Magnus mountains

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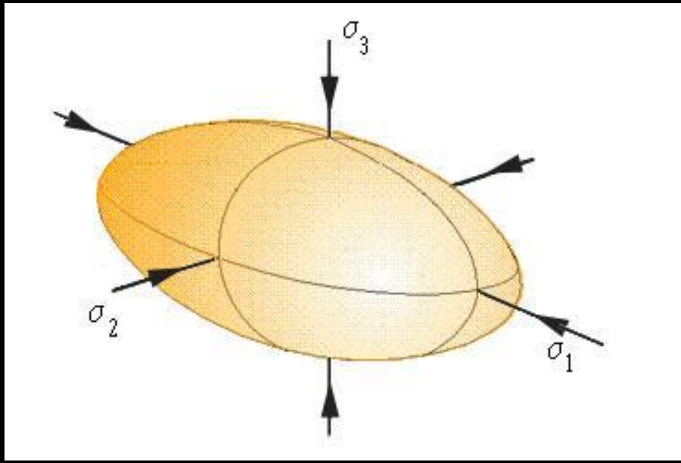
Based on Gangwar & DIJ, in preparation

Gravitational waves from mountains

A *triaxial* neutron star, rotating steadily, emits gravitational waves:



$$h = 3 \times 10^{-28} \left(\frac{\epsilon}{10^{-6}}\right) \left(\frac{f_{\rm spin}}{10\,{\rm Hz}}\right)^2 \left(\frac{1\,{\rm kpc}}{r}\right)$$

$$\epsilon = \frac{I_{yy} - I_{xx}}{I_{zz}}$$

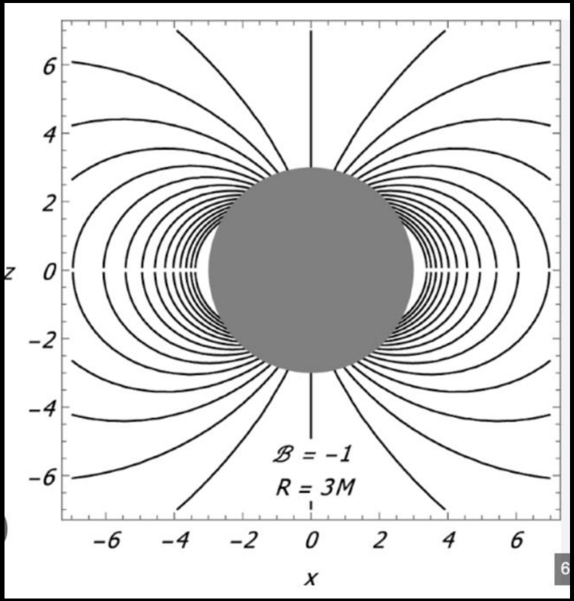
Dimensionless asymmetry in moment of inertia tensor

Spin frequency

Distance to source

Two types of mountain

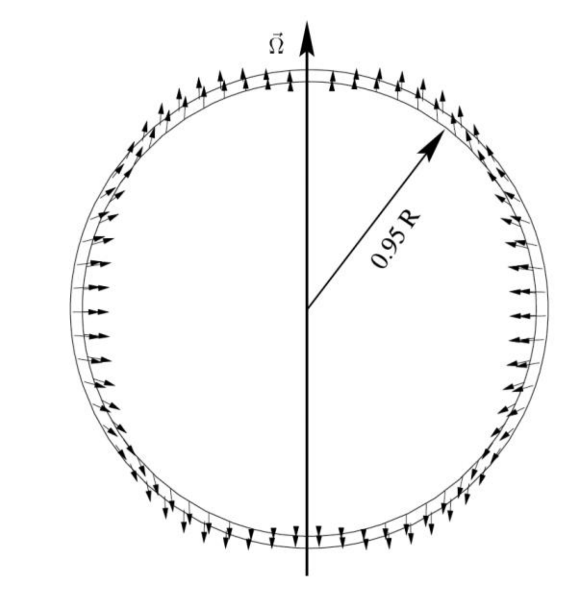
Magnetic Mountains



- Sustained by Lorentz ($\vec{j} \times \vec{B}$) forces
- Definitely exist, but likely small:

$$\epsilon \sim 10^{-12} \left(\frac{B}{10^{12}\,\mathrm{G}} \right)^2$$

Elastic Mountains



- Sustained by elastic forces in solid crust
- Size limited by strength of crust:

$$\epsilon_{\max} \approx 10^{-6} \left(\frac{u_{\mathrm{break}}}{10^{-1}} \right)$$

What *creates* an elastic mountain?

