Entrainment effects in the inner crust and outer core of a neutron star

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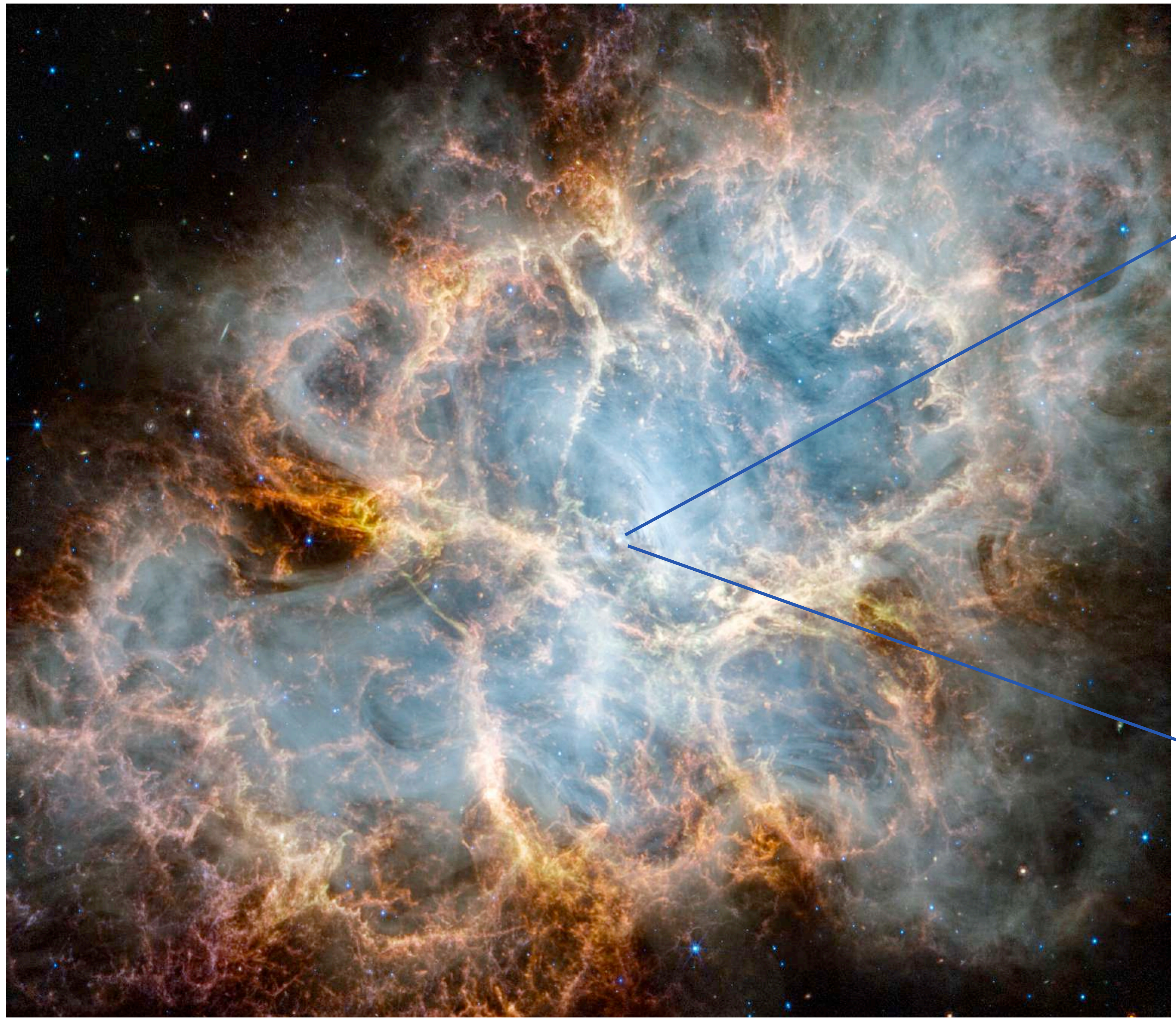
Neutron stars are cool

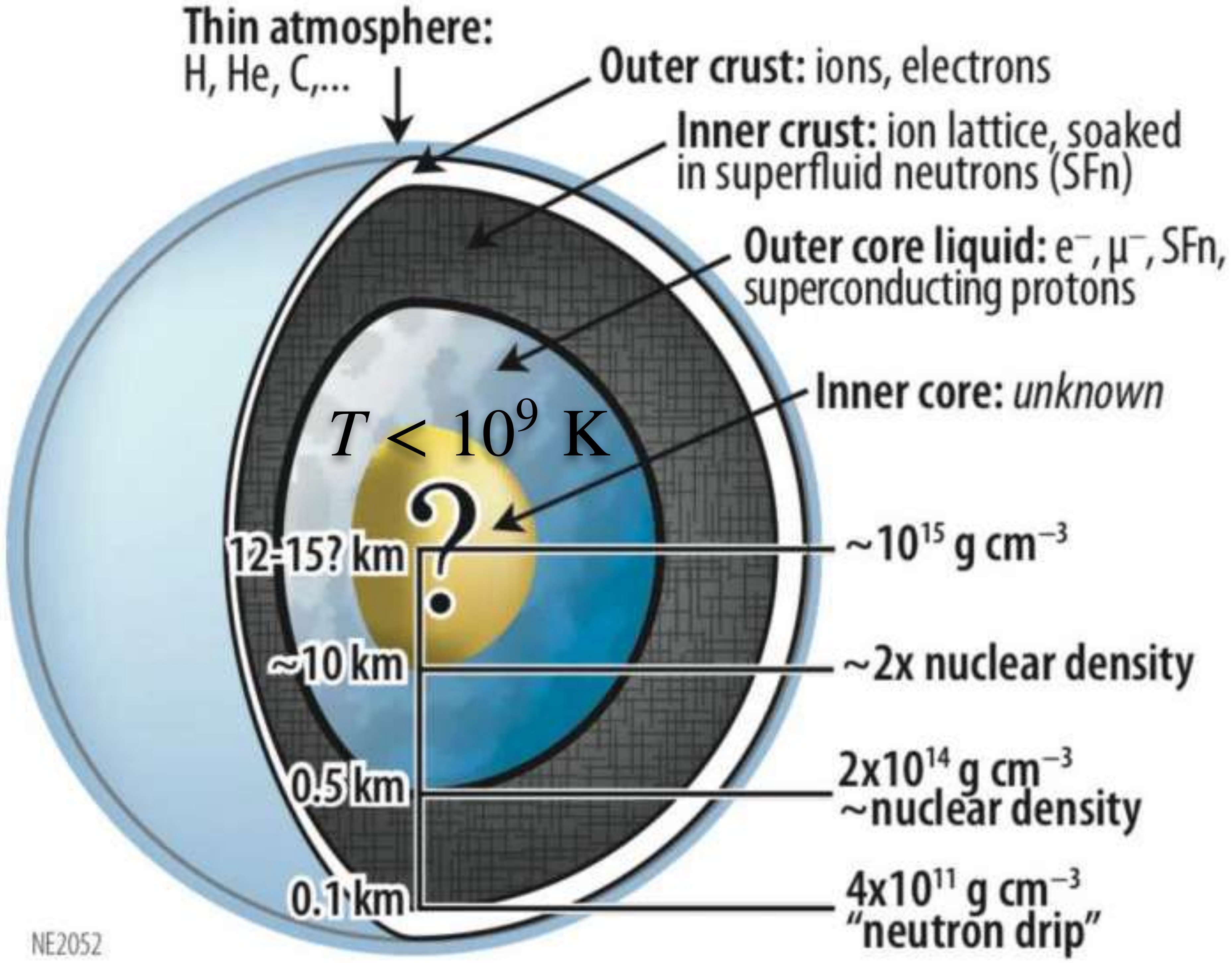
Neutron stars are the extremely compact remnants of gravitational core-collapse supernova explosions.
Initially very hot ($T \sim 10^{12}$ K), they cool down to 10^9 K within days.



NASA, ESA, CSA, STScI, T. Tamim (Princeton University)

$1 - 2\ M_{\odot}$

$A \sim 10^{57}$



Gendreau et al., Proc. SPIE 8443, Space Telescopes and Instrumentation 2012: Ultraviolet to Gamma Ray, 844313; https://doi.org/10.1117/12.926396

$\Delta \sim 1$ MeV $(T_C \sim 10^{10}$ K$)$

$\varepsilon_F \sim 10 - 100$ MeV $(T_F \sim 10^{11} - 10^{12}$ K$)$

Super clocks with glitches

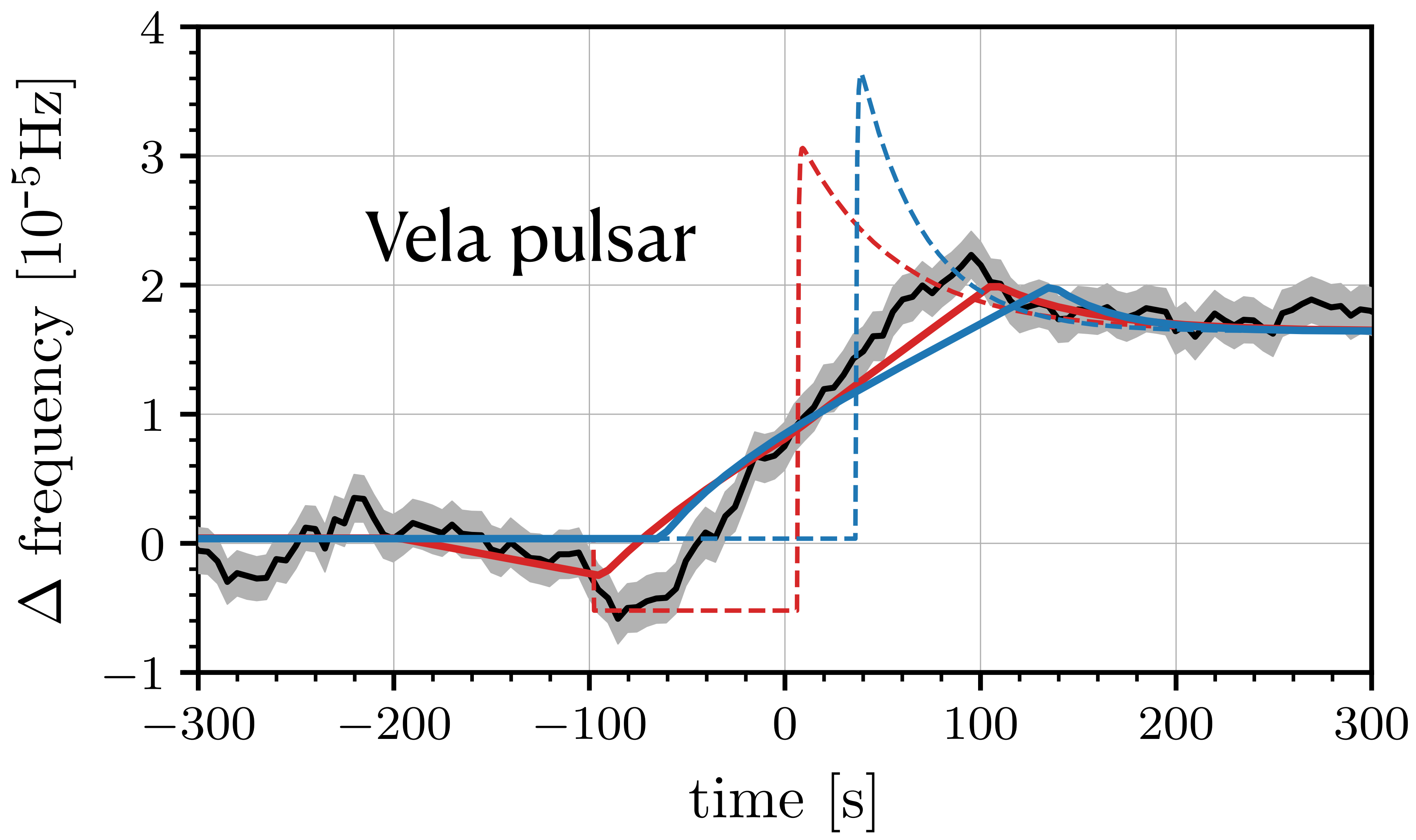
Pulsars are spinning very rapidly with extremely stable periods, $P \gtrsim 1$ ms and $\dot{P} \sim 10^{-21}$, outperforming the best atomic clocks!

