Hydrodynamics, black holes and black hole microstates

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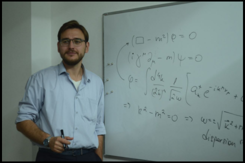

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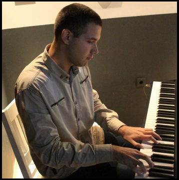

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1 String theory, holography and what to do with them

String theory: from high to low energies

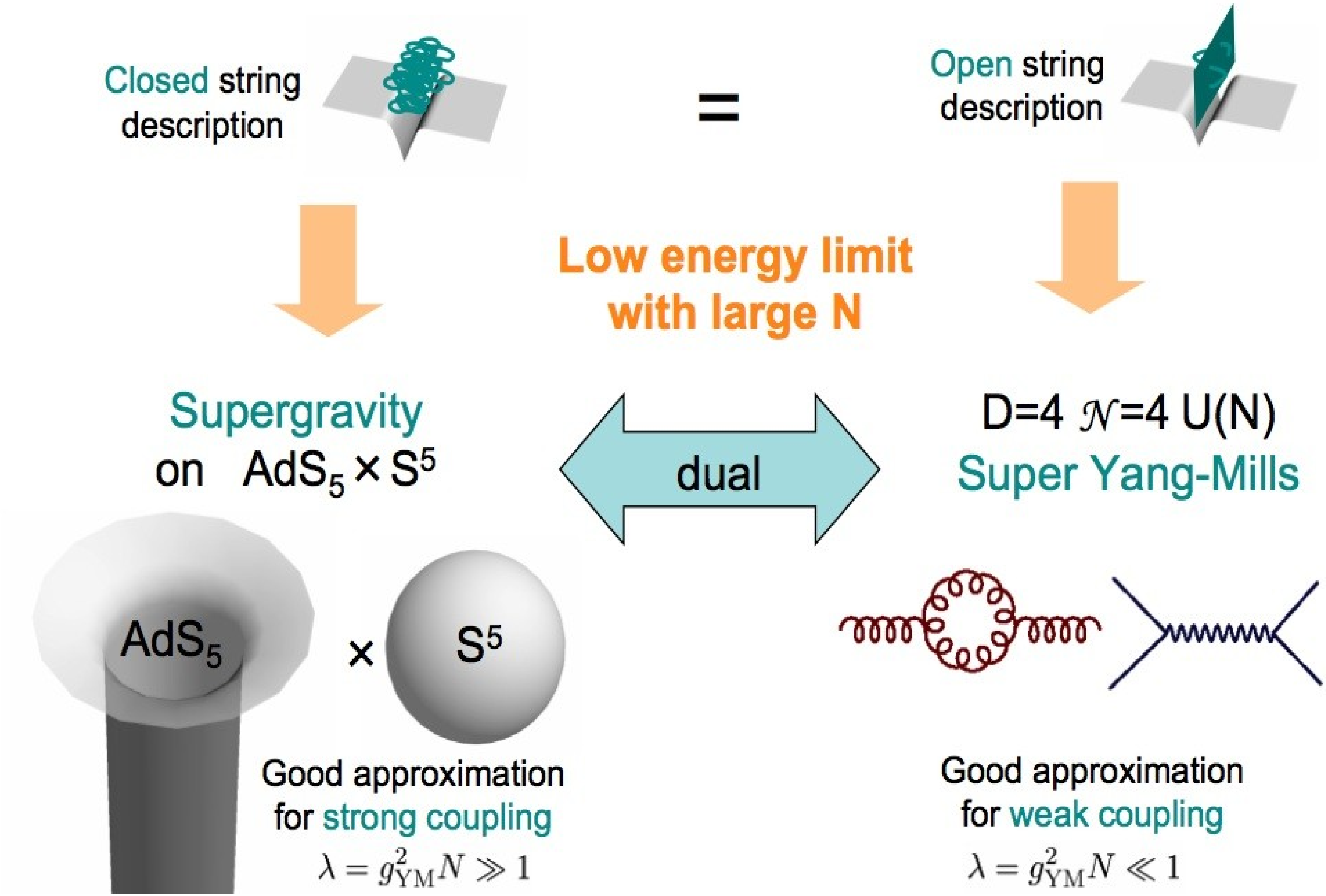
- Modern string theory aims to understand quantum gravity $\Rightarrow$ extremely high energies
- More modest beginnings in the 60's: try to understand Regge trajectories and hadron masses (bootstrap program, S-matrix theory etc)
- Last ~ 25 years: again employed to understand nuclear matter, neutron star matter, superfluids, superconductors, metals

- The idea: holography (AdS/CFT correspondence, gauge/gravity duality)

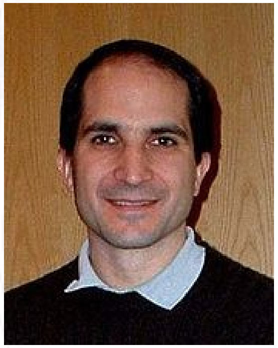
Holographic duality

- The essence of holography: string theory at coupling $1/g$ in $D+1$ spacetime dimensions is dual to a field theory at coupling g in D space dimensions
- The field theory can be an effective field theory (hydrodynamics, Landau theory, nuclear model...)
- For $g \gg 1$: strongly coupled field theory dual to weakly coupled string theory $\approx$ classical gravity
- Punchline: model strogly coupled quantum many-body systems by solving classical Einstein equations of gravity (can be much easier)

