Probing the Stability of Neutron Star Magnetic Fields through Configurational Entropy

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Overview and definitions

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Configurational entropy:

- From Gleiser and Stamatopoulos (2012), consider f(x) ∈ $L^2(R^d)$:

$$F(\mathbf{k}) = \int \mathrm{d}^d x\, f(\mathbf{x}) e^{-i\mathbf{k}\cdot\mathbf{x}} \quad \Rightarrow \quad f(\mathbf{k}) = \frac{|F(\mathbf{k})|^2}{\int \mathrm{d}^d k' \, |F(\mathbf{k}')|^2} \quad \Rightarrow \quad \boxed{S_C[f] = -\int d^d k\, f(\mathbf{k}) \ln f(\mathbf{k})}$$

Fourier transform — Modal fraction — CE

Magnetic field model:

- Following Haskell et al. (2008) → analytical solution of the hydro-magnetic equilibrium

$$\nabla \times \left[\frac{(\nabla \times \mathbf{B}) \times \mathbf{B}}{\rho}\right] = 0 \quad \rightarrow \quad A = \frac{B_k R^2}{(\lambda^2 - 1)^2 y} \left\{ 2\pi \frac{\lambda y \cos(\lambda y) - \sin(\lambda y)}{\pi\lambda\cos(\pi\lambda) - \sin(\pi\lambda)} + \left[\left(1 - \lambda^2\right) y^2 - 2\right] \sin y + 2y \cos y \right\}$$

$$\mathbf{B} = \left(\frac{2A\cos\theta}{r^2}, -\frac{\sin\theta}{r}\frac{dA}{dr}, \frac{\pi\lambda A \sin\theta}{Rr}\right)$$

with

$$\pi\lambda\left(\lambda^2 - 1\right)\cos(\pi\lambda) - \left(3\lambda^2 - 1\right)\sin(\pi\lambda) = 0 \quad \text{and} \quad \lambda \neq 1$$

M. Gleiser and N. Stamatopoulos, Phys. Lett. B, vol. 713, pp. 304-307, July 2012
B. Haskell et al., *MNRAS*, vol. 385, pp. 531-542, Mar. 2008