- Still, more than 200 pulsars **suddenly spin up** (< minute) in some cases multiple times
- This is observed in radio timing as **frequency glitches**

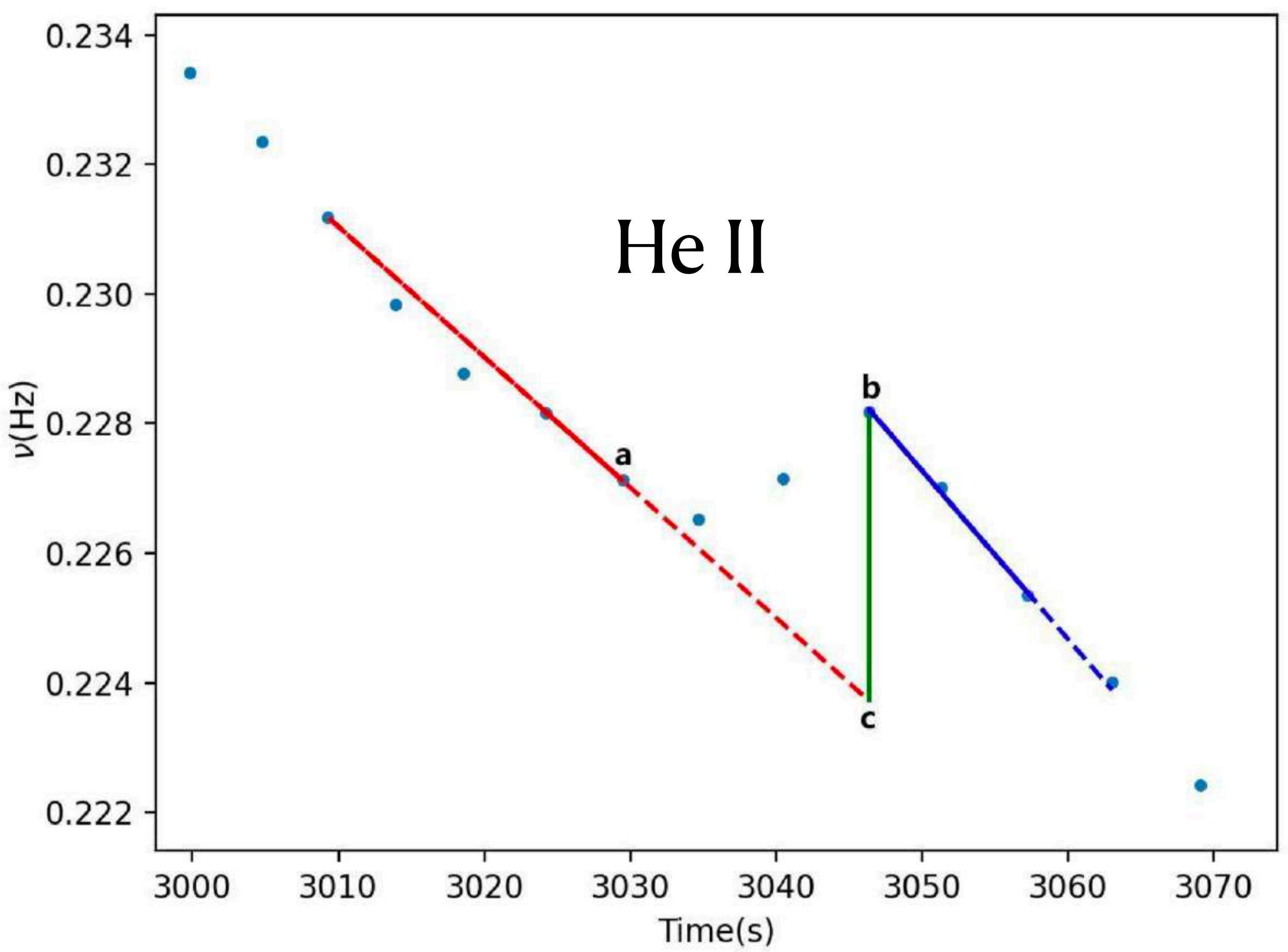
Antonopoulou, Haskell, Espinoza, Rep. Prog. Phys. 85, 126901 (2022); Zhou et al., Universe 8, 641 (2022)

Similar phenomena have been **experimentally observed in superfluid helium**

J.S. & S.J. Tsakadze, JLTP 39, 649 (1980)



Ashton et al.,Nature Astronomy 3, 1143 (2019)



Haoyang Zhou & R. Zieve (2025)

Pulsar glitches provide the most compelling evidence for the superfluid dynamics

Mutual entrainment in superfluid neutron-star cores

Neutron and proton superfluids are mutually entrained by **nondissipative effects** of the kind originally discussed by Andreev & Bashkin for ^{4}He-^{3}He superfluid mixtures.

Andreev & Bashkin, Sov. Phys. JETP 42, 164 (1975)

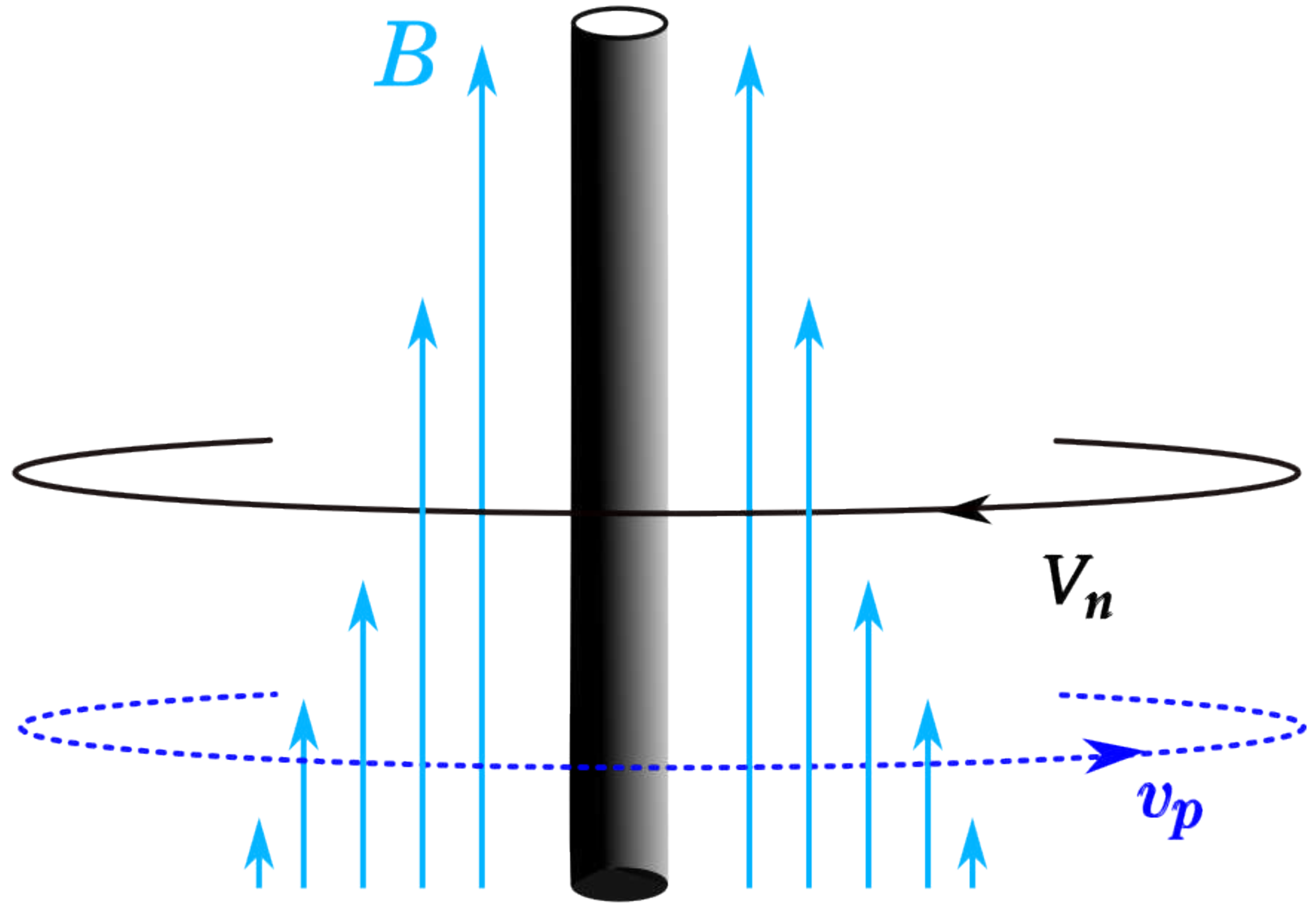
The mass currents $\boldsymbol{\rho_q}$ (q=n,p) are not aligned with the superfluid velocities $\boldsymbol{V_q}$ (in the normal-fluid frame):

$$\boldsymbol{\rho_n} \equiv \rho_n \boldsymbol{v_n} = \rho_{nn} \boldsymbol{V_n} + \rho_{np} \boldsymbol{V_p}$$
$$\boldsymbol{\rho_p} \equiv \rho_p \boldsymbol{v_p} = \rho_{pn} \boldsymbol{V_n} + \rho_{pp} \boldsymbol{V_p}$$

Neutron vortices carry a fractional magnetic quantum flux:

$$\Phi_\star = \int \boldsymbol{B}\cdot d\boldsymbol{S} = \frac{\rho_{pn}}{\rho_{pp}}\Phi_0 \qquad \text{with} \qquad \Phi_0 = \frac{hc}{2e}$$

Sedrakyan &Shakhabasyan, Astrofizika 8, 557 (1972); ibid. 16, 727 (1980)



Electrons scattering off the magnetic field leads to a **dissipative drag force** between the neutron superfluid and all the charged particles

Alpar, Langer, Sauls, ApJ 282, 533 (1984)

However, the induced magnetic field may exceed the critical one leading to **clusters of quantised proton fluxoids**

Sedrakian & Sedrakian, ApJ 447, 305 (1995)

With implications for pulsar glitches

Sourie & Chamel, MNRAS 493, L98 (2020)

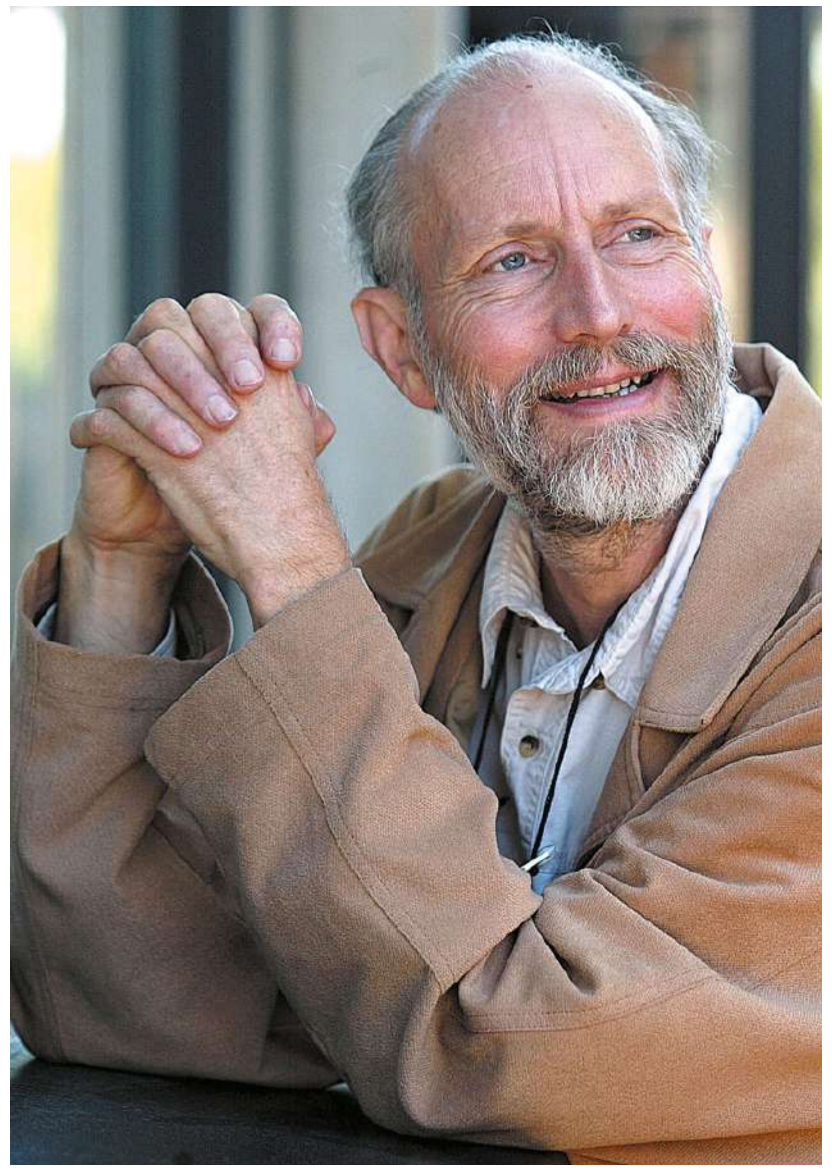
Mutual entrainment in superfluid neutron-star cores

At the global scale of a rotating neutron star, the averaged angular momenta can be decomposed as

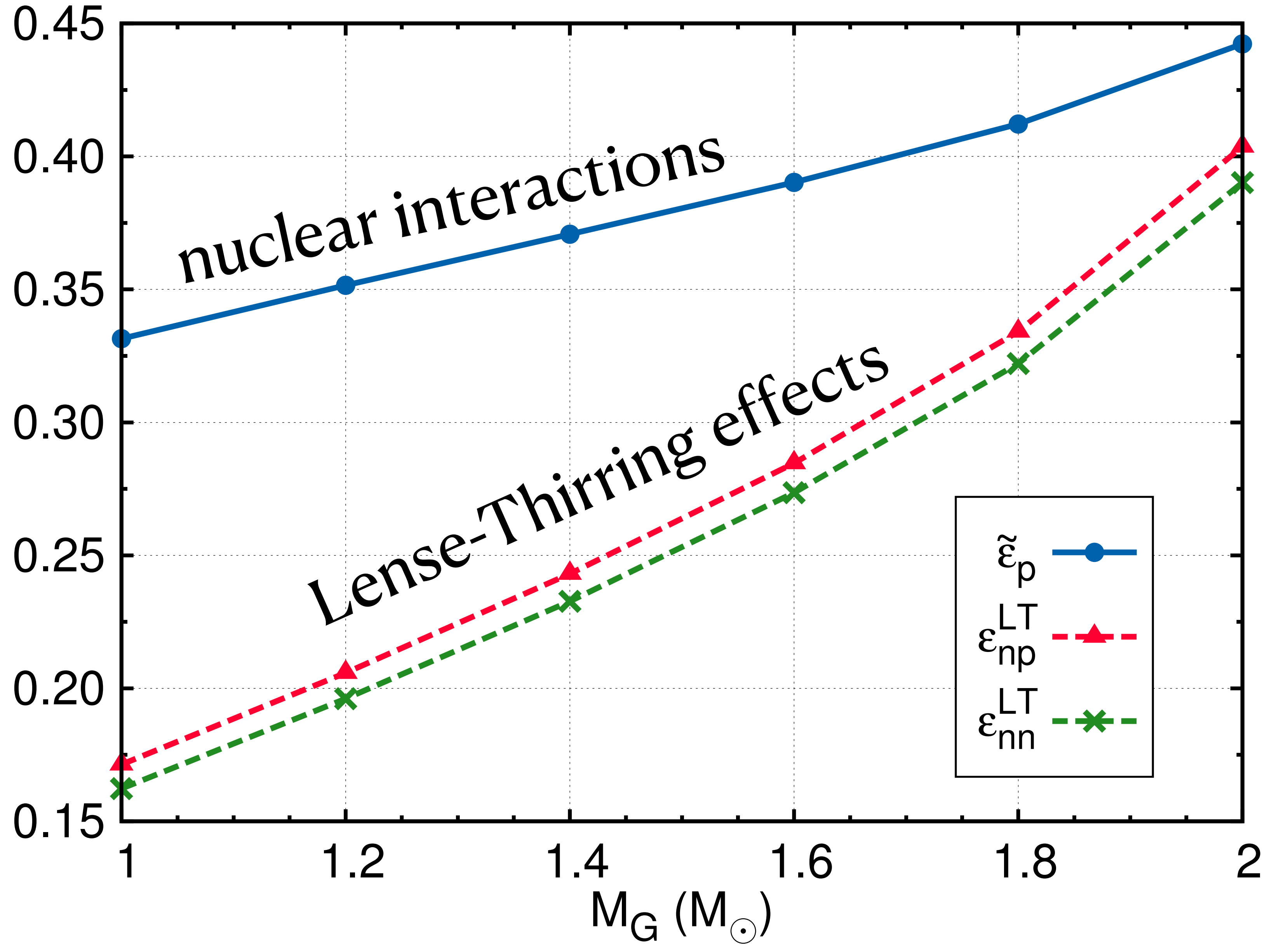
$$J_n = I_{nn}\Omega_n + I_{np}\Omega \qquad I_n = I_{nn} + I_{np}$$
$$J_p = I_{pn}\Omega_\mathrm{n} + I_{pp}\Omega \qquad I_p = I_{pn} + I_{pp}$$
$$I_{np} = I_{pn}$$

