Vortex-nucleus interaction in the inner crust of neutron star

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22nd June 2026, SCALES 1st General Meeting, Coimbra

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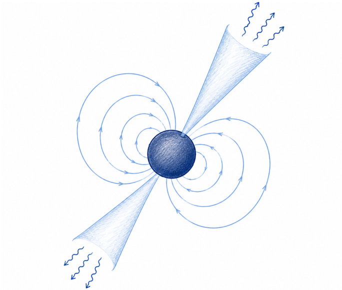
Nicolas Chamel

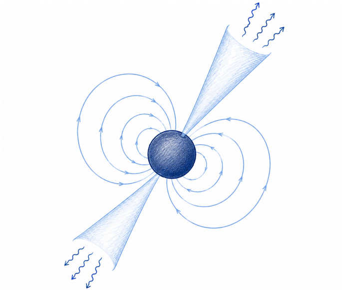
LUMI   PL GRID   NARODOWE CENTRUM NAUKI

PLGrid LUMI PLL/2022/03/016433 ★ NCN 2024/55/D/ST2/01516



$M_{\mathrm{NS}} \approx 2\,M_{\odot}$

$R_{\mathrm{NS}} \approx 12\,\mathrm{km}$



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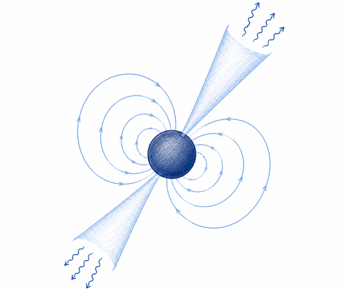
$$\bar{\rho} = \frac{3M}{4\pi R^3} \approx 5.5 \times 10^{17}\,\mathrm{kg\,m^{-3}}$$

$$k_B T_{\mathrm{NS}} \sim 10\,\mathrm{keV}$$

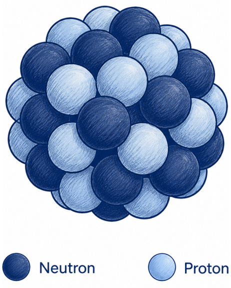


$$M_{\mathrm{NS}} \approx 2\,M_\odot$$

$$R_{\mathrm{NS}} \approx 12\,\mathrm{km}$$

$$M_{\mathrm{nuc}} \sim 200\,\mathrm{GeV}/c^2$$

$$R_{\mathrm{nuc}} \sim 10\,\mathrm{fm}$$



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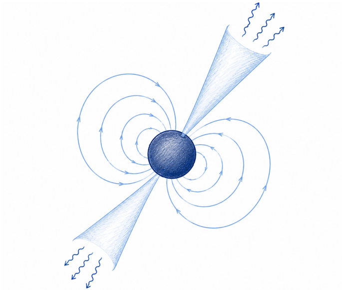

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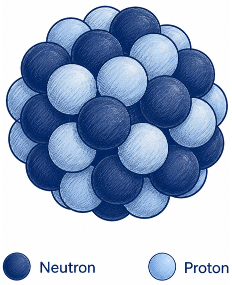

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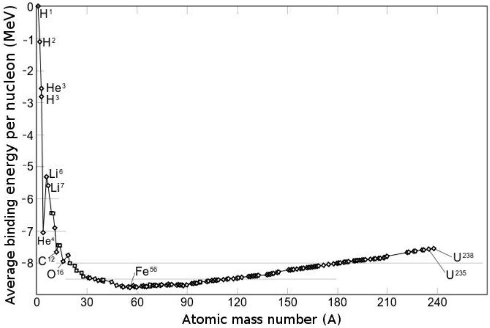
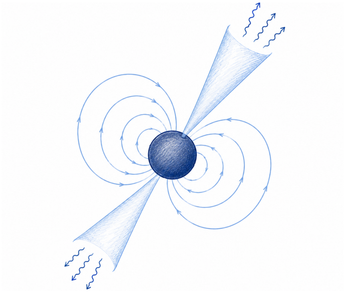
Binding energy: $E/A \approx 8\,\rm MeV$
Nuclear forces are attractive





