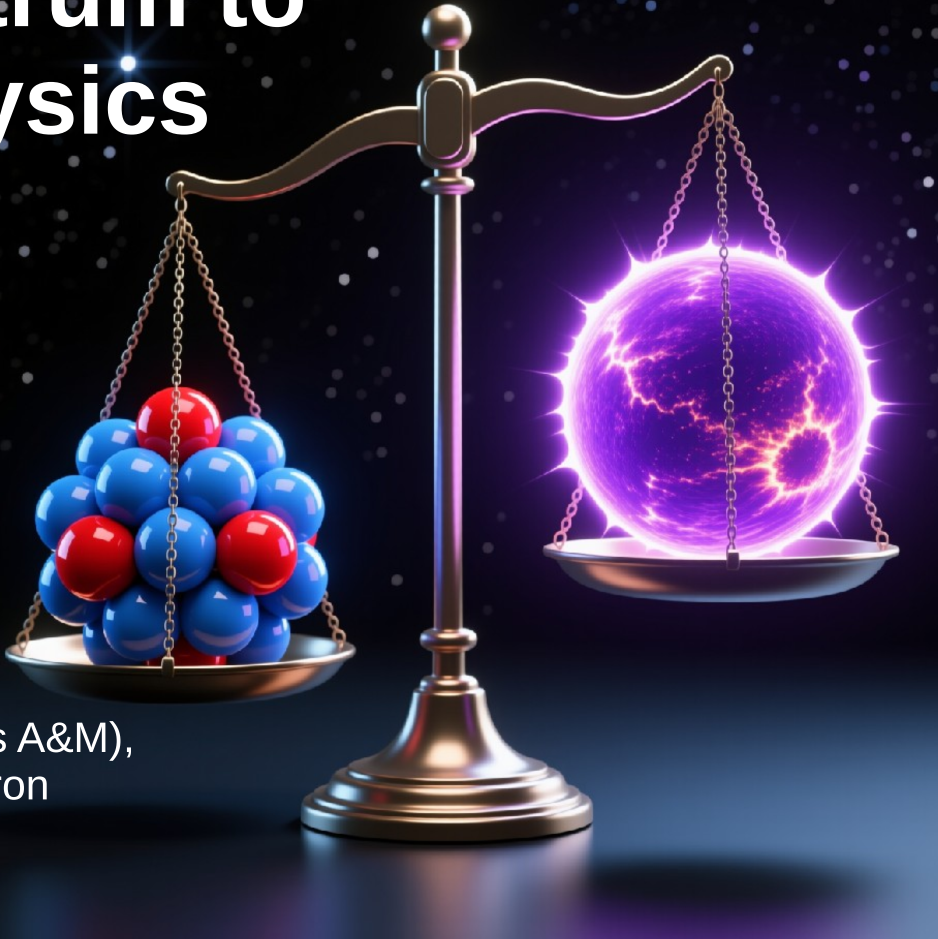
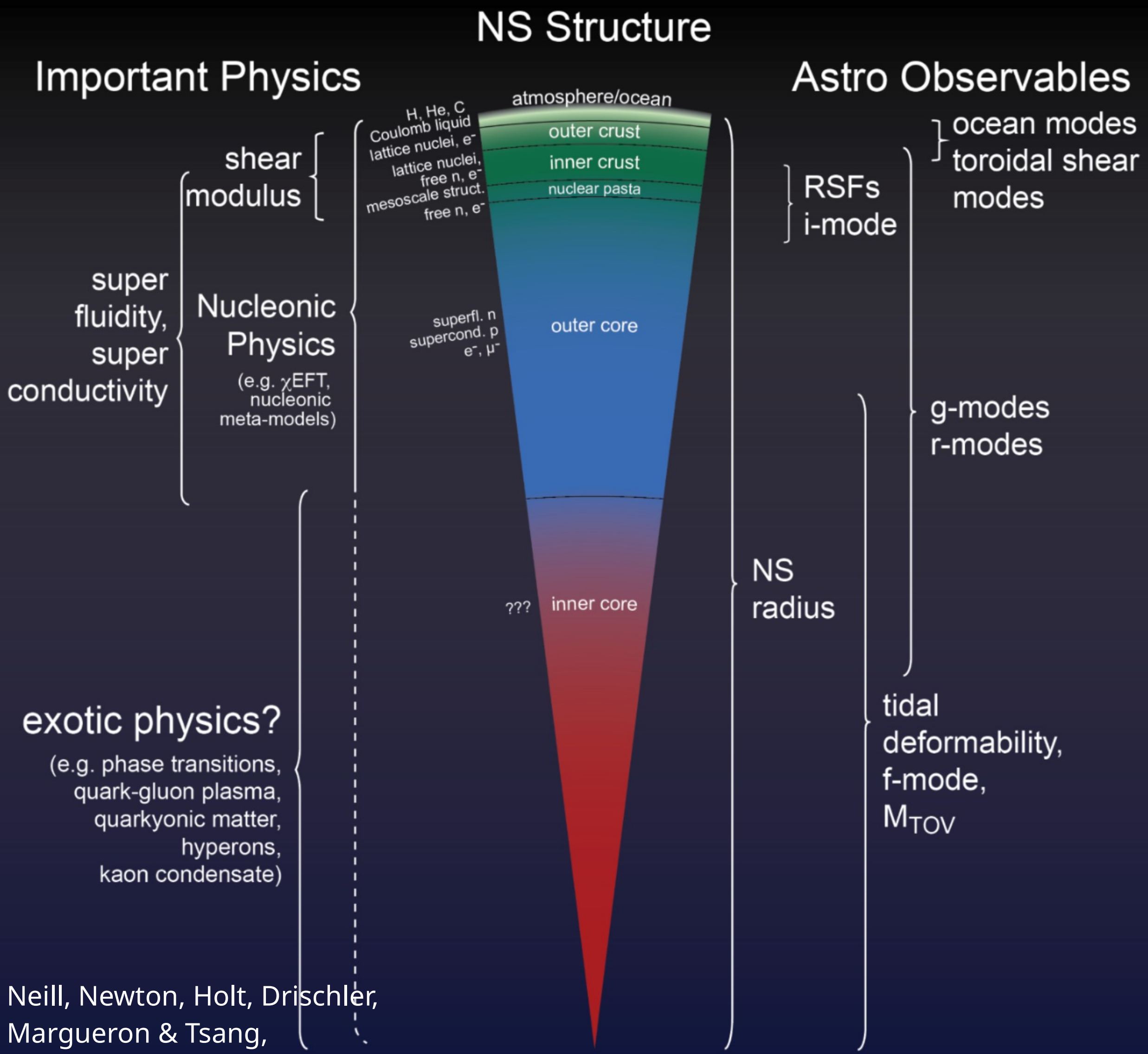
Self-consistently connecting the neutron star mode spectrum to fundamental nuclear physics



Duncan Neill*, University of Bath

With William Newton (East Texas A&M), Jeremy Holt (Texas A&M), Christian Drischler (Ohio University/FRIB), Jérôme Margueron (CNRS/FRIB) & David Tsang (University of Bath)

*dn431@bath.ac.uk



Neill, Newton, Holt, Drischler, Margueron & Tsang, Submitted (arXiv:2606.09621)

The NS M-R relationship is one-to-one with the equation of state (EOS)

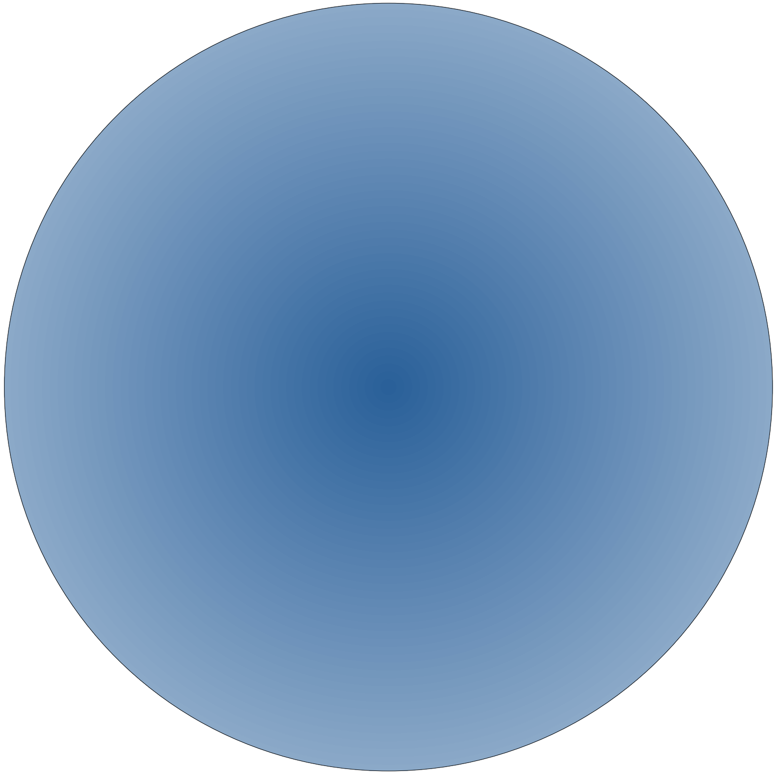
But using this to constrain the EOS requires many radius measurements for NSs with various masses

And there is a lot more interesting physics to study in NSs

How else can we probe NSs? Asterseismology!

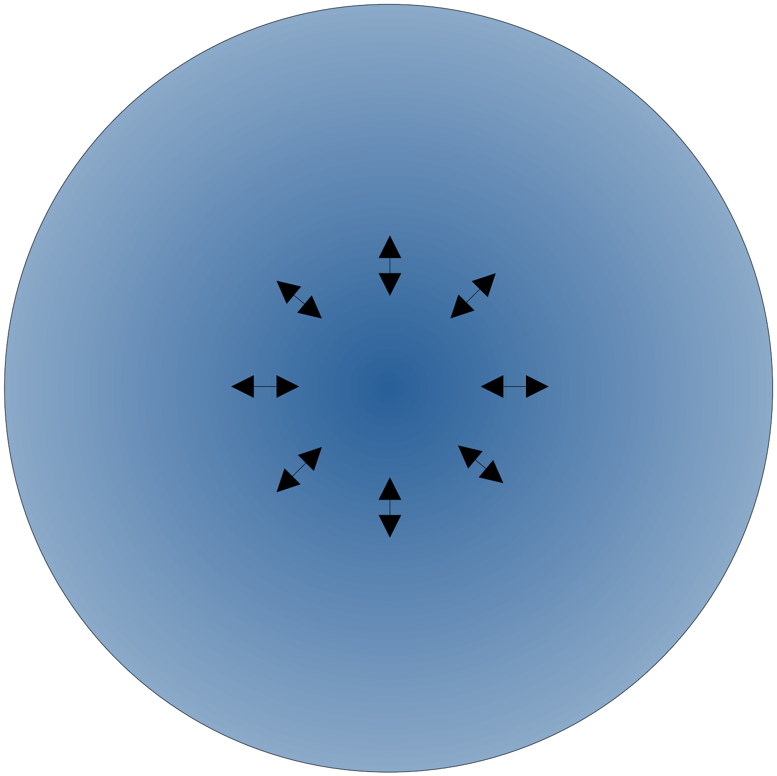
Modes

- Fluid sphere with a pressure gradient:
  fundamental (f)-modes
  pressure (p)-modes
- ...



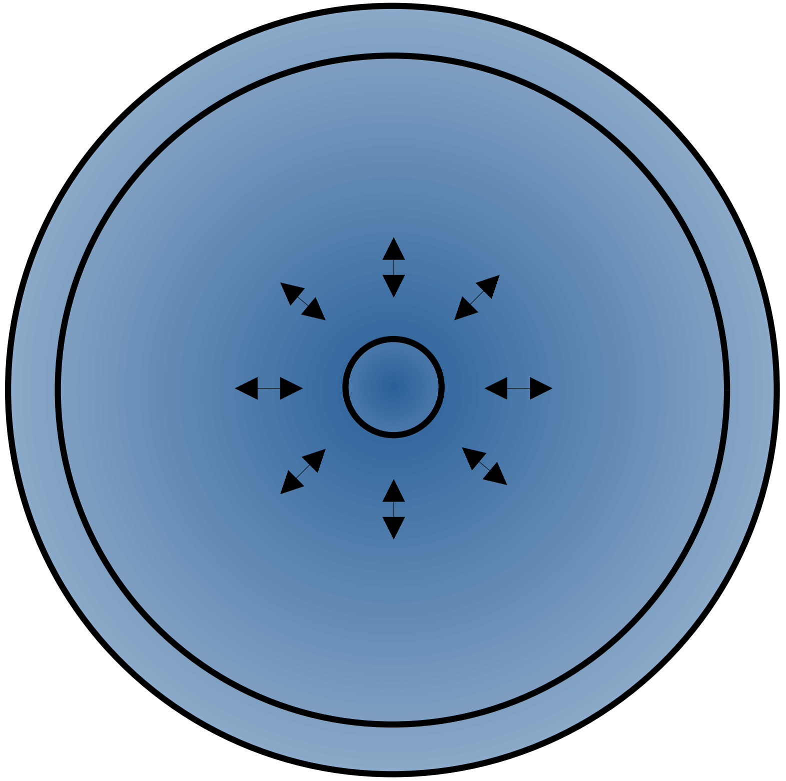
Modes

- Fluid sphere with a pressure gradient:
  fundamental (f)-modes
  pressure (p)-modes
- Add compositional gradient:
  gravitational (g)-modes
- ...



