Vortex Avalanches in Neutron Stars

Gary (I-Kang) Liu

Julie Thomas

Carlo Barenghi

Andrew Baggaley

Toby Wood

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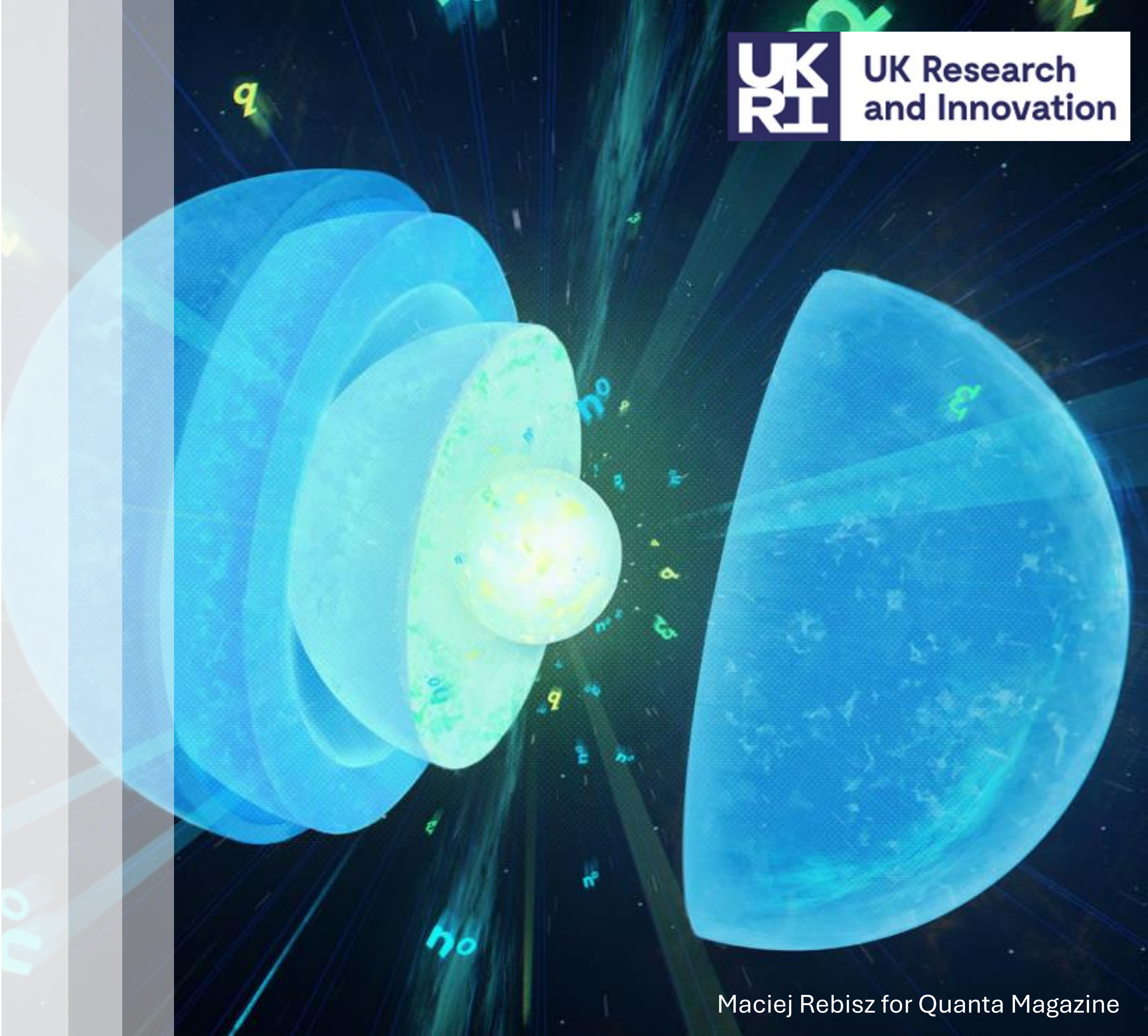
ROYAL HOLLOWAY UNIVERSITY OF LONDON

Newcastle University

UK Research and Innovation



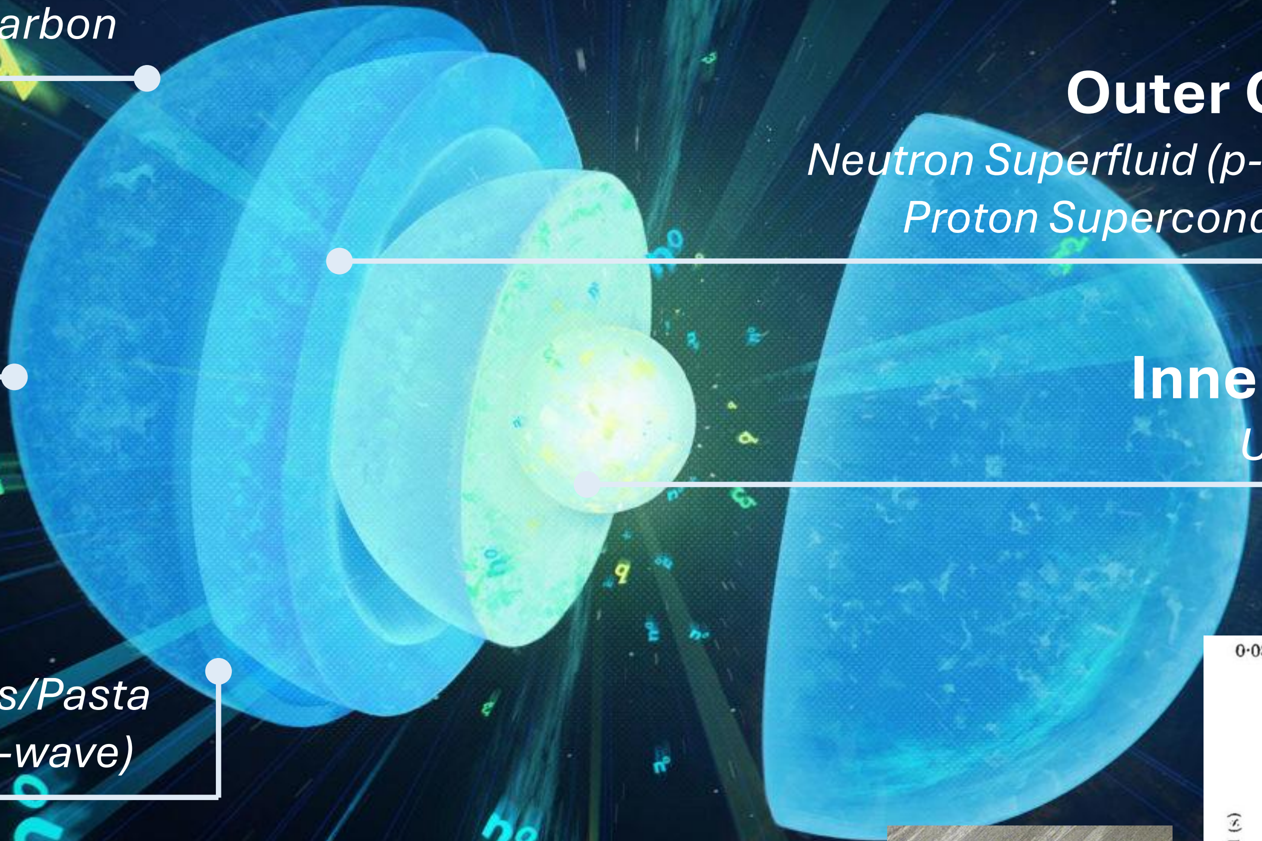
Maciej Rebisz for Quanta Magazine

Neutron Star Interior

Maciej Rebisz for Quanta Magazine

$M \approx 1.4 M_{\odot}$

$R \approx 10\ \mathrm{km}$



Atmosphere
Hydrogen, Helium, Carbon

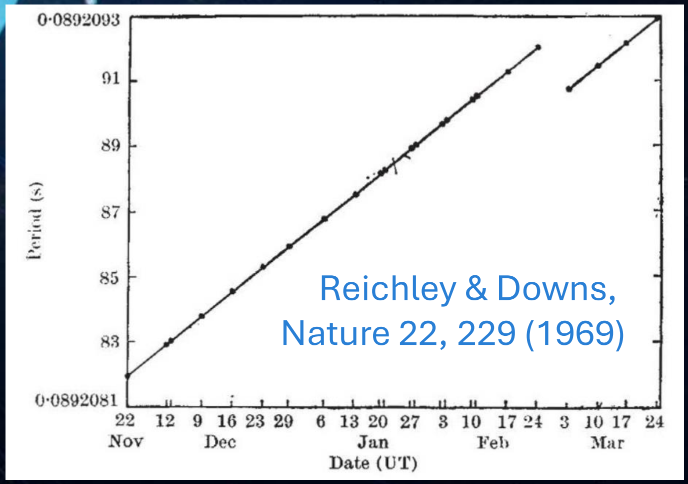
Outer Crust
Ions, Electrons

Inner Crust
Ions, Nuclear Lattices/Pasta
Neutron Superfluid (s-wave)

Outer Core
Neutron Superfluid (p-wave)
Proton Superconductor

Inner Core
Unknown

40. Neutron stars as cosmic laboratories for superfluid dynamics
Nils Andersson

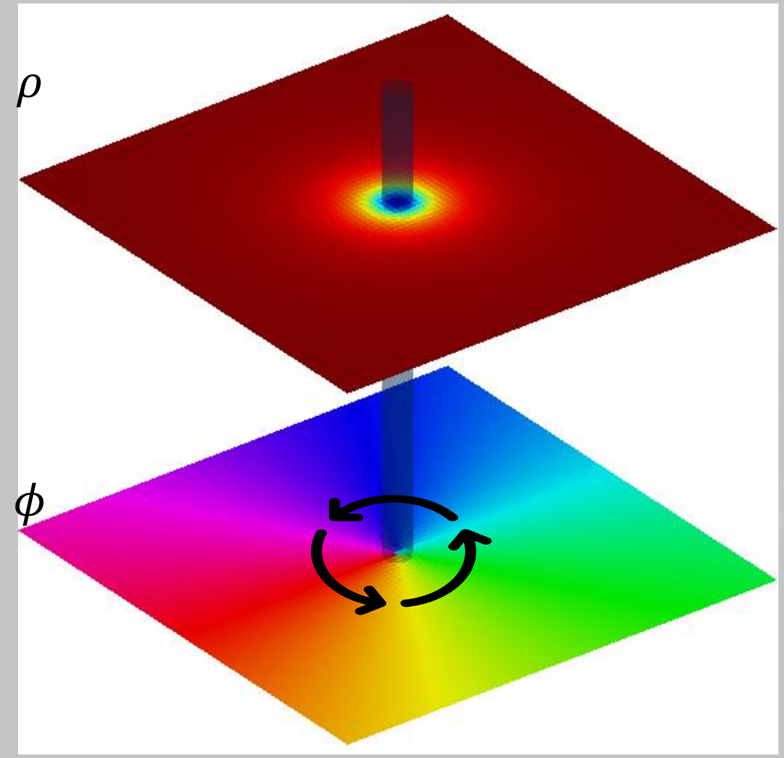


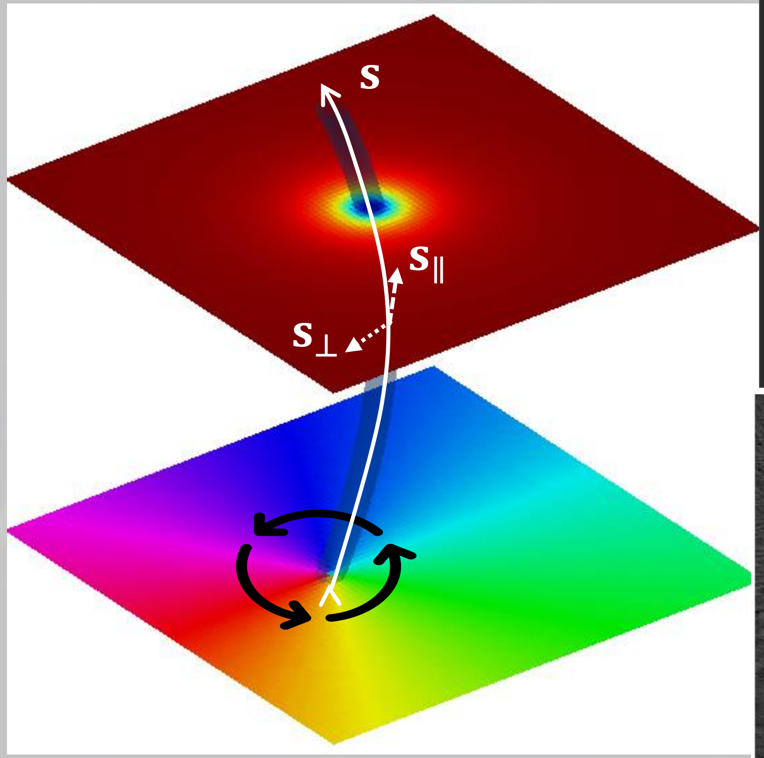
Reichley & Downs,
Nature 22, 229 (1969)

Quantum Vortices

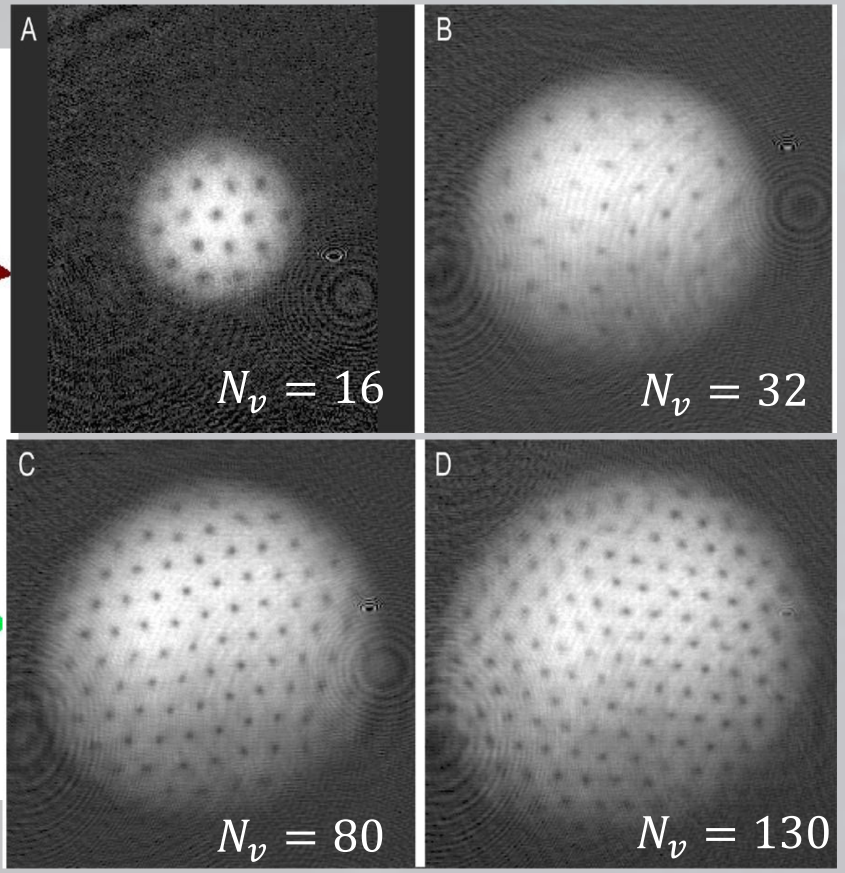
- Macroscopic superfluid wavefunction $\Psi(\mathbf{r},t) = \sqrt{n(\mathbf{r},t)}e^{i\phi(\mathbf{r},t)}$
- Irrotational, $\mathbf{v} = \frac{\hbar}{m}\nabla\phi \Rightarrow \nabla\times\mathbf{v} = 0$
- Phase defect, $\phi = \kappa\theta \Rightarrow \mathbf{v} = \frac{\hbar}{m}\frac{\kappa}{r}\hat{\boldsymbol{\theta}}$

More is Different



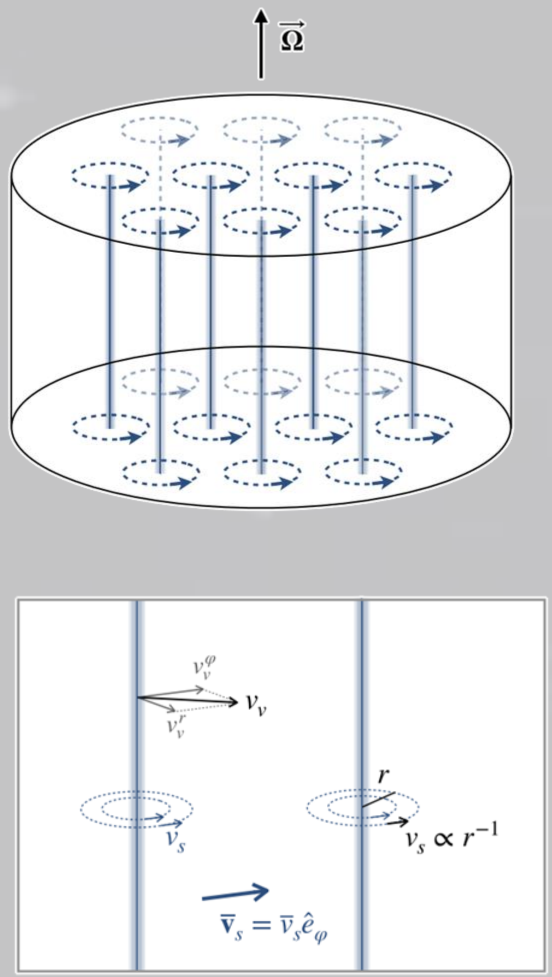


Abo-Shafer et al., Science 292, 476 (2001)



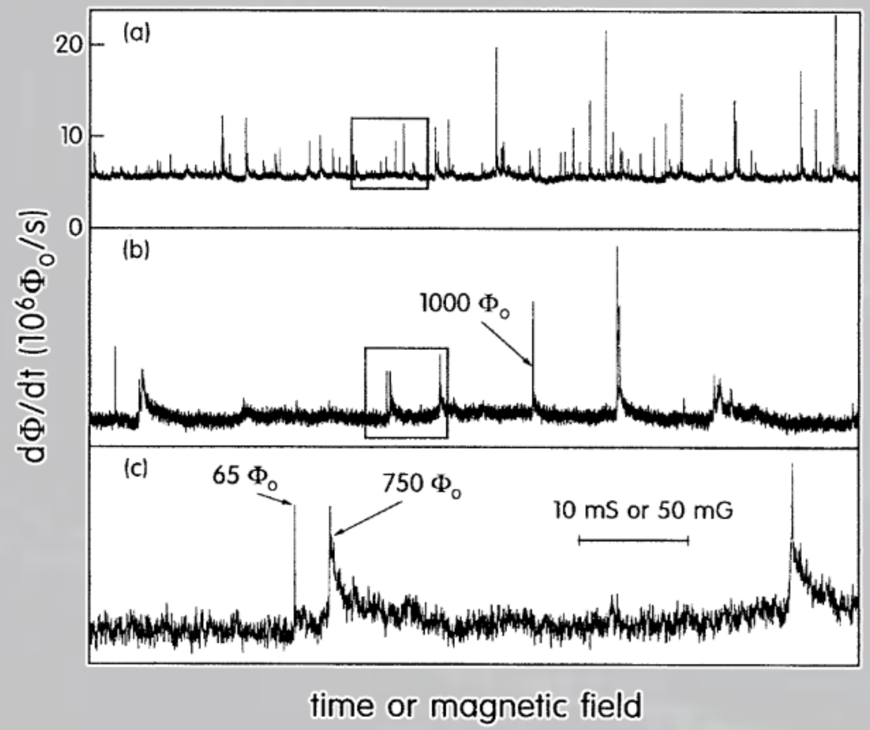
Glitches and Vortex Pinning and Avalanches

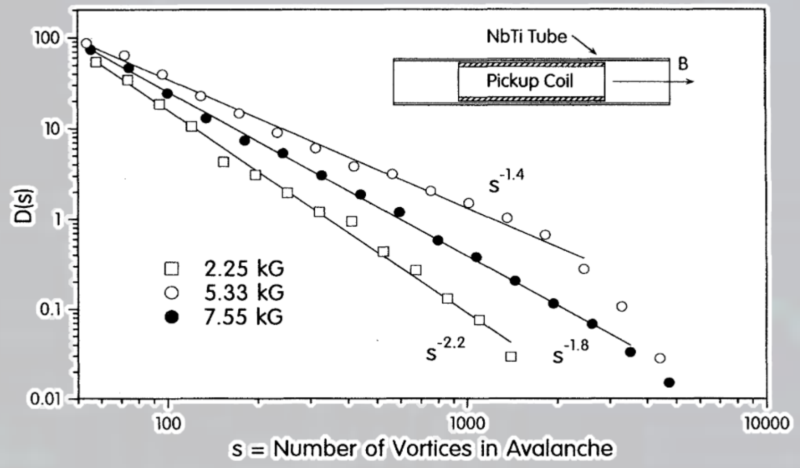
Analogy with type-II superconductor

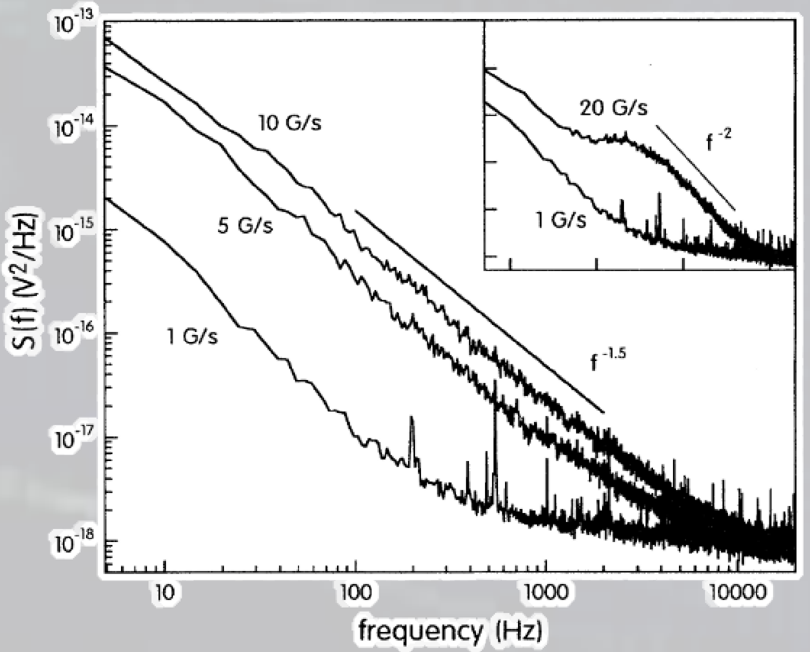


Antonopoulou, Haskell, Espinoza,
Rep. Prog. Phys. 85, 126901 (2022)

