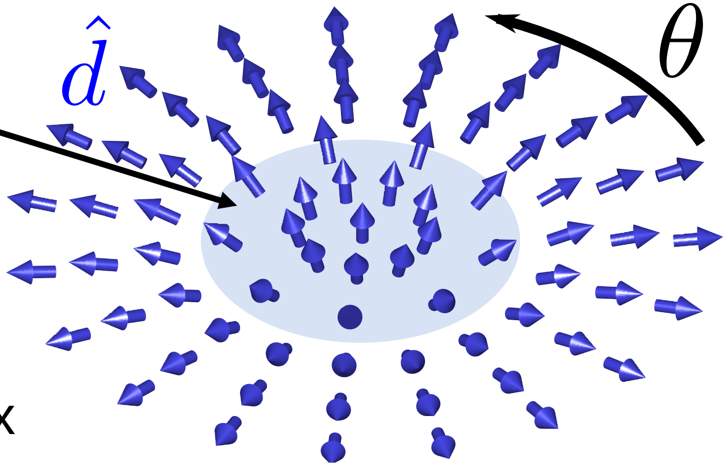
$\Delta\phi_\downarrow = 2\pi$ $\Delta\phi_\downarrow = 2\pi$ $\Delta\phi_\downarrow = 0$ $\Delta\phi_\downarrow = -2\pi$

Spin vortex

Non-singular, "**Soft core**"

No phase winding
$\hat{d}$ winds by 2π

Spin currents around vortex



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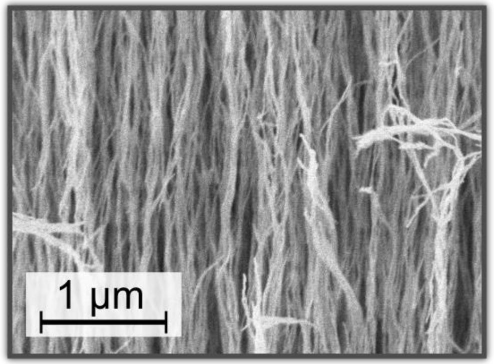

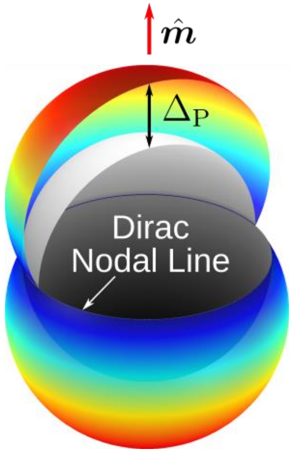

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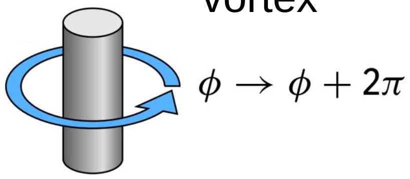
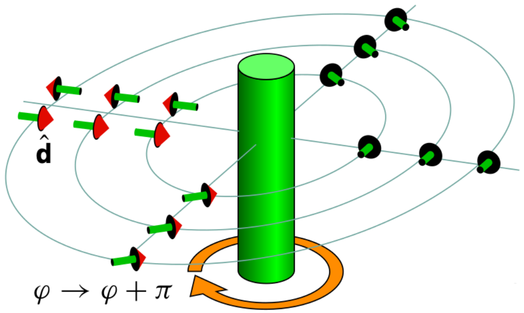
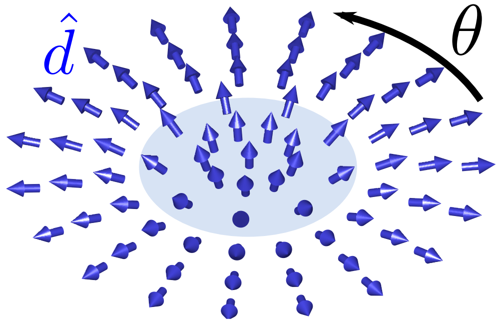
Single-quantum vortex



Half-quantum vortex



Spin vortex



Vortices in the B phase

Vortices in the B phase

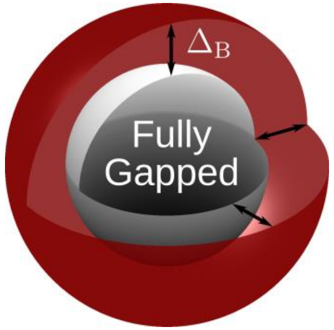
Cooper pair total angular momentum

$$J = L + S = 0$$

$$|S_z L_z\rangle = |+-\rangle, |00\rangle, |-+\rangle$$

Order parameter describes relative orientation of **spin** and **orbital** spaces

$$A_{\mu j} = \Delta_{\mathrm{B}} e^{i\phi} R_{\mu j}(\hat{n}, \theta)$$



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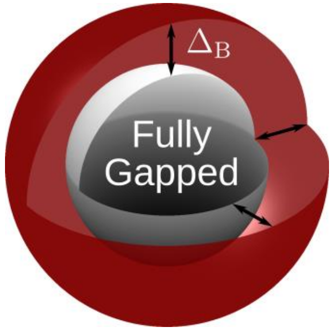

Leggett angle:

$$\theta = \cos^{-1}(-1/4)$$

$\hat{n}$ only weakly oriented by **magnetic field** and **surfaces**

A!

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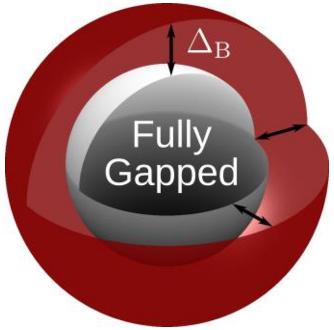

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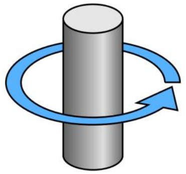
Two distinct single-quantum vortices:

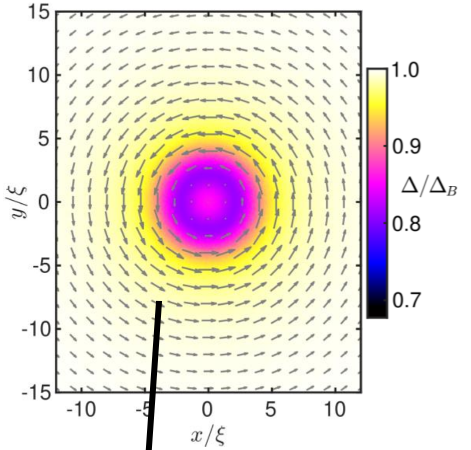


$\phi \rightarrow \phi + 2\pi$

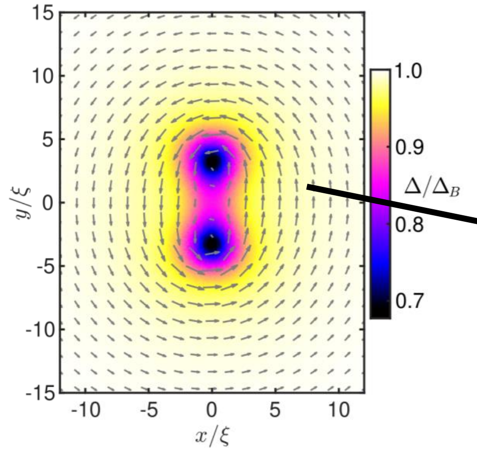
2π phase winding

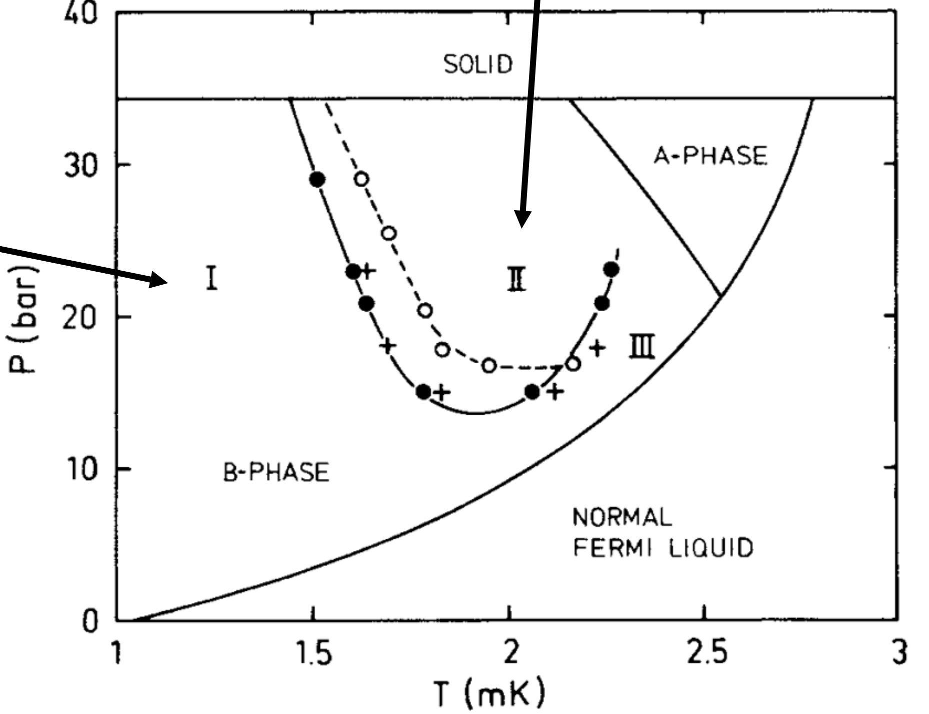
Different core structures

“A-phase-core” vortex



“Double-core” vortex





*Rantanen & Eltsov, PRR **6,** 043112 (2024)*

*Pekola et al., PRL **53**, 584 (1984)*

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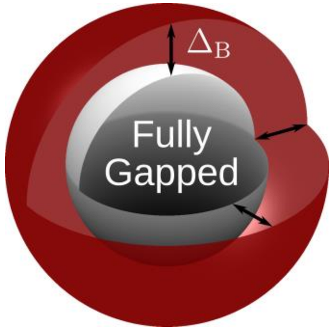

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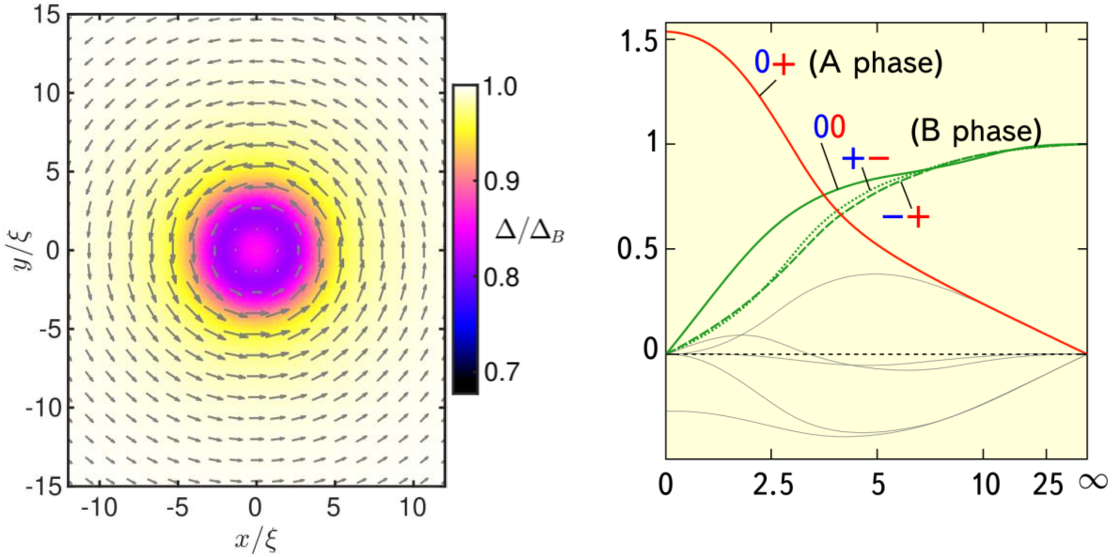

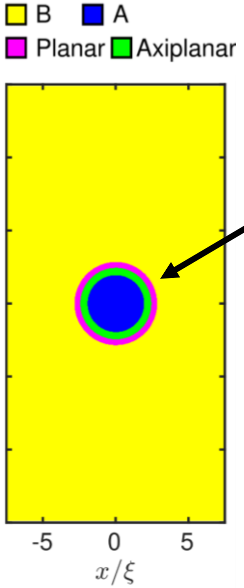

Axially symmetric, core filled with **A phase**

Interface between bulk B and core
A phase energetically expensive

Only stable at high temperatures and pressures

Rantanen & Eltsov, PRR ***6,*** *043112 (2024)*

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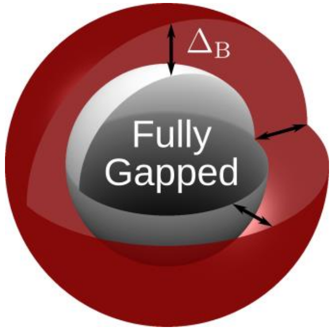

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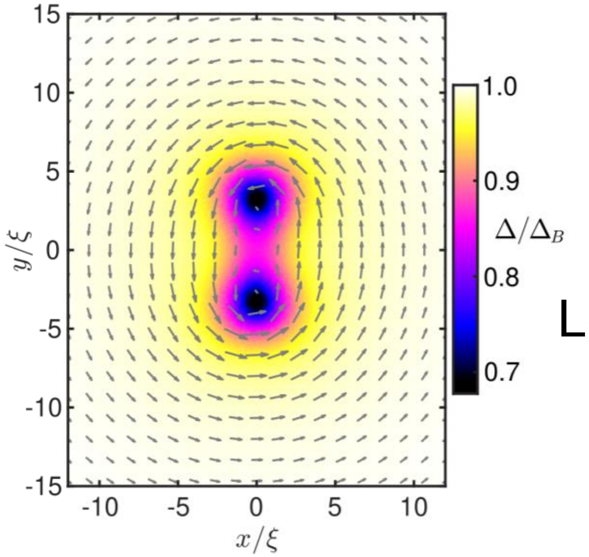


Core splits into two subcores

Can be thought of as **a bound pair of HQVs**

Lone HQVs are not stable in the B phase due to presence of **spin-0** component

Rantanen & Eltsov, PRR ***6,*** *043112 (2024)*

Vortices in the B phase

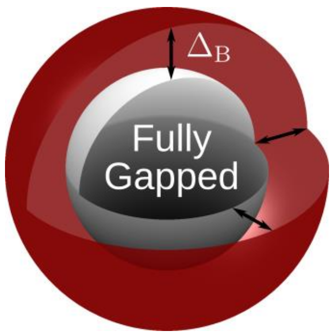
Cooper pair total angular momentum

$$J = {\color{red}L} + {\color{blue}S} = 0$$

$$|{\color{blue}S_z}{\color{red}L_z}\rangle = |{\color{blue}+}{\color{red}-}\rangle, |{\color{blue}0}{\color{red}0}\rangle, |{\color{blue}-}{\color{red}+}\rangle$$

Order parameter describes relative orientation of **spin** and **orbital** spaces

$$A_{{\color{red}\mu}{\color{blue}j}} = \Delta_\mathrm{B} e^{i\phi} R_{{\color{red}\mu}{\color{blue}j}}(\hat{n}, \theta)$$



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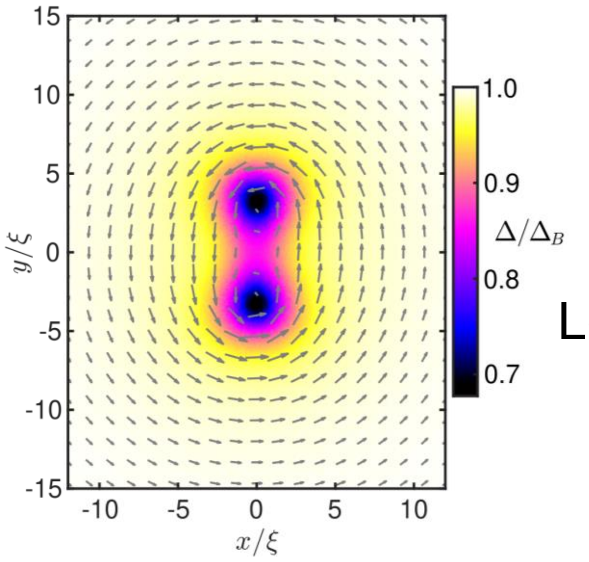

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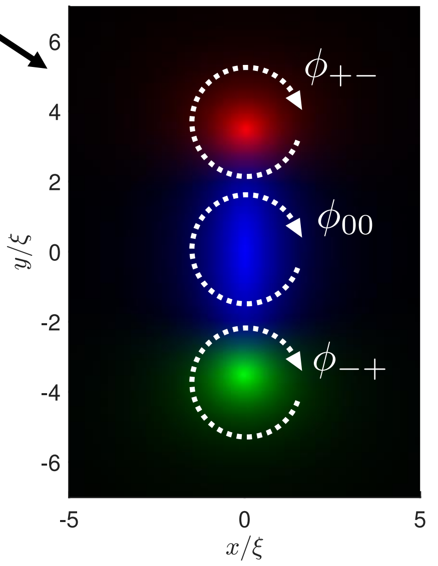
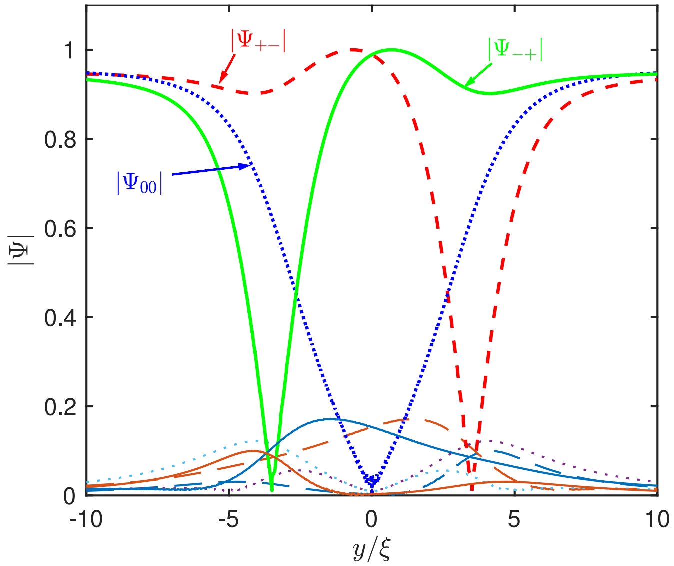
Colors indicate suppression

Spin-0 component acts as “glue” preventing further splitting

Three-core structure



$$|+-\rangle, |00\rangle, |-+\rangle$$



Rantanen, JLTP ***222,*** *59 (2026)*

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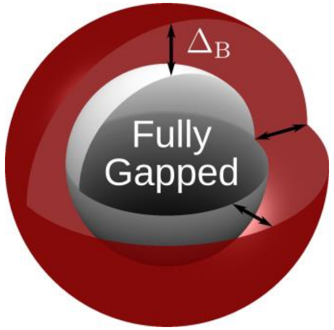

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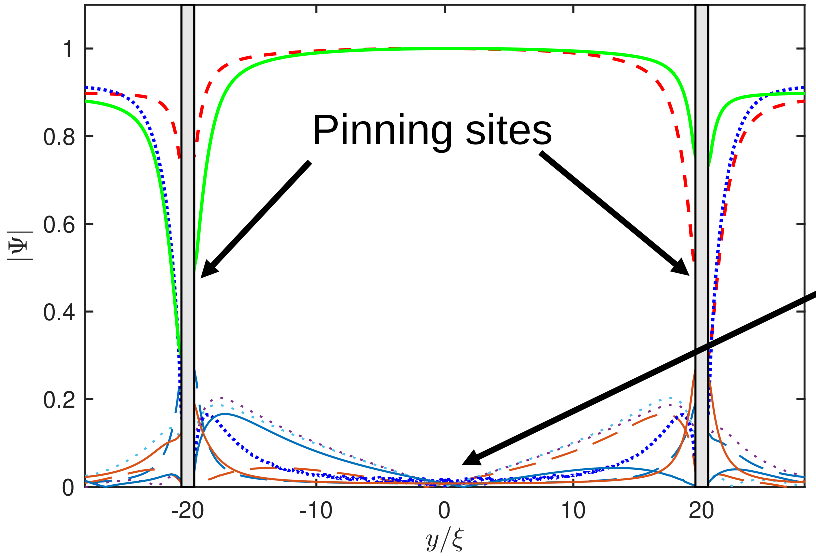


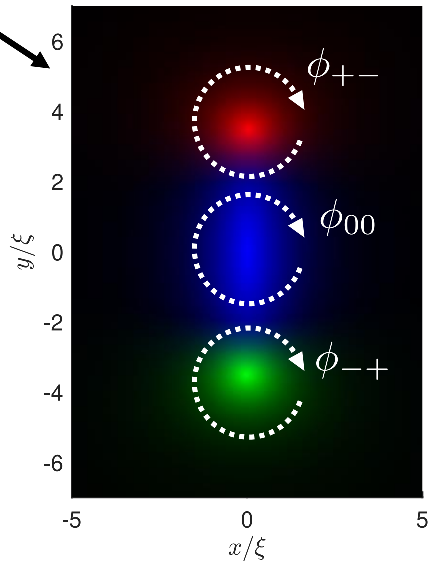
Extended structures possible with **pinning**

"Kibble wall" instead of point-like vortex in **spin-0** component

Colors indicate suppression

Spin-0 component acts as "glue" preventing further splitting

Three-core structure



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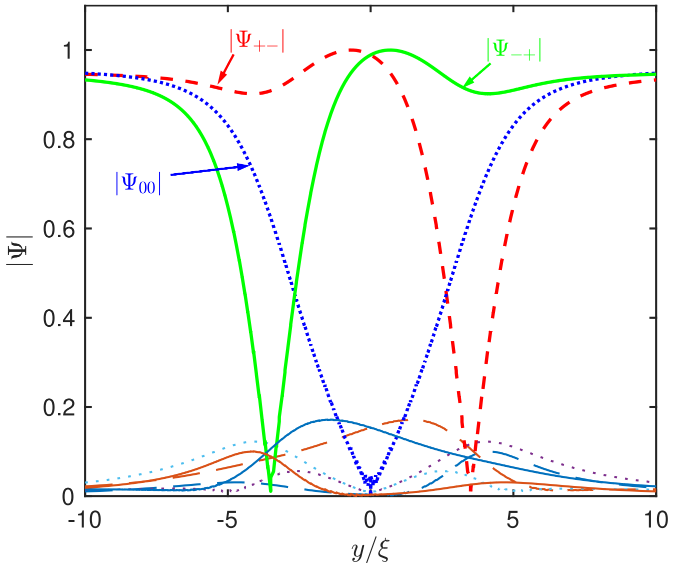

Rantanen, JLTP ***222,*** *59 (2026)*

Mäkinen et al., J. Phys.: Cond. Matt. ***35****, 214001 (2023)*

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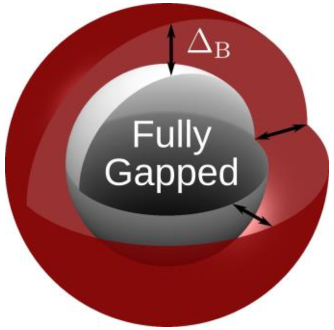

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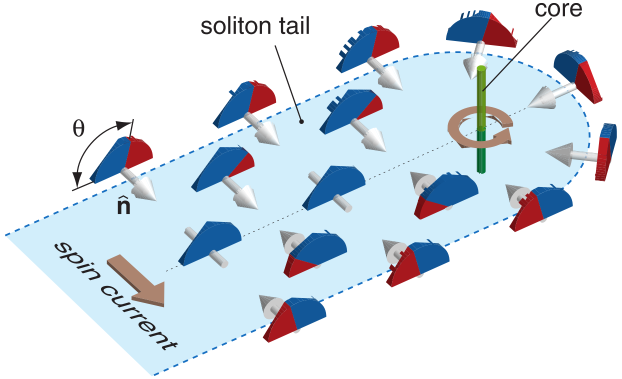
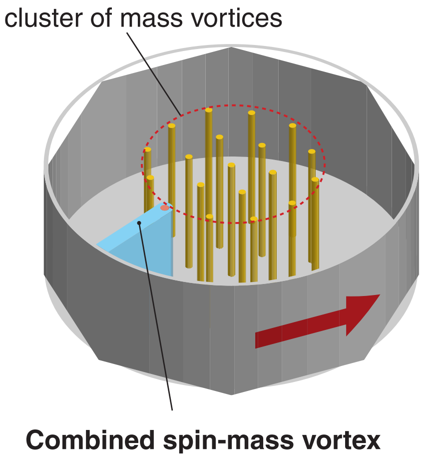
Spin and spin-mass vortices