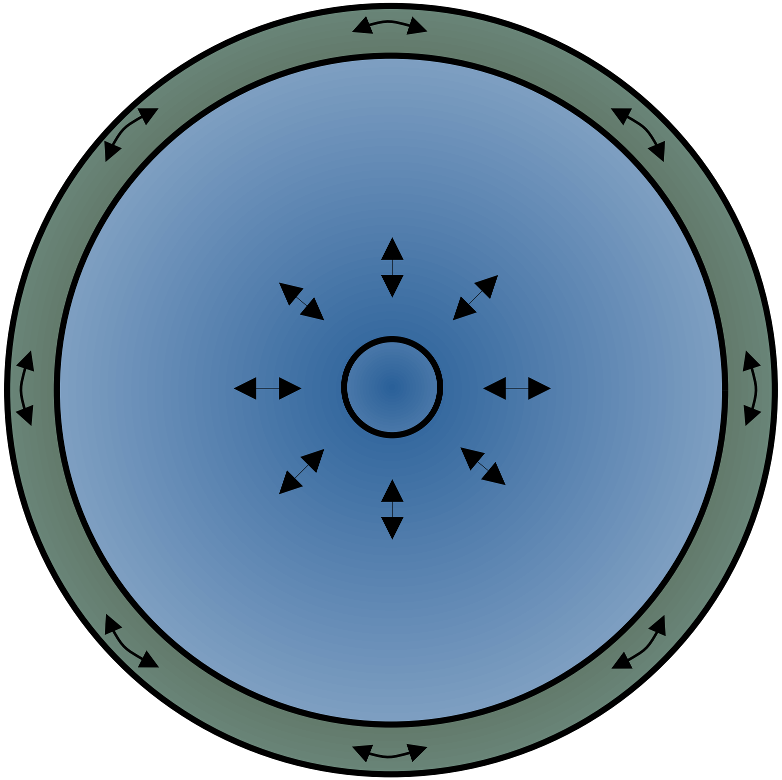
Modes

- Fluid sphere with a pressure gradient:
  fundamental (f)-modes
  pressure (p)-modes
- Add compositional gradient:
  gravitational (g)-modes
- Add phase transitions:
  interface (i)-modes
- ...



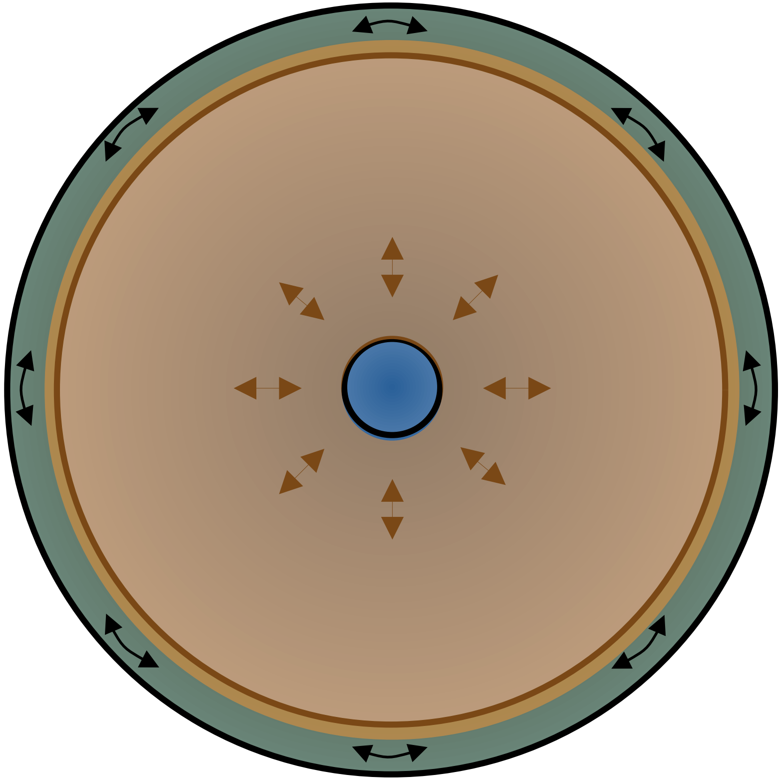
Modes

- Fluid sphere with a pressure gradient:
  fundamental (f)-modes
  pressure (p)-modes
- Add compositional gradient:
  gravitational (g)-modes
- Add phase transitions:
  interface (i)-modes
- Add shear in the solid crust:
  shear (s)-modes
- ...



Modes

- Fluid sphere with a pressure gradient:
  fundamental (f)-modes
  pressure (p)-modes
- Add compositional gradient:
  gravitational (g)-modes
- Add phase transitions:
  interface (i)-modes
- Add shear in the solid crust:
  shear (s)-modes
- Add superfluidity, superconductivity:
  counter- and co-moving mode splitting
- … (r-modes, w-modes, Alfvén modes, ...)



Perturbative mode calculation

- We solve for modes by adding small perturbations to the equations governing equilibrium NS structure, from which we obtain a set of coupled differential equations
- For a non-relativistic, non-rotating and non-superfluid star, neglecting perturbations of gravitational potential (the Cowling approximation):

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0 \, ,$$

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = \frac{1}{\rho} \nabla \cdot \boldsymbol{\sigma} - \nabla \Phi \, ,$$

$$\nabla^2 \Phi = 4\pi G \rho \, ,$$

$x \rightarrow x + \delta x$

$\delta x \delta y \approx 0$

$$(1 + \tilde{V}) \frac{dz_1}{dx} = -\left(1 + 2\frac{\alpha_2}{\alpha_3}\right) z_1 + \frac{1}{\alpha_3} z_2 + l(l+1) \frac{\alpha_2}{\alpha_3} z_3 \, ,$$

$$(1 + \tilde{V}) \frac{dz_2}{dx} = \left(-c_1 \tilde{V} \Omega^2 - 4\tilde{V} + \tilde{U}\tilde{V} + 12\Gamma_1 \frac{\alpha_1}{\alpha_3}\right) z_1 + \left(\tilde{V} - 4\frac{\alpha_1}{\alpha_3}\right) z_2 + l(l+1)\left(\tilde{V} - 6\Gamma_1 \frac{\alpha_1}{\alpha_3}\right) z_3 + l(l+1) z_4 \, ,$$

$$(1 + \tilde{V}) \frac{dz_3}{dx} = -z_1 + \frac{1}{\alpha_1} z_4 \, ,$$

$$(1 + \tilde{V}) \frac{dz_4}{dx} = \left(\tilde{V} - 6\Gamma_1 \frac{\alpha_1}{\alpha_3}\right) z_1 - \frac{\alpha_2}{\alpha_3} z_2 + \left[\!\left[-c_1 \tilde{V} \Omega^2 + \frac{2}{\alpha_3} \{[2l(l+1) - 1]\alpha_1 \alpha_2 + 2[(l(l+1) - 1]\alpha_1^2\} \right]\!\right] z_3 + (\tilde{V} - 3) z_4 \, ,$$

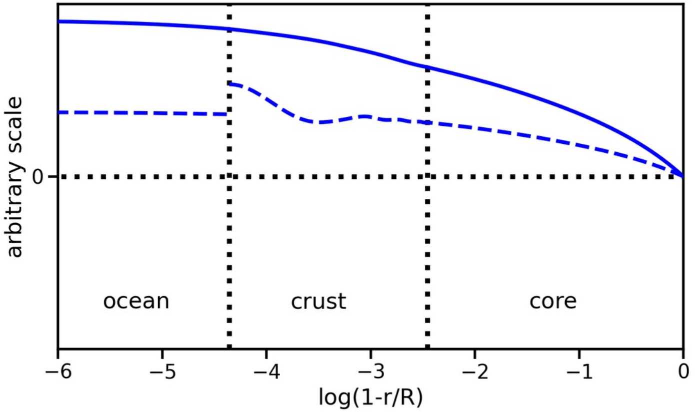
McDermott, Van Horn & Hansen, ApJ 325, 725 (1988).

We can add back in the elements neglected here: See e.g. Gittins & Andersson (2023) for slow rotation, Yoshida & Lee (2002) for relativistic equations, ...

