Beyond the Taylor Expansion:

Systematics in the phase of continuous gravitational wave

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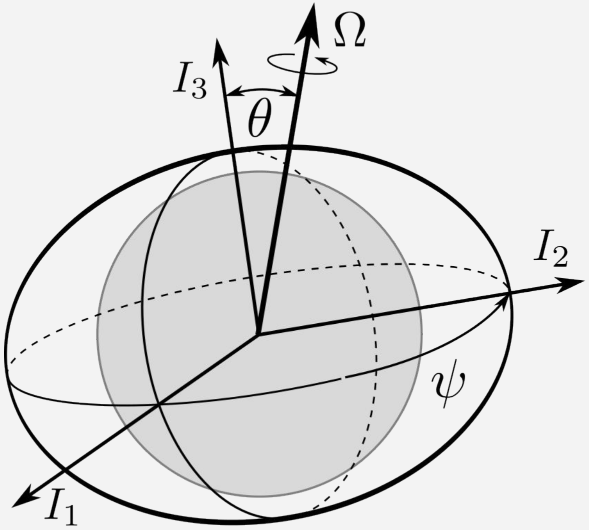
Continuous Gravitational Waves (CWs)

- CWs are expected from a time-varying nonaxisymmetric mass distribution in rotating neutron stars.
  - Formation of mountains due to strain in the elastic crust.
  - Accretion from a companion star.
  - Strong inner magnetic field.
  - Fluid oscillations, such as those due to r-modes.
- Interesting astrophysical sources are
  - Young neutron stars (e.g. in supernova remnants)
  - accreting objects (e.g. Sco X-1)



$$h_0 = \frac{4\Omega^2(I_3 - I_1)}{r} \qquad I_2 = I_1$$

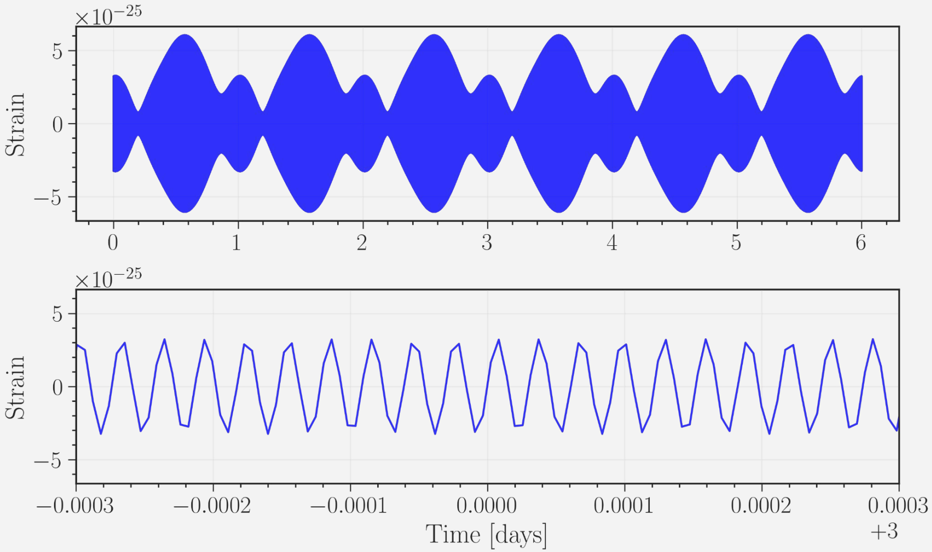
Image credits - Michał Bejger and Andrzej Królak

Continuous Gravitational Waves (CWs)

- Expected to last for long time (years), unless interrupted by internal mechanisms.
- Emitted at almost constant frequency (Low spindown).

$$h(t) = F_+(t)\, h_+(t) + F_\times(t)\, h_\times(t)$$

$$h_+(t) = A_+ \cos\widetilde{\phi}(t), \qquad A_+ = \frac{1}{2} h_0 (1 + \cos^2 \iota),$$
$$h_\times(t) = A_\times \sin\widetilde{\phi}(t) \qquad A_\times = h_0 \cos\iota$$



How is the phase modelled?

$$\widetilde{\phi}(t) = 2\pi \sum_{k=0}^{s} f_k \frac{t^{k+1}}{(k+1)!} + \frac{2\pi}{c} \mathbf{n}_0 \cdot \mathbf{r}_{\mathrm{d}}(t) \sum_{k=0}^{s} f_k \frac{t^k}{k!}$$