CE results

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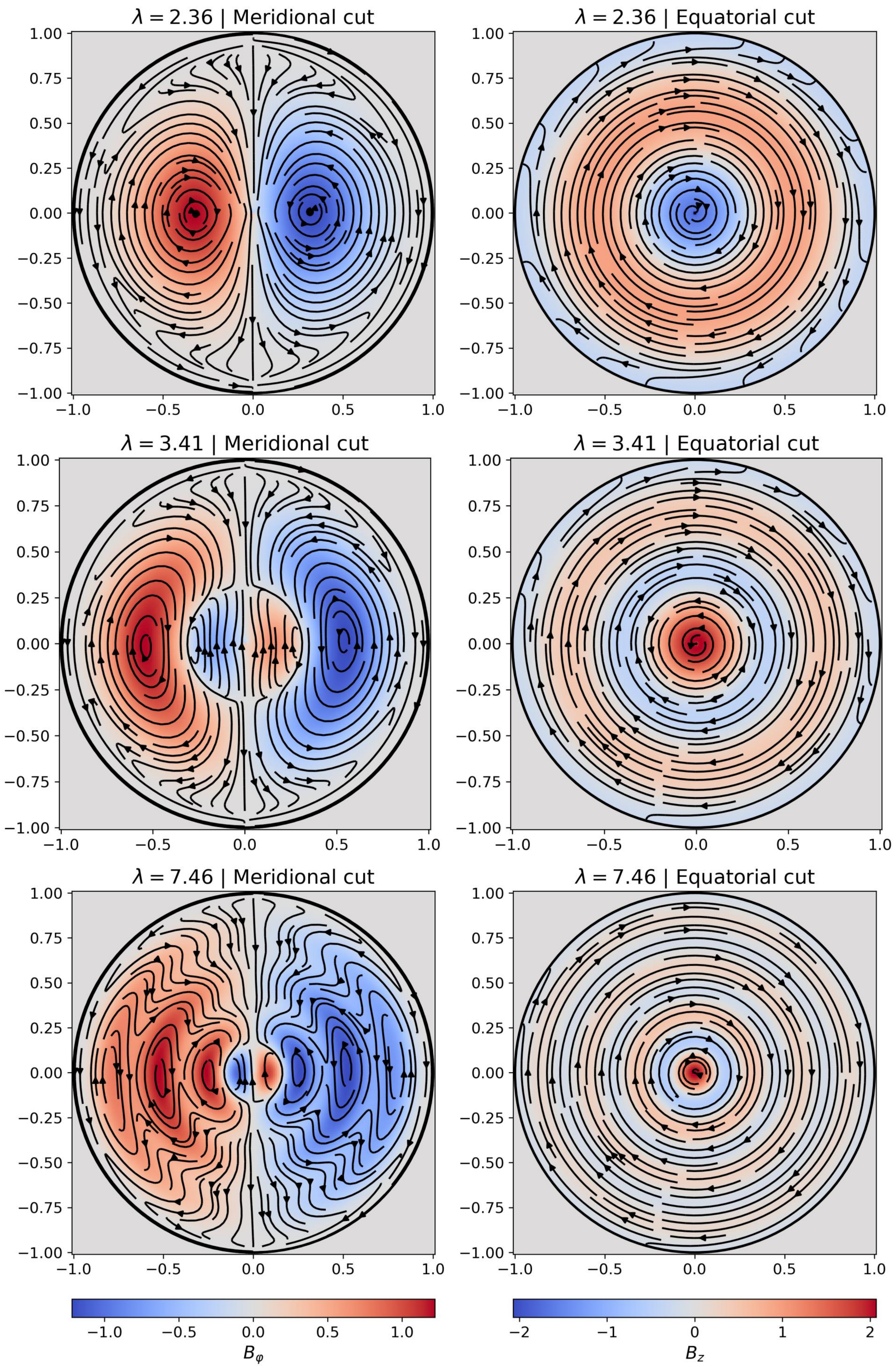
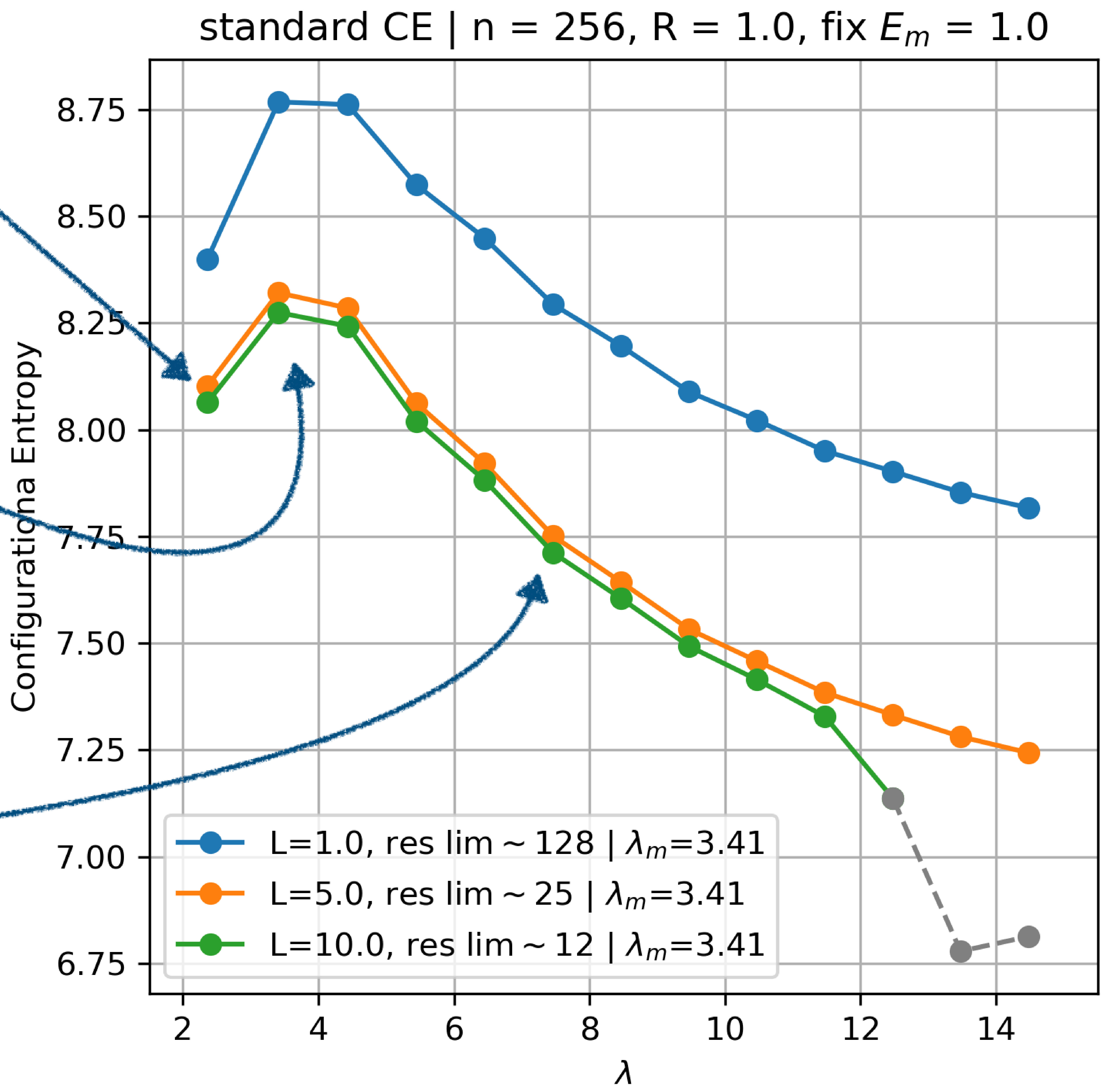