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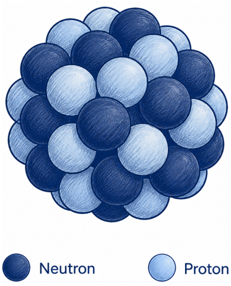

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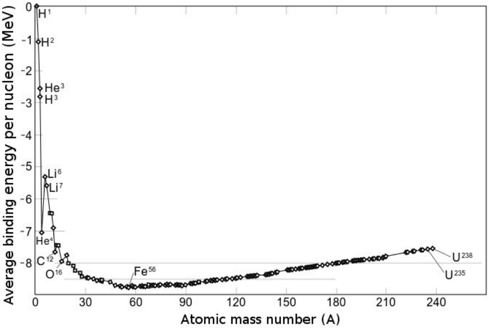
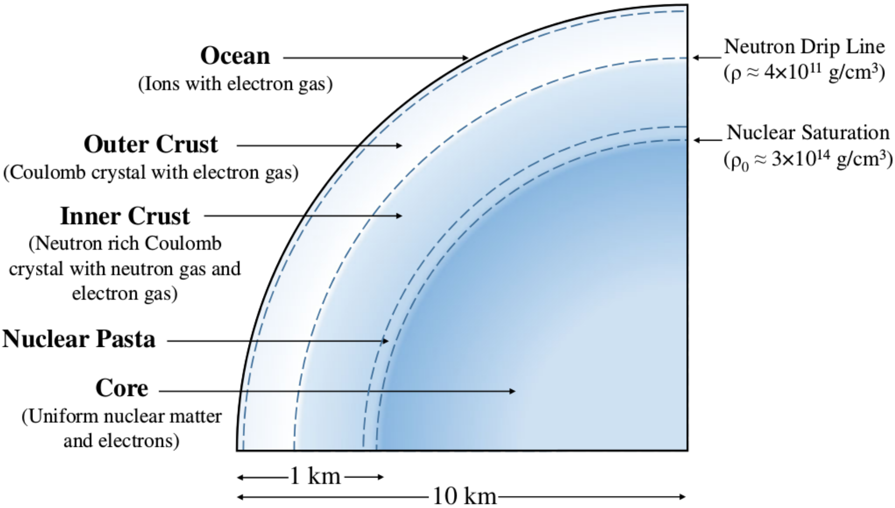
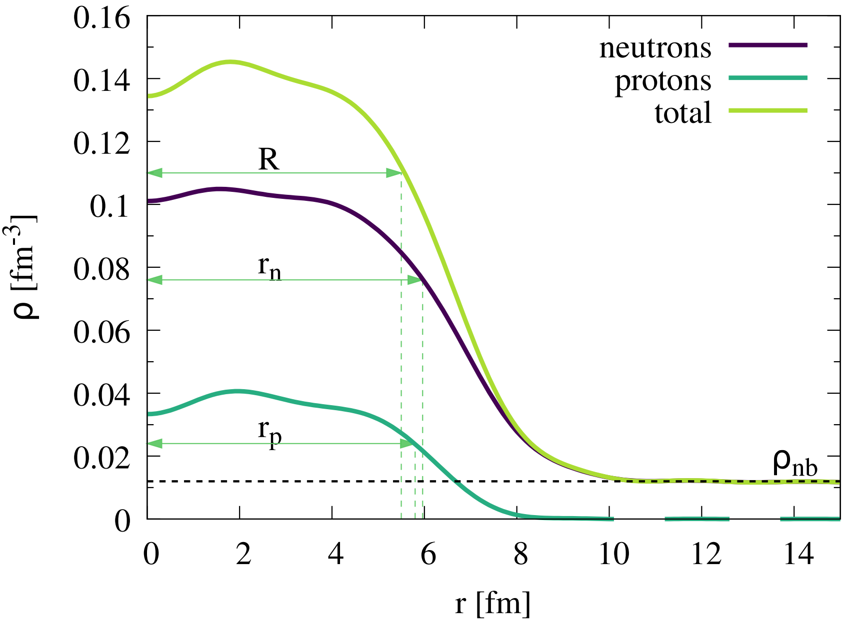
Superfluidity