General relativity induces additional entrainment couplings due Lense-Thirring effects

Carter, Ann. Phys. 95, 53 (1975)



Pulsar spinning at 11.19 Hz



$$\frac{I_{XY}}{I_X} = \frac{\tilde{\epsilon}_X - \epsilon^{\mathrm{LT}}_{YX}}{1 - \epsilon^{\mathrm{LT}}_{YX} - \epsilon^{\mathrm{LT}}_{XX}}$$

- $\tilde{\epsilon}_X$ is the entrainment effect caused by nuclear interactions
- $\epsilon^{\mathrm{LT}}_{XY}$ is the frame-dragging effect on fluid Y caused by fluid X

Sourie et al., Phys. Rev. D 93, 083004 (2016); MNRAS 464, 4641 (2017)

Nuclear superfluid dynamics within TDHFB

The dynamics of nuclear systems can be described by the **time-dependent Hartree-Fock-Bogoliubov equations (*q*=*n*,*p*)**

$$\begin{pmatrix} h_q(\boldsymbol{r},t)-\mu_q & \Delta_q(\boldsymbol{r},t) \\ \Delta_q(\boldsymbol{r},t)^* & -h_q(\boldsymbol{r},t)^*+\mu_q \end{pmatrix} \begin{pmatrix} \psi_1^{(q)}(\boldsymbol{r},t) \\ \psi_2^{(q)}(\boldsymbol{r},t) \end{pmatrix} = \mathrm{i}\hbar \frac{\partial}{\partial t} \begin{pmatrix} \psi_1^{(q)}(\boldsymbol{r},t) \\ \psi_2^{(q)}(\boldsymbol{r},t) \end{pmatrix}$$

These ressemble the **Bogoliubov - De Gennes equations** but the Hamiltonian is generally more complicated

$$h_q(\boldsymbol{r},t) \equiv -\boldsymbol{\nabla}\cdot \frac{\hbar^2}{2m_q^{\oplus}(\boldsymbol{r},t)}\boldsymbol{\nabla} + U_q(\boldsymbol{r},t) \underbrace{-\frac{\mathrm{i}}{2}\left\{\boldsymbol{I_q}(\boldsymbol{r},t),\boldsymbol{\nabla}\right\}}_{\text{contribution from the flow}} + \dots$$

$\dfrac{\hbar^2}{2m_q^{\oplus}(\boldsymbol{r},t)} = \dfrac{\delta E}{\delta \tau_q(\boldsymbol{r},t)}$ and $\boldsymbol{I_q}(\boldsymbol{r},t) = \dfrac{\delta E}{\delta \boldsymbol{j_q}(\boldsymbol{r},t)}$ arise from the same couplings in the energy because of Galilean invariance:

$$E = \dots C_+\left[(n_n+n_p)(\tau_n+\tau_p)-(\boldsymbol{j_n}+\boldsymbol{j_p})^2\right] + C_-\left[(n_n-n_p)(\tau_n-\tau_p)-(\boldsymbol{j_n}-\boldsymbol{j_p})^2\right]$$

corresponding to **effective 2-body contact interactions** with terms of the form

$p_{ij}^2\,\delta(\boldsymbol{r}_i-\boldsymbol{r}_j)+\delta(\boldsymbol{r}_i-\boldsymbol{r}_j)\,p_{ij}^2$ and $\boldsymbol{p}_{ij}\cdot\delta(\boldsymbol{r}_i-\boldsymbol{r}_j)\boldsymbol{p}_{ij}$ with $\boldsymbol{p}_{ij} = -i\hbar(\boldsymbol{\nabla}_i-\boldsymbol{\nabla}_j)/2$

If $C_\pm = 0$, $m_q^{\oplus}(\boldsymbol{r},t) = m_q$ and $I_q(\boldsymbol{r},t) = 0$

Entrainment effects from TDHFB

The **superfluid velocity** is defined from the pair potential

$$\Delta_q(\boldsymbol{r},t) = |\Delta_q(\boldsymbol{r},t)| e^{\mathrm{i}\phi_q(\boldsymbol{r},t)} \quad \text{by} \quad \boldsymbol{V_q}(\boldsymbol{r},t) = \frac{\hbar}{2m_q}\nabla\phi_q(\boldsymbol{r},t)$$

VS

The **mass current** is given by

$$\boldsymbol{\rho_q}(\boldsymbol{r},t) = \frac{m_q}{m_q^{\oplus}(\boldsymbol{r},t)}\hbar\boldsymbol{j_q}(\boldsymbol{r},t) + \rho_q(\boldsymbol{r},t)\frac{\boldsymbol{I_q}(\boldsymbol{r},t)}{\hbar}$$

In **homogeneous matter with stationary flows**, the TDHFB equations can be solved exactly at any temperature T

For $T \ll T_{cq}$ and $V_q \leq V_{Lq}$

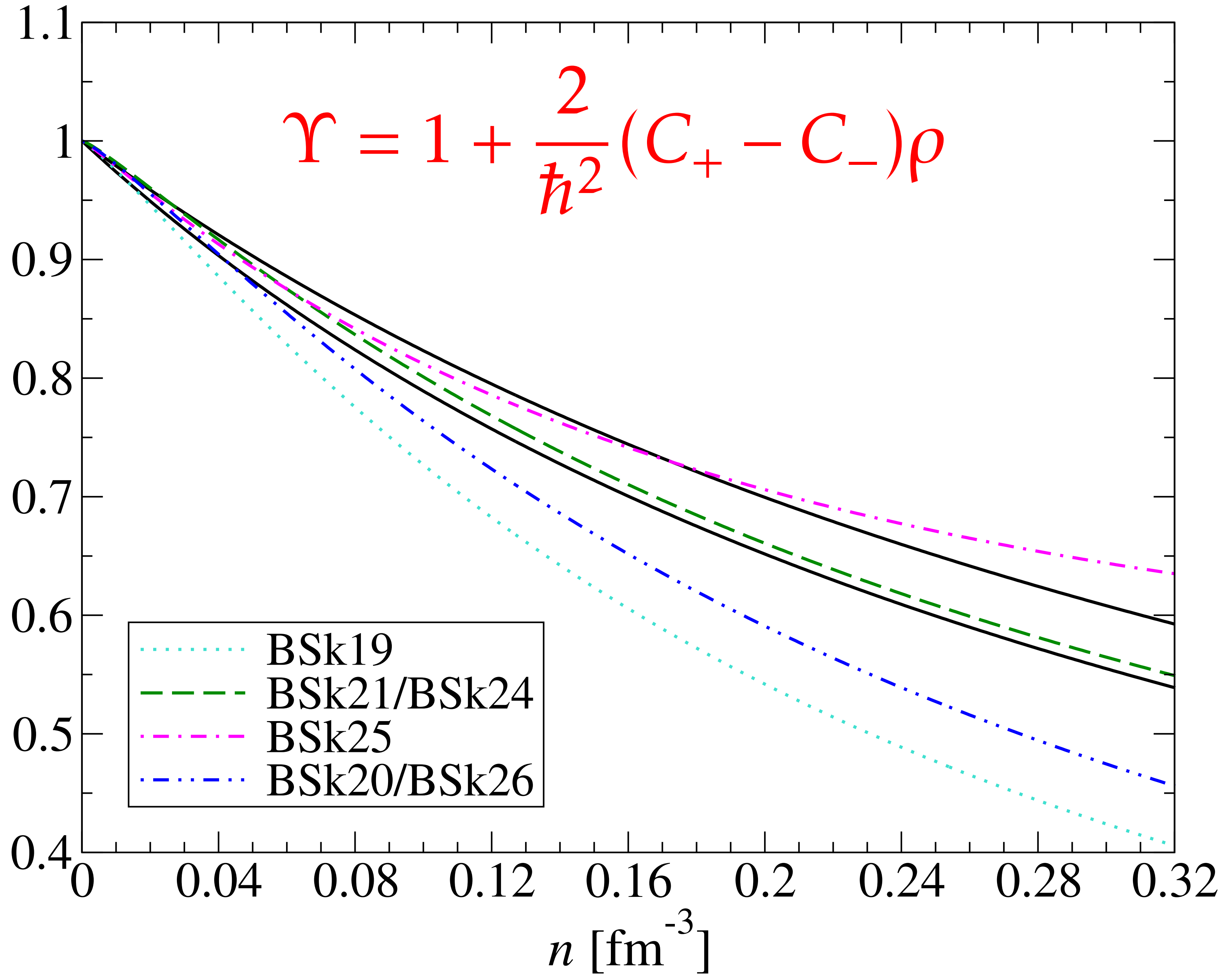
$$\rho_{nn} = \frac{1}{2}\rho(1+\eta) - \frac{1}{4}\rho\left(1-\eta^2\right)(1-\Upsilon)$$

$$\rho_{pp} = \frac{1}{2}\rho(1-\eta) - \frac{1}{4}\rho\left(1-\eta^2\right)(1-\Upsilon)$$

$$\rho_{np} = \frac{1}{4}\rho\left(1-\eta^2\right)(1-\Upsilon) = \rho_{pn}$$

$$\rho = \rho_n + \rho_p$$

$$\eta = (\rho_n - \rho_p)/\rho$$



Entrainment effects can be large independently of the pairing gaps

Chamel & Allard, PRC 100, 065801 (2019); Allard & Chamel, PRC 103, 025804 (2021); Universe 7(12), 470 (2021)

Entrainment in neutron-star crust

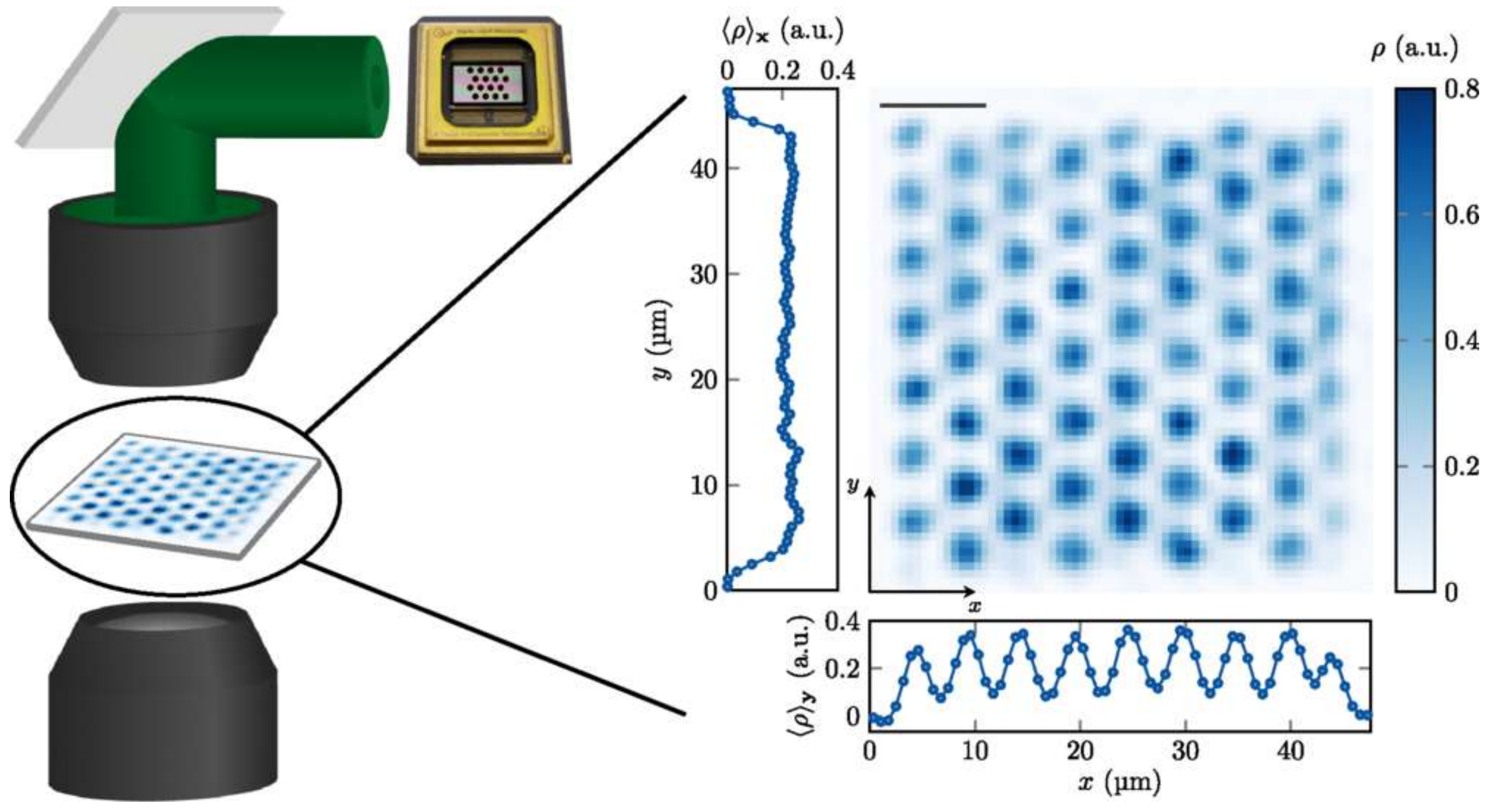
Whenever translational symmetry is broken, a superflow with velocity $\boldsymbol{V}$ induces a mass current $\bar{\boldsymbol{\rho}} = \rho_s \boldsymbol{V}$