Asymmetric cracking in spinning-up or spinning-down star

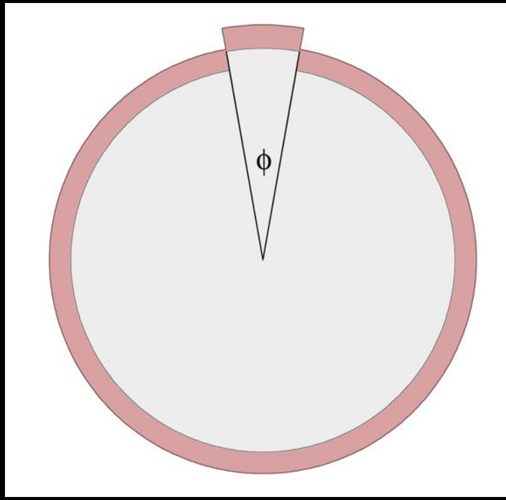


Image credit Fattoyev+ (2018)

Temperature asymmetries

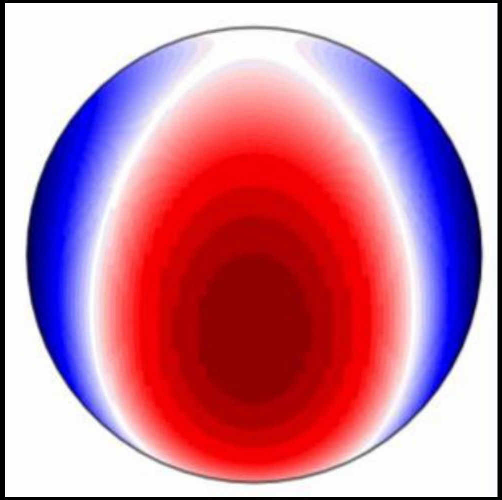
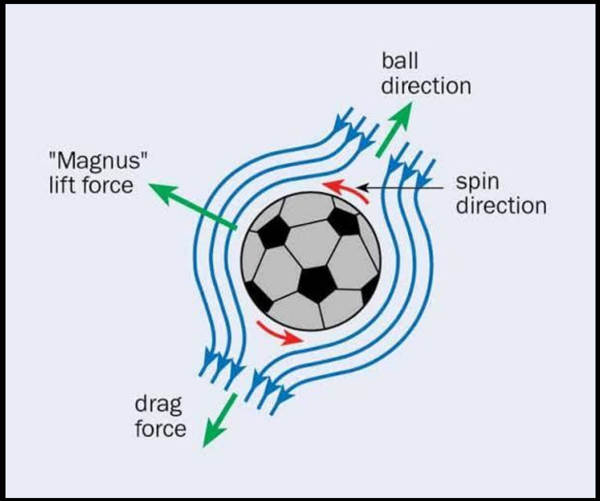


Image credit Hutchins (2023)

Asymmetric Magnus forces “Magnus mountain”



What *creates* an elastic mountain?