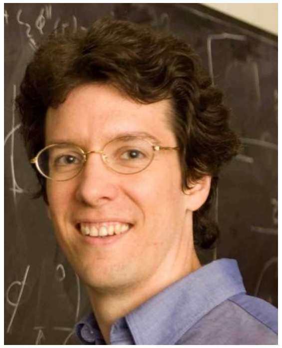
AdS/CFT correspondence



AdS/CFT in a nutshell



J. Maldacena



S. Gubser

I. Klebanov

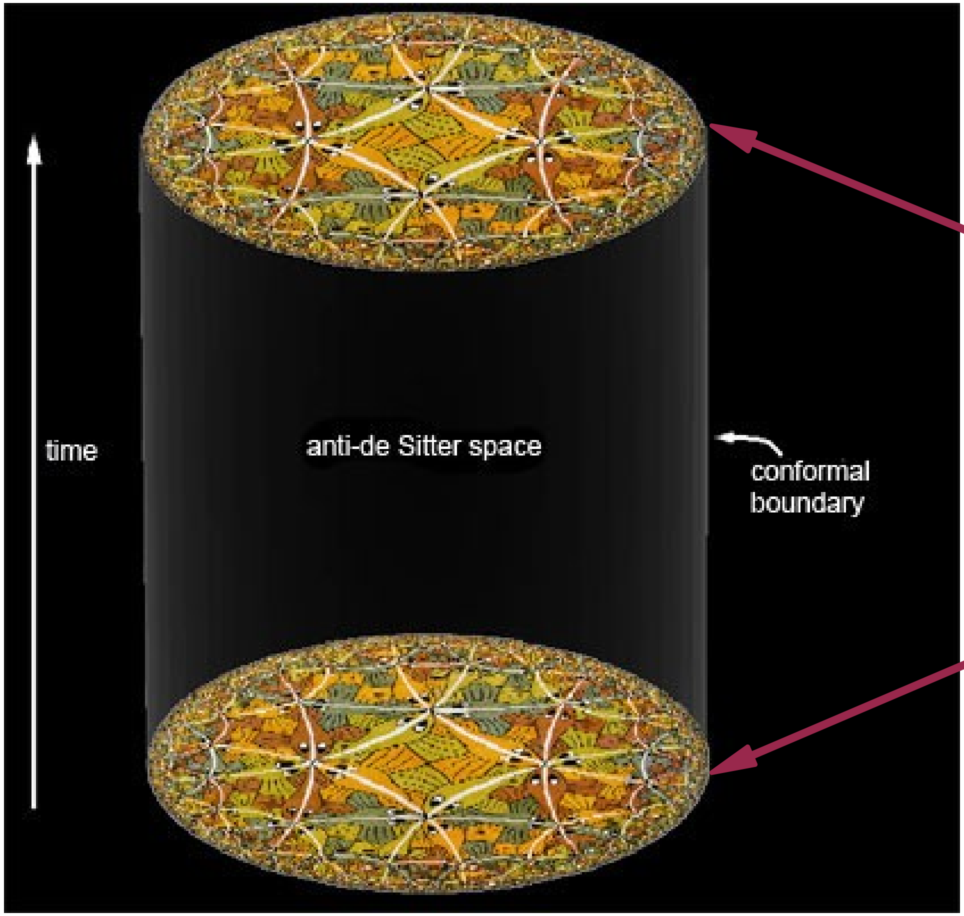
E. Witten

- Maldacena 1997, Gubser, Klebanov & Polyakov 1998, Witten 1998
- Stack of N parallel D3 branes: open vs. closed string description
- Open strings: $\mathcal{N}$=4 U(N) super Yang-Mills
- Closed strings: type IIB supergravity on anti-de Sitter space

