Hybrid star with the NJL model using a density-dependent vector coupling

Rafael P. Cardoso
PhD candidate

Collaborators: Milena Albino, Tuhin Malik, and Constança Providência

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Outline

QCD phase diagram

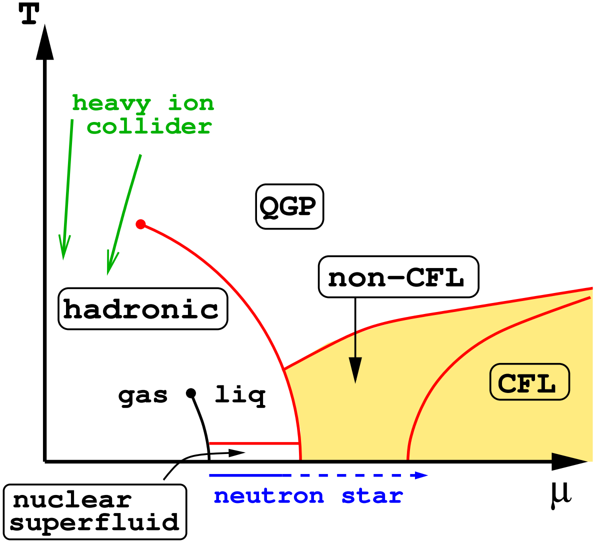
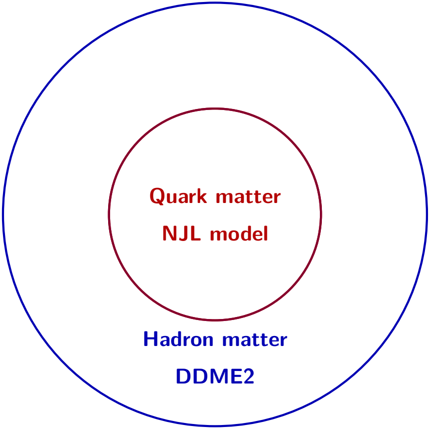


Figure: 🔗 Alford et al., Rev. Mod. Phys **80**, 1455-1515 (2008).

Outline

Hybrid star



- The quark matter is based on the NJL–SU(3) model with 4- and 8-quark interaction terms[1];
- The hadronic phase is described by the density-dependent meson-nucleon – DDME2 – model[2];
- The thermodynamical Maxwell construction is applied for the deconfinement;
- This hybrid star model is based on 🔗 Milena Albino et al., PRD **110**, 083037 (2024) .

[1] 🔗 Osipov, Brigitte & da Providência, Phys. Lett. B **634**, 48-54 (2006) .
[2] 🔗 Lalazissis et al., Phys. Rev. C **71**, 024312 (2005)

Quark matter: Nambu–Jona-Lasinio – NJL – SU(3) model

$$\mathcal{L}_{\mathsf{NJL}} = \overline{\psi}\left(i\partial\!\!\!/ - m + \mu\gamma^0\right)\psi + \frac{G}{2}\left[\left(\overline{\psi}\lambda_a\psi\right)^2 + \left(\overline{\psi}i\gamma^5\lambda_a\psi\right)^2\right] + \mathcal{L}_{\text{'t Hooft}} + \mathcal{L}_{\mathsf{I}} \ ,$$

the term which breaks the $U_A(1)$ symmetry is

$$\mathcal{L}_{\text{'t Hooft}} = \kappa\left\{\det\left[\overline{\psi}\left(1+\gamma_5\right)\psi\right] + \det\left[\overline{\psi}\left(1-\gamma_5\right)\psi\right]\right\} \ .$$