Asymmetric cracking in spinning-up or spinning-down star

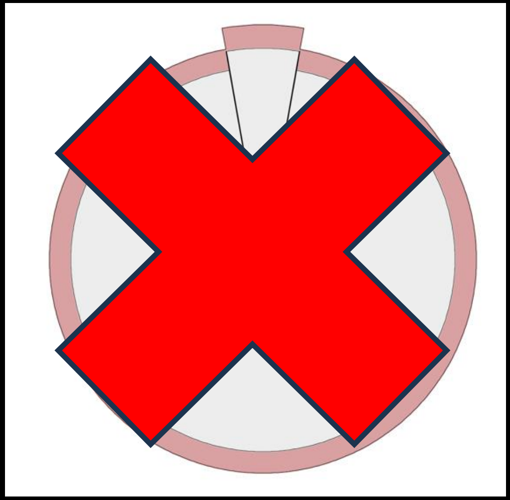
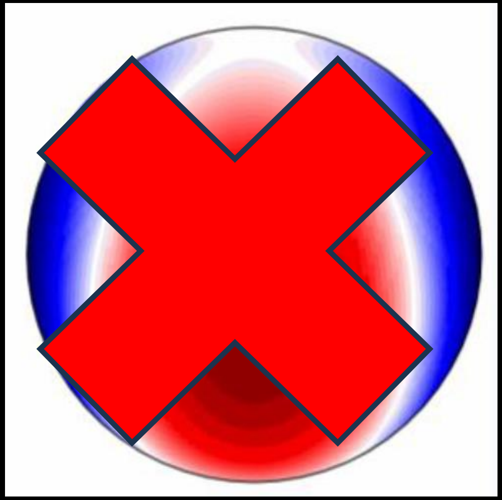


Image credit Fattoyev+ (2018)

Temperature asymmetries



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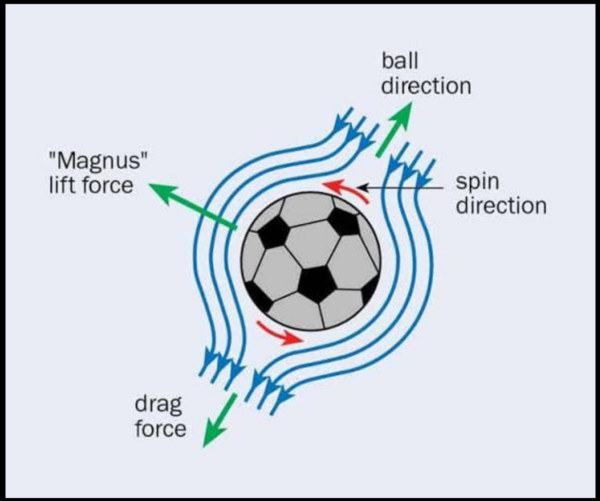
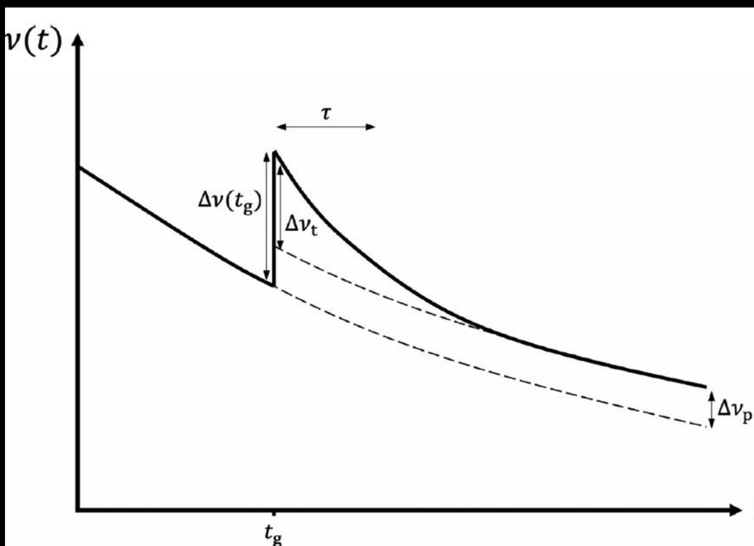
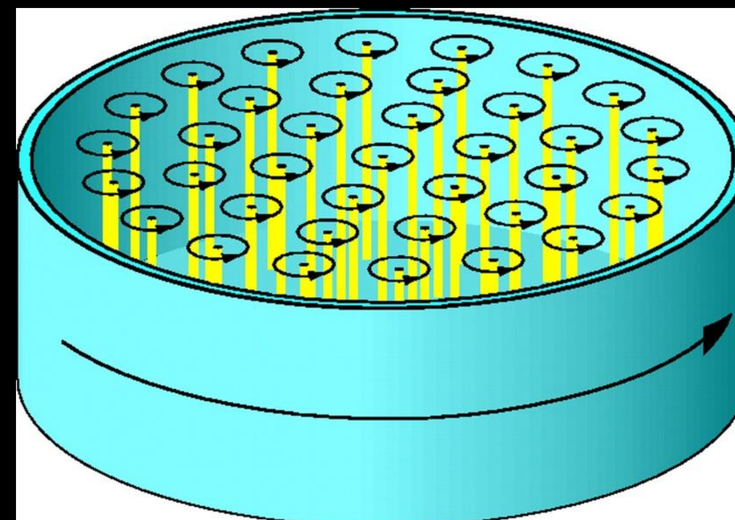
Asymmetric Magnus forces
“Magnus mountain”