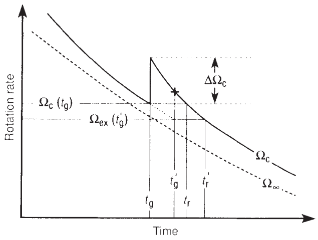


Caplan, M. E., and C. J. Horowitz, Reviews of Modern Physics 89, 041002 (2017)



DP, Zdanowicz, Chamel, Magierski, and Wlazłowski Physical Review X 14, 041054 (2024)

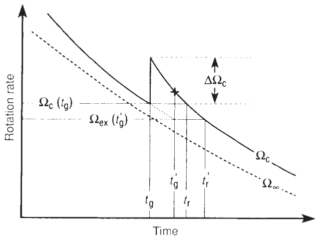
Rotation

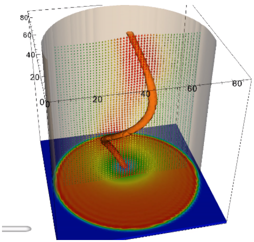


Rotation

Scale $10^4\,\mathrm{m}$

Macroscopic models
based on
precise observations





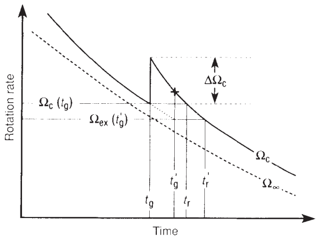
Rotation

Scale 10^4 m

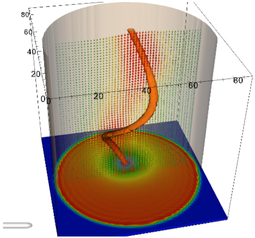
Macroscopic models
based on
precise observations

Scale 10^{-15} m

Microscopic theory
based on
nuclear experiments



Effective models



Rotation

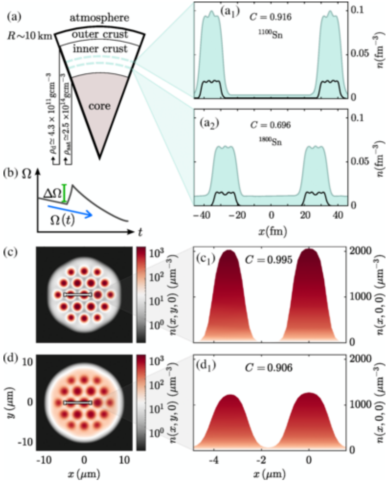
Scale 10^4 m

Macroscopic models
based on
precise observations

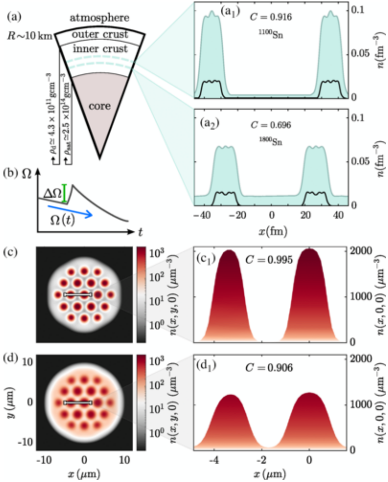
Scale 10^{-15} m

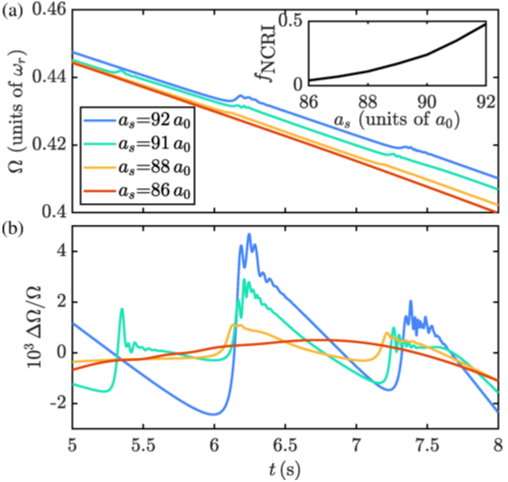
Microscopic theory
based on
nuclear experiments