Taylor Expansion

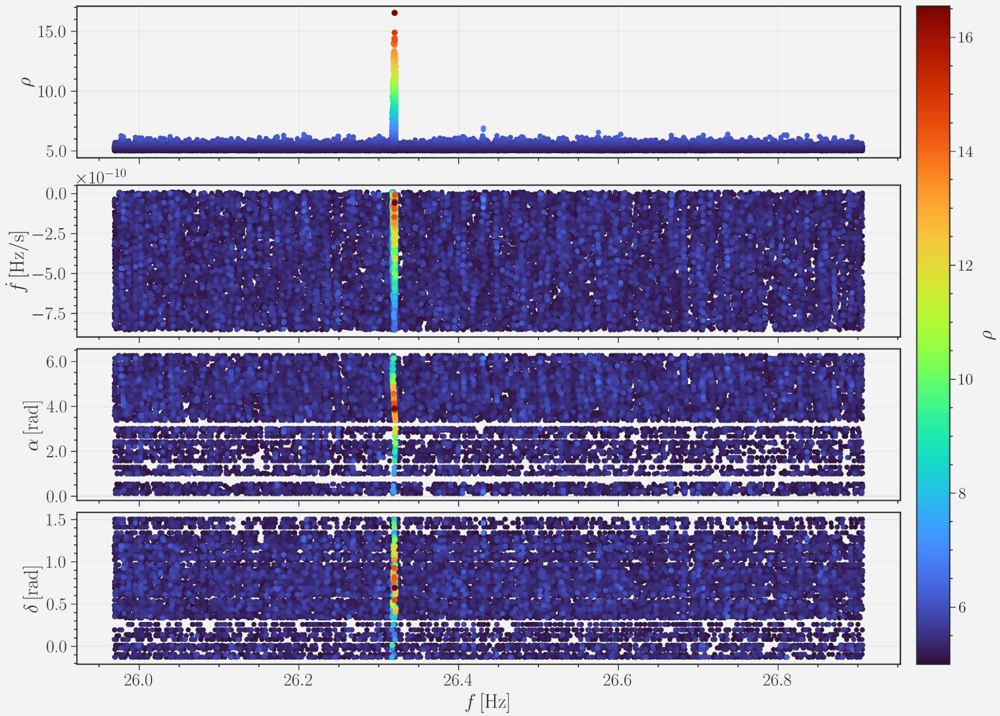
Doppler shifts due to Earth motion

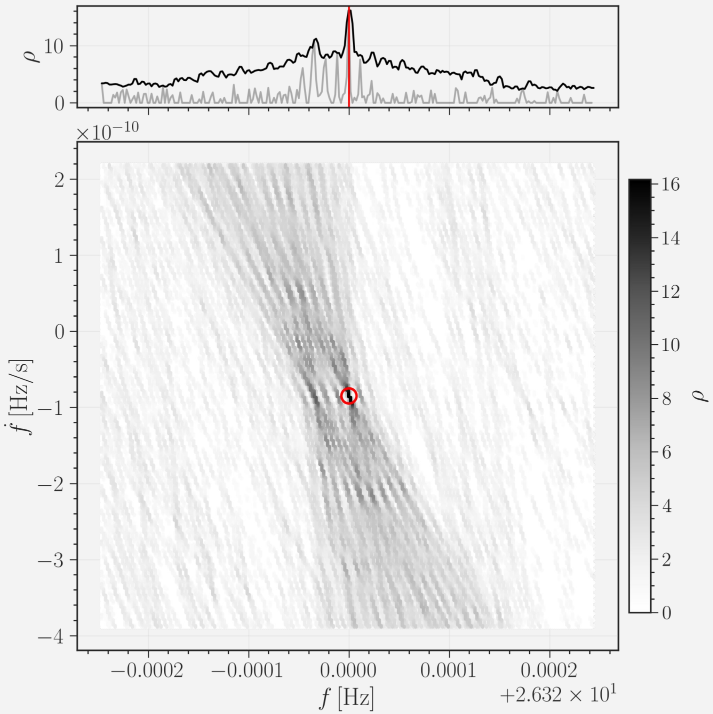
How do we detect them? - F statistic

(Obtained using TDFstat)

$$\rho = \sqrt{2\mathcal{F} - 4}$$

$$\mathcal{F} \approx \frac{2}{S_0 T_o}\left(\frac{|F_a|^2}{\langle a^2 \rangle} + \frac{|F_b|^2}{\langle b^2 \rangle}\right), \quad F_\gamma := \int_0^{T_o} x(t)\,\gamma(t)\exp[-i\widetilde{\phi}(t)]\,dt; \ \gamma = a, b$$





Additions to the Taylor Expansion - Unknown knowns

$$\widetilde{\phi}(t) = 2\pi \sum_{k=0}^{s} f_k \frac{t^{k+1}}{(k+1)!} + \frac{2\pi}{c} \mathbf{n}_0 \cdot \mathbf{r}_{\mathrm{d}}(t) \sum_{k=0}^{s} f_k \frac{t^k}{k!}$$