Field, Witt, Nori, PRL 74, 7 (1995)





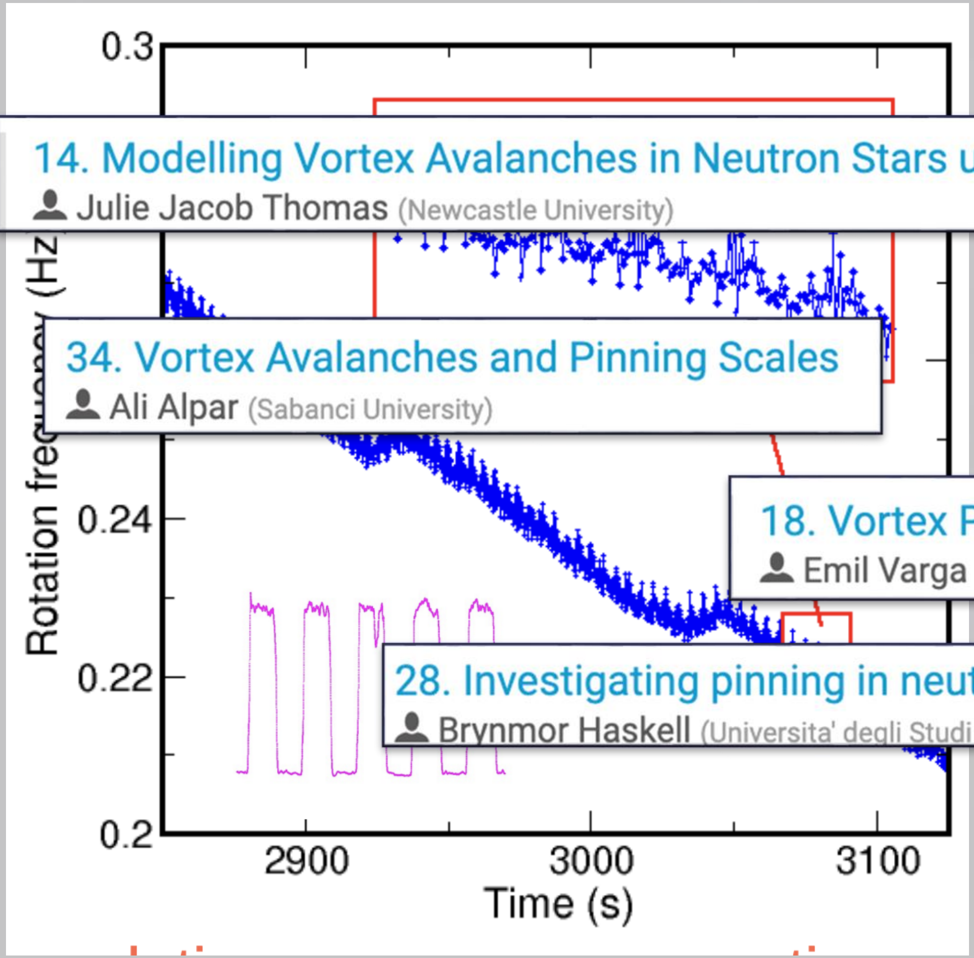


Glitches and Vortex Pinning and Avalanches

44. Glitch Studies in Superfluid Helium
Rena Zieve (University of California, Davis)

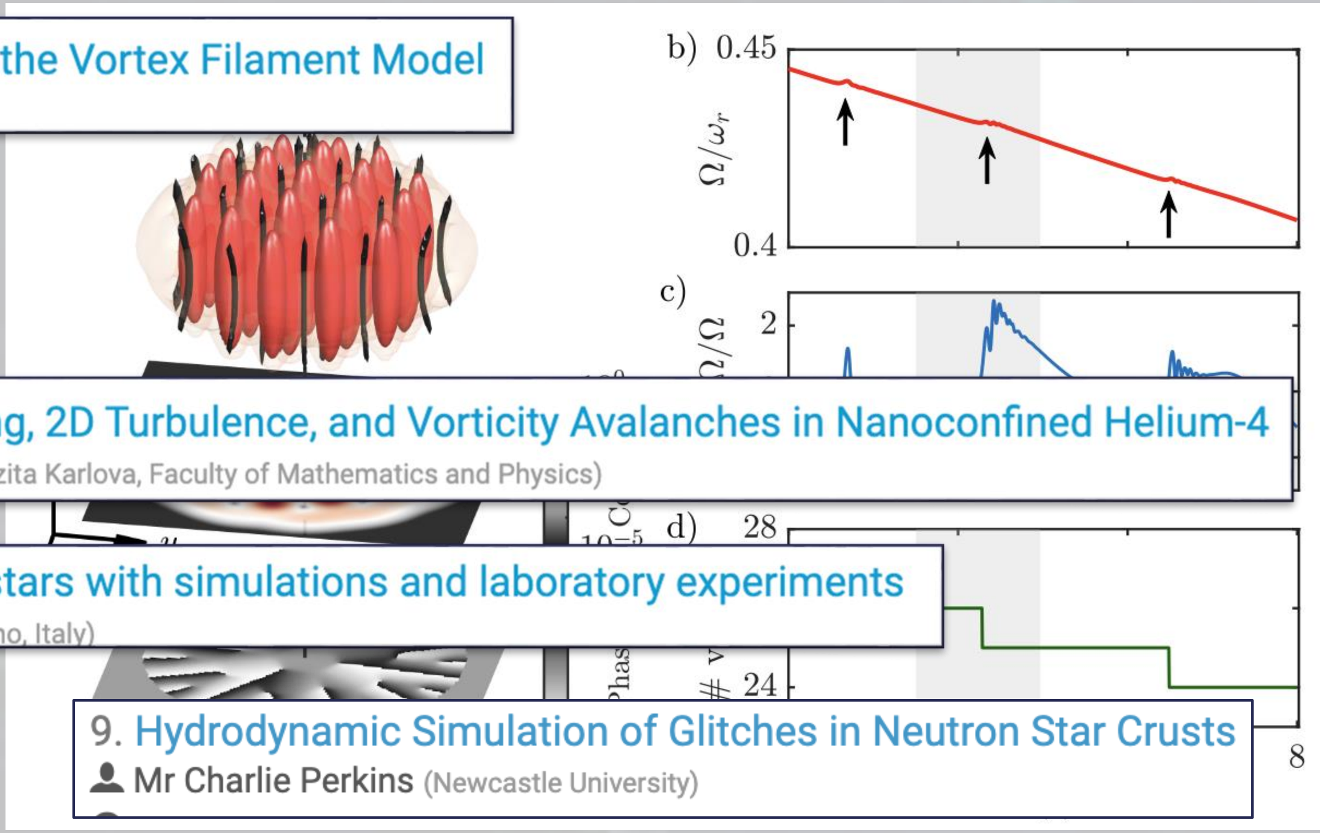
31. Vortices and glitches in rotating dipolar supersolids
Elena Poli (Pitaevskii BEC Center, CNR-INO, Trento, Italy)

36. Neutron star glitches in dipolar atoms: towards a layered structure
Héctor Briongos-Merino (University of Barcelona, ICCUB)



14. Modelling Vortex Avalanches in Neutron Stars using the Vortex Filament Model
Julie Jacob Thomas (Newcastle University)

34. Vortex Avalanches and Pinning Scales
Ali Alpar (Sabanci University)



18. Vortex Pinning, 2D Turbulence, and Vorticity Avalanches in Nanoconfined Helium-4
Emil Varga (Univerzita Karlova, Faculty of Mathematics and Physics)

28. Investigating pinning in neutron stars with simulations and laboratory experiments
Brynmor Haskell (Universita' degli Studi di Milano, Italy)

9. Hydrodynamic Simulation of Glitches in Neutron Star Crusts
Mr Charlie Perkins (Newcastle University)

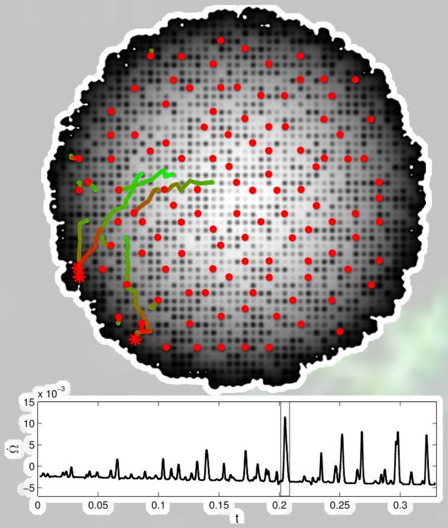
Superfluid Modelling

Sub-Mesoscopic

$$\xi \approx [10{,}100]\ \mathrm{fm}$$

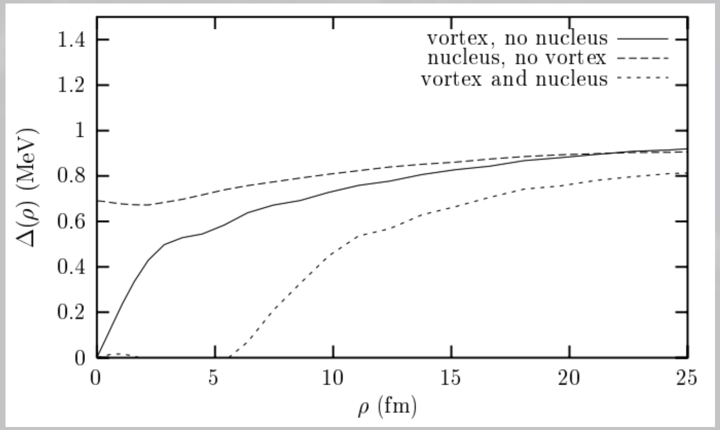
Gross-Pitaevskii Model

Warszawski & Melatos, MNRAS 415, 1611 (2011)



HFB/BdG Models

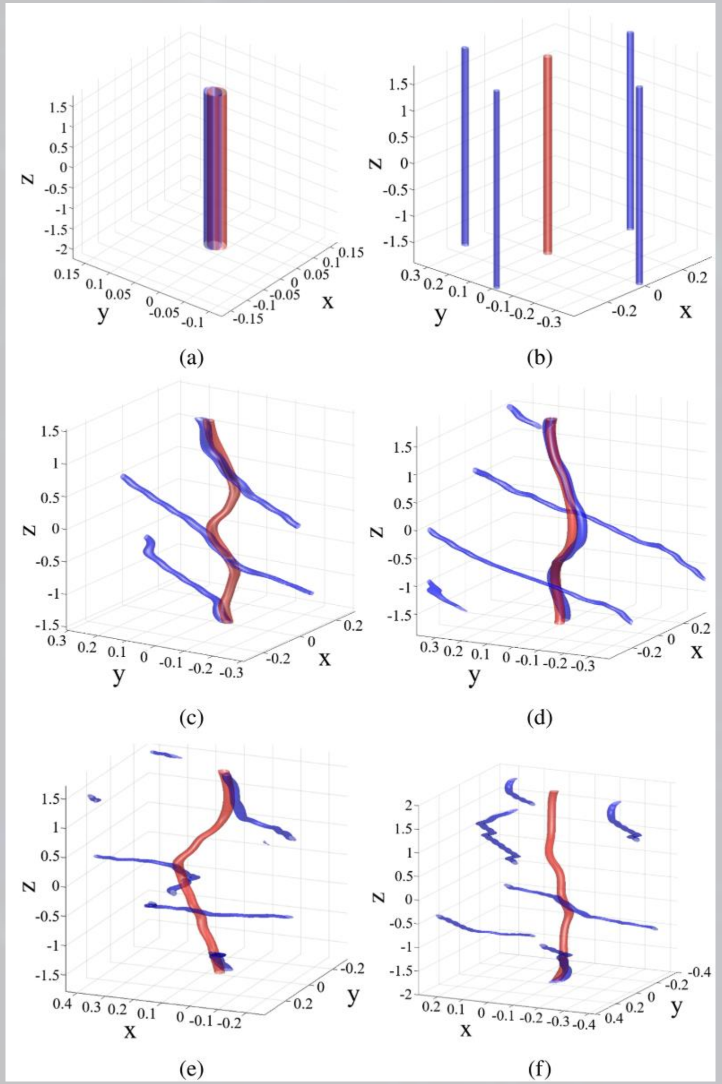
Ø. Elgarøy et al., AA 370, 939 (2001)



Ginzburg-Landau Model

Wood & Graber, Universe 8, 228 (2022)

Melatos' group, MNRAS (2017, 2018, 2024)



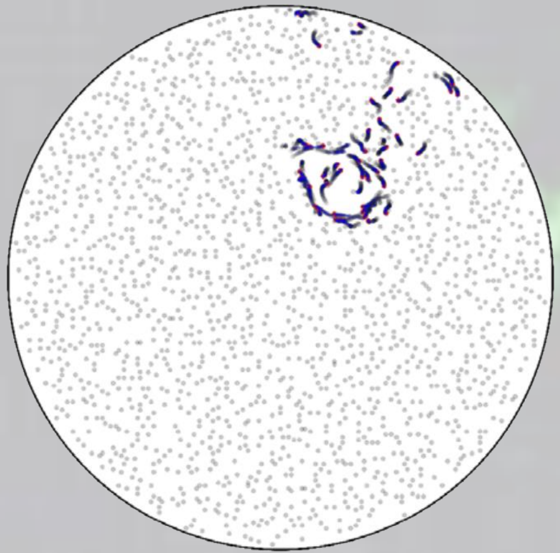
Superfluid Modelling

Mesoscopic

$$\xi \approx [10{,}100]\ \mathrm{fm} \ll R$$

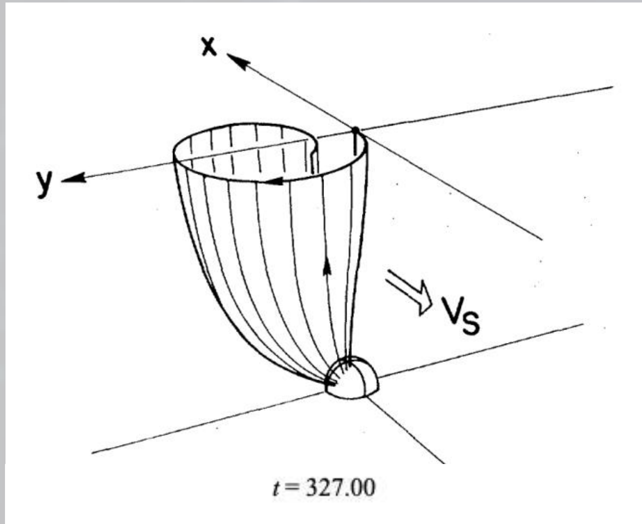
Point Vortex Model

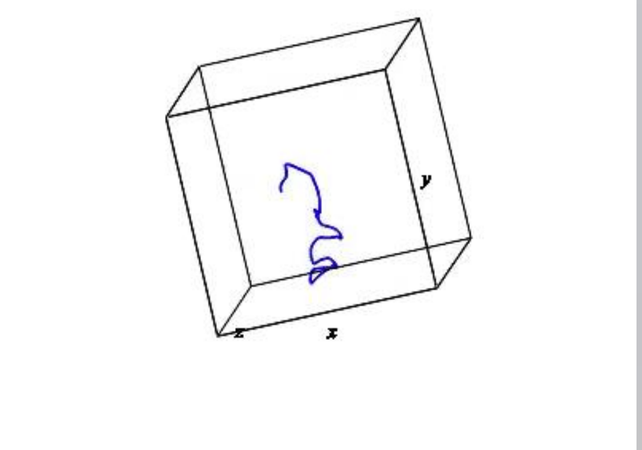
Howitt et al., MNRAS 498, 320 (2020)
S. V. Anantharaman & D. Bhattacharya, MRNAS 539, 3645 (2025)



Vortex Filament Method

Schwarz, PRL 47, 251 (1987)
Link & Levin, ApJ 941, 148 (2022)





Superfluid Modelling

Macroscopic

HBVK Model

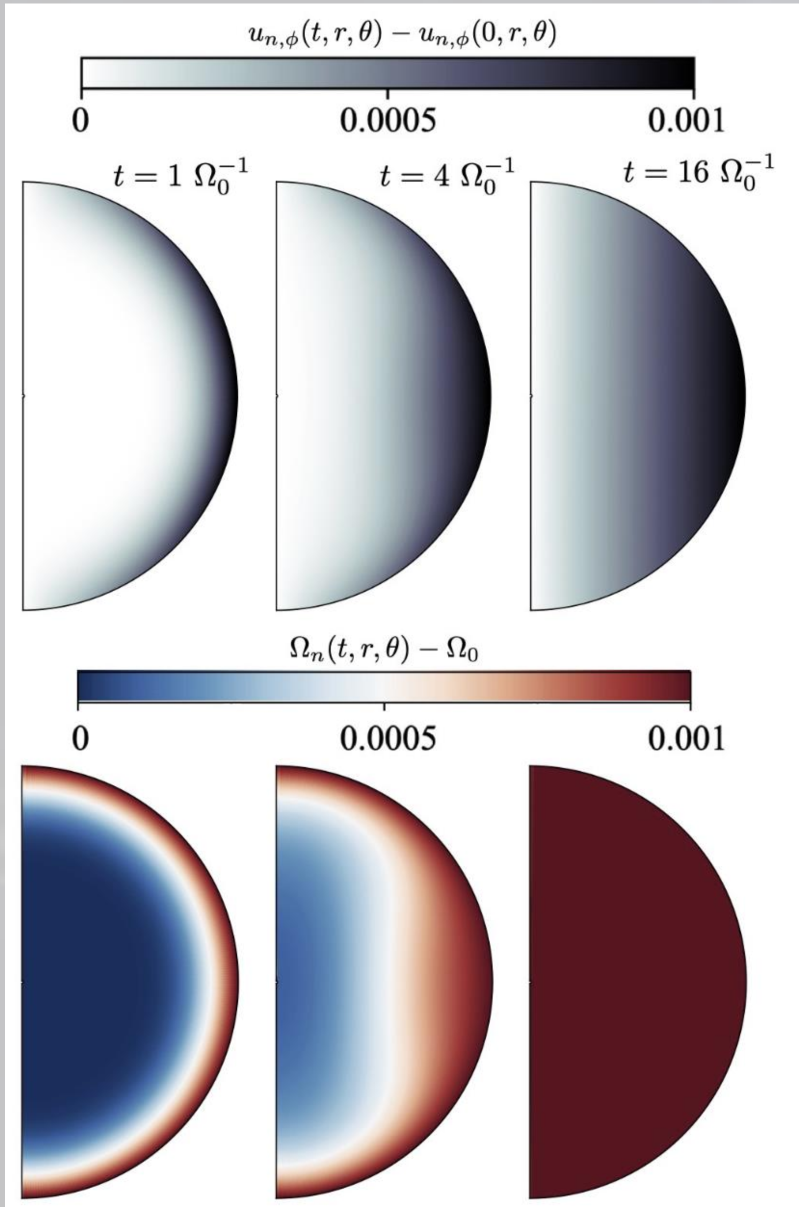
Fuentes & Graber, ApJ 974, 300 (2024)

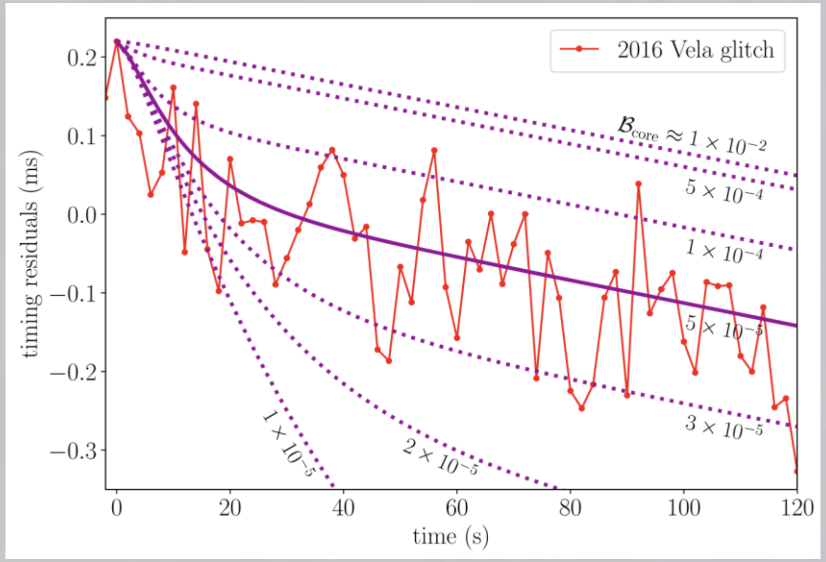
Multi-component Models

Graber+, ApJ 865, 23 (2018)

Pizzochero+, A&A 636, A101 (2020)

Tu & Li, arXiv:2510.20791





$$\dot{\Omega}_{\rm crust} = -\frac{N_{\rm ext}}{I_{\rm crust}} - \frac{I_{\rm sf}}{I_{\rm crust}}\dot{\Omega}_{\rm sf} - \frac{I_{\rm core}}{I_{\rm crust}}\dot{\Omega}_{\rm core}$$

$$\dot{\Omega}_{\rm sf} = 2B_{\rm sf}\Omega_{\rm sf}(\Omega_{\rm sf} - \Omega_{\rm crust})$$

$$\dot{\Omega}_{\rm core} = 2B_{\rm core}\Omega_{\rm core}(\Omega_{\rm core} - \Omega_{\rm crust})$$

Toward Larger Scales

$10^7 - 10^{18}$ Vortices for a glitch

Sub-Mesoscopic

Gross-Pitaevskii Model

Warszawski & Melatos, MNRAS 415, 1611 (2011)