Astrophysics meets ultracold gases



Astrophysics meets ultracold gases

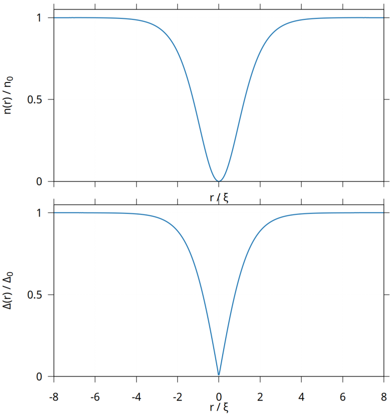




Poli, Elena, et al. "Glitches in rotating supersolids." Physical Review Letters 131, 223401 (2023)

Bose / BEC

$$\Delta_{\mathrm{BEC}} \sim \psi_{\mathrm{BEC}} \propto \sqrt{n}$$

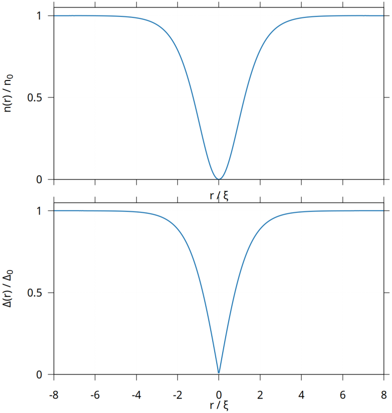


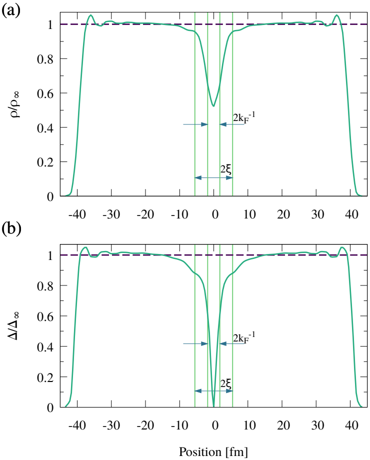
Bose / BEC

Fermi / BCS

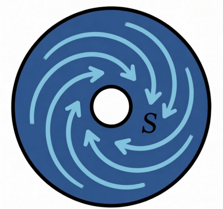
$$\Delta_{\mathrm{BEC}} \sim \psi_{\mathrm{BEC}} \propto \sqrt{n}$$

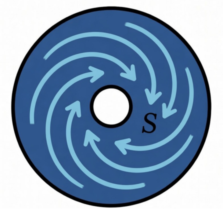
$$\Delta_{\mathrm{BCS}} \sim \psi_{\mathrm{Cooper}} \not\propto \sqrt{n}$$





DP, N. Chamel, P. Magierski, G. Wlazłowski PRC **104**, 055801 (2021)


S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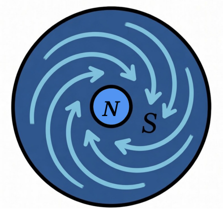
N
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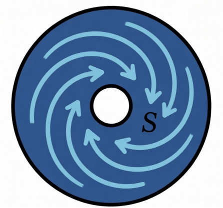