Taylor Expansion

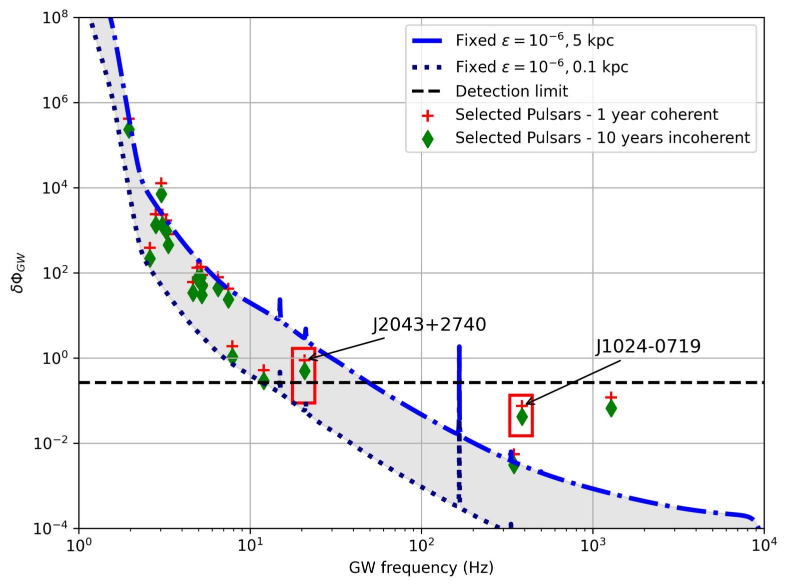
Doppler shifts due to Earth motion

There are **known knowns**; these are things we know that we know.
There are **known unknowns**; that is to say, there are things that we know we don't know.
But there are also **unknown unknowns**; there are things we don't know we don't know.

— *Donald Rumsfeld*

Additions to the Taylor Expansion - Unknown knowns

- More sensitive detectors and non detection of CWs is forcing us to reconsider the phase modelling.
- Neutron star effects such as
  - Timing noise,
  - Spin wandering.
- Astrophysical effects such as
  - Gravitational (micro)lensing.



Credit - Marco Antonelli, Avishek Basu, Brynmor Haskell (arXiv:2507.18439)

Timing Noise

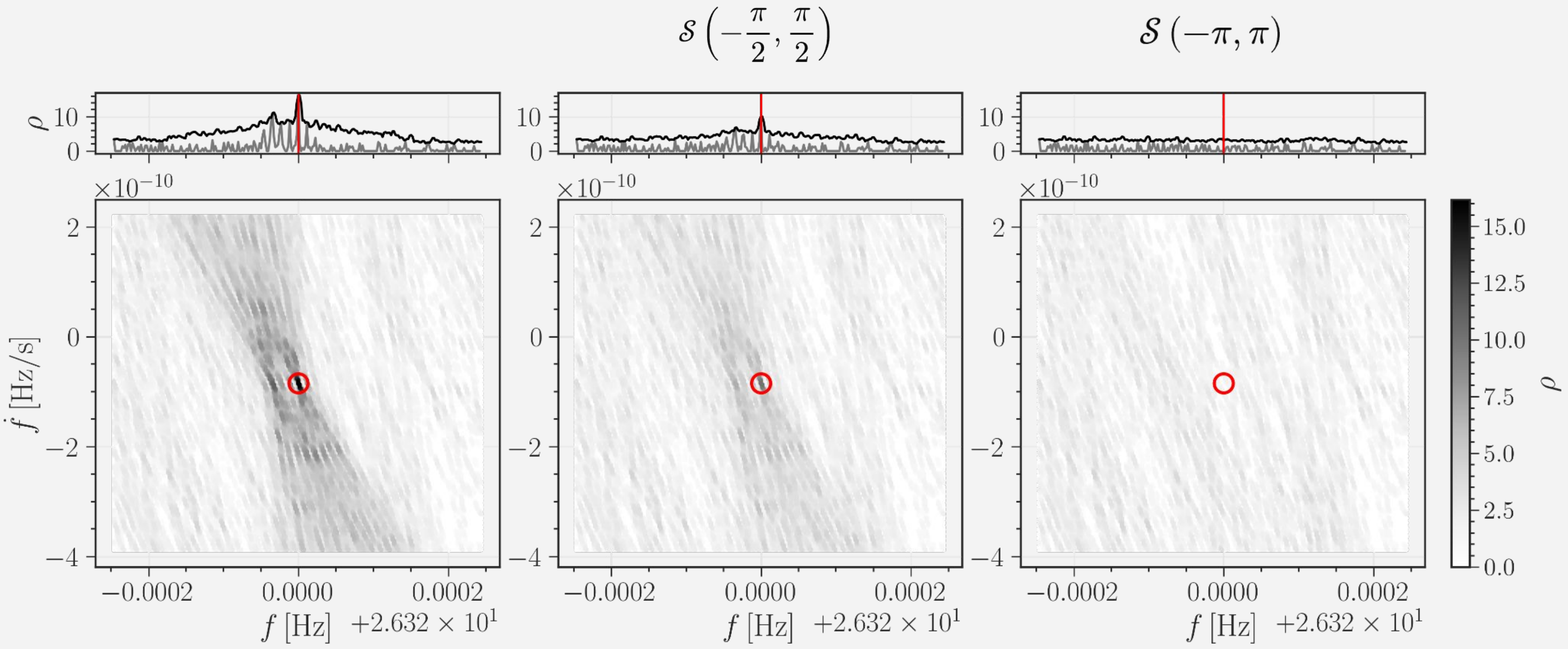
- Deviations from regular timing solution observed in pulsars' chronometry, resulting from, e.g.,
  - (super)fluid components rotating at different rates within the neutron star.
- From a multi-messenger perspective, timing noise is an important factor missing in the CW phase description.
- The phase change are stochastic in nature.