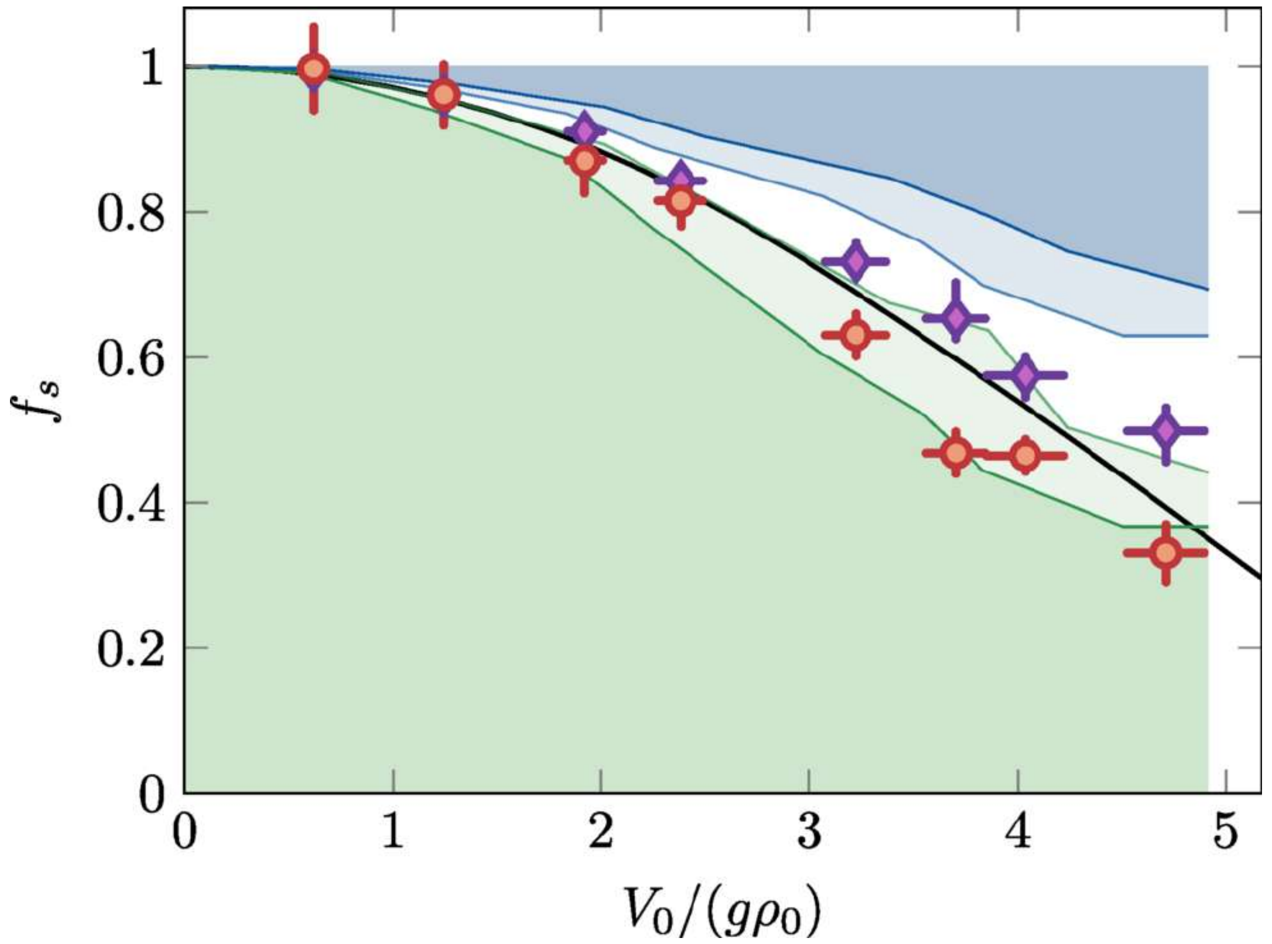
Superfluid fraction $f_s \equiv \dfrac{\rho_s}{\bar{\rho}} < 1$ Leggett, J. Stat. Phys. 93, 927 (1998)

Such depletion has been measured in bosonic condensates of cold atoms in optical traps and supersolids

Chauvet et al., PRL 130, 226003 (2023); Tao et al., PRL 131, 163401 (2023); Rabec et al., PRL 136, 133401 (2026); Biagioni et al., Nature 629, 773 (2024)

Rabec et al. (2026)





... and has been studied in fermionic condensates

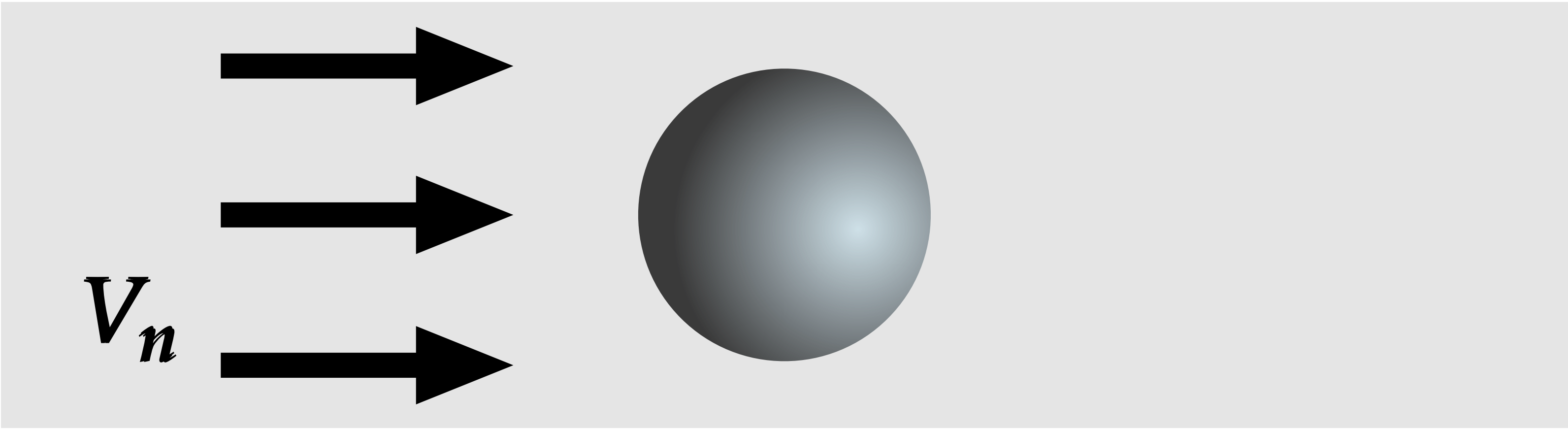
Orso, PRL 99, 250402 (2007)
Orso & Stringari, PRA 109, 023301 (2024)

A similar effect was predicted in the inner crust of a neutron star

Carter, Chamel, Haensel, Nucl. Phys. A748, 675 (2005)

Entrainment and effective masses

In the crust frame

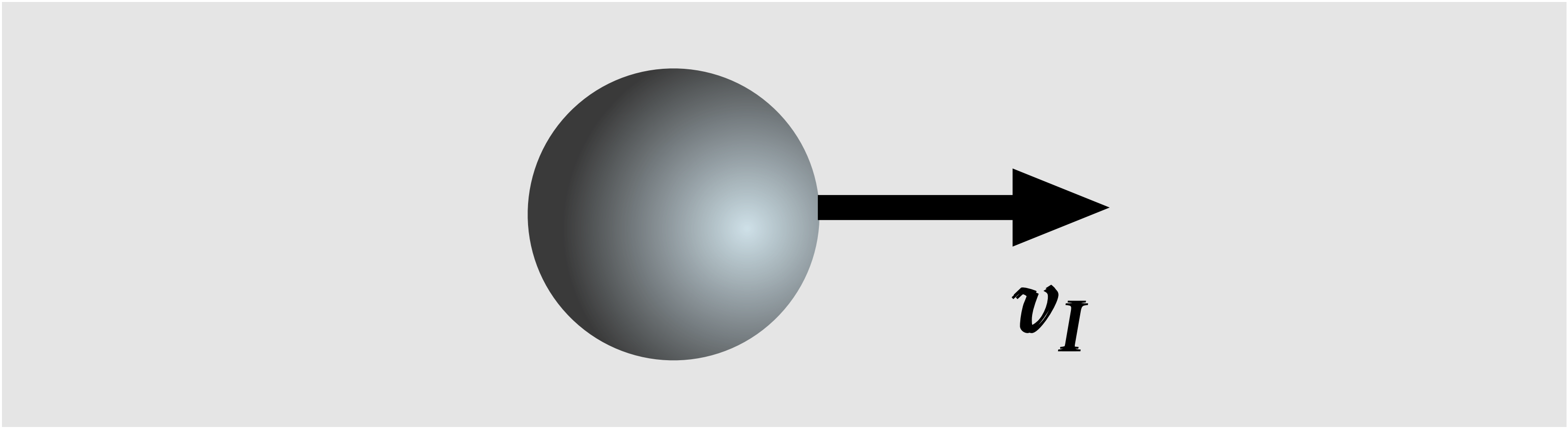


A neutron superflow with velocity $\boldsymbol{V_n}$ induces an average neutron mass current

$$\bar{\boldsymbol{\rho}}_{\boldsymbol{n}} = \rho_{n,s}\boldsymbol{V_n} = \bar{\rho}_n \frac{m_n}{m_n^\star}\boldsymbol{V_n}$$

It is as if neutrons had en **effective mass** $m_n^\star > m_n$

In the superfluid frame



Ions moving with a velocity $\boldsymbol{v_I} = -\boldsymbol{V_n}$ carry on average a mass current

$$\bar{\boldsymbol{\rho}}_{\boldsymbol{I}} = n_I m_I^\star \boldsymbol{v_I}$$

as if they had an **effective mass** $m_I^\star$

These are two different aspects of the same entrainment effects

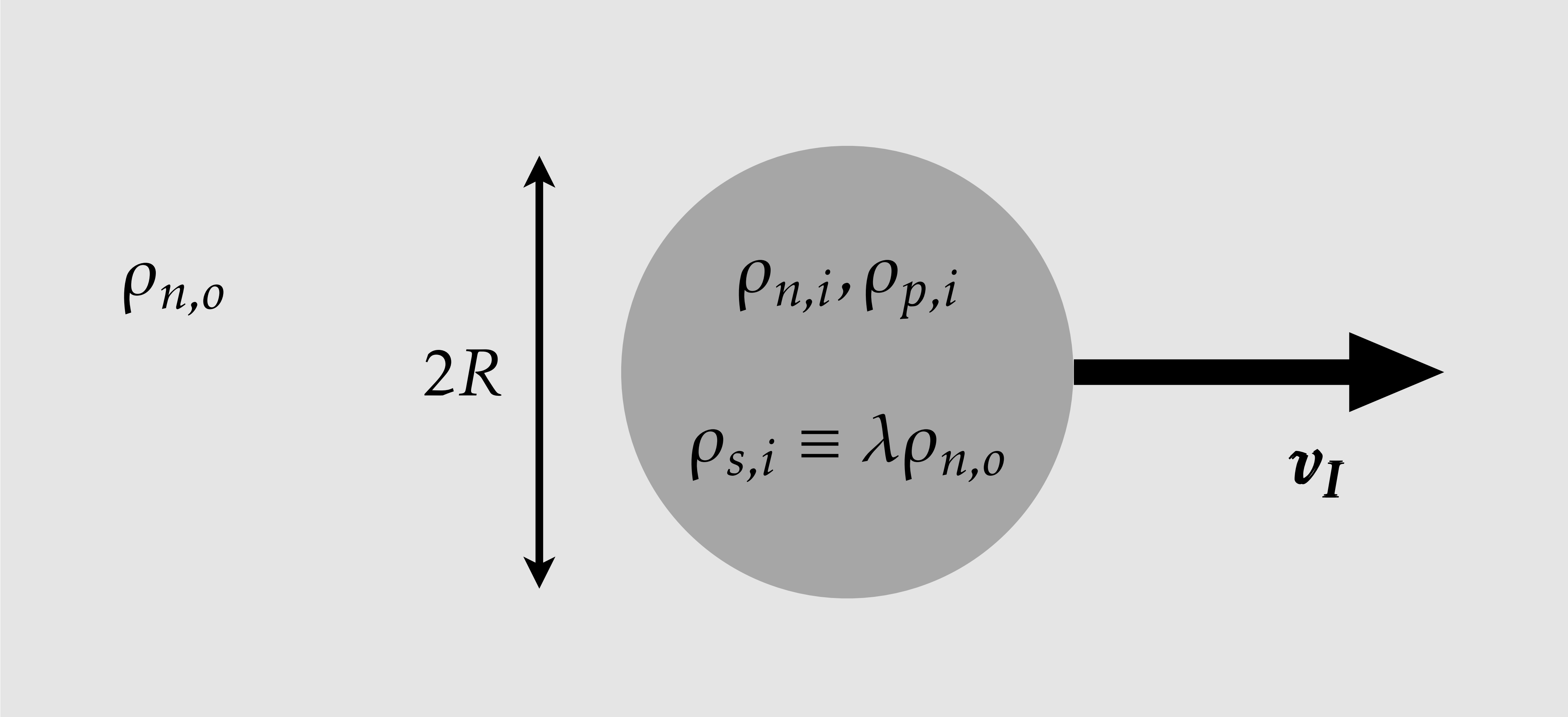
$$m_n^\star = m_n \frac{\bar{\rho}_n}{\rho_{n,s}} \qquad m_I^\star = \frac{\bar{\rho} - \rho_{n,s}}{n_I}$$