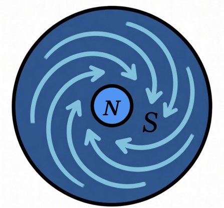

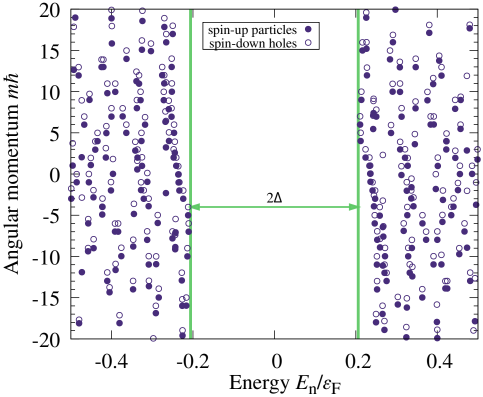

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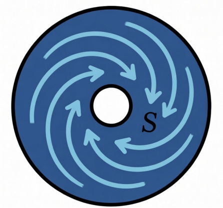

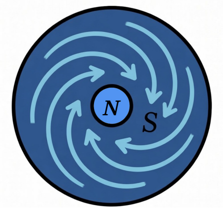

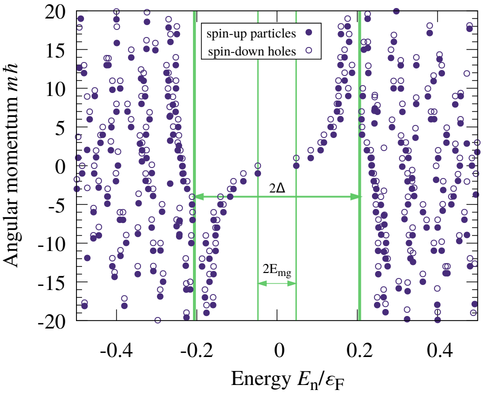

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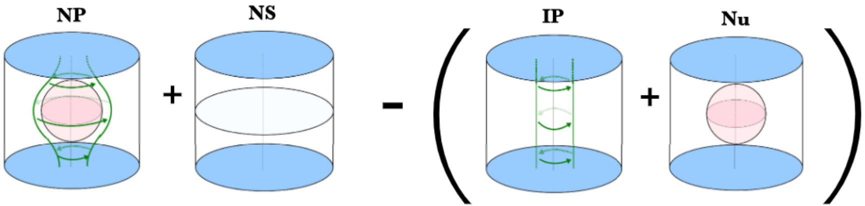
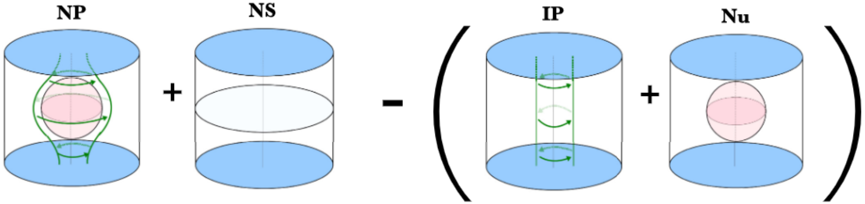


FIG. 1. Visual representation of (7). The binding energy is shown as the energy cost to move a vortex from its position on top of a nucleus to an infinite distance from it.



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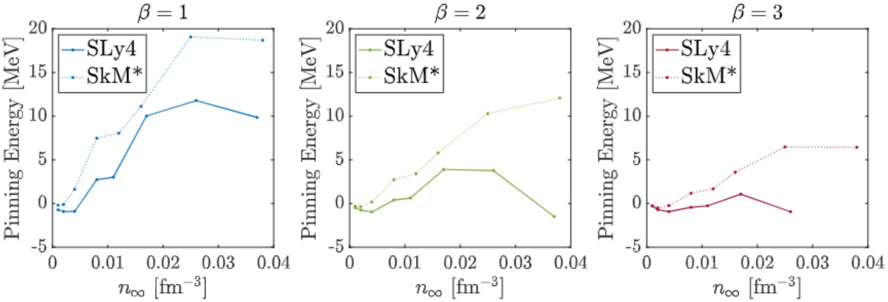