BdG Model

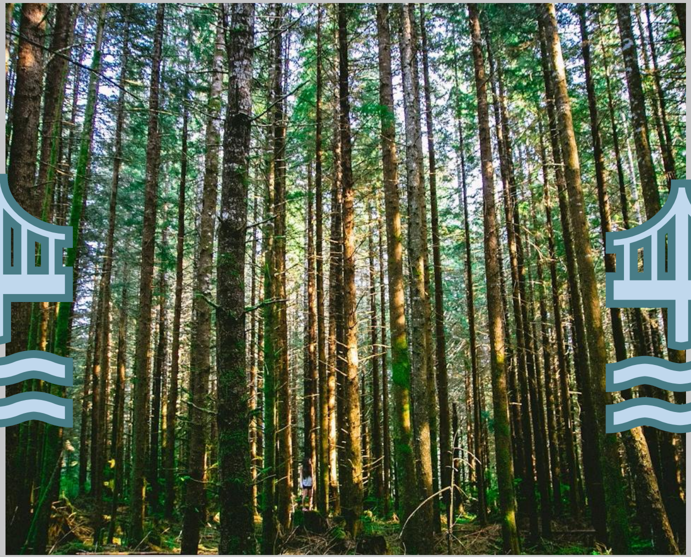
Avogadro et al., PRC 75, 912795(R) (2007)

Ø. Elgarøy et al., AA 370, 939 (2001)

Ginzburg-Landau Model

Wood & Graber, Universe 8, 228 (2022)

Mesoscopic



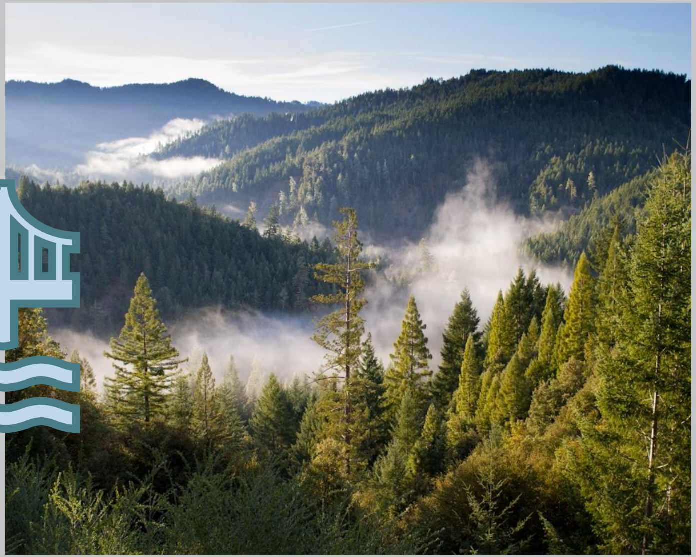
Point Vortex Model

Howitt et al., MNRAS 498, 320 (2020)

Vortex Filament Method

Schwarz, PRL 47, 251 (1987)

Macroscopic



HBVK Model

Fuentes & Graber, ApJ 974, 300 (2024)

Multi-component Models

Graber+, ApJ 865, 23 (2018)

Pizzochero+, A&A 636, A101 (2020)

Tu & Li, arXiv:2510.20791

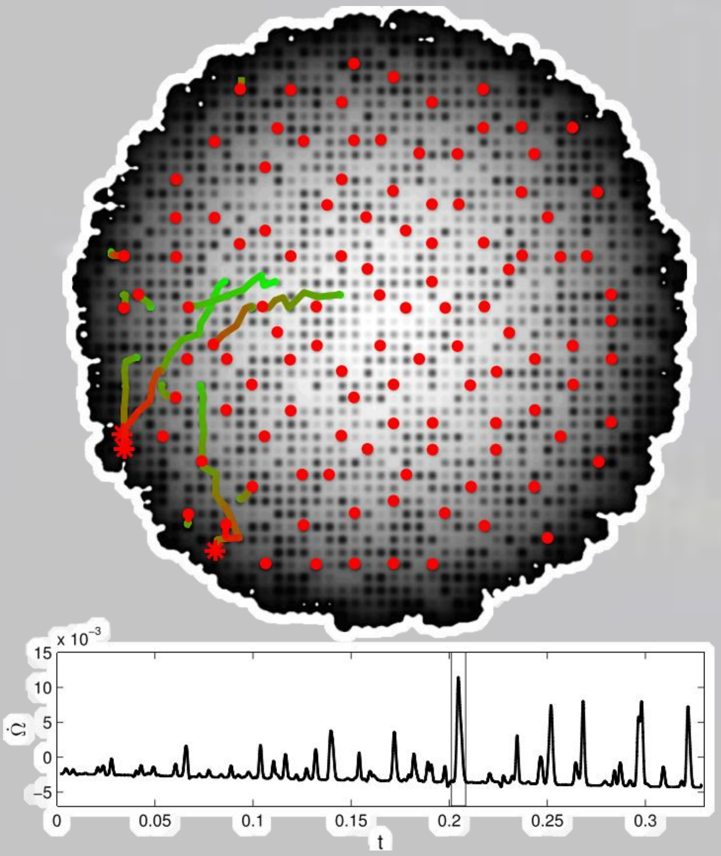
Simulation-Based Inference

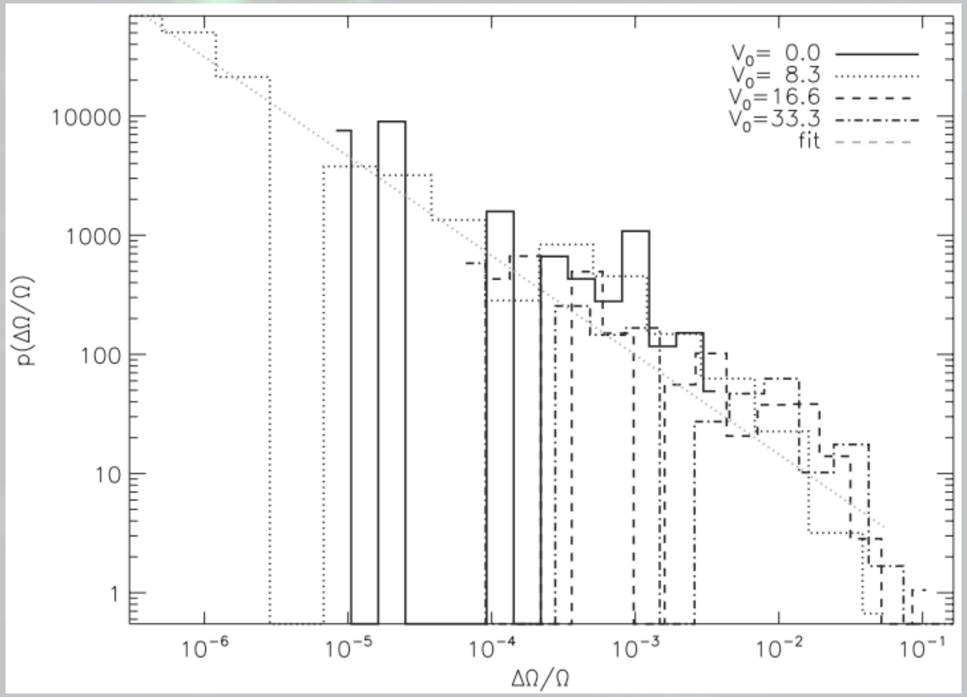
Glitches and Vortex Pinning and Avalanches

- Dissipative Gross-Pitaevskii model (dGPE)

$$i\hbar\partial_t\psi(\mathbf{r},t) = (1-i\gamma)\left[-\frac{\hbar^2\nabla^2}{2m} + V(\mathbf{r},t) + g|\psi(\mathbf{r},t)|^2 - \Omega(t)\hat{L}_z - \mu\right]\psi(\mathbf{r},t)$$

$$I_c\frac{\mathrm{d}\Omega}{\mathrm{dt}} = -\frac{\mathrm{d}\langle\hat{L}_z\rangle}{\mathrm{dt}} + N_{\mathrm{EM}}$$





$$\left(\frac{\Delta\Omega}{\Omega}\right)^{-a}$$

$a \in (0.8, 1.2)$

Warszawski & Melatos, MNRS 415, 1611 (2011)
Melatos, Douglas & Simula, Ap 807, 132 (2015)

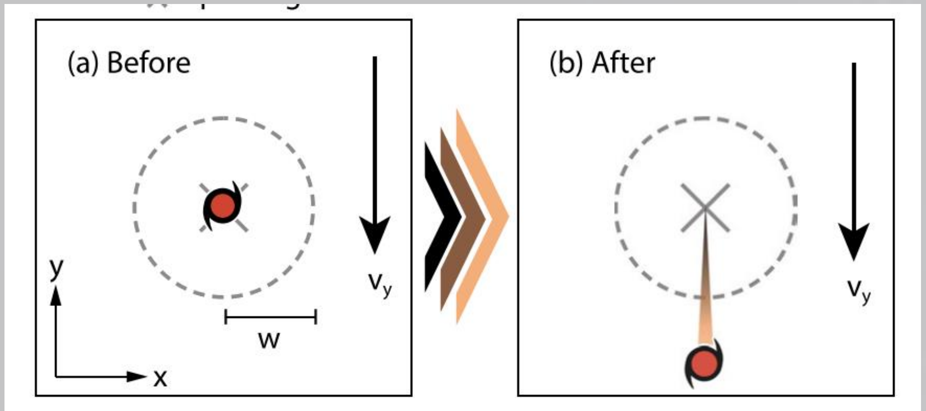
Glitches and Vortex Pinning and Avalanches

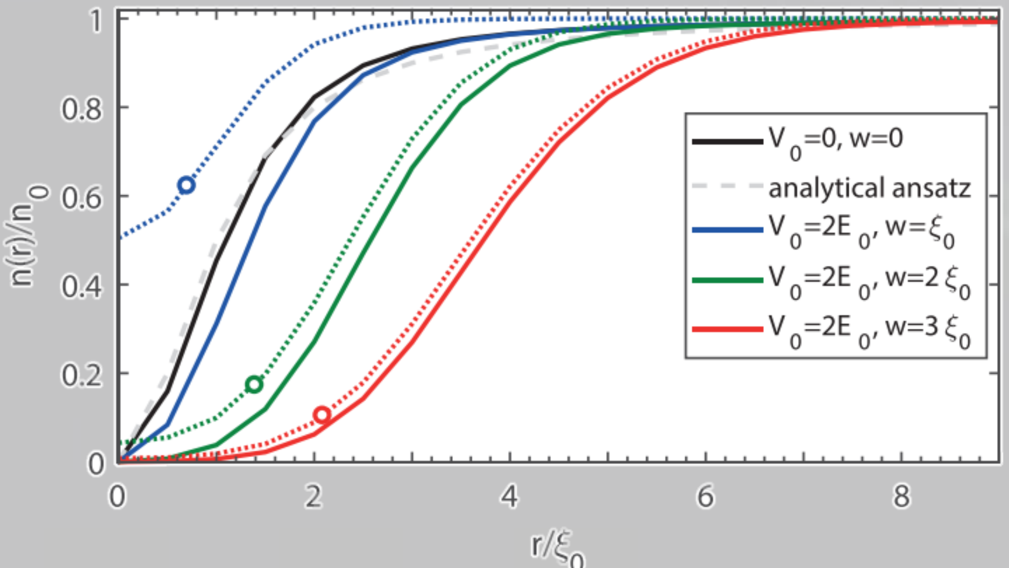
- 10. Bouquets of Topological Defects and Pinning in Neutron Stars
  Marco ANTONELLI (CNRS - LPC Caen)

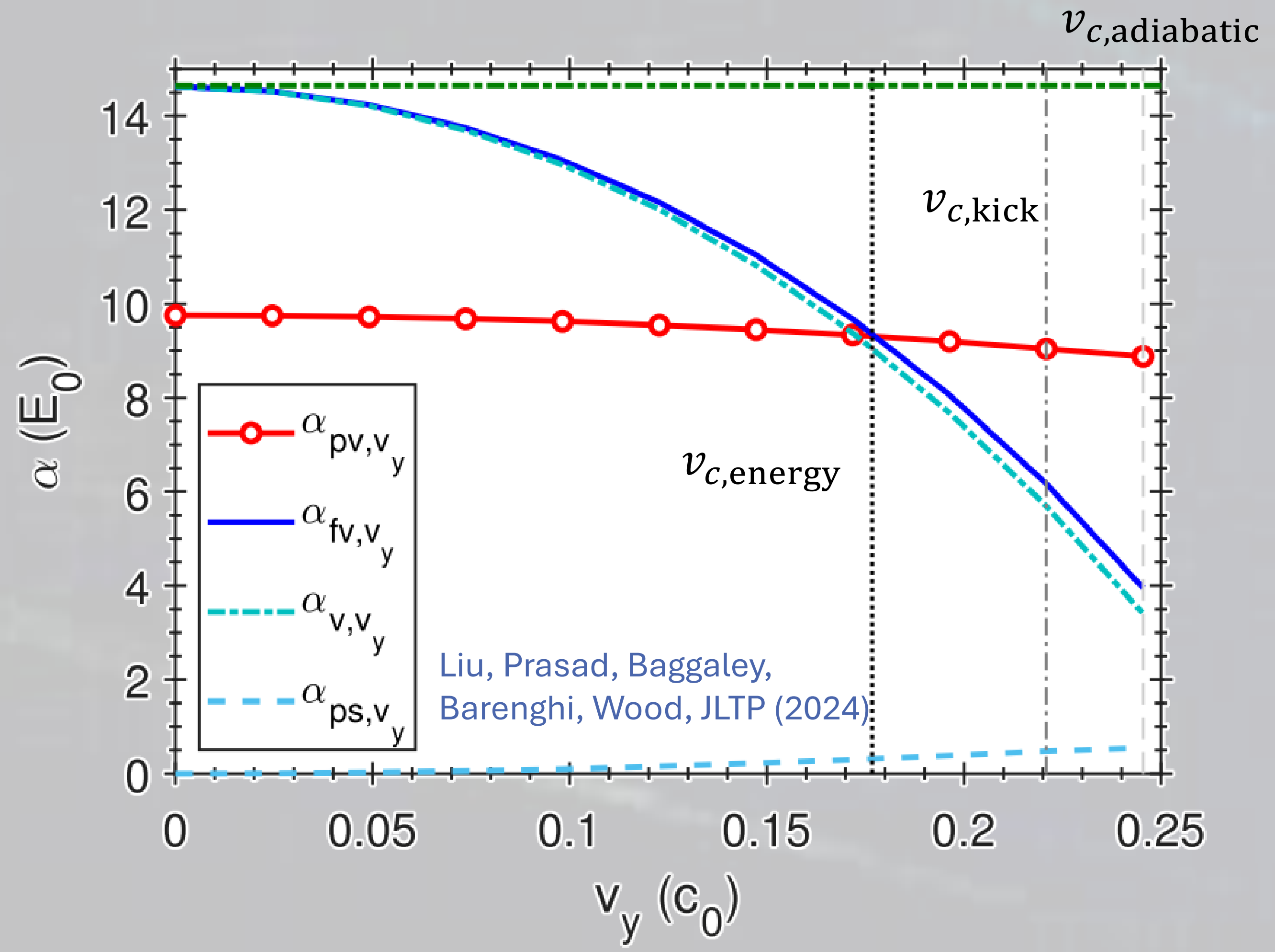
34. Vortex Avalanches and Pinning Scales
Ali Alpar (Sabanci University)

38. Vortex-nucleus interaction in the inner crust of neutron star
Daniel Pęcak (Institute of Physics, Polish Academy of Sciences)

29. Vortex pinning in a spin-triplet superfluid
Riku Rantanen (Aalto University)



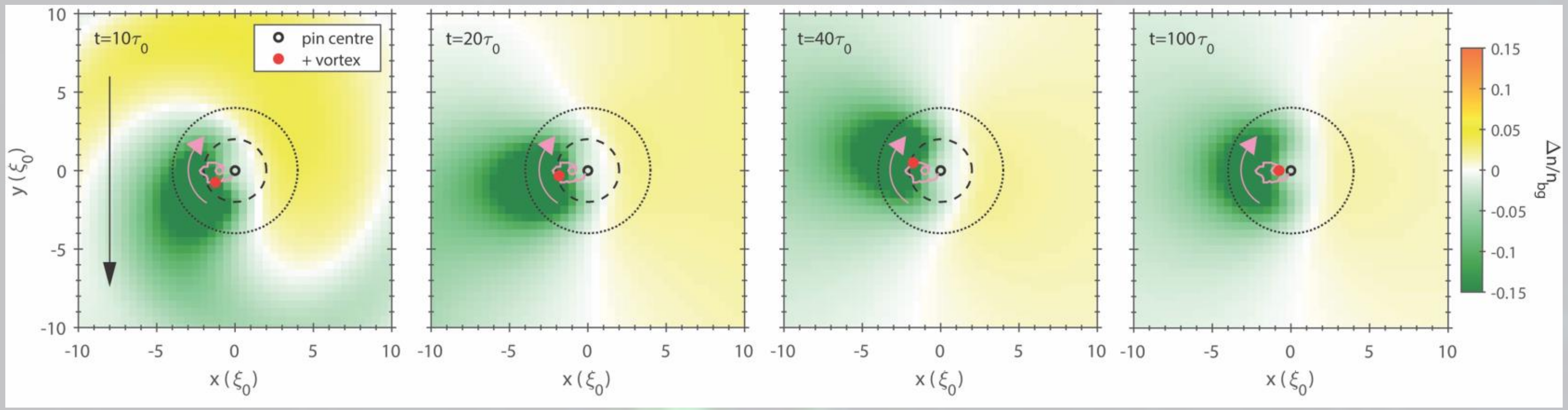




Glitches and Vortex Pinning and Avalanches

$v_y < v_c$

Micro vortex movement
And re-equilibration



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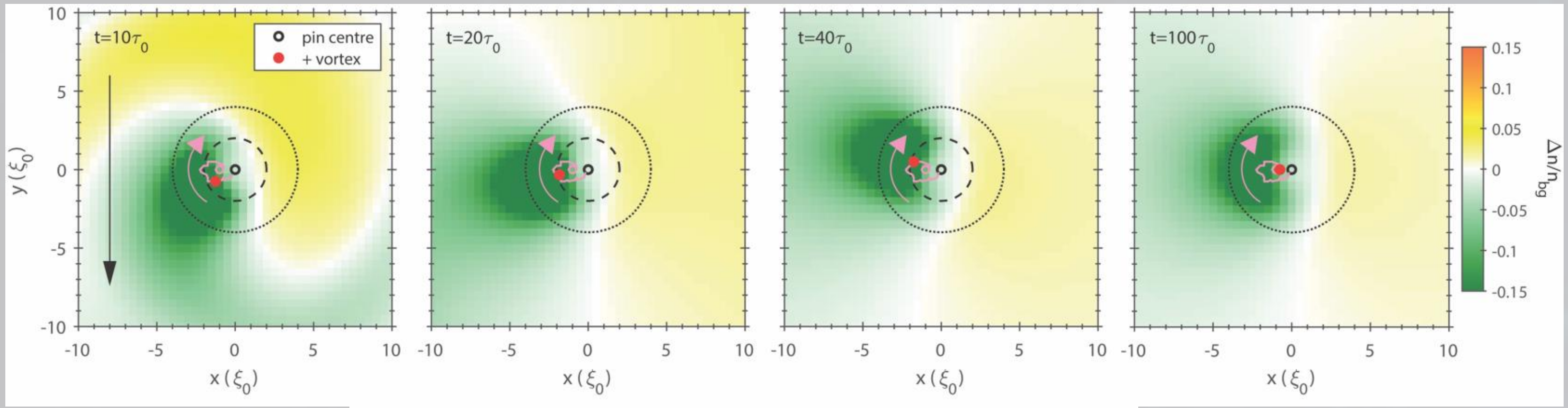
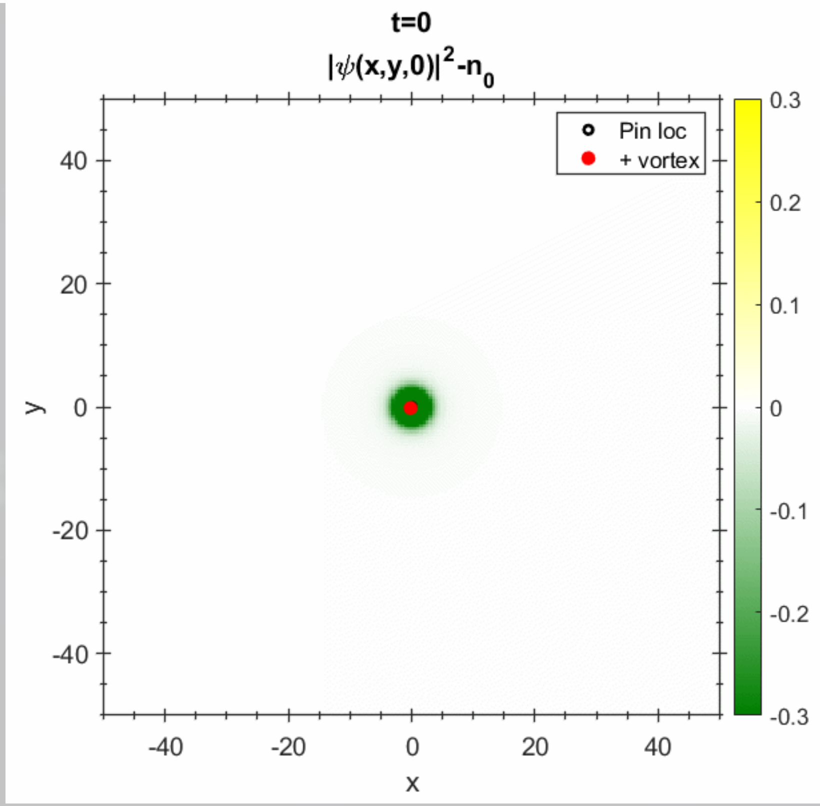