No phase winding, either θ or $\hat{n}$ (or both) rotate around axis



Spin vortex gets "pinned" on phase vortex

*Mäkinen et al., J. Phys.: Cond. Matt. **35**, 214001 (2023)*

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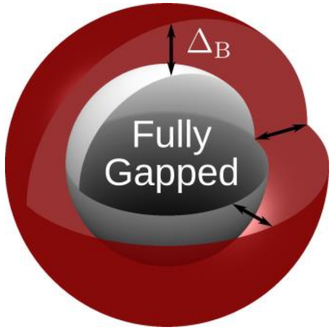

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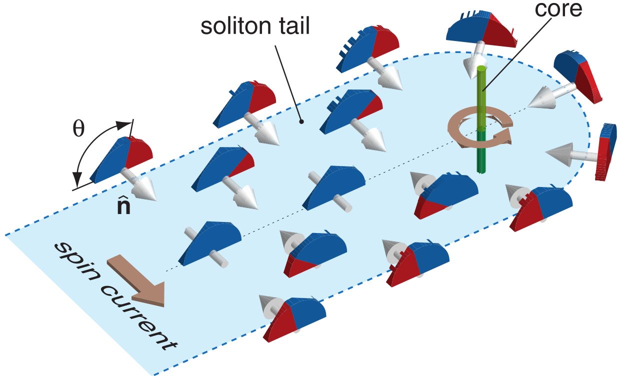

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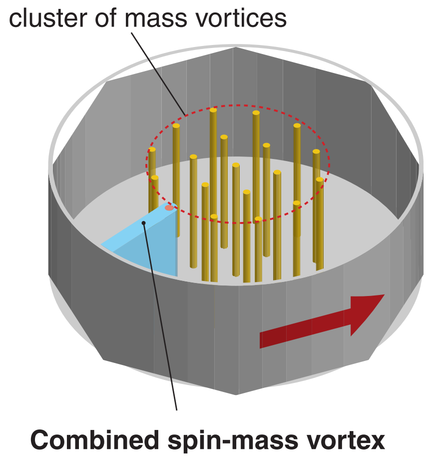

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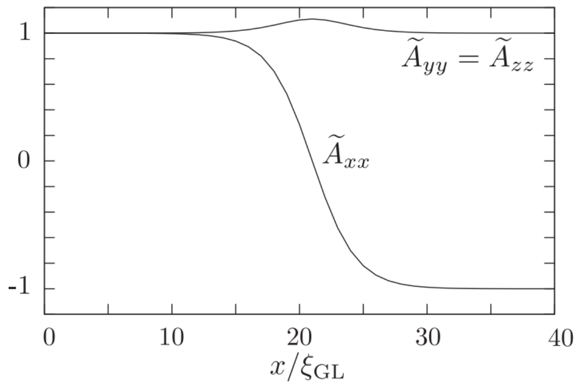
Domain walls

Name	n	ϕ	$\psi_\perp$	$\psi_\parallel$	$\widetilde{\mathsf{A}}^R$	H_P	Type	H_S
12	0	0	1	2	$\begin{bmatrix} +1 & 0 & 0 \\ 0 & +1 & 0 \\ 0 & 0 & +1 \end{bmatrix}$	∞, m, T	Bulk (no interface)	∞, m, T
10	2,3	π	-1	0	$\begin{bmatrix} +1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & +1 \end{bmatrix}$	$2_x, m_y, m_z, T, 2_y^o$	Single	$2_x, m_y, m_z, T, 2_y^o$
$\bar{1}2$	1	π	1	-2	$\begin{bmatrix} -1 & 0 & 0 \\ 0 & +1 & 0 \\ 0 & 0 & +1 \end{bmatrix}$	$\infty_x, m_y, m_z, T, m_x^o$	Mixed	T
$1\bar{2}$	6	0	1	-2	$\begin{bmatrix} +1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$	$\infty_x, m_y, m_z, T, m_x^o$	Double Texture	$2_x, m_y, m_z, T$ ∞_x, T
$\bar{1}0$	4,5	0	-1	0	$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & +1 \end{bmatrix}$	$2_x, m_y, m_z, T, 2_z^o$	Mixed double Mixed Texture	m_y, T $2_x, m_y, m_z$ m_z, T
$\overline{12}$	7	π	1	2	$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$	$\infty_x, m_y, m_z, T, m_x$	Mixed triple Mixed 2 Texture	T T ∞_x, m_y, m_z

*Silveri et al., PRB **90**, 184513 (2014)*

Variety of different types of domain walls are possible



Vortices in the A phase

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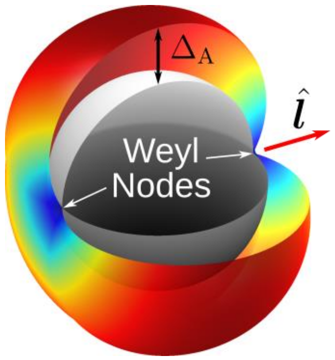
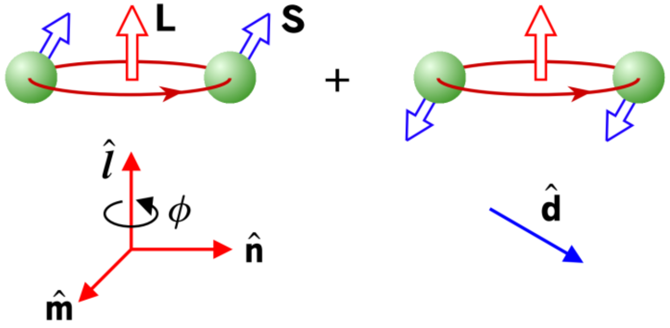


$$\hat{l} = \hat{m} \times \hat{n}$$

Chiral phase with defined angular momentum direction $|S_z L_z\rangle = |0+\rangle$

$$A_{\mu j} = \Delta_A \hat{d}_\mu (\hat{m}_j + i\hat{n}_j)$$



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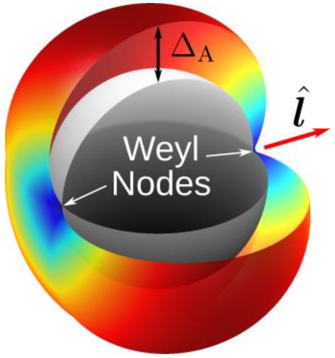

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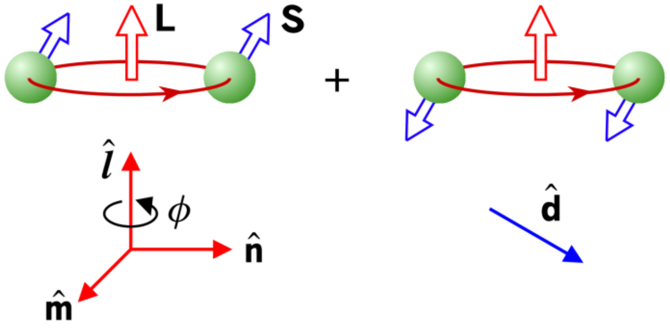
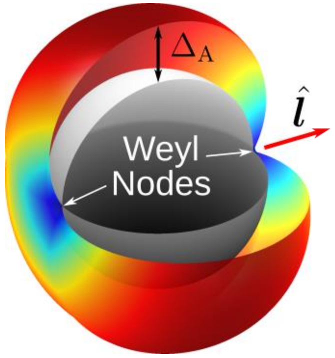


$$v_s = \frac{\hbar}{2m_3}\nabla\phi \longrightarrow \nabla \times v_s = 0$$

Vortices in the A phase



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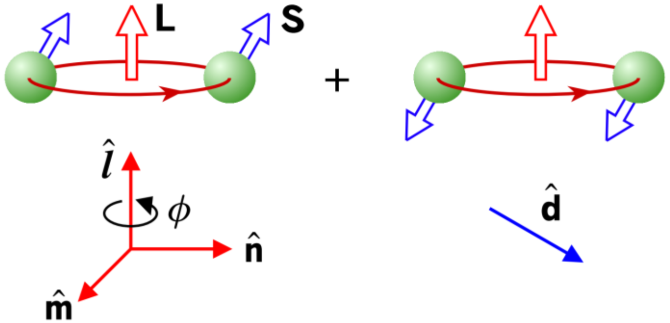


$$\xcancel{v_s = \frac{\hbar}{2m_3}\nabla\phi \quad \nabla \times v_s = 0}$$

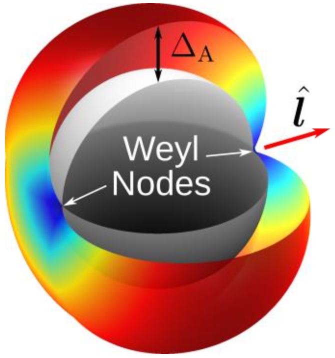
$$v_s = \frac{\hbar}{2m_3}\sum_i \hat{m}_i \nabla \hat{n}_i \longrightarrow \nabla \times v_s \neq 0$$

A!

Mermin-Ho relation:

$$(\nabla \times v_s)_i = \frac{\hbar}{2m_3}\epsilon_{ijk}\hat{l}\cdot(\nabla_j \hat{l} \times \nabla_k \hat{l})$$

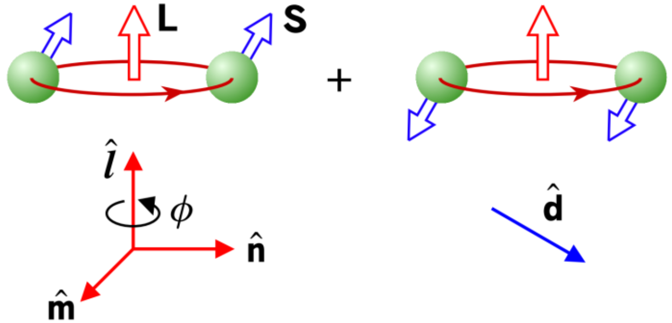
Vortices in the A phase



$$\color{red}{\hat{l} = \hat{m} \times \hat{n}}$$

Chiral phase with defined angular momentum direction $|S_z L_z\rangle = |0+\rangle$

$$A_{\mu j} = \Delta_A \hat{d}_\mu (\hat{m}_j + i\hat{n}_j)$$



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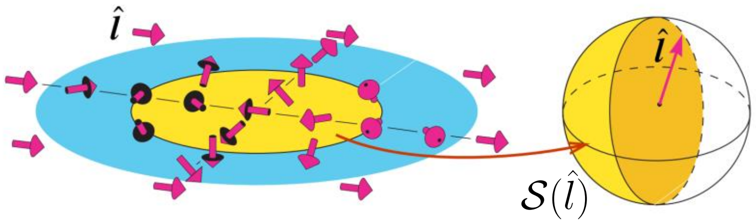

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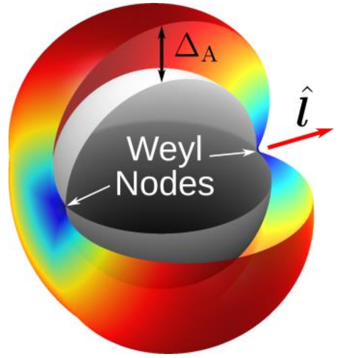
Vortices in the A phase



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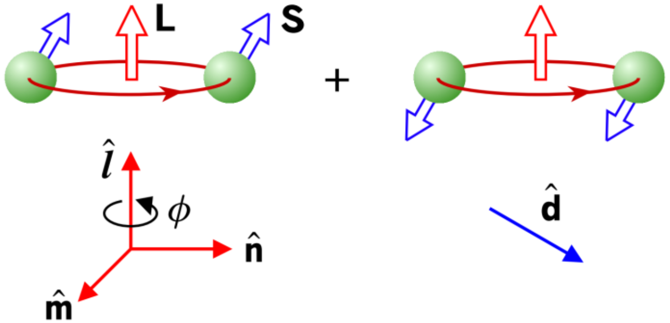

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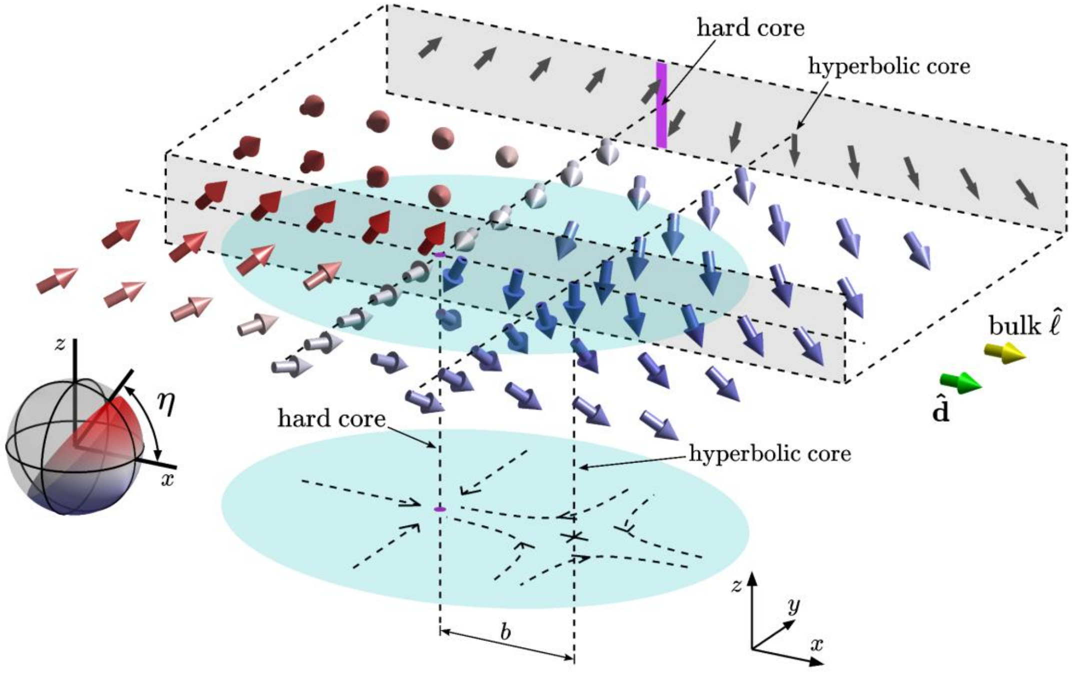
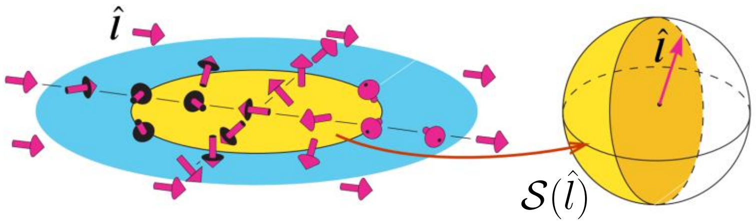
Single-quantum vortex:



*Rantanen et al., JLTP **220**, 88 (2025)*

Vorticity carried in $\hat{l}$ vector texture

Only observed when rotating during transition



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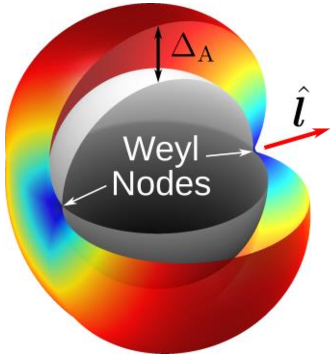
Vortices in the A phase



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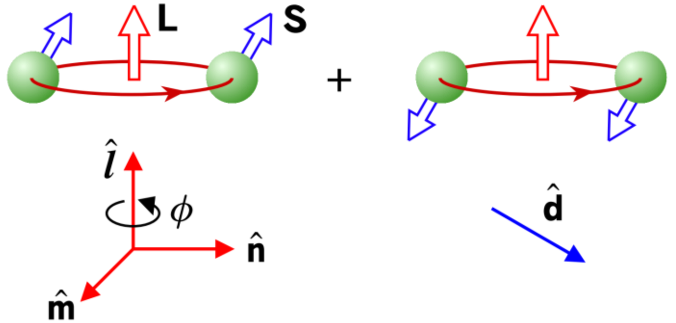

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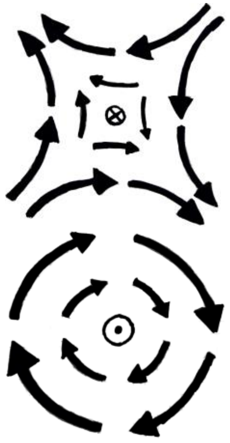
Double-quantum vortex:

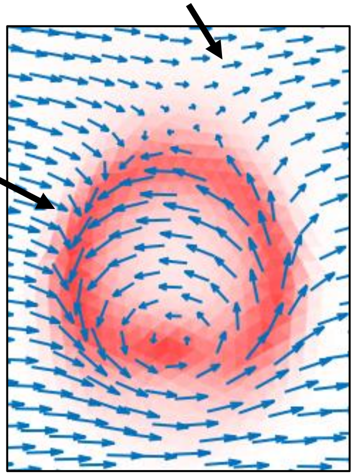


“merons”

Each meron carries one quantum of circulation

Arrows: $\hat{l}$

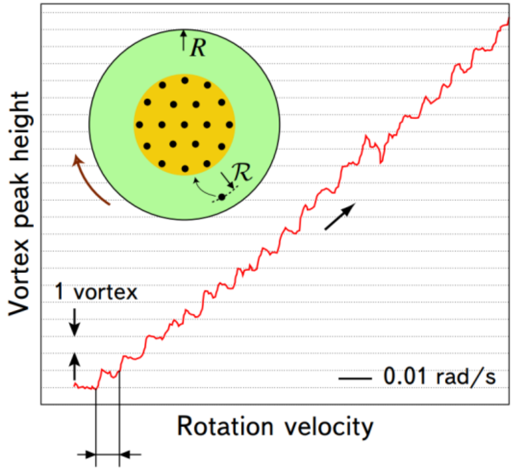
Color: $\nabla \times v_s$

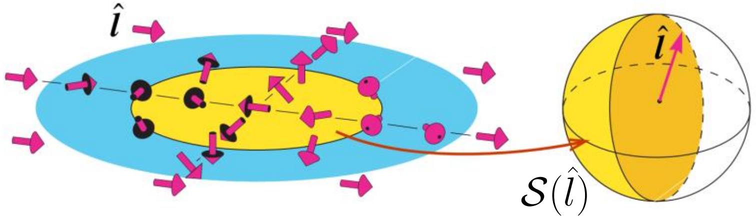


*Rantanen & Eltsov., PRB **107**, 104505 (2023)*

Higher energy than SQVs, but easier to nucleate

“**Skyrmion**” or “**Bimeron**”





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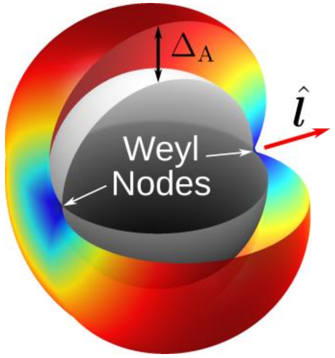
Vortices in the A phase



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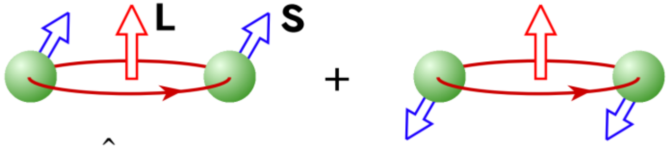

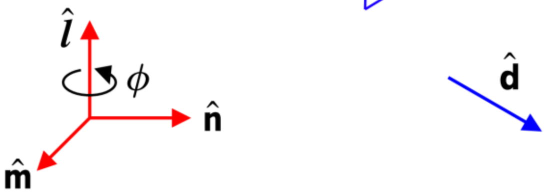

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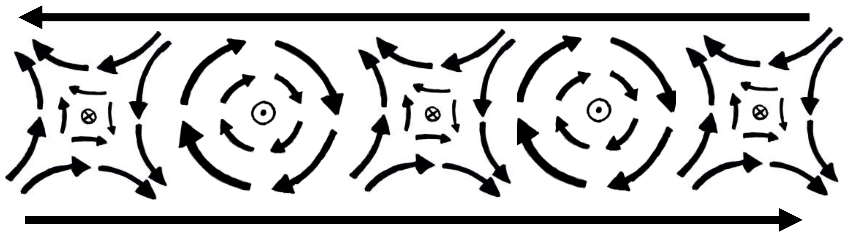
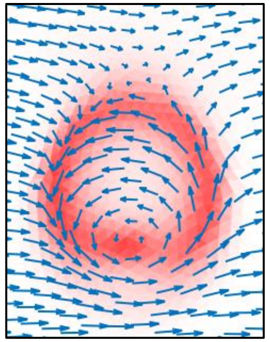
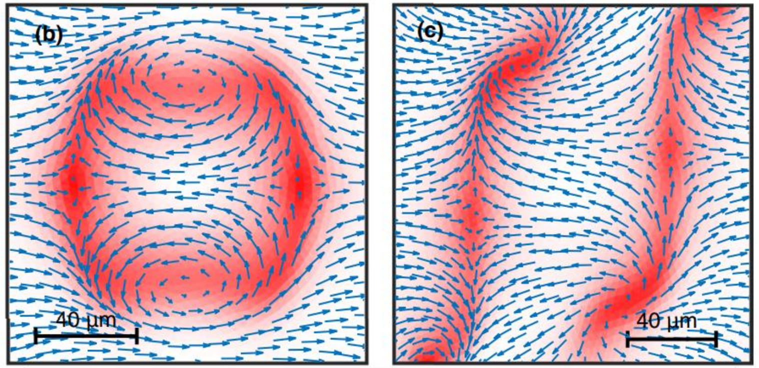
Vortex sheet: Planar object carrying vorticity