The 4- and 8-quark interaction term is

$$\begin{aligned}
\mathcal{L}_{\mathsf{I}} = & -G_\omega\left[\left(\overline{\psi}\gamma^\mu\lambda_0\psi\right)^2 + \left(\overline{\psi}\gamma^\mu\gamma_5\lambda_0\psi\right)^2\right] - G_\rho\sum_{a=1}^{8}\left[\left(\overline{\psi}\gamma^\mu\lambda^a\psi\right)^2 + \left(\overline{\psi}\gamma^\mu\gamma_5\lambda^a\psi\right)^2\right] \\
& - G_{\omega\omega}\left[\left(\overline{\psi}\gamma^\mu\lambda_0\psi\right)^2 + \left(\overline{\psi}\gamma^\mu\gamma_5\lambda_0\psi\right)^2\right]^2 \\
& - G_{\sigma\omega}\sum_{a=0}^{8}\left[\left(\overline{\psi}\lambda_a\psi\right)^2 + \left(\overline{\psi}i\gamma^5\lambda_a\psi\right)^2\right]\left[\left(\overline{\psi}\gamma^\mu\lambda_0\psi\right)^2 + \left(\overline{\psi}\gamma^\mu\gamma_5\lambda_0\psi\right)^2\right] \\
& - G_{\rho\omega}\sum_{a=1}^{8}\left[\left(\overline{\psi}\gamma^\mu\lambda_0\psi\right)^2 + \left(\overline{\psi}\gamma^\mu\gamma_5\lambda_0\psi\right)^2\right]\left[\left(\overline{\psi}\gamma^\mu\lambda_a\psi\right)^2 + \left(\overline{\psi}\gamma^\mu\gamma_5\lambda_a\psi\right)^2\right] \ .
\end{aligned}$$

Quark matter: Nambu–Jona-Lasinio – NJL – SU(3) model

$$\mathcal{L}_{\text{NJL}} = \overline{\psi}\left(i\partial\!\!\!/ - m + \mu\gamma^0\right)\psi + \frac{G}{2}\left[\left(\overline{\psi}\lambda_a\psi\right)^2 + \left(\overline{\psi}i\gamma^5\lambda_a\psi\right)^2\right] + \mathcal{L}_{\text{'t Hooft}} + \mathcal{L}_{\text{I}} \ ,$$

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& - G_{\omega\omega}\left[\left(\overline{\psi}\gamma^\mu\lambda_0\psi\right)^2 + \left(\overline{\psi}\gamma^\mu\gamma_5\lambda_0\psi\right)^2\right]^2 \\
& - G_{\sigma\omega}\sum_{a=0}^{8}\left[\left(\overline{\psi}\lambda_a\psi\right)^2 + \left(\overline{\psi}i\gamma^5\lambda_a\psi\right)^2\right]\left[\left(\overline{\psi}\gamma^\mu\lambda_0\psi\right)^2 + \left(\overline{\psi}\gamma^\mu\gamma_5\lambda_0\psi\right)^2\right] \\
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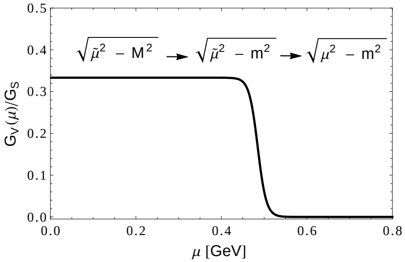
\end{aligned}$$

Chemical potential dependent vector coupling

🔗 Marcus B. Pinto, Phys. Rev. C **107**, 045807 (2023) introduced

$$G_V = \frac{G_V(0)}{1 + e^{(\mu - \mu_0)/\delta}} , \qquad (1)$$

where $G_V(0) = G_S/3$, $\mu_0 = [M(0) + \Lambda]/2$ and $\delta = 10$ MeV.



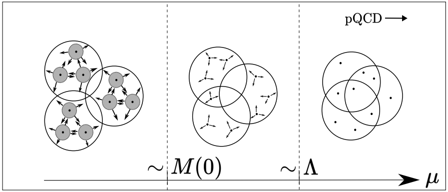


Figure: 🔗 M. B. Pinto, PRC **107**, 045807 (2023)

Density-dependent vector coupling

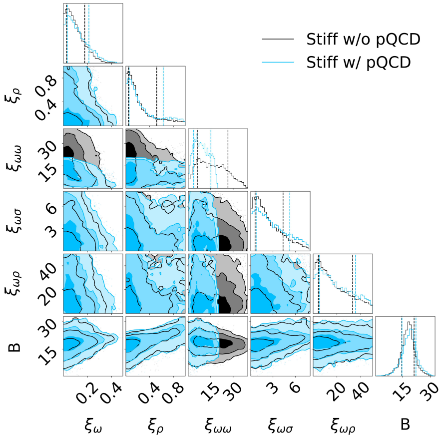


Figure: Milena Albino et al., PRD **110**, 083037 (2024) .