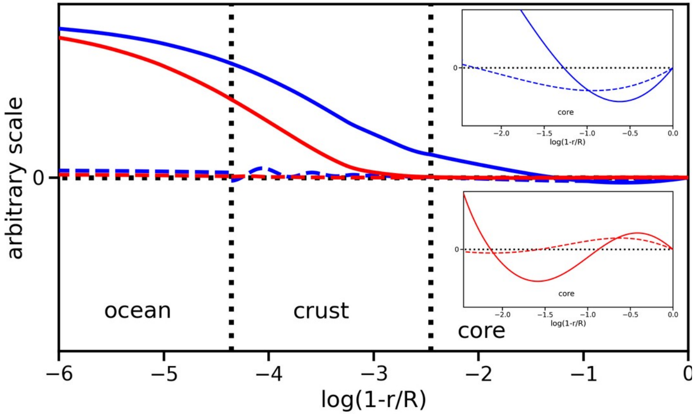
$l = 2$

Mode eigenfunctions

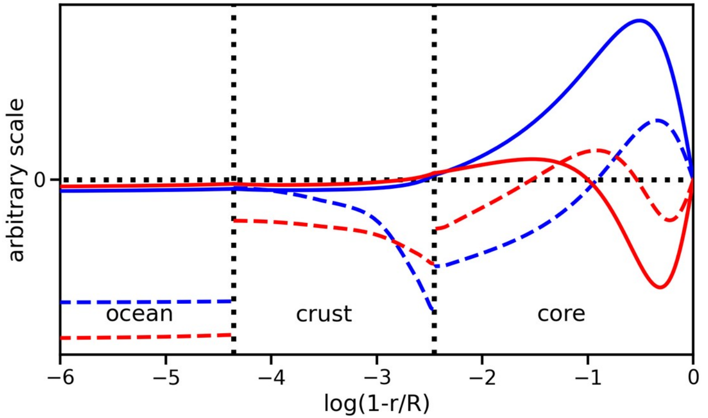
Fundamental mode



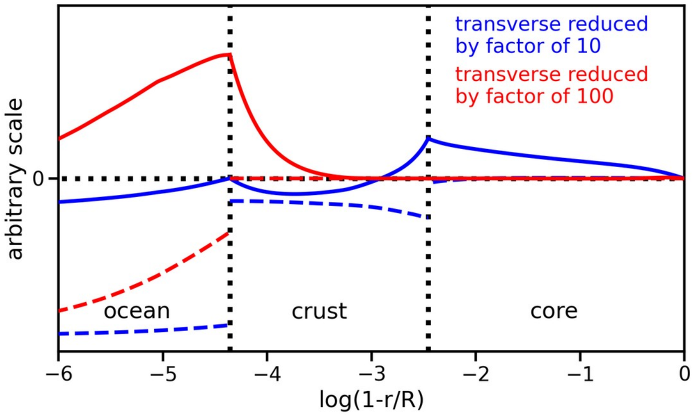
Pressure modes



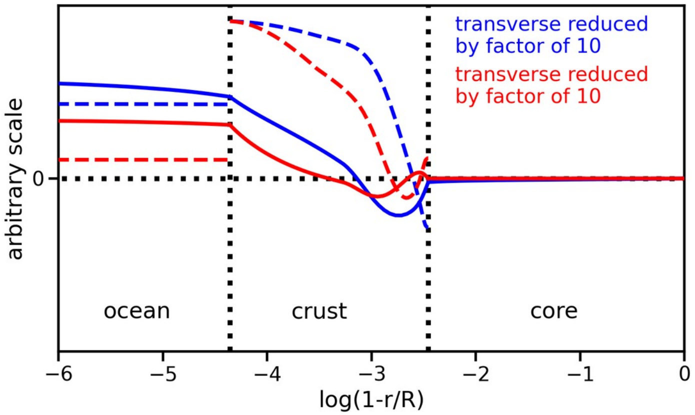
Gravitational modes



Interface modes



Shear modes



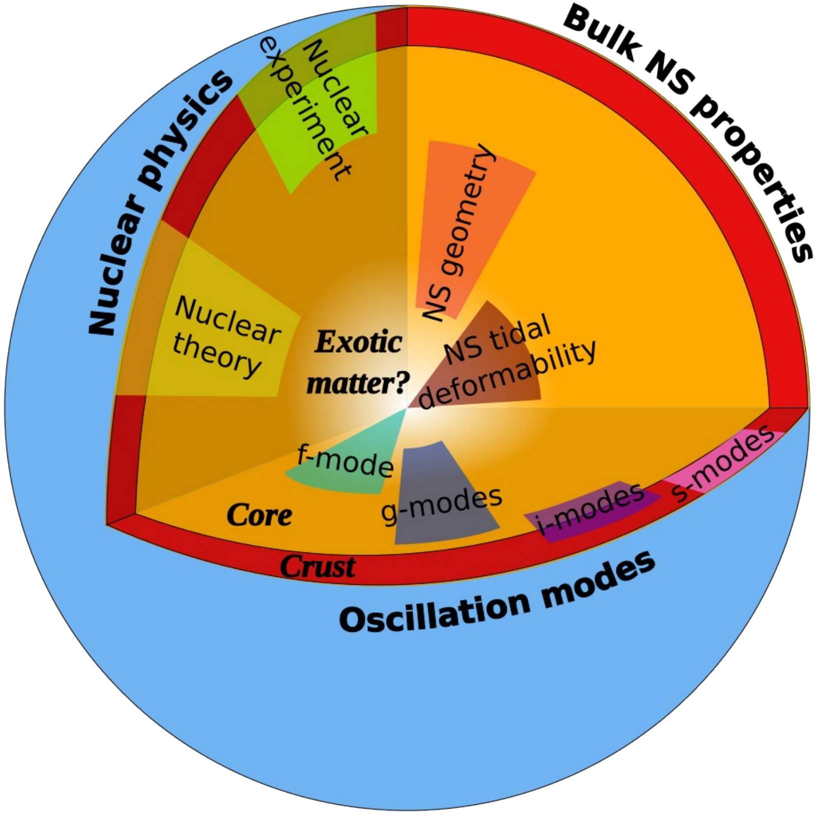
I have neglected:
Superfluidity/Superconductivity
Magnetic fields
Metric perturbations
Rotation
Temperature

NS Asteroseismology

- NS matter exists at densities not currently reached by terrestrial experiments or rigorously predicted with theory

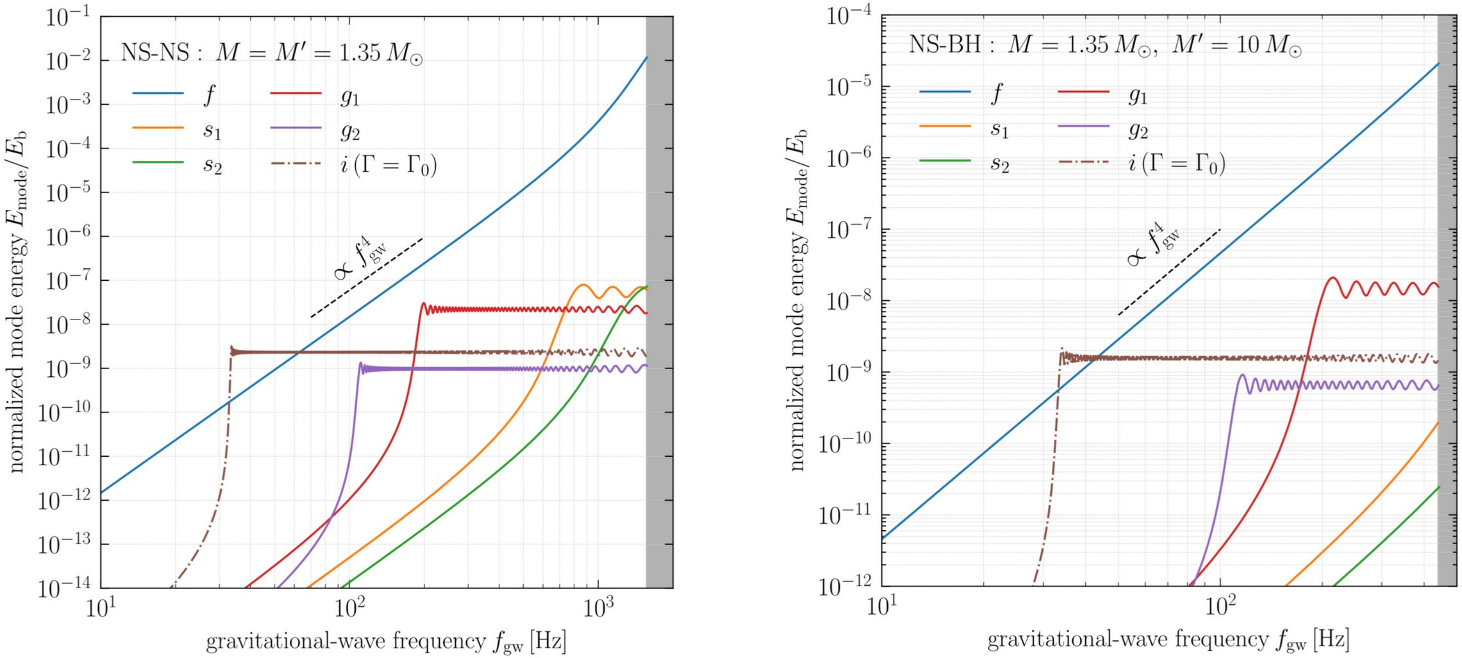
- Measurements of modes probe dynamics where their eigenfunctions are located, allowing for detailed insight into NS matter

- Unlike integrated (bulk) properties, such as radius and tidal deformability, modes can be highly sensitive to matter in a small region of the star



Asteroseismic GW observables

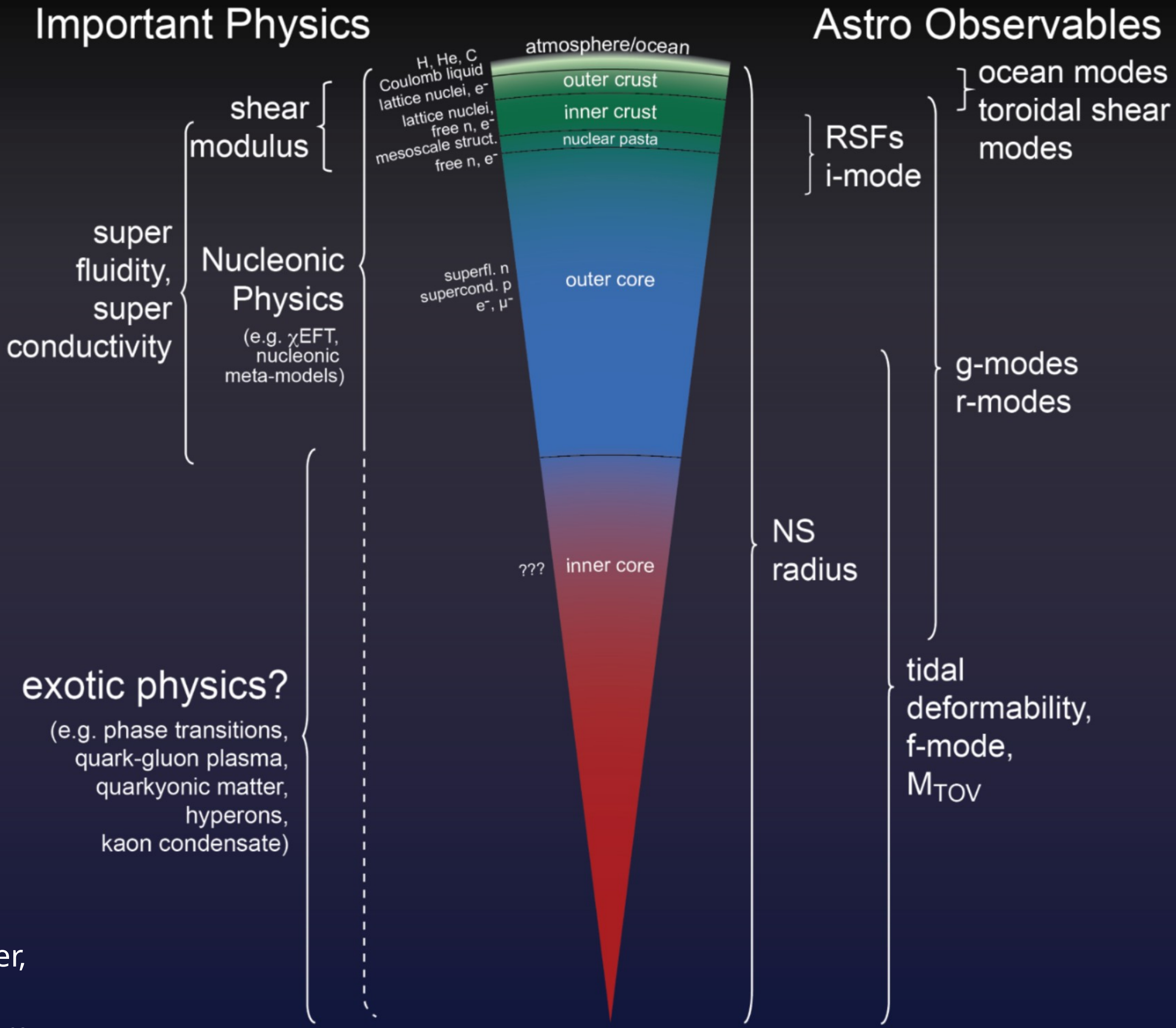
Energy is transferred to modes from binary orbit
→ phase shift / time advance of binary merger



Y..Gao et al., Phys. Rev. D 112, 123006 (2025).

What can we learn from mode frequencies and eigenfunctions?

NS Structure



Neill, Newton, Holt, Drischler, Margueron & Tsang, Submitted (arXiv:2606.09621)

(Cold) Infinite Nuclear Matter Equation of State

EOS:

$$E(n,\delta) = E(n,0) + E_{\rm sym}(n)\delta^2 + \mathcal{O}[\delta^4]$$

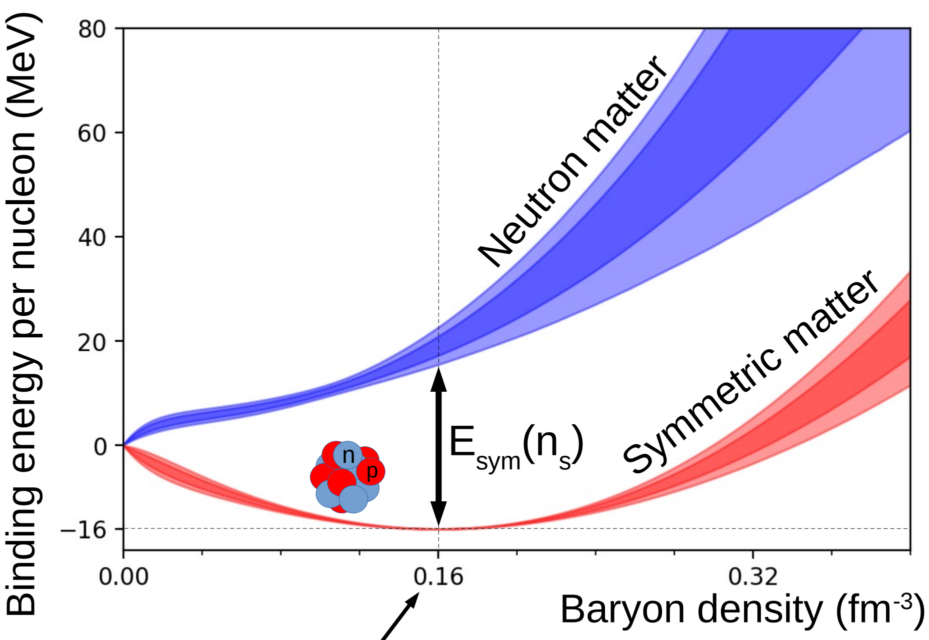
Expansion in nuclear asymmetry

$$E_{\rm sym}(n) = J + L\chi + \frac{1}{2}K_{\rm sym}\chi^2 + \mathcal{O}[\chi^3]$$