Chain of merons



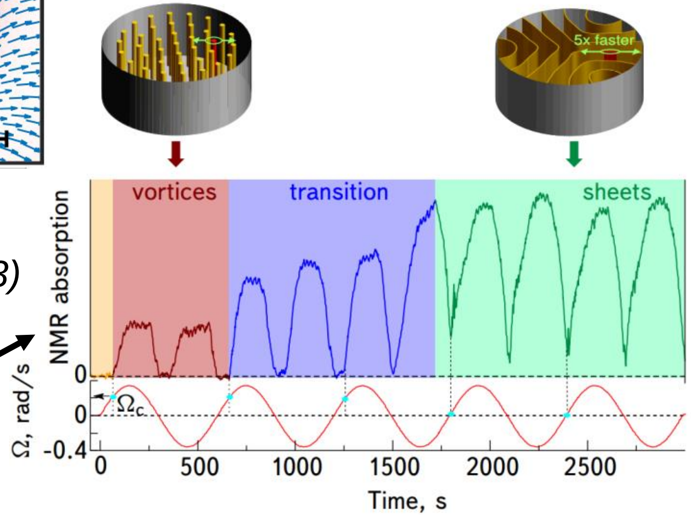
DQV: smallest sheet

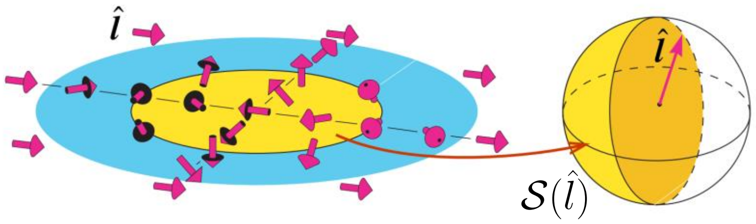


Closed Open

Rantanen & Eltsov., PRB ***107****, 104505 (2023)*

Created by rapidly oscillating rotation





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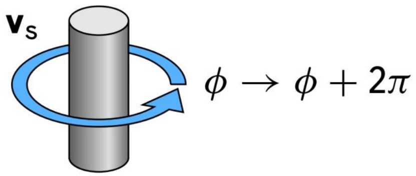
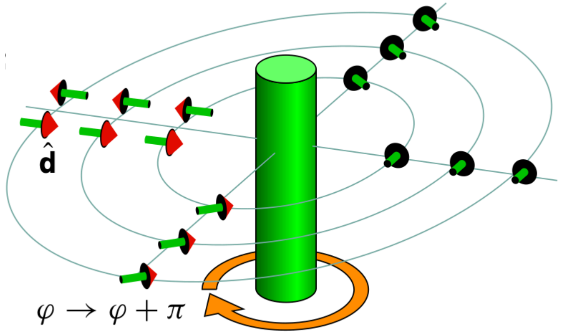
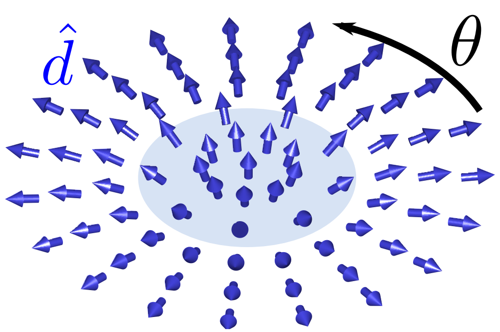
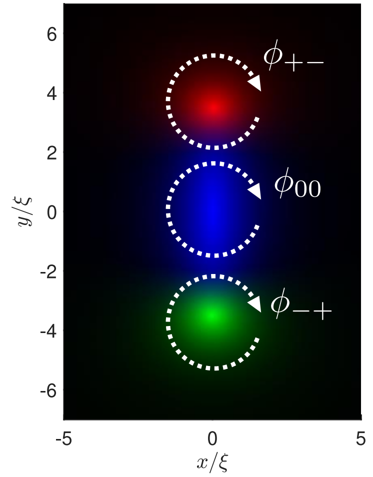
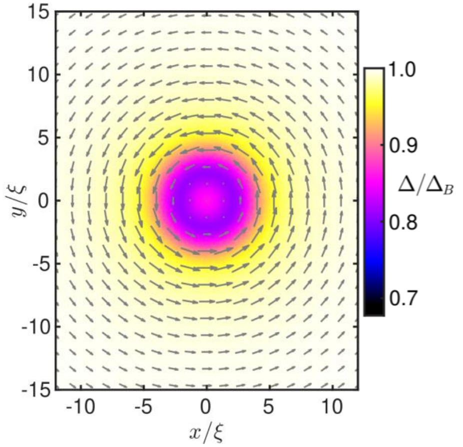
Vortices in superfluid ^{3}He

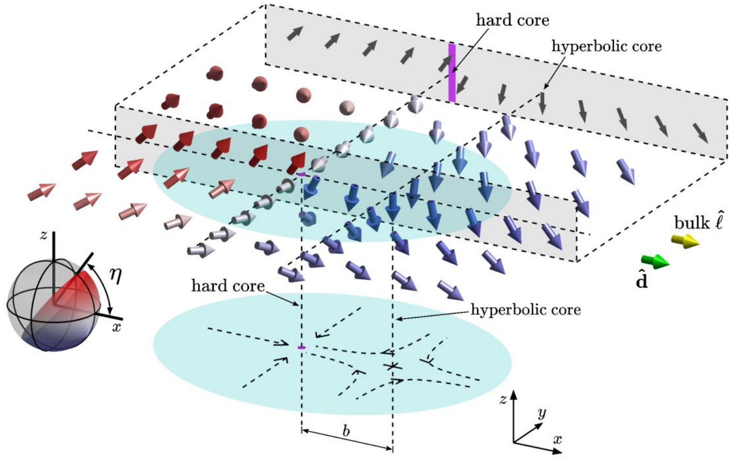


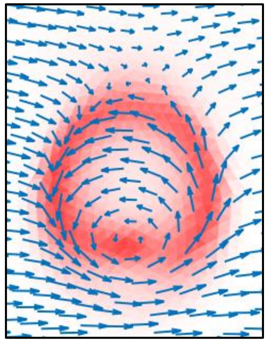


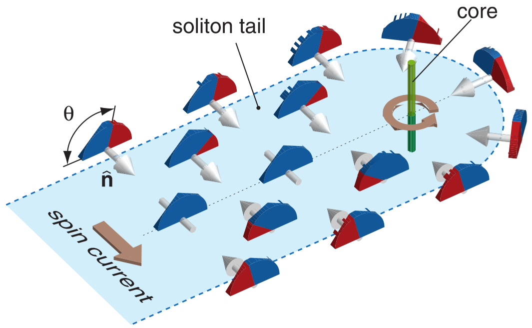


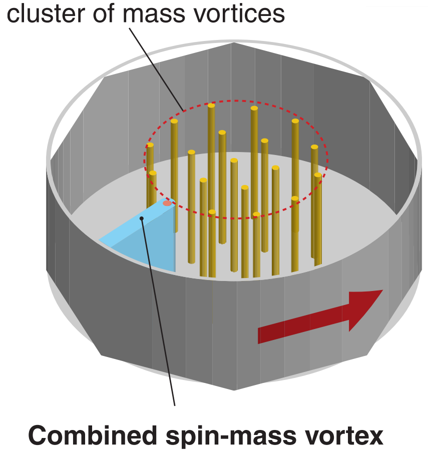


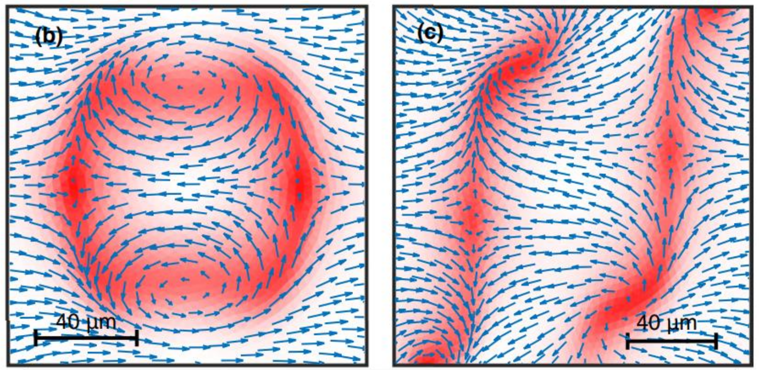












Part 2: Vortex pinning in superfluid ^{3}He

Vortex pinning

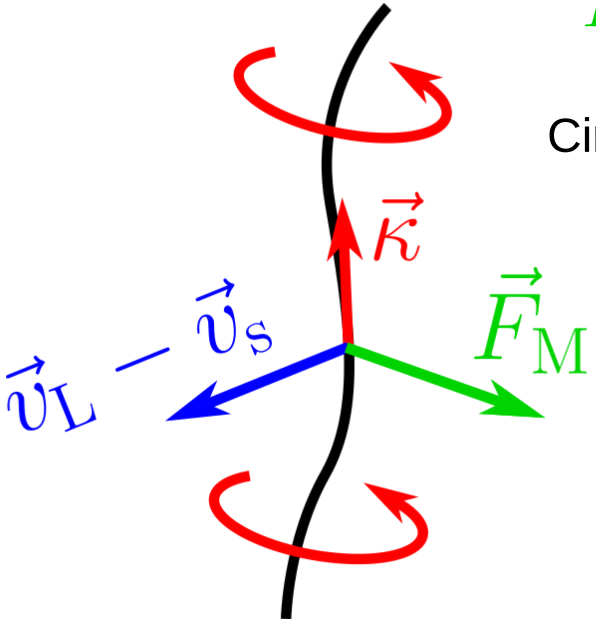
Free vortices move, pushed by the Magnus force

$$\vec{F}_{\mathrm{M}} = \rho_{\mathrm{s}} \vec{\kappa} \times (\vec{v}_{\mathrm{L}} - \vec{v}_{\mathrm{s}})$$

Circulation

Velocity of
vortex line

Superfluid
velocity



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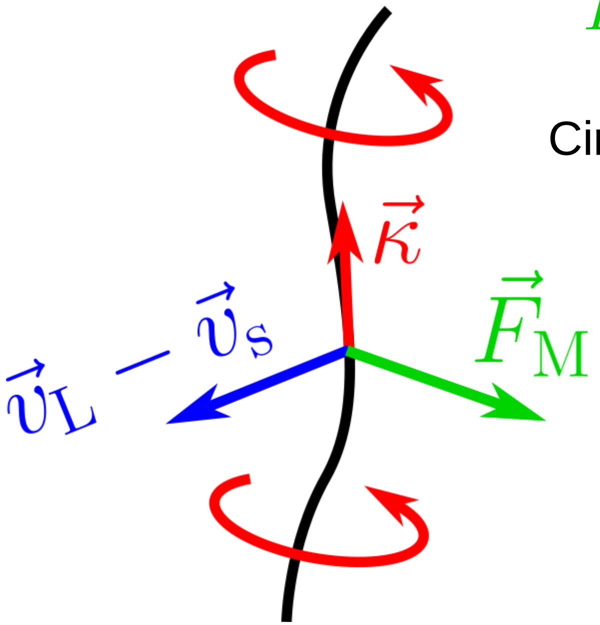
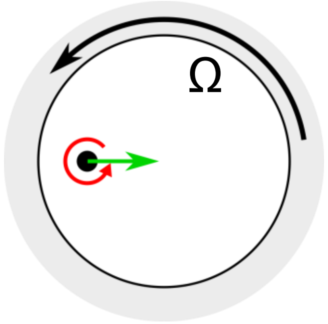
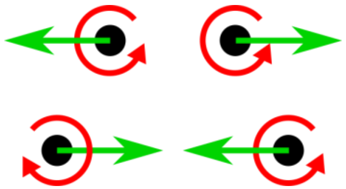
Superfluid velocity

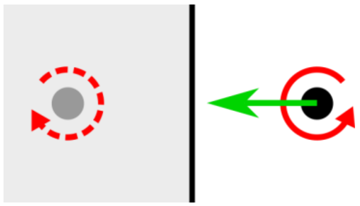




External rotation



Interaction between vortices



Attraction to images

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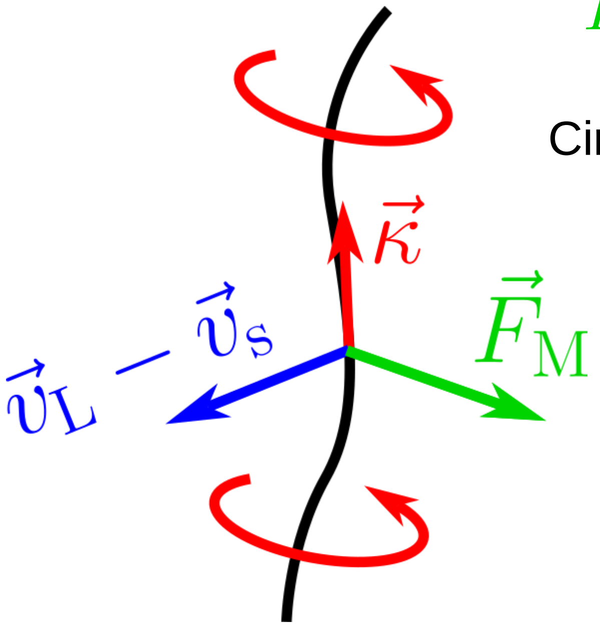

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Circulation

Velocity of
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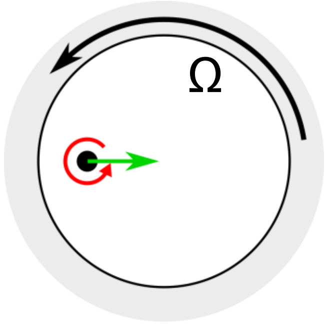

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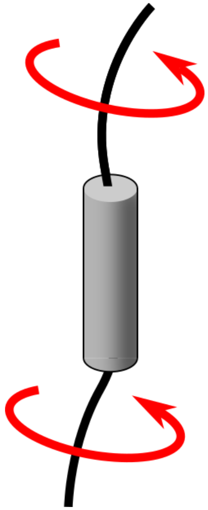
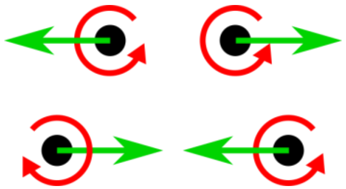


Vortices can be
pinned on objects

Vortex is unable to move until
force exceeds a critical threshold



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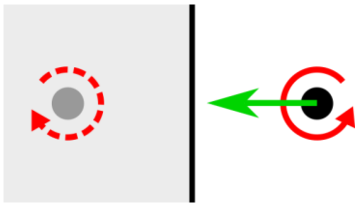

Attraction to images

Vortex pinning

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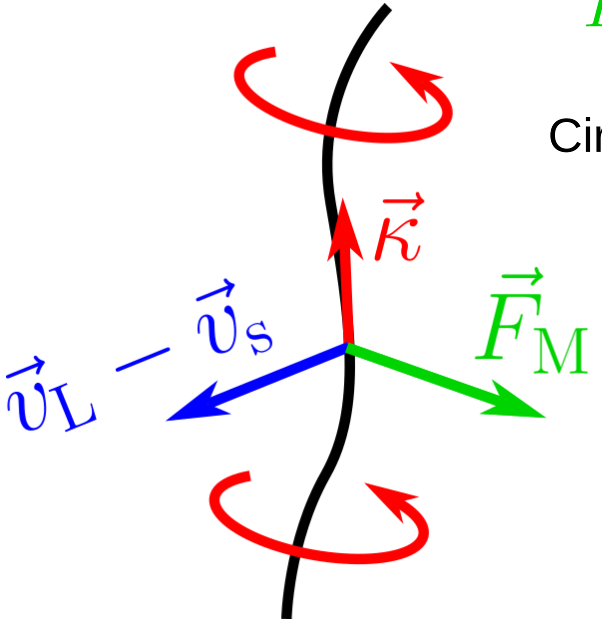
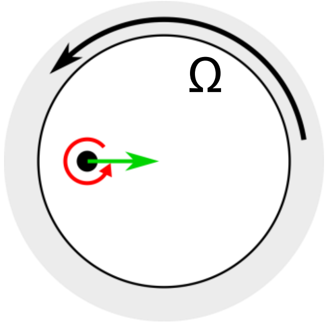
Circulation

Velocity of vortex line

Superfluid velocity





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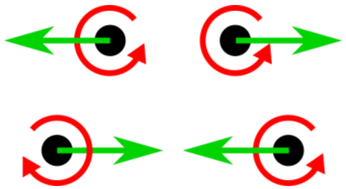

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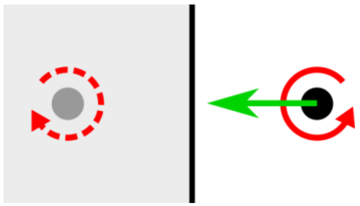
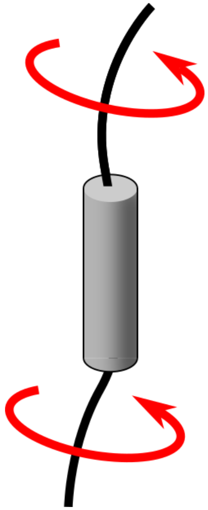


Attraction to images



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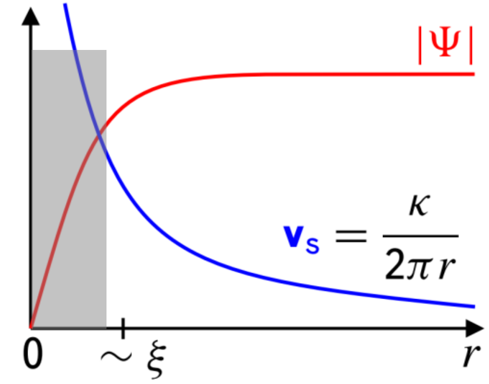

Simple picture: Part of the core is displaced by pinning site

Vortex pinning

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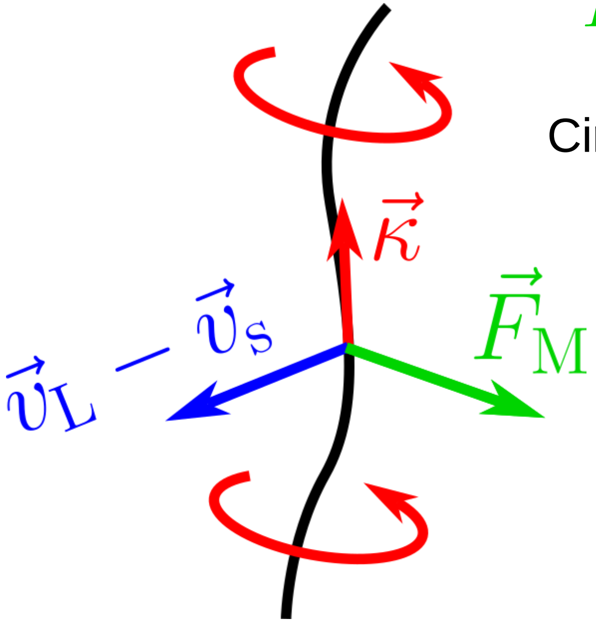
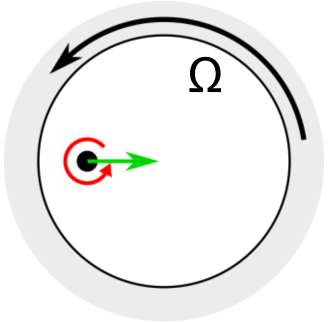
Circulation

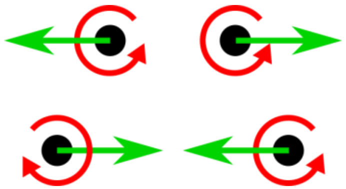
Velocity of vortex line

Superfluid velocity





External rotation



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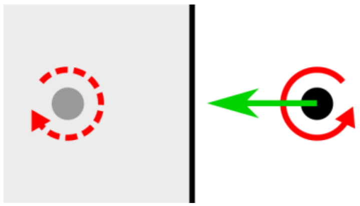
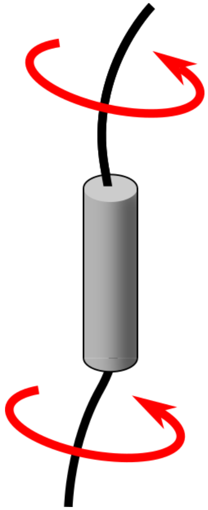


Attraction to images



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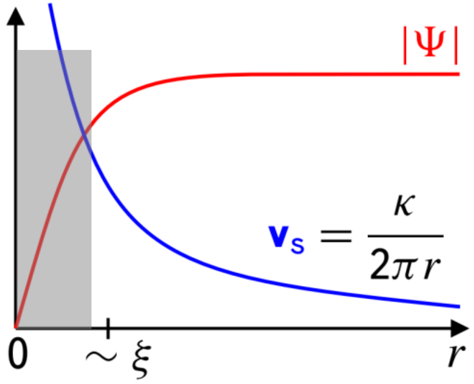
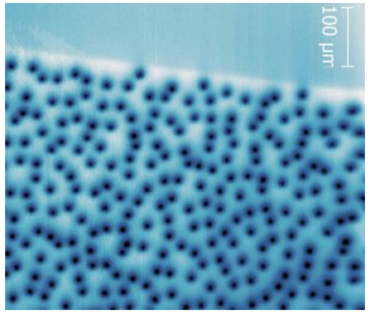
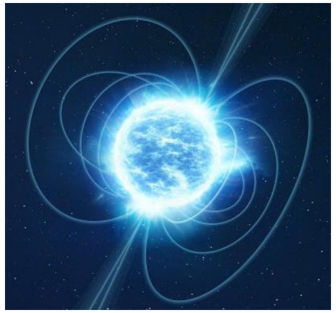


Simple picture:
Part of the core is displaced by pinning site

Relevance:



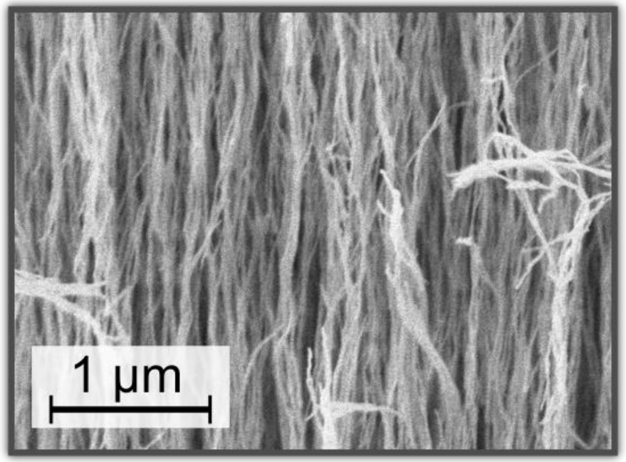
Superconductors



Neutron star glitches

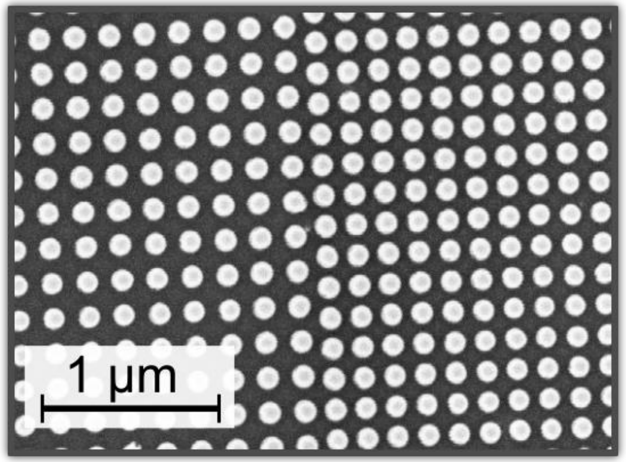
Pinning in ^{3}He

Microscopic:
Chemically grown nanomaterials

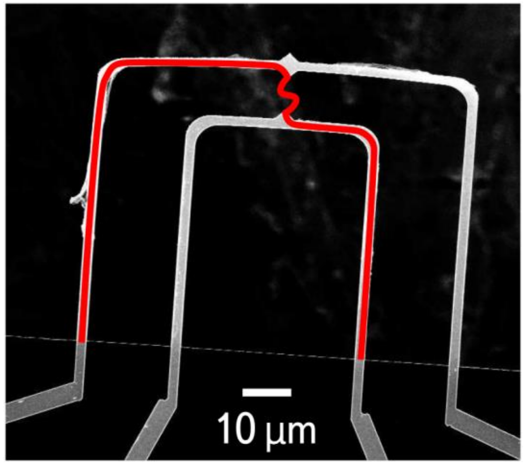


“**nafen**” Al_2O_3 strands
with ~8 nm diameters

Mesoscopic:
Nanofabricated substrates and devices



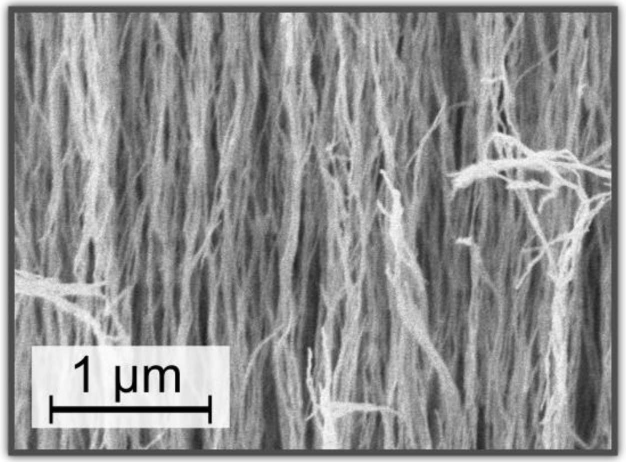
Silicon pillars less than
100 nm in diameter