$\Rightarrow \xi_{\omega\omega}$ is strongly constrained by pQCD!

$$G_V = \frac{G_V(0)}{1 + e^{(\mu-\mu_0)/\delta}} , \tag{2}$$

We propose:

$$G_{\omega\omega}(\rho_B) = \frac{G_{\omega\omega}}{1 + e^{(\rho_B-\rho_c)/\delta}} , \tag{3}$$

and solve $G_{\omega\omega}$ ($\xi_{\omega\omega}$), ρ_c and δ with Bayesian inference.

Another analogous density-dependent proposal was made on Geng et al., PRD **113**, 103002 (2026).

- We do a mean-field approach (MFA) taking into account $G_{\omega\omega}(\rho_B)$;
- We calculate the Ω, the thermodynamical potential at $T = 0$;
- We get the effective quark mass, $\tilde{m}_f$, and the effective quark chemical potential, $\tilde{\mu}_f$, $f \in u, d, s$;
- $\tilde{m}_f$ and $\tilde{\mu}_f$ are written in terms of the condensates σ_f and ρ_f;

The condensates are given by $(T = 0)$

$$\sigma_f = -\frac{3}{\pi^2} \int_{k_{F\,f}}^{\Lambda} dp\, p^2 \frac{\tilde{m}_f}{E_f} \ , \ \rho_f = \frac{1}{\pi^2} {k_{F\,f}}^3 \ , \tag{4}$$

where $k_{F\,f} = \sqrt{\tilde{\mu}_f^2 - \tilde{m}_f^2}$ and $E_f = \sqrt{p^2 + \tilde{m}_f^2}$. Calculating

$$\frac{d\Omega}{d\sigma_f} = 0 \ , \ \frac{d\Omega}{d\rho_f} = 0 \ , \tag{5}$$

we get the gap equations and guarentee thermodynamical consistence[3]. We add a bag constant $P \to P + B$ to allow that the chiral symmetry breaking can happen at the same baryon density as the deconfiment.

[3] 🔗 Buballa, Phys. Rept. **407**, 205-376 (2005) .

Outline

Priors

Λ (MeV)	$m_{u,d}$ (MeV)	m_s (MeV)	$G\Lambda^2$	$\kappa\Lambda^5$
623.58	5.70	136.60	3.34	-13.67

Parameters	Units	min	max
ξ_ω	...	0	0.5
ξ_ρ	...	0	1
$\xi_{\omega\omega}$	...	0	30
$\xi_{\sigma\omega}$	...	0	8
$\xi_{\rho\omega}$	...	0	50
B	MeV.fm^{-3}	0	30
ρ_c	fm^{-3}	0	1.5
δ	fm^{-3}	0.1	1.0

The adimensional coupling constants are defined as

$$\xi_i = 2\frac{G_i}{G} \ , \ G_i = \{G_\omega, G_\rho\} \ , \ \xi_{ij} = 16\frac{G_{ij}}{G^4} \ , \ G_{ij} = \{G_{\omega\omega}, G_{\sigma\omega}, G_{\rho\omega}\} \ . \quad (6)$$

NJL

$$(\xi_\omega, \xi_\rho, \xi_{\omega\omega}, \xi_{\sigma\omega}, \xi_{\rho\sigma}, B)$$

NJL–pQCD constrained

$$(\xi_\omega, \xi_\rho, \xi_{\omega\omega}, \xi_{\sigma\omega}, \xi_{\rho\sigma}, B)$$

NJL–$G_{\omega\omega}(\rho_B)$

$$(\xi_\omega, \xi_\rho, \xi_{\omega\omega}, \xi_{\sigma\omega}, \xi_{\rho\sigma}, B, \rho_c, \delta)$$

- The range of phase transition around $1.7\rho_0$, $\rho_0 = 0.16$ fm^{-3};
- EoS of hadron and quark phase have an intersection;
- Mass-radius diagram satisfying the NICER data from pulsars J0030+0451, J0740+6620 and J0437+4715;
- Gravitational wave data from GW170817 from LIGO/Virgo;
- For the NJL we also imposed pQCD constrains[4].