Expansions in density $\left(\chi = \frac{n-n_s}{3n_s}\right)$

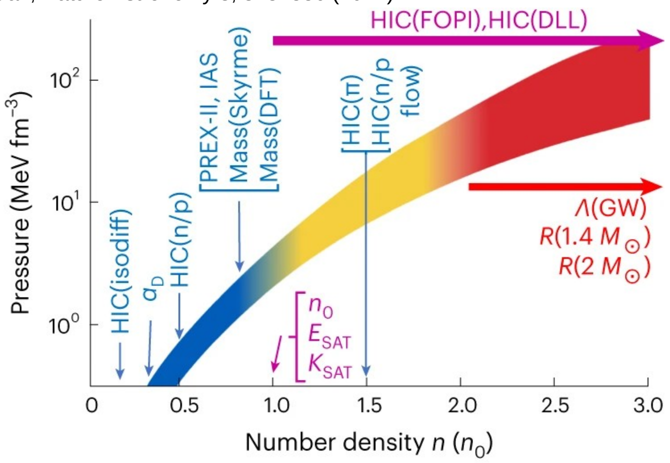
$$E_0(n,0) = E_0 + \frac{1}{2}K_0\chi^2 + \mathcal{O}[\chi^3]$$

The EOS can be described by
E_0, n_s, K_0, …, J, L, K_{sym}, …

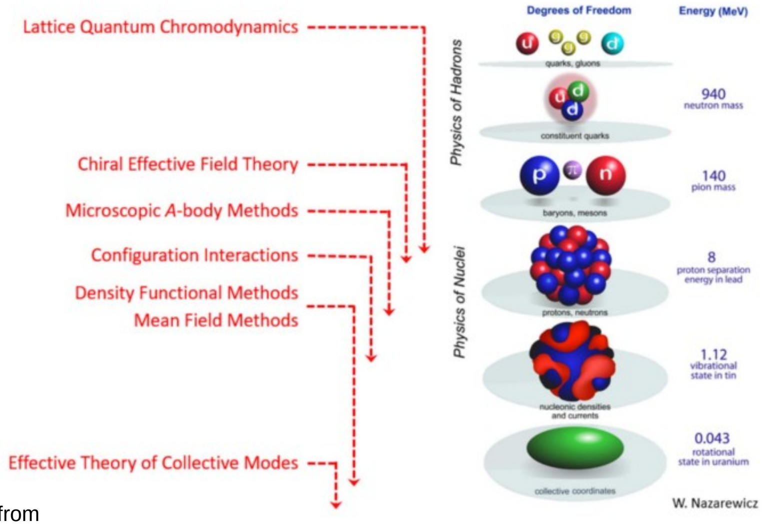


Nuclear saturation density (n_s) ≈ 0.16 fm^{-3}

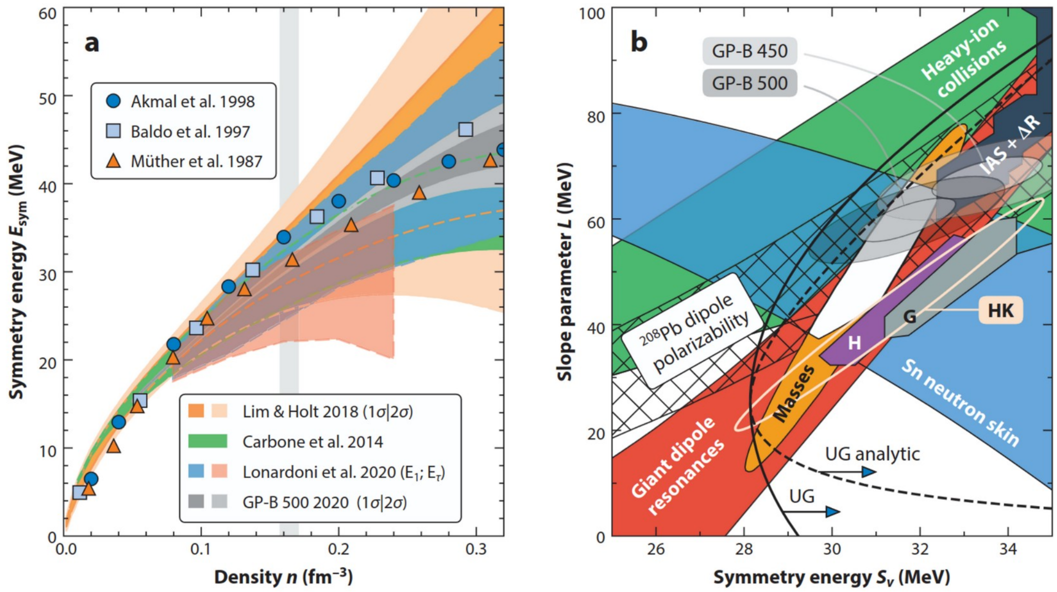
C. Y. Tsang et al., Nature Astronomy 8, 328–336 (2024)



Nuclear matter EOS constraints

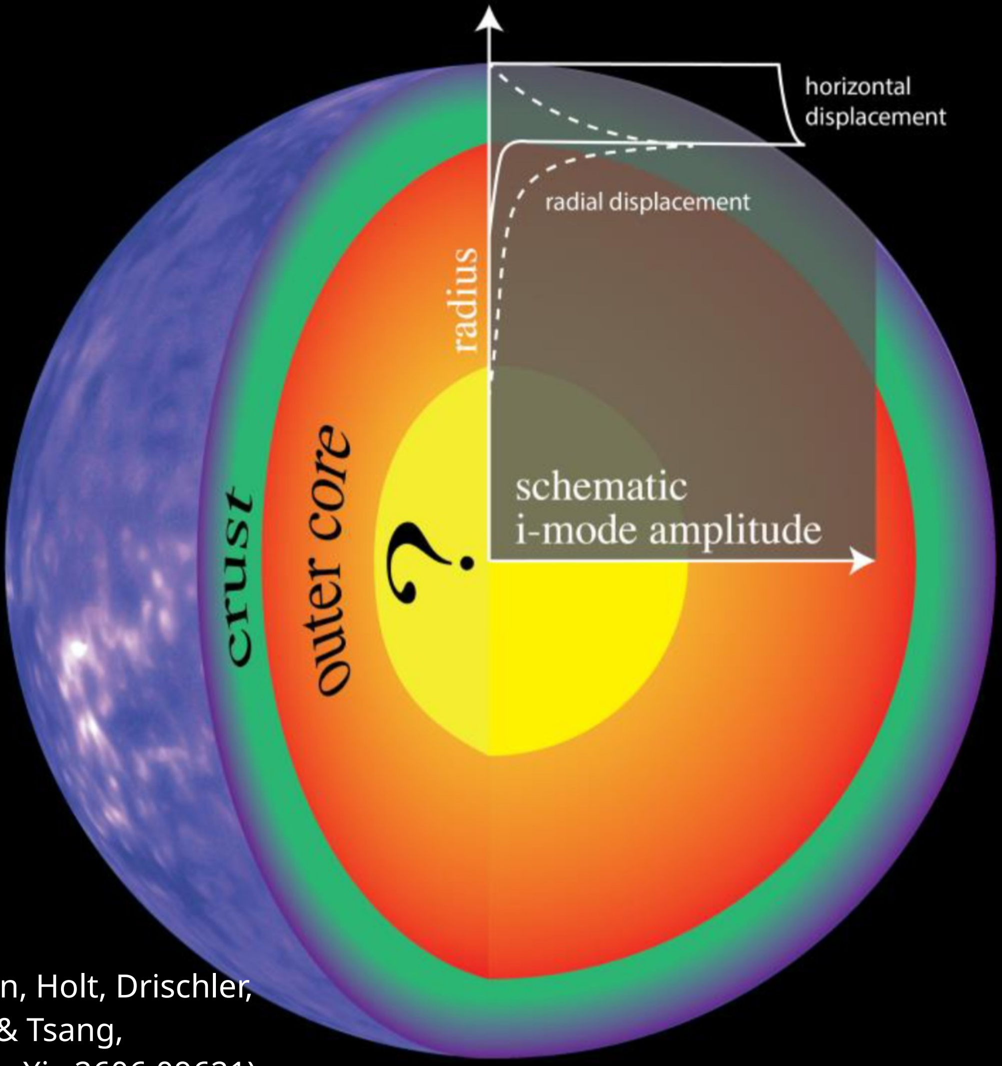


Illustation from
https://indico-tdli.sjtu.edu.cn/event/2821/attachments/5277/8738/20241210_SJTU_bnlu.pdf,
adapted from
https://archive.int.washington.edu/NNPSS/2009_school/talks/nazarewicz/nazarewicz_handouts2.pdf



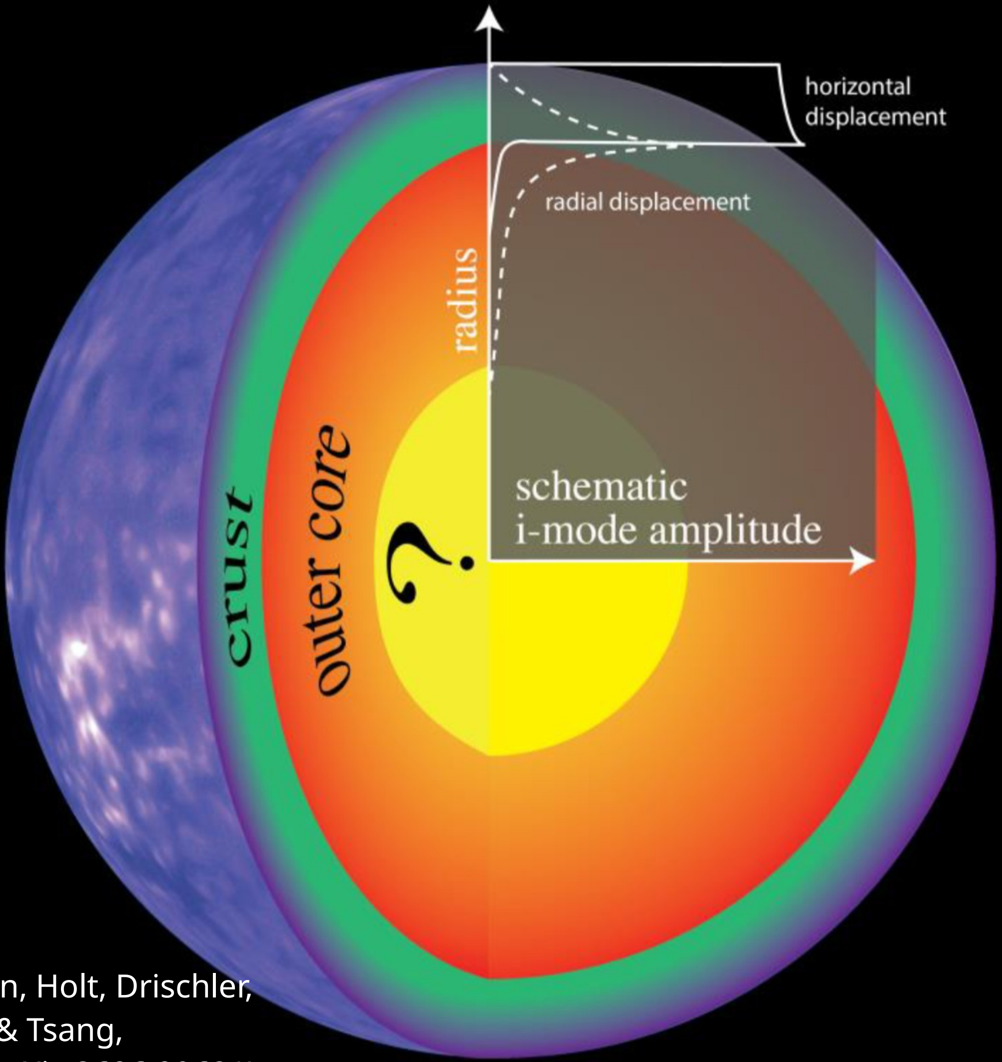
C. Drischler, J. W. Holt & C. Wellenhofer, Annu. Rev. Nucl. Part. Sci. 71, 403–432 (2021) (and see references therein)