The fraction $1 - \rho_{n,s}/\bar{\rho}_n$ of neutrons are **entrained** by the crust

Carter, Chamel, Haensel, Int. J. Mod. Phys. D15, 777 (2006)

Effective ion mass from hydrodynamics

The effective mass of a single ion can be calculated from **classical hydrodynamics assuming potential flow**



Effective ion mass $m_I^\star = m_I + \frac{(\lambda - 1)^2}{\lambda + 2} m_d$

Mass of displaced neutrons $m_d = \frac{4}{3}\pi R^3 \rho_{n,o}$

Mass of nucleons transported by ions $m_I = m_I^0 - \frac{4}{3}\pi R^3 \rho_{s,i}$

Mass of nucleons located inside ions $m_I^0 = \frac{4}{3}\pi R^3 (\rho_{n,i} + \rho_{p,i})$

Epstein & Baym, ApJ 328, 680 (1988); 333, 880 (1988)

The mass density $\rho_{s,i} \equiv \lambda \rho_{n,o}$ of nucleons participating to the superfluid motion be determined by hydrodynamics alone

- Impenetrable ions $\rho_{s,i} = 0$ Sedrakian, Astr. Sp. Sci. 236, 267(1996)

  $m_I = Z m_p + N m_n$

- All nucleons are superfluid $\rho_{s,i} = \rho_{n,i} + \rho_{p,i}$ Magierski & Bulgac, Nucl. Phys. A738, 143 (2004)

  $m_I = 0$

- Permeable ions $\rho_{s,i} = \rho_{n,o}$ Martin & Urban, PRC 94, 065801 (2016)

  $m_I = Z m_p$

A more microscopic approach is needed

Superfluid density and Bragg scattering

Band theory of solids

For neutrons flowing in a **periodic potential** $U_n(\boldsymbol{r})$ with velocities $V_n \ll V_{Ln}$ in the limit of **weak pairing** $\Delta \ll \varepsilon_F$

$$\rho_{n,s} \approx \frac{m_n^2}{12\pi^3\hbar^2} \sum_\alpha \int_{\rm BZ} {\rm d}^3\boldsymbol{k}\, \delta(\varepsilon_{\alpha\boldsymbol{k}} - \varepsilon_F) |\nabla_{\boldsymbol{k}} \varepsilon_{\alpha\boldsymbol{k}}|^2 = \frac{m_n^2}{12\pi^3\hbar^2} \sum_\alpha \int_{\rm F} |\nabla_{\boldsymbol{k}} \varepsilon_{\alpha\boldsymbol{k}}| {\rm d}\mathcal{S}^{(\alpha)}$$

Carter, Chamel, Haensel, Nucl. Phys. A748, 675 (2005)

Same expression as obtained for a dilute Fermi superfluid in a 1D optical lattice

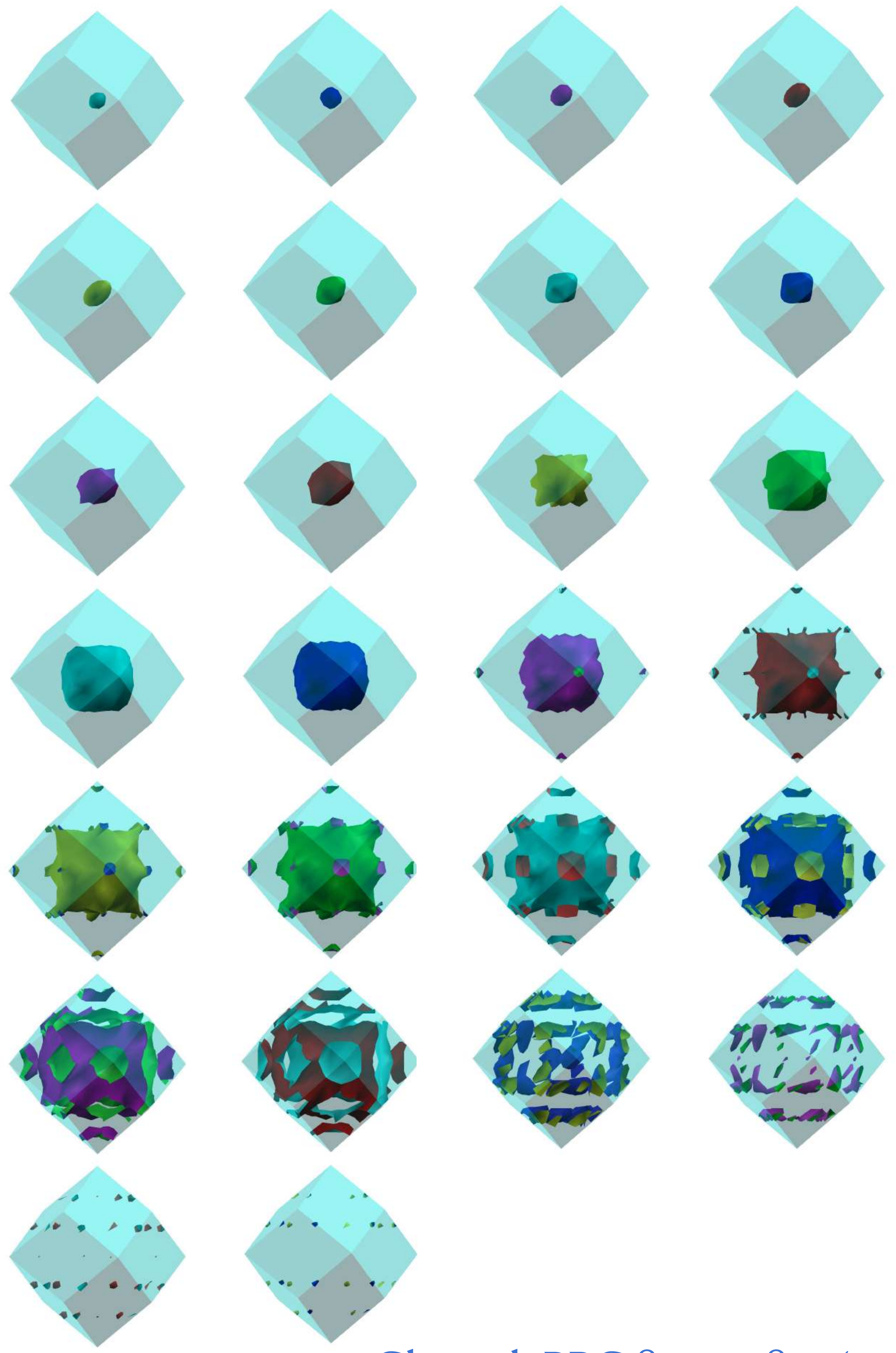
Pitaevskii, Stringari, Orso, PRA71, 053602 (2005)

In intermediate layers at densities ~0.005-0.04 fm^{-3}
Bragg scattering leads to **strong distorsions of the Fermi surface**

- locally flat bands $|\nabla_{\boldsymbol{k}} \varepsilon_{\alpha\boldsymbol{k}}| \sim 0$
- shrinking of Fermi surface $\mathcal{S}_F \ll 4\pi k_F^2$

The superfluid fraction is suppressed $\rho_{n,s} \sim 10\% \bar{\rho}_n$

Chamel, Nucl. Phys. A747, 109 (2005)



Chamel, PRC 85, 035801 (2012)

Superfluid density and pairing

Including constant BCS pairing

$$\rho_{n,s} = \frac{m_n^2}{24\pi^3\hbar^2}\sum_\alpha \int_{\rm BZ} {\rm d}^3\boldsymbol{k}\, |\nabla_{\boldsymbol{k}}\varepsilon_{\alpha\boldsymbol{k}}|^2 \frac{\Delta^2}{\sqrt{(\varepsilon_{\alpha\boldsymbol{k}}-\varepsilon_F)^2+\Delta^2}^3}$$

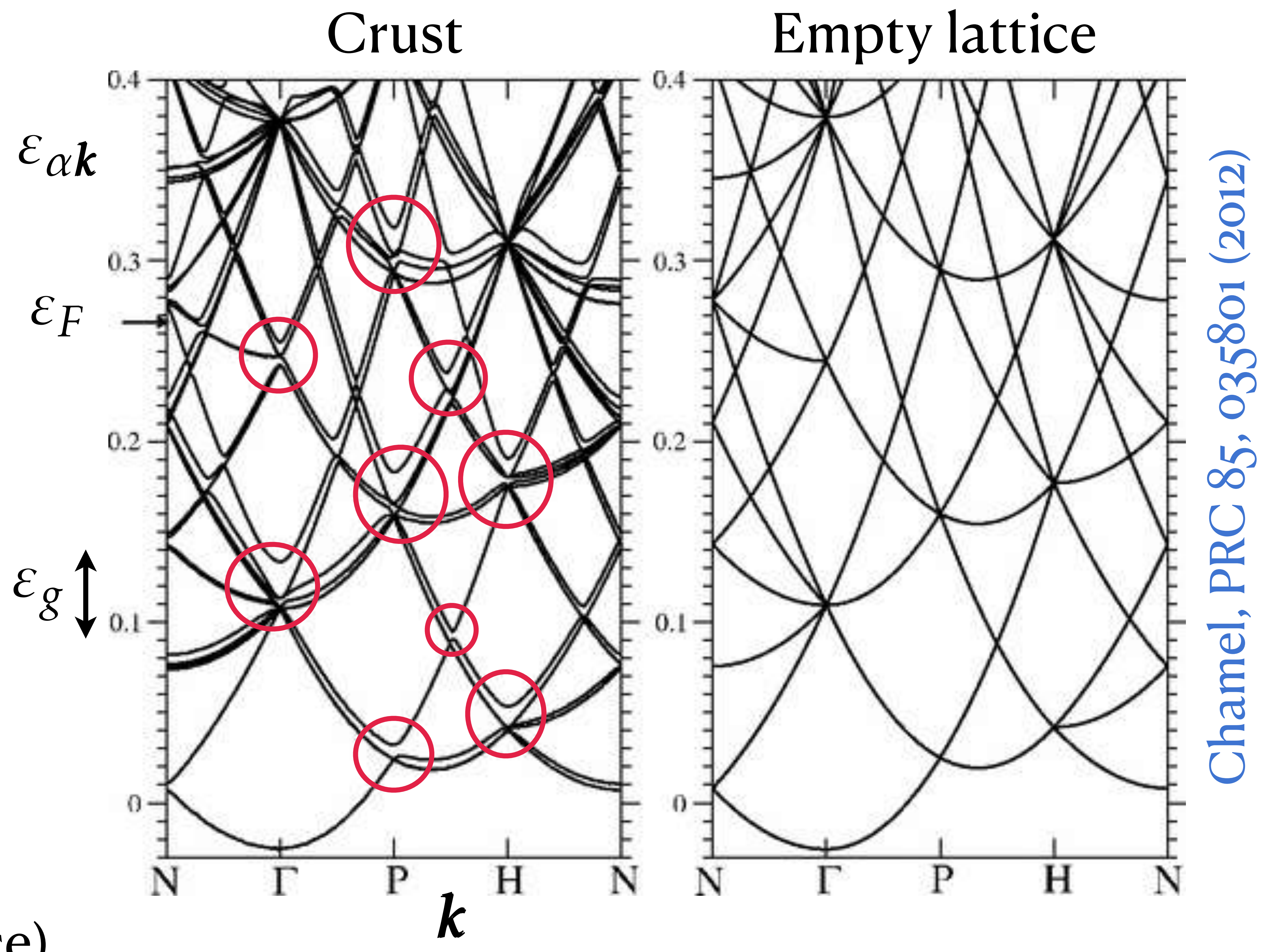
Carter, Chamel, Haensel, Nucl. Phys. A759, 441 (2005)

Extension to $h_n(\boldsymbol{r}) = -\nabla\cdot\frac{\hbar^2}{2m_n^\oplus(\boldsymbol{r})}\nabla + U_n(\boldsymbol{r}) - \frac{i}{2}\left[\boldsymbol{I_n}(\boldsymbol{r})\cdot\nabla + \nabla\cdot\boldsymbol{I_n}(\boldsymbol{r})\right]$

$$\rho_{n,s} = \frac{m_n^2}{24\pi^3\hbar^2}\sum_\alpha \int_{\rm BZ} {\rm d}^3\boldsymbol{k}\left(1-\frac{\varepsilon^0_{\alpha\boldsymbol{k}}}{\sqrt{(\varepsilon^0_{\alpha\boldsymbol{k}}-\varepsilon_F)^2+\Delta^2_{\alpha\boldsymbol{k}}}}\right)\nabla_{\boldsymbol{k}}\cdot\nabla_{\boldsymbol{Q}}\varepsilon_{\alpha\boldsymbol{k}+\boldsymbol{Q}}$$

intraband coupling

Chamel, PRC 111, 045803 (2025)