We utilized the `PyMultinest` as nested sampling method with `BILBY`.

[4] 🔗 Komoltsev & Kurkela, Phys. Rev. Lett. **128**, 202701 (2022).

Outline

The pressure is $P = -\Omega$, and for $T = 0$,

$$\varepsilon = -P + \sum_{f=u,d,s} \mu_f \, \rho_f \ , \tag{7}$$

the speed of sound is given by

$$c_s^2 = \frac{dP}{d\varepsilon} \ , \tag{8}$$

trace anomaly[5],

$$\Delta = \frac{1}{3} - \frac{P}{\varepsilon} \ , \tag{9}$$

the measure of conformability[6]

$$d_c = \sqrt{\Delta^2 + \Delta'^{\,2}} \, , \ \Delta' = \frac{d\Delta}{d\varepsilon} \ , \tag{10}$$

and the polytropic index

$$\gamma = \frac{d \ln P}{d \ln \varepsilon} \ . \tag{11}$$

[5] Fujimoto et al., Phys. Rev. Lett. **129**, 252702 (2022) .
[6] Annala et al., Nature Commun. **14**, 8451 (2023) .

Outline

Equation of state

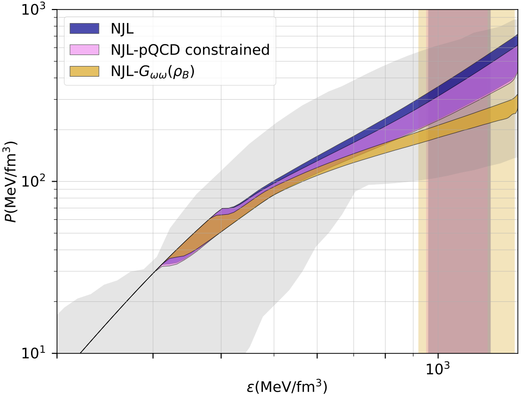


Figure: The equations of state (EoS) distributions as function of energy density for different models. Sets are shown at a 90% credible intervals (CIs). Gray plots from 🔗 Annala et al., Nature Phys. **16**, 907-910 (2020) . Vertical bands are 90% CIs for maximum ε at maximum NS mass.

Speed of sound

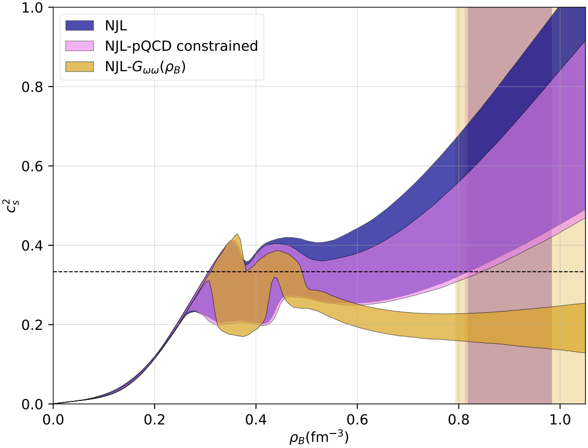


Figure: The speed of sound distribution as a function of the baryon density. Vertical lines are 90% CIs central density at maximum NS mass.