THIS TALK!

Axisymmetric pinning: pulsar glitches

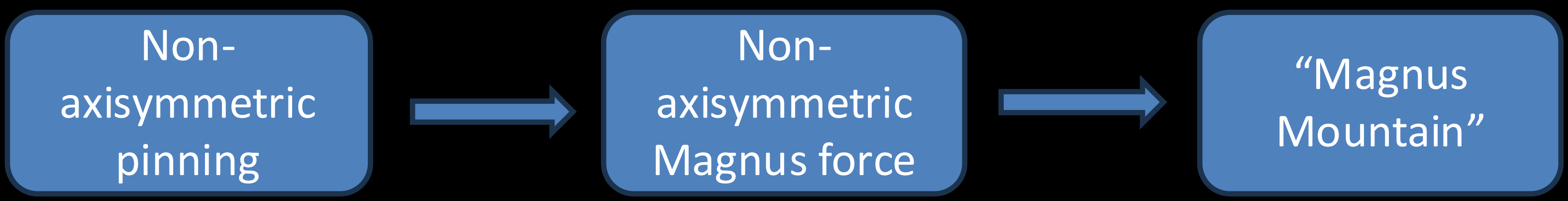




Velocities of both neutrons and crust assumed axisymmetric ⟹ Magnus force axisymemtric

$$\vec{F}^{\mathrm{mag}} = \rho_n \left(\nabla \times \vec{v}_n\right) \times \left(\vec{v}_n - \vec{v}_c\right)$$

Non-axisymmetric pinning: Magnus mountains

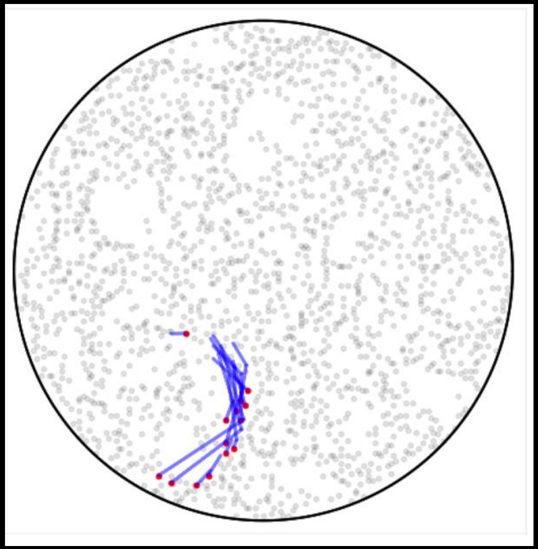


First proposed in DIJ (2002), with back-of-the-envelope estimate based on pinning in the inner crust:

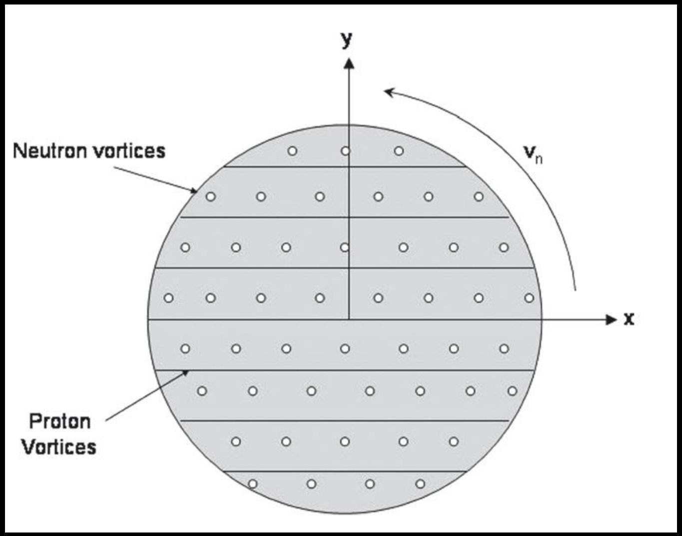
$$\epsilon \sim 5 \times 10^{-8} \left(\frac{\omega}{0.1\mathrm{Hz}}\right) \left(\frac{f}{\mathrm{kHz}}\right) \left(\frac{R}{10^{6}\,\mathrm{cm}}\right)^{3} \left(\frac{\delta M}{0.01 M_{\odot}}\right) \left(\frac{1.4 M_{\odot}}{M}\right)^{2} \left(\frac{f_{\mathrm{pin}}}{1}\right)$$

Why would the pinning be non-axisymmetric?

Asymmetric unpinning/re-pinning at glitches,
e.g. Cheunchitra+ (2024):



Asymmetric interaction between vortices and magnetic fluxtubes,
e.g. Sidery & Alpar (2009):

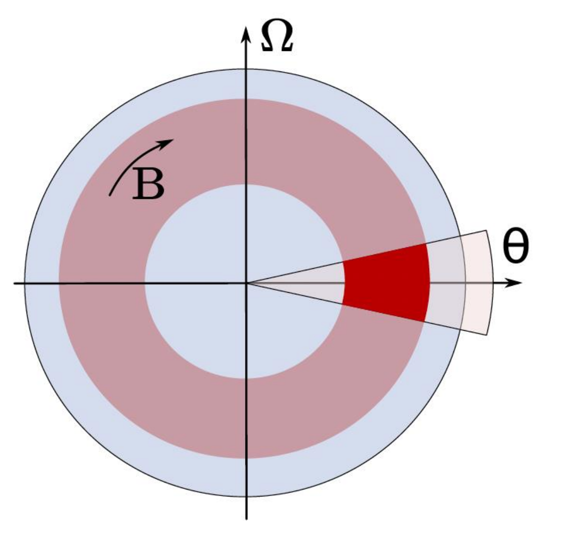


State-of-the-art: Haskell+ (2022)

Haskell, Antonelli & Pizzochero (2022) considered a spherical incompressible star.

Assumed perfect pinning in two diametrically opposed regions.

Found mass and mass-current quadrupoles were zero, but nevertheless used pressure perturbation to estimate:



$$\epsilon \approx -10^{-9}\left(\frac{B}{10^8 G}\right)^{1/2}\left(\frac{\nu}{100\,\mathrm{Hz}}\right)$$