$$\mathrm{AdS}_{D+1} \sim \mathrm{CFT}_{D}$$

gravity in D+1 dimensions = field theory in D dimensions

Deformed AdS/broken CFT



Pure AdS geometry ⇔ ***N***=4 U(N) super-Yang-Mills

N=4 SYM has a conformal (quantum critical) point

Deform away from AdS ⇔ deform away from conformality and supersymmetry

Holographic RG flow

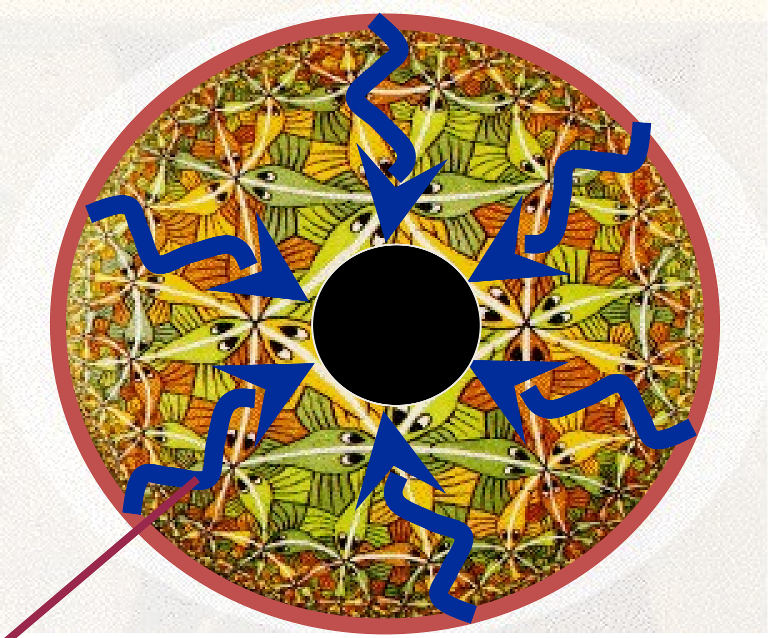
AdS space – maximally symmetric space with uniform negative curvature

$$ds^2 = r^2(-dt^2 + d\vec{x}^2) + \frac{dr^2}{r^2}$$

Scale invariance $ds \rightarrow ds$ with respect to

$$t \rightarrow \lambda t, \vec{x} \rightarrow \lambda \vec{x}, r \rightarrow r/\lambda$$

Therefore: pure AdS ⇔ full conformal invariance



Matter propagates from AdS boundary to the horizon

Radial distance („extra dimension") in AdS ~ RG scale in CFT

Holographic dictionary

- Gauge symmetry ⟷ Global symmetry
- Radial electric field F_{z0} ⟷ Charge density ρ
- Transverse electric field F_{i0} ⟷ Electric field E_i
- Radial magnetic field F_{zi} ⟷ Current density j_i
- Transverse magnetic field F_{ij} ⟷ Magnetic field $\varepsilon_{ijk} B_k$
- Scalar mass m^2 ⟷ Conformal dimension $\Delta = D/2 \pm \sqrt{D^2/4 + m^2}$
- Spinor mass m ⟷ Conformal dimension $\Delta = D/2 + m$

Holographic duality: what is it good for?

- So far: qualitative insights, scaling exponents, quantum critical behavior, universal bounds
- Connects different fields of physics and helps us understand how they connect
- Quantitative accuracy tends to be low – hard to get the right Hamiltonian on field theory side

Holographic superfluids

- Action:

$$S = \int d^5x\sqrt{-g}\left[R - 2\Lambda - \frac{1}{4}F^2 - |D\psi|^2 - M^2|\psi|^2\right]$$

- Superfluid order parameter encoded by ψ, interactions encoded by curvature R and cosmological constant Λ
- Bulk EM field strength $F_{\mu\nu}$ encodes for chemical potential and magnetic field
- As promised: simple bulk action (quadratic!) but complex and strongly interacting superfluid
- Superfluid phase corresponds to a hairy black hole (with nonzero profile of ψ at the horizon)
- Time-dependent and non-equilibrium configurations of vortices, vortex creation and annihilation (Landsteiner, Arean, Donos...)

Holographic superfluids: variations on a theme

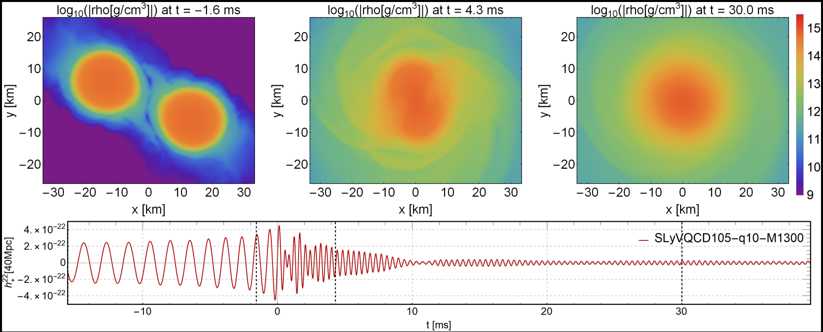
- Superfluidity of different particles (atoms, neutrons...): additional bulk scalars with different boundary values:

$$S \mapsto S - \int d^5x \sqrt{-g} \left[|D\phi|^2 - m^2 |\phi|^2 \right]$$

- Transition to quark-gluon plasma: Hawking-Page transition between the hairy black hole and the Reissner-Nordstrom ("bald" black hole)

Holographic neutron stars

- Ingredients: holographic QCD (large-number-of-colors approximation to QCD) + holographic superfluid (Kiritsis et al)
- Static: neutron star equation of state
- Dynamic (numerical GR): neutron star mergers, prediction of the gravity wave signal (van der Schee et al, PRD 101, 103006 (2020))



2 Holographic Fermi surface compendium

What is the essence of Fermi liquids?

- Renormalized Fermi gas with weakly interacting quasiparticles
- Landau theory: arguably the first Effective Field Theory ever written

- Litmus tests:
  – free energy scaling $F \propto T^2$
  – entanglement entropy log law $S_E \sim \log \ell$
  – zero sound $\omega = ck + \dots$
  – resistivity $\rho \propto T^2$ – ...