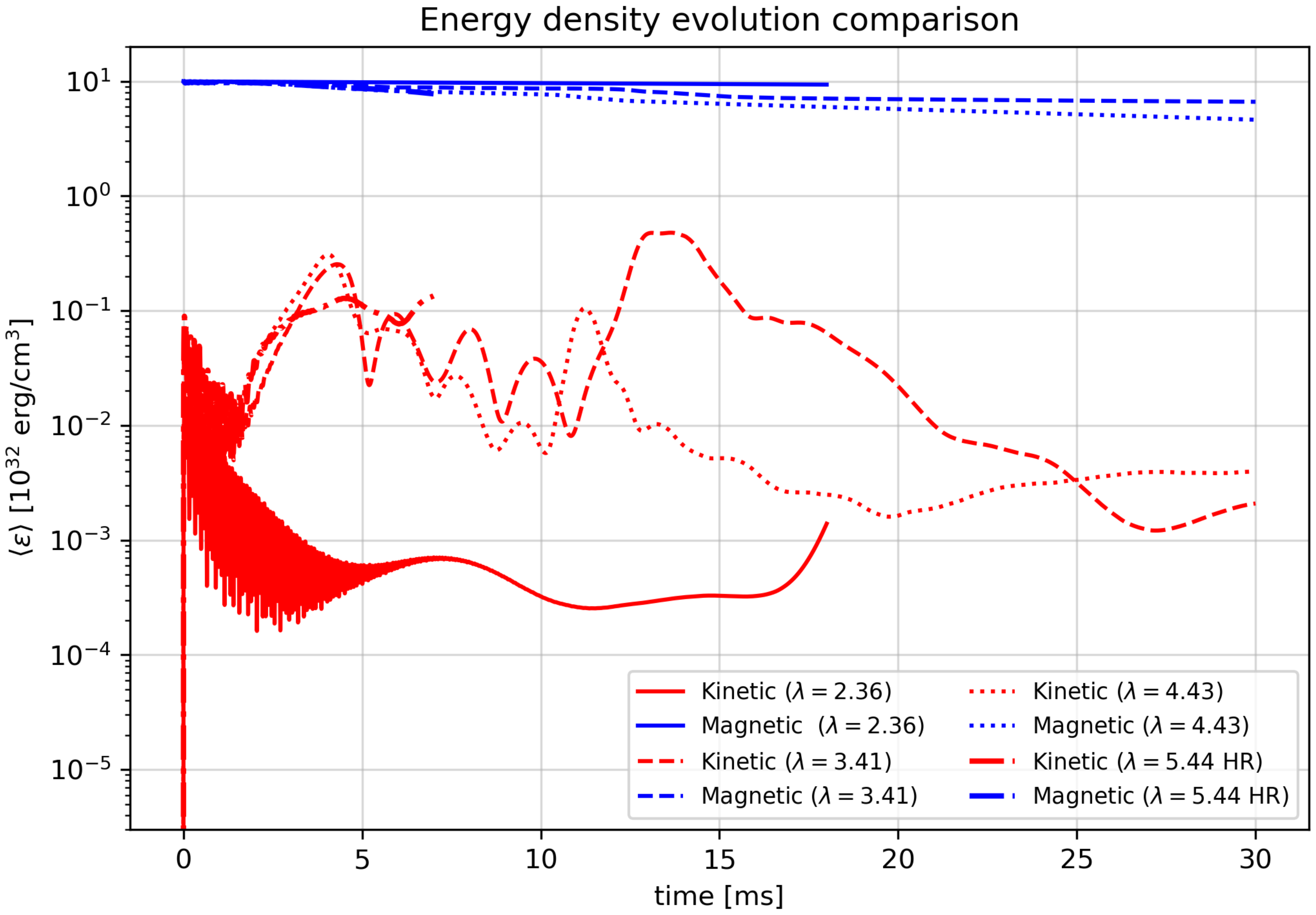
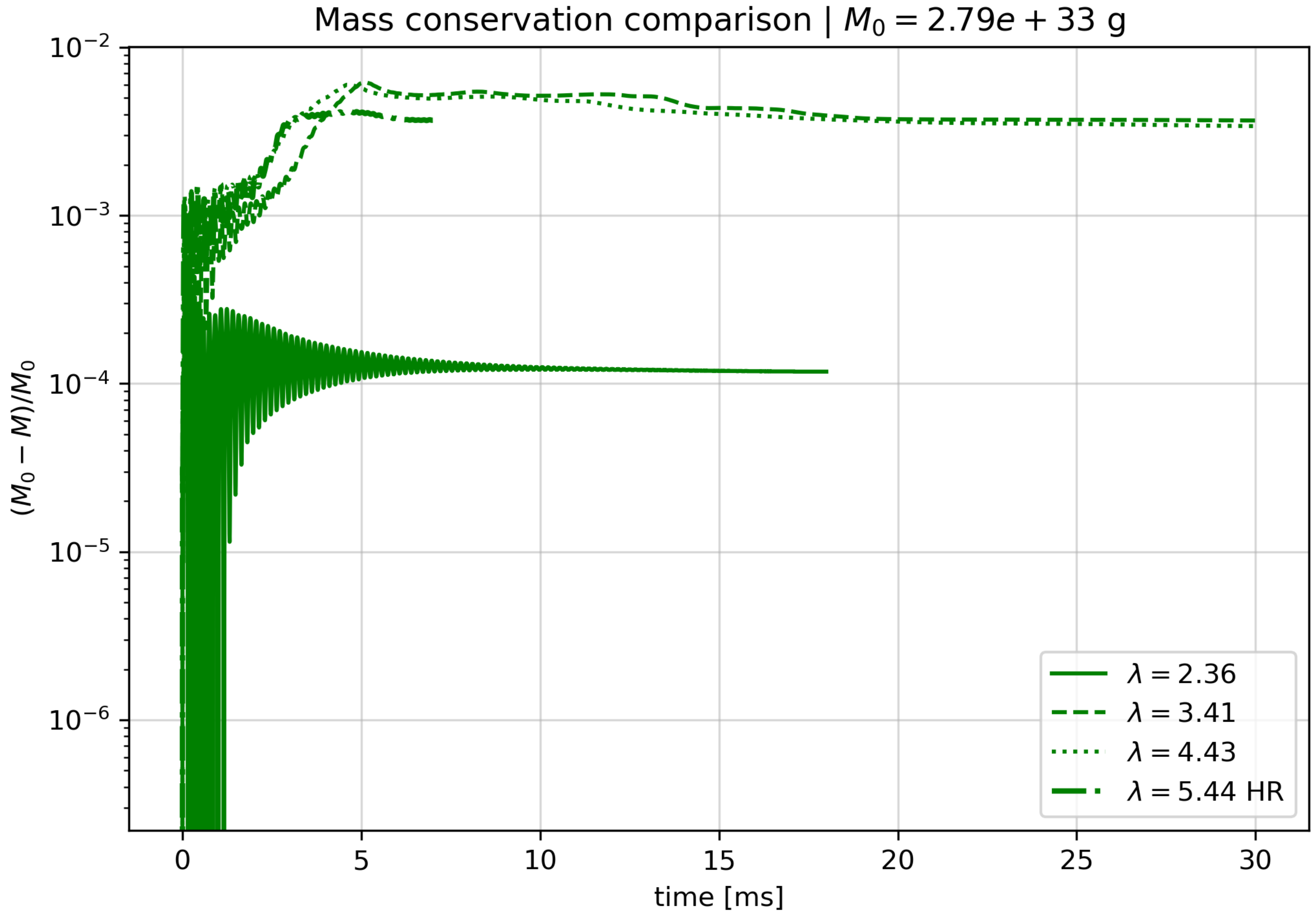