Klausner et al. PRC, 108, 035808 (2023).

$$\varepsilon(\rho, \vec{\nabla}\rho, \nu, \tau, \boldsymbol{j}) = \frac{\hbar^2}{2M}\tau + \varepsilon_\rho(\rho) + \varepsilon_\tau(\rho, \tau, \boldsymbol{j}) + \varepsilon_{\Delta\rho}(\rho, \vec{\nabla}\rho) + \varepsilon_\pi(\rho, \vec{\nabla}\rho, \nu)$$

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$$h(r) = \frac{\delta\varepsilon}{\delta\rho} - \nabla\frac{\delta\varepsilon}{\delta\tau}\nabla - \frac{i}{2}\left\{\frac{\delta\varepsilon}{\delta\boldsymbol{j}}, \nabla\right\}$$

$$\Delta(r) = \frac{\delta\varepsilon}{\delta\nu}$$

Superfluid **L**ocal **D**ensity **A**pproximation

A. Bulgac, Physical Review A **76**, 040502 (2007)

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Hartree-Fock-Bogoliubov equations

$$\begin{pmatrix} h(r) & \Delta(r) \\ \Delta^*(r) & -h^*(r) \end{pmatrix} \begin{pmatrix} u_k(r) \\ v_k(r) \end{pmatrix} = \epsilon_k \begin{pmatrix} u_k(r) \\ v_k(r) \end{pmatrix}$$

$$\varepsilon(\rho, \vec{\nabla}\rho, \nu, \tau, \boldsymbol{j}) = \frac{\hbar^2}{2M}\tau + \varepsilon_\rho(\rho) + \varepsilon_\tau(\rho, \tau, \boldsymbol{j}) + \varepsilon_{\Delta\rho}(\rho, \vec{\nabla}\rho) + \varepsilon_\pi(\rho, \vec{\nabla}\rho, \nu)$$

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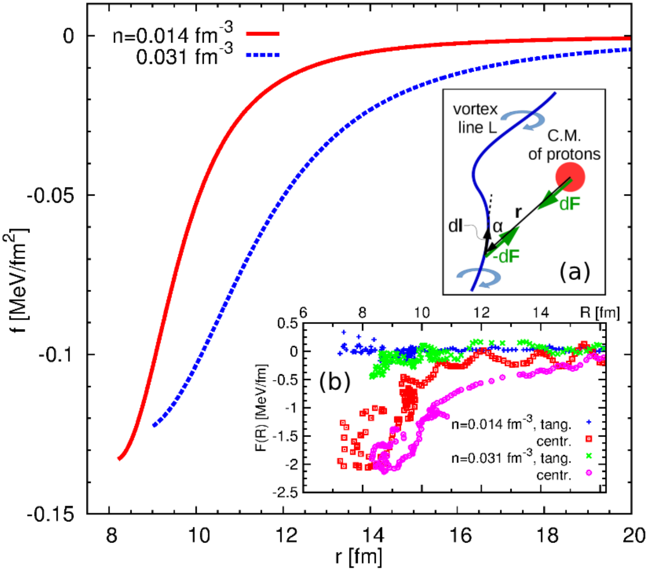
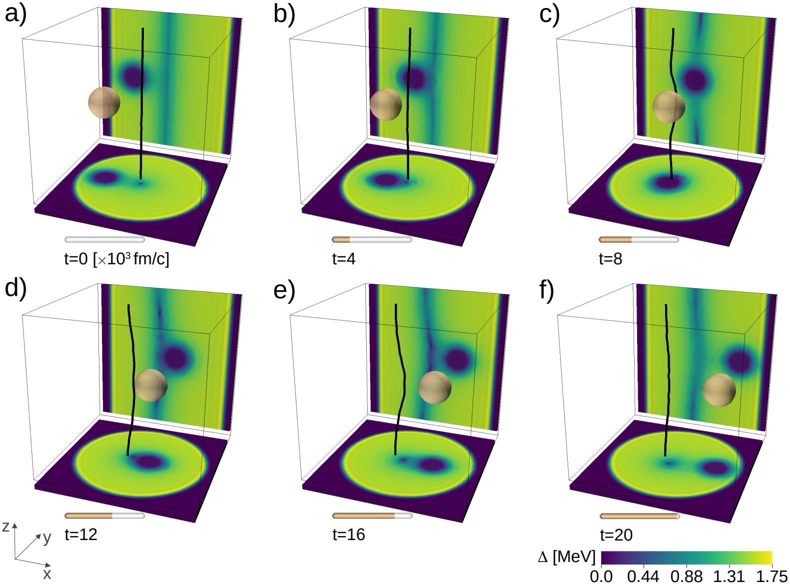
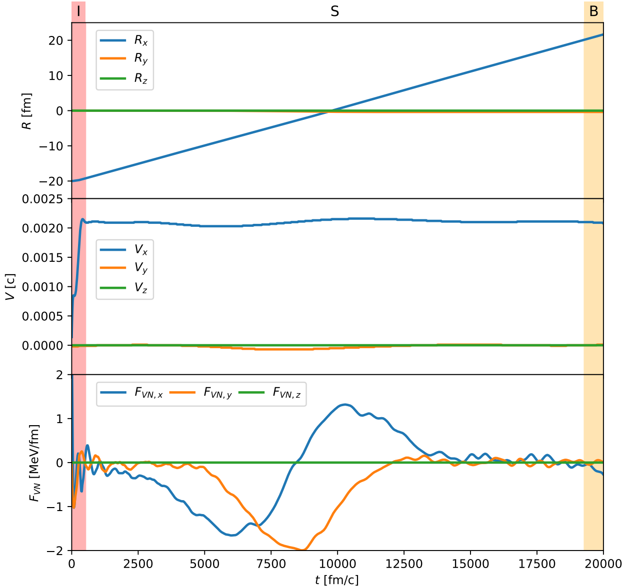
Quality of results highly depends on the quality of density functional!