Nanomechanical devices
micrometers in size

Pinning in ^{3}He

Microscopic:
Chemically grown nanomaterials



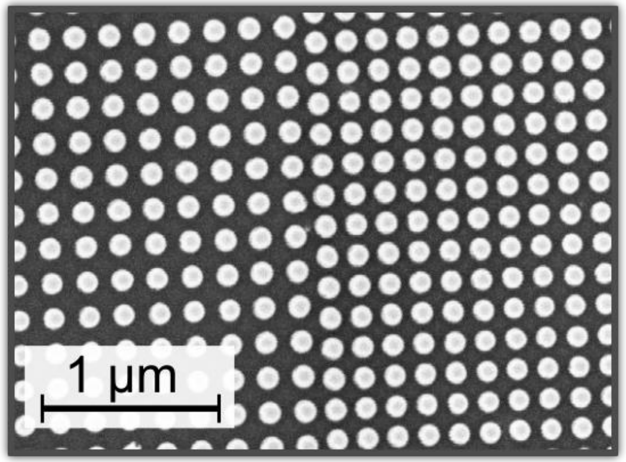
“**nafen**” Al_2O_3 strands
with ~8 nm diameters

Energy from
point impurities: $A_{\alpha i} I_{ij}(\vec{r}) A^*_{\alpha j}$

$$I_{ij} \propto \sigma_\perp(\delta_{ij} - \hat{n}_i \hat{n}_j) + \sigma_\parallel \hat{n}_i \hat{n}_j$$

Scattering cross sections Strand direction

Mesoscopic:
Nanofabricated substrates and devices



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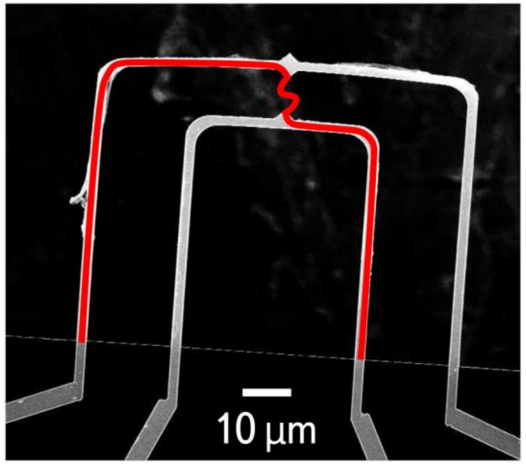

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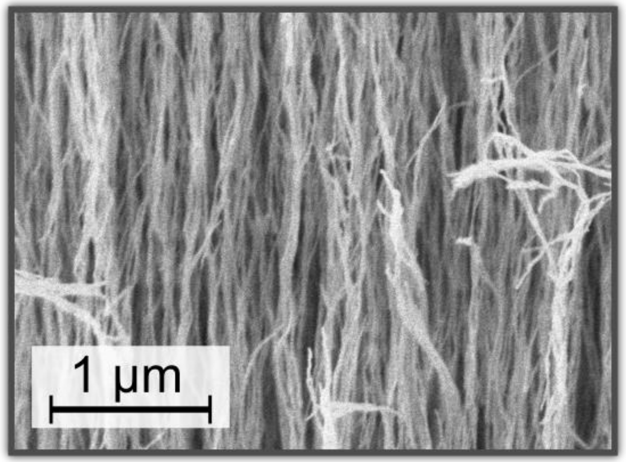

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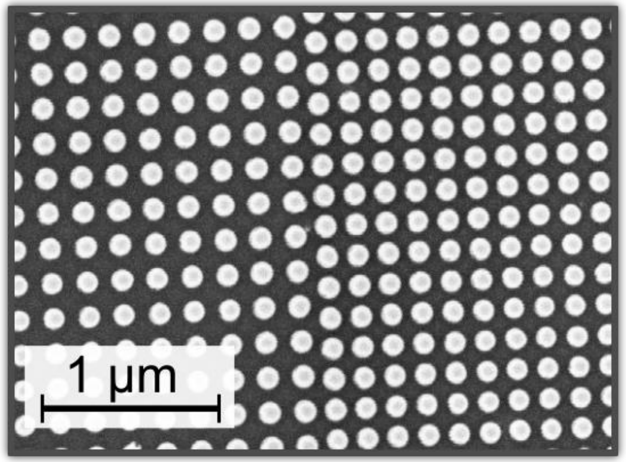

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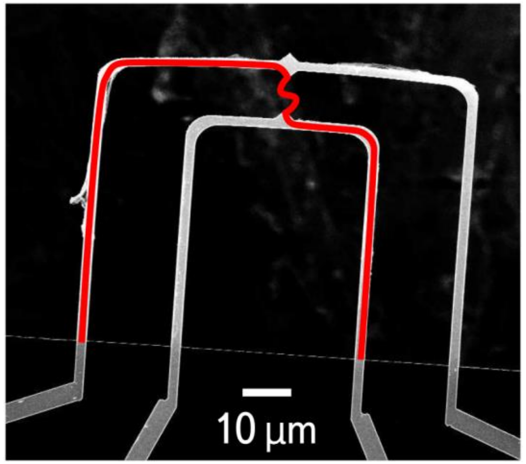

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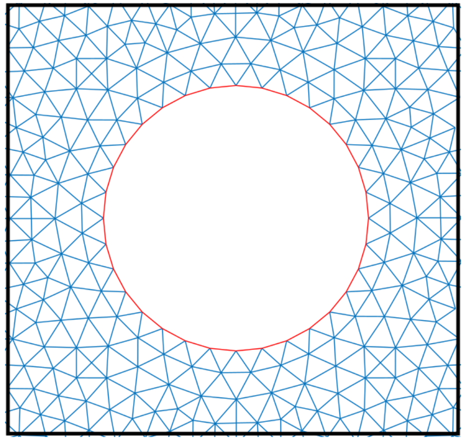


Objects modeled as
actual geometry

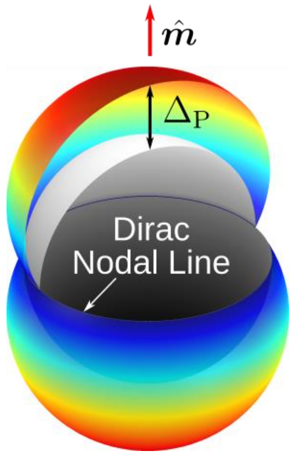
Specular boundary
conditions

Point impurity model

Randomly generated impurities

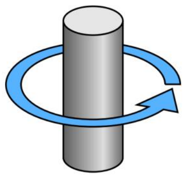
Denser regions can
pin topological objects

$$A_{\mu j} = \Delta_{\mathrm{P}} e^{i\phi} \hat{d}_{\mu} \hat{m}_{j}$$

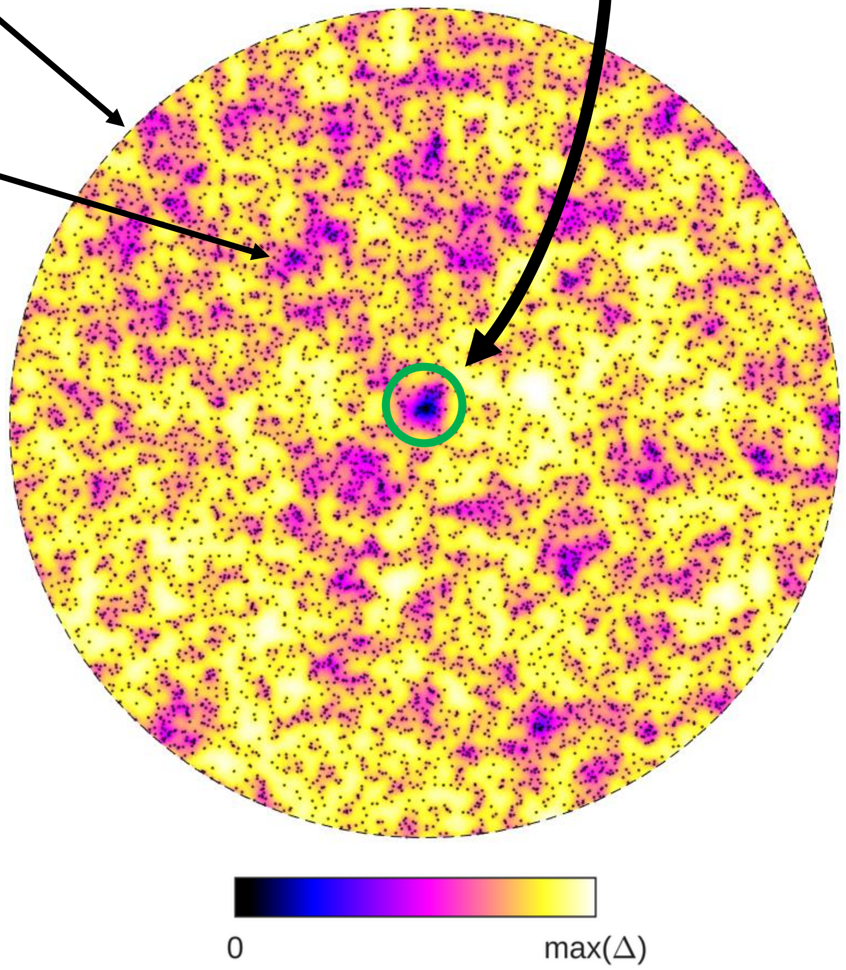


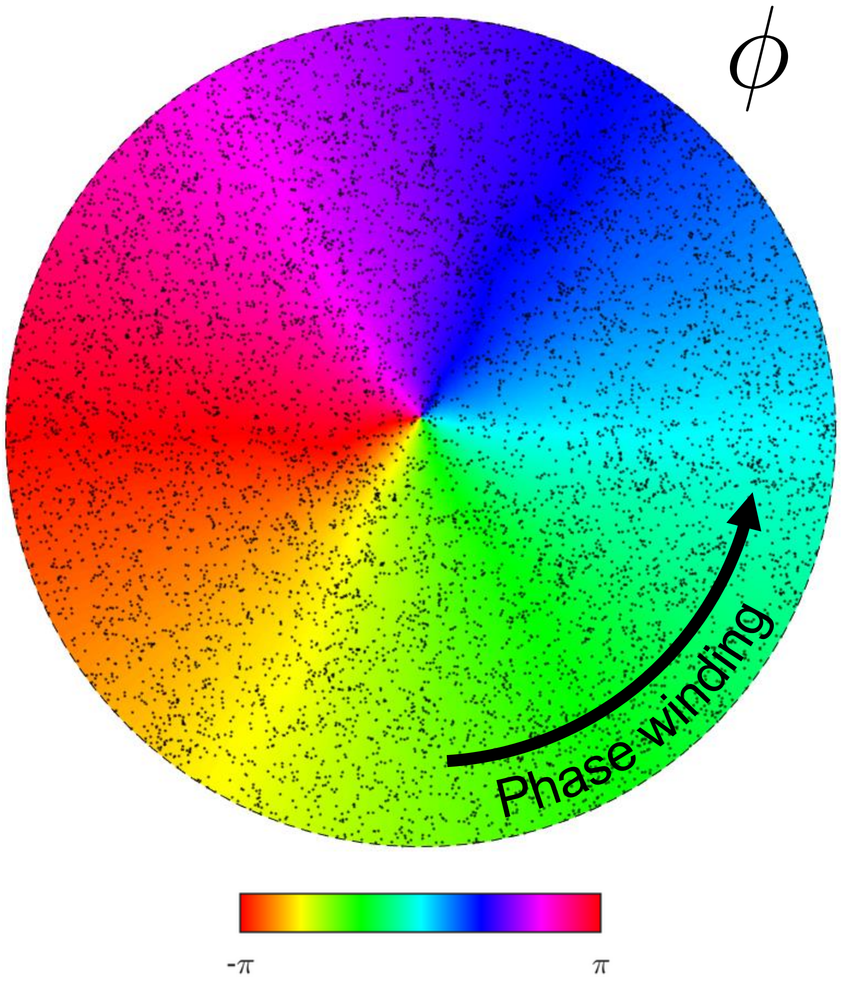
Polar phase

Single-quantum
vortex



$\phi \to \phi + 2\pi$

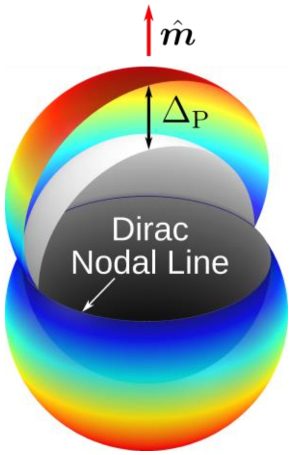




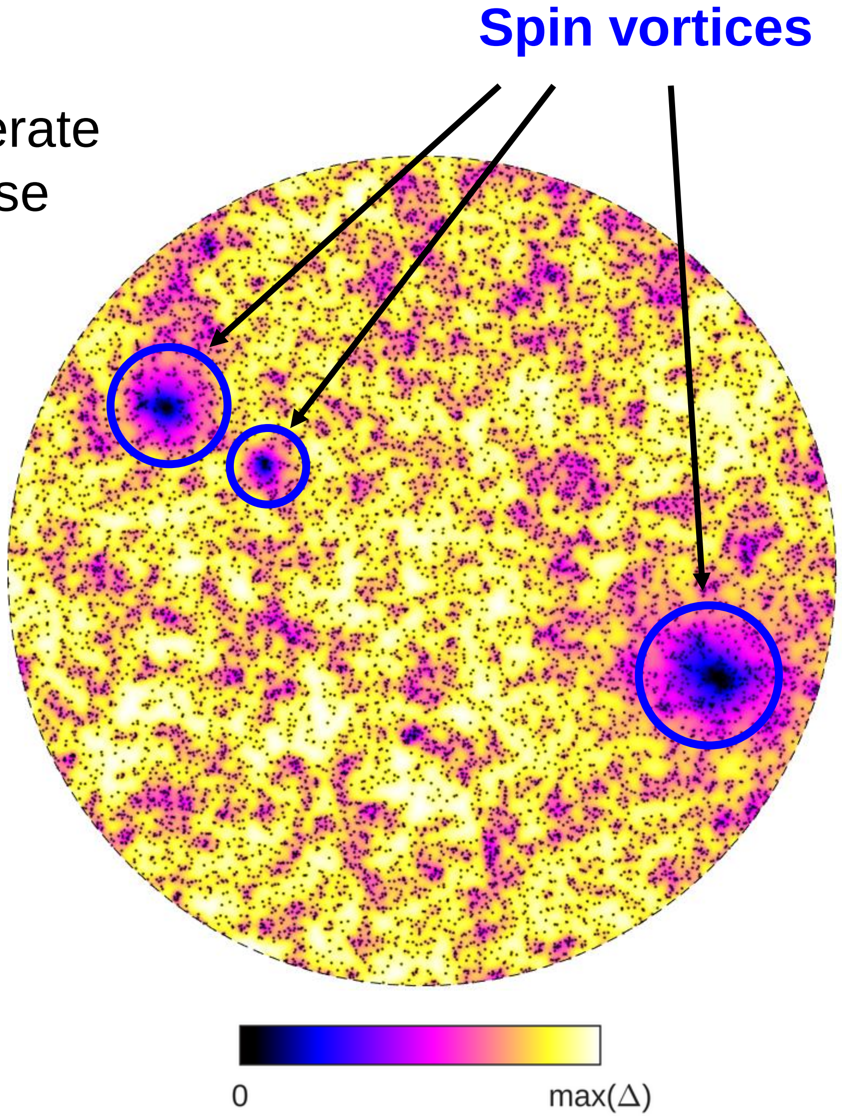
Point impurity model

Phase transitions can generate **spin vortices** in Polar phase

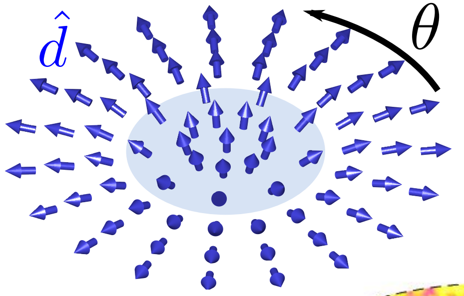
$$A_{\mu j} = \Delta_{\mathrm{P}} e^{i\phi} \hat{d}_\mu \hat{m}_j$$

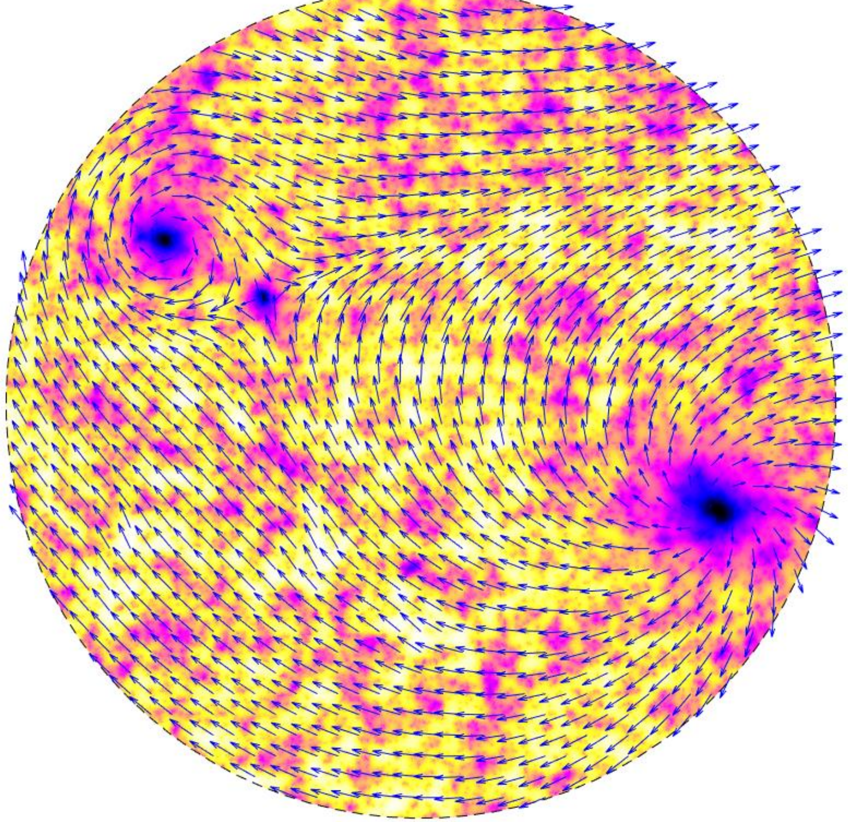


Polar phase



Spin vortices





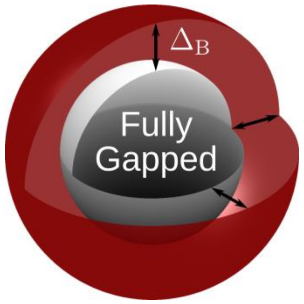
Arrows show direction of $\hat{d}$

Point impurity model

After cooling to B phase, **domain walls** form

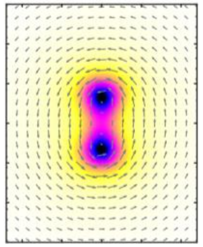
Just like vortices, domain walls are **strongly pinned**

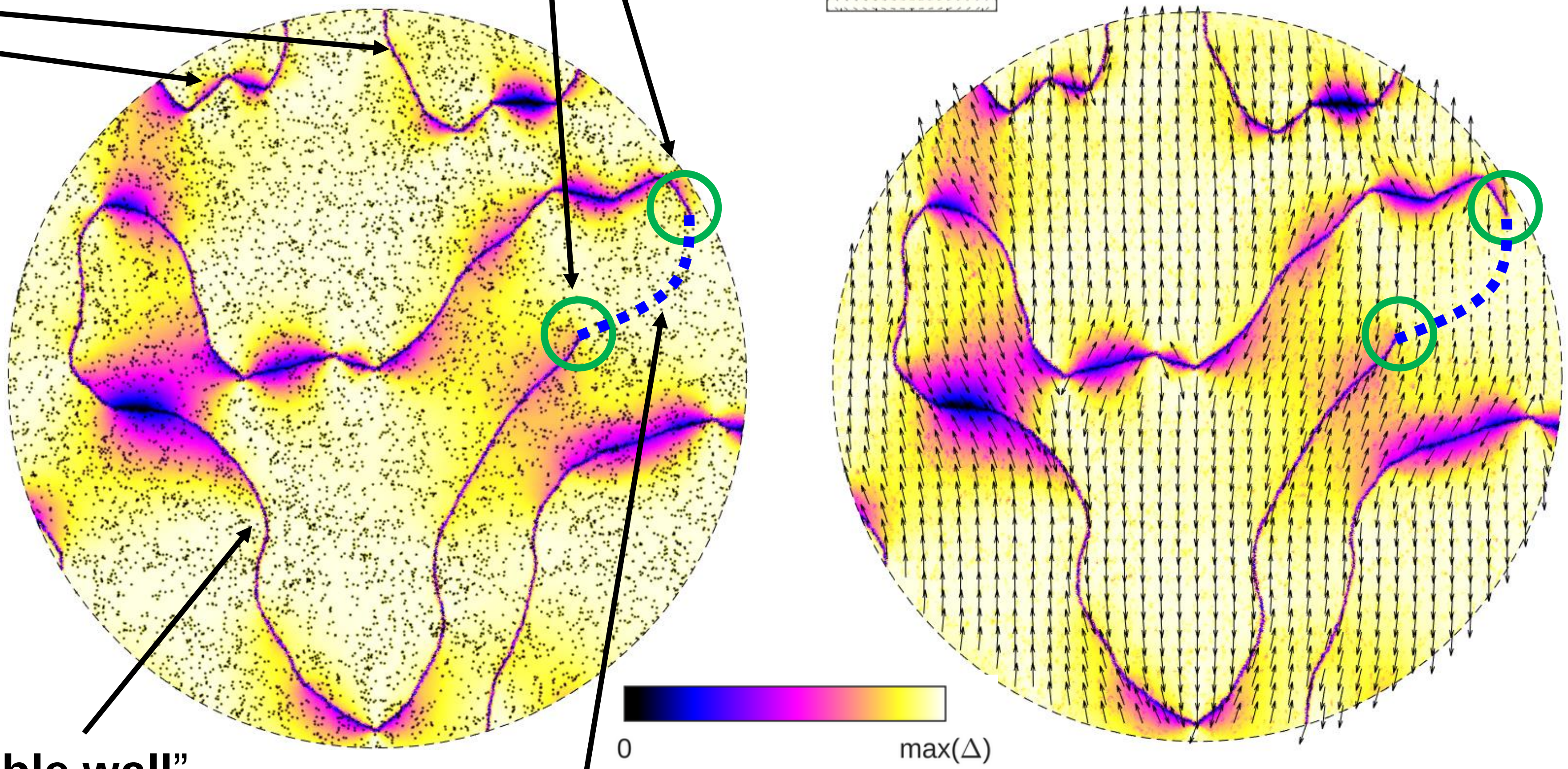
$$A_{\mu j} = \Delta_{\mathrm{B}} e^{i\phi} R_{\mu j}(\hat{n}, \theta)$$