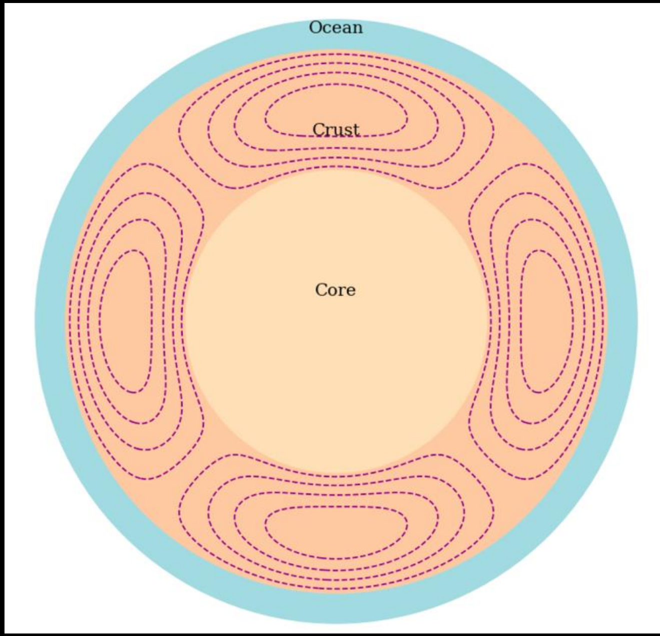
Going further: Yashaswi Gangwar's PhD

Modelled two-component compressible self-gravitating rotating cylinder.

Outer part of proton fluid has non-zero shear modulus (i.e. the crust).

By hand, added a velocity perturbation to the neutron superfluid:

$$\delta \vec{v_{\mathrm{n}}} \sim e^{2i\phi}$$



Find unexpected cancellation, no mass-current quadrupole.

Balancing the forces on the neutron fluid

For the superfluid neutrons:

$$2\vec{\Omega} \times \delta\vec{v}_n = -\nabla\delta\mu_n + \delta\vec{F}^{mag},$$

Find Coriolis force exactly balances Magnus force, leaving

$$\nabla\delta\mu_n = 0$$

⇒ no density perturbation in the neutron fluid

⇒ no mass quadrupole from neutron superfluid

Balancing the forces on the crust

For the crust:

$$\nabla^a \delta\tau_{ab} - \frac{\delta\rho_c}{\rho_c}\nabla^a \tau_{ab} - \delta F_b^{mag} = 0,$$

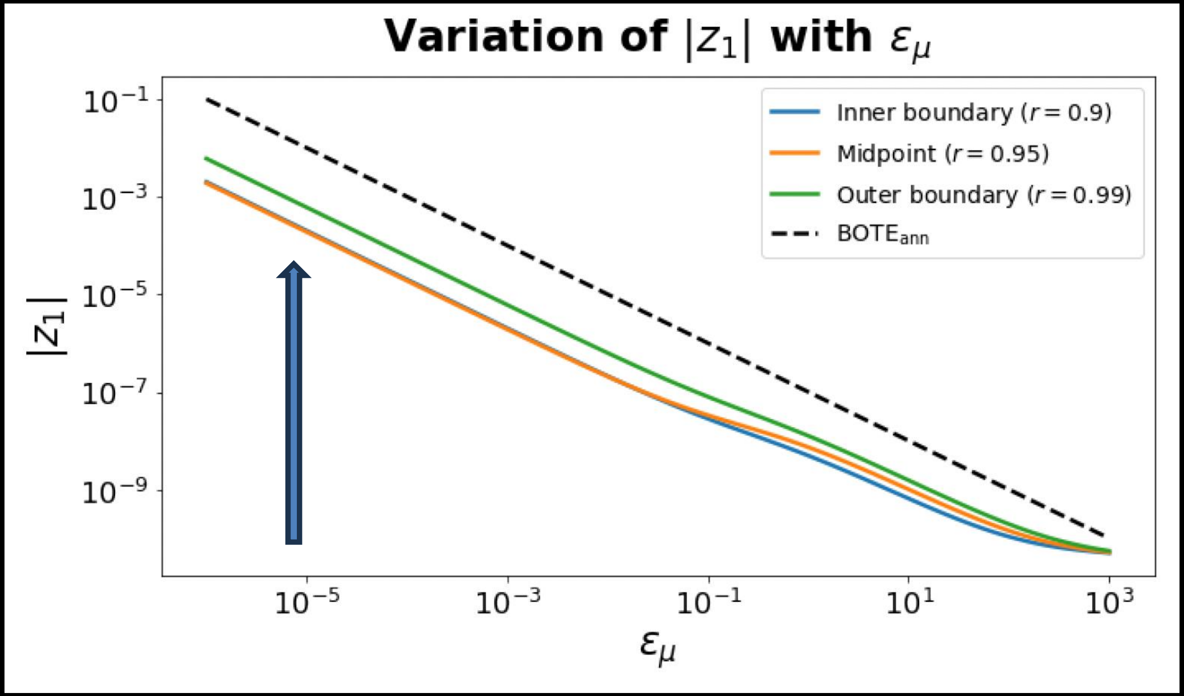
where τ_{ab} is the stress tensor, so that