Mass-radius diagram

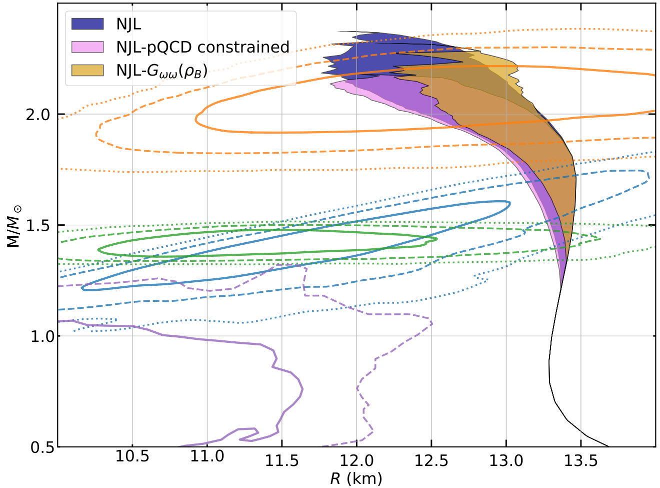
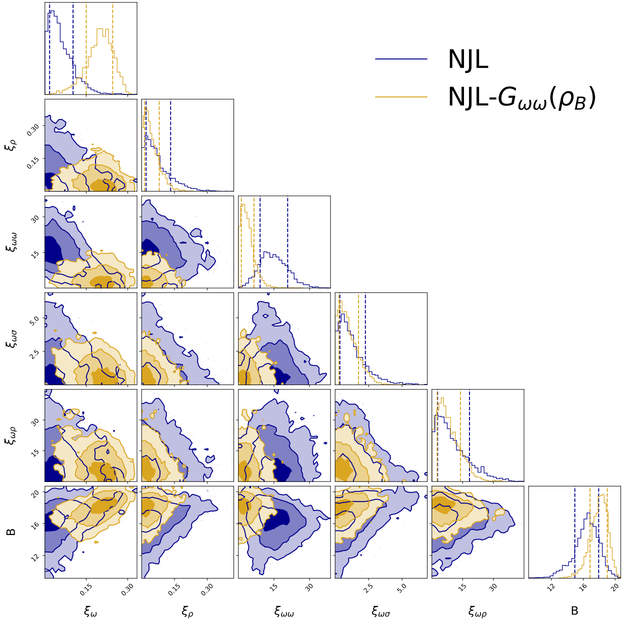


Figure: Mass-radius diagram distributions. Observational data from J0030 (blue), J0740 (orange), J0437 (green) and HESS (purple). Sets are shown at a 90% (CL).


NJL
NJL-$G_{\omega\omega}(\rho_B)$
ξ_ρ
$\xi_{\omega\omega}$
$\xi_{\omega\sigma}$
$\xi_{\omega\rho}$
B
ξ_ω
0.15
0.30
15
30
2.5
5.0
12
16
20

Selected corner plots

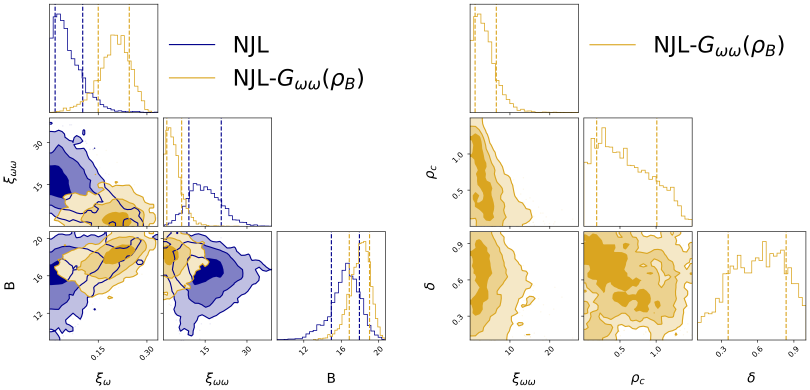


Figure: **Left:** Corner plots of the parameters $(\xi_\omega, \xi_{\omega\omega}, B)$ for the two models – NJL and NJL-$G_{\omega\omega}(\rho_B)$. **Right:** Corner plots of the parameters $(\xi_{\omega\omega}, \rho_c, \delta)$ of the NJL-$G_{\omega\omega}(\rho_B)$ model.