B phase

Two **half-quantum vortices**





“Kibble wall” connecting HQVs

Spin soliton between HQVs

Arrows show direction of $\hat{n}$

Point impurity model

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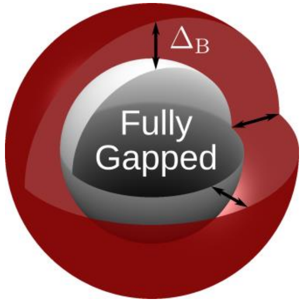
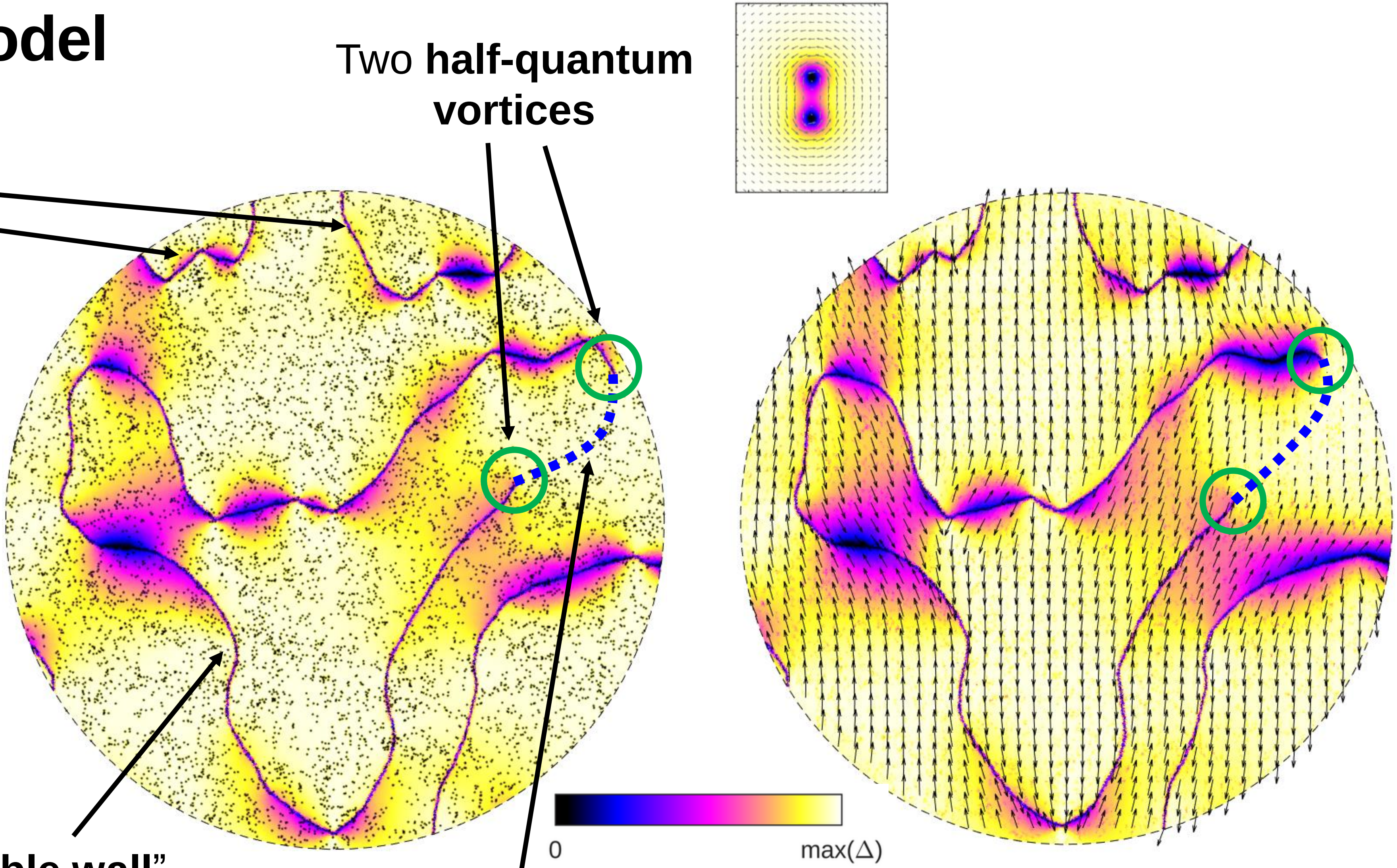


B phase



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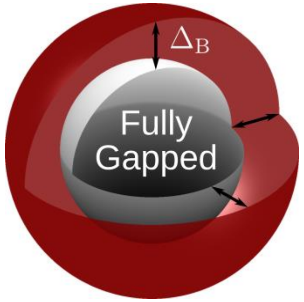

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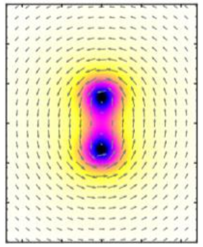

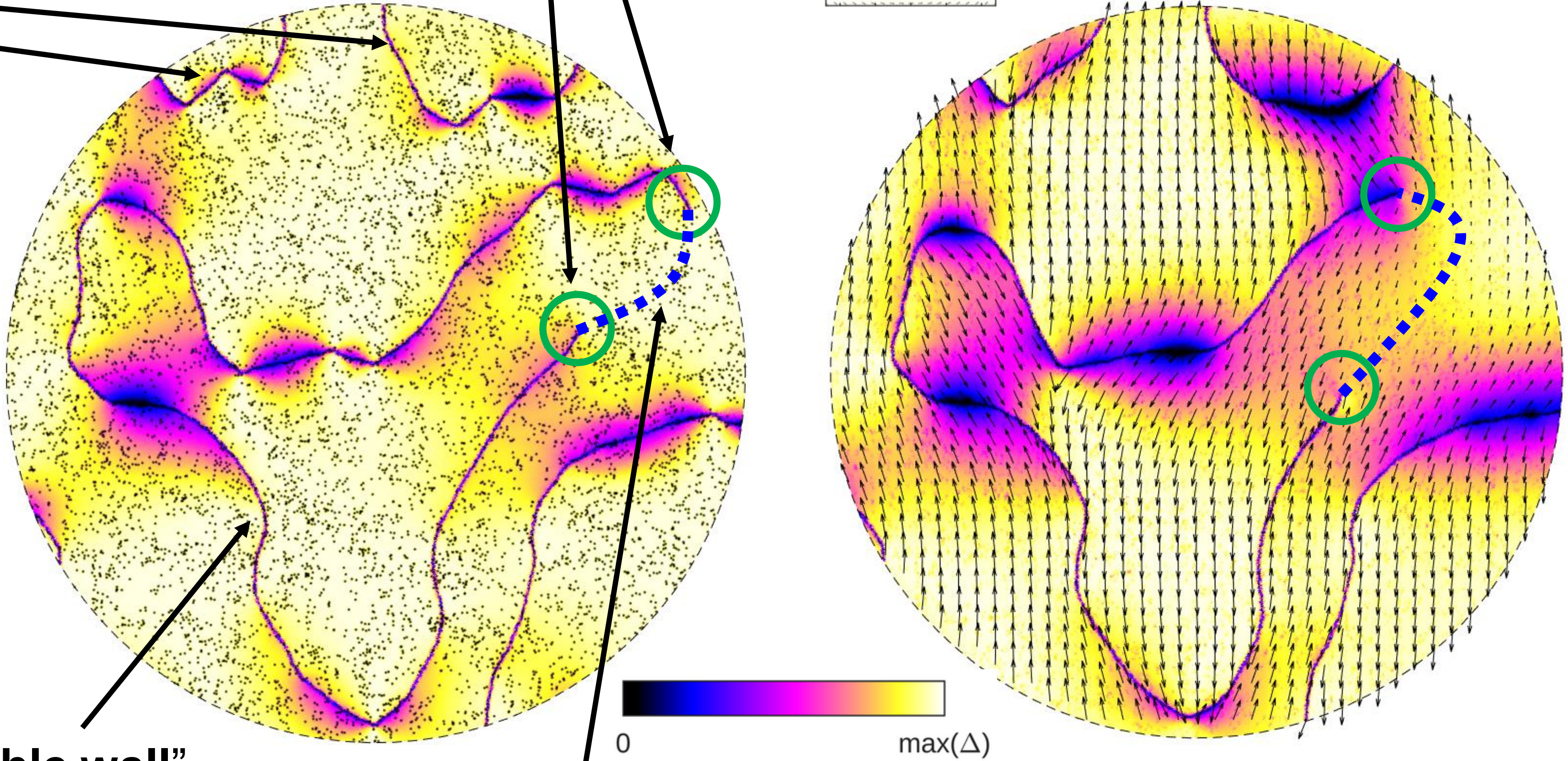

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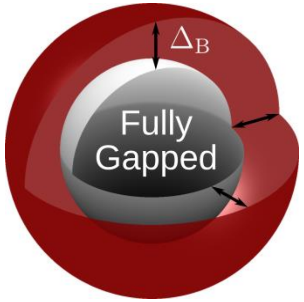

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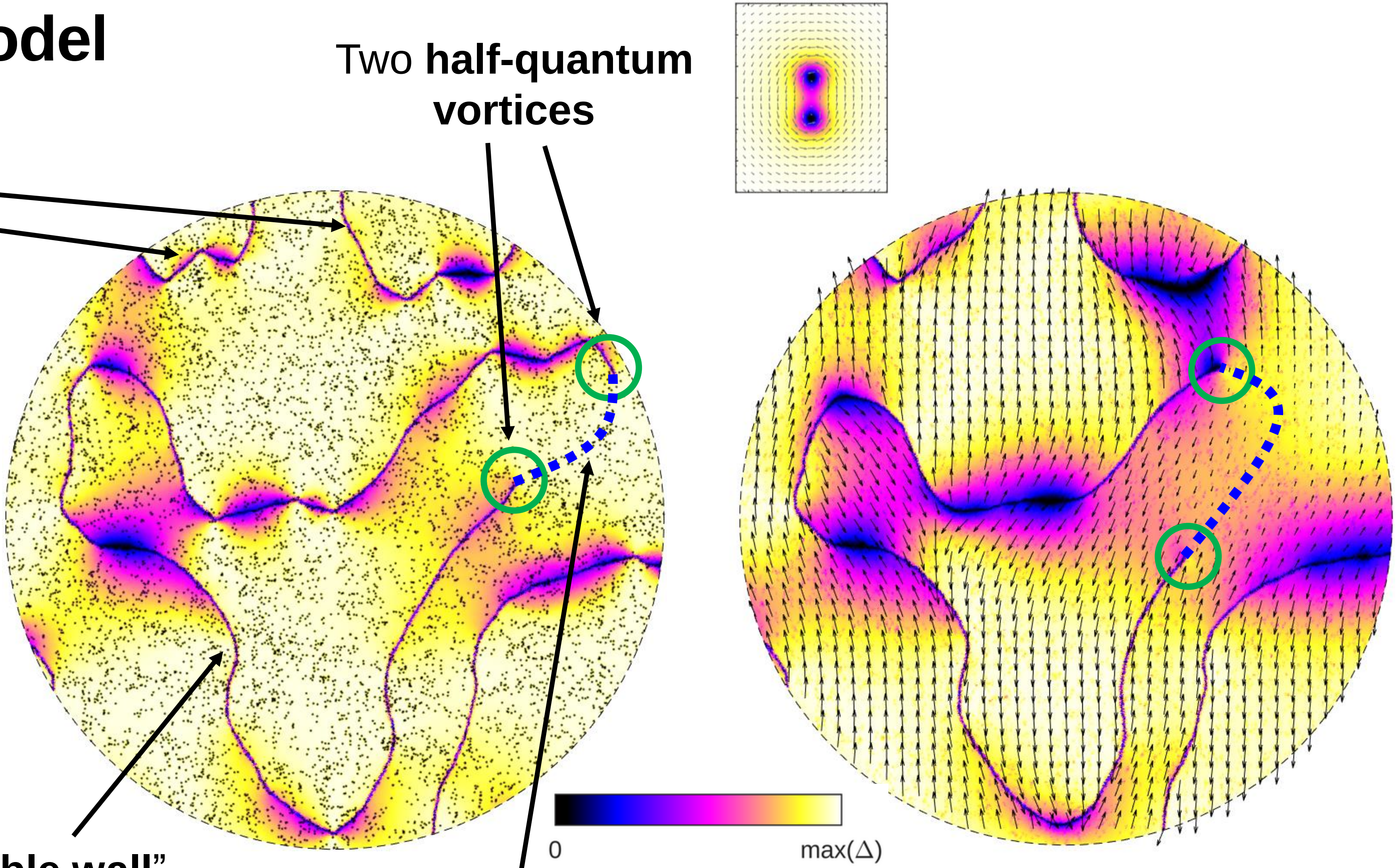

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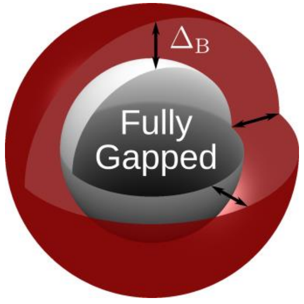

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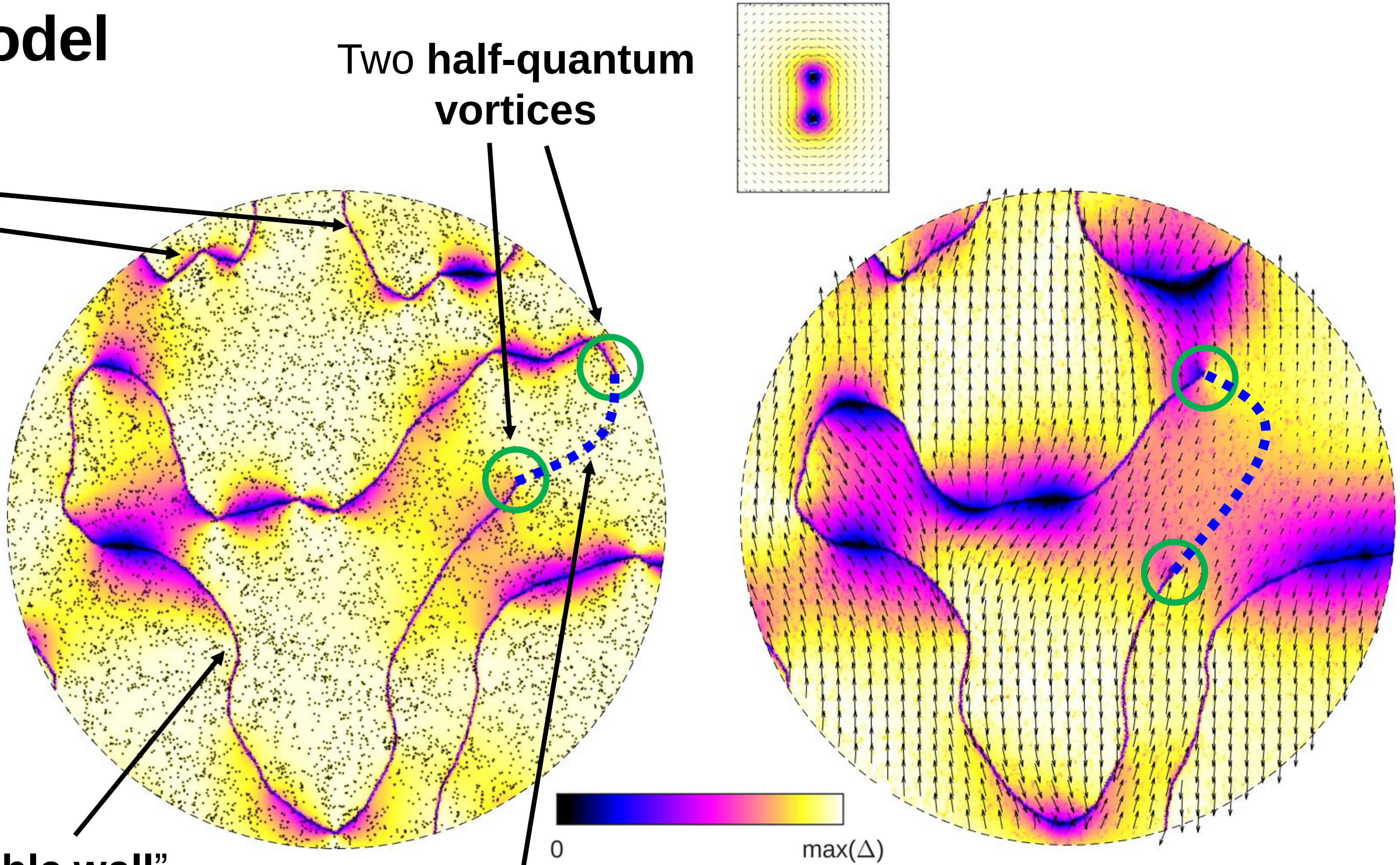

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Spin soliton between HQVs

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A!

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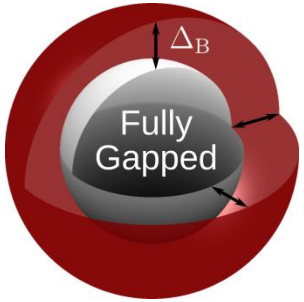

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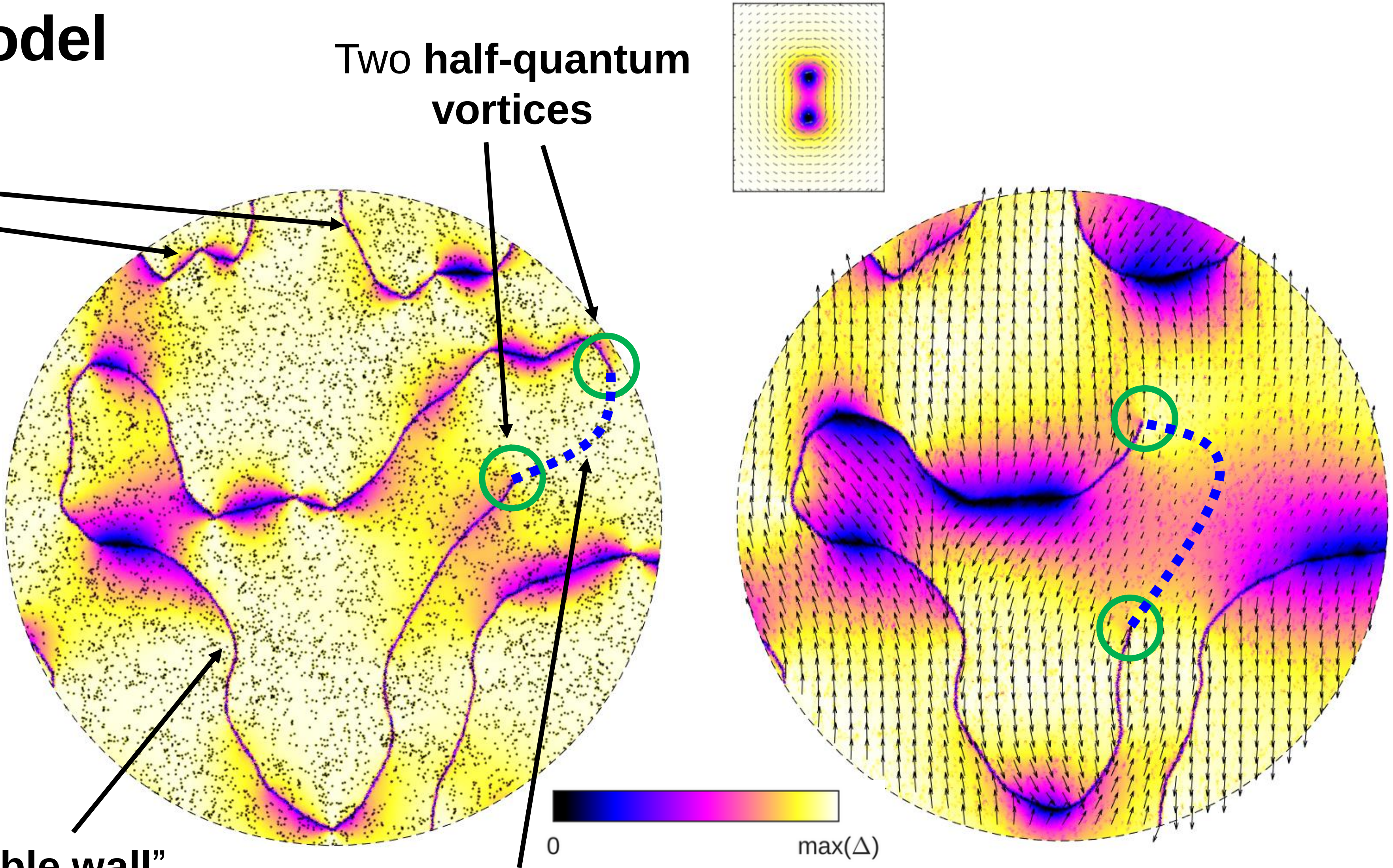

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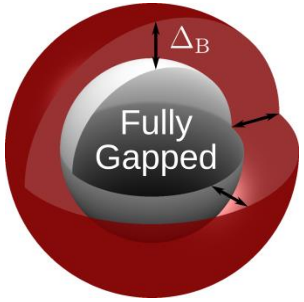

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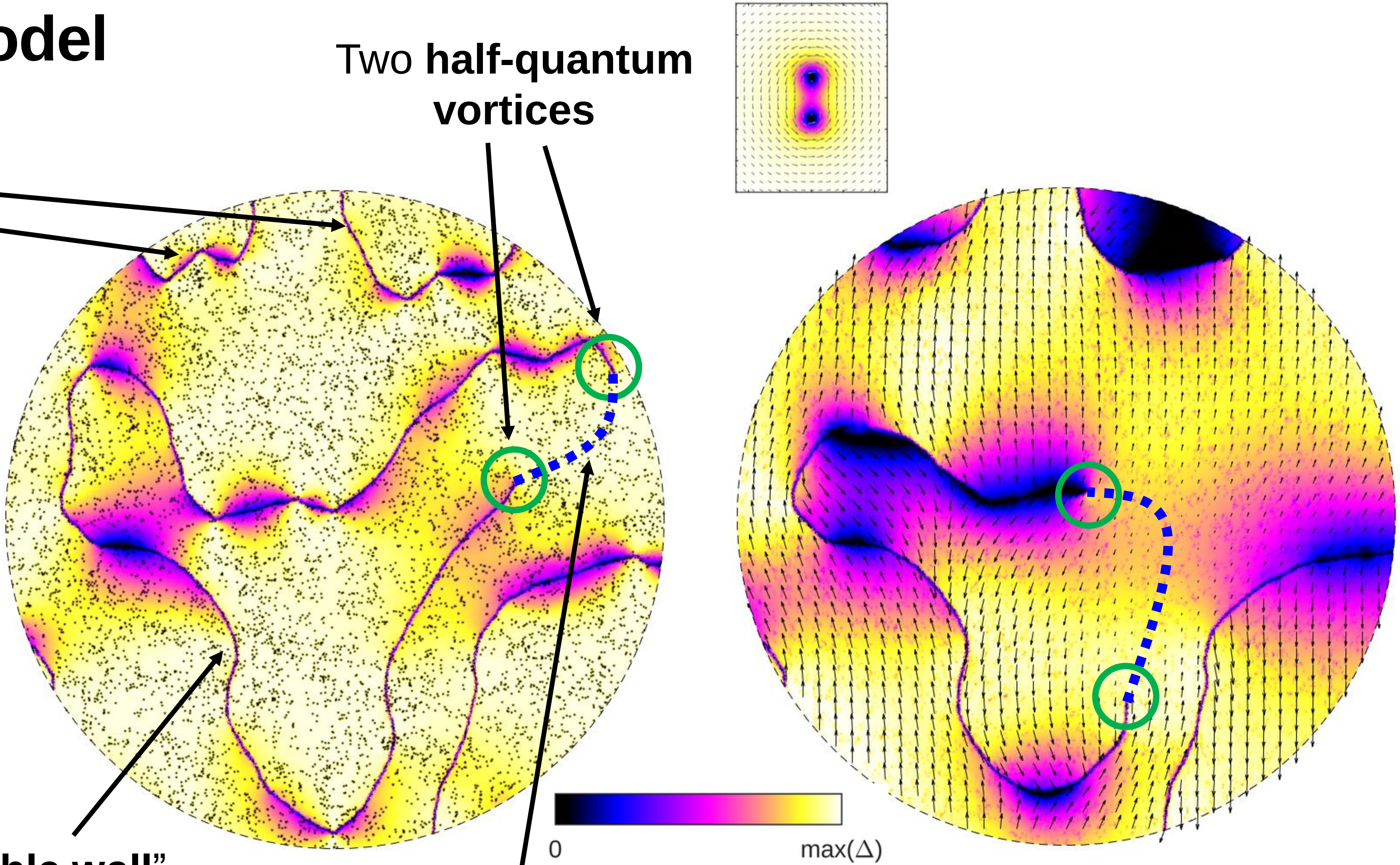

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“Kibble wall”
connecting HQVs

Spin soliton
between HQVs

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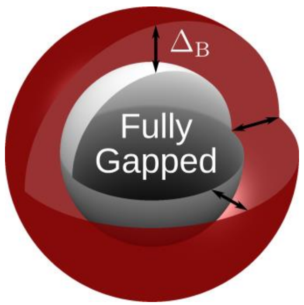