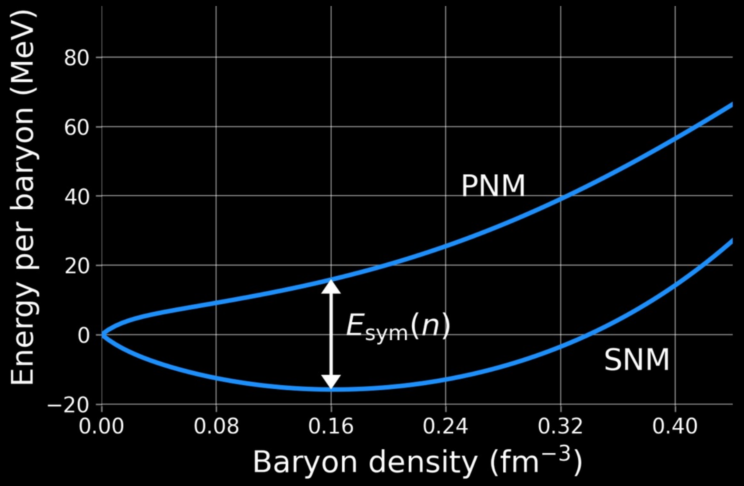
Crust-core interface mode

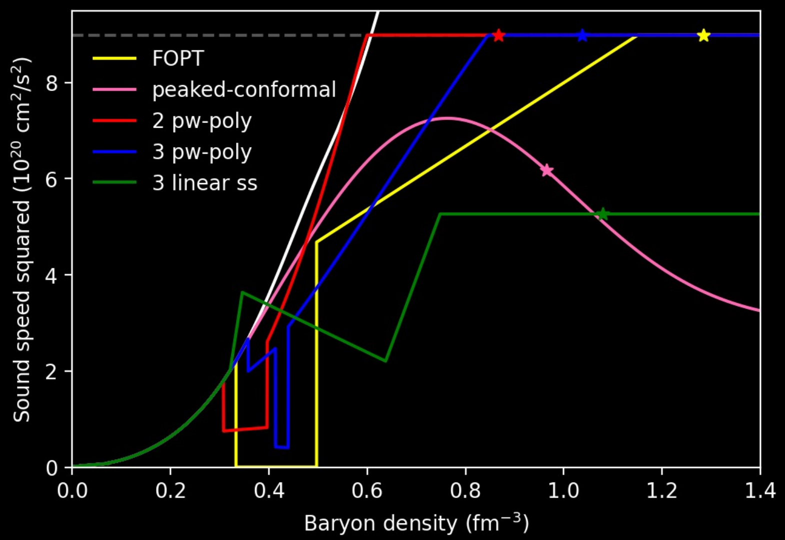


- The i-mode oscillates around the crust-core transition, restored by elasticity and compositional buoyancy
- Elasticity and buoyancy in-turn depend on the structure and composition of neutron star matter
- NS structure and composition are primariy determined by the equation of state of (cold) infinite nuclear matter

Neill, Newton, Holt, Drischler, Margueron & Tsang, Submitted (arXiv:2606.09621)

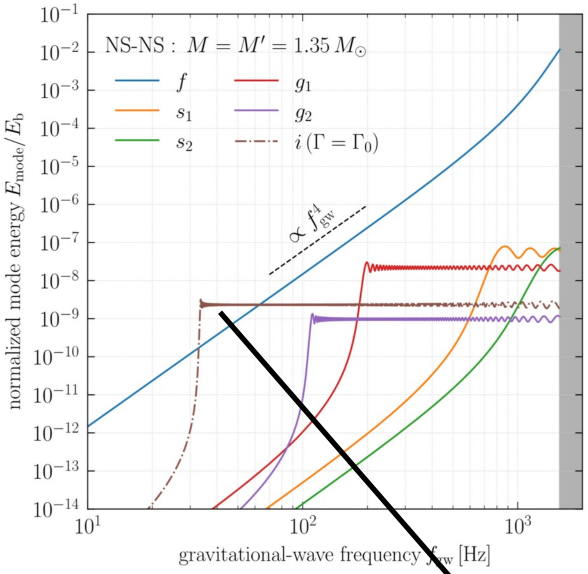




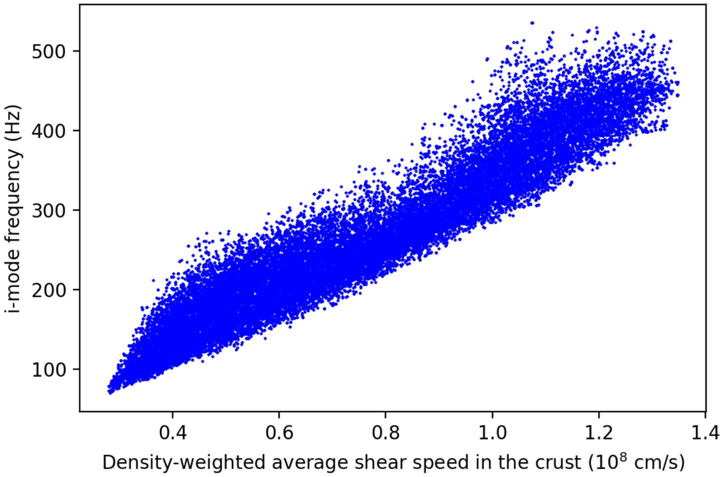


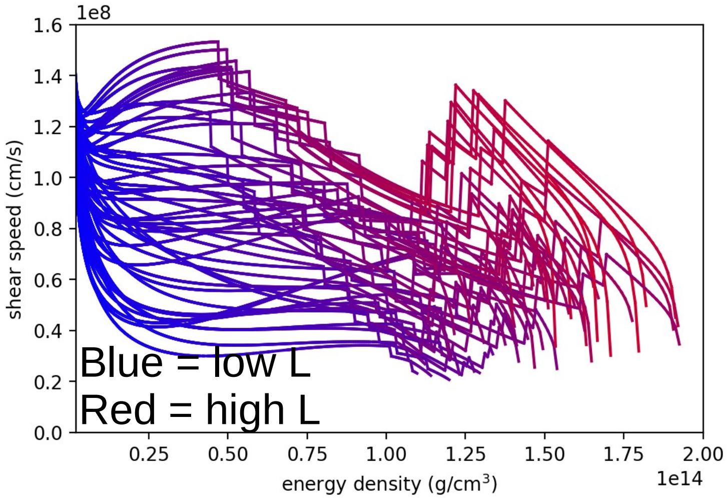
Neill, Newton, Holt, Drischler,
Margueron & Tsang,
Submitted (arXiv:2606.09621)

Constraining nuclear physics using i-mode frequency



Y..Gao et al., Phys. Rev. D 112, 123006 (2025).





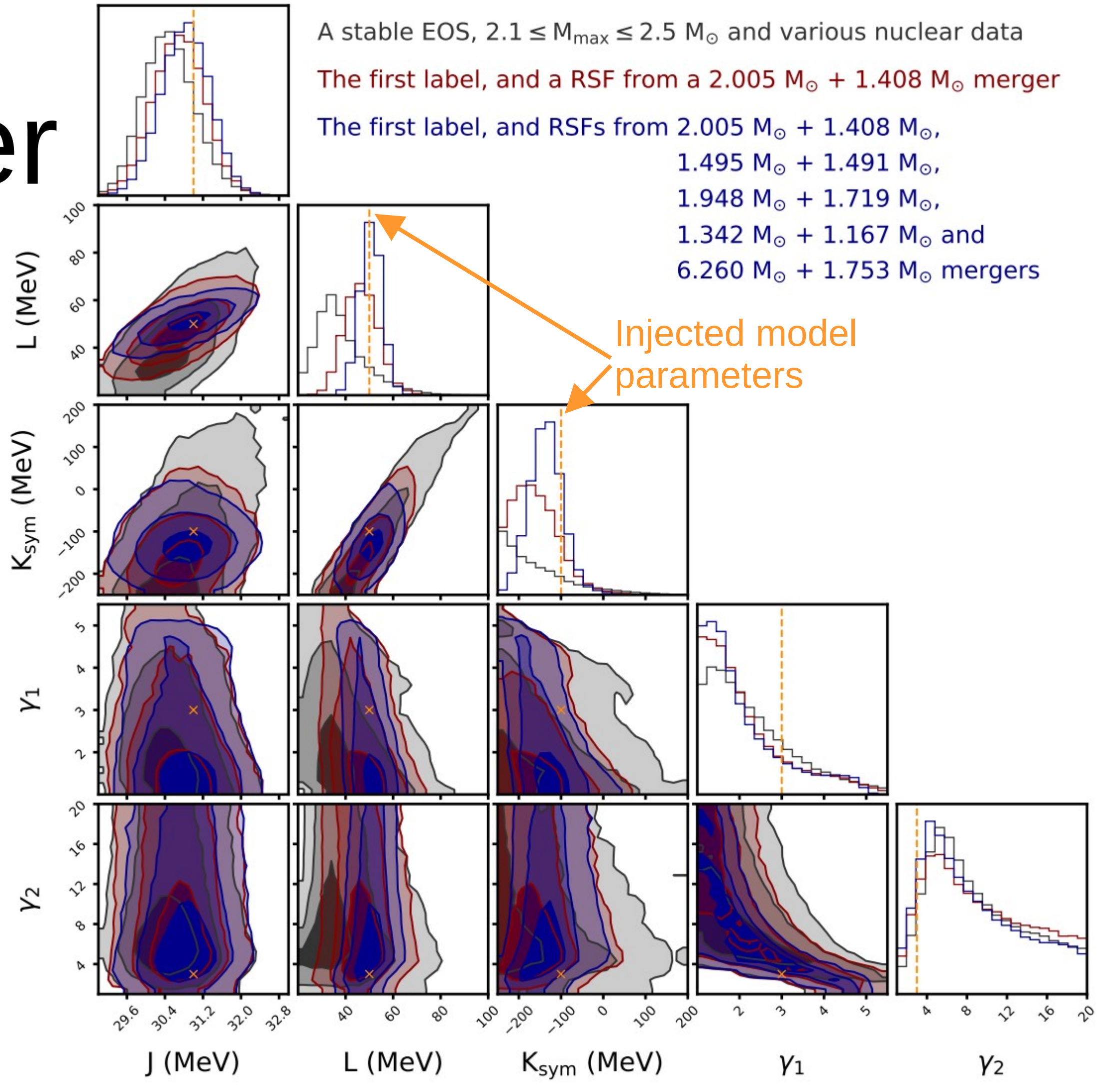
Neill, Newton & Tsang, MNRAS 504, 1129–1143 (2021).

GW (and possible EM counterpart) observation → i-mode frequency → Composition of the NS crust → Fundamental nuclear physics