$v_y > v_c$
Vortex depinning



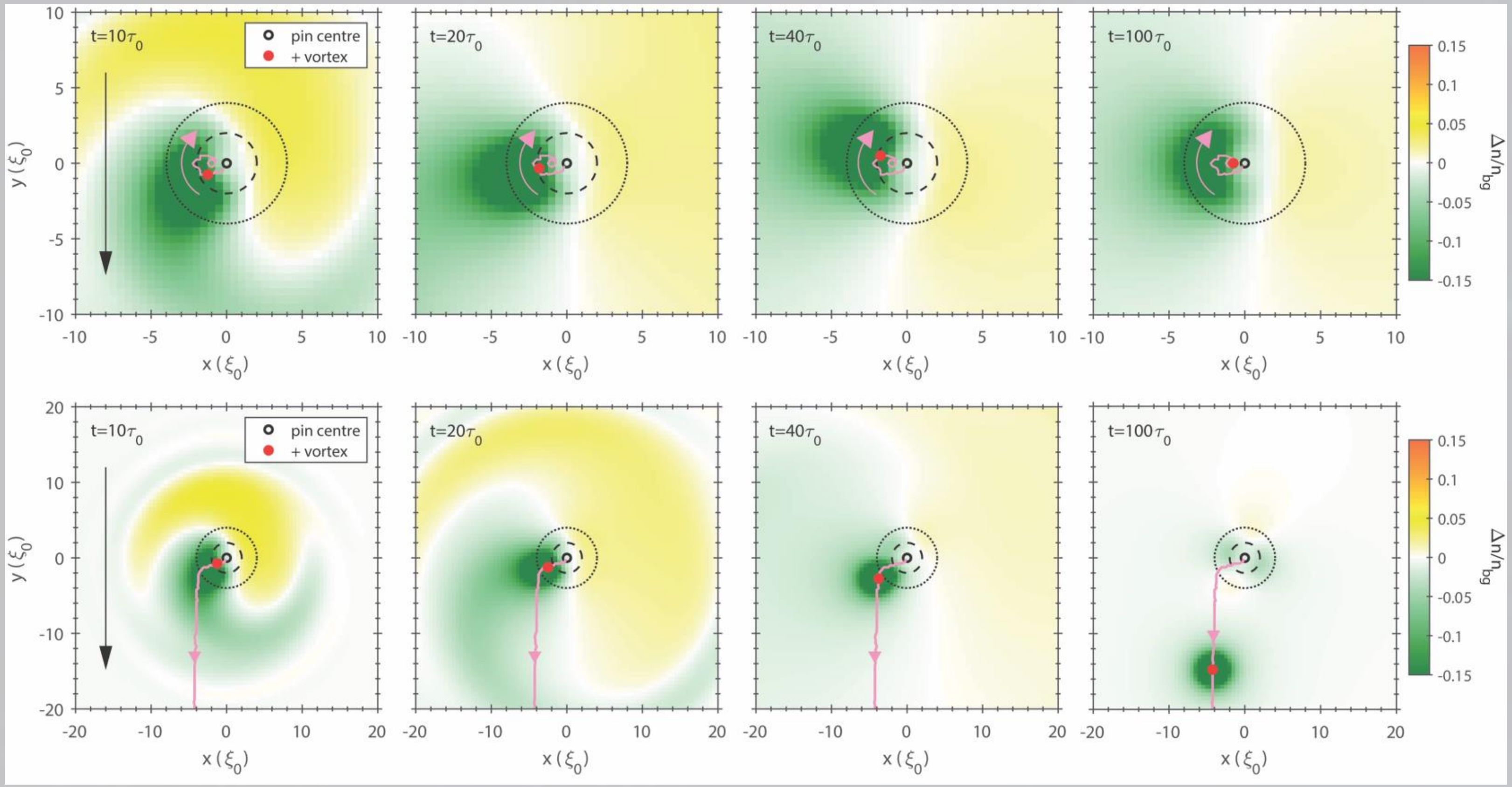
Glitches and Vortex Pinning and Avalanches

$v_y < v_c$

Micro vortex movement
And re-equilibration

$v_y > v_c$

Vortex depinning



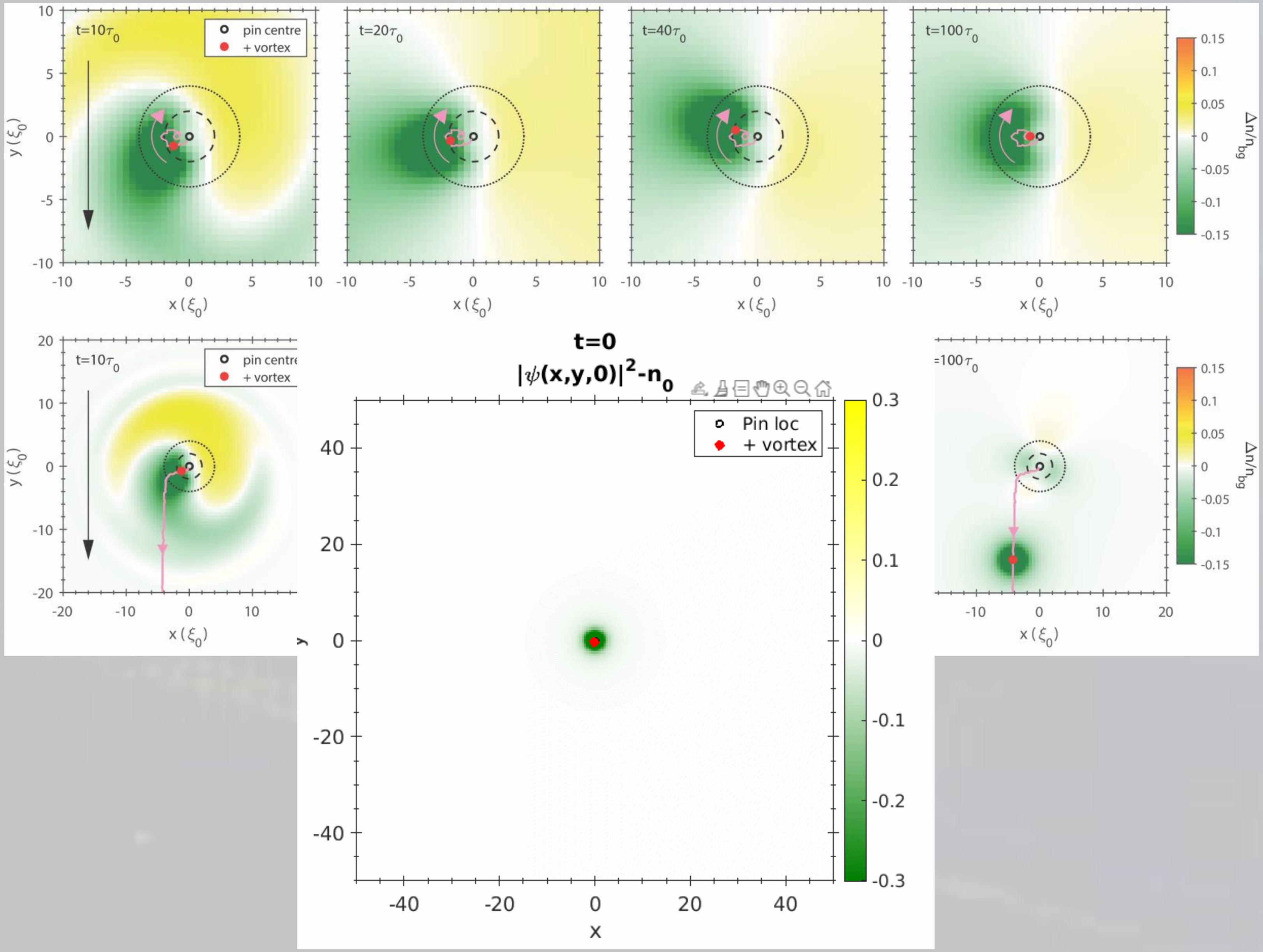
$v_y \gg v_c$ / Strong Pinning
Vortex Pair Nucleation

Glitches and Vortex Pinning and Avalanches

$v_y < v_c$
Micro vortex movement
And re-equilibration

$v_y > v_c$
Vortex depinning

$v_y \gg v_c$ / Strong Pinning
Vortex Pair Nucleation

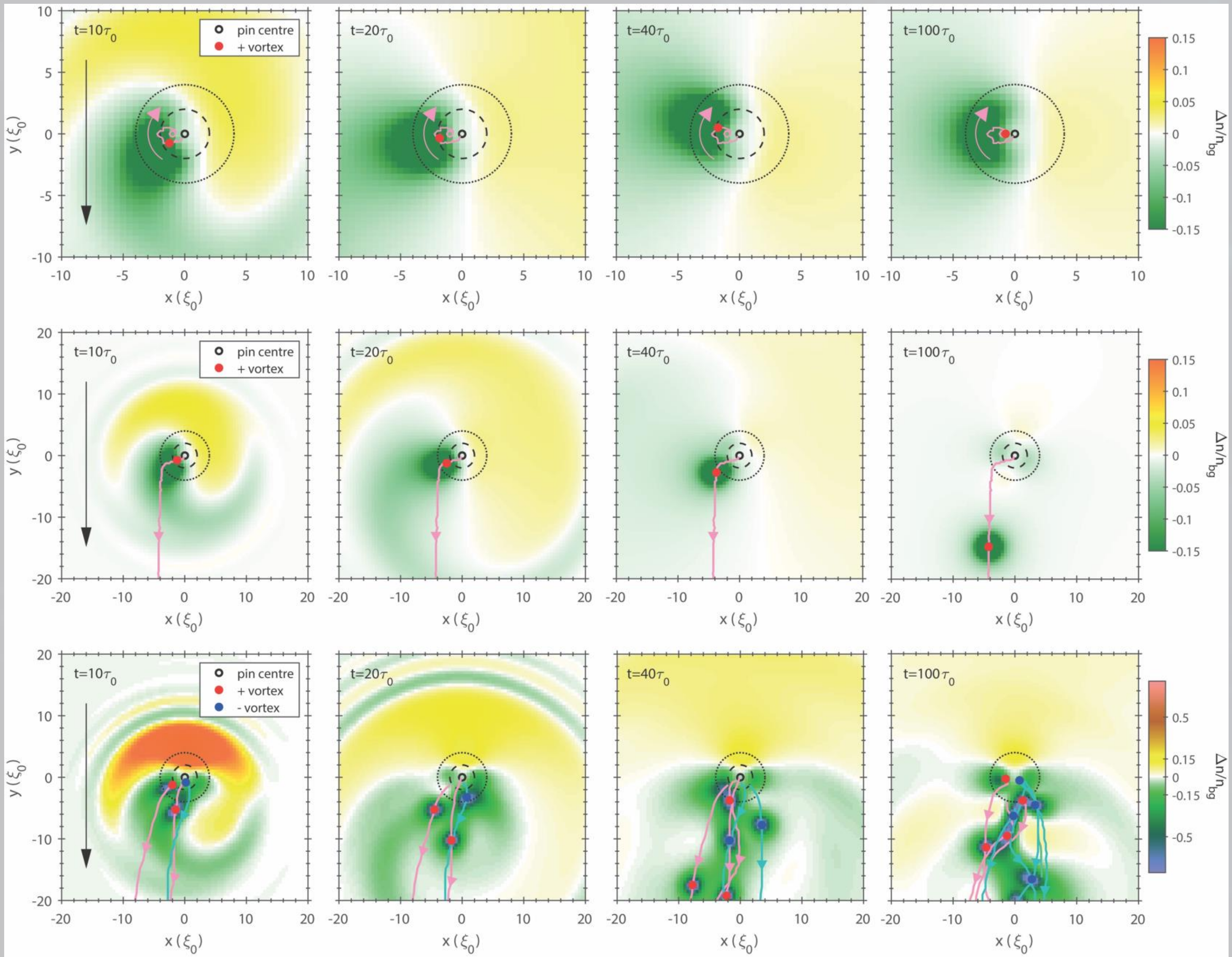


Glitches and Vortex Pinning and Avalanches

$v_y < v_c$
Micro vortex movement
And re-equilibration

$v_y > v_c$
Vortex depinning

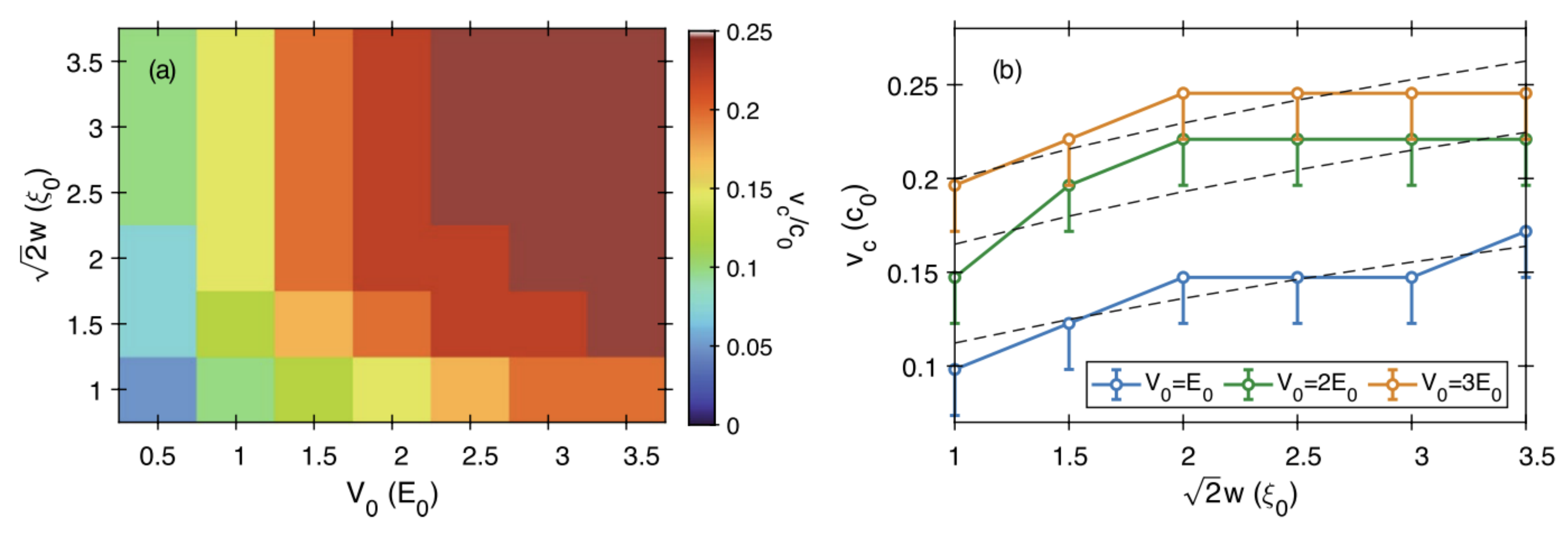
$v_y \gg v_c$ / Strong Pinning
Vortex Pair Nucleation



Glitches and Vortex Pinning and Avalanches

- Critical Vortex Depinning Velocity

Liu, Prasad, Baggaley, Barenghi, Wood, JLTP (2024)



$$\frac{v_c}{c_0} \approx a \ln\left[1 + \frac{V_p}{\mu}\left(\frac{b w_p}{\xi_0} + c\right)\right]$$

$$a \approx 0.1, b \approx 1.1, c \approx 1.2$$

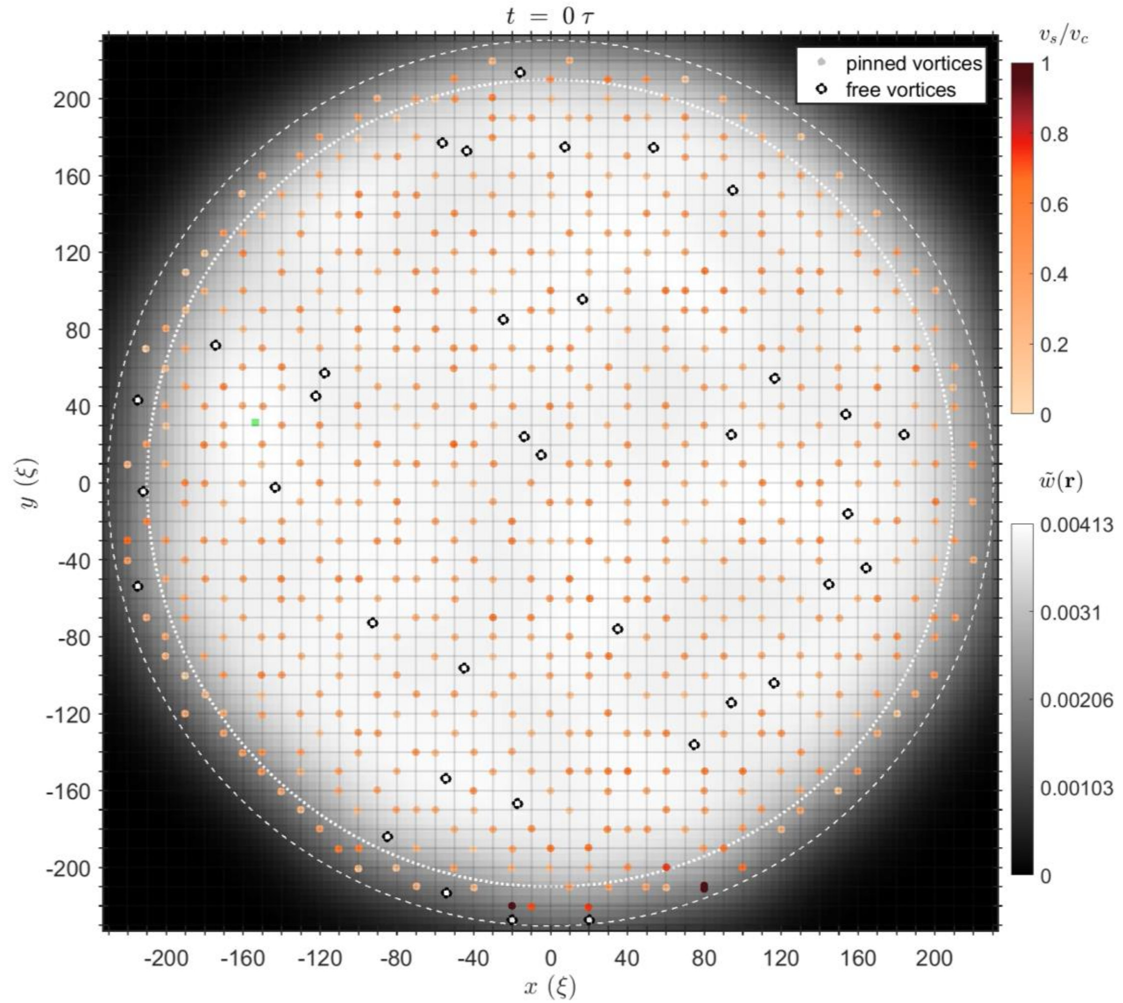
(20% higher is possible according the adiabatic probe)

Superfluid Modelling

- Gross-Pitaevskii model

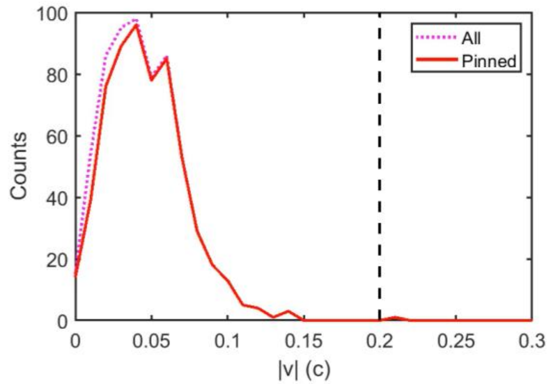
Slowly linearly decreasing

$$i\hbar\partial_t\psi(\mathbf{r},t) = (1-i\gamma)\left[-\frac{\hbar^2\nabla^2}{2m} + V(\mathbf{r},t) + g|\psi(\mathbf{r},t)|^2 - \Omega(t)\hat{L}_z - \mu\right]\psi(\mathbf{r},t)$$



Pinning vs Magnus forces

$$\mathbf{F}_M = \boldsymbol{\kappa} \times \left(\mathbf{v}_s - \frac{d\mathbf{x}}{dt}\right)$$

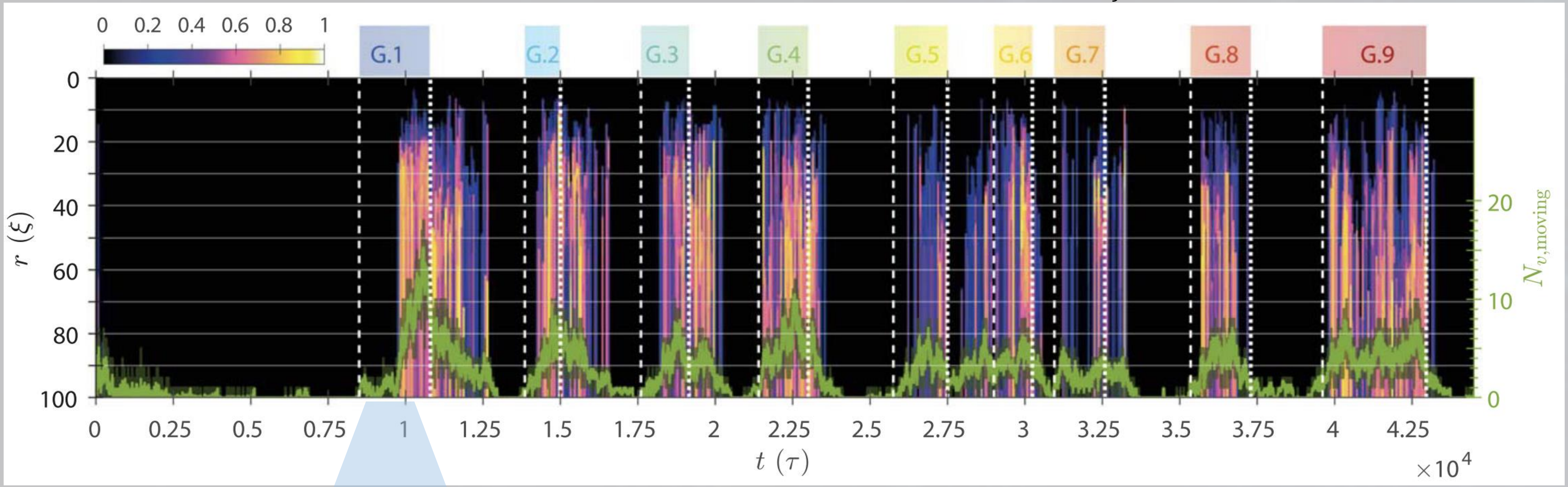


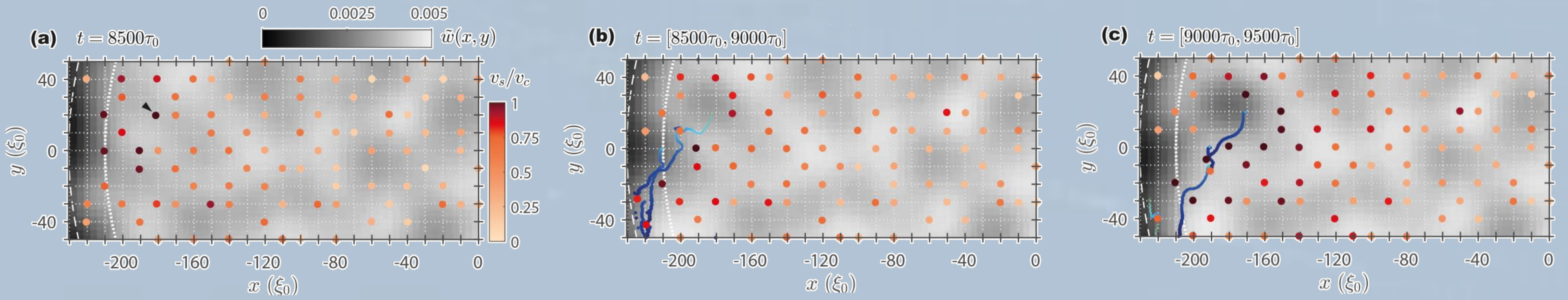
$$\mathbf{v}_{s,j} = \sum_{k\neq j} \frac{\boldsymbol{\kappa}_k \times (\mathbf{x}_j - \mathbf{x}_k)}{|\mathbf{x}_j - \mathbf{x}_k|^2} - \Omega \times \mathbf{x}_j$$