Holographic Fermi liquids?

- Why should we be interested in building a holographic model?
  – To understand Fermi liquids within the broader context of EFTs
  – To understand the limits of holography
  – To understand how they form in quantum phase transitions, e.g. in heavy fermion systems

- History: started from simple-minded holographic models, got enthusiastic, then got demoralized, now trying to came back in the true spirit of EFT (no microscopic fermions!)

FL perversions: weakly or strongly coupled?

- Textbook formulation: effectively *weakly coupled quasiparticles* but can be very *strongly renormalized* from the bare particles (up to 100 times in heavy fermions, up to 10^3 times in He3)
- This indicates that classical gravity dual exists

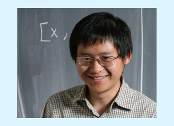
- But quasiparticle interactions are always perturbative!
- This indicates there is no classical gravity dual!

3 Atlas of holographic Fermi surfaces

Semilocal quantum liquids

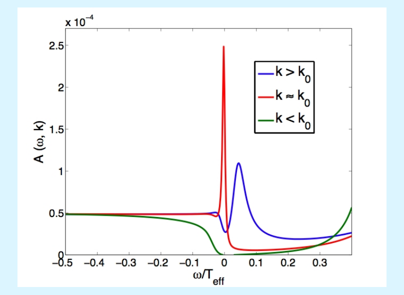

Jan Zaanen


Koenrad Schalm


Hong Liu

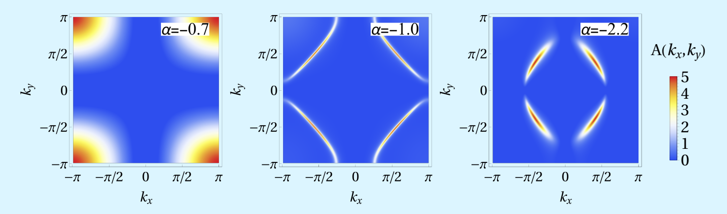
- Gravity side: the simplest possible system – charged black hole with charge $Q \sim$ electron density
- Holographic ARPES: robust non-Fermi-liquid peaks (Čubrović, Zaanen, Schalm, Science 325, 439 (2009)) + linear resistivity (Faulkner, Iqbal, Liu, McGreevy, Vegh, Science 329, 1043 (2010))

Semilocal quantum liquids



- Fractionalized phase – no Luttinger theorem, no log violation of entanglement scaling
- Finite $T = 0$ entropy – but this can be cured (Gubser-Rocha)
- Linear resistivity; smells like strange metals but not much
- Can be unstable to backreaction! What is the true ground state?

Holographic fermions on lattices



MDCs on a holographic lattice with a different Mott-like coupling α (dipole coupling in the bulk). Can be fit to QMC results for the Hubbard model (Herček, Gecin, Čubrović SciPost Phys. 6, 027 (2023)).

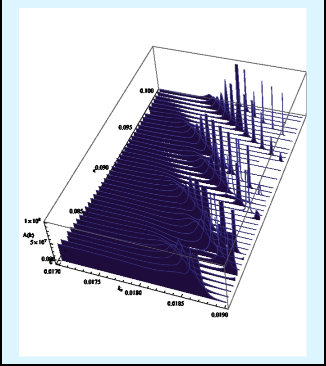
- Black holes with corrugated horizons lead to modulating chemical potential ~ lattice
- A wealth of new physics: signs of Mottness, Hubbard-like physics...
- The highlight: fit to measured ARPES MDCs (Nature Communications, Leiden-Utrecht-Amsterdam-Nijmegen, Nat. Com. 15, 4581 (2024))

Electron stars

- Add "fermionic hair" to the charged black hole ⇒ black hole discharges into the fermionic cloud
- Tractable in the Tolman-Openheimer-Volkov (fluid) approximation:

$$
\begin{aligned}
S &= S_{\mathrm{g}} + S_{\mathrm{EM}} + S_{\mathrm{Dirac}} + S_{\mathrm{ct}}, \\
S_{\mathrm{Dirac}} &\rightarrow S_{\mathrm{fluid}} = \int \mathrm{d}^4x\sqrt{-g}\left(-\rho\left(\sigma\right) + \sigma u^{\mu}\left(\partial_{\mu}\phi + A_{\mu}\right) + \lambda\left(u^2+1\right)\right)
\end{aligned}
$$

- Sharp, long-living quasiparticle peaks in the spectral function



Electron star diffusion

- Holographic dictionary: solve the bulk fluctuation equations
- Transverse sector: expect a diffusive channel $\omega \sim -\imath D k^2$

$$
\begin{aligned}
g_{\mu\nu} &\mapsto g_{\mu\nu} + \delta g_{\mu\nu}, \quad \delta g_{\mu\nu} \sim L^2 e^{ikx - i\omega t} h_{\mu\nu}(r) \\
A_\mu &\mapsto A_\mu + \delta A_\mu, \quad \delta A_\mu \sim \frac{eL}{\kappa} e^{ikx - i\omega t} a_\mu(r)
\end{aligned}
$$