B phase

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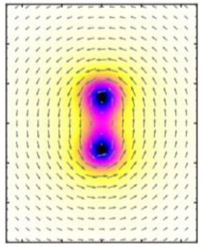

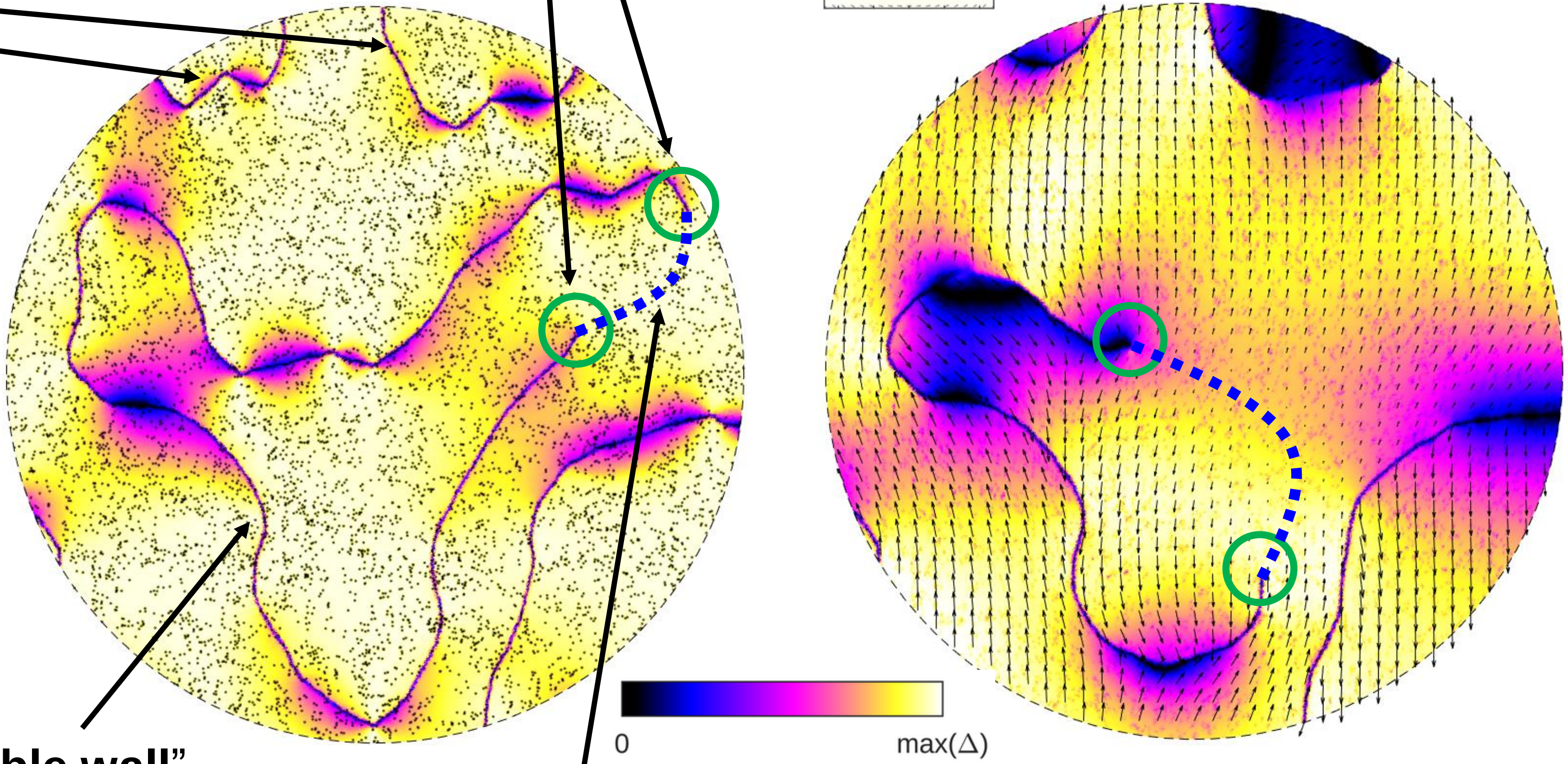

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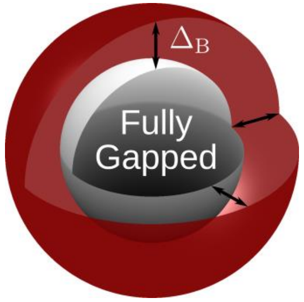

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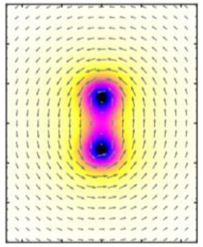

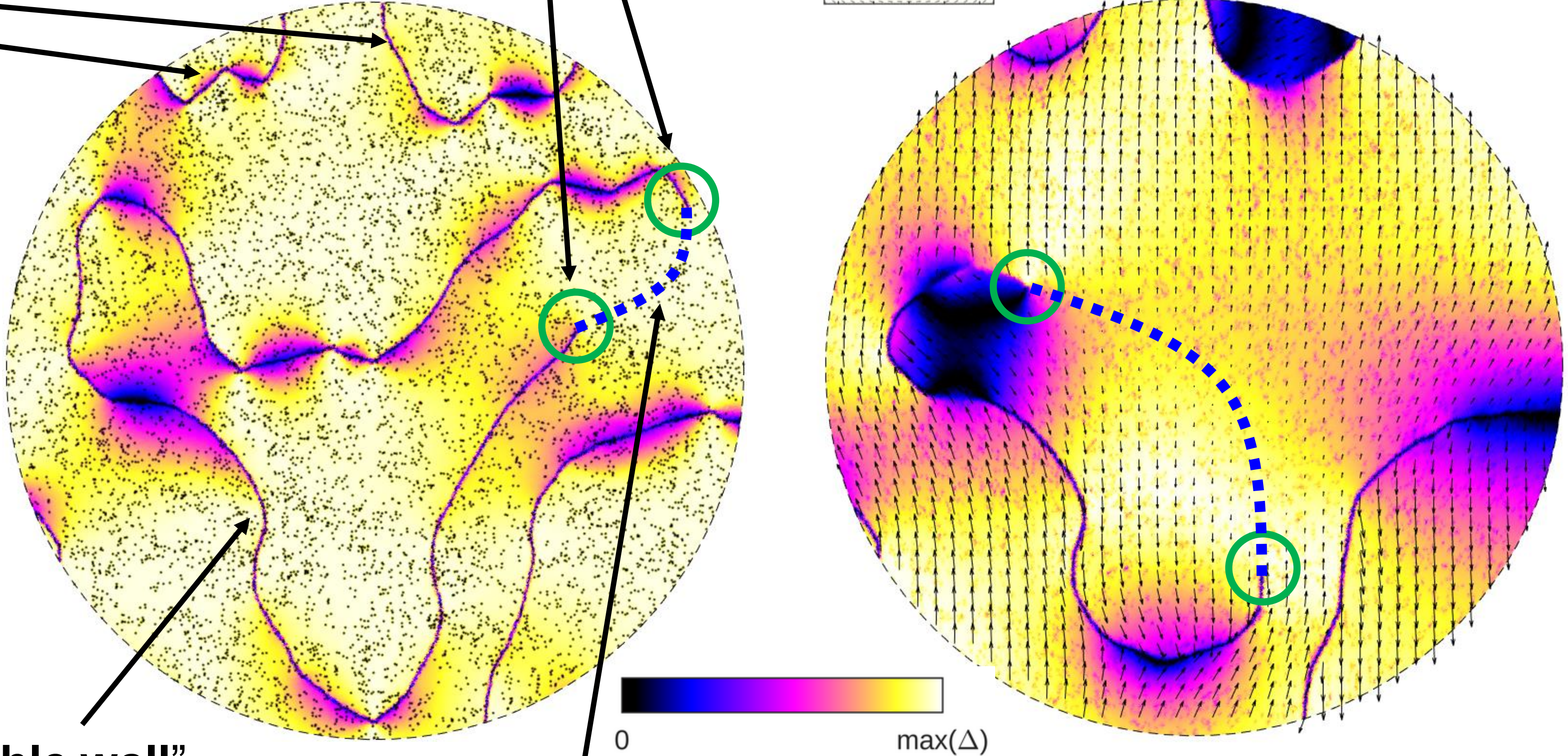

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Spin soliton between HQVs

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Point impurity model

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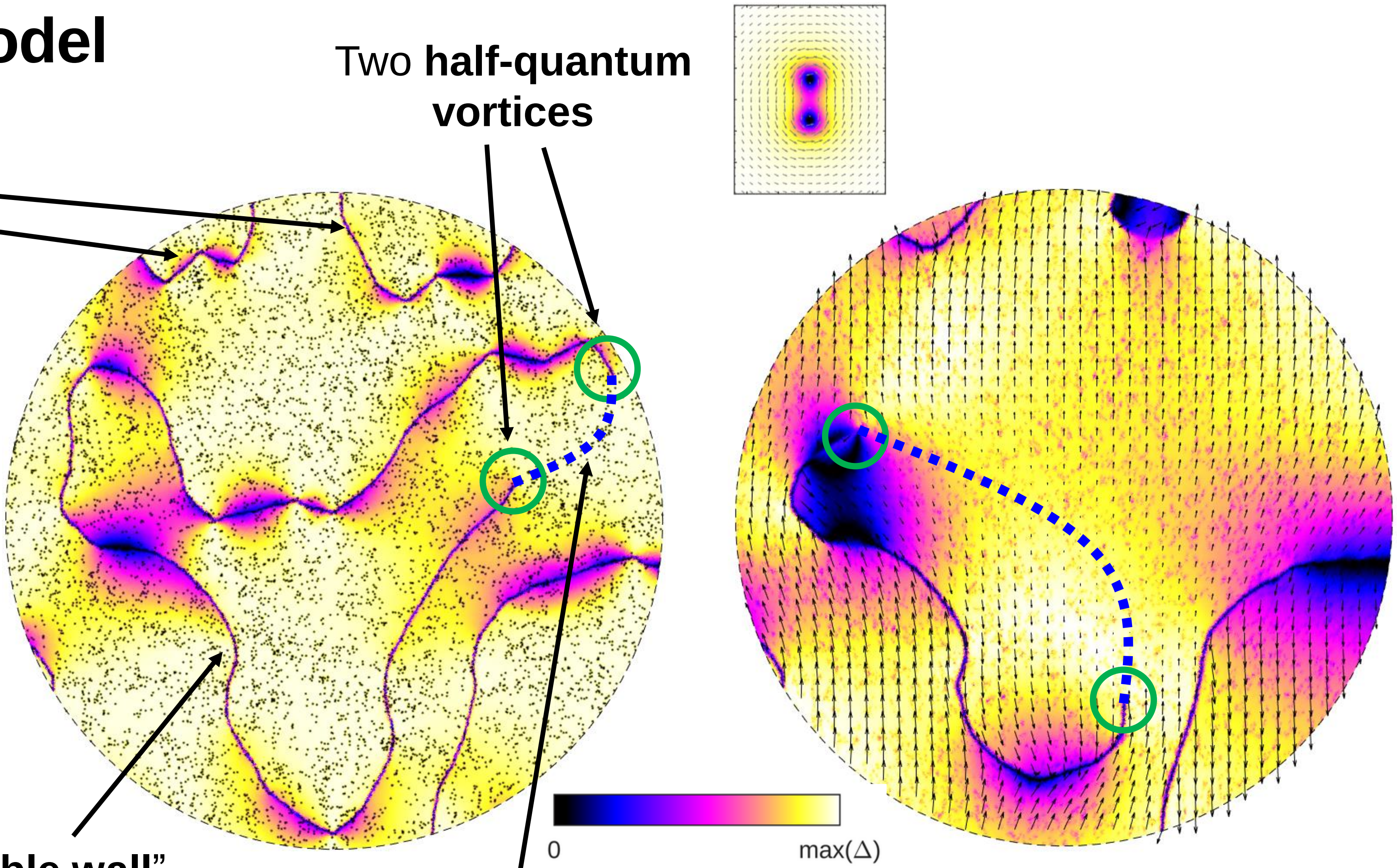
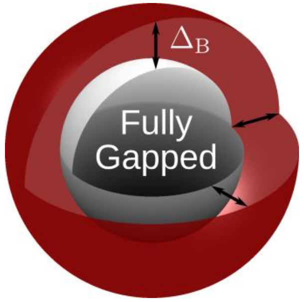


Two **half-quantum vortices**

"Kibble wall" connecting HQVs

Spin soliton between HQVs

Arrows show direction of $\hat{n}$



B phase

Point impurity model

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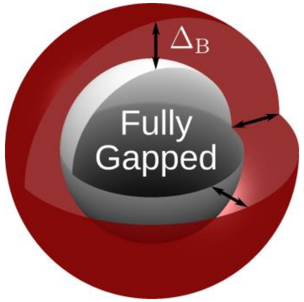

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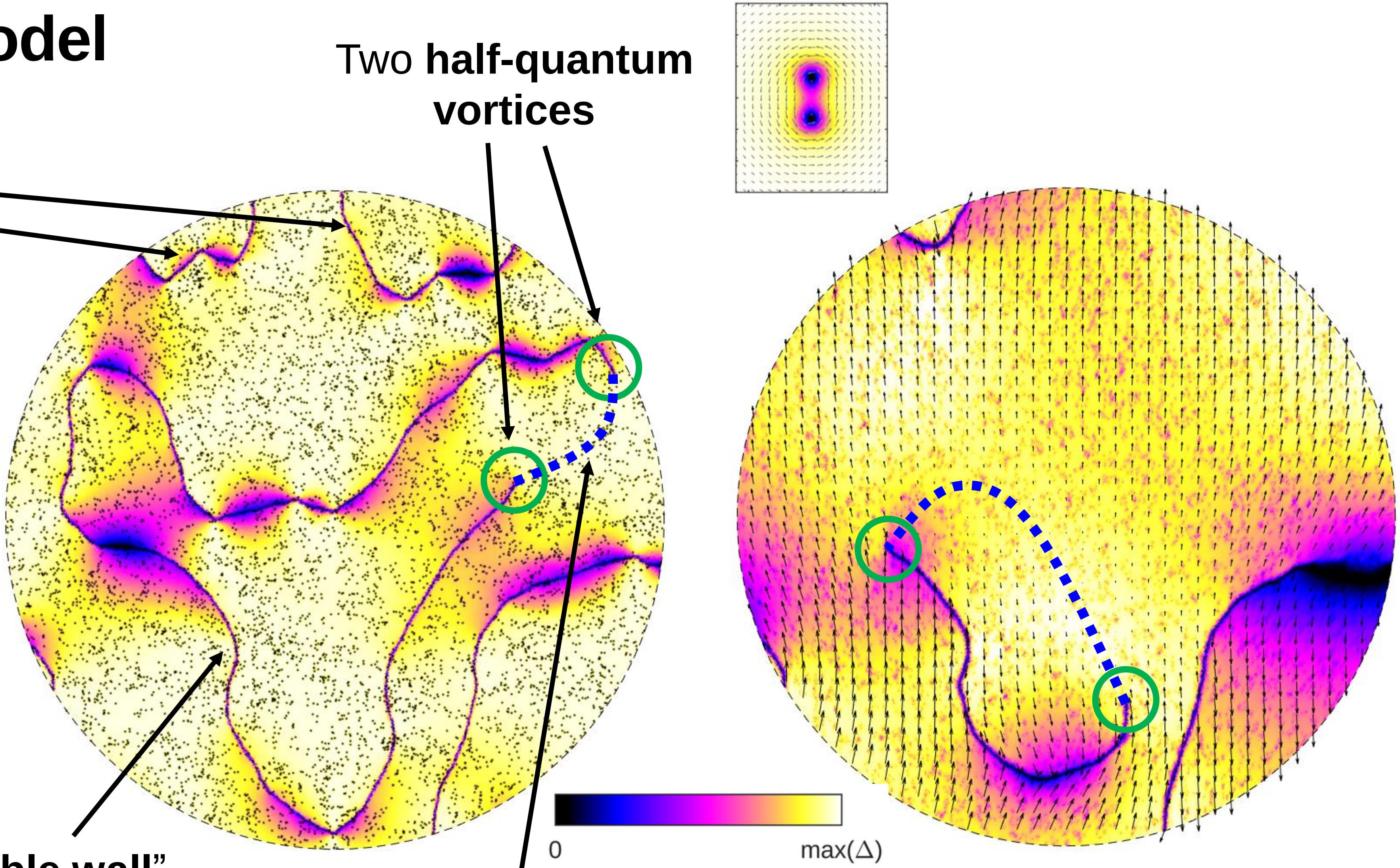

Two **half-quantum vortices**

“Kibble wall” connecting HQVs

Spin soliton between HQVs

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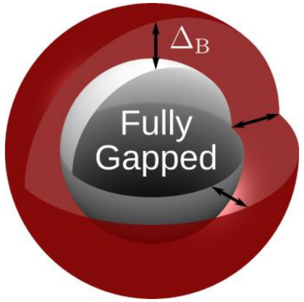

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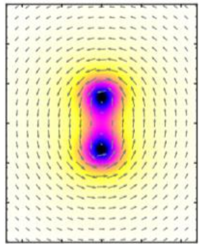
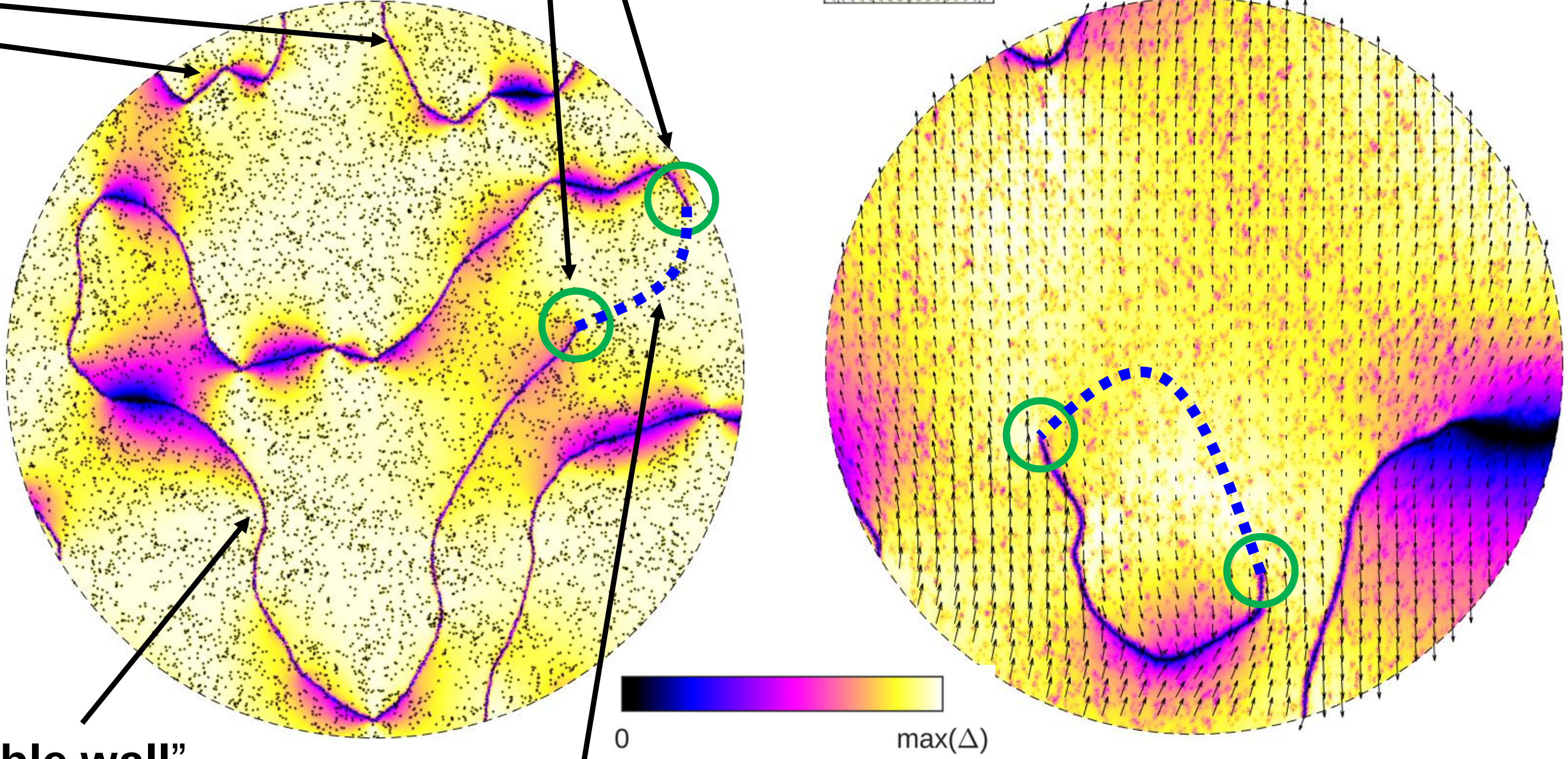




“Kibble wall” connecting HQVs

Spin soliton between HQVs

Arrows show direction of $\hat{n}$

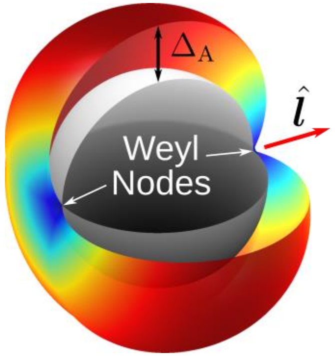
A!