ApJ 984, 83 (2025)

Vortex Collective Motion

$$\varphi(r,t) = \left\langle \frac{1}{\mathcal{N}_{j,r}(t)} \sum_{\{j,k\}_r} \vec{v}_k(t) \right\rangle_j$$





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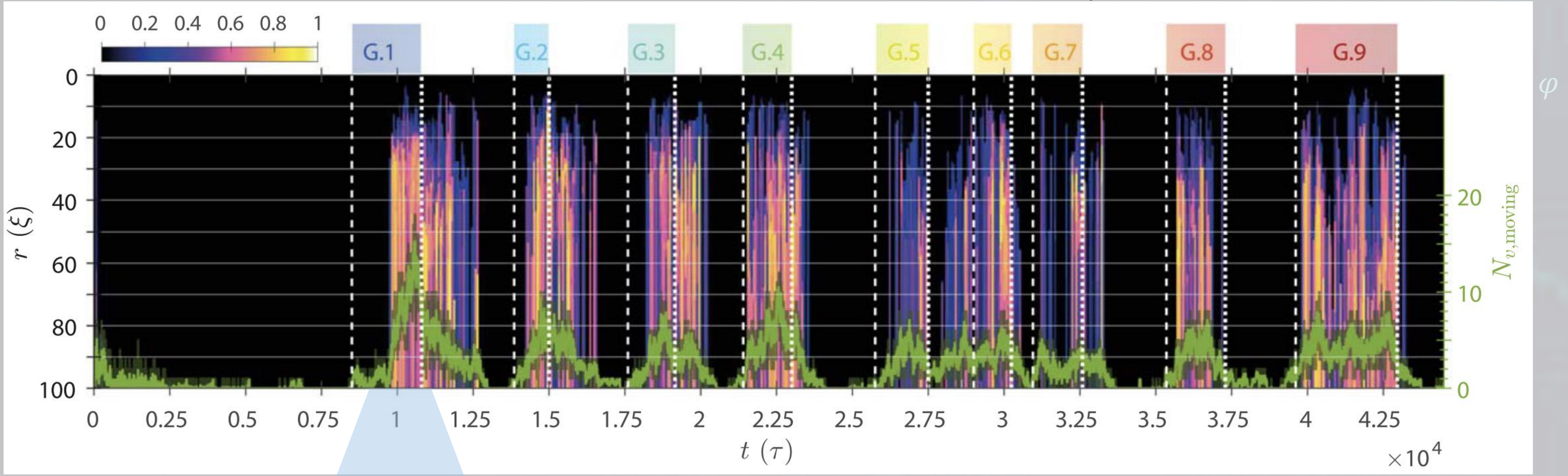

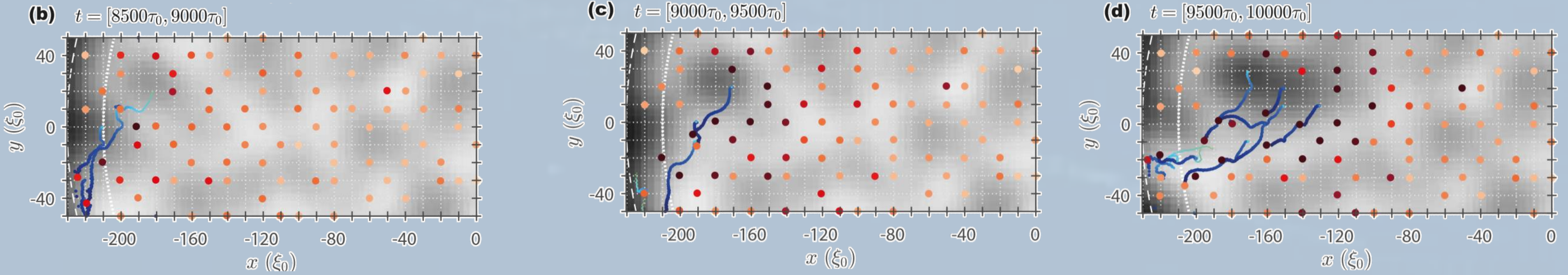

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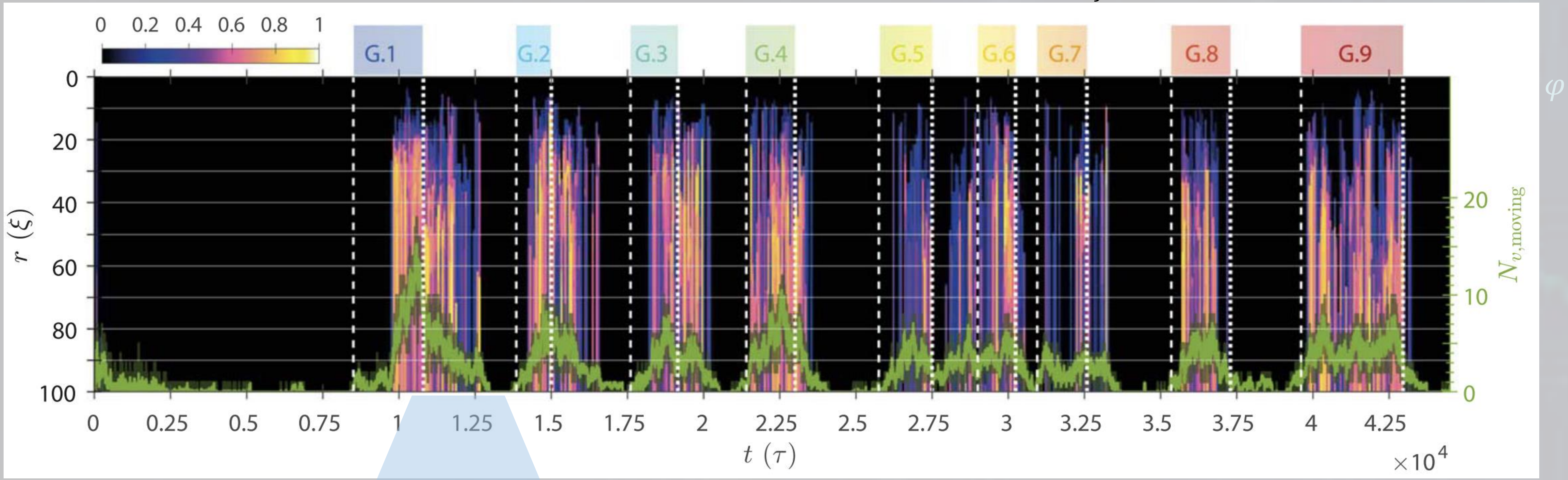
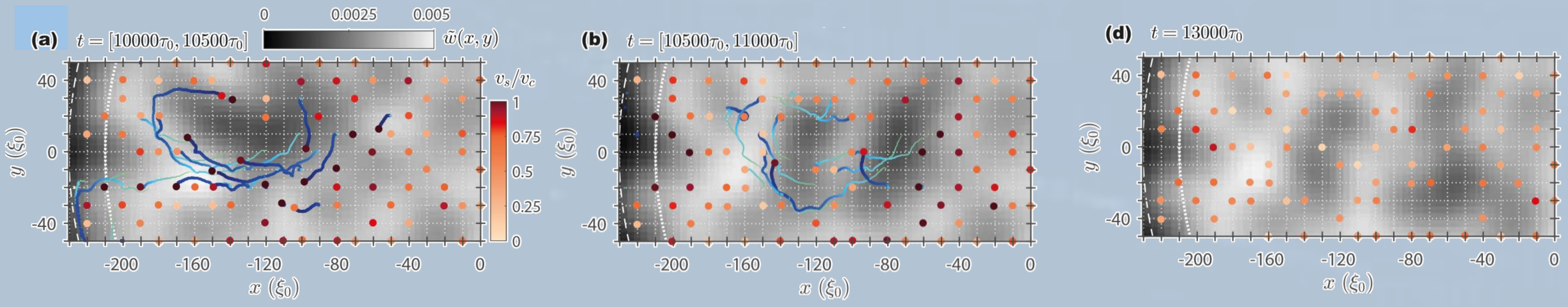
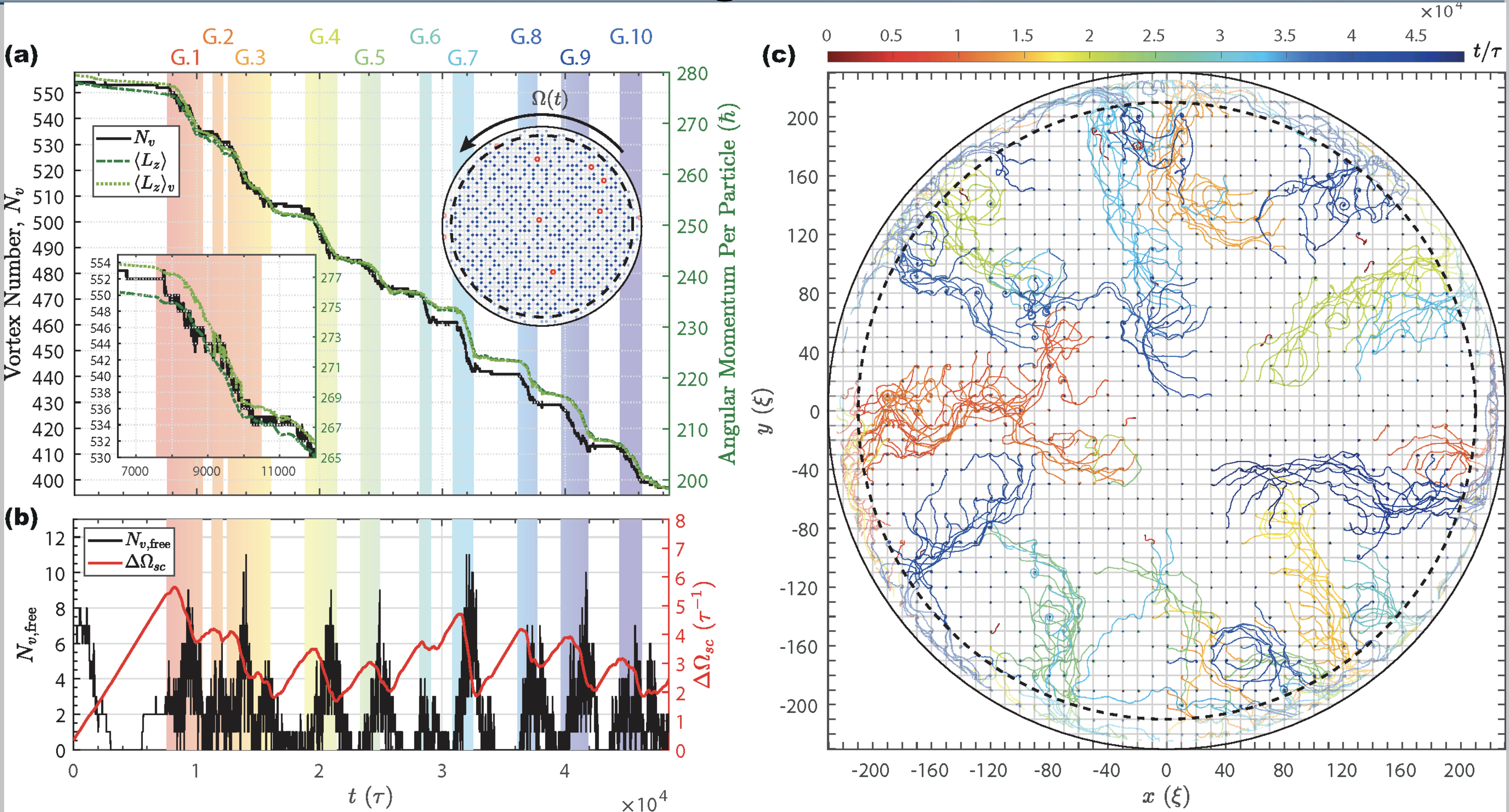


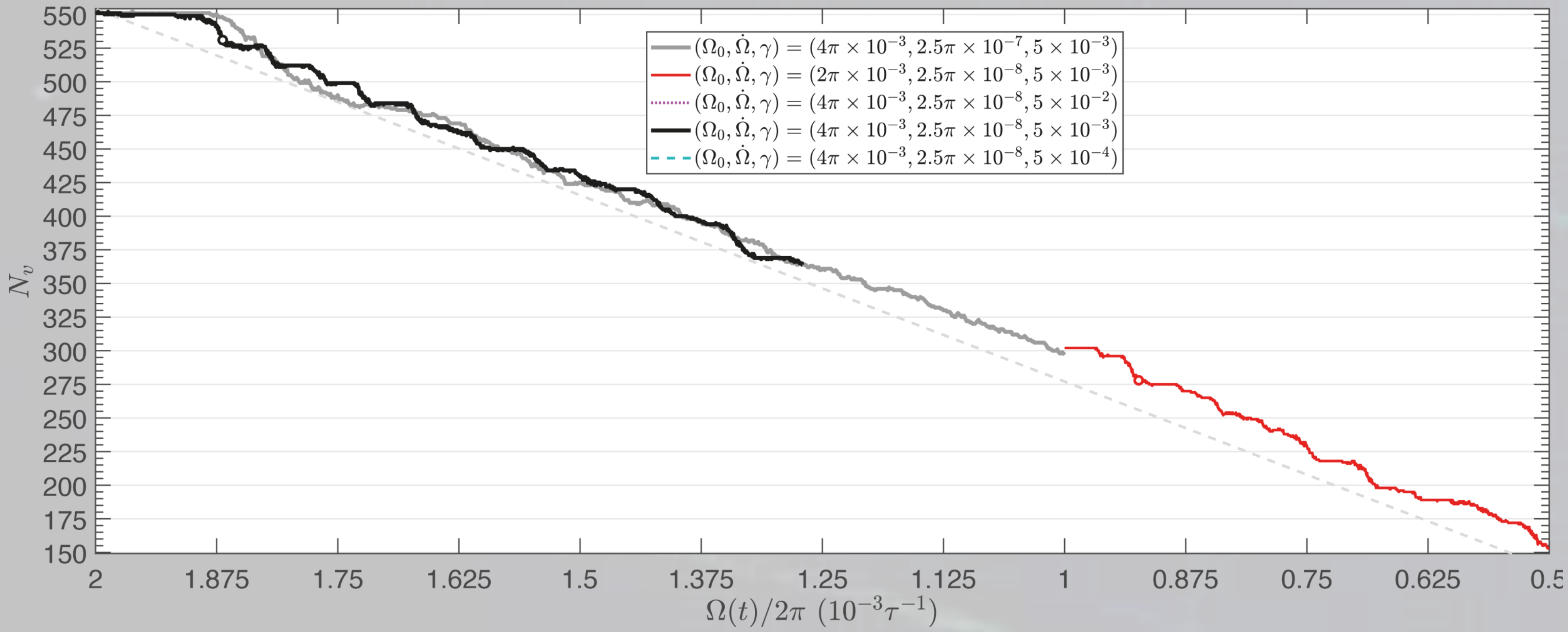


Glitches and Vortex Pinning and Avalanches



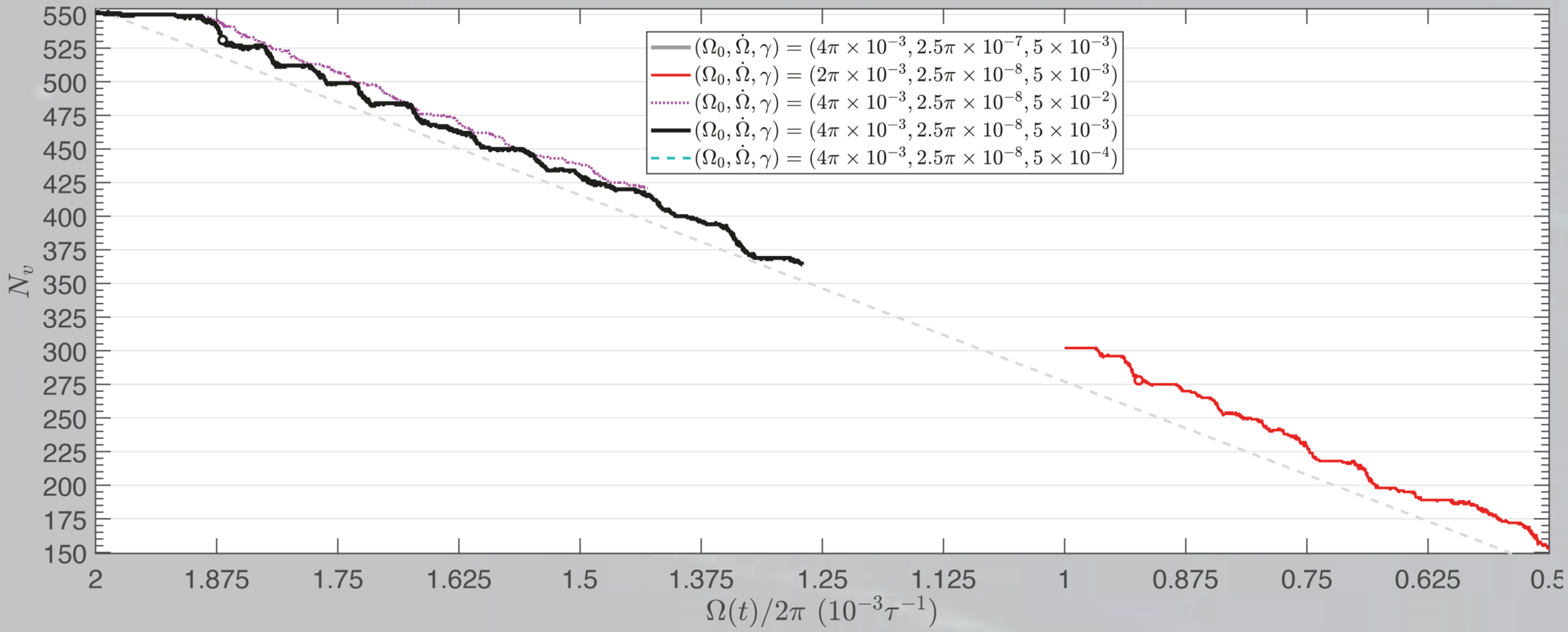
on

Parameter sensitivity



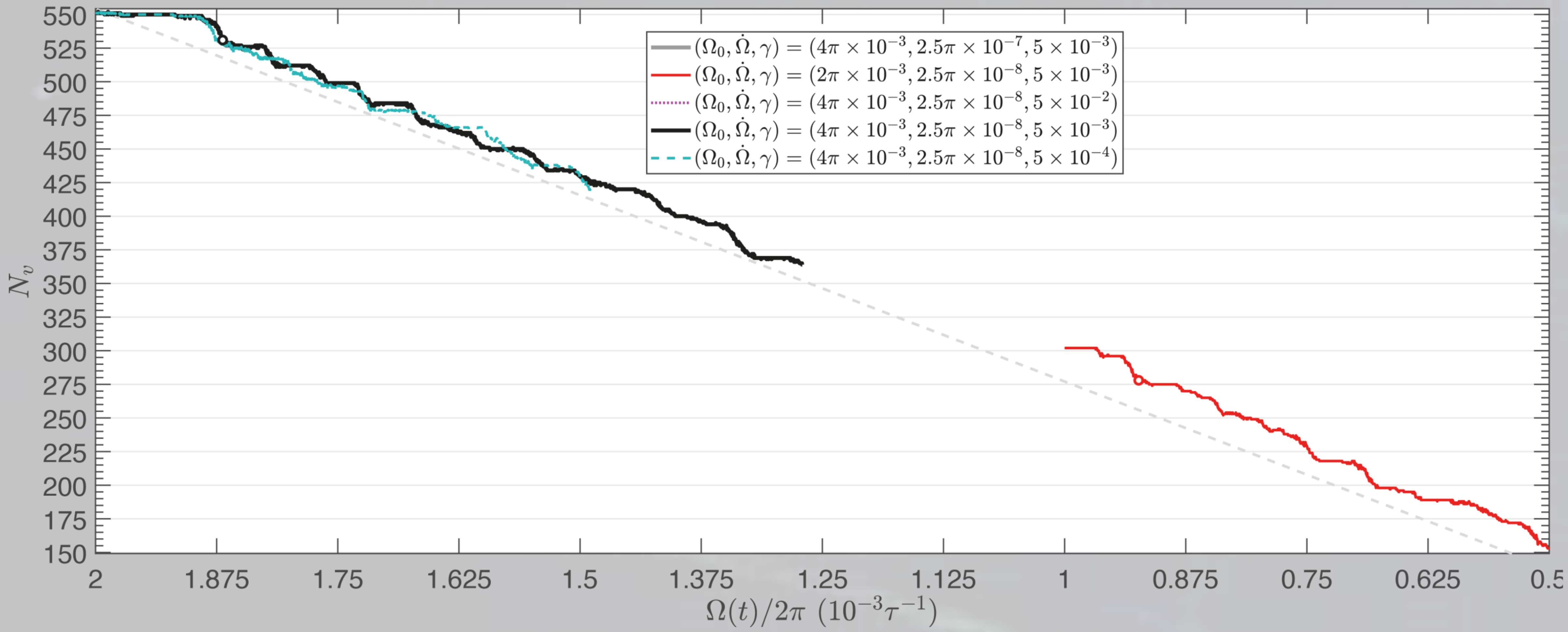
Slower spinning down rate to create step-like drops in vortex number