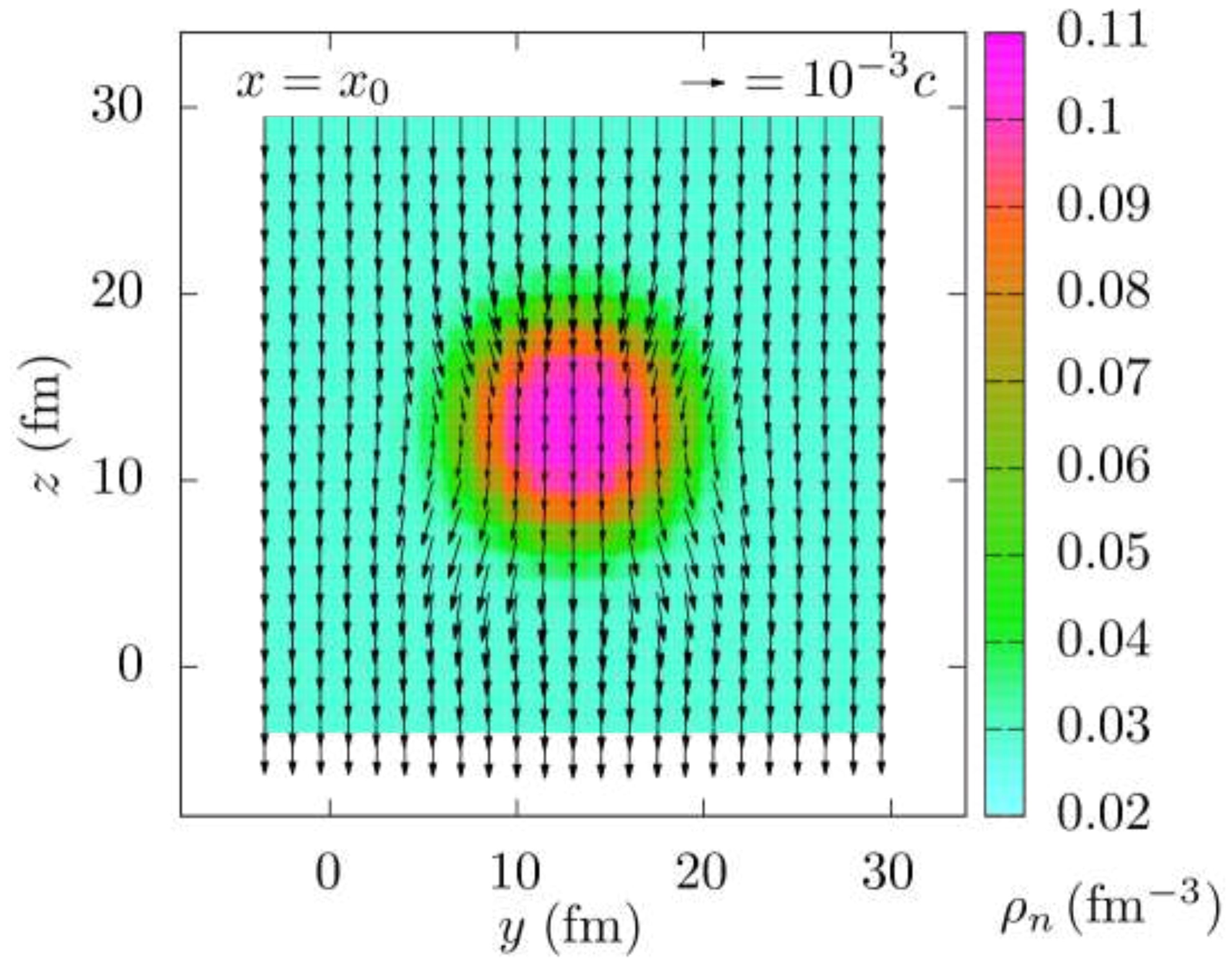
- This converges to the **exact result in homogeneous matter** (empty lattice)
- The superfluid fraction is almost independent of $\Delta_{\alpha\boldsymbol{k}}$
- This expression is valid when pairing gaps $\Delta_{\alpha\boldsymbol{k}} \ll$ band gaps $\varepsilon_g \sim 10$ keV

Watanabe & Pethick, PRL119, 062701 (2017); Almirante & Urban, PRL135, 132701 (2025)

However this condition is generally not fulfilled except for the shallowest layers

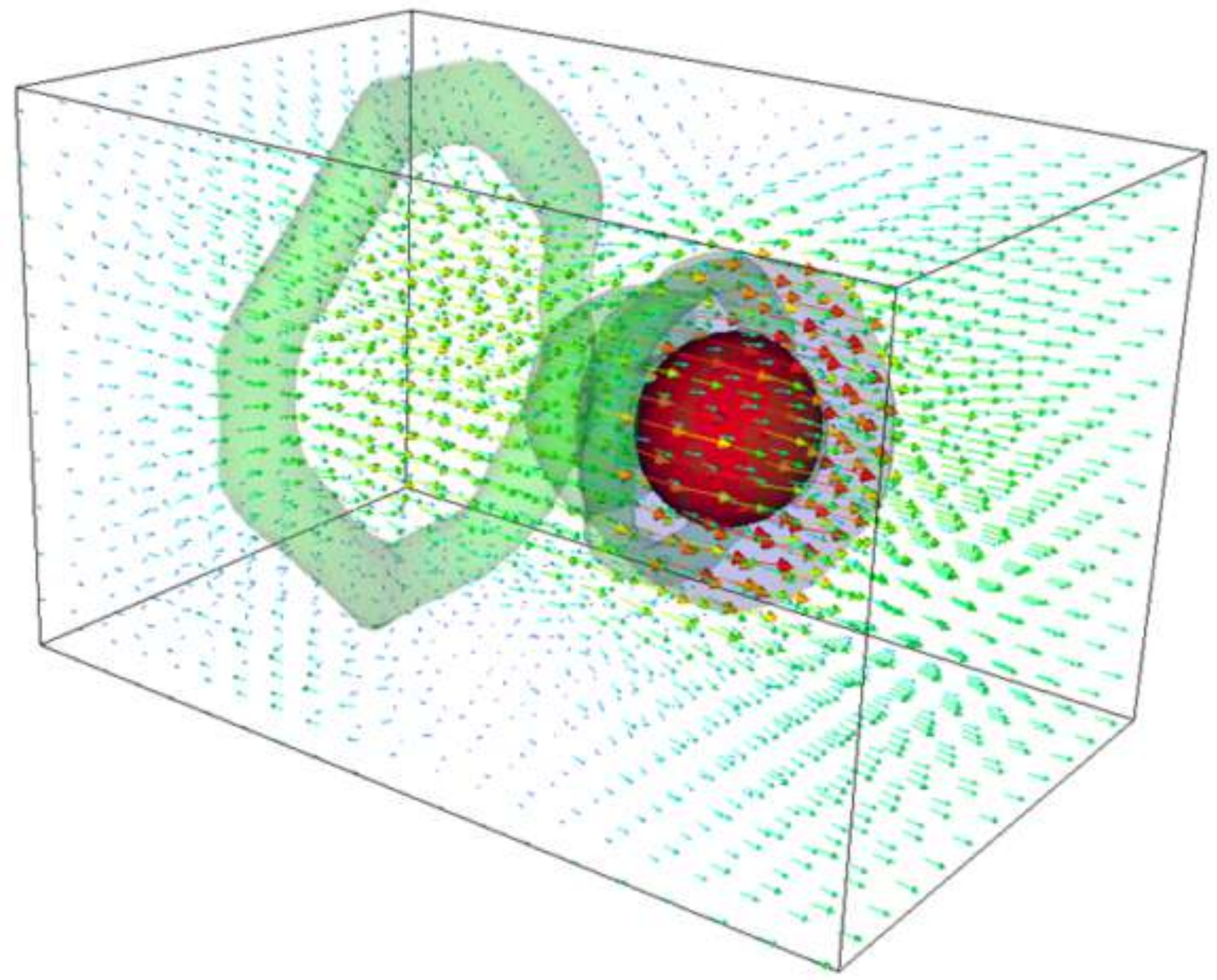
(TD)HFB in the crust

$\rho_{n,s}$ from stationary superflow through a lattice



Almirante, Kaskitsi, Urban (2026)

$m_I^\star$ from motion of a single ion in a superfluid



Peçak et al., PRX 14, 041054 (2024)

Solving numerically the TDHFB equations is very challenging:
- coupled equations for both neutrons and protons
- the number of nucleons per lattice site can be huge ~10^3
- full 3D (TD)HFB equations must be considered
- very different scales (lattice spacing ~100 fm vs cluster size ~10 fm, coherence length)
- calculations must be repeated for different superfluid velocities

Unlike atoms in periodic traps, each crustal layer is a different material with unknown composition and structure

Linear response theory within TDHFB

Approximations:

- crustal composition from 4th order extended Thomas-Fermi method with shell corrections (fast HFB approx.)
- BCS pairing (proximity effects tend to smooth out pairing fluctuations)
- Stationary flow with an average superfluid velocity $\bar{\boldsymbol{V}}_{\boldsymbol{n}} \equiv \frac{1}{\Omega}\int \mathrm{d}^3\boldsymbol{r}\,\boldsymbol{V}_{\boldsymbol{n}}(\boldsymbol{r}) = \frac{\hbar \boldsymbol{Q}}{m_n}$

In the linear current response $\bar{V}_n \ll V_{n,c} \sim \frac{\Delta_{F,n}}{\hbar k_{F,n}}$ (likely a very good approximation for most isolated pulsars)

$$\rho_{n,s} = \frac{1}{\Omega}\int \mathrm{d}^3\boldsymbol{r}\,\rho_n(\boldsymbol{r})\left[\frac{m_n}{m_n^{\oplus}(\boldsymbol{r})} + \frac{m_n}{3\hbar^2}\nabla_{\boldsymbol{Q}}\cdot \boldsymbol{I}_{\boldsymbol{n}}(\boldsymbol{r})\right]$$

entrainment effects similar to those in the core

$$+\frac{m_n^2}{24\pi^3}\sum_{\alpha\neq\beta}\int \mathrm{d}^3\boldsymbol{k}\,\frac{\xi^0_{\alpha\boldsymbol{k}}\xi^0_{\beta\boldsymbol{k}} - E^0_{\alpha\boldsymbol{k}}E^0_{\beta\boldsymbol{k}} + \Delta^0_{\alpha\boldsymbol{k}}\Delta^0_{\beta\boldsymbol{k}}}{E^0_{\alpha\boldsymbol{k}}E^0_{\beta\boldsymbol{k}}(E^0_{\alpha\boldsymbol{k}}+E^0_{\beta\boldsymbol{k}})}\boldsymbol{v}^0_{\beta\boldsymbol{k},\alpha\boldsymbol{k}}\cdot\left\{\boldsymbol{v}^0_{\alpha\boldsymbol{k},\beta\boldsymbol{k}} + \frac{1}{6\hbar^2}\sum_{\sigma}\int \mathrm{d}^3\boldsymbol{r}\,\varphi^0_{\alpha\boldsymbol{k}}(\boldsymbol{r},\sigma)^*[\nabla_{\boldsymbol{Q}}\cdot\boldsymbol{I}_{\boldsymbol{n}}(\boldsymbol{r})\boldsymbol{p} + \boldsymbol{p}\nabla_{\boldsymbol{Q}}\cdot\boldsymbol{I}_{\boldsymbol{n}}(\boldsymbol{r})]\varphi^0_{\beta\boldsymbol{k}}(\boldsymbol{r},\sigma)\right\}$$

interband coupling **entrainment effects from inhomogeneities**

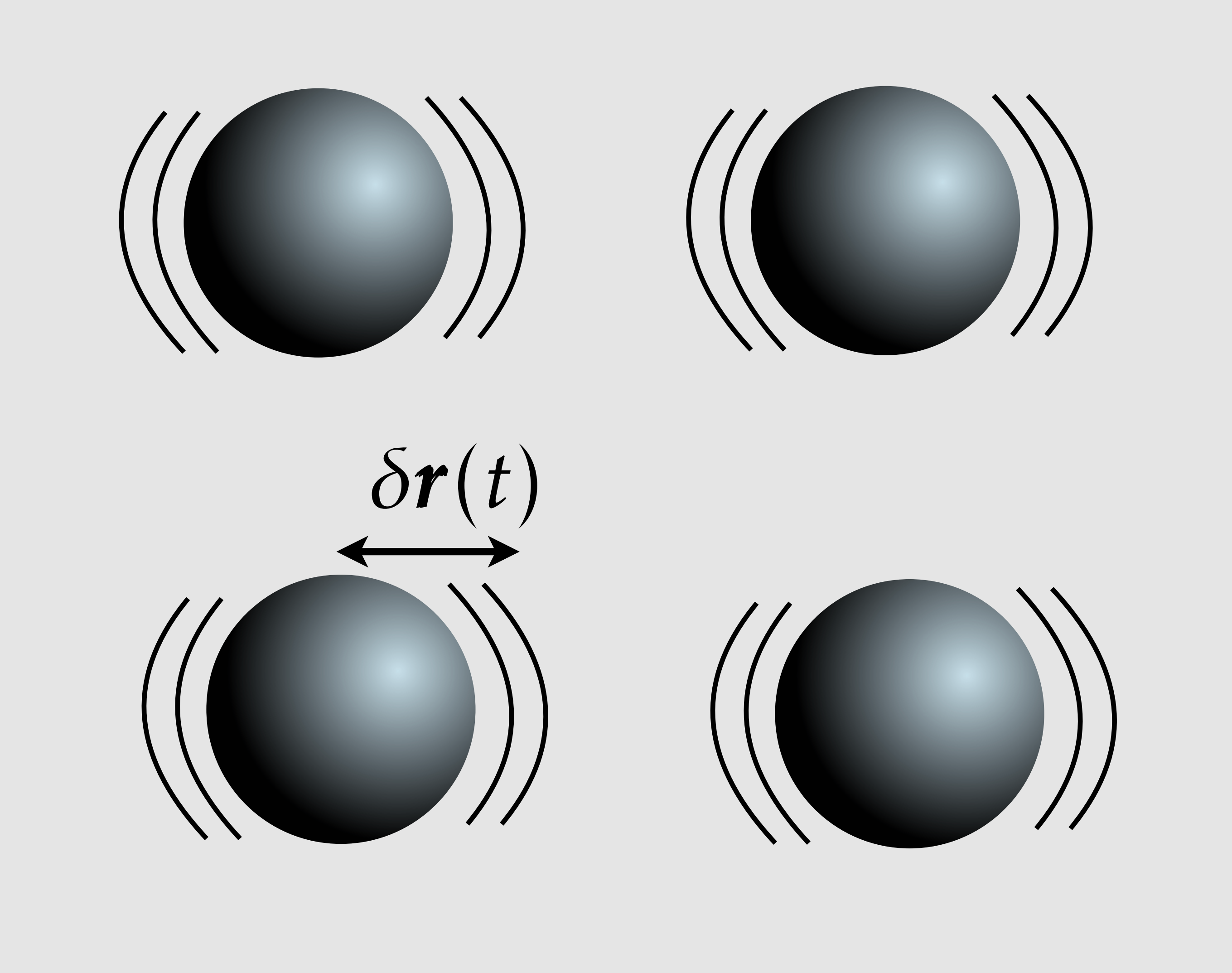
where $E^0_{\alpha\boldsymbol{k}} = \sqrt{\varepsilon^0_{\alpha\boldsymbol{k}} + |\Delta^0_{\alpha\boldsymbol{k}}|^2}$ and $\nabla_{\boldsymbol{Q}}\cdot\boldsymbol{I}_{\boldsymbol{n}}(\boldsymbol{r}) = \int \mathrm{d}^3\boldsymbol{r}'\, f[\boldsymbol{r},\nabla_{\boldsymbol{Q}}\cdot\boldsymbol{I}_{\boldsymbol{n}}(\boldsymbol{r}')]$

nonlocal

Chamel, arXiv:2605.25600 (2026)