- Redefine the fields:

$$
X = k h^y{}_t + \omega h^x{}_y, \quad Y = a_y, \quad U = \frac{r}{k^2 r^2 f - \omega^2} \sqrt{\frac{f}{g}} \left(\frac{X'}{r^2} + 2kh' Y \right)
$$

- The fluctuation equations become

$$
U'' + \frac{1}{2}\left(\frac{f'}{f} - \frac{g'}{g}\right) U' - \left(g \left(k^2 r^2 - \frac{\omega^2}{f} \right) + \frac{1}{2r}\left(\frac{f'}{f} - \frac{g'}{g}\right)\right) U + 2krh' \sqrt{\frac{g}{f}} Y = 0
$$

$$
Y'' + \frac{1}{2}\left(\frac{f'}{f} - \frac{g'}{g}\right) Y' - \left(g \left(k^2 r^2 - \frac{\omega^2}{f} \right) + \frac{2h'^2}{f} + \frac{\sqrt{f} g \hat{\sigma}}{h} \right) Y + krh' \sqrt{\frac{g}{f}} U = 0
$$

Electron star diffusion

- Obtained in Gecin & Čubrović JHEP10 (2025) 152
- In a nutshell: good diffusive hydro at low temperatures and at high temperatures
- No diffusive pole at intermediate temperatures
- Intuition:
- At high T we are almost in the RN black hole (semilocal quantum liquid) regime – this is known to have good hydro (Parnachev, Davison)
- At low T we have Lifshitz hydrodynamics which also has a diffusive pole (Kiritsis, Gouteraux, Gursoy)
- At intermediate T both the fermions and the BH contribute and interact $\Rightarrow$ complicated mess

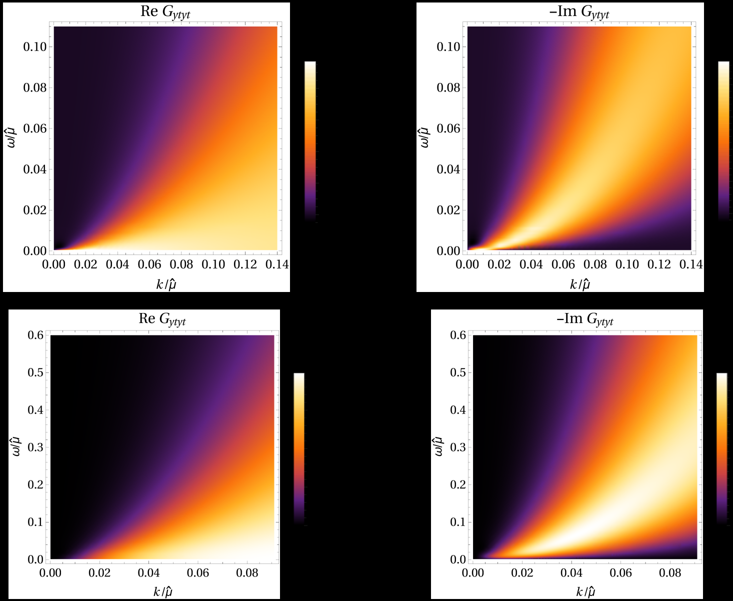


Figure: Low-T regime: density plots of G_{ytyt} for $z = 4$, for $T/T_c = 0$ (top) and $T/T_c = 0.23$ (bottom).

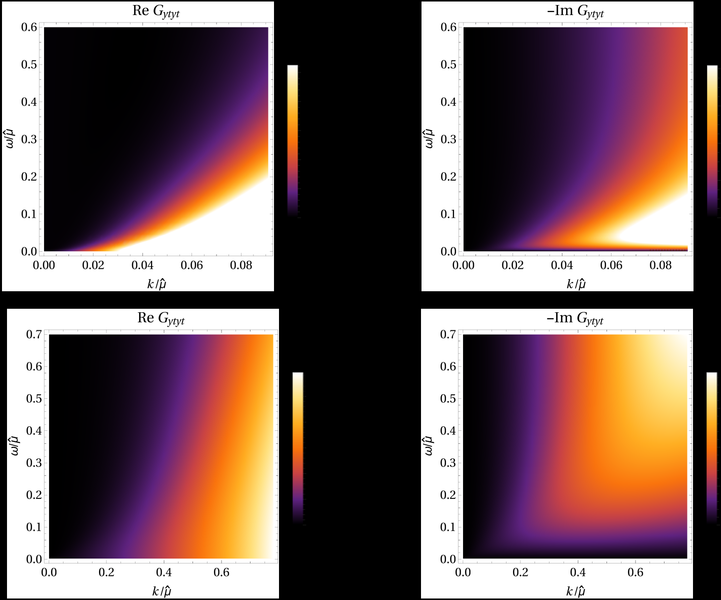


Figure: Intermediate-T regime: density plots of G_{ytyt} for $z = 4$, for $T/T_c = 0.35$ (top) and $T/T_c = 0.65$ (bottom).

