Real-Time Browser-Based PIC Simulation of Pulsar Magnetospheres

Particle-in-Cell · Yee FDTD · Kerr Spacetime · QED Pair Production · Monte Carlo Photons

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MOTIVATION & WHY IT MATTERS

Self-consistent pulsar magnetosphere modelling couples Maxwell's equations, relativistic particle dynamics, radiation, and QED pair production — demanding 10^4–10^6 CPU-hours on HPC clusters (Philippov & Spitkovsky 2014; Cerutti+ 2016).

This project demonstrates the **same physical pipeline** running interactively in a browser:

- **Rapid exploration** of magnetospheric regimes before committing HPC time
- **Intuition building** for gap formation, cascades, current sheets, spin-down
- **Education & outreach** with zero installation (single HTML file)
- **Cross-disciplinary dialogue** — observers, theorists, students share one tool

PIC · YEE FDTD · KERR GR · QED PAIRS · MC PHOTONS · BORIS PUSH

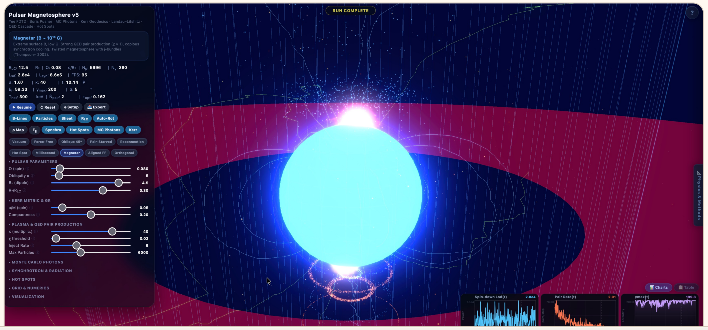
COMPUTATIONAL PIPELINE

1 CIC Current Deposition Bilinear weighting to grid	**2 Maxwell FDTD Solve** $\partial B/\partial t=-\nabla\times E$, $\partial E/\partial t=\nabla\times B-4\pi J$
3 Boundary Conditions Conducting R_*, absorbing outer	**4 Boris Particle Push** Symplectic relativistic integrator
5 Radiation Reaction Landau–Lifshitz synchrotron drag	**6 MC Photon Propagation** Kerr null geodesics + τ
7 QED Pair Creation $\gamma+B\rightarrow e\pm$, $T(\chi)$ Erber rate	**8 Hot Spot Emission** Polar cap X-ray seeds

3 sub-steps per frame × 60 FPS = 180 physics steps/s

Each PIC cycle executes this entire sequence. CIC deposition maps particle charge/current to the grid; Maxwell advances fields; Boris integrates trajectories; radiation reaction decelerates emitting particles; MC photons carry energy along Kerr geodesics; QED pair creation closes the cascade loop.

LIVE SIMULATION



SELF-CONSISTENT PAIR CASCADE

Particle in $E_\parallel$ gap (acceleration zone) → Gains $\gamma \gg 1$ (relativistic energy) → Emits MC photon (synchrotron / curvature)

Photon propagates (Kerr geodesic + τ) → χ_γ > threshold (QED parameter) → $\gamma+B \rightarrow e^+e^-$ (pair creation)

Daughters accelerate in $E_\parallel$ → cascade amplifies → modifies magnetosphere self-consistently

$\kappa = n_\pm/n_{GJ}$ **emerges from the physics**, not imposed. Distinguishes PIC from force-free. Cascade multiplicity, gap screening, and current sheet structure are all simulation outputs.

CORE EQUATIONS

$\partial B/\partial t = -\nabla\times E \;\;|\;\; \partial E/\partial t = \nabla\times B - 4\pi J$ (Maxwell on Yee grid)

$ds^2 = -\alpha^2dt^2 + (\Sigma/\Delta)dr^2 + \Sigma d\theta^2 + \varpi^2(d\varphi-\omega dt)^2$ (Kerr metric, Boyer–Lindquist)

$L_{sd} = \mu^2\Omega^4/c^3\cdot(1+\sin^2\alpha) \;\;|\;\; \rho_{GJ} = -\Omega\cdot B/(2\pi c)$ (Spin-down & GJ density)

$T(\chi) \propto \chi\cdot\exp(-8/3\chi) \;\;|\;\; \chi = (\varepsilon/mc^2)(B_\perp/B_{cr})$ (QED pair rate, Erber 1966)

WHAT THE SIMULATION COMPUTES

A **Particle-in-Cell code** running in the browser via JavaScript. It evolves electromagnetic fields on a 2.5D (r,θ) staggered Yee grid, pushes relativistic e± with a Boris integrator, propagates Monte Carlo synchrotron/curvature photon packets along Kerr null geodesics, and creates new pairs via QED magnetic pair production (Erber 1966). The Kerr spacetime provides exact lapse, frame-dragging, and gravitational redshift through analytic Boyer–Lindquist functions.

Nothing is precomputed or replayed. All fields, particles, photons, and pair cascades are generated live at 60 FPS — 180 physics steps per second.

Key Computational Methods

- **Maxwell (Yee FDTD):** ∇×E, ∇×B on staggered grid — ∇·B=0 to machine precision
- **Boris pusher:** Symplectic relativistic e± integration in self-consistent E, B
- **CIC deposition:** Cloud-in-Cell bilinear charge/current mapping
- **Kerr metric:** Exact lapse $\alpha=\sqrt{(\Delta\Sigma/A)}$, frame-dragging $\omega=2Mar/A$
- **QED pairs:** $\gamma+B\rightarrow e^+e^-$ with rate $T(\chi) \propto \chi\cdot\exp(-8/3\chi)$
- **MC photons:** Synchrotron/curvature packets along Kerr null geodesics
- **Radiation reaction:** Continuous Landau–Lifshitz synchrotron drag

DIAGNOSTICS & OUTPUTS

Six Real-Time Charts

- **$L_{sd}(t)$:** Spin-down luminosity at R_{LC}
- **$N(\gamma)$:** Particle energy spectrum (25 log bins)
- **$P_{syn}(\nu)$:** Synchrotron power spectrum
- **$N(\varepsilon)$:** Escaped photon energy spectrum
- **$\dot{N}_{pair}(t)$:** Pair creation rate history
- **$E_{\parallel max}(t)$:** Maximum parallel E-field

Time-history table (10 quantities per sample) + JSON/TXT/CSV export for reproducibility.

LIMITATIONS & SCOPE

PHYSICAL APPROXIMATIONS

- 2.5D axisymmetric (r,θ + 3D momenta)
- Compressed R_*/R_{LC} ~ 0.2
- ~6000 particles, ~2000 MC photons
- Simplified radiative transfer

COMPUTATIONAL CONSTRAINTS

- Browser limits grid & particle count
- Current sheet ~ cell size
- γ_{max} capped at ~200
- No photon splitting / inv. Compton

SUITABLE FOR

- Education and outreach
- Rapid hypothesis testing
- Conference demonstrations
- Magnetospheric intuition

NOT INTENDED FOR

- Precision fitting (NICER, Fermi-LAT)
- Production-scale PIC science
- Converged parameter studies
- Resolved kinetic microphysics

Refs: Goldreich & Julian 1969 · Spitkovsky 2006 · Philippov & Spitkovsky 2014, 2018 · Cerutti+ 2016 · Erber 1966 · Muslimov & Tsygan 1992 · Cruz+ 2021 · Kerr 1963 · Harding & Muslimov 2001 · Boris 1970