Work done by Marco Antonelli et al. 2025, was done for lower sampling rates, for CW modelling, we use a higher sampling rate.

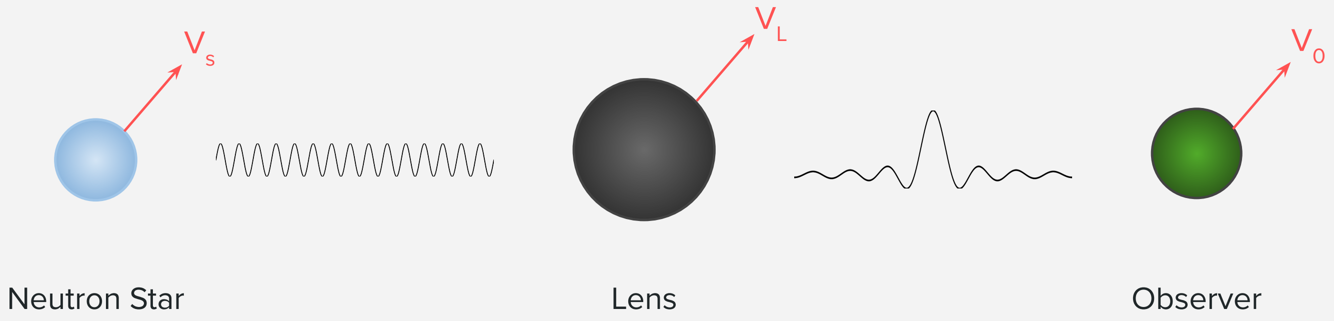
In this presentation, I will show how stochastic noise can affect the CW signal phasing.

Timing Noise

$$\mathcal{S}\left(\theta; t\right) = \mathrm{Random}(\theta)$$



Astrophysical additions - gravitational (micro)lensing



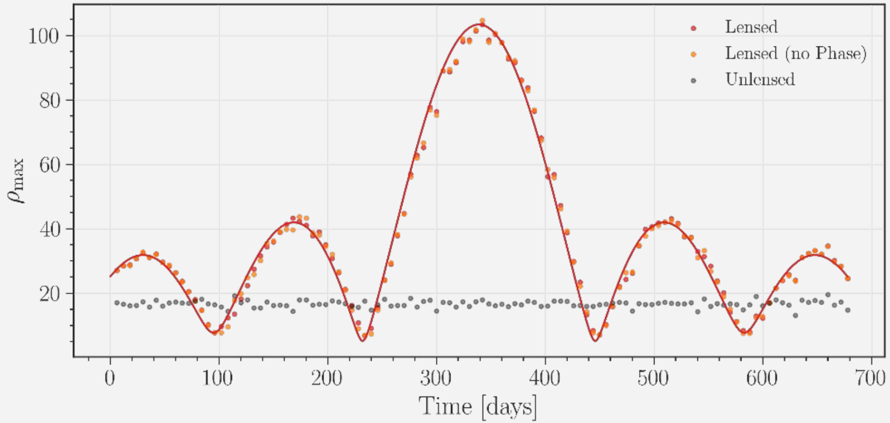
$$\vec{\nu}_{eff} = \vec{\nu}_s - \frac{D_s}{D_l}\vec{\nu}_L + \left(\frac{D_s}{D_l} - 1\right)\vec{\nu}_o$$

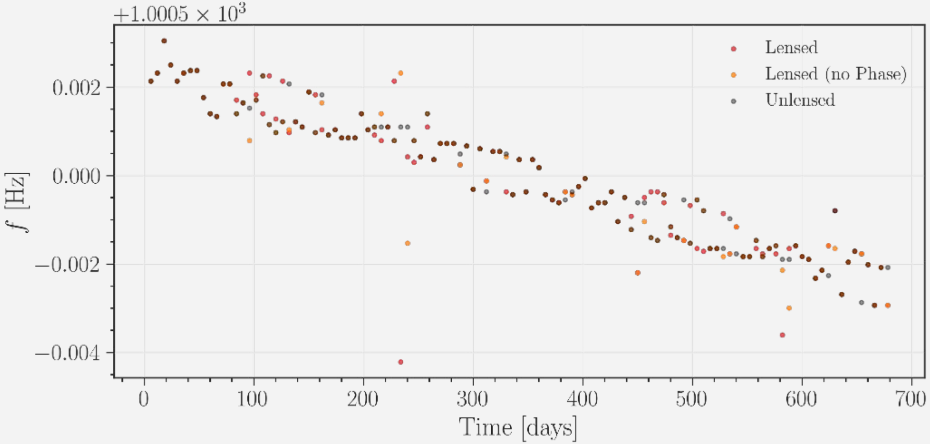
Lensing induces measurable amplitude and phase changes