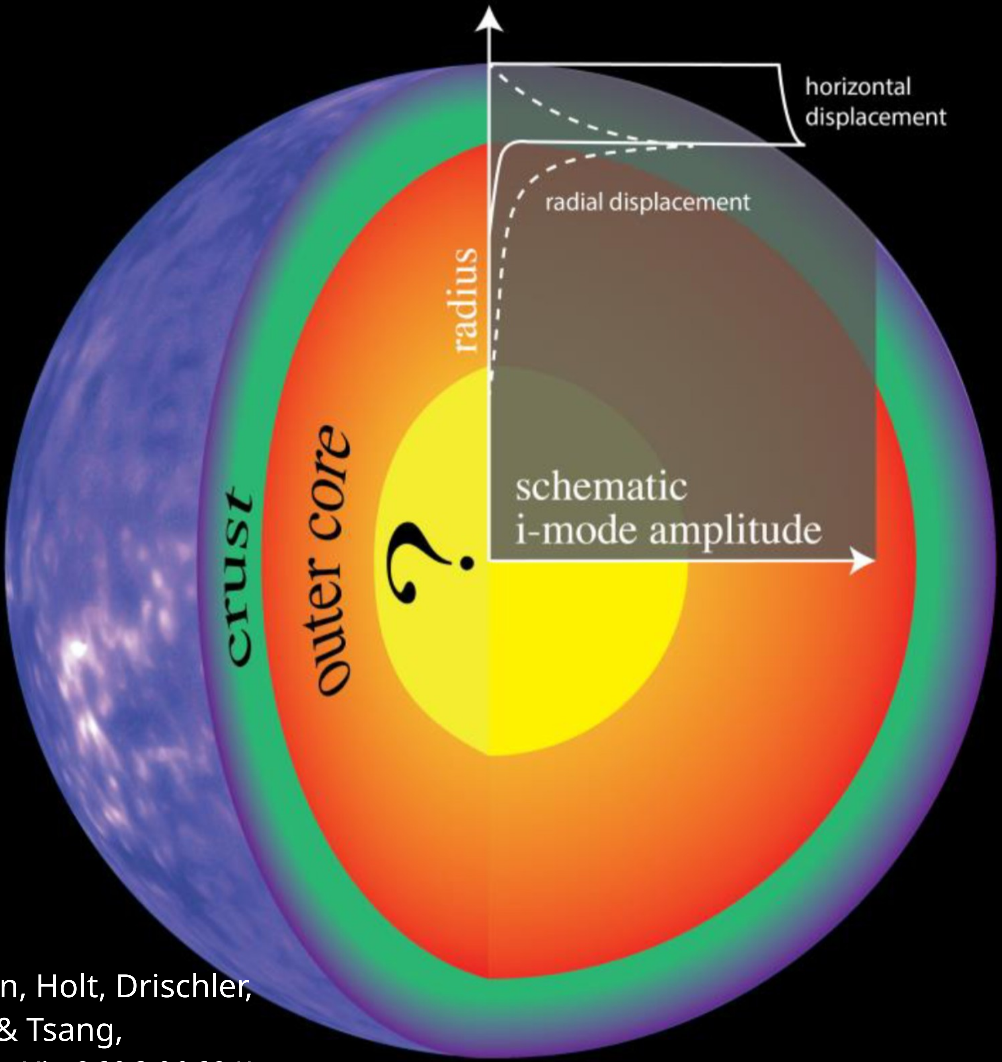
Bayesian parameter inference

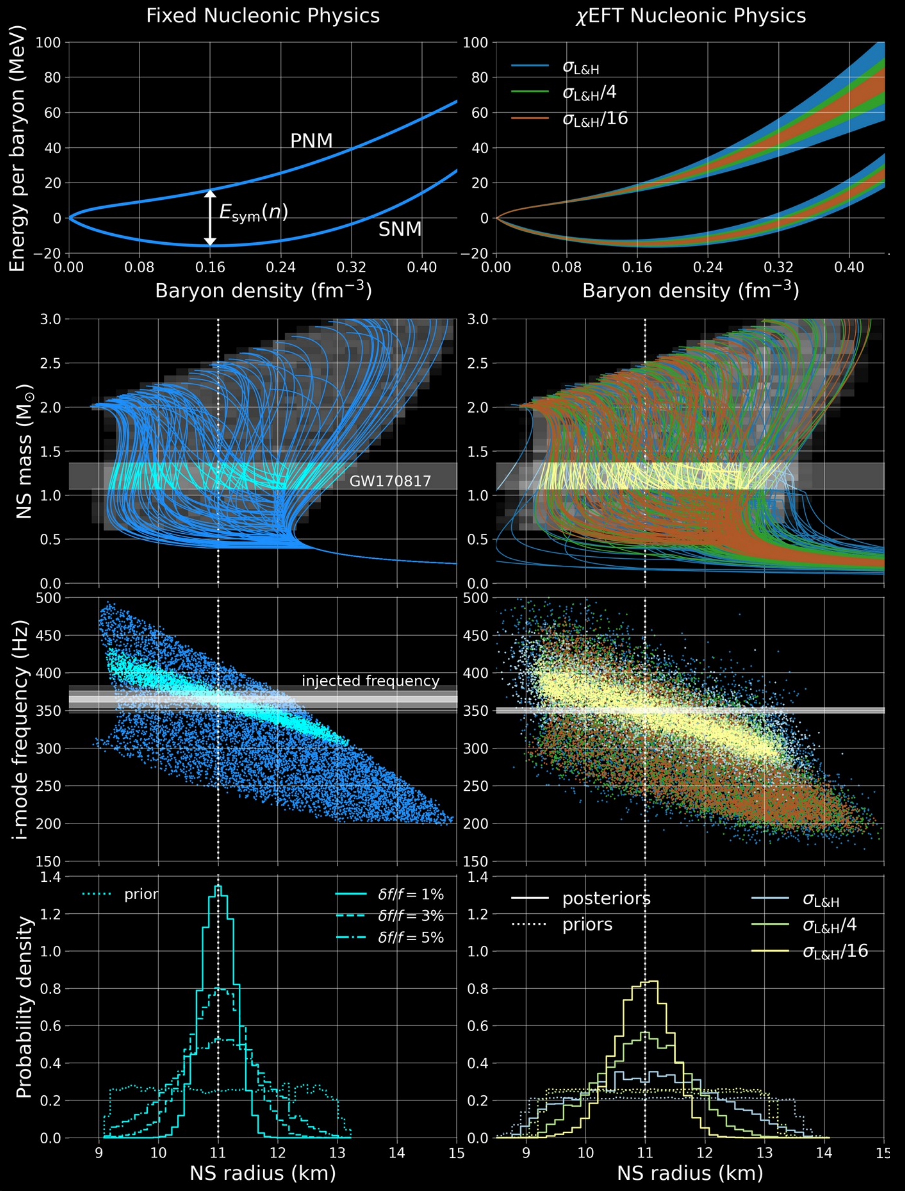
- We inject crust-core i-mode frequency measurements, conservative to what could be obtained from RSFs.

- The nucleonic matter parameters that determine the composition structure of matter around the crust-core transition (mainly L) are recovered well.



D. Neill, D. Tsang & W. G. Newton, MNRAS 532, 827–840 (2024).



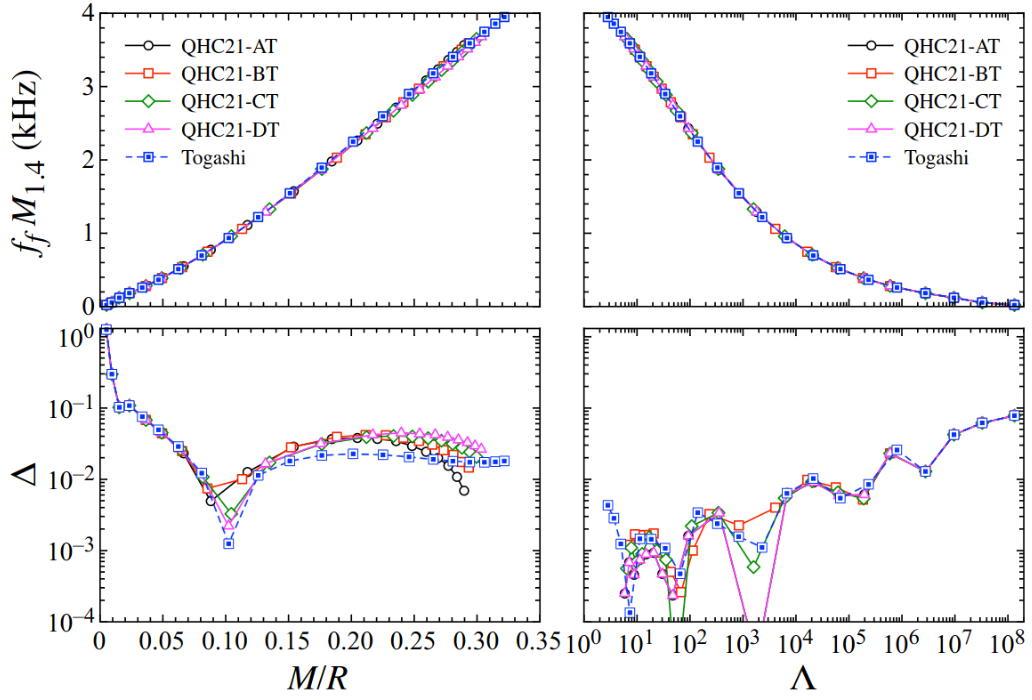


Neill, Newton, Holt, Drischler,
Margueron & Tsang,
Submitted (arXiv:2606.09621)

f-mode

- The dominant mode – easily detected with next-generation GW facilities

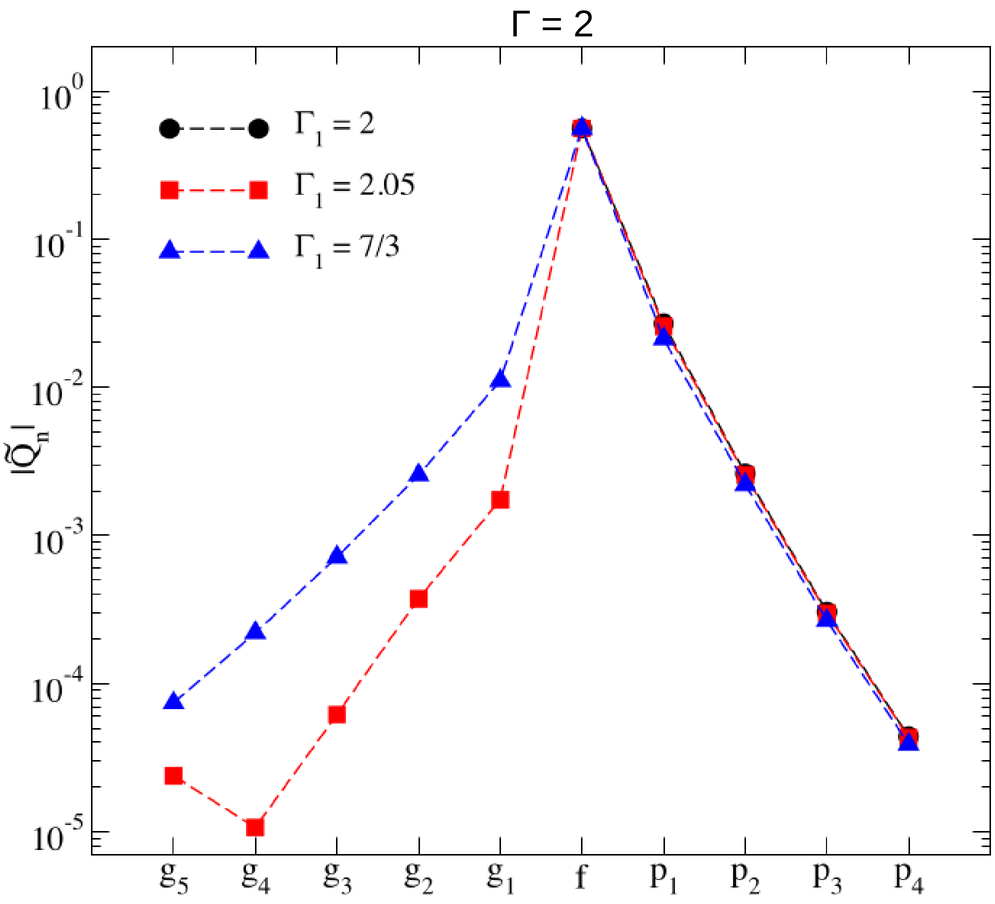
- Similar information to tidal deformability (stellar compactness, or the EOS weighted towards the core)



H. Sotani & T. Kojo, Phys. Rev. D 108, 063004 (2023).

g-modes

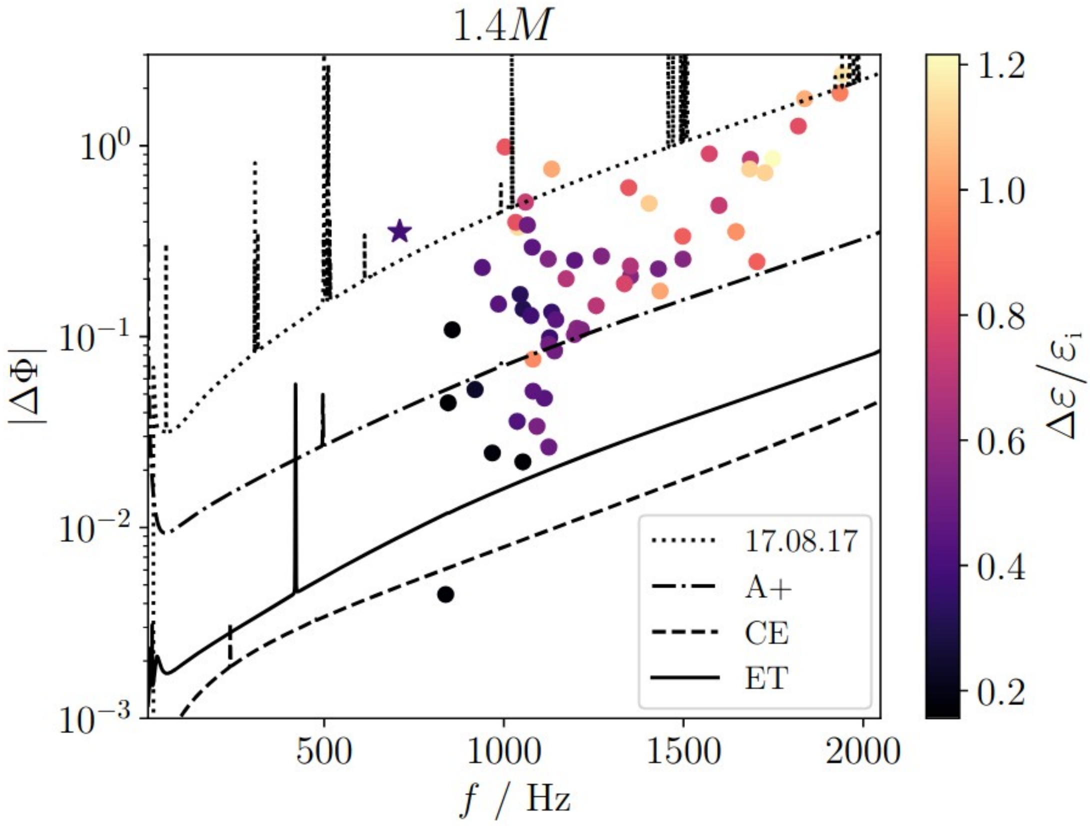
- Restored by (compositional) buoyancy
- Contains information about the compositional gradient
- The n-th g-mode has n nodes – measuring multiple g-modes informs us of composition throughout the NS core



Passamonti, Andersson & Pnigouras, MNRAS 504, 1273–1293 (2021).