Entrainment and lattice vibrations

Unlike optical traps in laboratory experiments, the crust of a neutron star is not perfectly rigid



The superfluid motion couples to crustal vibrations

Pethick, Chamel, Reddy, PTPS 186, 9 (2010)

Quantum zero-point motion of ions can be included through the Debye-Waller factor

$$\widetilde{U}_n(\boldsymbol{G}) \equiv \frac{1}{\Omega}\int d^3\boldsymbol{r}\, U_n(\boldsymbol{r})\exp(-i\,\boldsymbol{G}\cdot\boldsymbol{r}) \rightarrow \widetilde{U}_n(\boldsymbol{G})\exp\left(-\frac{1}{6}G^2\langle\delta\boldsymbol{r}(t)^2\rangle\right)$$

Bragg scattering is reduced

Self-consistency problem

- Lattice vibrations influence the superfluid motion and the ion effective mass $m_I^\star = \dfrac{\bar{\rho}-\rho_{n,s}}{n_I}$
- In turn, this changes $\langle\delta\boldsymbol{r}(t)^2\rangle \approx \dfrac{3\hbar\langle\omega_I^\star/\omega_{ph}\rangle}{2m_I^\star\omega_I^\star}$ where $\omega_I^\star = \sqrt{\dfrac{4\pi Z^2e^2n_I}{m_I^\star}}$

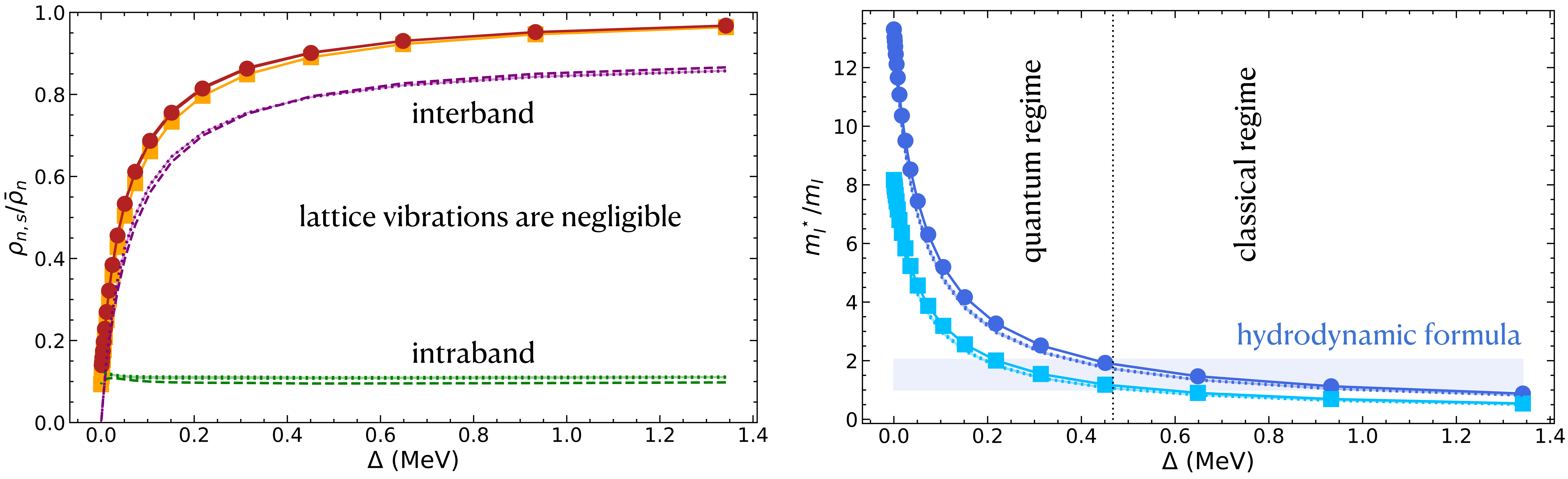
Chamel, PRC 111, 055803 (2025)

Superfluid fraction and effective ion mass

3D band-structure calculations in different crustal layers:

- ~10^3 bands
- ~10^3 Bloch wave vectors
- spatial grid with up to 25x25x25 points

$\bar{n} = 0.04\ \mathrm{fm}^{-3}$



Chamel, arXiv:2605.25600 (2026)

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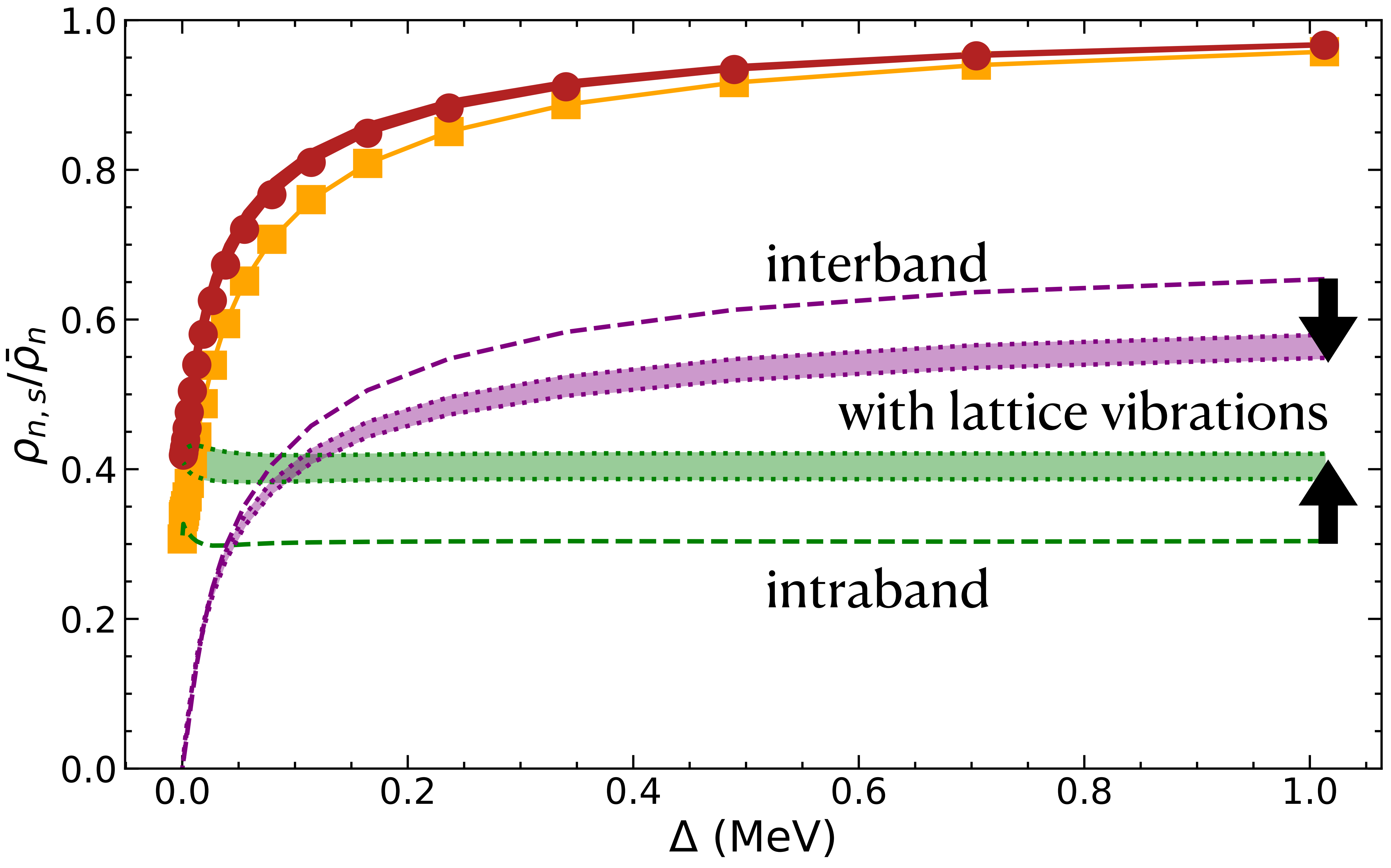

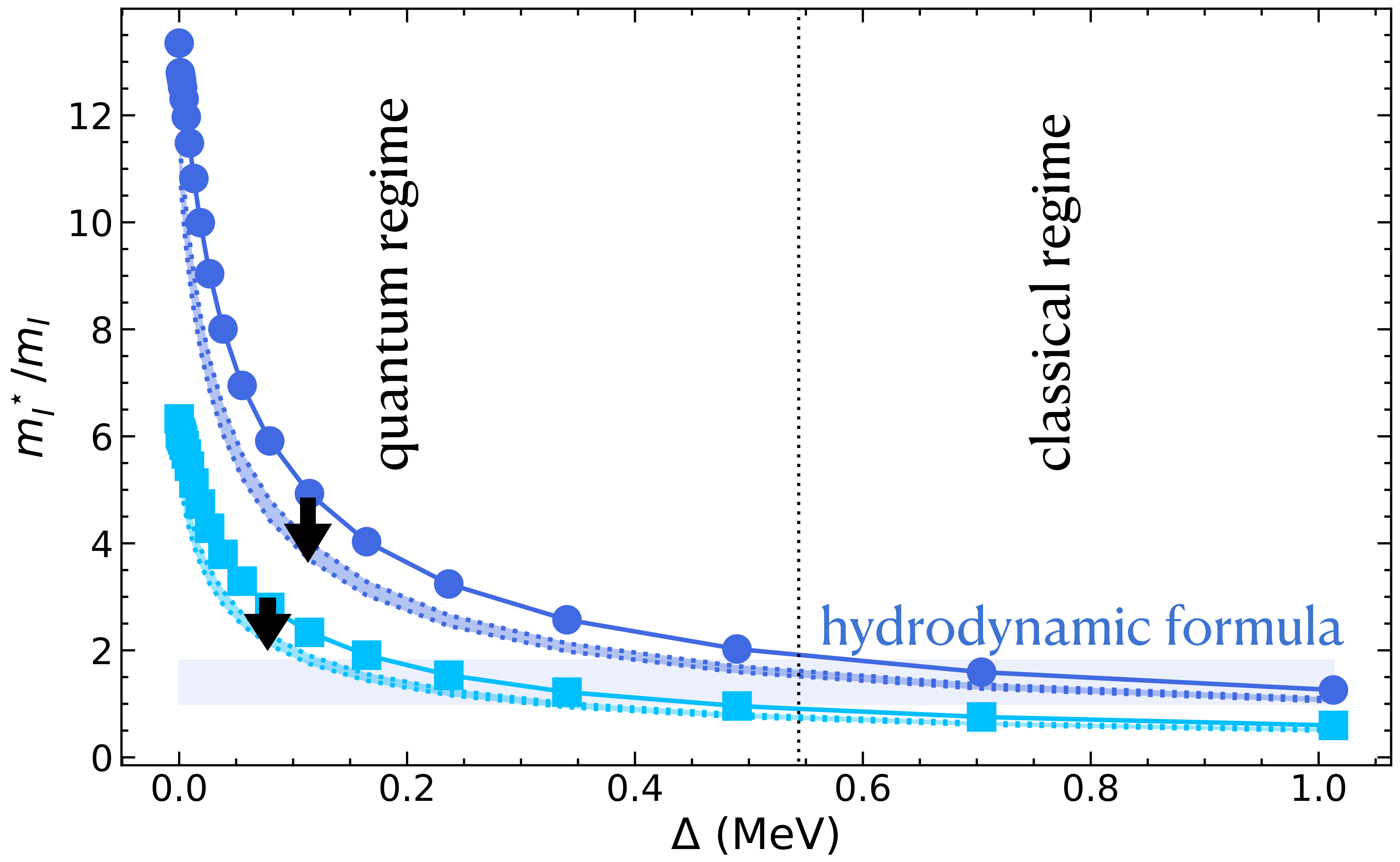

Chamel, arXiv:2605.25600 (2026)