Wlazłowski, Gabriel, et al. PRL 117, 232701 (2016).



$$\boldsymbol{F}_{\rm VN}(t) = M\ddot{\boldsymbol{R}}_p(t) - \boldsymbol{F}_{\rm ext}(t) \tag{1}$$



I
S
B
R_x
R_y
R_z
R [fm]
V_x
V_y
V_z
V [c]
$F_{VN,x}$
$F_{VN,y}$
$F_{VN,z}$
F_{VN} [MeV/fm]
t [fm/c]

Warsaw University of Technology | W-SLDA Toolkit
W-BSk Toolkit

W-SLDA Toolkit

Self-consistent solver of mathematical problems which have structure formally equivalent to Bogoliubov-de Gennes equations.

static problems: st-wslda

$$\begin{pmatrix} h_a(\boldsymbol{r})-\mu_a & \Delta(\boldsymbol{r}) \\ \Delta^*(\boldsymbol{r}) & -h_b^*(\boldsymbol{r})+\mu_b \end{pmatrix} \begin{pmatrix} u_n(\boldsymbol{r}) \\ v_n(\boldsymbol{r}) \end{pmatrix} = E_n \begin{pmatrix} u_n(\boldsymbol{r}) \\ v_n(\boldsymbol{r}) \end{pmatrix}$$

time-dependent problems: td-wslda

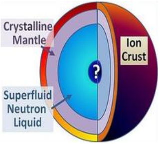
$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u_n(\boldsymbol{r},t) \\ v_n(\boldsymbol{r},t) \end{pmatrix} = \begin{pmatrix} h_a(\boldsymbol{r},t)-\mu_a & \Delta(\boldsymbol{r},t) \\ \Delta^*(\boldsymbol{r},t) & -h_b^*(\boldsymbol{r},t)+\mu_b \end{pmatrix} \begin{pmatrix} u_n(\boldsymbol{r},t) \\ v_n(\boldsymbol{r},t) \end{pmatrix}$$

Extension to nuclear matter in neutron stars

Unified solvers for static and time-dependent problems

Dimensionalities of problems: 3D, 2D and 1D

Extension to nuclear matter in neutron stars



The W-SLDA Toolkit has been expanded to encompass nuclear systems, now available as the W-BSk Toolkit.