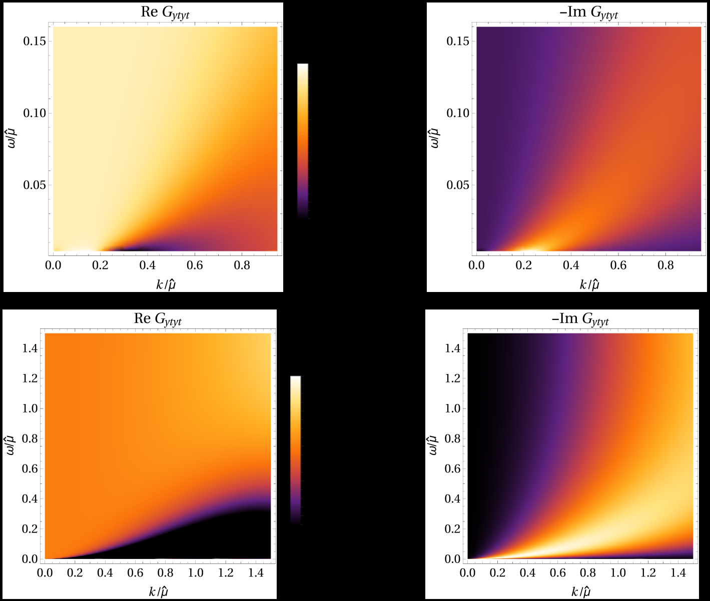


Figure: High-T regime: density plots of G_{ytyt} for $z = 4$ and $T/T_c = 0.95$ (top) and for the RN case at $T/T_c = 0.95$ (bottom).

The anticlimax – electron star instability

- IR expansion leads to an analytic result:

$$\omega = \imath D_1 |k| - \imath D_2 k^2 + \mathcal{O}(k^3); \quad D_1, D_2 > 0$$

- The diffusion poles do exist at low temperatures along those in the upper half-plane but...
- For small k the linear term dominates and leads to a pole in the upper half-plane!
- Far IR instability, around $k = 0 \Rightarrow$ alarming, unstable to small perturbations!
- No instability at $T = 0$: $D_1(T = 0) = 0$
- The linear $|k|$ term is nonanalytic – where the hell did this come from?

The instability: a numerical check

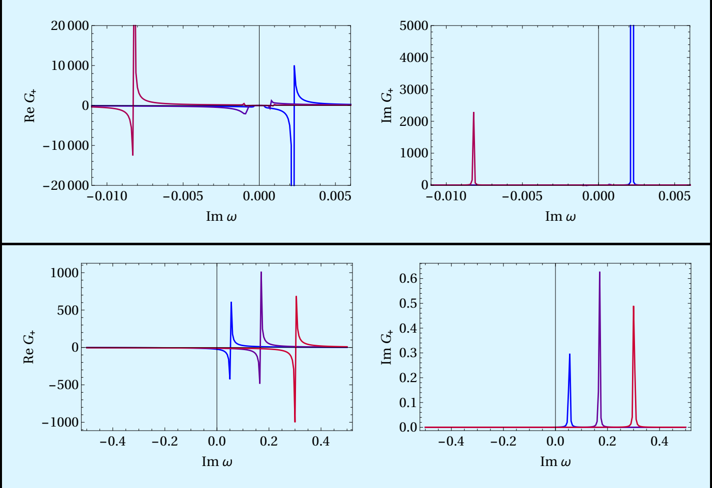


Figure: The poles of G_+ for $T/T_c = 0.95$, for $z = 2.5$ and $k = 0.1, 0.2, 0.3$ (from blue to red; top) and $T/T_c = 0.65$, for $z = 4$ and $k = 0.1, 0.3, 0.5$ (from blue to red; bottom).

4 Toward holographic Fermi liquids

The holographic portfolio

"...I don't know of a holographic model that captures everything though. Inevitably, it seems like one needs to make a choice which features one insists on keeping..."
B. Gauteraux, in the email from Dec 1 2025.

- We want an EFT with:
  - correct scaling $F \propto T^2$
  - correct entanglement entropy law
  - correct quasiparticle form ($\Im\Sigma \propto \omega^2$)
  - correct zero sound
- It might not exist!

The QFT portfolio

- Back to Landau: start from the original 1953 papers, make a mean-field theory of a Fermi liquid
- The appropriate language: patch bosonization (Houghton & Marston 1990's, Haldane 2000's, S. S. Lee 2010's)
- Related but not equivalent to the loop group approach of Senthil & Else and the WZW approach of Delacretaz (2020's)
- Divide the Fermi surface into linear (1+1)d patches, each patch can be bosonized as it is (1+1)d
- Each patch is a CFT but interpatch interaction destroys conformal invariance

Patch-bosonized field theory

- Bosonization on a patch **S**:

$$\psi(t,\mathbf{x}) = \sqrt{\Omega}\exp\left(\imath\mathbf{k_S}\cdot\mathbf{x}\right)\exp\left(\imath\sqrt{2}\phi\left(t,\mathbf{x}\right)\right)$$