Core interface modes

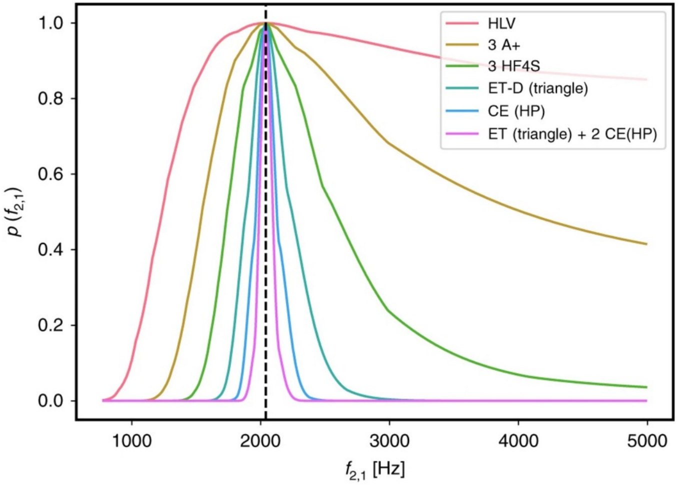
- Mode arising from a phase transition in the inner core
- Indicates the presence of non-nucleonic matter
- Properties of the mode inferred from its effect on GWs (or its non-detection?) inform us of the nature of the transition



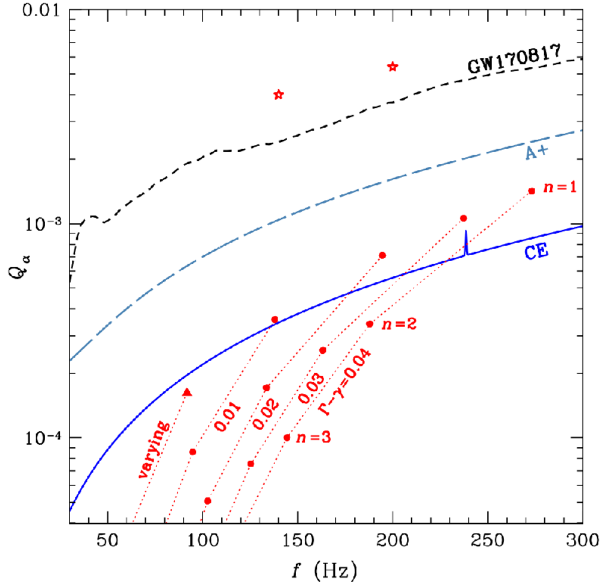
A. R. Counsell et al., Phys. Rev. Lett. 135, 081402 (2025).

Detectability with GWs

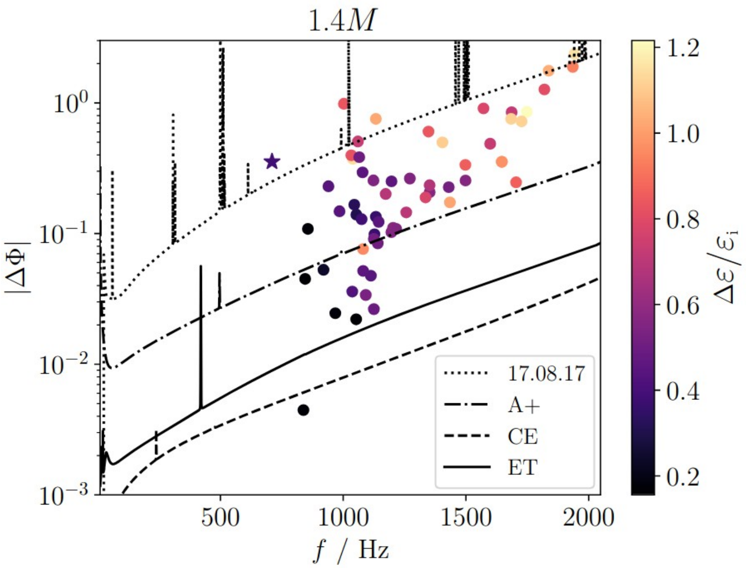
- CE/ET will make GW170817-like constraints on NS mass common, allowing for frequent multimessenger observations
- Next-generation facilities will also enable the detection of resonant and off-resonant excitation in GW signals alone



G. Pratten, P. Schmidt & T. Hinderer, Nature Communications 11, 2553 (2020).



W. C. G. Ho & N. Andersson, Phys. Rev. D 108, 043003 (2023).



A. R. Counsell et al., Phys. Rev. Lett. 135, 081402 (2025).

Holistic NS mode spectrum

- Typically, we build models that focus on NS properties relevant to one mode family
- Next-generation GW facilities will allow us to observe many modes at once
- What do we need to include in our models to get a holistic view of the NS mode spectrum?
- What can we learn from relationships between modes?

Adding superfluidity

- We can also account for neutron and proton superfluidity: 2 fluid formalism (either superfluid and non-superfluid components, or charged and non-charged components)

Rau & Wasserman. MNRAS 481, 4427–4444 (2018):

The motion of the two fluids is described by the relativistic Euler equations (Carter & Langlois 1998; Andersson & Comer 2007)

$$0 = u_{\rm n}^{\rho}\nabla_{\rho}(\mu_{\rm n}u_{\sigma}^{\rm n}) + \nabla_{\sigma}\mu_{\rm n} - 2u_{\rm n}^{\rho}\nabla_{[\rho}(\mu_{\rm n}\epsilon_{\rm n}W_{\sigma]}), \tag{31}$$

$$0 = u_{\rm q}^{\rho}\nabla_{\rho}(\mu_{\rm q}u_{\sigma}^{\rm q}) + \nabla_{\sigma}\mu_{\rm q} + (\mu_{\mu} - \mu_{\rm e})\nabla_{\sigma}f + 2u_{\rm q}^{\rho}\nabla_{[\rho}(\mu_{\rm n}\epsilon_{\rm p}W_{\sigma]}), \tag{32}$$

and the continuity equations

$$\nabla_{\rho}(n_{\rm n}u_{\rm n}^{\rho}) = 0, \tag{33}$$

$$\nabla_{\rho}(n_{\rm q}u_{\rm q}^{\rho}) = 0. \tag{34}$$

where $W_{\sigma} = u_{\sigma}^{\rm n} - u_{\sigma}^{\rm q}$. $\epsilon_{\rm n}$ and $\epsilon_{\rm p}$ are defined to parameterize the entrainment, and are related by

$$n_{\rm q}\epsilon_{\rm p} = n_{\rm n}\epsilon_{\rm n}. \tag{35}$$

→

Additional inputs:
- Densities at which neutrons & protons are superfluid
- Entrainment

Effects:
- Splitting of regular and superfluid modes

Neutron Superfluidity: questions for you!

- How can we incorporate superfluidity into a holistic view of NS dynamics? (Purely phenomenological, fit to results, calculations grounded in nuclear interactions, … ?)
- What properties need to be considered when calculating oscillations? (Entrainment, interaction with magnetic field, rotation, … ?)
- How might we detect effects of superfluidity on modes (excitation of superfluid modes by glitches, effects of superfluid phenomena on regular fluid modes, … ?)

Conclusions

dn431@bath.ac.uk

- Different modes are sensitive to different physics in different regions of the NS, making them powerful probes of particular physics
- The crust-core i-mode, for example, provides information about both nuclear matter at densities around the crust-core transition, and the radius of the NS
- Unlike the mass-radius relationship, which requires many NSs to be measured, constraining many modes for a single NS would provide strong insights into dense matter in these compact objects
- How can we best incorporate as much physics as possible into our mode calculations? What is the balance between self-consistency and computational tractability?

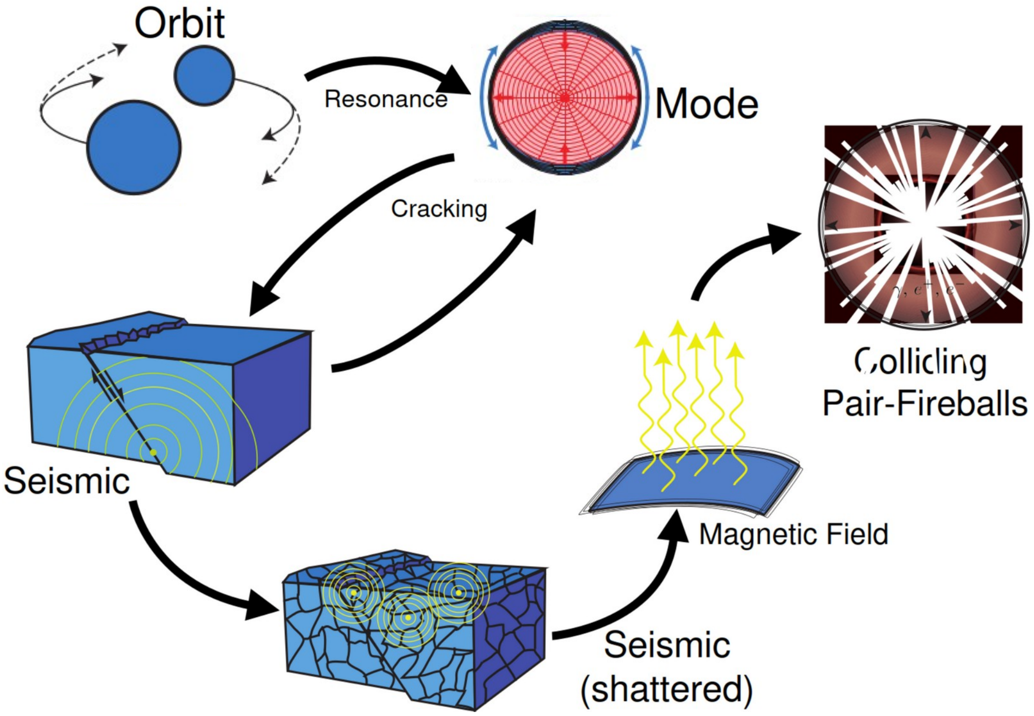
Neill, Preston, Newton & Tsang, Phys. Rev. Lett. 130, 112701 (2023)

Neill, Tsang & Newton, MNRAS 532, 827–840 (2024)

Neill, Newton, Holt, Drischler, Margueron & Tsang, Submitted (arXiv:2606.09621)