- Stellar-mass black holes, the Galactic centre, dark matter clumps, or globular clusters.

$$F(f) = \frac{D_S R_E^2}{D_L D_{LS}} \frac{f(1+z_l)}{i} \int d^2x \exp[2\pi i f t_d(\mathbf{x}, \mathbf{y})]$$

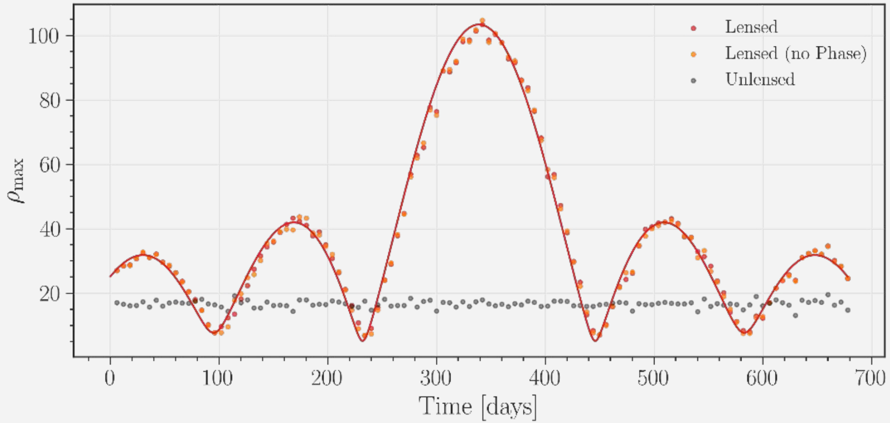
Astrophysical additions - gravitational (micro)lensing

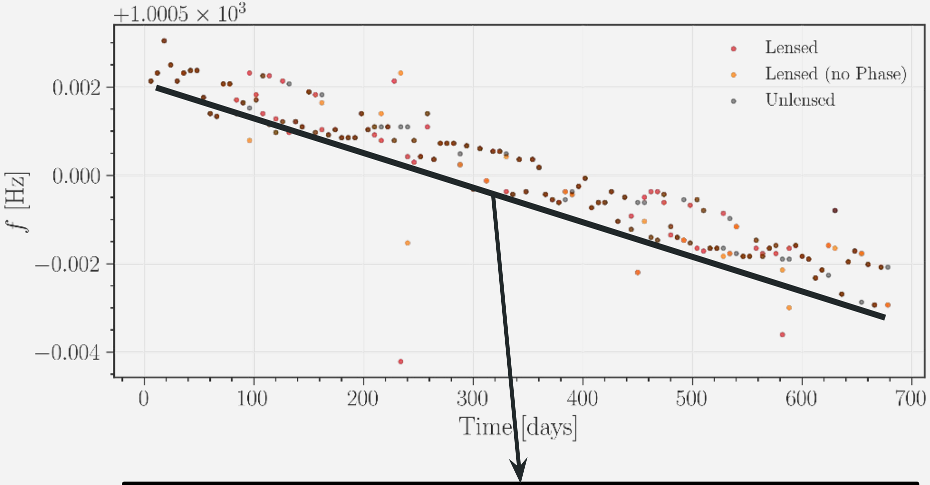




- Slight drop of SNR due to different phasing, during the deamplification, rapid phase changes are showing systematics in frequency.
- The lens solution is obtained from the point mass approximation from GWLENS by Sreekanth Harikumar, CAMK PAN

Astrophysical additions - gravitational (micro)lensing





Comes from different systematics

Known Knowns - Doppler effects

$$\widetilde{\phi}(t) = 2\pi \sum_{k=0}^{s} f_k \frac{t^{k+1}}{(k+1)!} + \frac{2\pi}{c} \mathbf{n}_0 \cdot \mathbf{r}_{\mathrm{d}}(t) \sum_{k=0}^{s} f_k \frac{t^k}{k!}$$