Trace anomaly

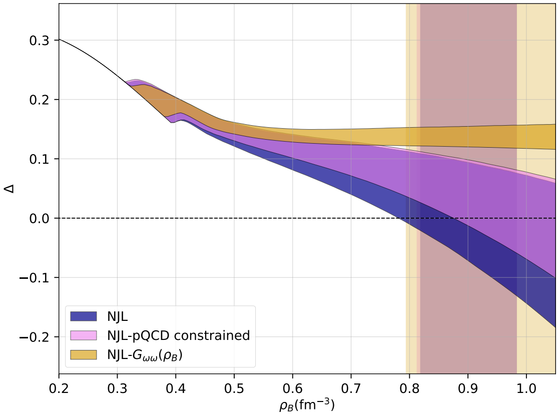


Figure: Trace anomaly distribution as a function of the baryon density.

Measure of conformability

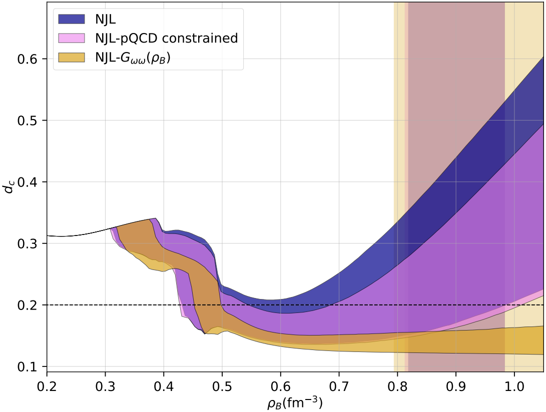


Figure: Measure of conformability as function of the baryon density.

Polytropic index

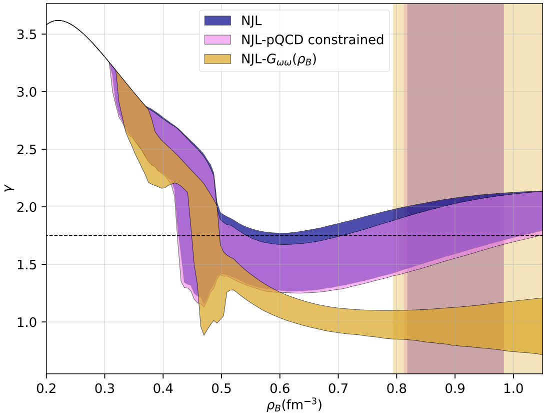


Figure: Polytropic index as function of the baryon density.

Outline

Summary

- Hybrid stars with the NJL-$G_{\omega\omega}(\rho_B)$ satisfy observational data;
- We achived speed of sound $c_s^2 < 1$ until $7\rho_0$;
- Besides the peaks of the deconfinement and the quark s appearance, c_s^2 is conformal ($\rho_B \leq 7\rho_0$);
- $\Delta > 0$ is a signature of conformabillity[7];
- The measure of conformability $d_c < 0.2$ for $\sim 2.8\rho_0$–$7\rho_0$;
- $d_c < 0.2$ is a signature of quark matter[8];
- The polytropic index $\gamma < 1.75$ (from $\sim 3\rho_0$) is a signature of deconfinement[9];
- In the future we can apply this density-dependent model in another terms;
- Also include another contributions, a.k.a, magnetic fields, temperature, rotation, etc.

[7] Fujimoto et al., Phys. Rev. Lett. **129**, 252702 (2022) .
[8] Annala et al., Nature Commun. **14**, 8451 (2023) .
[9] Annala et al., Nature Phys. **16**, 907-910 (2020)

Collaborators:

Milena Albino (UC)
Tuhin Malik (UC)
Constança Providência (UC)
William Rafael Tavares (UERJ/UC)
Sidney dos Santos Avancini (UFSC)
Ricardo Luciano Sonego Farias (UFSM)

rafaelpachecocardoso@gmail.com

Obrigado!