- The descendant of the boson is a (1+1)d conformal field of weight $(1,0)$ *up to a finite UV cutoff* $a \sim 1/\Lambda$:

$$\begin{aligned} G_\phi(\mathbf{S},\mathbf{x}) &= \frac{\Omega^2}{4\pi}\log\frac{\imath a}{\mathbf{n_S}\cdot\mathbf{x}+\imath a} \\ G_\psi(\mathbf{S},\mathbf{x}) &= \imath\left(\frac{\Lambda}{2\pi}\right)^{D-1}\frac{\exp\left(\imath\mathbf{k_S}\cdot\mathbf{x}\right)}{\mathbf{n_S}\cdot\mathbf{x}+\imath a} \end{aligned}$$

- Phase invariance for ψ becomes gauge invariance for ϕ: $\phi \mapsto \phi + \partial\lambda$ $\Rightarrow$ formal magnetic field (not the phyiscal magnetic field!)
- This is all just for a single patch!

Patch-bosonized field theory

- Hamiltonian for the whole Fermi surface:

$$H = \int d^3x d^3y \sum_{\mathbf{S},\mathbf{S}'} V_c(\mathbf{S},\mathbf{S}';\mathbf{x}-\mathbf{y}) \left(\mathbf{n}_{\mathbf{S}} \cdot \nabla\phi(\mathbf{S};\mathbf{x})\right) \left(\mathbf{n}_{\mathbf{S}'} \cdot \nabla\phi(\mathbf{S}';\mathbf{y})\right)$$

$$V_c(\mathbf{S},\mathbf{S}';\mathbf{q}) = \frac{v_F(\mathbf{S})}{2\Omega}\delta_{\mathbf{S}\mathbf{S}'} + F(\mathbf{S},\mathbf{S}';\mathbf{q}).$$

- $\mathbf{S}$ – patch number
- $\mathbf{n}_{\mathbf{S}}$, $v_F(\mathbf{S})$ – unit normal and Fermi velocity at the patch
- $F(\mathbf{S},\mathbf{S}';\mathbf{q})$ – Landau FL interaction
- In the semiclassical regime $\mathbf{S}$ is just a coordinate, like $\mathbf{x}$!

Toward a holographic dual

- There is a SUGRA background that has the correct F scaling and correct entanglement entropy: AdS_3
- QFT dual: CFT (quantum critical point) on a line, i.e. (1+1)d
- Need to embed it into *two* additional dimensions: transverse spatial coordinate $\mathbf{y}$ and the momentum on the Fermi surface $\mathbf{S} \mapsto \theta$
- For a Fermi surface in d spatial dimensions: need $2(d-1)$ extra dimensions

- It exists!

Towards a holographic dual

- AdS_{2n+1} Chern-Simons SUGRA with AdS_3 throat in the interior (IR)
- Old story by Kraus and D'Hokker: 0908.3875, 0911.4518 – AdS_5 for simplicity but should work in any *odd* dimension (Chern-Simons term requires odd d!)
- Field content: $g_{\mu\nu}$, $U(1)$ field A_μ, dilaton ϕ
- Action – Einstein-Hilbert-Maxwell + Chern-Simons for the gauge field:

$$S = \int d^5x \left(R - \Lambda + F^2\right) + \oint d^4x + S_{\mathrm{bnd}} + \oint d^4x A \wedge F \wedge F$$

- In progress (Gecin, Čubrović, Gouteraux): look for zero sound and diffusion modes

5 Hydrodynamics without temperature?

Black hole microscopics

- Celebrated Hawking-Bekenstein formula for black hole entopy: $S_{\mathrm{BH}} = \frac{1}{4}A$
- The conflict: macroscopic entropy vs. only 3 numbers – mass, charge, angular momentum
- Resolution for very special models (supersymmetric, near-extremal): make a black hole from strings wrapping D-branes
- Strominger & Vafa 1997: D1-D5-P black hole entropy reproduced by counting open strings starting and ending on D1 and D5 branes (combinatorics)
- Supersymmetric black holes don't radiate

Page curve and the factorization puzzle

- What about radiating black holes?
- Hawking radiation poses a new puzzle – the Page curve
- Page curve reproduced in 2019 – replica islands and the factorization puzzle
- Factorization puzzle – are black holes ensemble averages after all?

Fuzzballs

- Smooth solutions – no singularities, no horizons and thus $T = 0$
- Should look like a black holes for a distant observer
- Conjectured to yield a black hole after "smearing out" or averaging
- Idea: test how this works for a low-frequency, low-momentum response in the holographic hydro theory

LLM solution

- A very simple, toy fuzzball – but highly tractable
- Lin, Lunin & Maldacena 2004: "bubbling anti-de-Sitter"
- Very simple dual matrix model (Berenstein 2004): free fermion in 2D with a constraint, solution specified by the Fermi surface
- Two 3-spheres (Ω_3 and $\tilde{\Omega}_3$) times the (t, x_1, x_2, ξ) manifold:

$$ds^2 = \frac{1}{h^2}\Big[-(dt + V_a dx^a)^2 + h^4 \left(d\xi^2 + dx_a dx^a\right) + \\ + \left(\frac{1}{2} - z\right) d\tilde{\Omega}_3^2 + \left(\frac{1}{2} + z\right) d\Omega_3^2 \Big]$$

$$h^2 = \frac{1}{\xi}\sqrt{\frac{1}{4} - z^2}, \quad \partial_\xi V_a = \frac{\epsilon_{ab}\partial_b z}{\xi}$$