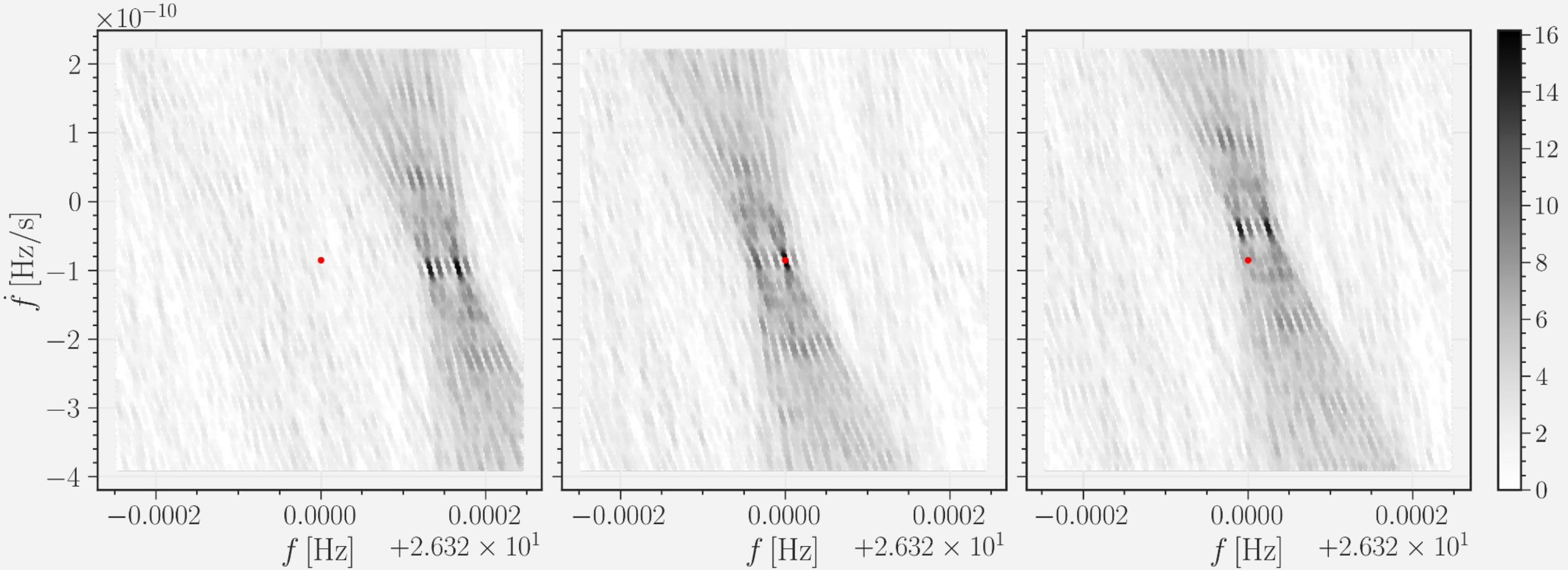
Doppler shifts due to Earth motion around the Sun

- Signals recovery depends on searched sky position: “wrong” right ascension (α) and declination (□) leads to systematics in frequency and spindown