Parameter sensitivity



Glitches prefer lower dissipation

Parameter sensitivity



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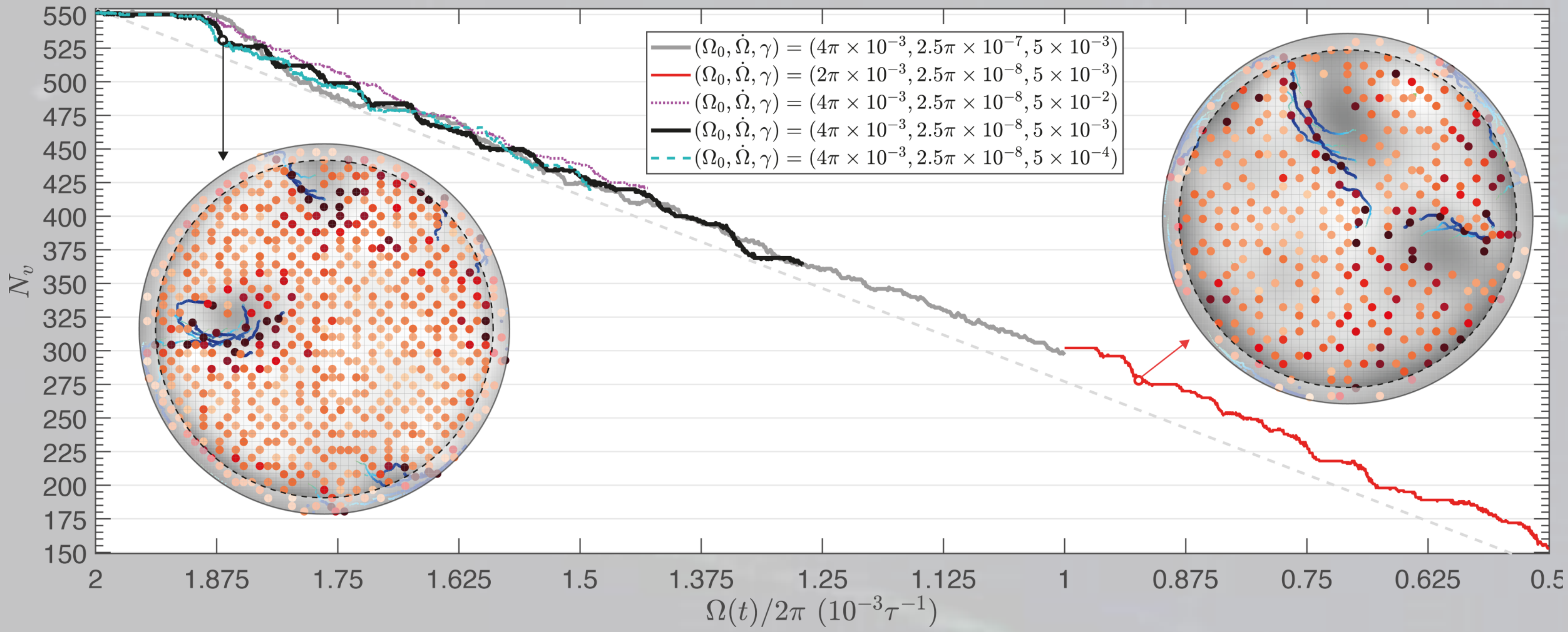
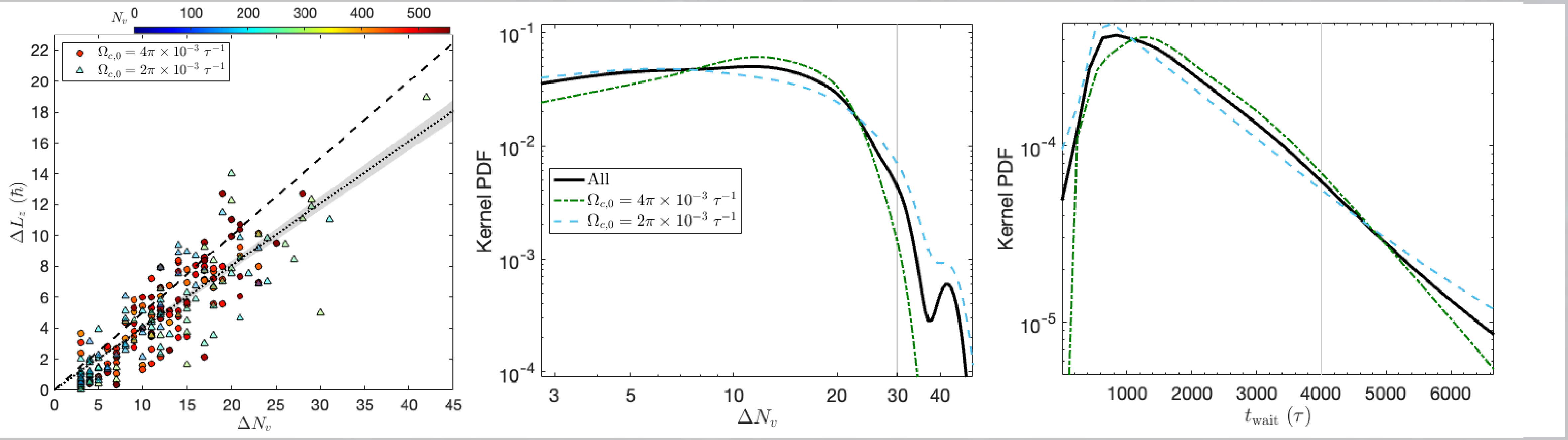


Vortex Avalanche Statistics

- Preliminary Results $\mathrm{N}_{avl} \approx 255$

Liu et al., in preparation



$$L_z = \hbar \sum_j 1 - \frac{r_{v,j}^2}{R^2}$$

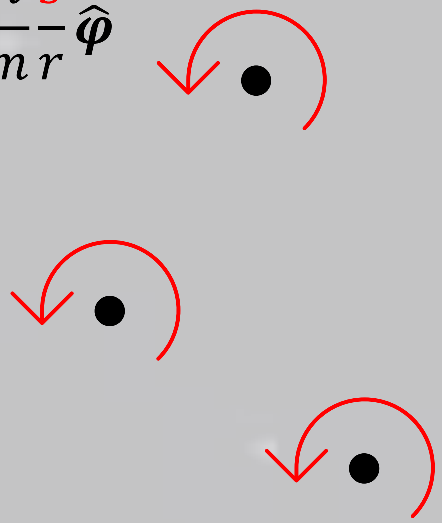
A. L. Fetter, Phys. Rev. 138, 42 (1965)

Better statistic is needed....

Mesoscopic Simulation

- Point-Vortex Model

$$-\frac{\hbar}{m}\frac{s}{r}\hat{\boldsymbol{\varphi}}$$



$$z_k = x_k + i y_k$$

$$v_{s,k} = i\frac{\hbar}{m}\sum_{j\neq k}\frac{s_j(z_k - z_j)}{|z_k - z_j|^2} - i\frac{\hbar}{m}\sum_{j}\frac{s_j(z_k - z_j')}{|z_k - z_j'|^2} + iF_{\mathrm{pin}}(z_k) + i\Omega_c(t) z_k$$

$$F_{\mathrm{pin}}(z) = -\nabla_z V_{\mathrm{pin}}(z)$$

$$U_{\mathrm{pin}}(z) = -V_p \exp\left[-\frac{d_p}{w_p}\left(\sin^2\frac{\pi x}{d_p} + \sin^2\frac{\pi y}{d_p}\right)\right]$$

B. Link and Y. Levin, ApJ 941, 148 (2022)

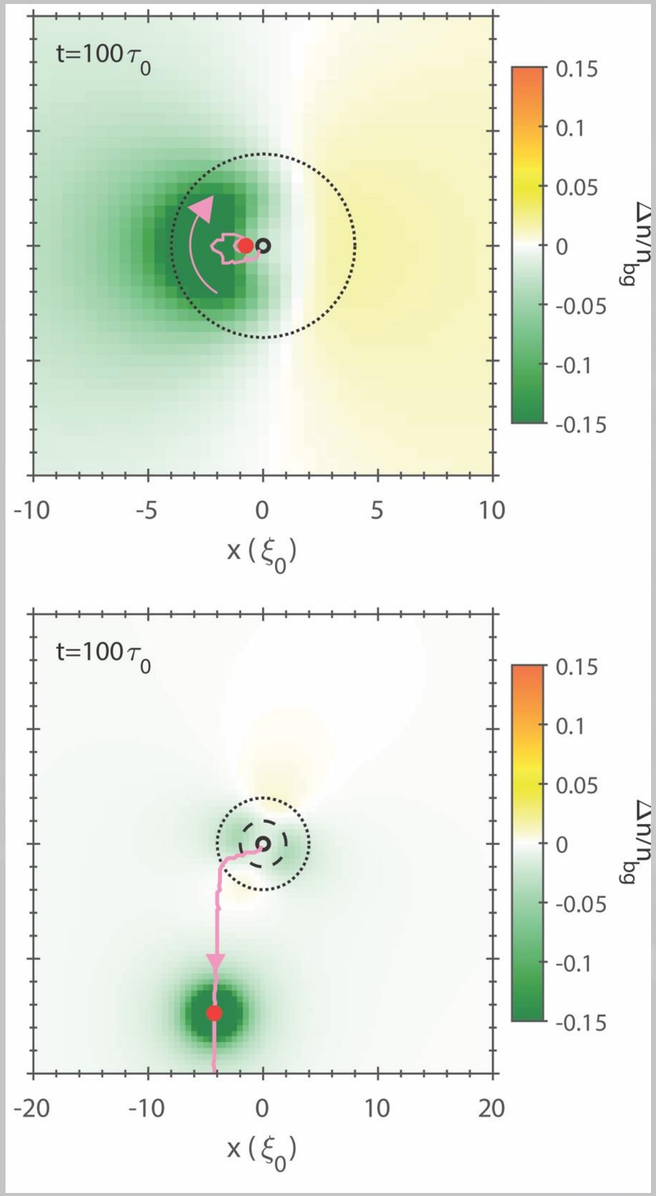
$$\frac{dz_k}{dt} = (1-\alpha')v_{s,k} - i\alpha v_{s,k}$$

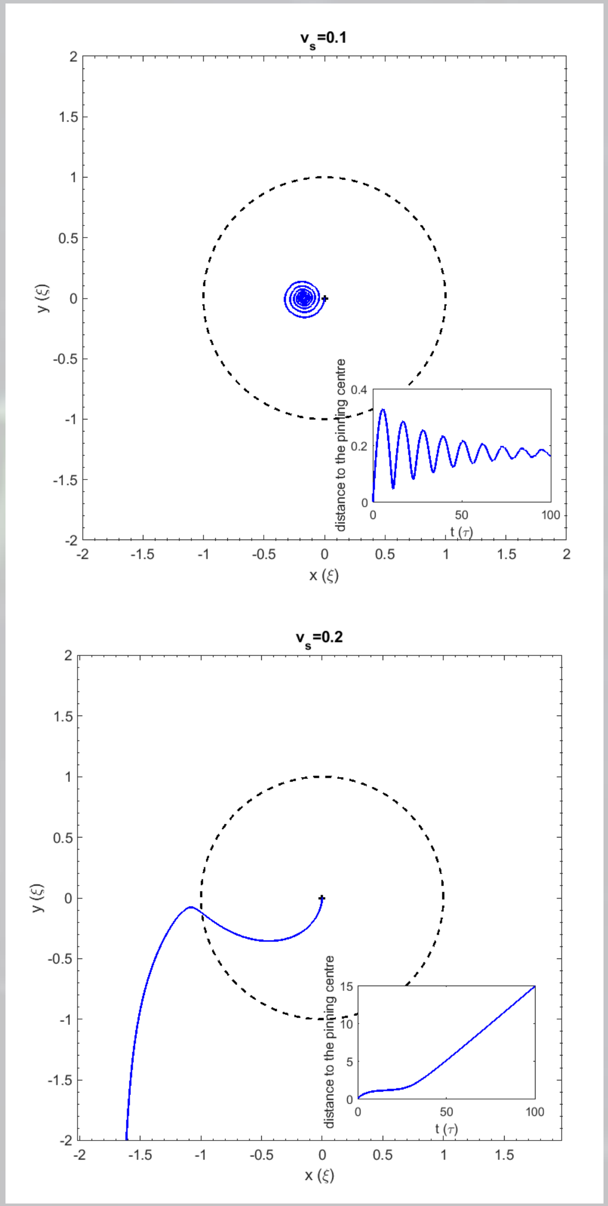
$$\frac{d\boldsymbol{z}_k}{dt} = \boldsymbol{e}^{-i\phi}\boldsymbol{v}_{s,k}$$

$$\alpha = \sin\phi$$

$$\alpha' = 1 - \cos\phi$$

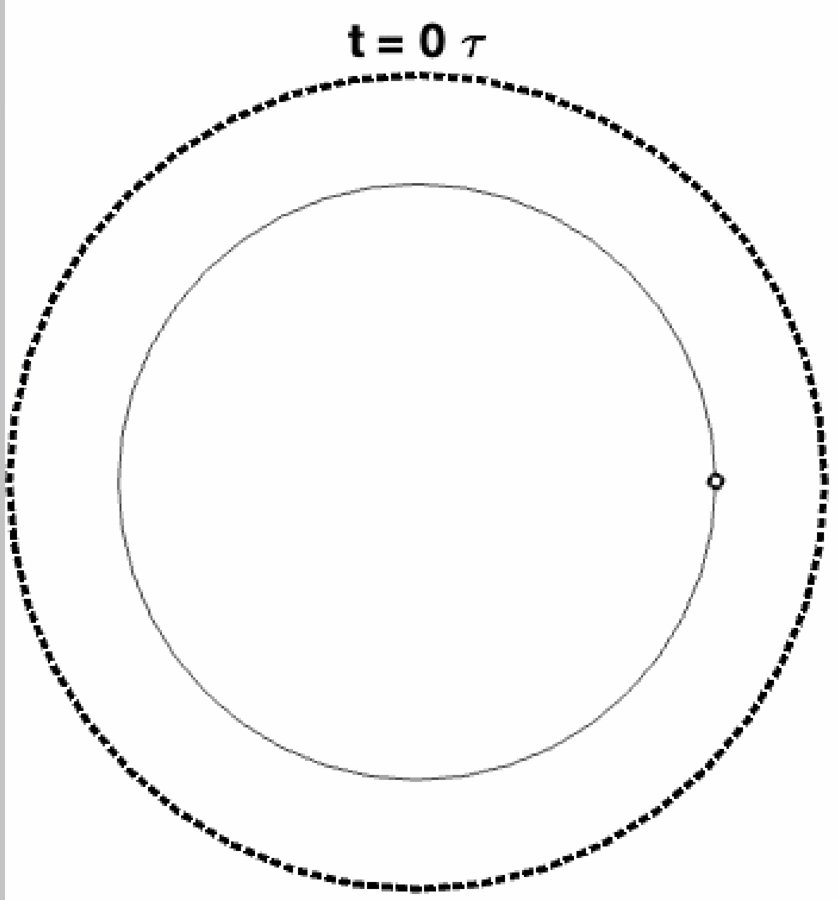
Mesoscopic Simulation





Mesoscopic Simulation

- PVM mapping



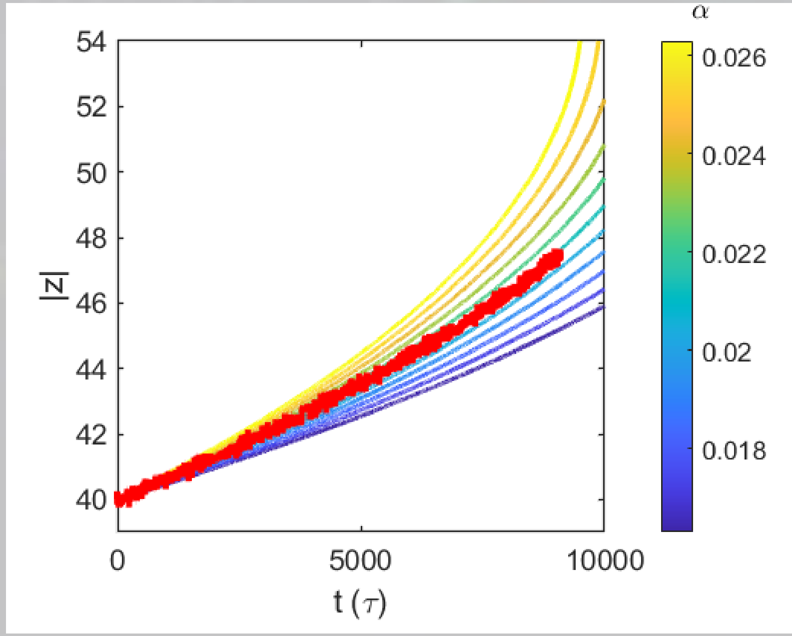
dGPE: $\gamma = 5 \times 10^{-3}$

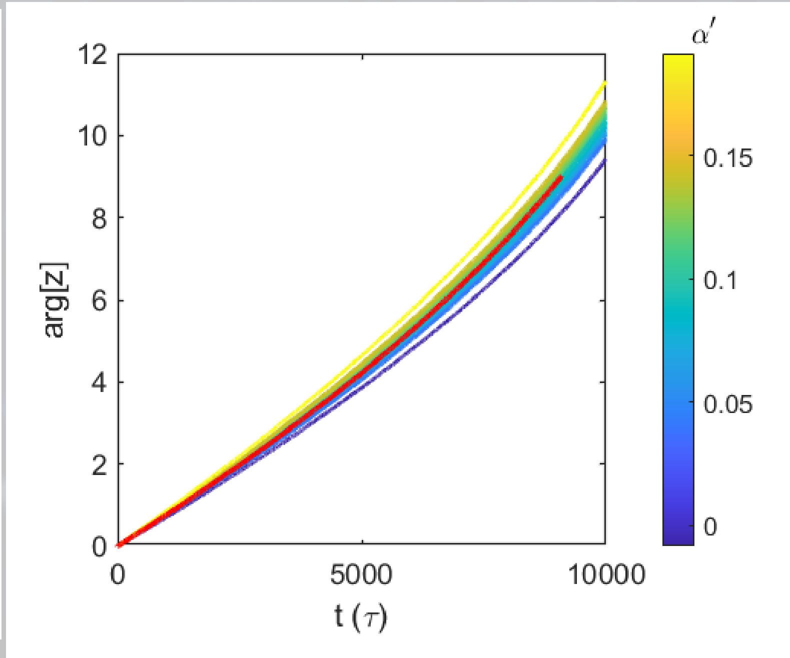
$$\frac{dz}{dt} = \frac{\hbar}{m}\left[i(1-\alpha')\frac{z}{R^2-|z|^2} + \alpha\frac{z}{R^2-|z|^2}\right]$$

$$\frac{d|z|}{dt} = \alpha\frac{|z|}{R^2-|z|^2}$$

$$\Longrightarrow R\ln|z(t)| - \frac{|z(t)|^2}{2} = \frac{\alpha\hbar}{m}t + \text{Const.}$$

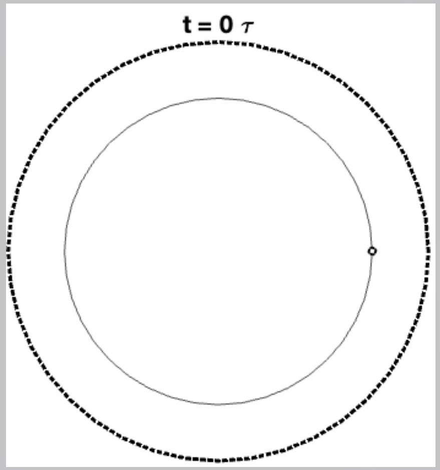
$$\frac{d\arg[z]}{dt} = (1-\alpha')\frac{\hbar}{m}\frac{1}{R^2-|z|^2}$$