Extra Slides

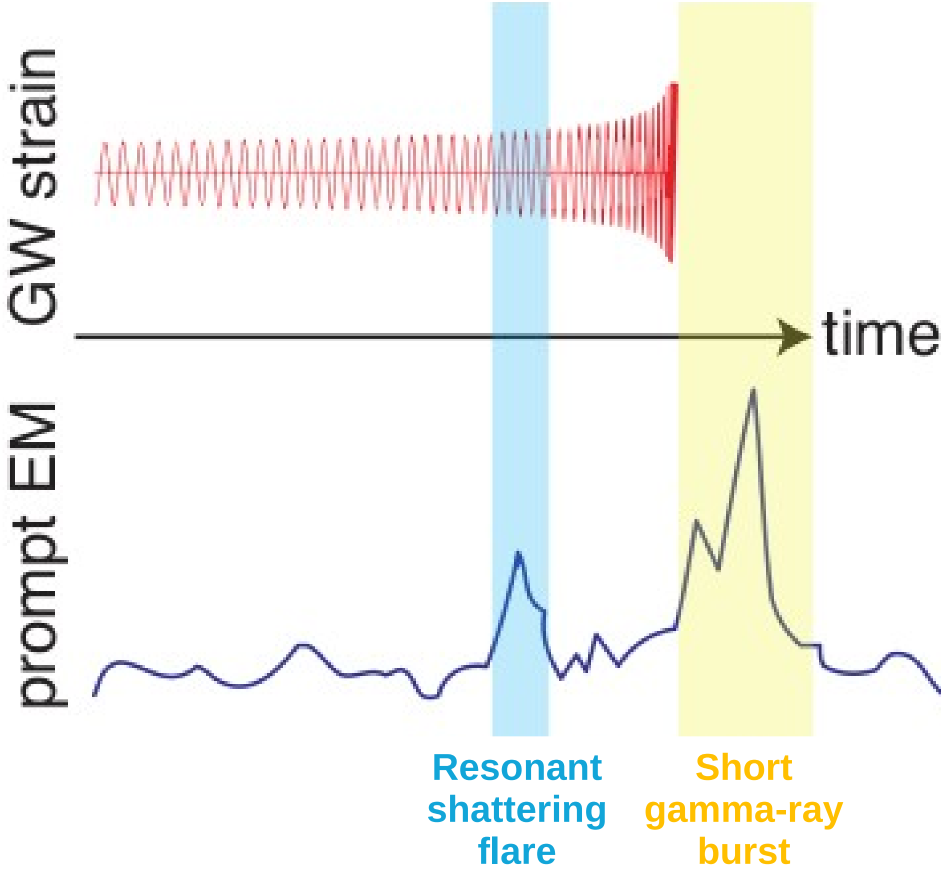
Multimessenger detectability: Resonant shattering flares



- Resonant shattering flares (RSFs) could be detected for nearby mergers

D. Tsang et al., Phys. Rev. Lett. 108, 011102 (2012).

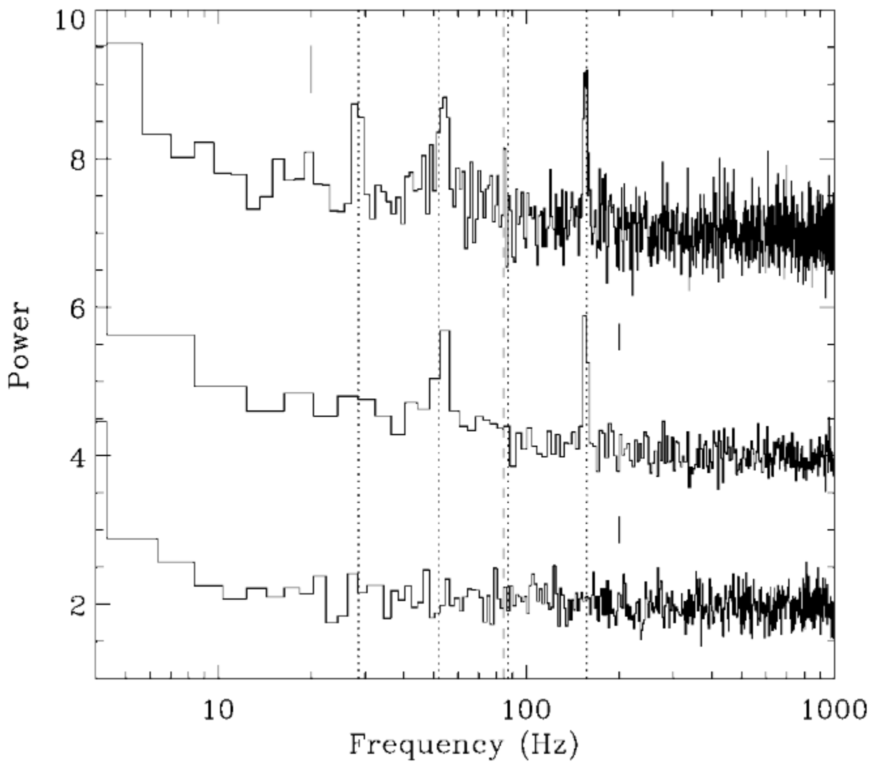
Multimessenger detectability: Resonant shattering flares



- Resonant shattering flares (RSFs) could be detected for nearby mergers
- Measuring the GW frequency at the time of an RSF gives us the frequency of the resonantly excited mode
- This makes frequency measurements possible using current observatories

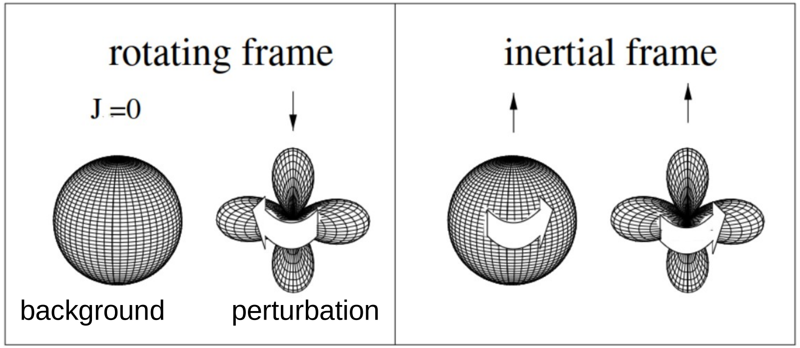
Mode detection

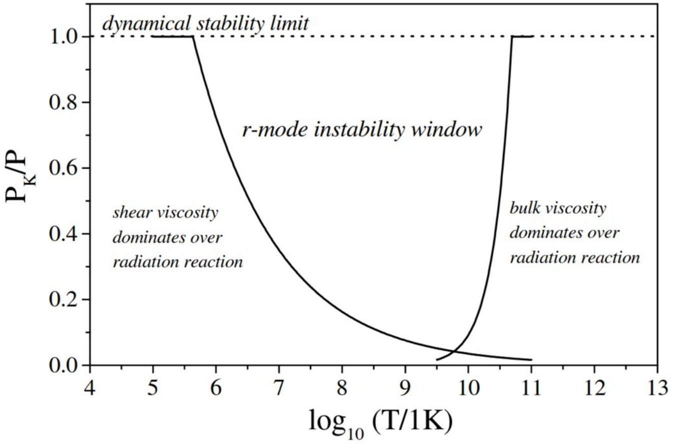
QPOs in SGR giant flares
→ toroidal shear modes



Strohmayer & Watts, ApJL, 632, L111 (2005)

GW damping instability in rotating NSs
→ r-modes





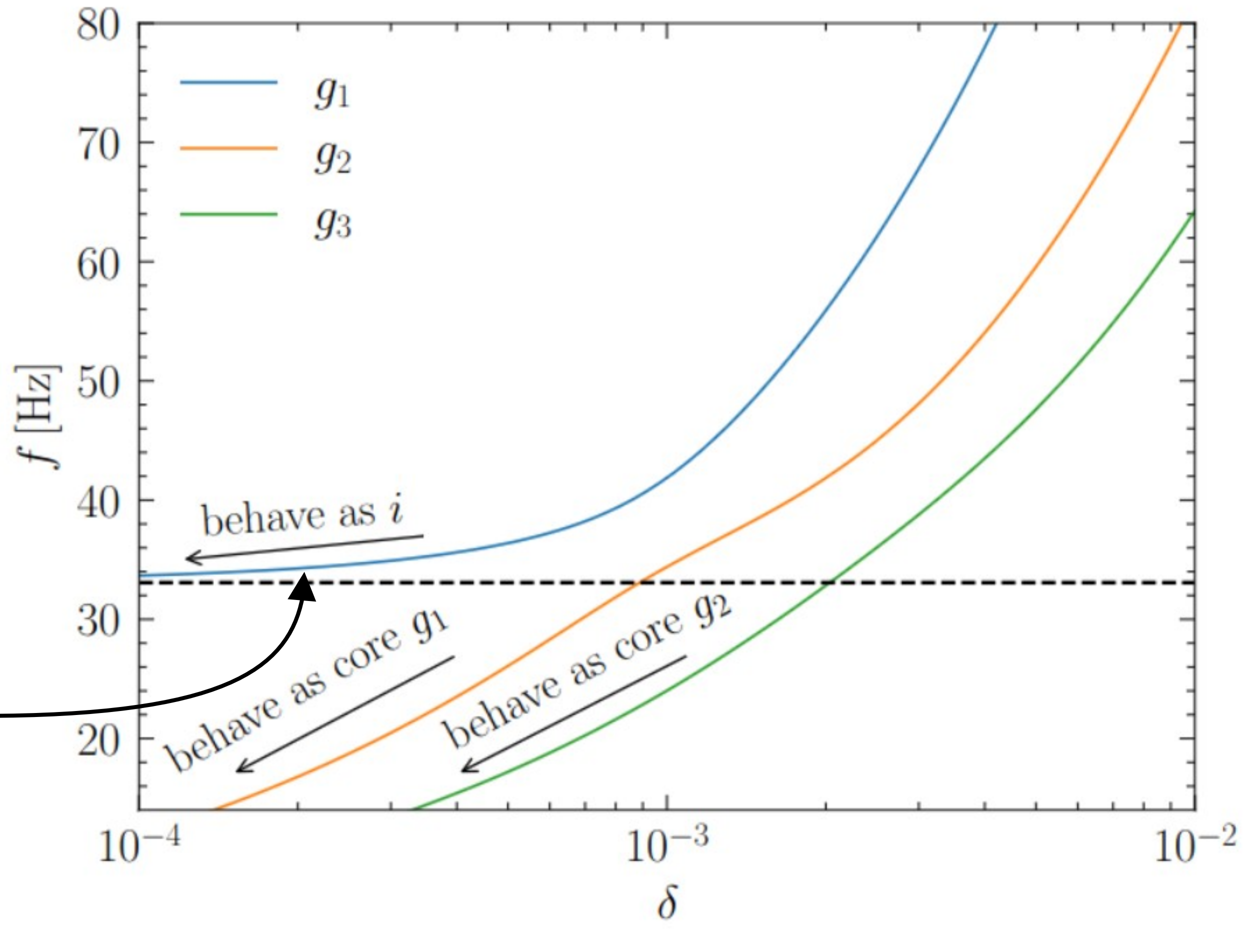
Andersson & Kokkotas, Int. J. Mod. Phys. D 10, 381–441 (2001)

A note on mode catagorisation

i-mode and g-mode features mix for highly stratified stars, such that there is no clearly defined i-mode.

For RSFs, it would therefore be better to analyse whether each mode can push the crust to its elastic limit for any given NS model, rather than assuming a particular mode.

In this work, when we say i-mode, we mean the mode which smoothly becomes the i-mode in the zero-stratification limit, which may appear more like the g1-mode is some cases



Y. Gao et al., Phys. Rev. D 112, 123006 (2025).

Fitting the i-mode frequency

$$f_{2i} = \mathrm{f}_1(M_{\mathrm{NS}},\ R_{\mathrm{NS}},\ N') + \mathrm{f}_2(M_{\mathrm{NS}},\ R_{\mathrm{NS}},\ N') + \cdots$$

$$N' = \frac{3}{\pi R_{cc}^3} \int_0^{R_{cc}} \frac{R_{cc} - r}{R_{cc}} N^2 d^3 r$$

(N = Brunt frequency)

- Allow indices -3,-2,-1,0,1,2,3
- Take the fitting formula that produces the best fit for a large sample of NS models that share the same nuclear matter EOS, but that have different inner-cores
- Repeat for several nuclear matter EOSs, and find the fitting formula that is, on average, best for all of them

Using mass, radius and buoyancy →

$$f_{2\mathrm{i}} = a\frac{1}{R_{\mathrm{NS}}} + b\frac{M_{\mathrm{NS}}}{R_{\mathrm{NS}}^2} + c\frac{N'}{R_{\mathrm{NS}}} + d\frac{N' M_{\mathrm{NS}}^2}{R_{\mathrm{NS}}}$$

Using mass and radius →

$$f_{2\mathrm{i}} = a\frac{1}{M_{\mathrm{NS}} R_{\mathrm{NS}}^3} + bR_{\mathrm{NS}} + cR_{\mathrm{NS}}^2 + d\frac{M_{\mathrm{NS}}^3}{R_{\mathrm{NS}}^3}$$

Using mass →

$$f_{2\mathrm{i}} = a + bM_{\mathrm{NS}} + cM_{\mathrm{NS}}^2 + dM_{\mathrm{NS}}^3$$

Fits for other fixed nuclear matter EOSs

Weakly stratified
(low buoyancy)

Strongly stratified
(high buoyancy)

Experimental constraints on nuclear matter

Impact of different uncertainties on radius recovery