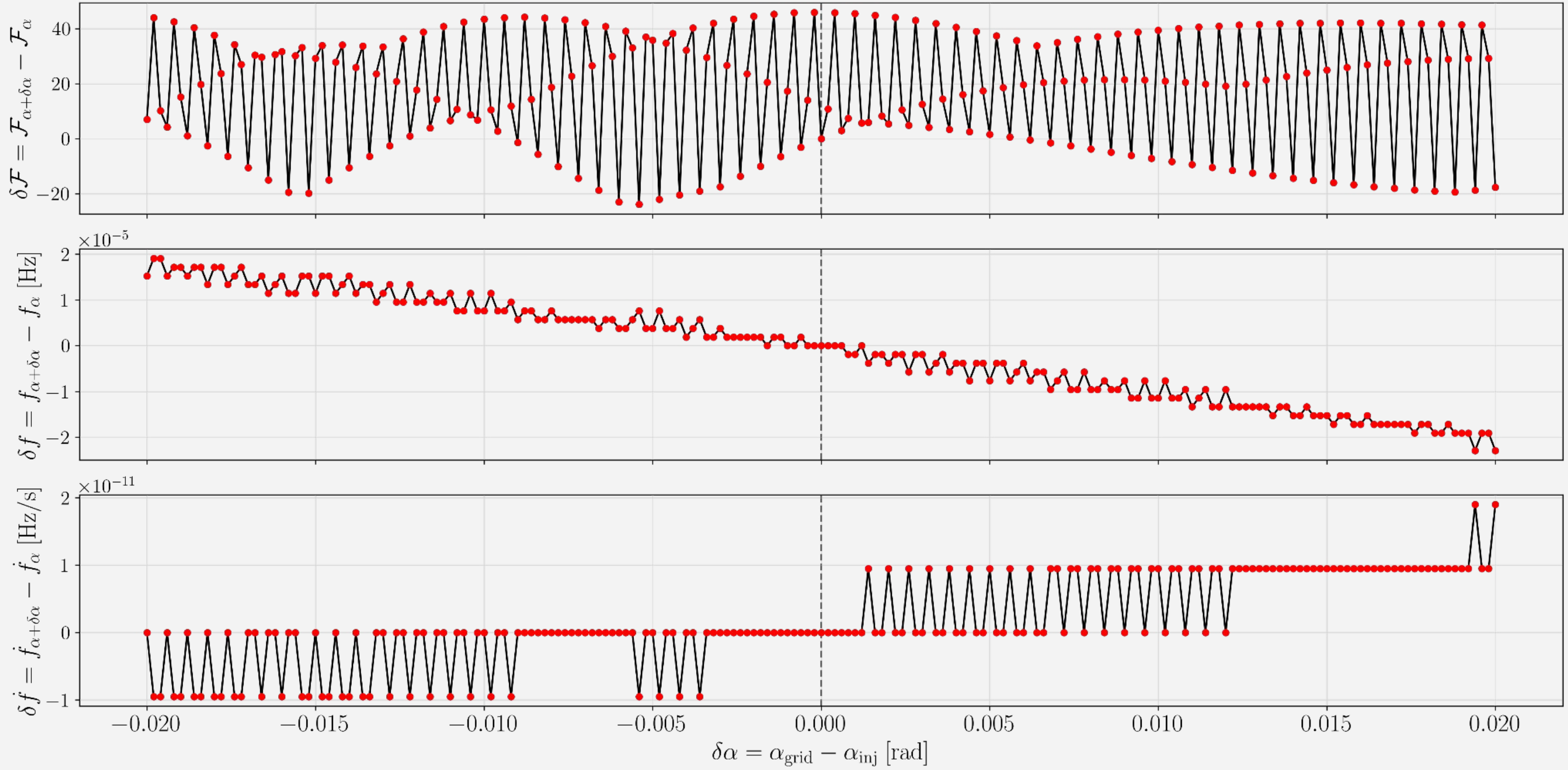
Known Knowns - Doppler effects

$\Delta\alpha = 0.1$ rad $(\sim 5.73^\circ)$

$\Delta\delta = 0.1$ rad $(\sim 5.73^\circ)$



Known Knowns - Doppler effects

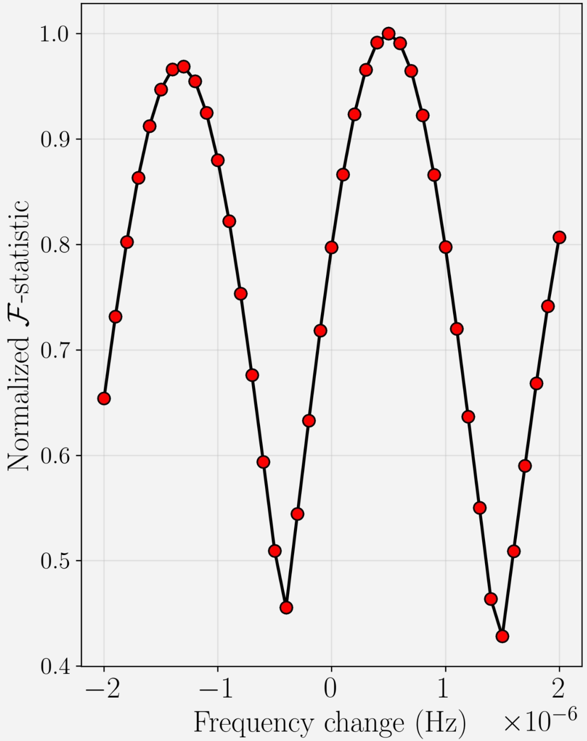


$$\dot{f} \approx \mathcal{O}(1\,\mathrm{grid})$$

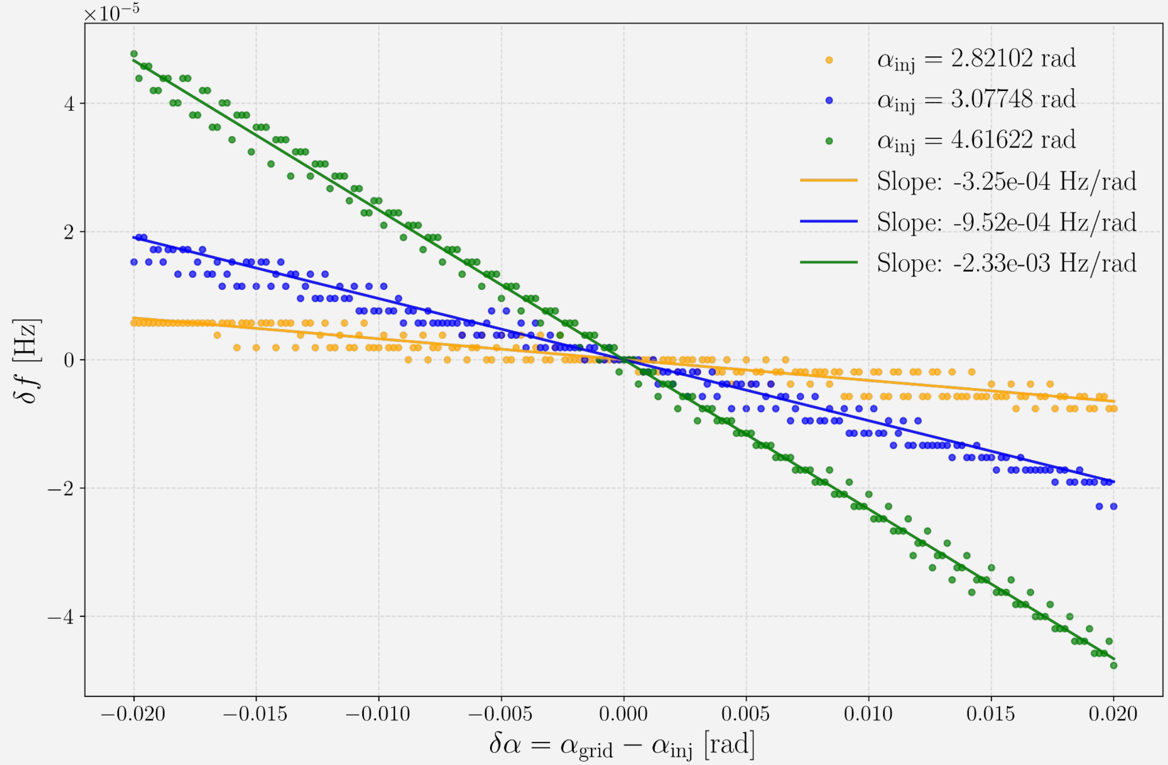
Changes in the strength of F-Statistic

- The change in F-Statistic can be attributed to two contributions.
  - Frequency lying between the FFT bins, thus leading to lowered F-Statistic.
    Can be upto 36.3% SNR loss [Pia Astone et al. 2010]
    Currently being studied independently.
  - Change in sky-position i.e, change in right ascension.

- These contributions need to be studied individually to distinguish the contributions from each of them.