Mesoscopic Simulation

- Point-Vortex Simulations

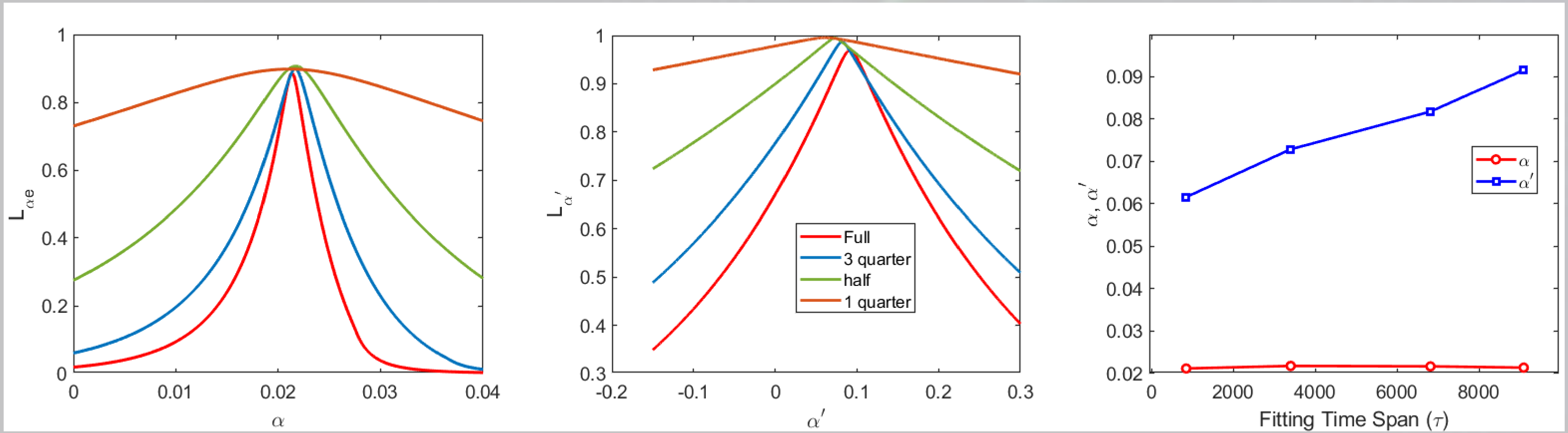


$$\frac{dz}{dt} = \frac{\hbar}{m}\left[i(1-\alpha')\frac{z}{R^2-|z|^2} + \alpha\frac{z}{R^2-|z|^2}\right]$$

$$\frac{d|z|}{dt} = \alpha\frac{|z|}{R^2-|z|^2}$$

$$\Longrightarrow R\ln|z(t)| - \frac{|z(t)|^2}{2} = \frac{\alpha\hbar}{m}t + \text{Const.}$$

$$\frac{d\arg[z]}{dt} = (1-\alpha')\frac{\hbar}{m}\frac{1}{R^2-|z|^2}$$

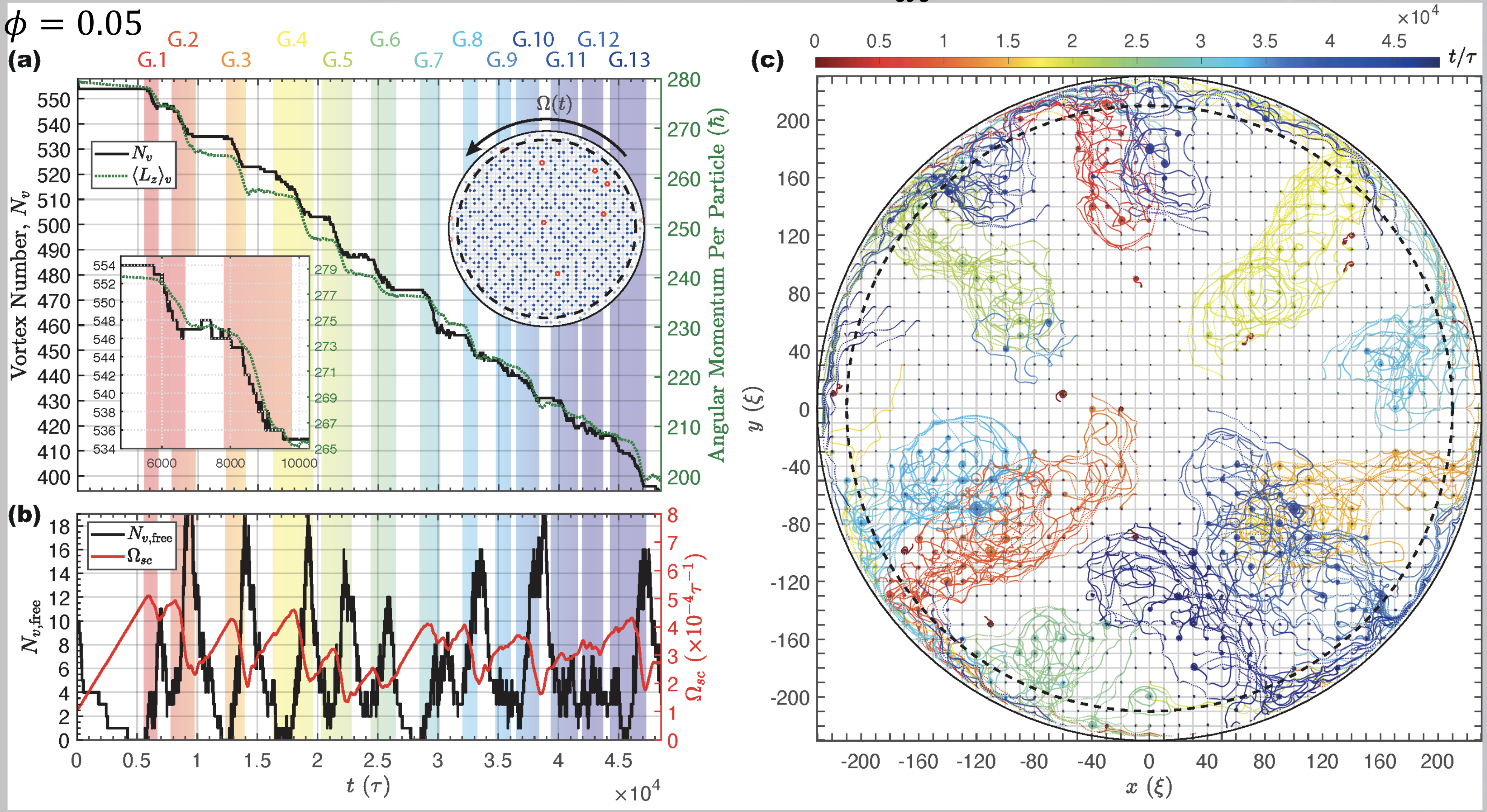


Superfluid Modelling (Preliminary)

- PVM: Mutual Friction Coefficients
  - Howitt's approach Howitt et al., MNRAS 498, 320 (2020)

$$\frac{dz_k}{dt} = e^{-i\phi} v_{s,k}$$

$\phi = 0.05$



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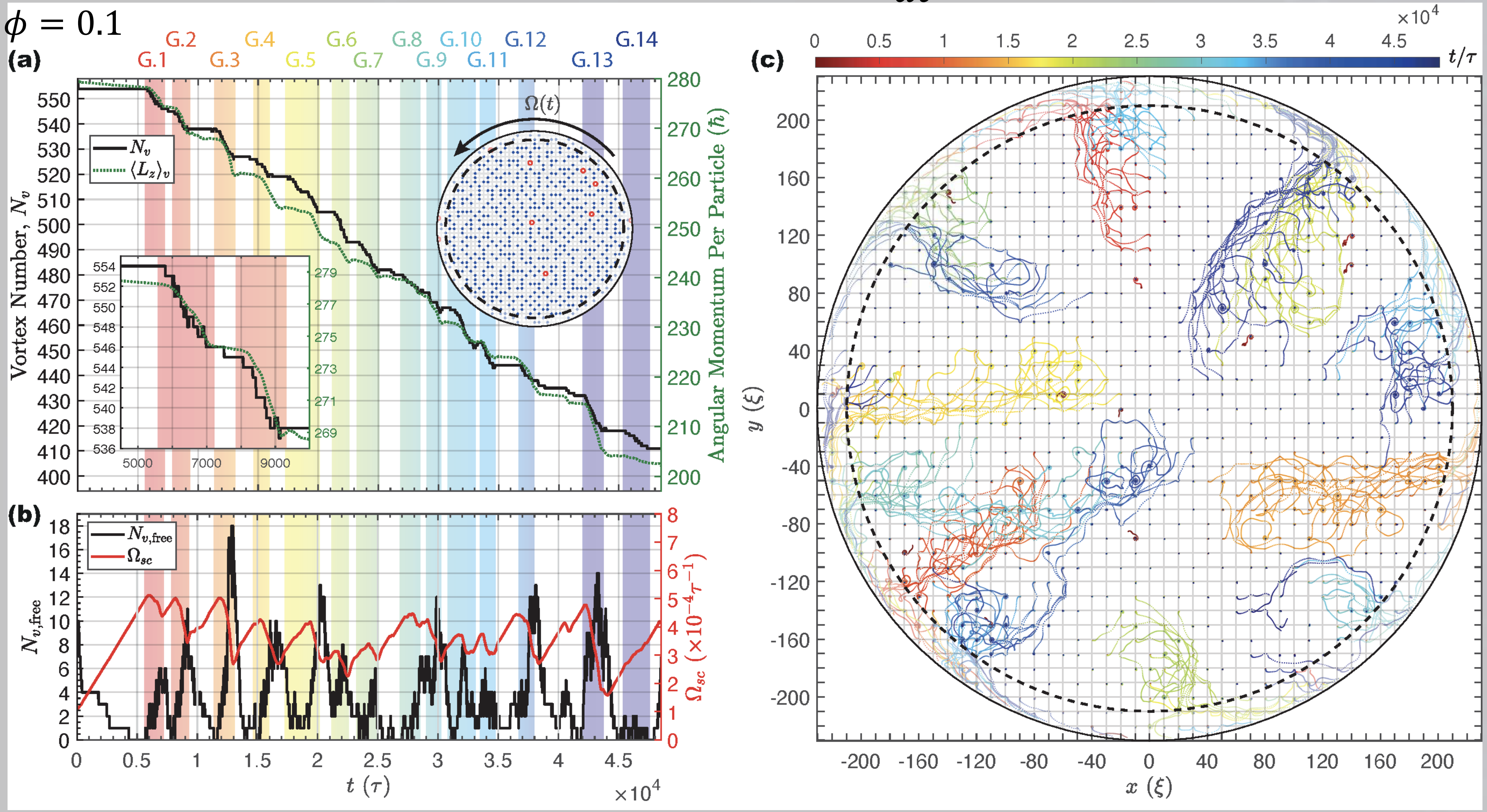
$\phi = 0.1$



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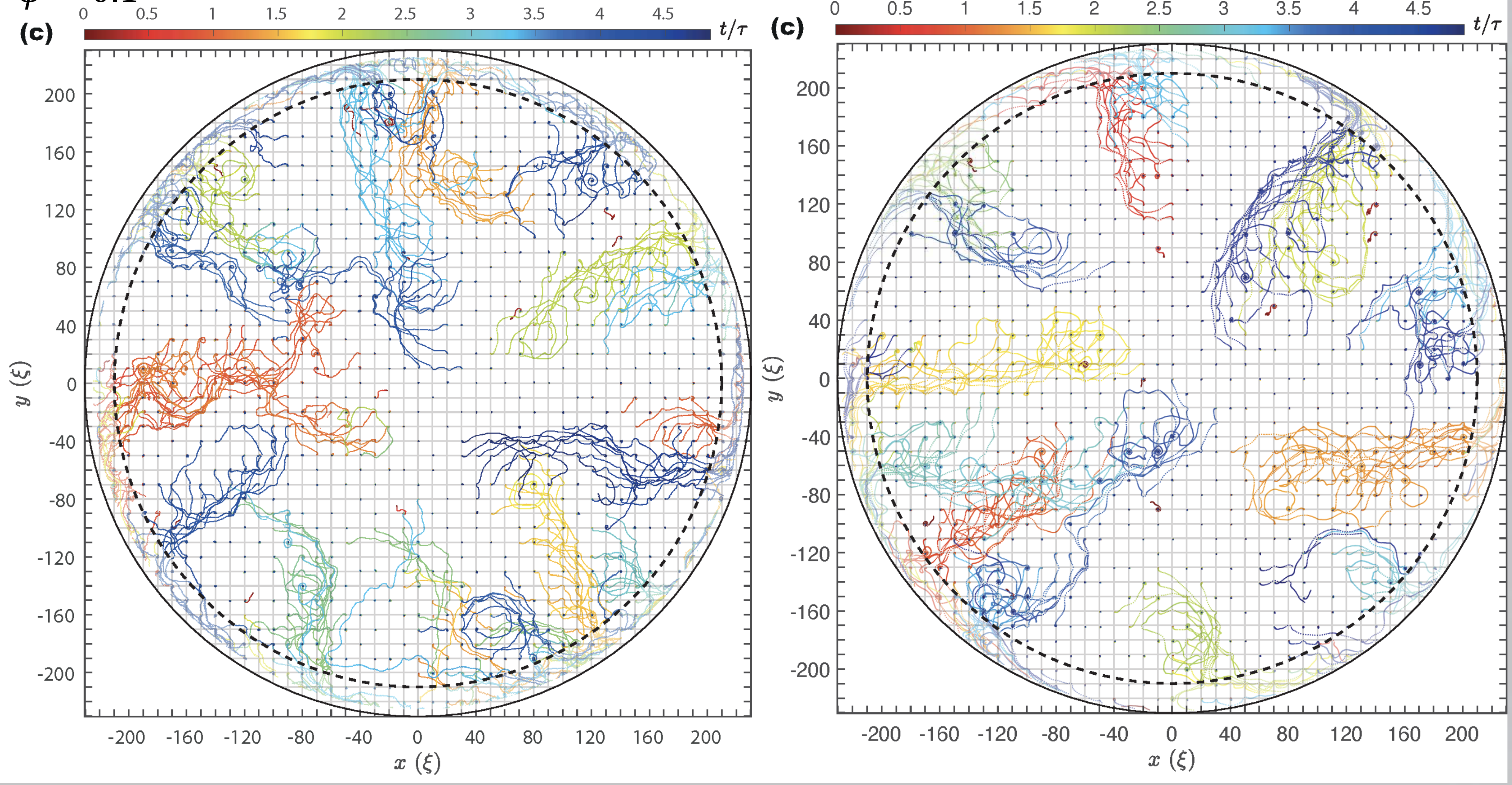

Superfluid Modelling (Preliminary)

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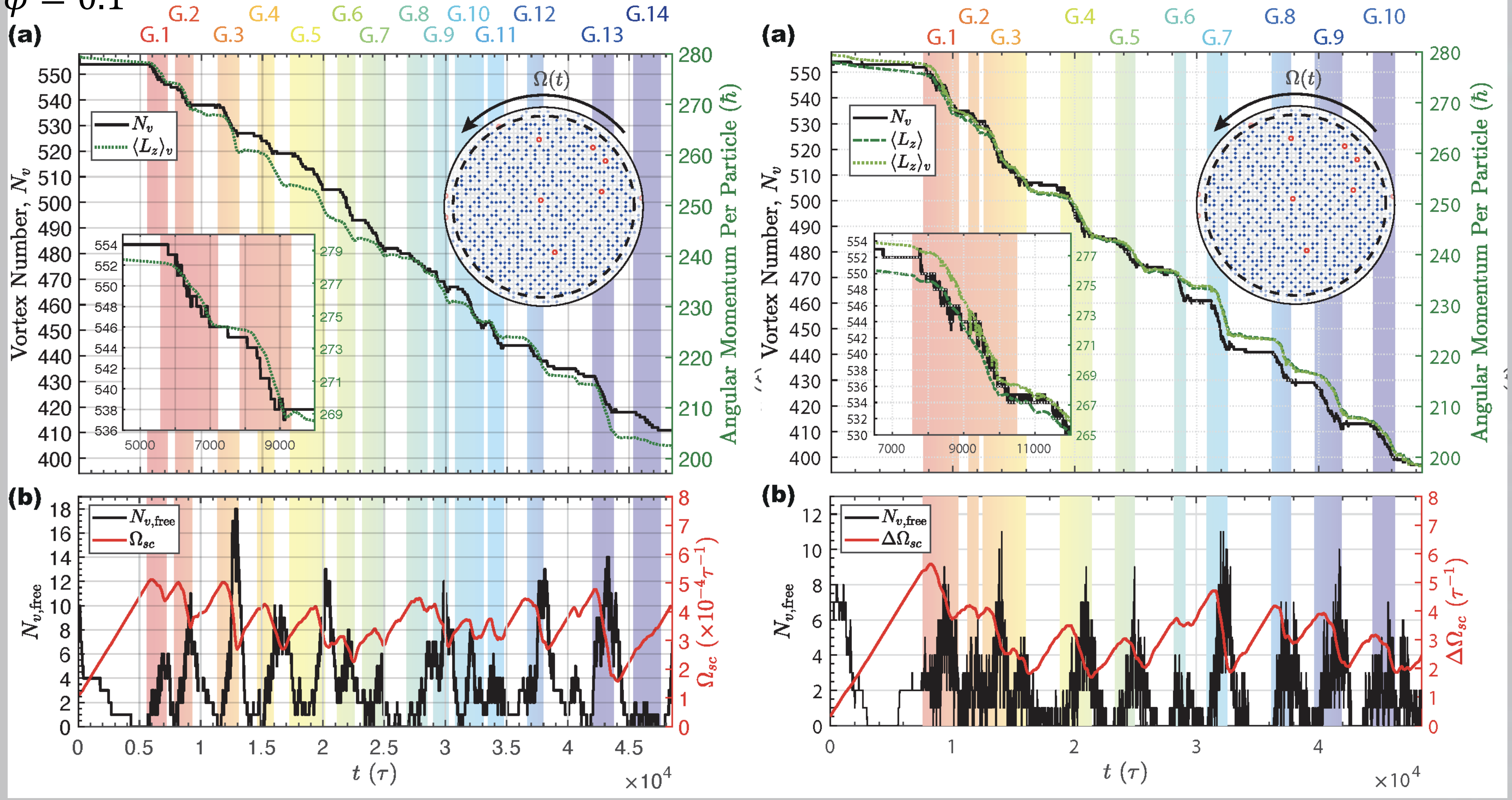
$\phi = 0.1$



Superfluid Modelling (Preliminary)

- PVM: Mutual Friction Coefficients
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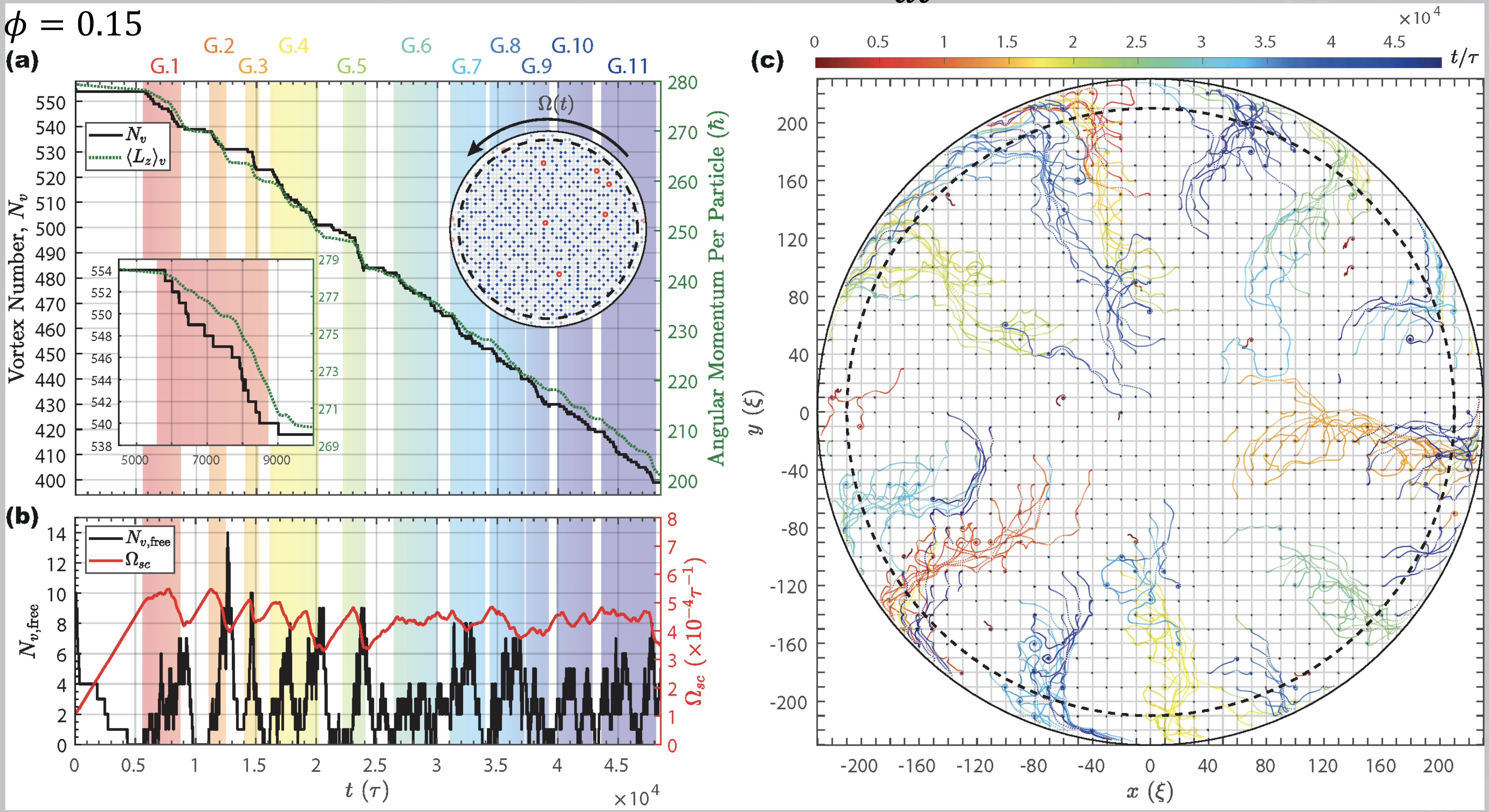
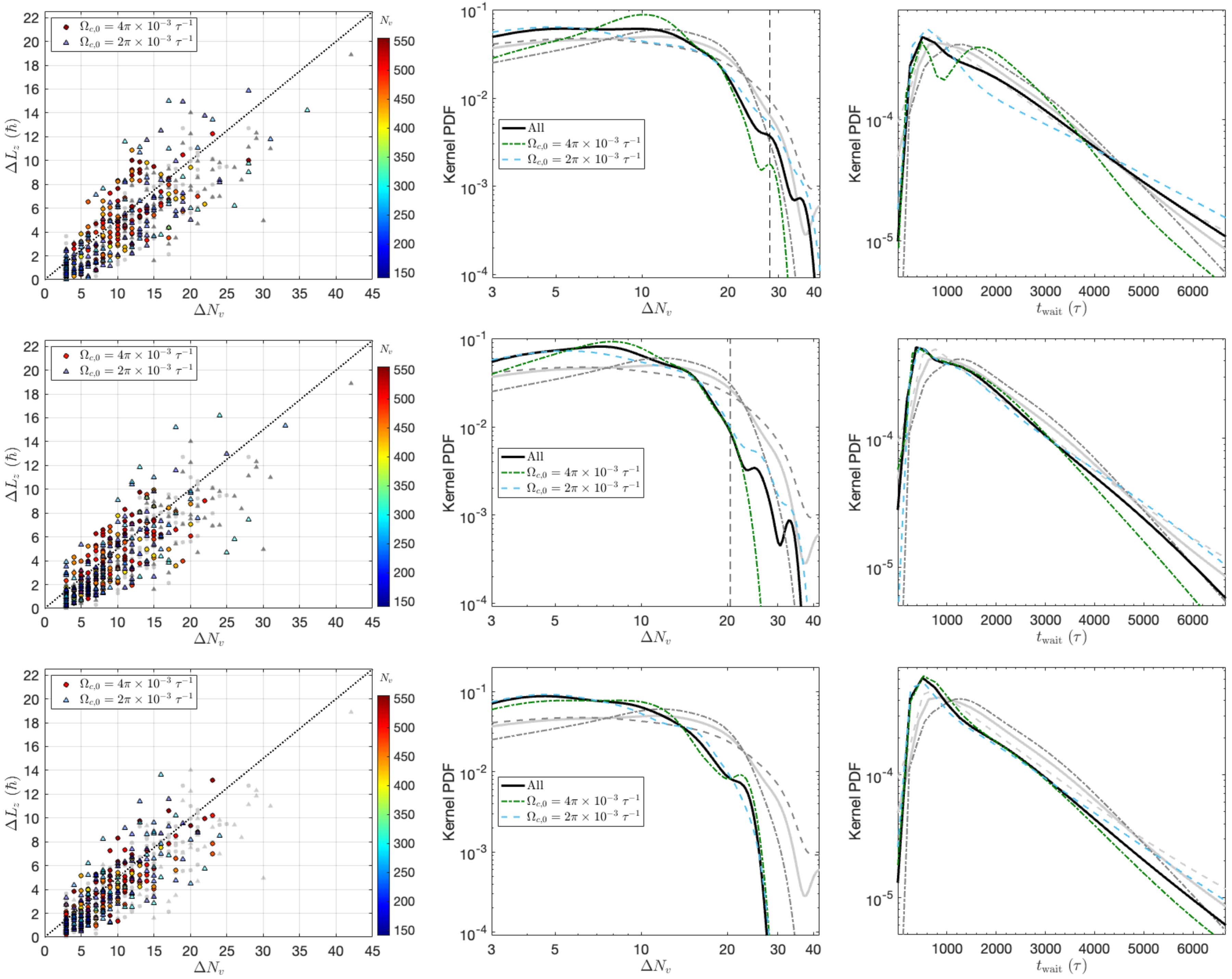
$\phi = 0.15$