ALL FUNCTIONALITIES →

Integration with VisIt: visualization, animation and analysis tool

Getting the code

DOWNLOAD

The W-SLDA & W-BSk Toolkits are free to download. It is published as open source under GNU GPL License. In order to get W-SLDA or W-BSk Toolkit click "Read more" and follow instructions.

READ MORE →

» Contributors

» How to cite W-SLDA

» Requirements & Documentation

Thank you!

Muito obrigado!

Brussels-Montreal (BSk) functional

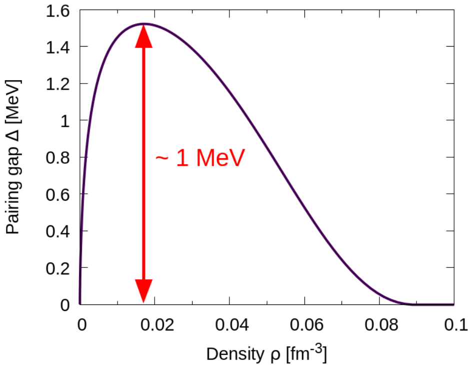
Experimental data

- atomic masses
- nuclear charge radii
- symmetry energy
- incompressibility

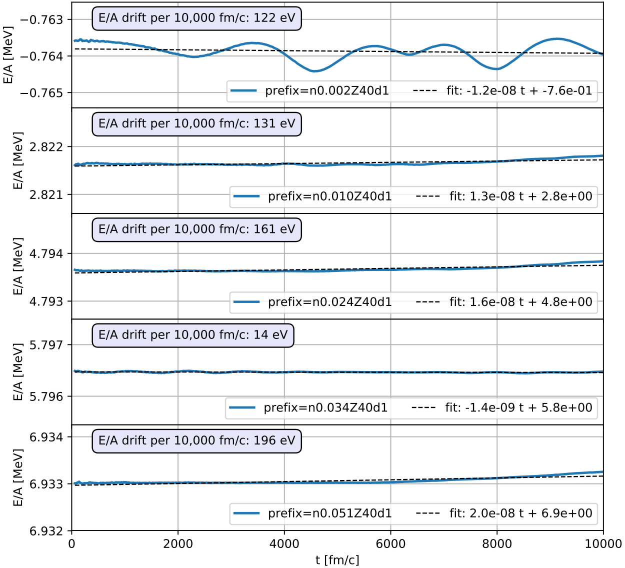
N-body calculations

- EoS of pure neutron matter
- $^1S_0$ pairing gaps in nuclear matter
- effective masses in nuclear matter

Chamel et al., Phys. Rev. C **80**, 065804 (2009)
Goriely et al., Phys. Rev. Lett. **102**, 152503 (2009)
Goriely et al., Phys. Rev. C **93**, 034337 (2016)



$$\varepsilon(\rho_q, \vec{\nabla}\rho_q, \nu_q, \tau_q, \boldsymbol{j}_q)$$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fit: -1.2e-08 t + -7.6e-01
E/A drift per 10,000 fm/c: 131 eV
prefix=n0.010Z40d1
fit: 1.3e-08 t + 2.8e+00
E/A drift per 10,000 fm/c: 161 eV
prefix=n0.024Z40d1
fit: 1.6e-08 t + 4.8e+00
E/A drift per 10,000 fm/c: 14 eV
prefix=n0.034Z40d1
fit: -1.4e-09 t + 5.8e+00
E/A drift per 10,000 fm/c: 196 eV
prefix=n0.051Z40d1
fit: 2.0e-08 t + 6.9e+00
E/A [MeV]
t [fm/c]