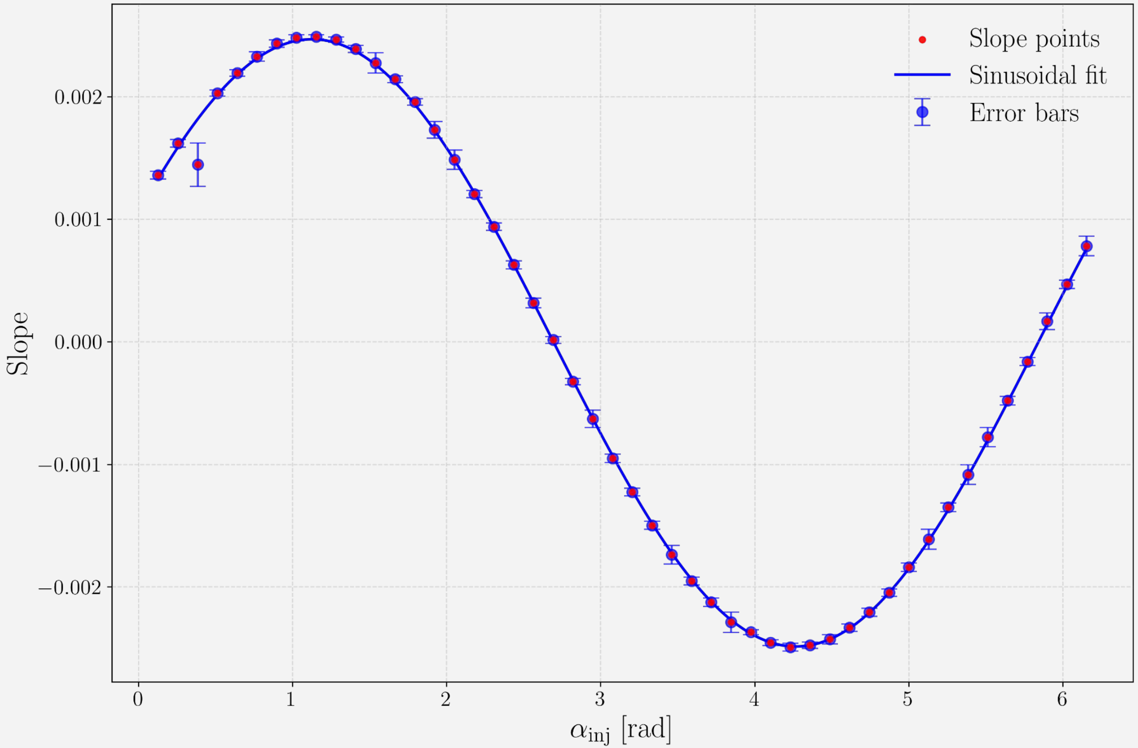
Known Knowns - Doppler effects



Known Knowns - Doppler effects

$$\text{slope} = A \sin[\alpha + \phi] + c$$

$$A = (2.4770 \pm 0.0117) \times 10^{-3},$$
$$\phi = (4.4433 \pm 0.0048) \times 10^{-1}\,\text{rad},$$
$$c = (-8.9260 \pm 8.3617) \times 10^{-6}.$$



Conclusion

- We expect that in the upcoming GW detectors such as Einstein Telescope, the effects such as
    - Timing noise, spin wandering, lensing, incorrect recovery of sky positions,

  can induce strong systematics and can affect our coincidence search methods.
- More detailed systematics should be carried out using parameter estimation methods (Fisher & Bayesian Analysis).
- Inclusion of internal mechanisms of Neutron stars in the template model can help us mitigate systematics.

Thank you