Simulations summary

Eigenvalue	Resolution	$\tau_{A,0}$	t_{stop}	$\lvert\Delta\varepsilon_m\rvert/\varepsilon_{m,0}$	$\lvert\Delta M\rvert/M_0$	Stable
$\lambda = 2.36$	75x40x40	1.75 ms	18 ms	0.066	0.12×10^{-3}	✓
$\lambda = 3.41$	75x40x40	1.51 ms	30 ms	0.338	3.67×10^{-3}	✗
$\lambda = 4.43$	75x40x40	1.46 ms	30 ms	0.538	3.40×10^{-3}	✗
$\lambda = 5.44$	110x32x64	1.40 ms	7 ms	0.223	3.69×10^{-3}	✗



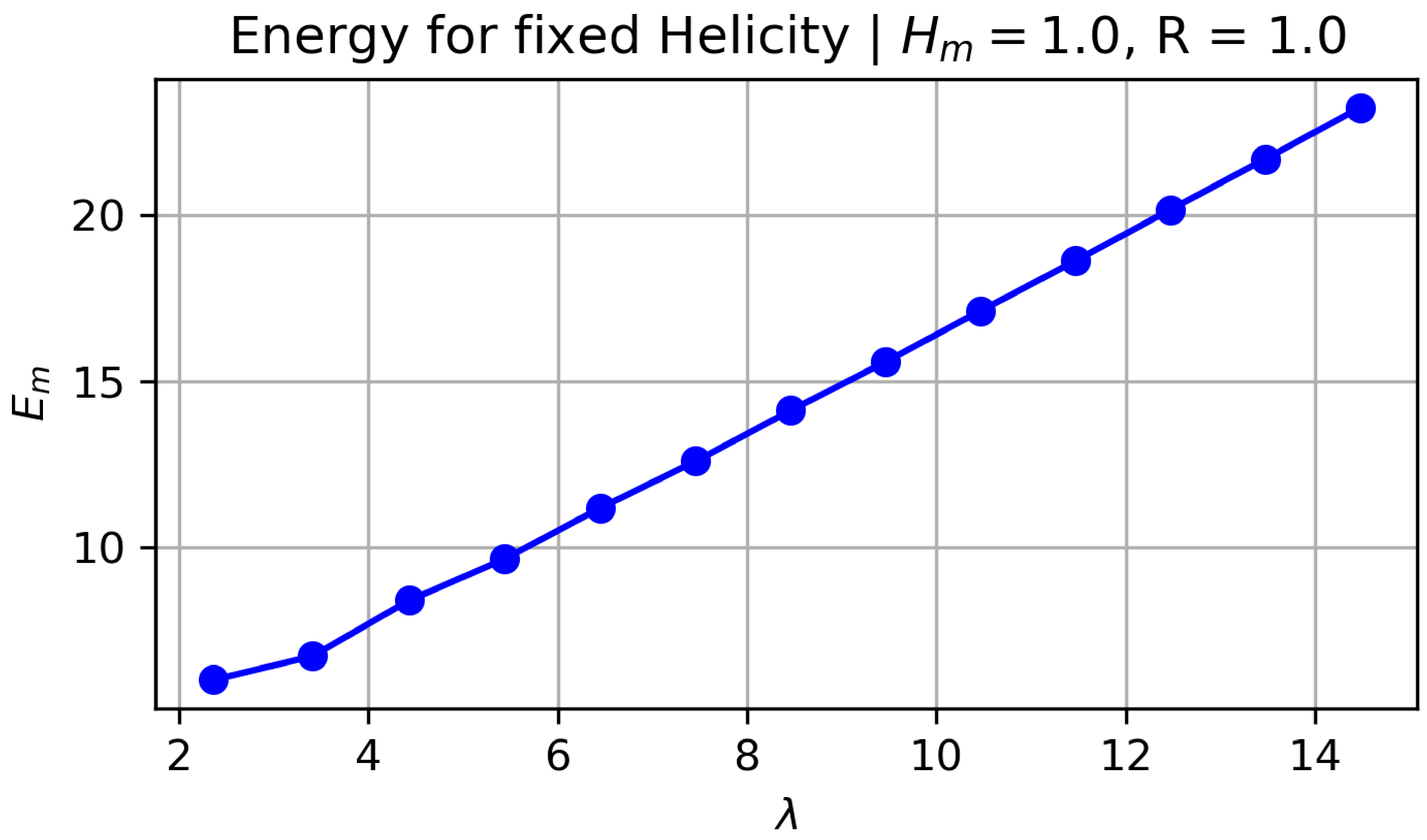


Discussion and conclusions

- Consistent with **Taylor relaxation**:

Magnetised plasmas in the force-free limit evolve towards a state that minimizes energy for a given magnetic helicity

➡ λ = 2.36 represents this minimum energy state



- **CE able to capture** the transition between stable and unstable configurations

➡ useful tool to study more complex **numerical models** from simulations

- Toroidal component of the field **cannot be arbitrarily** strong or complex → likely results in instabilities

➡ of interest for CW astronomy

J. B. Taylor, Phys. Rev. Lett., vol. 33, pp. 1139-1141, Nov. 1974