- The BPS conditions yields a *linear* equation for z:

$$\partial_a^2 z + \xi \partial_\xi \left(\frac{\partial_\xi z}{\xi}\right) = 0$$

- LLM (black and white) plane - (x_1, x_2) plane at $\xi = 0$

Black & white patterns and bubbling AdS

- Finite curvature requires $z = +1/2$ ("black") or $z = -1/2$ ("white") in the LLM plane $\xi = 0$: dual to particles/holes in 2D Fermi liquid
- Geometry of black & white patterns:
  - Black disk – AdS
  - Multi-disk patterns – bubbling AdS
  - Black half-plane – pp-wave limit
  - Small deformations (rings, droplets etc) – small fluctuations
- Disk + concentric thin ring ≈ giant graviton excitation on AdS

Grayscale solutions

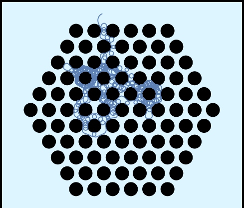
- $-1/2 < z < 1/2 \Rightarrow$ naked singularity
- $g_{\mu\nu} \sim h^2 \sim \sqrt{1/4 - z^2}/\xi \Rightarrow R \sim 1/\xi^3$
- "Good" singularity – finite potentials so can be enclosed by a horizon $\Rightarrow$ "small black hole"

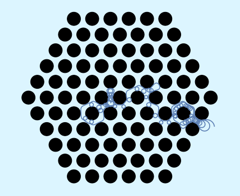
- Geometry-wise: very close to superstar solutions (Myers, Tafjord 2001)
- Matrix-wise: coarse-grained Young tableaux – smoothen the edges (Balasubramanian, Berenstein Levkowycz, Miller, Parrikar 2019)

- Natural arena for coarse-graining: we expect to get grayscale physics by averaging over small deformations of black & white solutions

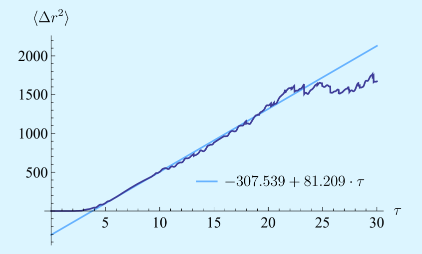
Lattice in the LLM plane

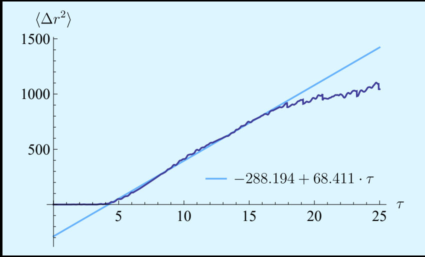
- Geodesic in the lattice ~ Lorentz gas
- Famous toy model in foundational works deriving diffusion and viscosity coefficients from the microscopics (Prigogine, Sinai, Gaspard...)
- Not quite the same as Lorentz gas as the metric is curved i.e. potential is nonzero also between the disks
- Naive expectation: ballistic or some other non-diffusive transport because of nontrivial potential





Diffusion in the LLM system





- Contrary to expectations, we get normal diffusion!
- Diffusion coefficient in the bulk behaves as $D = \alpha \frac{c}{s}$ – inversely proportional to entropy density s and proportional to central charge (coupling strength) c
- What does it tell us about the field theory side?
- We know that conformal (quantum-critical) 2D fluids have

$$D = \frac{1}{2\pi T}, \quad s = \frac{\pi}{3} c T \quad \Rightarrow \quad D = \frac{c}{6} \frac{1}{s}$$

Diffusion in the LLM system

- We know that conformal (quantum-critical) 2D fluids have

$$D = \frac{1}{2\pi T}, \quad s = \frac{\pi}{3} c T \quad \Rightarrow \quad D = \frac{1}{6}\frac{c}{s}$$

- Our result looks like a zero-temperature generalization where $\frac{1}{6} \mapsto \alpha$
- Viscosity from the gravity stress-energy tensor:

$$\eta = \frac{1}{4\pi} = \text{const.}$$

- Same universal value as for thermal quark-gluon plasmas!
- Outcome: healthy conformal hydrodynamics at zero temperature but in presence of statistical averaging. It is the ground state of some system. Which one?
- Candidates: strange metals again (also have $D \propto 1/s$), some deformation of quark-gluon plasma...

Conclusions

- Holography cannot (at present) describe the microscopics of a realistic quantum many-body system but it can provide phenomenological/EFT descriptions at strong coupling
- Should there be a classical gravity dual for Fermi liquids, or even any stable Fermi surfaces?
- Employ the existing machinery of off-equilibrium holography to find more realistic models of neutron star mergers, quantum critical fluid dynamics etc.