Point impurity model

In the A phase, impurities orient $\hat{l}$

Defects in $\hat{l}$ texture don't carry phase winding, but are pinned by impurities

$$A_{\mu j} = \Delta_A \hat{d}_\mu (\hat{m}_j + i\hat{n}_j)$$



A phase

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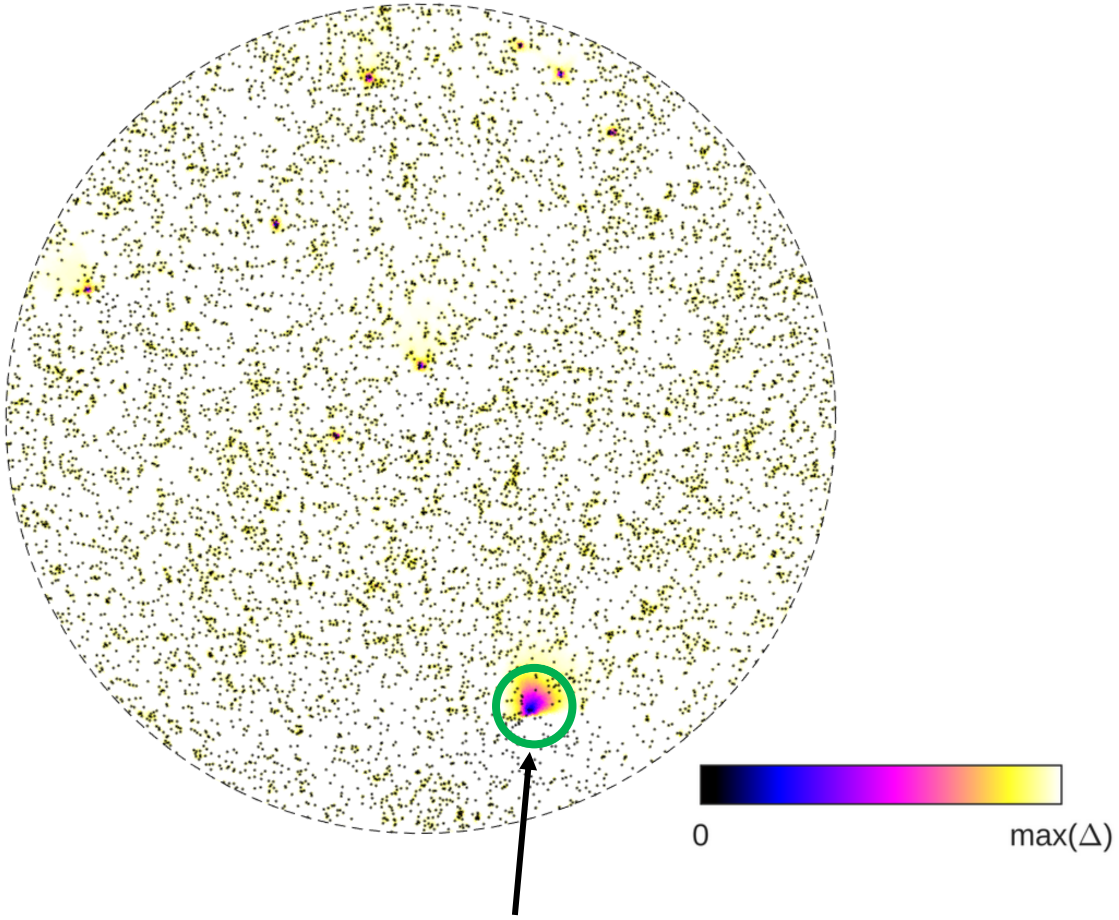

Single-quantum vortex

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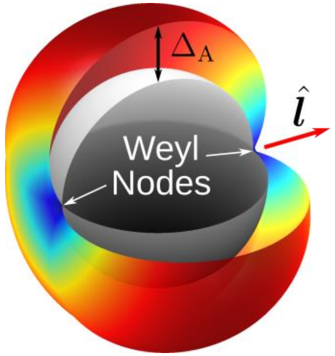
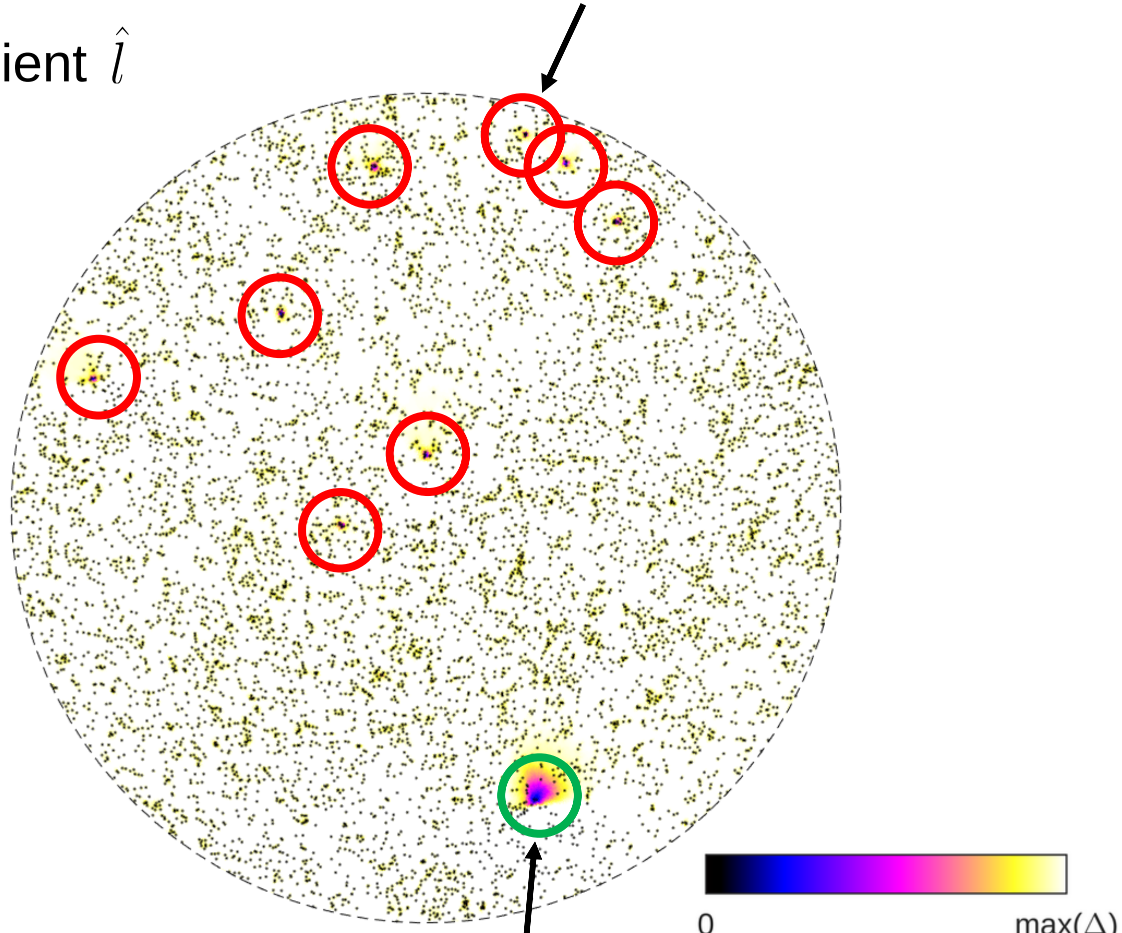


A phase

A!



"Disgyrations"

Single-quantum vortex

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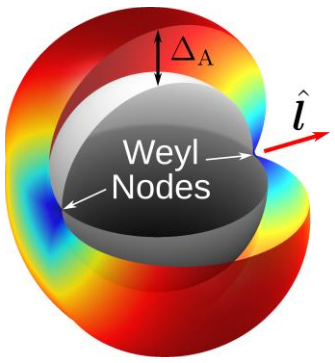
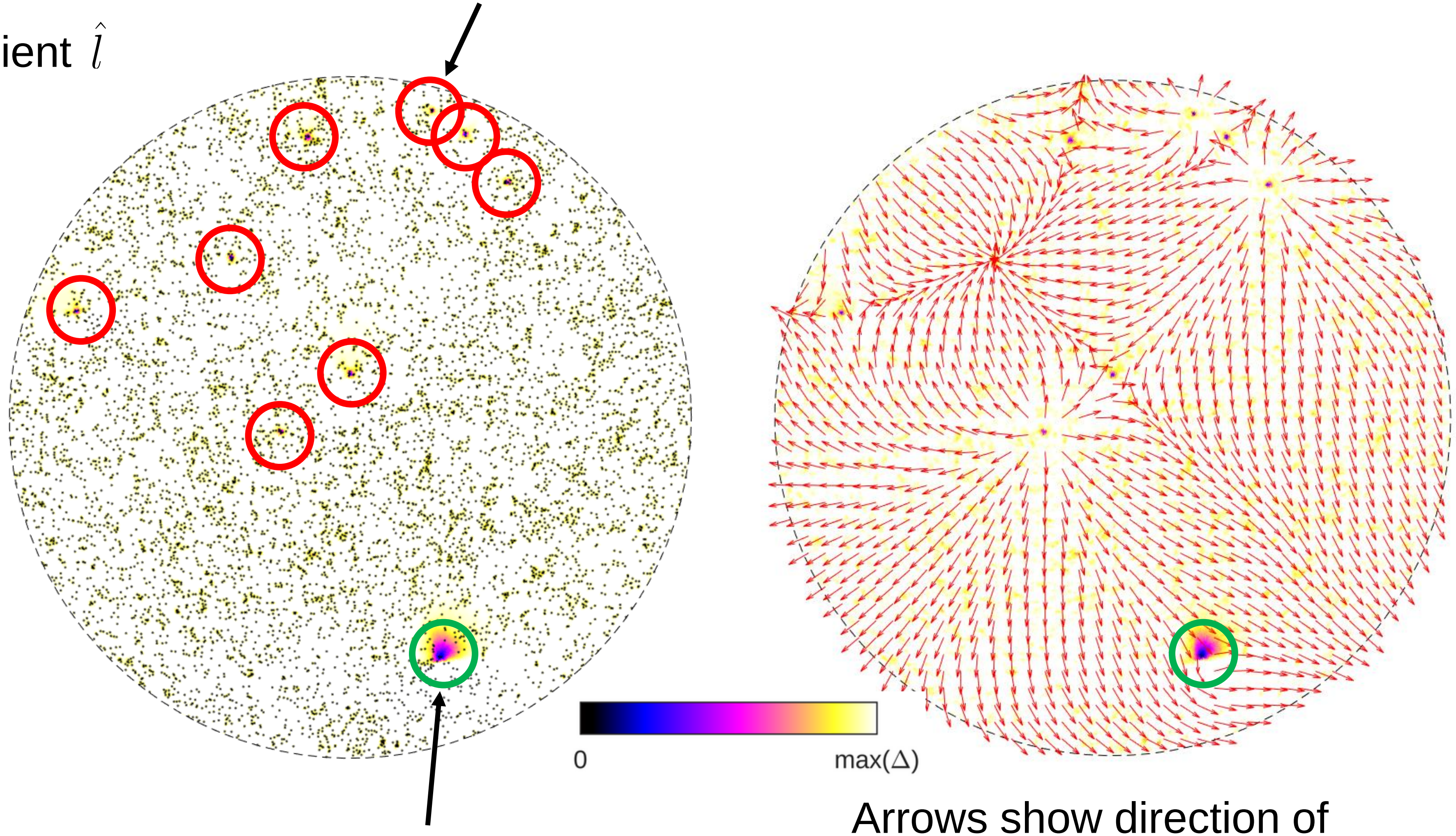


A phase

A!

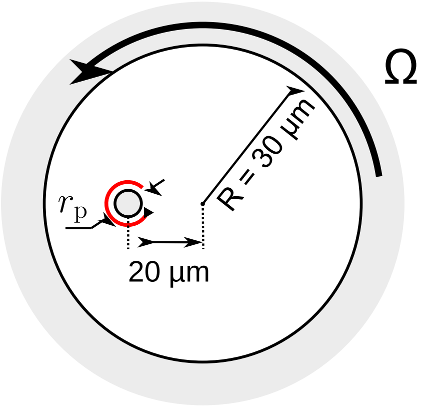


"Disgyrations"

Single-quantum vortex

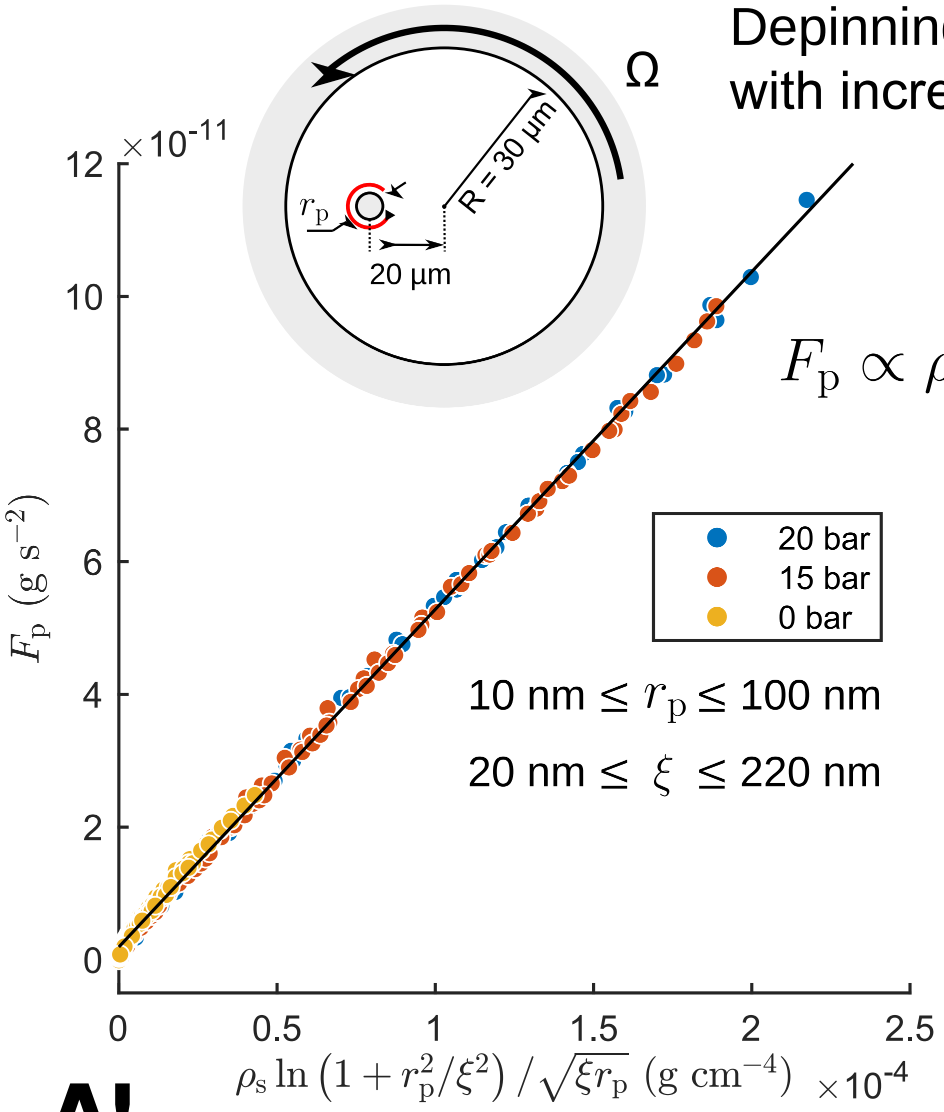
Arrows show direction of $\hat{l} = \hat{m} \times \hat{n}$

Mesoscopic pinning



Depinning from a single pillar with increasing Ω

Mesoscopic pinning



10 nm ≤ r_p ≤ 100 nm

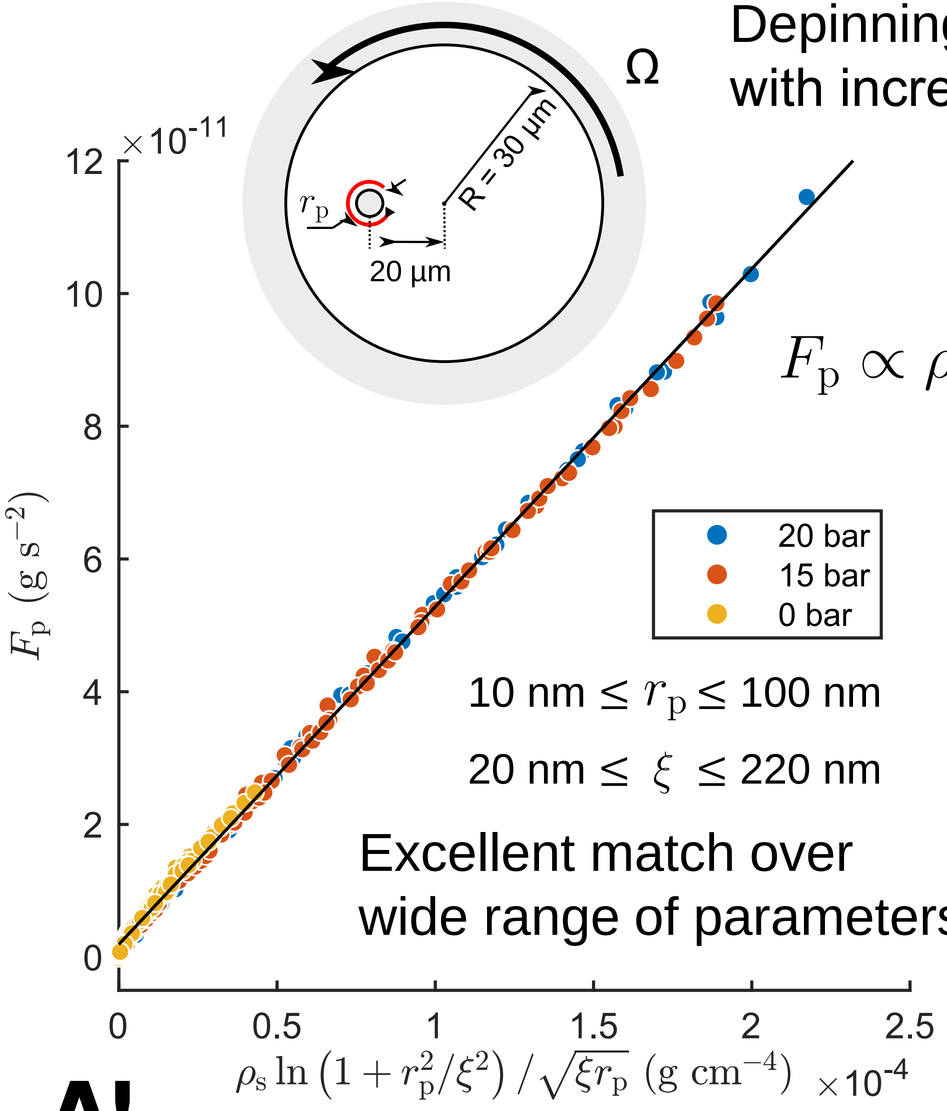
20 nm ≤ ξ ≤ 220 nm

Depinning from a single pillar with increasing Ω

$$F_\mathrm{p} \propto \rho_\mathrm{s} \frac{\ln\left(1 + r_\mathrm{p}^2/\xi^2\right)}{\sqrt{\xi r_\mathrm{p}}}$$

Mesoscopic pinning

Depinning from a single pillar with increasing Ω



$$F_\mathrm{p} \propto \rho_\mathrm{s} \frac{\ln\left(1 + r_\mathrm{p}^2/\xi^2\right)}{\sqrt{\xi r_\mathrm{p}}}$$

10 nm ≤ r_p ≤ 100 nm

20 nm ≤ ξ ≤ 220 nm

Excellent match over wide range of parameters!

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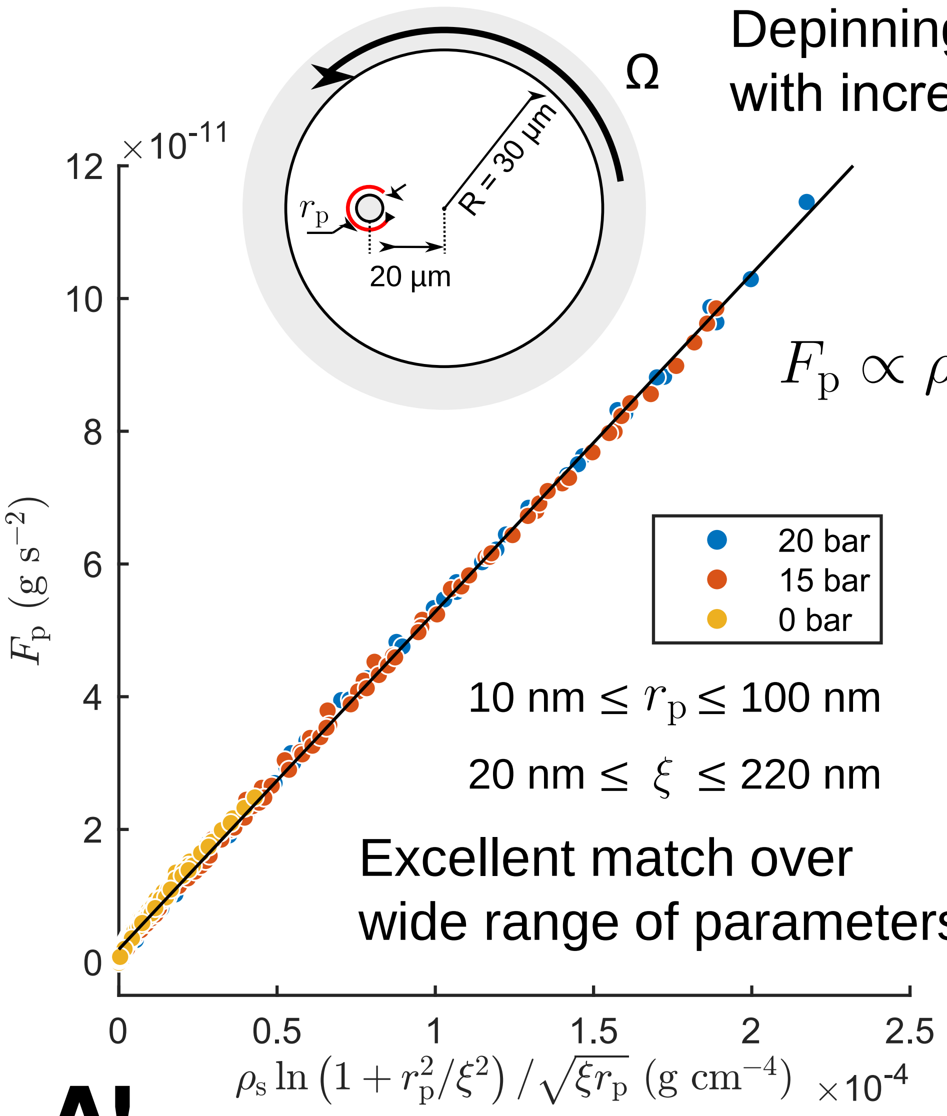

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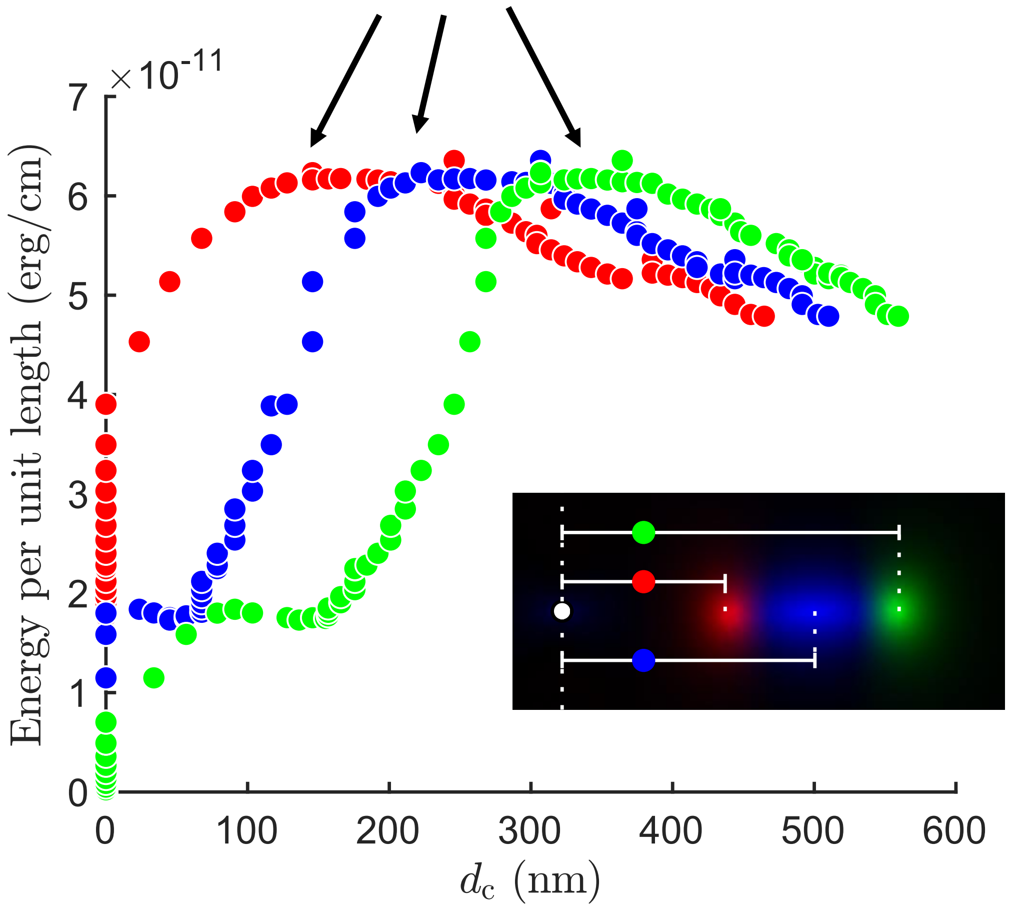
Calculated energy barriers using the **Nudged Elastic Band method**