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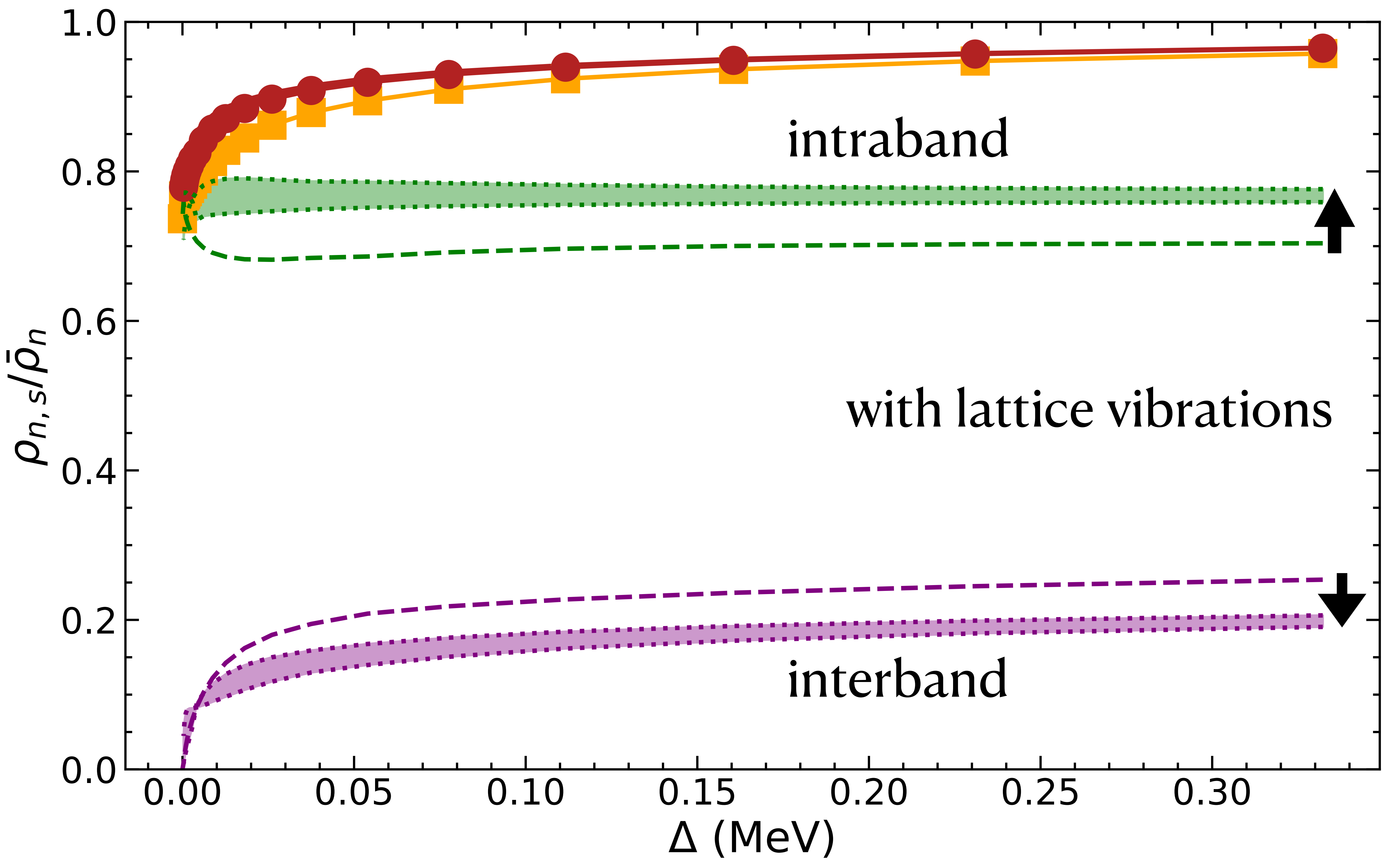
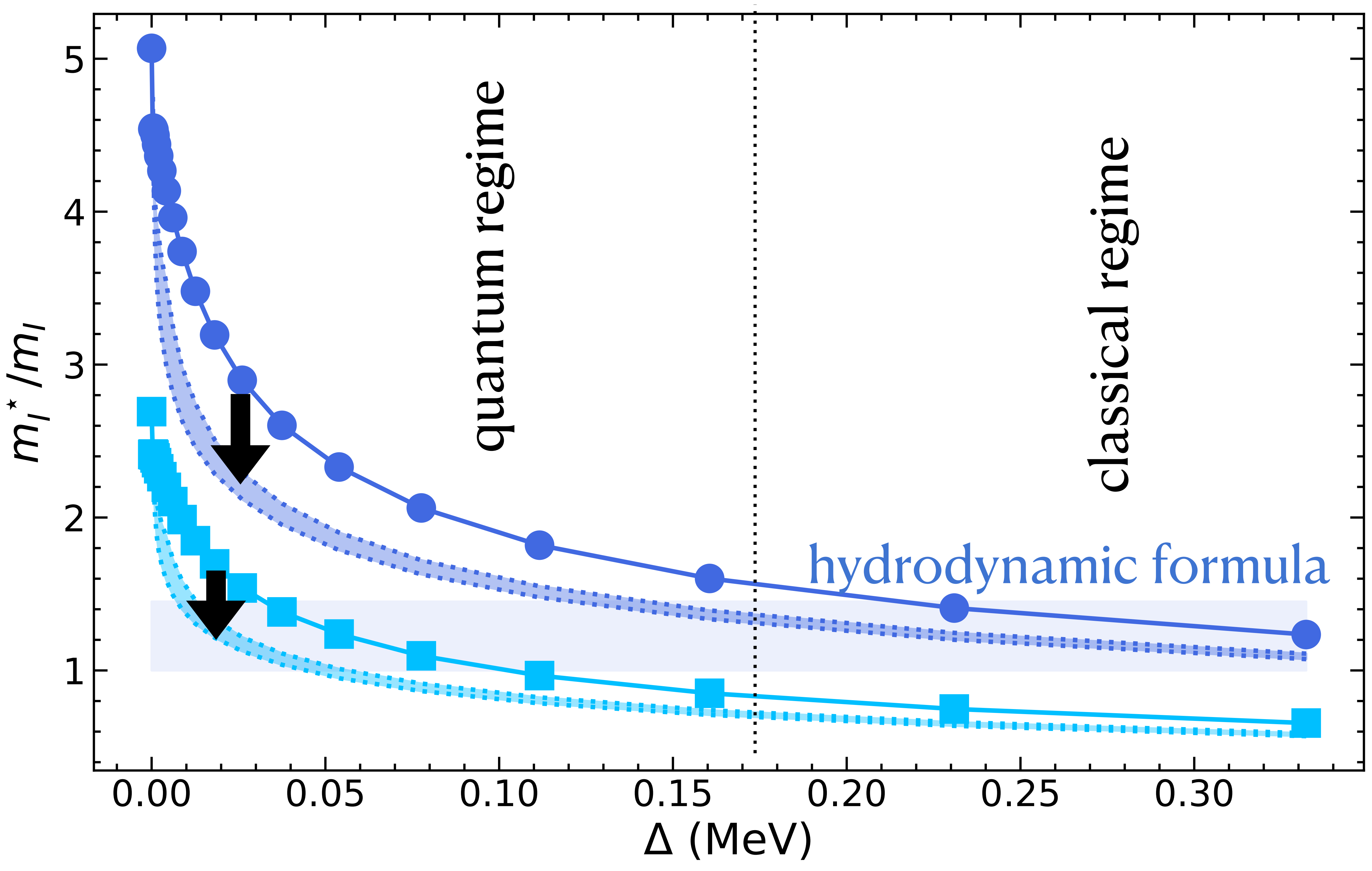
- spatial grid with up to 25x25x25 points

$\bar{n} = 0.07$ fm^{-3}

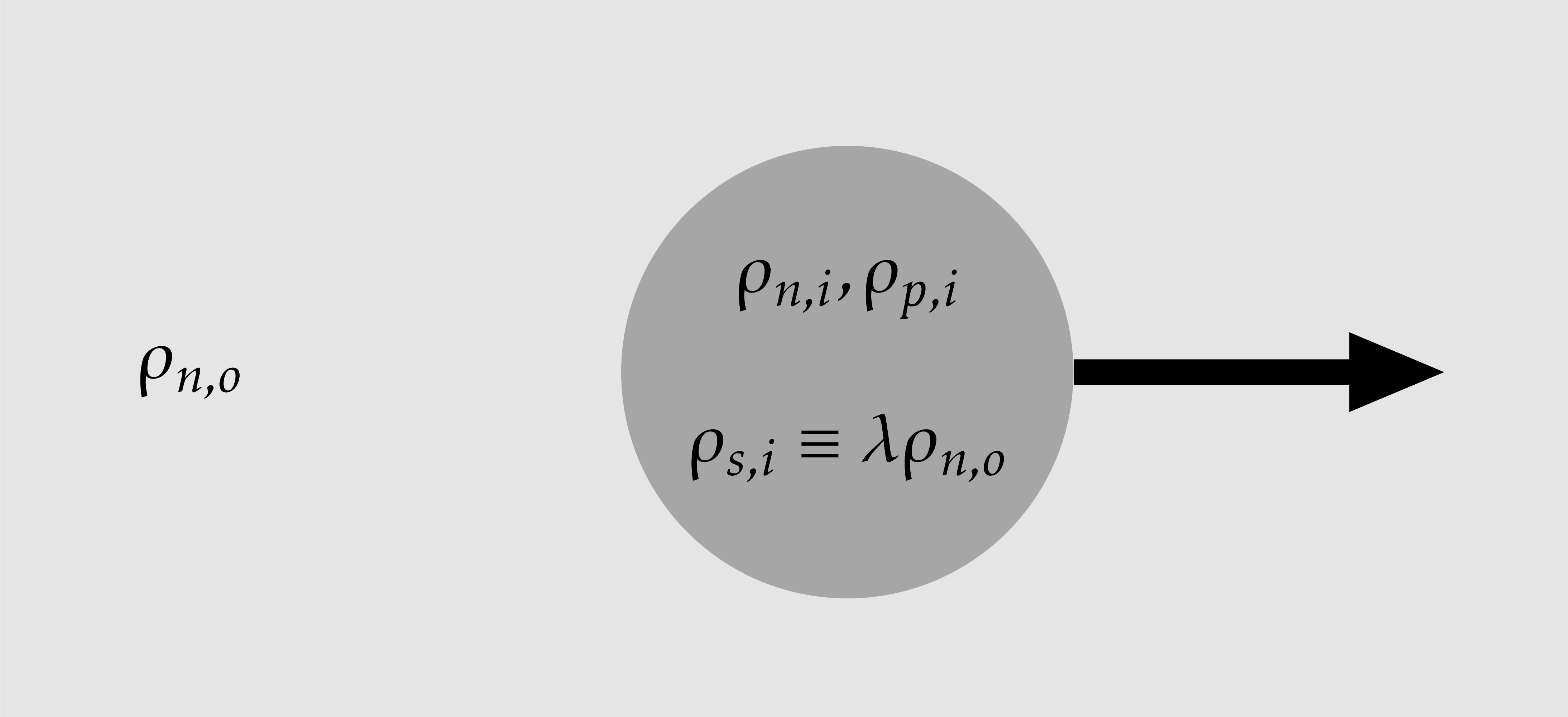




Chamel, arXiv:2605.25600 (2026)

Effective ion mass from hydrodynamics

The effective mass of a single ion can be calculated from **classical hydrodynamics assuming potential flow**



Effective ion mass $$\frac{m_I^\star}{m_I} = 1 + \frac{(\lambda - 1)^2}{\lambda + 2} \frac{\rho_{n,o}}{\rho_{p,i} + \rho_{n,i} - \lambda \rho_{n,o}}$$

Epstein & Baym, ApJ 328, 680 (1988); 333, 880 (1988)

$\rho_{s,i}$ must be **fined-tuned in each layer** to reproduce band-structure results in the strong pairing (hydrodynamic) limit

- at low densities $\rho_{s,i} \ll \rho_{n,i}$ (impenetrable ions)
- near the crust-core interface $\rho_{s,i} \sim \rho_{n,i}$ (fully permeable ions)

This ambiguity can be avoided by solving instead the hydrodynamic equations with **realistic nucleon distributions**

Almirante, Kaskitsi, Urban (2026)

This implicitly assumes $\xi \ll R$, but this can only be checked from HFB calculations...

Conclusions

Entrainment effects in neutron-star crust and core can be described consistently within the (TD)HFB framework

In the core:

- Entrainment arises from the **momentum-dependence of** the neutron-proton (contact) interactions

$$p_{ij}^2\,\delta(\boldsymbol{r}_i-\boldsymbol{r}_j)+\delta(\boldsymbol{r}_i-\boldsymbol{r}_j)\,p_{ij}^2 \quad \text{and} \quad \boldsymbol{p}_{ij}\cdot\delta(\boldsymbol{r}_i-\boldsymbol{r}_j)\boldsymbol{p}_{ij} \quad \text{with} \quad \boldsymbol{p}_{ij}=-i\hbar(\boldsymbol{\nabla}_i-\boldsymbol{\nabla}_j)/2$$

In the crust:

- Entrainment arises additionally from **translational symmetry breaking**
- The superfluid fraction and effective ion mass are two facets of the same effects: $m_I^\star=\dfrac{\bar{\rho}-\rho_{n,s}}{n_I}$
- **The superfluid fraction can be large** $\rho_{n,s}/\bar{\rho}_n \gtrsim 90\%$ due to **interband response** in agreement with Almirante & Urban
- But entrainment is **sensitive to pairing gaps** $\Delta_{\alpha\boldsymbol{k}}$
- $m_I^\star \gg m_I$ for small $\Delta_{\alpha\boldsymbol{k}}$ and in any case $m_I^\star > Zm_p$
- Entrainment is **slightly reduced by lattice vibrations**

More systematic calculations using different functionals and including shallower layers are still needed