$\phi = 0.05$

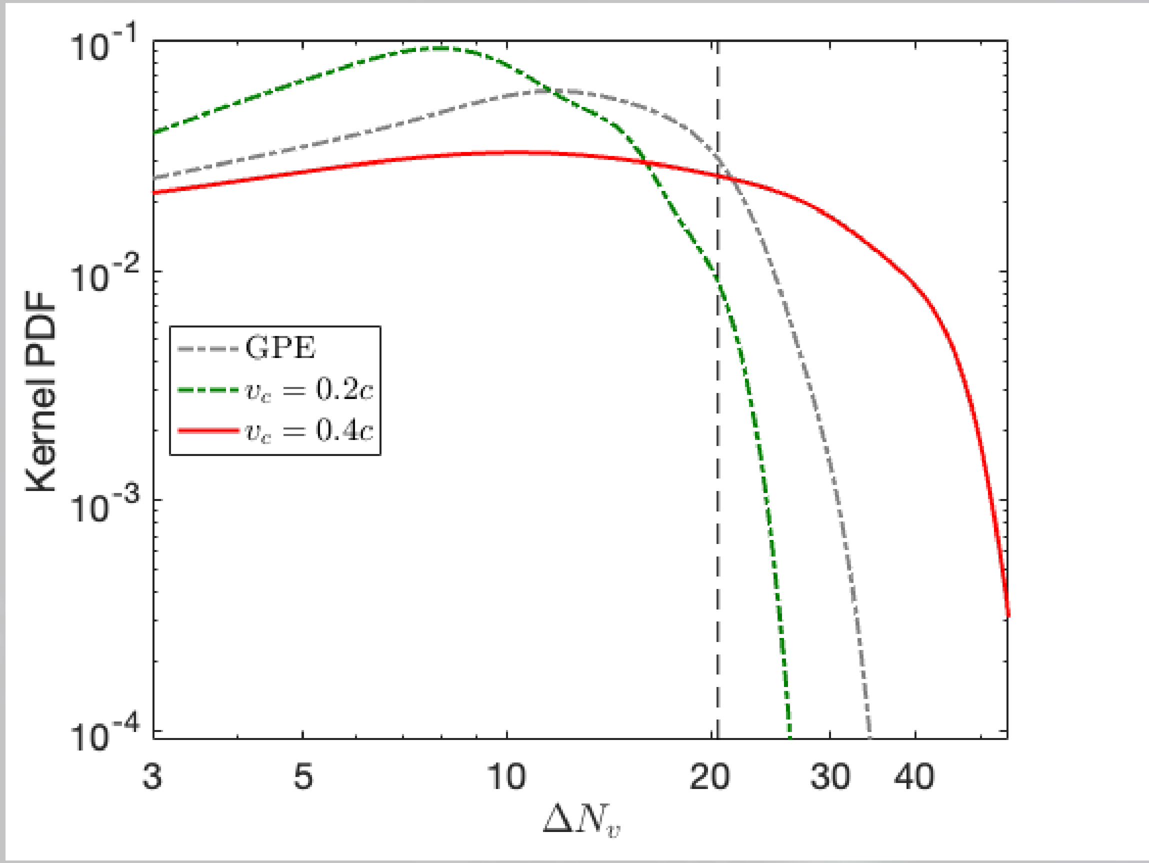
$\phi = 0.1$

$\phi = 0.15$



Superfluid Modelling (Preliminary)

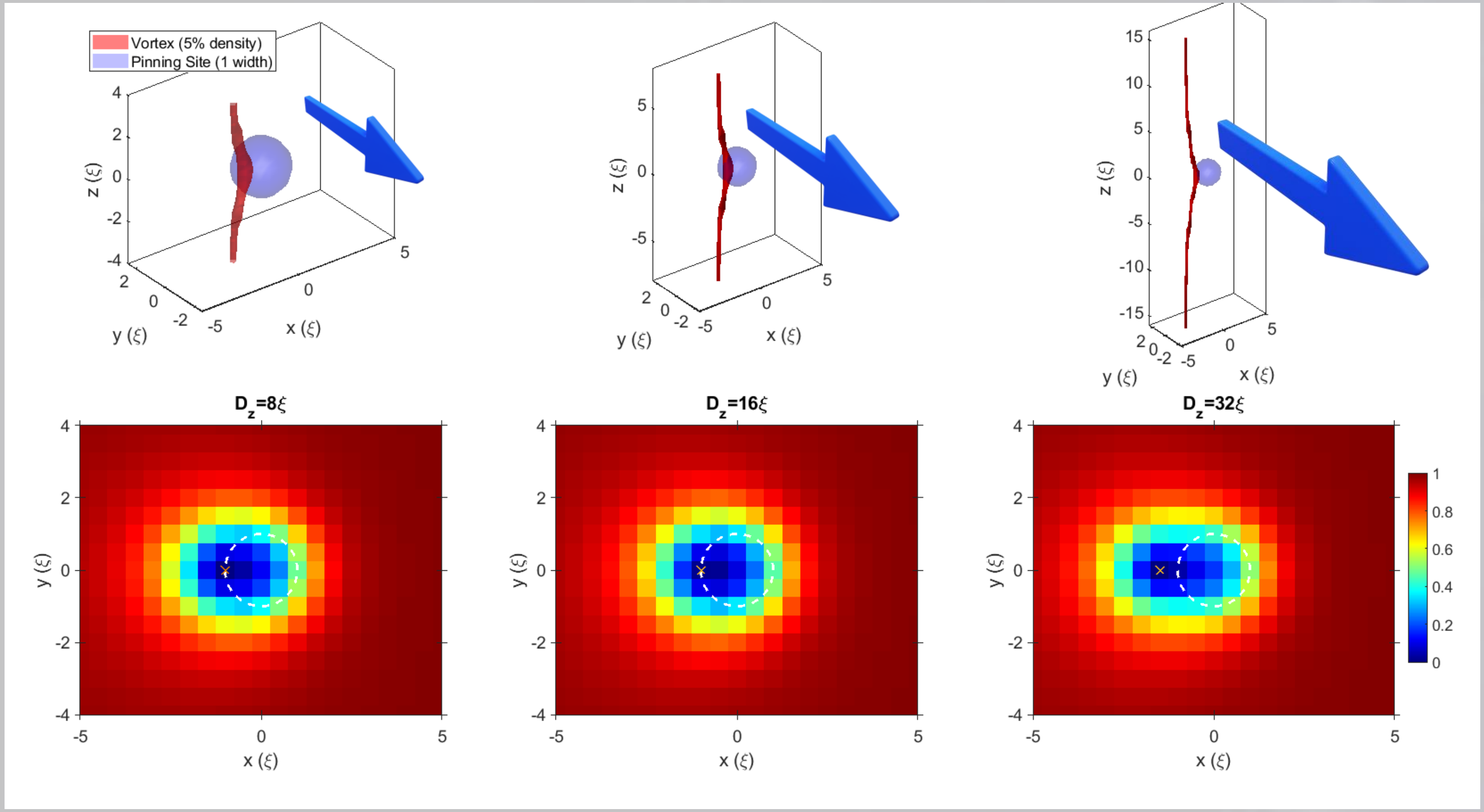
- PVM – Finding larger glitches



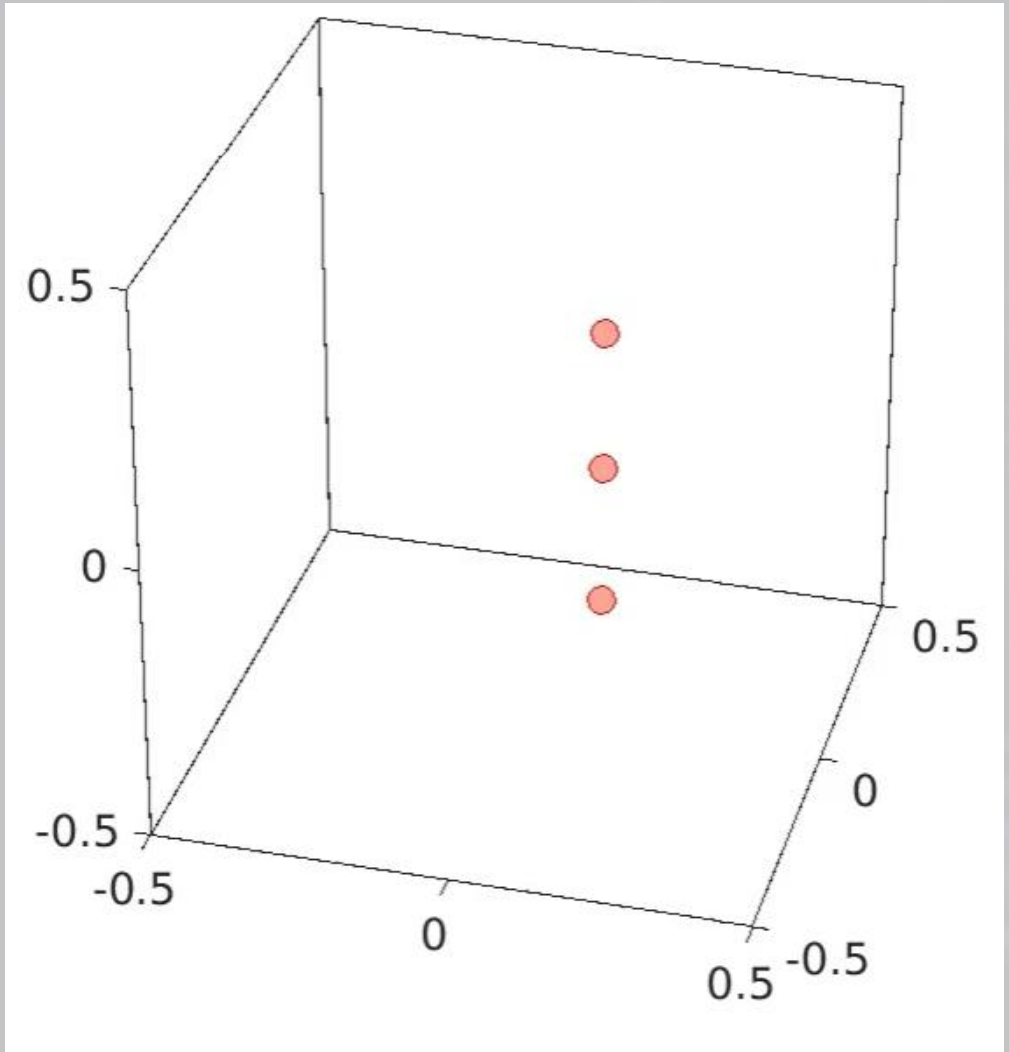
On-going works: 3D Simulation

- Vortex Depinning in 3D

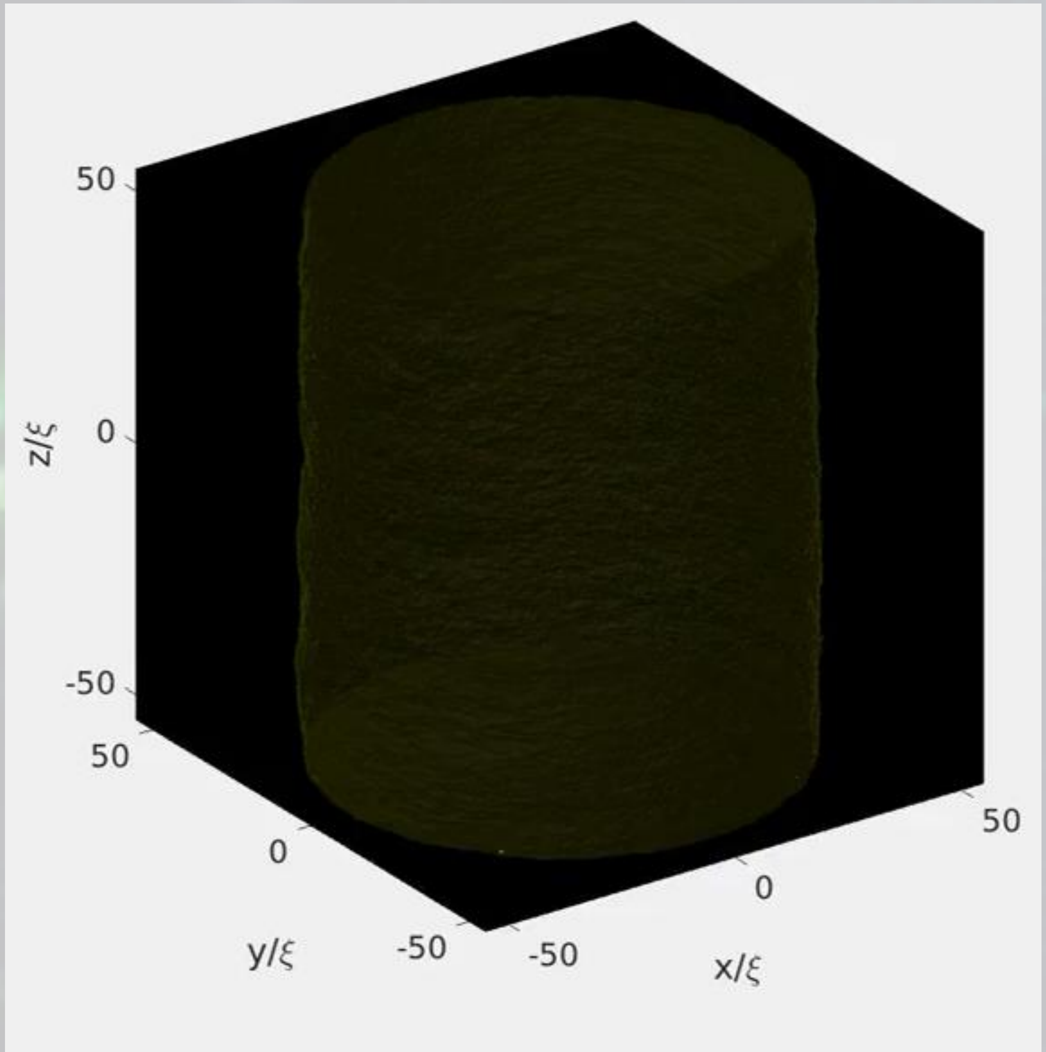
Liu et al., in progress



On-going works: 3D Simulation

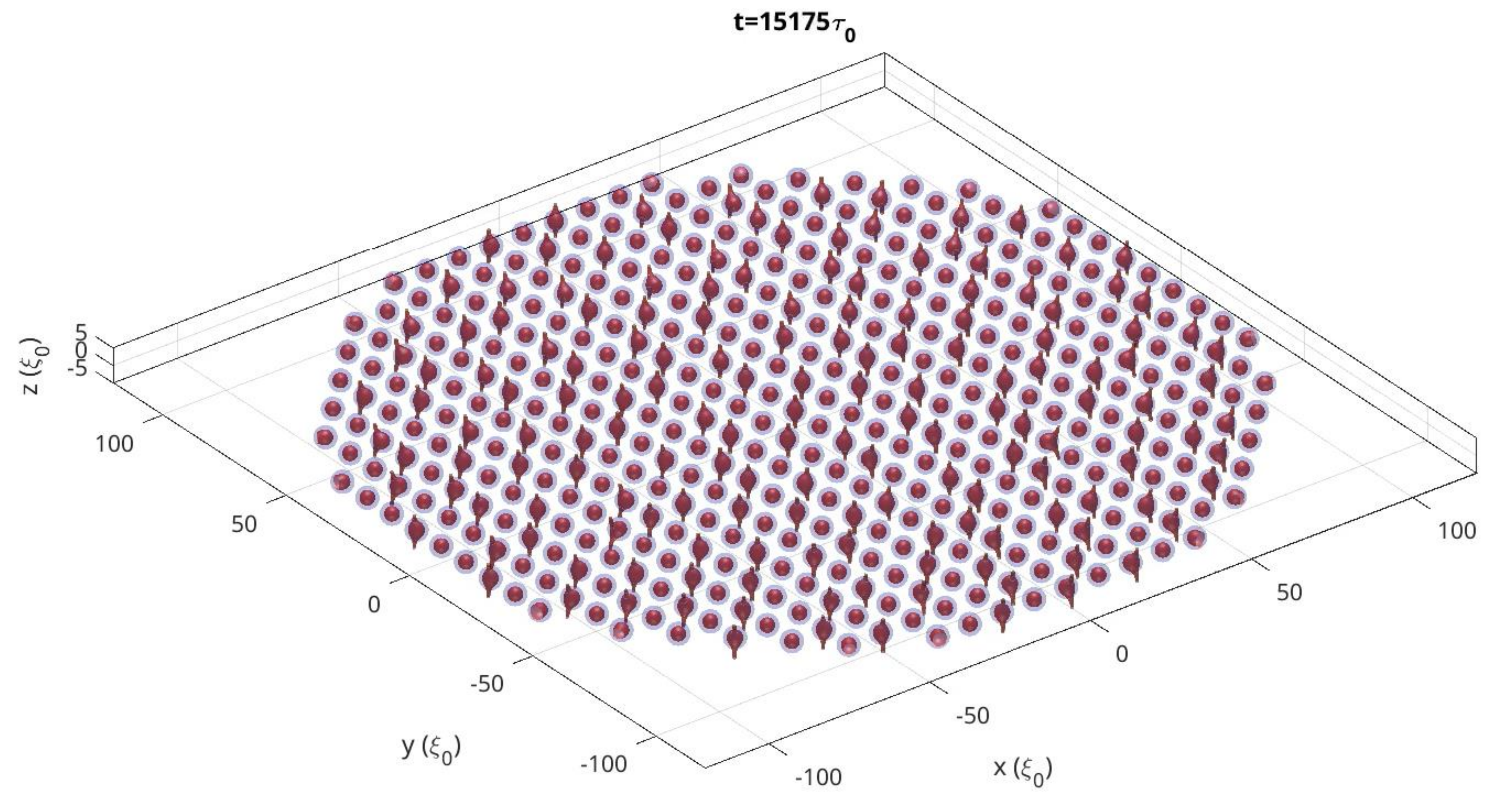


J. Thomas et al., in preparation



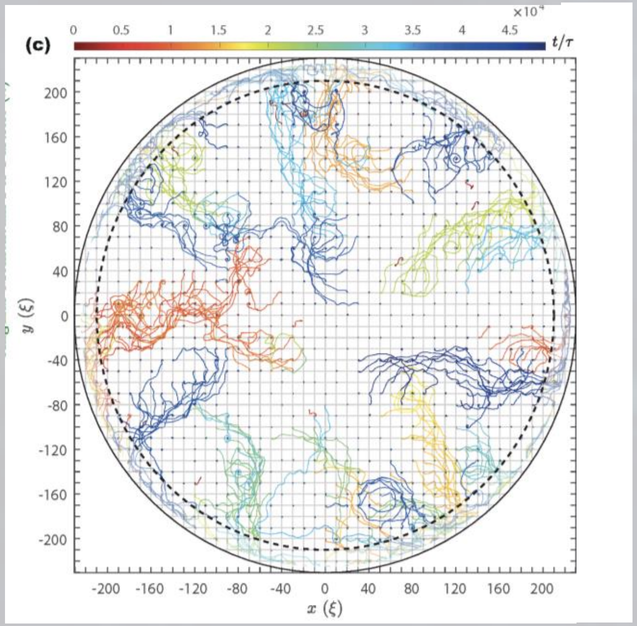
N. A. Keepfer et al., Phys. Rev. B **102**, 144520 (2020)

On-going works: 3D Simulation



Concluding Remarks

- Vortices can be depinned due to the Magnus force.
- Vortex avalanches occur locally and temporally with the emergence of vortex channels during glitches.
- The post-glitch dynamics smooth the vortex density.
- Reproduction of GPE-like dynamics in the PVM with similar statistic behaviours can be achieved with fine-tunings.
- A simple PVM tends to have smaller glitches compared with that in GPE.
- Larger glitches can be found when the pinning is stronger.



In progress...

- Larger glitches/avalanches
- Vortex depinning analysis
- 3D simulation

Thank You

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