Vortex leaves in **fractions**



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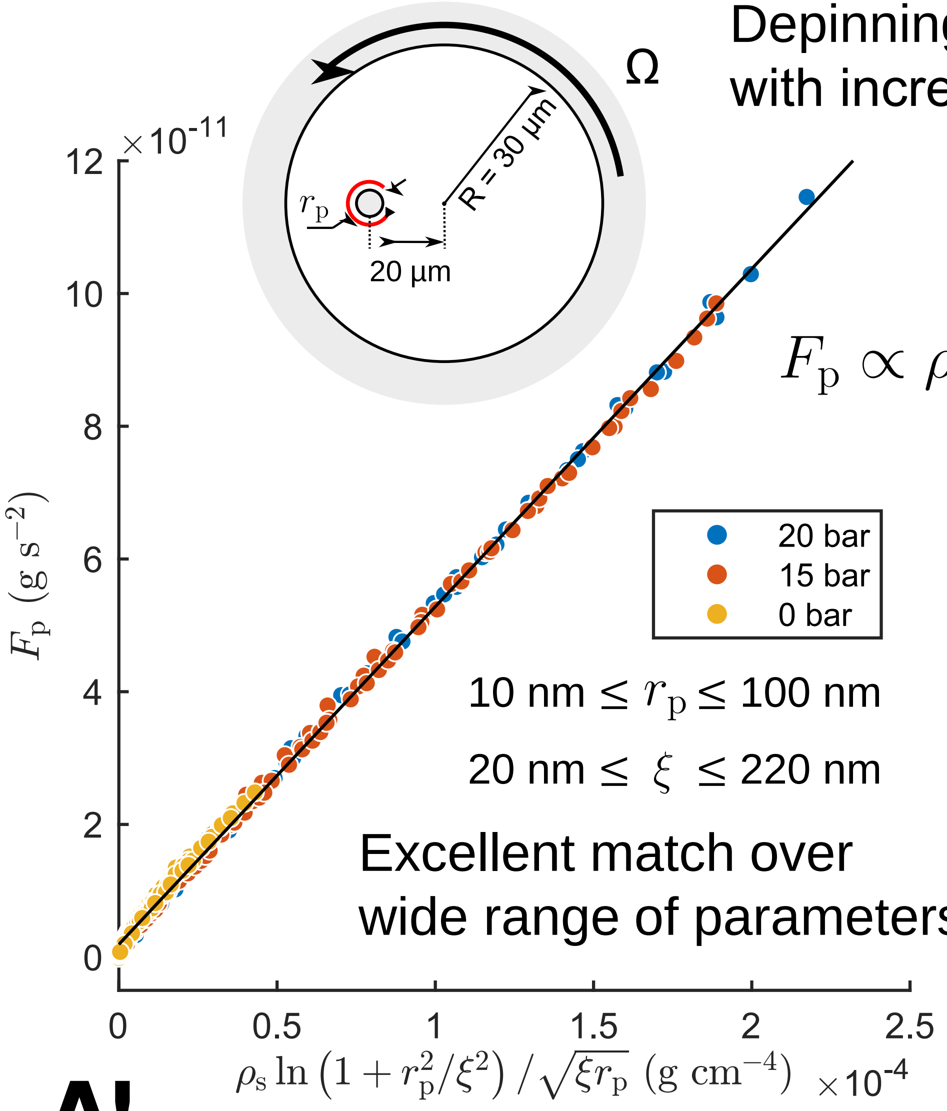

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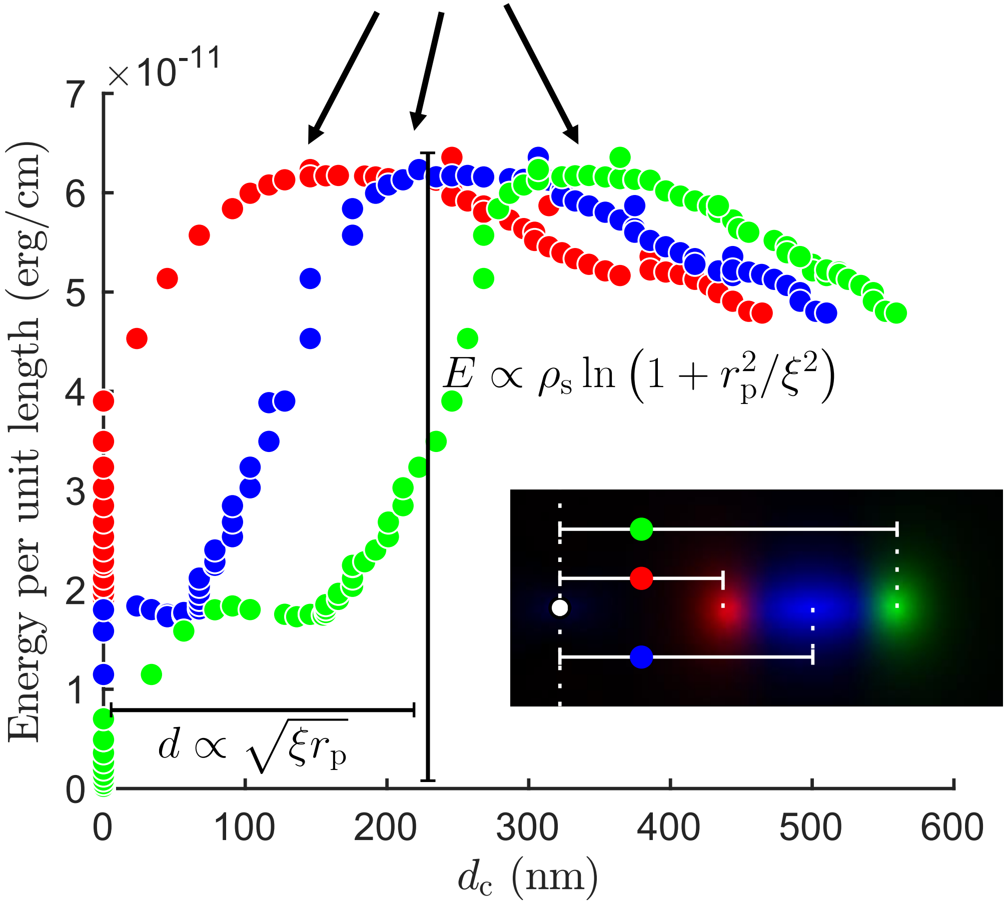

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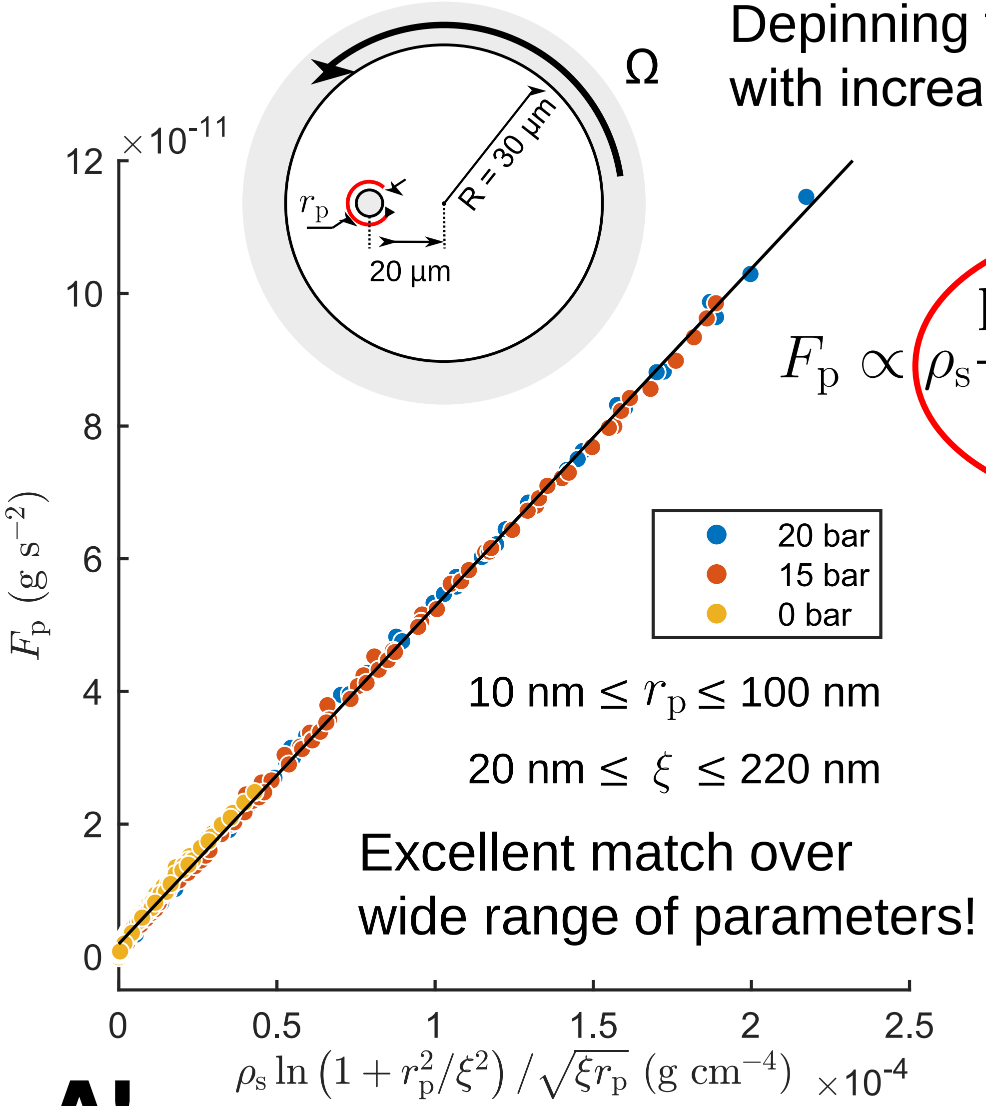

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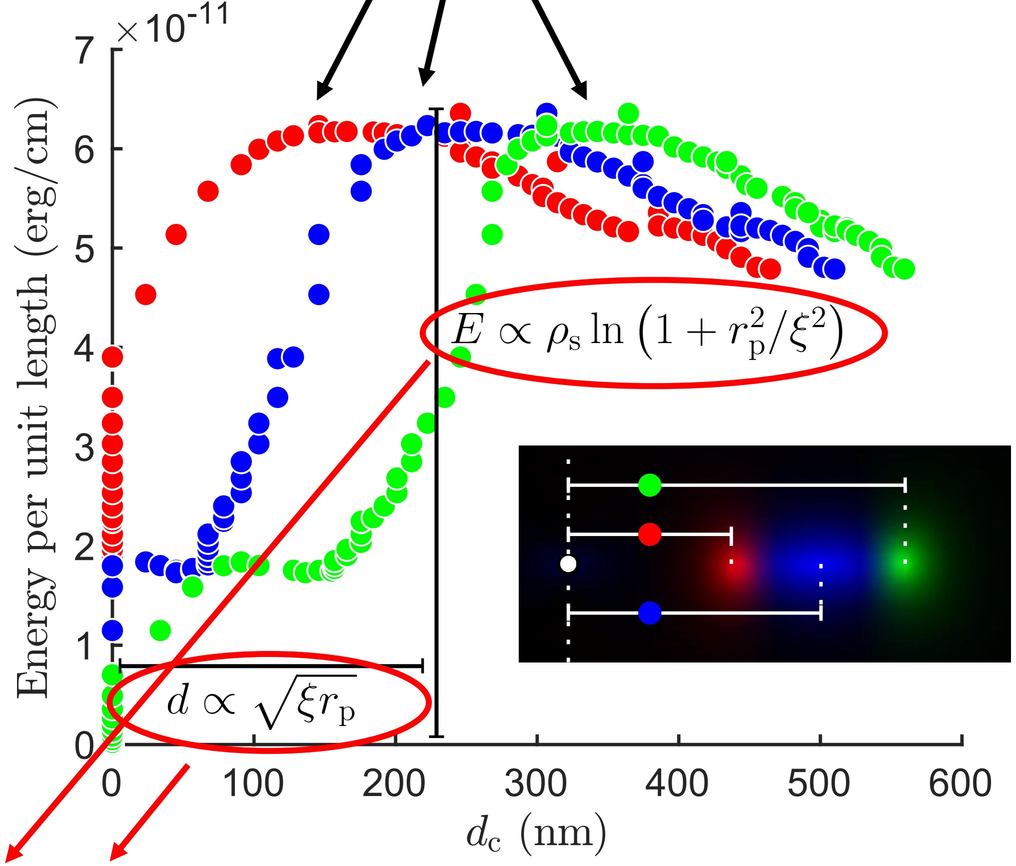


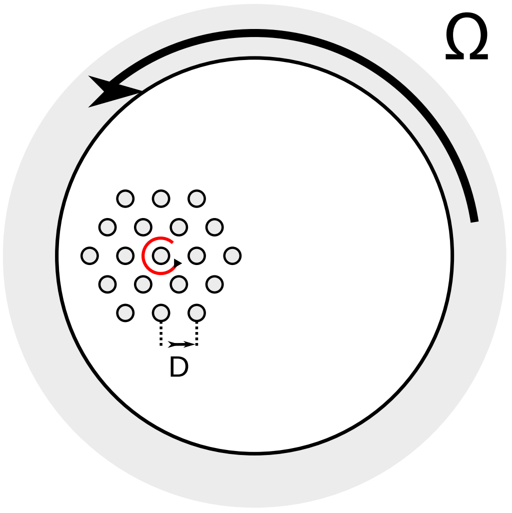
$E \propto \rho_\mathrm{s} \ln\left(1 + r_\mathrm{p}^2/\xi^2\right)$

$d \propto \sqrt{\xi r_\mathrm{p}}$

$$F_\mathrm{p} \propto E/d$$

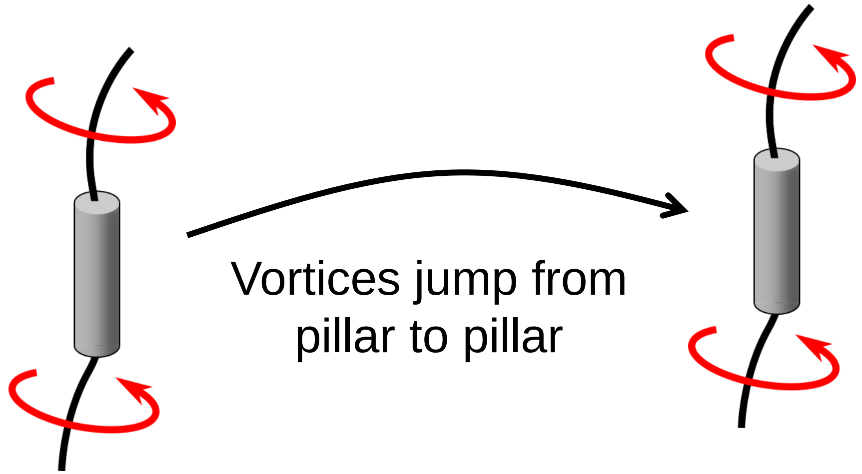
Strange scaling for length scale, triplet physics?

Mesoscopic pinning

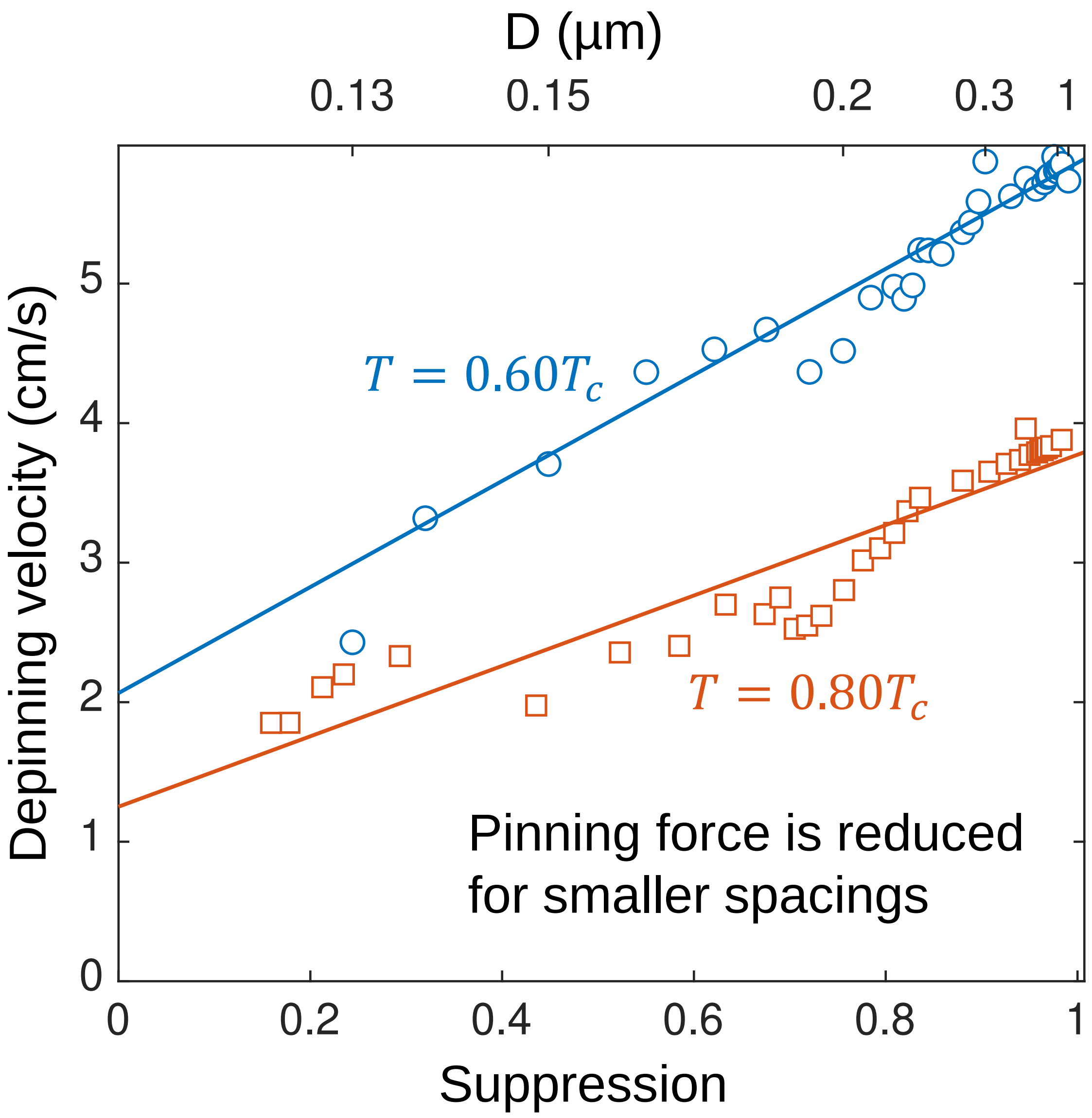


Depinning in an array of pillars with spacing D

Decreasing pillar spacing suppresses gap in plane



Vortices jump from pillar to pillar

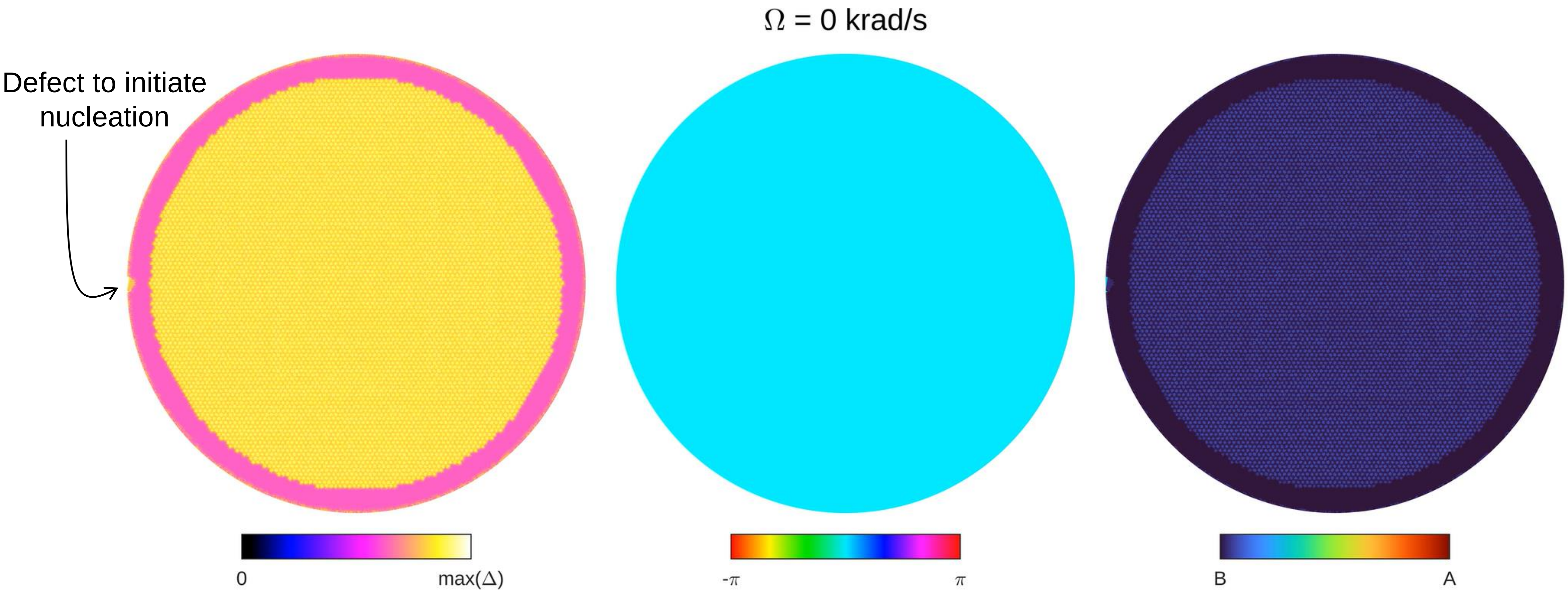


Pinning force is reduced for smaller spacings

Rotation sweep

Cylinder filled with ~**8000 pillars**

Increasing rotation rapidly up to 20 krad/s, then **decreasing** back to 0 krad/s



Defect to initiate nucleation

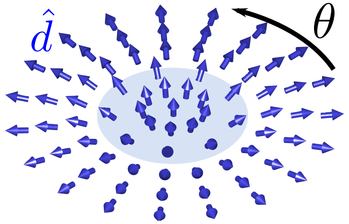
Amplitude of 00 component

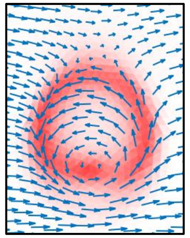
Phase of 00 component

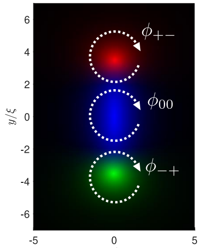
Blue = **B phase**
Red = **A phase**

Summary

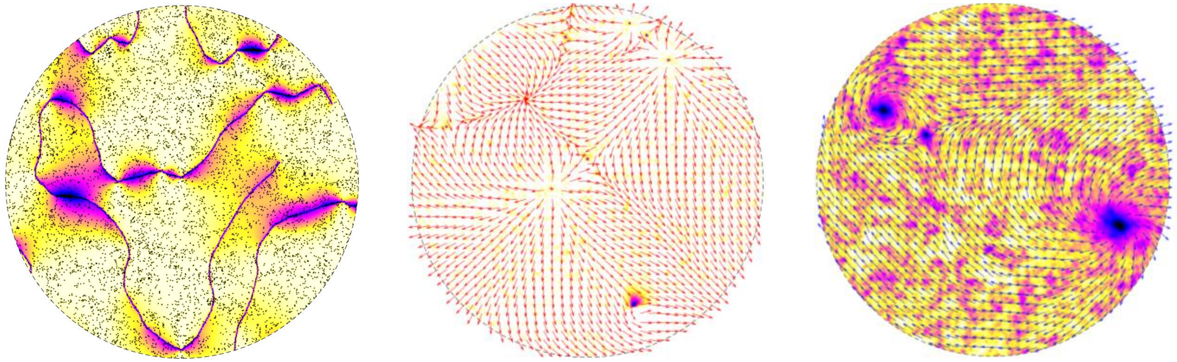
- Superfluid ^{3}He is an excellent model system for studying topology
- A multicomponent order parameter can support a **wide variety of different vortex structures**







- Impuritities can pin **other kinds of topological objects** in addition to regular vortices



- **Pinning force** depends on coherence length and size of object:

$$F_{\mathrm{p}} \propto \rho_{\mathrm{s}} \frac{\ln\left(1 + r_{\mathrm{p}}^2/\xi^2\right)}{\sqrt{\xi r_{\mathrm{p}}}}$$