$$\delta\tau_{ab} = -\rho_c \delta\tilde{\mu}_c g_{ab} + \mu\left(\nabla_a \xi_b + \nabla_b \xi_a - \frac{2}{3} g_{ab} \nabla^c \xi_c\right)$$

We solved numerically to obtain the density perturbation $\delta\rho_c$ and the crustal displacement $\vec{\xi}$.

Results: varying the crust's shear modulus

$$z_1 = \frac{\xi_r}{r}$$

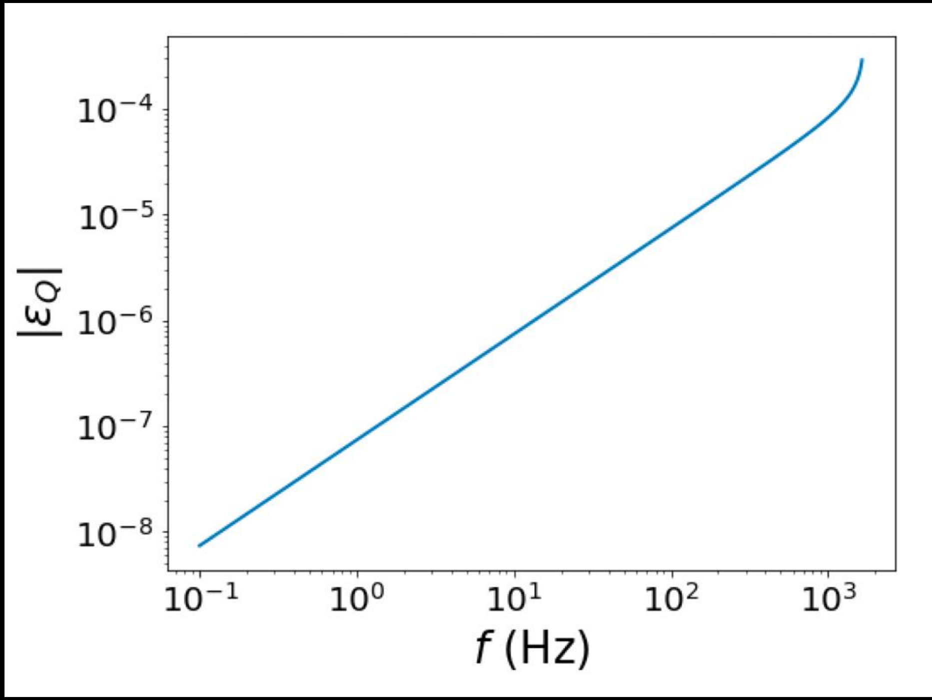


$$\epsilon_\mu \sim \frac{\mu}{GM^2/R^4}$$

Chose $\delta v_n \sim 10^4$ cm/s , based on Seveso+ (2015)

Stiffer crusts lead to smaller mountains, as expected.

Main result: the ellipticity



Plot has:

$$\epsilon_\mu = 10^{-5}$$

$\delta v_n \sim 10^4$ cm/s

Ellipticity approximately linear in spin frequency, as expected.

Reaches $\epsilon_Q \sim 10^{-5}$ for fast (cylindrical!) stars.

Summary

- Calculations so far highly idealized.
- Areas for improvement include:
    1. Spherical geometry with compressible fluids
    2. First-principles calculation of vortex asymmetry
    3. Proper 3-d vortex model, allowing for bending
    4. Calculation in general relativity
    5. Use of realistic equations of state
- Magnus mountains promising in terms of gravitational wave emission